# MATH 30, 4/10/2020: L'HÔPITAL, CONT'D.

Last time: When you get "$\frac{0}{0}$" or "$\frac{\infty}{\infty}$" in a limit, it means "I need to do more work." At the beginning of the semester, we factorized and canceled. Now we have a new technique: L'Hôpital's Rule.

We can also use L'Hôpital's Rule for limits of *products* where we get the indeterminate form "$0 \cdot \infty$," which is also meaningless.

**Example.** $\lim_{x \to 0^+} x \ln x =$? It looks like "$0 \cdot \infty$," which really means "I need to do more work."

We rewrite it as:

$$\begin{aligned}
\lim_{x \to 0^+} x \ln x &= \lim_{x \to 0^+} \frac{\ln x}{\left(\frac{1}{x}\right)} && \text{but now it looks like "}\tfrac{-\infty}{\infty}\text{,"} \\
&= \lim_{x \to 0^+} \frac{(1/x)}{(-1/x^2)} && \text{which means we can use L'Hôpital's Rule as before} \\
&= \lim_{x \to 0^+} (-x) = 0.
\end{aligned}$$

**Example.** $\lim_{x \to \infty} x^3 e^{-x^2} =$? Again it looks like "$\infty \cdot 0$," which means "I need to do more work." Rewrite it as:

$$\begin{aligned}
\lim_{x \to \infty} x^3 e^{-x^2} &= \lim_{x \to \infty} \frac{x^3}{e^{x^2}} && \text{but now it looks like "}\frac{\infty}{\infty}\text{,"} \\
&= \lim_{x \to \infty} \frac{3x^2}{2xe^{x^2}} && \text{which means we can use L'Hôpital's Rule} \\
&= \lim_{x \to \infty} \frac{3x}{2e^{x^2}} \\
&= \lim_{x \to \infty} \frac{3}{4xe^{x^2}} = 0 && \text{and use it again....}
\end{aligned}$$

**Application:** We can use L'Hôpital's Rule to help with curve sketching.

On the next page I have a worksheet for you to try.

(1) Sketch the graph of the function $f(x) = (x^2+x+1)e^{-x}$. Start by finding critical points, where it is increasing & decreasing, where it has local max & min, and what it does as $x \to \pm\infty$.

(2) Sketch the graph of the function $f(x) = \sqrt{x}\ln x$ over $[0, \infty)$. Start by finding critical points, where it is increasing & decreasing, where it has local max & min, and what it does as $x \to 0$ and $x \to \infty$.

[You will need another piece of paper. ☺]