MATH 30, SPRING 2020: PRACTICE EXAM 3

(1) We say the function g is the "rooverse" of the function f if

$$g(f(x)) = \sqrt{x} \qquad \text{for all } x > 0.$$

Suppose $f(4) = 3$ and $f'(4) = 5$. Find the value of $g'(3)$. Show your work and explain each step.

(2) Sarah Sellers is jogging around a circular track of radius 50 meters. In a coordinate system with origin at the center of the circle, Sarah's x-coordinate is changing at a rate of $-\frac{5}{4}$ meters per second when her coordinates are $(40, 30)$. Find $\frac{dy}{dt}$ at this moment. Make a sketch to illustrate your answer.

(3) Use linear approximation to estimate $(16.5)^{1/4}$. Write your answer as a fraction.

(4) For the function $f(x) = x^6 - 9x^4$, determine
   (a) intervals where f is increasing or decreasing,
   (b) local minima and maxima of f,
   (c) intervals where f is concave up and concave down, and
   (d) the inflection points of f.
   (e) Also, sketch the graph of f.

(5) A rectangle has one corner on the positive x-axis, one corner on the positive y-axis, one corner at the origin, and one corner on the line $y = -5x + 4$. Which rectangle of this type has greatest area? Show your work.

Here is one example of such a rectangle. Draw a few more, including the optimal rectangle.