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CSC 28 Fall 2020

Relations (Total 150 points)

1.1) From the given ordered pairs (3, 5); (8, 4); (6, 6); (9, 11); (6, 3); (3, 0); (1, 2) find the following relations. Also, find the **domain and range** of each relation.

(a) Is greater than (2 points)

Greater than = {(8,4), (6,3), (3,0)}

Domain = {8, 6, 3}, Range = {4, 3, 0}

Looking for greater than 0, 3-5=-2, 8-4=4, 6-6=0, 9-11=-2, 6-3=3, 3-0=3, 1-2=-1

(b) Is less than (2 points)

Less than = {(3,5), (9,11), (1,2)}
Domain = {3, 9, 1}, Range = {5, 11, 2}

Looker for less than 0, 3-5=-2, 8-4=4, 6-6=0, 9-11=-2, 6-3=3, 3-0=3, 1-2=-1

(c) Is less than, by at least two (2 points)

Less than = {(3,5), (9,11)}
Domain = {3, 9}, Range = {5, 11}

Looking for less than -1, 3-5=-2, 8-4=4, 6-6=0, 9-11=-2, 6-3=3, 3-0=3, 1-2=-1

(d) Is equal to (2 points)

Equal to = {(6,6)}
Domain = {6}, Range = {6}

Looking for 0, 3-5=-2, 8-4=4, 6-6=0, 9-11=-2, 6-3=3, 3-0=3, 1-2=-1

1.2)

(a) Determine the domain and range of the relation R defined by

R = {x + 2, x - 3}; x ∈ {6, 7, 8, 9} (domain and range: 4 points) and (b) draw the arrow diagram for relation R. (4 points)

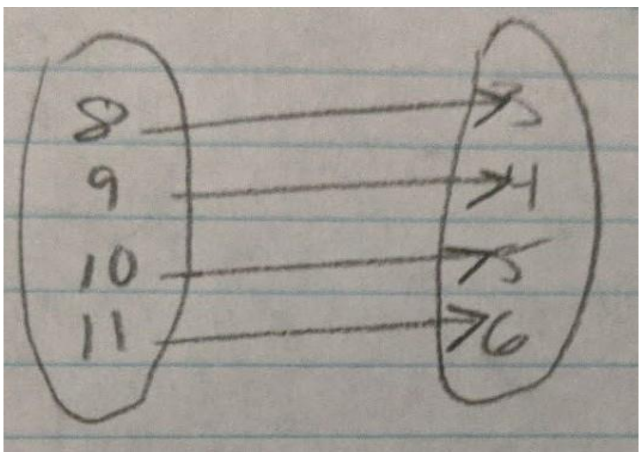
x=6, x + 2 = 0 > 6 + 2 = 8; x - 3 = 0 > 6 - 3 = 3

x=7, x + 2 = 0 > 7 + 2 = 9; x - 3 = 0 > 7 - 3 = 4

x=8, x + 2 = 0 > 8 + 2 = 10; x - 3 = 0 > 8 - 3 = 5

x=9, x + 2 = 0 > 9 + 2 = 11; x - 3 = 0 > 9 - 3 = 6

Domain = {8, 9, 10, 11}, Range = {3, 4, 5, 6}



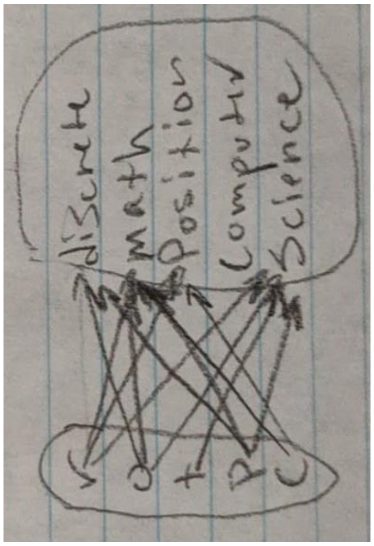
(c) What is the range of the relation R if the co-domain includes additional values 7, 12? (1 points)

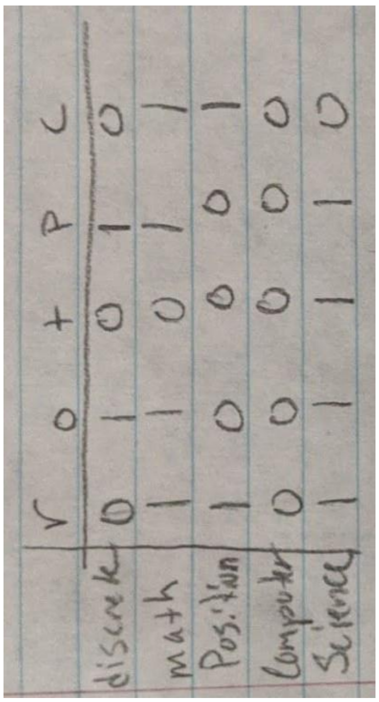
Range = {3, 4, 5, 6}

2) Draw the **<u>arrow diagram</u>** and the **<u>matrix representation</u>** for the following two relations:

(a) Define the set A = {r, o, t, p, c} and B = {discrete, math, position, computer, science}. Define the relation R ⊆ A × B such that (letter, word) is in the relation if that letter doesn't occur somewhere in the word. Be careful - recheck your answer again to see if you by mistake reversed the definition for any pair (if you mistakenly used 'does occur' for 'doesn't occur'). Even if you know the answer psychological distraction can play a role. It can happen to me too!

(Arrow Diagram 5 points + Matrix representation 5 points)



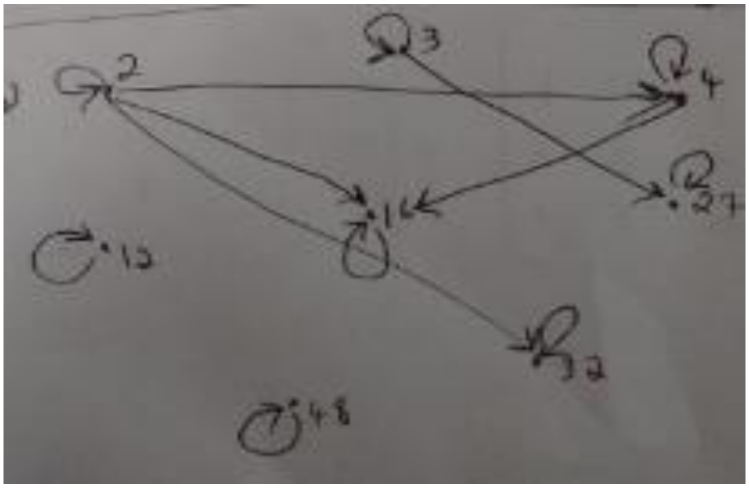


	r	o	t	p	c
discrete	0	1	0	1	0
math	1	1	0	1	1
Position	1	0	0	0	1
Computer	0	0	0	0	0
Science	1	1	1	1	0

Please Note: {r, o, t, p, c} are the rows and {discrete, math, position, computer, science} are in the columns to show that the pairs are (r, math), (r, position) and so on and not (math, r), (position, r).
Please view the top arrow diagram horizontally like in 1.2 (a)!

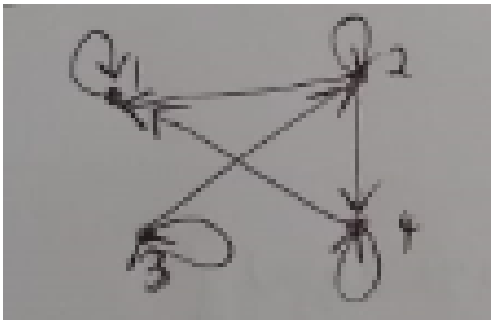
(b) The domain for the relation M is the set {2, 3, 4, 12, 16, 27, 32, 48}. For x, y in the domain, xMy if there is a positive integer n such that x power n = y. Be careful - include only (x, y) pairs (not (x, n) or n, y) pairs). Even if you know the answer, psychological distraction can play a role. It can happen to too!

(Arrow Diagram 5 points + Matrix representation 5 points)


2
3
4
27
16
12
32
48

3) Give the arrow diagram for each of the two matrices, then express the relation as a set of ordered pairs (The rows and columns are numbered 1 through 3 or 4. In other words, the domain and co domain contain the row and column numbers.)

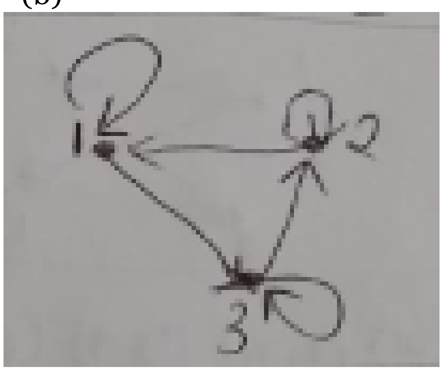
(a)



R = {(1,1), (2,1), (2,2), (2,4), (3,2), (3,3), (4,1), (4,4)}

Arrow Diagram (8 points) + Relation as ordered pairs (8 points) + correct domain and co-domain (1 point)

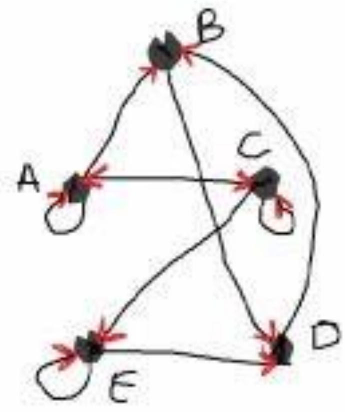
(b)



R = {(1,1), (1,3), (2,1), (2,2), (3,2), (3,3)}

4) Give the matrix representation for the relation depicted in each arrow diagram. Then express the relation as a set of ordered pairs. Assume the domain and co-domain for each graph contain the letters in the corresponding graph.

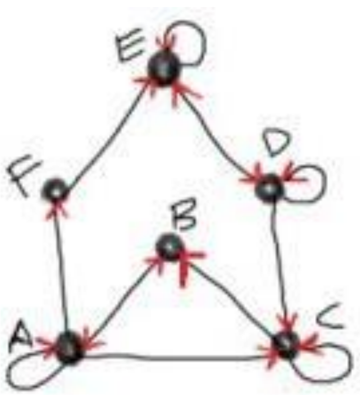
(a)



	A	B	C	D	E
A	1	1	1	0	0
B	0	0	0	1	0
C	1	0	1	0	1
D	0	1	0	0	0
E	0	0	0	1	1

Ordered pair
{(A, A) (A,B), (A,C),(B,D), (C,A), (C,C),(C,E),(D,B),(E,D),(E,E)}

(matrix representation 10 points) + (relation as ordered pairs 10 points)
(b)



(b)

	A	B	C	D	E	F
A	1	1	1	0	0	1
B	1	0	0	0	0	0
C	0	1	1	0	0	0
D	0	0	1	1	1	0
E	0	0	0	1	1	0
F	0	0	0	0	1	0

Ordered pair is
{(A,A),(A,B),(A,C),(A,F),(B,A),(C,B),(C,C),(D,C),(D,D),(D,E),(E,D),(E,E),(F,E)}

5) What are the types of Relations, Explain each with ***<u>one example</u>*** on your own. (3 * 2 points)

A reflexive relation is when each element in the domain has a matching element in the range with the same value. Ex. R = {(1,1), (2,2), (3,3), (5,5), (7,7), (9,9)}

A symmetric relation is when each ordered pair has another ordered pair with its X and Y values swapped. Ex. S = {(1,2), (2,1), (1,3), (3,1)}

An asymmetric relation is when each ordered pair does not have another ordered pair with its X and Y value swapped. Ex. A = {(1,2), (2,2), (1,3), (3,2)}

A transitive relation is when there is a path from one element to another element in a set. Ex. T = {(1,2), (2,3), (1,3)}

An equivalence relation is when a relation is reflexive, symmetric and transitive.
Ex. E = {(1,1), (1,3), (1,5), (2,2), (3,3), (3,1), (4,4), (5,1), (5,5)}

6) True/ False (To get full marks explain the answer in one or two sentences) (5 * 1 points)

(a) If R is the set of all females in a family, then the relation "is sister of" is reflexive over R.

False, the relation 'is sister of' is not reflexive over the set R of all females in a family because you cannot be your own sister.

b) If R is the set of all females in a family, then the relation "is sister of" is not symmetric over R.

False, the relation 'is sister of' is symmetric over the set R of all females in a family because two peoples' relation does not change if taken from different perspectives. Female A is sister to female B and female B is sister to female A.

(c) If R is the set of mothers and S is the set of children in a family then a relation F on RxS is a symmetric relation.
False, the mothers cannot be mother to themselves, therefore they cannot reappear in the range to make this relation symmetric.

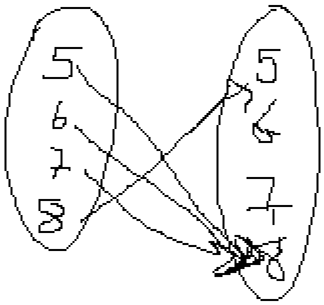
(d) "Is the same height as" is a reflexive relation.
True, 'is the same height as' is a reflexive relation because you will always be the same height as yourself.

(e) "Is descendent of" is a transitive relation.
True, 'is descendent of' is a transitive relation because there is always a path down a person'sz ancestry that connects all the elements.

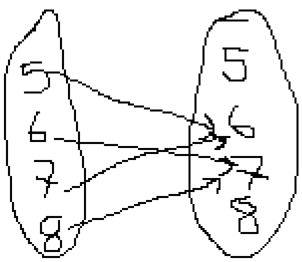
7) Let A= {5, 6, 7, 8}, Let R: A-> A and S: A-> A. Let R= {(5, 6) (6, 8) (7, 6) (8, 7)} and S= {(5, 5) (6, 8) (7, 5) (8, 8)}. Find the answers for the following and draw the arrow diagram for each result.

(a) R o S (5 points)

{(5,8),(6,8) and (7,8), (8,5)}



(b) S o R (5 points)



(5,6),(6,7), (7,6)and (8,7).

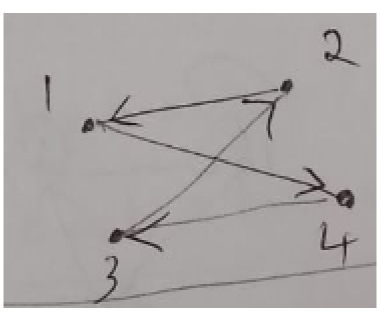
8) Below is the arrow diagram (graph) for relation R with the domain {1, 2, 3, 4}.

Define a new relation A to be R o R

(a) Express relation A as a set of related pairs. (5 points)

{(1,3),(2,4),(3,1),(4,2)}

(b) Draw the arrow diagram for R o A. (5 points)



9) What are the three types of closures in relation and **<u>explain with an example</u>**. (3 * 2 points)

Symmetric closure is when you add ordered pairs to a relation to make it symmetric. Ex. R = {(1,1), (2,2), (3,4)} is not symmetric, add (4,3)

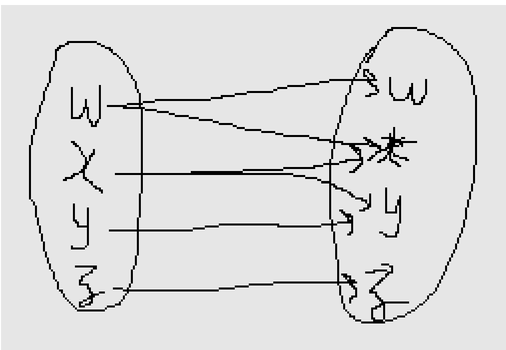
Transitive closure is when you add a path between unconnected elements to make the relation transitive. Ex. R = {(1,2), (2,3)} is not transitive, add (1,3)

Reflexive closure is when you add ordered pairs to a relation to make it reflexive.

Ex. R = {(1,1), (2,2), (3,4), (5,5)} is not reflexive, add (3,3), (4,4)

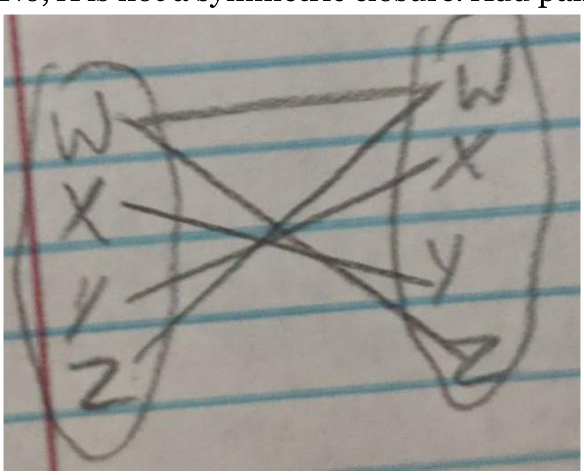
10) Given R= {w, x, y, z} and relation A= {(w, w) (x, y) (w, z)}.

(a) Is relation A reflexive closure? if not what are the pairs that should be added to make it a proper reflexive closure? Explain in detail and then draw arrow diagram for the result. (2 points) No, A is not a reflexive closure. Add pairs (x,x), (y,y), (z,z) to make it reflexive.



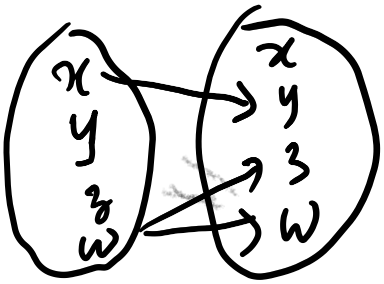
Every element that is connected to something needs to have a connection to itself or else it is not reflexive

(b) Is relation A symmetric closure? if not what are the pairs that should be added to make it a proper symmetric closure? Explain in detail and then draw arrow diagram for the result. (2 points)
No, A is not a symmetric closure. Add pairs (y,x) and (z,w)



Every x element in the set needs to have a y element. Each ordered pair has to have an inverse

(c) Is relation A transitive closure? if not what are the pairs that should be added to make it a proper transitive closure? Explain in detail and then draw arrow diagram for the result. (2 points)
No, relation A is not transitive. Add pair (w, z) since (w, w) & (w, z) => (w, z) and (w, z) is already in A!



To make this relation transitive, there must be a path between each element