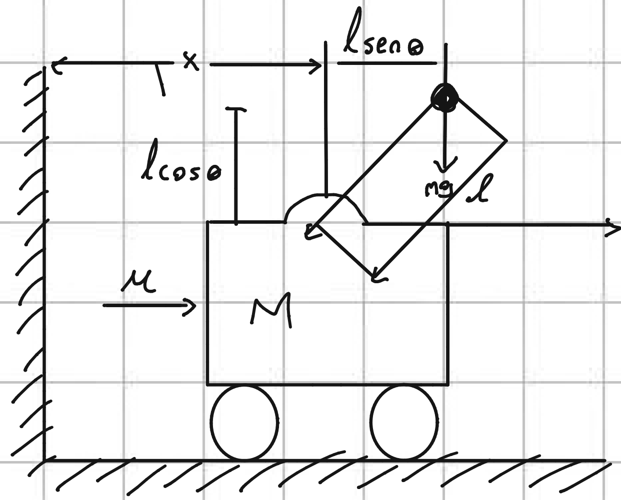
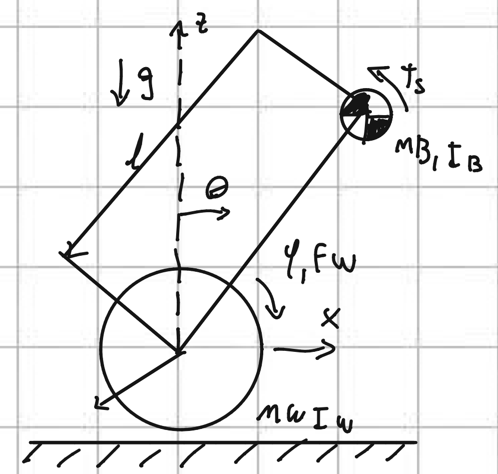
Tarea 4 - Santiago Andres Gomez Barbon 20211005034

Modelo pendulo invertido





X - axis   M = Masa del carro   m = masa del Pendulo

$F = u - b\dot{x}$ x = Posición del carro $x_p = x + l\,sen\,\theta$ -> Posición pendulo

$F = ma \rightarrow u - b\dot{x} = m \cdot a$ $u - b\dot{x} = M \cdot \ddot{x} + m\ddot{x}_p$

$u - b\dot{x} = M\ddot{x} + m\dfrac{d^2(x + l\,sen\,\theta)}{dt^2}$ $u - b\dot{x} = M\ddot{x} + m\ddot{x} + m\dfrac{d^2}{dt^2}l\,sen\,\theta$

$u - b\dot{x} = (M+m)\ddot{x} + ml\dfrac{d^2}{dt^2}sen\,\theta$ $\dfrac{d}{dt}sen\,\theta = \cos\theta \cdot \dot{\theta}$

$\dfrac{d}{dt}\cos\theta \cdot \dot{\theta} = -\dot{\theta}^2 sen\,\theta + \cos\theta \cdot \ddot{\theta}$

$u - b\dot{x} = (M+m)\ddot{x} + ml(-\dot{\theta}^2 sen\,\theta + \cos\theta \cdot \ddot{\theta})$

$\rightarrow u - b\dot{x} = (M+m)\ddot{x} + ml\cos\theta \cdot \ddot{\theta} - ml\dot{\theta}^2 sen\,\theta$

y - axis

$F = mg \cdot sen\,\theta$ $x_p = x + l\,sen\,\theta$ $y_p = l\cos\theta$

$F = ma \rightarrow mg \cdot sen\,\theta = m\cos\theta\dfrac{d^2}{dt^2}x_p - m\,sen\,\theta\dfrac{d^2}{dt^2}y_p$

$mg\,sen\,\theta = m\cos\theta\dfrac{d^2}{dt^2}(x + l\,sen\,\theta) - m\,sen\,\theta\dfrac{d^2}{dt^2}(l\cos\theta)$

$mg\,sen\,\theta = m\cos\theta(\ddot{x} + l\cos\theta \cdot \ddot{\theta} - l\,sen\,\theta \cdot \dot{\theta}^2) - ml\,sen\,\theta \cdot (-sen\,\theta \cdot \ddot{\theta} - \cos\theta \cdot \dot{\theta}^2)$

$mg \cdot sen\,\theta = m\ddot{x}\cos\theta + ml\cos^2\theta \cdot \ddot{\theta} - ml\,sen\,\theta \cdot \cos\theta\,\dot{\theta}^2 + ml\,sen^2\theta\,\ddot{\theta} + \underbrace{ml\,sen\,\theta}_{\cdot\cos\theta \cdot \dot{\theta}^2}$

Se divide sobre la masa y se considera que:

$\cos^2\theta + sen^2\theta = 1$

quedando la ecuación de tal forma:

$g \cdot sen\,\theta = \ddot{x}\cos\theta + l\ddot{\theta}$

Linealización

linealizando alrededor de θ cerca de 0 grados se pueden hacer las siguientes aproximaciones:

$sen\,\theta \approx \theta$ $\cos\theta \approx 1$

$u - b\dot{x} = (M+m)\ddot{x} - ml\,sen\,\theta \cdot \dot{\theta}^2 + ml\cos\theta \cdot \ddot{\theta}$

$\rightarrow u - b\dot{x} = (M+m)\ddot{x} - ml\theta \cdot \dot{\theta}^2 + ml\ddot{\theta}$

Se toma $\dot{\theta}^2 = 0$ y se simplifica la ecuación

$u - b\dot{x} = (M+m)\ddot{x}\cos\theta + x\ddot{\theta} \rightarrow u - b\dot{x} = (M+m)\ddot{x} + x\ddot{\theta}$

Por ultimo

$g \cdot sen\,\theta = \ddot{x}\cos\theta + l\ddot{\theta} \rightarrow g\theta = \ddot{x} + l\ddot{\theta}$

Representación en el espacio de estados

$u - b\dot{x} = M\ddot{x} + m\ddot{x} + ml\ddot{\theta}$ $u - b\dot{x} = M\ddot{x} + m(\ddot{x} + l\ddot{\theta})$

$u - b\dot{x} = M\ddot{x} + mg\theta$ $x_1 = \theta$

$M\ddot{x} = u - b\dot{x} - mg\theta$ $\ddot{x} = \dfrac{u}{M} - \dfrac{b}{M}\dot{x} - \dfrac{mg}{M}\theta$ $x_2 = \dot{\theta}$

$x_3 = x$

$\dot{x}_4 = \dfrac{u}{M} - \dfrac{b}{M}x_4 - \dfrac{mg}{M}x_1$ $x_4 = \dot{x}$

$g\theta = \ddot{x} + l\ddot{\theta} \rightarrow g\theta = \dfrac{u}{M} - \dfrac{b}{M}\dot{x} - \dfrac{mg}{M}\theta + l\ddot{\theta} \rightarrow l\ddot{\theta} = g\theta - \dfrac{1}{M}u + \dfrac{b}{M}\dot{x} + \dfrac{mg}{M}\theta$

$\ddot{\theta} = \dfrac{g}{l}(1 + \dfrac{m}{M})\theta - \dfrac{u}{Ml} + \dfrac{b}{Ml}\dot{x}$ $\ddot{\theta} = g\dfrac{M+m}{Ml}\theta - \dfrac{u}{Ml} + \dfrac{b}{Ml}\dot{x}$

$\dot{x}_2 = g\dfrac{M+m}{Ml}x_1 - \dfrac{u}{Ml} + \dfrac{b}{Ml}x_4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ g\frac{M+m}{Ml} & 0 & 0 & \frac{b}{Ml} \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & -\frac{b}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{Ml} \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]\,u$$