Math 30, Friday May 1, 2020

1pm class

Questions?

Quiz 9: please submit your work by 11:59pm.

Q: about sigma notation

$$\sum_{j=0}^{15} a_j = a_0 + a_1 + a_2 + \cdots + a_{15}$$

Ex.

$$\sum_{k=3}^{7} k^2 = 3^2 + 4^2 + 5^2 + 6^2 + 7^2$$

$$= 9 + 16 + 25 + 36 + 49$$

$$= 135 \ (?)$$

k "dummy index"

Ex. $\displaystyle\sum_{k=3}^{6} k = 3+4+5+6$

$= 18$

Ex. $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ "Gauss's formula" (?)

Fund. Thm. of Calc. Part I:

"differentiating an integral":

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$$

Fund. Thm. of Calc. Part II:

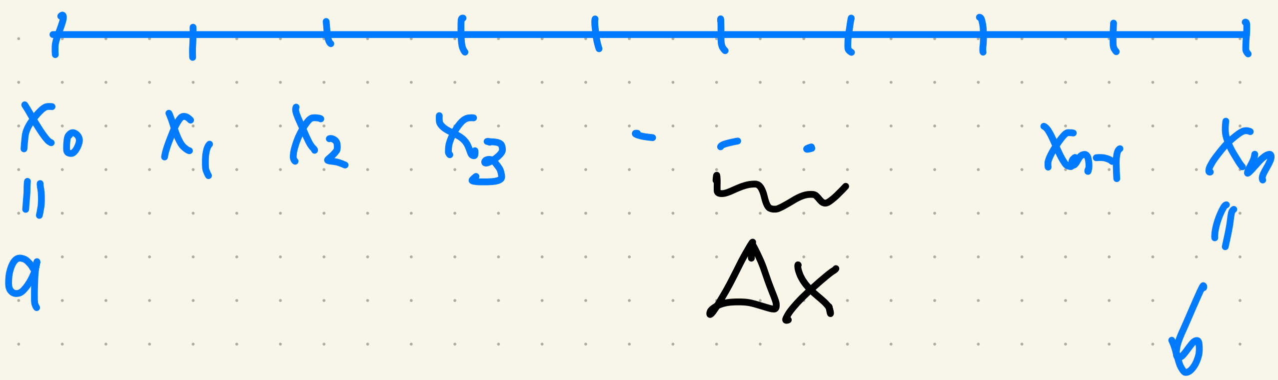
"integrating a derivative"

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Reason: Apply linear approximation over & over:

$$f(x) - f(a) \approx f'(a)\cdot(x-a)$$

Repeat it: equally spaced points



$$f(x_1) - f(a) \approx f'(a)\Delta x$$

linear approx. on first subinterval

$$f(x_2) - f(x_1) \approx f'(x_1)\Delta x$$

lin. approx. on second subinterval

$$f(x_3) - f(x_2) \approx f'(x_2)\Delta x$$

$$\vdots$$

Now add them up...

$$f(b) - f(x_{n-1}) \approx f'(x_{n-1})\Delta x$$

$f(x_1) - f(a) \approx f'(x_0)\Delta x$ linear approx. on first subinterval $(x_0 = a)$

$f(x_2) - f(x_1) \approx f'(x_1)\Delta x$ lin. approx. on second subinterval

$f(x_3) - f(x_2) \approx f'(x_2)\Delta x$

$\vdots$

Now add them up...

$f(b) - f(x_{n-1}) \approx f'(x_{n-1})\Delta x$

Add left sides: (have a lot of cancellation)

$$f(b) - f(a) \approx \sum_{j=0}^{n-1} f'(x_j)\Delta x$$

add right sides

Now take limit as $n \to \infty$.

Two things happen: 1. it becomes an equality

2. RHS becomes $\int_a^b f'(x)\,dx$

Summary: $$\int_a^b f'(x)\,dx = f(b) - f(a)$$

Common notation:

$$[f(x)]_a^b \quad \text{and} \quad f(x)\Big|_a^b$$

both stand for $f(b) - f(a)$.

So $$\int_a^b f'(x)\,dx = [f(x)]_a^b$$

$$= f(b) - f(a)$$

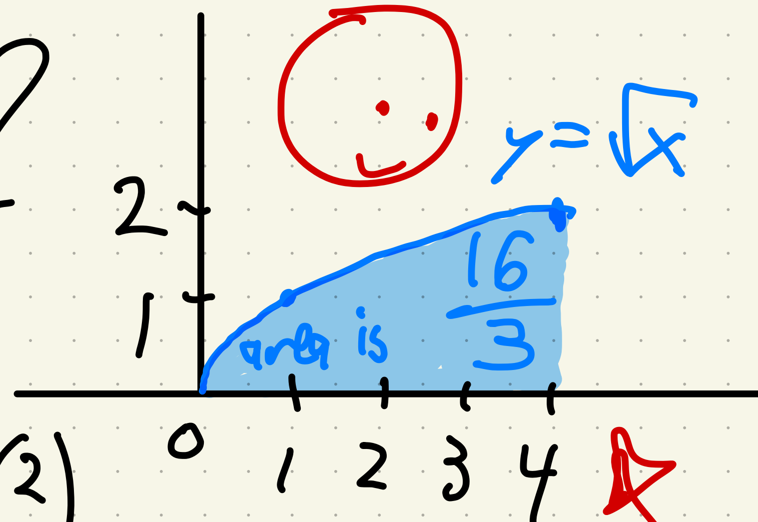
Now an example or two...

F.T. of C. Part II is great because it makes certain integrals fast & easy to calculate.

$4^{3/2} = 8$

Find an antideriv. & eval. at endpts:

Ex. $\displaystyle\int_0^4 \sqrt{x}\, dx = ?$



Here $f'(x) = \sqrt{x}\ \left(= x^{1/2}\right)$

Can use antiderivative: $f(x) = \frac{2}{3} x^{3/2}$ (check using Power Rule)

So F.T. of Calc. Part II says:

$$\int_0^4 \sqrt{x}\, dx = \left[\frac{2}{3} x^{3/2}\right]_0^4 = \frac{2}{3} \cdot 8 - 0 = \boxed{\frac{16}{3}}$$

check:

Done w/ lecture part early.

- Please work on "Corona Quiz 9" due before 11:59pm tonight

- Please do the worksheet on Canvas under "Files", May 1.

Questions?

Have a nice weekend!