MATH 30, 4/8/2020: CORONA QUIZ 7 SOLUTION

Related Rates. The radius and height of a circular cylinder are changing with time in such a way that the volume remains constant at 1000 cubic centimeters. If, at a certain time, the radius is 4 centimeters and is increasing at a rate of $\frac{1}{2}$ centimeter per second, what is the rate of change of the height?

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(1) **Read the problem carefully.**

(2) **Draw a picture of a cylinder.**

(3) **Introduce notation:** Let $r(t)$ denote the radius at time t and $h(t)$ the height at time t.

(4) **Express the given information mathematically:** The volume is a constant:

$$V = \pi r(t)^2 h(t) = 1000\,\mathrm{cm}^3.$$

We want to know: At a time when $r(t) = 4$ and $\dfrac{dr}{dt} = \dfrac{1}{2}$ (centimeters per second), what is $\dfrac{dh}{dt}$?

(5) **Write an equation that relates the various quantities:** Oops, I already did that:

$$V = \pi r(t)^2 h(t) = 1000\,\mathrm{cm}^3.$$

(6) **Use the Chain Rule.** Since V is constant, we have

$$0 = \frac{dV}{dt} = 2\pi r(t) r'(t) h(t) + \pi r(t)^2 h'(t). \qquad (*)$$

(Here we used the Chain Rule and the Product Rule.)

(7) **Substitute into the resulting equation and solve for the related rate.** We are interested in a time when $r(t) = 4$ and $r'(t) = \frac{1}{2}$. But also at that time we can use the equation $V = \pi r^2 h = 1000$ to find that $h = \frac{125}{2\pi}$ at that time. Substitute all these values into equation $(*)$ to find that

$$h'(t) = -\frac{125}{8\pi} \text{ centimeters per second.}$$

at that time.