STAT 50 HW #11

Section 4.5 #'s 1, 3, 5, 6, 9, 13, 15, 23, 27

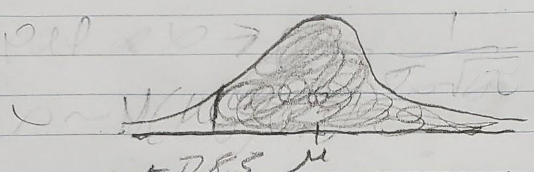
1.

Find the area under the normal curve

a. **To the right of z = −0.85.**

Find the area under the normal curve

a) To the right of $z = -0.85$



$\mu = 0$

$\sigma = 1$

From the z table, the area to the left of -0.85 is 0.1977.

So the area to the right is:

$1 - 0.1977 = \boxed{0.8023}$

b. **Between z = 0.40 and z = 1.30.**

b) Between $z = 0.40$ and $z = 1.30$

From z table, area left of 0.40 is 0.6554

From z table, area left of 1.30 is 0.9032

$0.9032 - 0.6554 = \boxed{0.2478}$

c. **Between z = −0.30 and z = 0.90.**

c) Between $z = -0.30$ and $z = 0.90$

Area left of -0.30 is 0.3821

Area left of 0.90 is 0.8159

$0.8159 - 0.3821 = \boxed{0.4338}$

d. **Outside z = −1 .50 to z = −0.45.**

d) Outside $z=-1.5$ to $z=-0.45$
Area left of -1.5 is 0.0668 $\quad 1-0.3264=0.6736$
Area left of -0.45 is 0.3264

$0.0668+(1-0.3264)$
$0.0668+0.6736=\boxed{0.7404}$

3.

Let Z ~ N(0, 1). Find a constant c for which

a. P(Z ≥ c) = 0.1587

3. Let $Z \sim N(0,1)$. Find a constant c for which
a) $P(Z \geq c)=0.1587$
$1-0.1587=0.8413$ ← area to left of 1!
* Work backwards in Z-table!

$\boxed{c=1.0}$

b. P(c ≤ Z ≤ 0) = 0.4772

b) $P(c \leq Z \leq 0)=0.4772$
0.5
What value sets us 0.228 to the left of c?
$0.5-0.4772=0.228$
So, $\boxed{c=-2.0}$

c. P(−c ≤ Z ≤ c) = 0.8664

c) $P(-c \leq Z \leq c)=0.8664$
$1-0.8664=\frac{0.1336}{2}=0.0668$ ← to left of $-c$!

$\boxed{c=1.5}$

d. P(0 ≤ Z ≤ c) = 0.2967

d) $P(0 \le Z \le c) = 0.2967$

$\downarrow$ 0.5

$0.5 + 0.2967 = 0.7967$ ← Value that gives us this area left of c?

$\boxed{c = 0.83}$

e. **P(|Z| ≤ c) = 0.1470**

e) $P(|Z| \le c) = 0.1470$ $1 - 0.1470 = 0.853$

$(-c \le |Z| \le c) = 0.1470 \rightarrow \frac{0.1470}{2} = 0.0735$

So, $\boxed{c = 1.45}$

area to left of $-c$

$1 - 0.0735 = 0.9265$ ← area to left of c

5.

A process manufactures ball bearings with diameters that are normally distributed with mean 25.1 mm and standard deviation 0.08 mm.

a. **What proportion of the diameters are less than 25.0 mm?**

$\mu = 25.1\text{mm}$ $X \sim N(\mu, \sigma^2)$

$\sigma = 0.08\text{mm}$ $X \sim N(25.1, 0.08^2)$

a) $P(X < 25.0\text{mm})$? $P\left(\frac{X - \mu_X}{\sigma_X} < \frac{25 - \mu_X}{\sigma_X}\right)$

$P\left(Z < \frac{25 - 25.1}{0.08}\right) = P(Z < -1.25)$

$\downarrow$ 0.1056

$\boxed{P(X < 25.0) = 0.1056}$

b. **Find the 10th percentile of the diameters.**

b) 10th percentile of diameters?

0.10 → -1.28 → $P(Z \le -1.28)$

$\frac{x-25.1}{0.08} = -1.28$

$x - 25.1 = -0.1024$

$+25.1 \quad +25.1$

→ $\boxed{x = 24.997 \text{ mm}}$

c. **A particular ball bearing has a diameter of 25.2 mm. What percentile is its diameter on?**

c) Percentile of $x = 25.2$ mm?

$\frac{25.2-25.1}{0.08} = 1.25 \rightarrow 0.8944$

$\boxed{\text{89th percentile}}$

d. **To meet a certain specification, a ball bearing must have a diameter between 25.0 and 25.3 millimeters. What proportion of the ball bearings meet the specification?**

d) $25 \le x \le 25.3$ ← Proportion in between?

$\frac{25-25.1}{0.08} = -1.25 \rightarrow 0.1056$

$\frac{25.3-25.1}{0.08} = 2.5 \rightarrow 0.9938$

$0.9938 - 0.1056 = \boxed{0.8882}$

6.

Depths of pits on a corroded steel surface are normally distributed with mean 822 µm and standard deviation 29 µm.

a. **Find the 10th percentile of pit depths.**

$\mu = 822\ \mu m$ $\quad X \sim N(\mu, \sigma^2)$

$\sigma = 29\ \mu m$ $\quad X \sim N(822, 29^2)$

a) 10th percentile?

$0.10 \rightarrow -1.28$

$\frac{x-822}{29} = -1.28 \cdot 29$

$x - 822 = -37.12$ $\quad \boxed{x = 784.88\ \mu m}$

$+822$

b. **A certain pit is 780 µm deep. What percentile is it on?**

b) Percentile of 780 μm?

$\frac{780-822}{29} = -1.45 \rightarrow 0.0735$

$\boxed{\text{7th percentile}}$

c. **What proportion of pits have depths between 800 and 830 µm?**

c) Proportion in between? $\rightarrow (800 \leq x \leq 830)$

$\frac{800-822}{29} = -0.76 \rightarrow 0.2236$

$\frac{830-822}{29} = 0.28 \rightarrow 0.6103$

$0.6103 - 0.2236 = \boxed{0.3867}$

9.

The lifetime of a lightbulb in a certain application is normally distributed with mean μ = 1400 hours and standard deviation σ = 200 hours.

a. **What is the probability that a lightbulb will last more than 1800 hours?**

$\mu = 1400$ hrs $\quad X \sim N(\mu, \sigma^2)$ $\quad 0.9772$

$\sigma = 200$ hrs $\quad X \sim N(1400, 200^2)$

a) $P(X > 1800)$? $\quad P\left(\frac{X-\mu_X}{\sigma_X} > \frac{1800-\mu_X}{\sigma_X}\right)$

$1 - 0.9772 = 0.0228$ $\quad \left(Z > \frac{1800-1400}{200}\right) = P(Z > 2)$

$\boxed{P(X > 1800) = 0.0228}$

b. **Find the 10th percentile of the lifetimes.**

b) 10th percentile? 0.10 → -1.28

$\frac{x-1400}{200} = -1.28$

$x - 1400 = 256$
$+1400 \quad +1400$
$= 1144$

$\boxed{x = 1144 \text{ hrs}}$

c. **A particular lightbulb lasts 1645 hours. What percentile is its lifetime on?**

c) Percentile of $x = 1645$?

$\frac{1645-1400}{200} = 1.23 \rightarrow 0.8907$

$\boxed{89\text{th percentile}}$

d. **What is the probability that the lifetime of a light-bulb is between 1350 and 1550 hours?**

d) $1350 \le X \le 1550$ ← probability?

$\frac{1350-1400}{200} = -0.25 \rightarrow 0.4013$

$\frac{1550-1400}{200} = 0.75 \rightarrow 0.7734$

$0.7734 - 0.4013 = \boxed{0.3721}$

e. **Eight lightbulbs are chosen at random. What is the probability that exactly two of them have lifetimes between 1350 and 1550 hours?**

e) 8 chosen. Prob that exactly 2 have widths between 1350 and 1550 ?

$P(1350 \le X \le 1550) = 0.3721$

$P(Y=2) \rightarrow Y \sim Bin(8, 0.3721)$

$P(Y=2) = \binom{8}{2}(0.3721)^2(1-0.3721)^6$

${}_8C_2 = \frac{8!}{(8-2)!\,2!} = \frac{8 \cdot 7 \cdot 6!}{6!\,2!} = 28$

$28(0.1385)(0.0613) = \boxed{0.2376}$

13.

A cylindrical hole is drilled in a block, and a cylindrical piston is placed in the hole. The clearance is equal to one-half the difference between the diameters of the hole and the piston. The diameter of the hole is normally distributed with mean 15 cm and standard deviation 0.025 cm, and the diameter of the piston is normally distributed with mean 14.88 cm and standard deviation 0.015 cm. The diameters of hole and piston are independent.

a. **Find the mean clearance.**

Clearance $= \frac{1}{2}$ (dia. of hole - dia. of pist)

Dia. of hole	Dia. of pis	Independent!
$\mu_h = 15$ cm	$\mu_p = 14.88$ cm	
$\sigma_h = 0.025$ cm	$\sigma_p = 0.015$ cm	

a) μ of clearance?

$15 - 14.88 = 0.12/2 = \boxed{0.06 \text{ cm}}$

b. **Find the standard deviation of the clearance.**

b) σ of clearance?

$\sigma = \sqrt{V(0.5h - 0.5P)}$

$= \sqrt{0.5^2 V(h) - 0.5^2 V(P)}$

$= \sqrt{0.5^2 \sigma_h^2 - 0.5^2 \sigma_P^2}$

$= \sqrt{(0.25(0.025)^2 + (0.25)(0.015)^2)}$

$= \boxed{0.01458}$

c. **What is the probability that the clearance is less than 0.05 cm?**

c) $P(X < 0.05)$? $\mu = 0.06$, $\sigma = 0.01458$

$z < \frac{0.05 - 0.06}{0.01458} \rightarrow -0.69$

$P(z < -0.69) = \boxed{0.2451}$

d. **Find the 25th percentile of the clearance.**

d) 25th percentile of clearance? $0.25 \rightarrow -0.67$

$\frac{x - 0.06}{0.01458} = -0.67$

$x - 0.06 = -0.0097$
$+0.06 \quad +0.06$

$\boxed{x = 0.0502}$

e. **Specifications call for the clearance to be between 0.05 and 0.09 cm. What is the probability that the clearance meets the specification?**

e) $P(0.05 \le x \le 0.09)$?

$\frac{0.05 - 0.06}{0.01458} = -0.69 \rightarrow 0.2451$

$\frac{0.09 - 0.06}{0.01458} = 2.06 \rightarrow 0.9803$

$0.9803 - 0.2451 = \boxed{0.7352}$

f. **It is possible to adjust the mean hole diameter. To what value should it be adjusted so as to maximize the probability that the clearance will be between 0.05 and 0.09 cm?**

$\mu = 15.02$

$\downarrow$

Prob: 0.8294

① What should the mean ... be adjusted to so that ... is between 0.03 and 0.09 cm for the highest probability possible?

Probability maximized if mean is exactly in middle between the 2 boundaries of 0.03 and 0.09

$$\frac{0.03+0.09}{2} = 0.07$$

$$\mu_C = \frac{1}{2}\mu_X + \left(-\frac{1}{2}\right)\mu_Y$$

$$\mu_X = 2\mu_C + \mu_Y$$

$$= 2(0.07) + 14.88$$

$$\boxed{\mu = 15.02\text{cm}}$$

15.

The fill volume of cans filled by a certain machine is normally distributed with mean 12.05 oz and standard deviation 0.03 oz.

a. **What proportion of cans contain less than 12 oz?**

15. $\mu = 12.05$

$\sigma = 0.03$

$X \sim N(\mu, \sigma^2)$

$X \sim N(12.05, 0.03)$

a) $P(X < 12)$?

$$P\left(\frac{X-\mu_X}{\sigma_X} < \frac{12-\mu_X}{\sigma_X}\right) = 0.0475$$

$$P\left(Z < \frac{12-12.05}{0.03}\right) = P(Z < -1.67)$$

$$\boxed{P(X<12) = 0.0475}$$

b. **The process mean can be adjusted through calibration. To what value should the mean be set so that 99% of the cans will contain 12 oz or more?**

b) $P(\bar{x} < x_0) = 0.99$

~~$P(\bar{x} \le x_0) = 0.01$~~

~~$\bar{x}_1 = \mu + \frac{\sigma}{\sqrt{n}} z = 12.05 + \frac{0.03}{\sqrt{\;}}$~~

$P(-x_0 \le \mu_x \le x_0) = 0.99$ → as close as possible on Z table

$\frac{12 - x_0}{0.02} = 2.33 \cdot 0.02$, 2.33 → use negative! 0.9901

$1 - 0.99 = 0.01 = 0.005$ above and below

$12 - x_0 = -0.0466$

$-12 \qquad -12$

$-x_0 = -12.0466 \rightarrow x_0 = 12.07$

$\boxed{\mu = 12.07\text{ oz}}$

c. If the process mean remains at 12.05 oz, what must the standard deviation be so that 99% of the cans will contain 12 oz or more?

c) $\mu = 12.05$ 99% ≥ 12 oz or more

$\sigma = ?$

$\frac{12 - 12.05}{\sigma_x} = -2.326$ * between 2.32 and 2.33 for accuracy

use negative

$\frac{12 - 12.05}{\sigma_x} = -2.326 \cdot \sigma_x$

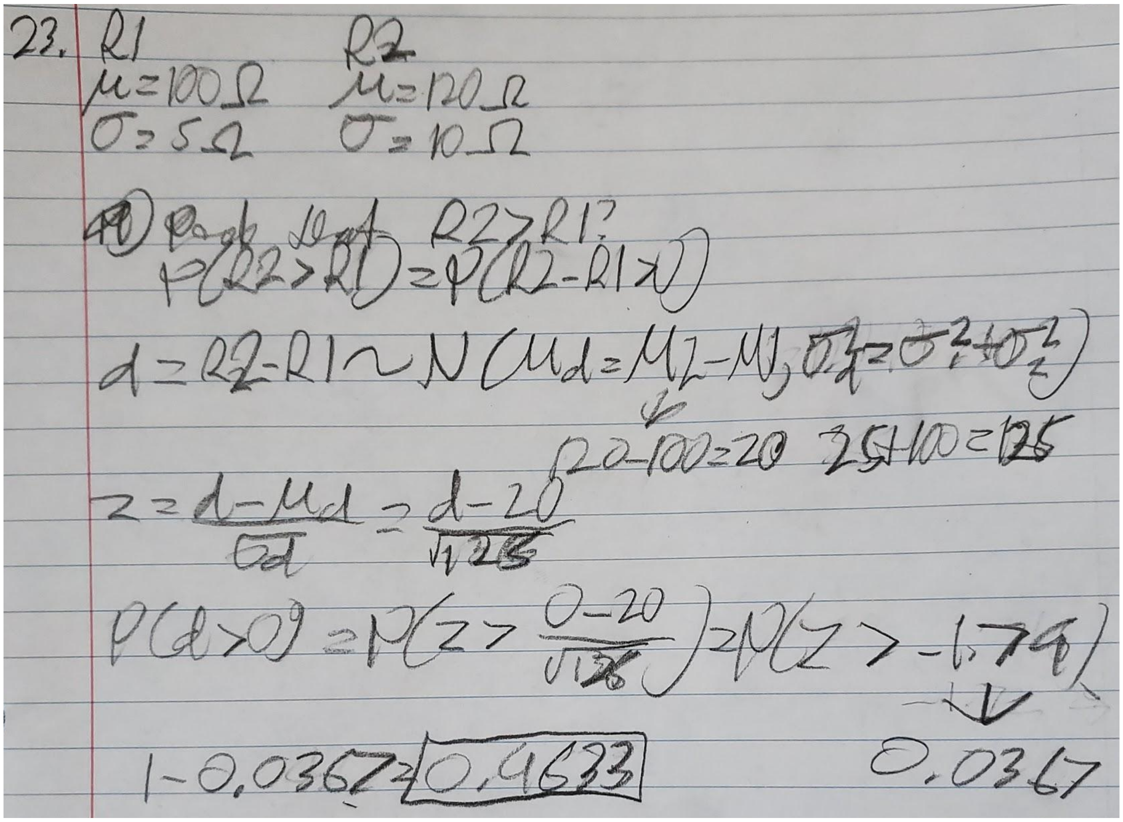
$\frac{-0.05}{-2.326} = \frac{-2.326 \cdot \sigma_x}{-2.326}$

$\boxed{\sigma_x = 0.0215}$

23.

Two resistors, with resistances R1 and R2 , are connected in series. R1 is normally distributed with mean 100 Ω and standard deviation 5 Ω, and R2 is normally distributed with mean 120 Ω and standard deviation 10 Ω.

a. **What is the probability that R2 > R1?**



23. R1: $\mu = 100\,\Omega$, $\sigma = 5\,\Omega$ R2: $\mu = 120\,\Omega$, $\sigma = 10\,\Omega$

a) Prob that $R2 > R1$?

$P(R2 > R1) = P(R2 - R1 > 0)$

$d = R2 - R1 \sim N(\mu_d = \mu_2 - \mu_1, \sigma_d^2 = \sigma_1^2 + \sigma_2^2)$

$120 - 100 = 20$ $25 + 100 = 125$

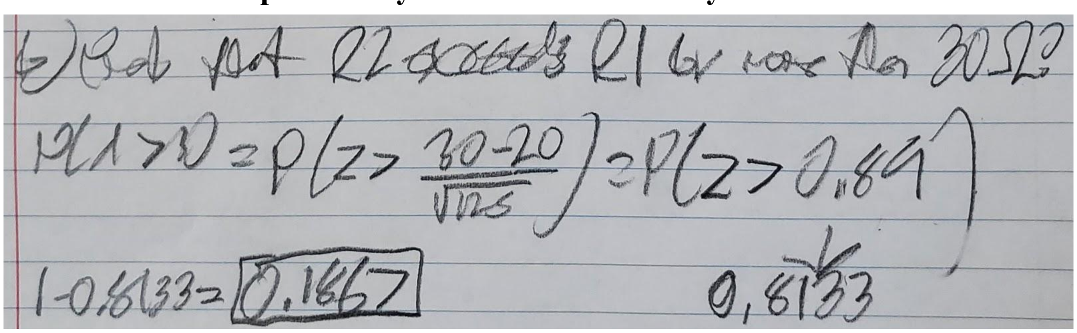
$z = \frac{d - \mu_d}{\sigma_d} = \frac{d - 20}{\sqrt{125}}$

$P(d > 0) = P\left(z > \frac{0 - 20}{\sqrt{125}}\right) = P(z > -1.79)$

0.0367

$1 - 0.0367 = \boxed{0.9633}$

b. **What is the probability that R2 exceeds R1 by more than 30 Ω?**



b) Prob that R2 exceeds R1 by more than $30\,\Omega$?

$P(d > 30) = P\left(z > \frac{30 - 20}{\sqrt{125}}\right) = P(z > 0.89)$

0.8133

$1 - 0.8133 = \boxed{0.1867}$

27.

A company receives a large shipment of bolts. The bolts will be used in an application that requires a torque of 100 J. Before the shipment is accepted, a quality engineer will sample 12 bolts and measure the torque needed to break each of them. The shipment will be accepted if the engineer concludes that fewer than 1% of the bolts in the shipment have a breaking torque of less than 100 J.

a. **If the 12 values are 107, 109, 111, 113, 113, 114, 114, 115, 117, 119, 122, 124, compute the sample mean and sample standard deviation.**

Breaking torque of 100J
Sample of 12 bolts
Accepted if ≤ 1% have a breaking torque less than 100J

a) 107, 109, 111, 113, 113, 114, 114, 115, 117, 119, 122, 124
Compute mean and std dev.

$$\frac{107+109+111+113+113+114+114+115+117+119+122+124}{12} = 114.83$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
107	107 − 114.83 = −7.83	61.36
109	109 − 114.83 = −5.83	34.03
111	111 − 114.83 = −3.83	14.69
113	113 − 114.83 = −1.83	3.36
113	113 − 114.83 = −1.83	3.36
114	114 − 114.83 = −0.83	0.69
114	114 − 114.83 = −0.83	0.69
115	115 − 114.83 = 0.17	0.03
117	117 − 114.83 = 2.17	4.69
119	119 − 114.83 = 4.17	17.36
122	122 − 114.83 = 7.17	51.36
124	124 − 114.83 = 9.17	84.03

$\boxed{\mu = 114.83 \quad \sigma = 5.006}$

$$\sigma = \sqrt{\frac{275.6}{11}} = 5.006$$

b. **Assume the 12 values are sampled from a normal population, and assume that the sample mean and standard deviation calculated in part (a) are actually the population mean and standard deviation. Compute the proportion of bolts whose breaking torque is less than 100 J. Will the shipment be accepted?**

b) $P(x<100)$? $P\left(\frac{x-\mu_x}{\sigma_x} < \frac{100-114.83}{5.006}\right) = P(z < -2.96) \rightarrow 0.0015$

$P(x<100) = 0.0015$

Yes. Only 0.15% of bolts would have breaking torques less than 100J

c. **What if the 12 values had been 108, 110, 112, 114, 114, 115, 115, 116, 118, 120, 123, 140? Use the method outlined in parts (a) and (b) to determine whether the shipment would have been accepted.**

c)

$$\frac{108+110+112+114+114+115+115+116+118+120+123+140}{12} = 117.08\overline{3}$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
108	$108 - 117.08\overline{3} = -9.08\overline{3}$	82.5069
110	$110 - 117.08\overline{3} = -7.08\overline{3}$	50.1736
112	$112 - 117.08\overline{3} = -5.08\overline{3}$	25.8403
114	$114 - 117.08\overline{3} = -3.08\overline{3}$	9.5069
114	$114 - 117.08\overline{3} = -3.08\overline{3}$	9.5069
115	$115 - 117.08\overline{3} = -2.08\overline{3}$	4.3403
115	$115 - 117.08\overline{3} = -2.08\overline{3}$	4.3403
116	$116 - 117.08\overline{3} = -1.08\overline{3}$	1.1736
118	$118 - 117.08\overline{3} = 0.91\overline{6}$	0.8403
120	$120 - 117.08\overline{3} = 2.91\overline{6}$	8.5069
123	$123 - 117.08\overline{3} = 5.91\overline{6}$	35.0069
140	$140 - 117.08\overline{3} = 22.91\overline{6}$	525.1736

$\mu = 117.08\overline{3}$

$\sigma = 6.2982$

$$\sigma = \sqrt{\frac{256.91\overline{6}}{11}} = 6.29$$

$$P(x < 1000)?\quad P\left(\frac{x-\mu_x}{\sigma_x} < \frac{110-117.083}{6.29}\right) = P(z < -2.06)$$

0.0197

$0.0197 \approx 2\%$

$\boxed{\text{No. About 2\% of bolts would have breaking points less than 1000, so the shipment would be rejected.}}$

d. **Compare the sets of 12 values in parts (a) and (c). In which sample are the bolts stronger?**

d) $\boxed{\text{The bolts in part C are stronger.}}$

e. **Is the method valid for both samples? Why or why not?**

e) $\boxed{\text{The method may not be valid in part C as the sample contains an outlier which may skew the results so that a normal distribution should not be used.}}$