

$x$	$y$	$f$
0	0	A
0	1	B
1	0	C
1	1	D

$x$	$y$	
	0	1
0	A	B
1	C	D

① Let  $A=1, B=1, C=0, D=0$

$\bar{x}$	0	1
0	1	1
1	0	0

$$BE = \bar{x}\bar{y} + \bar{x}y$$

$$= \bar{x}(\bar{y} + y) = \bar{x}$$

② Let  $A=0, B=0, C=1, D=1$

$\bar{x}$	0	1
0	0	0
1	1	1

$$BE = x\bar{y} + xy$$

$$= x(\bar{y} + y) = x$$

③ Let  $A=1, C=1, B=D=0$

$\bar{y}$	0	1
0	1	0
1	1	0

$$BE = \bar{y}$$

④  $B=D=1, A=C=0$

$y$	0	1
0	0	1
1	0	1

$$BE = y$$

⑤

	$\bar{y}$	$y$
$\bar{x}$	1	0
$x$	0	1

$x$	$y$	$f$
0	0	A
0	1	B
1	0	C
1	1	D

we cannot make a loop because A D are not adjacent.

$BE = \bar{x}\bar{y} + xy$  . Also this cannot be simplified.

⑥

0	1
1	0

So this when  $B = C = 1$

$$BE = \bar{x}\bar{y} + \bar{x}y$$

observation: In the expression, only one literal can have one complimented.

check out ①  $\bar{x}\bar{y} + \bar{x}\underline{y}$  ✓

②  $\underline{x}\bar{y} + \underline{x}\underline{y}$  ✓

③  $\underline{\bar{x}}\bar{y} + \underline{\bar{x}}\underline{y}$  ✓

⑤  $\bar{x}\bar{y} + xy$  ✗

① How about  $A=0, B=C=D=1$

Page 3

0	1
1	1

Groups must contain  $1, 2, 4, \dots, 2^n$  cells

The cells must be adjacent.

So group of 3 is not allowed. we can split this as

0	1
1	1

WRONG

	$\bar{Y}$	$Y$
$\bar{X}$	0	1
$X$	1	1

Two groups

$$\bar{X}Y + XY \quad (\text{See 4})$$

$$X\bar{Y} + XY \quad (\text{See 2})$$

By combining ② and ④, we get  $x+y$ .

Cross check by simplifying  $\bar{X}Y + \underline{XY} + X\bar{Y} + \underline{XY}$

$$= X\bar{Y} + XY + \bar{X}Y$$

$$= X(\bar{Y} + Y) + \bar{X}Y$$

$$= X + \bar{X}Y = (X + \bar{X})(X + Y) = X + Y.$$

⑧ let  $A=1, B=1, C=1, D=0$

Page 4

1	1
1	0

In this, we could group as

1	1
1	0

$$\bar{x}\bar{y} + \bar{x}y \quad \text{and}$$

$$x\bar{y} + x\bar{y}$$

from ① we have  $\bar{x}$ .

from ③ we have  $\bar{y}$ .

By combining (spell), we have  $\bar{x} + \bar{y}$ .

$$\begin{aligned} \text{cross check} \quad & \bar{x}\bar{y} + \bar{x}y + \bar{x}\bar{y} + x\bar{y} \\ &= \bar{x}(\bar{y} + y) + \bar{y}(\bar{x} + x) \\ &= \bar{x} + \bar{y} \end{aligned}$$

we can group cells as large as possible, but it has to be adjacent to each other and can overlap, not diagonal.

In case 3 variables,  $X Y Z$  can table is Page 5

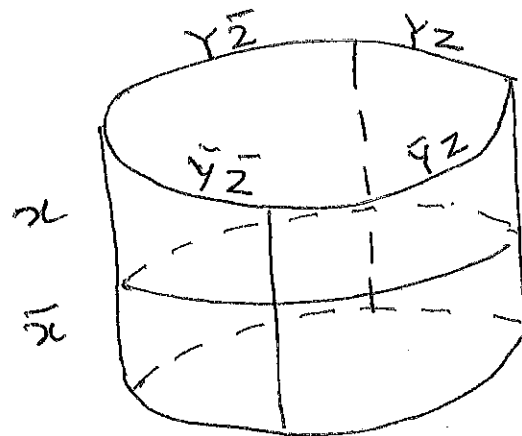
$X$	$Y$	$Z$	$F$
0	0	0	A
0	0	1	B
0	1	0	C
0	1	1	D
1	0	0	E
1	0	1	F
1	1	0	G
1	1	1	H

The corresponding map is

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	$YZ$	$Y\bar{Z}$
$\bar{x}$	A	B	D	C
$x$	E	F	H	G

Note how B and D are adjacent in the map, but not in the table. So is F and H.

The rules governing formation of groups still applies. Also, you have to visualize that the map wraps around making the first and last column as adjacent.



⑨ let  $A = B = D = C = 1$

	$\bar{y}\bar{z}$	$\bar{y}z$	$y\bar{z}$	$yz$
$\bar{x}$	1	1	1	1
$x$	0	0	0	0

we can form a group

$\bar{x}$	1	1	1	1
$x$	0	0	0	0

Boolean expression  $\bar{x}\bar{y}\bar{z} + \bar{y}z\bar{x} + \bar{x}y\bar{z} + \bar{x}yz$

$$= \bar{x}(\bar{y}\bar{z} + \bar{y}z) + \bar{x}(y\bar{z} + yz)$$

$$= \bar{x}\bar{y}(\bar{z} + z) + \bar{x}y(\bar{z} + z)$$

$$= \bar{x}\bar{y} + \bar{x}y = \bar{x}(\bar{y} + y) = \bar{x}$$

In other words, the top row depends on  $\bar{x}$  but not any values of  $y, \bar{y}, z, \bar{z}$ .

⑩ How about

$$E = F = H = G = 1, B = A = C = D = 0$$

$\bar{x}$	0	0	0	0
$x$	1	1	1	1

As noted above, the value depends entirely on  $x$ , but not any values of  $y, \bar{y}, z, \bar{z}$

11. other possible groups are

Page 7

$\bar{x}$	1	1	0	0
$x$	1	1	0	0

$\bar{y}$

12.

0	0	1	1
0	0	1	1

$y$

13.

0	1	1	0
0	1	1	0

$z$

14

1	0	0	1
1	0	0	1

$\bar{z}$

15.

$\bar{x}$	1	1	1	1
$x$			1	1

combine (9) and (12)  
 $\bar{x} + y$

1	1	1	1
		1	1

incorrect because  
 we need them as large  
 as possible.

(16)

0	0	1	1
0	0	0	1

we need two overlapping groups

		YZ	Y $\bar{Z}$
$\bar{X}$	0	0	1
X	0	0	1

$$\bar{x}yz + \bar{x}y\bar{z} \text{ and}$$

$$\bar{x}y\bar{z} + x y \bar{z}$$

which yields

$$\bar{x}y + y\bar{z} = y(\bar{x} + \bar{z})$$

(17)

1	1	0	0
1	0	0	0

again we need two overlapping groups

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$		
$\bar{X}$	1	1	0	0
X	1	0	0	0

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z \text{ and}$$

$$\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z}$$

which when combined

$$= \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z}$$

$$= \bar{x}\bar{y} + \bar{y}\bar{z} = \bar{y}(\bar{x} + \bar{z})$$