1. 15 → 0.95 chance of working in 24 hours

Need 14 must work for success $\boxed{0.290}$

a) Prob of success:
$X \sim$ Poisson(15) $X \sim$ Bin(15, 0.95) 0.347046

4L - do not need to calculate integrals, just set them up!

→ $P(X \geq 14)$

$1 - \boxed{P(X = 14) + P(X = 15)} = \binom{15}{14}(0.95)^{14}(0.05)^1 = 0.367$

$\binom{15}{15}(0.95)^{15}(0.05)^0 = 0.46$

b) Expected to work

$15 * 0.95 = 14.25 = \boxed{14}$

c) * Gamma $\Gamma(\alpha, \lambda)$

$T \sim \Gamma(2, 15)$

$P(X = 5)$

$\frac{e^{-2} 2^5}{5!} = \boxed{0.036,\ X \sim \text{Poisson}(2)}$

2. reg X_1 +\$0.52 $\mu_1 = 1200$ $\sigma_1 = 140$

mid X_2 +\$0.61 $\mu_2 = 350$ $\sigma_2 = 80$

prem X_3 -\$0.15 $\mu_3 = 200$ $\sigma_3 = 80$

$P = 0.52X_1 + 0.61X_2 - 0.15X_3$

a) Find $E(P)$

$1200 \cdot 0.52 = 624$
$350 \cdot 0.61 = 213.5$
$200 \cdot -0.15 = -30$

$\boxed{E(P) = \$807.50}$

$E(P) = 624 + 213.5 - 30$

b) Assuming indep. Find std dev of profit

σ_P $0.52^2(140^2) + 0.61^2(80^2) + 0.15^2(80^2)$

~~5299.84 + 2381.44 + 144 = 68.724~~

$= 82.96$

$\sqrt{7825.28} = \sqrt{68.724} = \boxed{82.96}$

c) $P(X > 900)$?

$$\frac{900 - 807.50}{82.96} = 1.05$$

$\downarrow$

$\boxed{0.8531}$

3. $X \sim \text{Poisson}(4)$

a) T = time until 3rd

$T \sim \Gamma(3,4)$ $P(T>2) = 1 - P(T \le 2)$

$1 - \sum_{i=0}^{2} e^{-4 \cdot 3} \frac{(4 \cdot 1)^i}{i!} =$ $1 - 1.1010^{-4} = 0.99$

$e^{-12} \left(\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \cancel{\frac{4^3}{3!}} \right) = 1.1010^{-4}$

$\boxed{T \sim \Gamma(3,4),\ 0.99}$

b) $60 \times 1/4 = 15$ $\quad 1 - P(X \le 1)$

$\boxed{X \sim \text{Poisson}(1) \text{ and } 0.26}$

$\frac{e^{-1} 1^0}{0!} = 0.36$

$\frac{e^{-1} 1^1}{1!} = 0.36$

$1 - (0.36 + 0.36) = 0.26$

c) $P(T > 2)$

$\boxed{\int_2^\infty x^2 e^{-4x}\,dx}$

4. a) $f_X(x) = \int_0^{\infty} f(x,y)\,dy$

$$\boxed{\begin{array}{l} f_X(x) = \int_0^2 \frac{1}{5}(x+3y)\,dy \\ f_Y(x) = \int_0^1 \frac{1}{5}(x+3y)\,dx \end{array}}$$

b) Indep.?

No

$\int_0^1 \frac{1}{5}(x+y)\,dy$

$\frac{1}{3}\int_0^1 (x+3y)\,dy = \frac{3}{2}$

$(x+3y - x)$

$0+3$

$\frac{1}{3} 3 = 1$

$1+3y^3 - 3y$

$\frac{1}{3}\int_0^1 (x+3y)\,dx = \frac{1}{3}$

$\frac{1}{3}(1)(0) \frac{1}{3}$

$\boxed{\text{No, because } f(x,y) \neq f_X(x) \,\&\, f_Y(y)}$

c) $\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$

$$\boxed{\mathrm{Cov}(X,Y) = \int_0^2\int_0^1 \frac{1}{5}(x+3y)\,dx\,dy - \int_0^2 \frac{1}{5}(x+3y)\,dy \int_0^1 \frac{1}{5}(x+3y)\,dx}$$

5. $\mu = 1.25$
Std dev $= 0.3$

$P(\bar{x} < 1.15)$

$$\frac{1.15 - 1.25}{0.3} = -0.33 \downarrow 0.3707$$

$\bar{x} \sim N(1.25, 0.3)$
Normal because sample > 30!
0.3707

6. $T \sim Exp(2)$

a) $P(X > 1)$ $\quad 1 - P(X \leq 1)$

$\lambda = 2$ $\quad 1 - (1 - e^{-1(1)}) = e^{-1} = \boxed{0.3678}$

b) $P(Y > 1 \mid X = 2)$

$T \geq t + s \mid T > s = P(T > t)$

$P(T > 1)$

$1 - P(T \leq 1) = 1 - F(1)$

$e^{-2(1)} = 0.135$

$P(Y > 1 \mid X = 2)$, 0.135

The problem is asking that what are the odds that the next pothole is as a mile away when you haven't encountered one. In other words the road may be expected to be clear if you haven't found a pothole.

7.

a) $\boxed{H \sim (\overset{N}{500}, \overset{R}{100}, \overset{n}{20})}$

b) $P(X \geq 19) = \dfrac{\binom{100}{20}\binom{500-100}{20-19}}{\binom{80}{20}}$

$= \boxed{\dfrac{\dfrac{100!}{5!(95)!} \cdot \dfrac{400!}{1!19!}}{\dfrac{500!}{20!480!}}}$

8. $X_1, X_2, X_3, X_4 \sim MN(20, 0.5, 0.1, 0.25, 0.15)$

$= \boxed{\dfrac{20!}{8!3!5!4!}(0.5)^8(0.1)^3(0.25)^5(0.15)^4}$