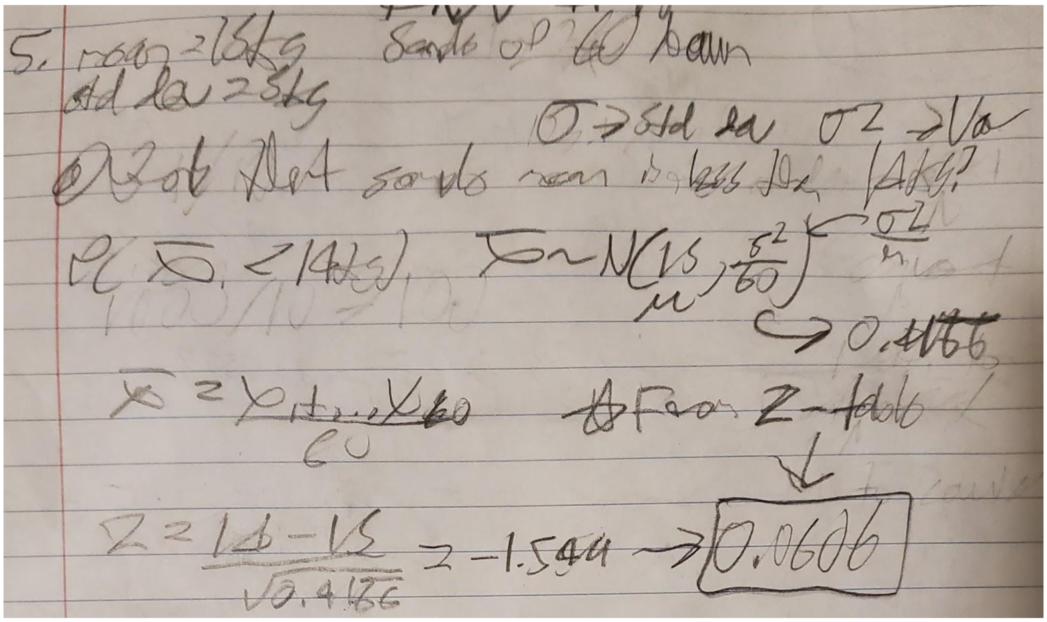
STAT 50 HW #13

Section 4.11 #'s 5, 6, 7, 8, 18

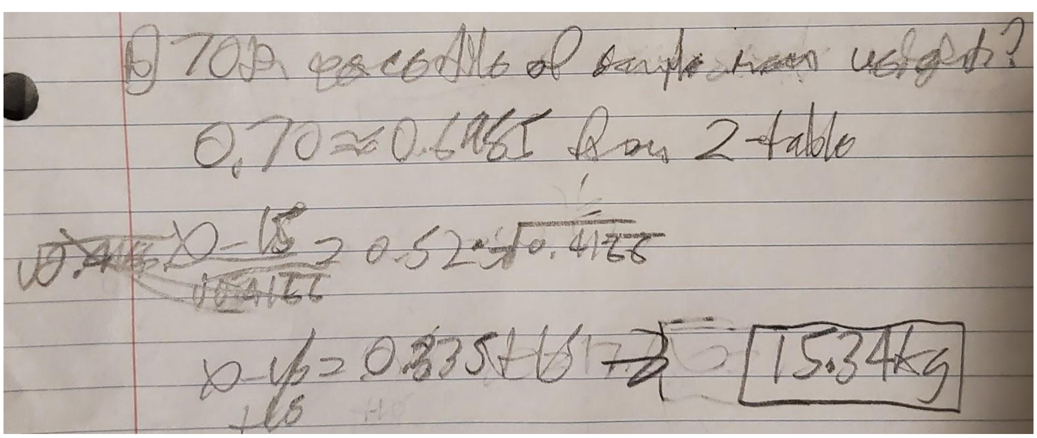
5.

Bags checked for a certain airline flight have a mean weight of 15 kg with a standard deviation of 5 kg. A random sample of 60 bags is drawn.

a. **What is the probability that the sample mean weight is less than 14 kg?**


5. mean = 15kg   Sample of 60 drawn
std dev = 5kg
$\sigma \to$ std dev $\sigma^2 \to$ Var
a) Prob that sample mean is less than 14kg?
$P(\bar{X} < 14\text{kg})$ $\bar{X} \sim N(\underset{\mu}{15}, \frac{5^2}{60})$ ← $\frac{\sigma^2}{n}$
→ 0.4166
$\bar{X} = \frac{X_1 + \dots + X_{60}}{60}$ From Z-table
$Z = \frac{14-15}{\sqrt{0.4166}} = -1.549 \to \boxed{0.0606}$

b. **Find the 70th percentile of the sample mean weights.**


b) 70th percentile of sample mean weights?
$0.70 \approx 0.6985$ from Z-table
$\frac{x-15}{\sqrt{0.4166}} = 0.52 \cdot \sqrt{0.4166}$
$x - 15 = 0.3356 + 15 \to \boxed{15.34\text{kg}}$

c. **How many bags must be sampled so that the probability is 0.01 that the sample mean weight is less than 14 kg?**

c) Sample size needed for prob 0.01 that sample mean weight is less than 14 kg?

Z table

$0.01 \to 0.0049 \to -2.33$ $\quad P(\bar{X} < 14)$

$Z = \frac{14-15}{\sqrt{5}/\sqrt{n}} = -2.33 \to \frac{-1}{5/\sqrt{n}} = -2.33$

$\sqrt{\frac{s^2}{n}} = \frac{s}{\sqrt{n}}$

$\frac{-1}{\frac{5}{\sqrt{n}}} \cdot \sqrt{n} \quad \cancel{\Rightarrow} -\frac{1}{5}\sqrt{n} = -2.33 \cdot 5$

$\sqrt{n} = 11.65$

$n = 11.65^2$

$\boxed{136} \leftarrow 135.7225$

6.

The amount of warpage in a type of wafer used in the manufacture of integrated circuits has mean 1.3 mm and standard deviation 0.1 mm. A random sample of 200 wafers is drawn.

a. **What is the probability that the sample mean warpage exceeds 1.305 mm?**

6. mean = 1.3 mm $\quad$ random sample of 200

std dev = 0.1 mm

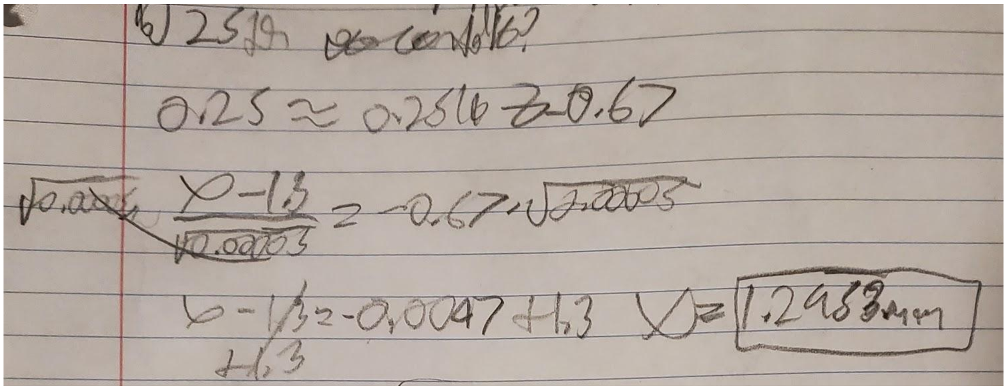
a) Prob that sample mean exceeds 1.305 mm?

$P(\bar{X} > 1.305)$ $\quad \bar{X} \sim (1.3, \frac{0.1^2}{200}) \to 0.00005$

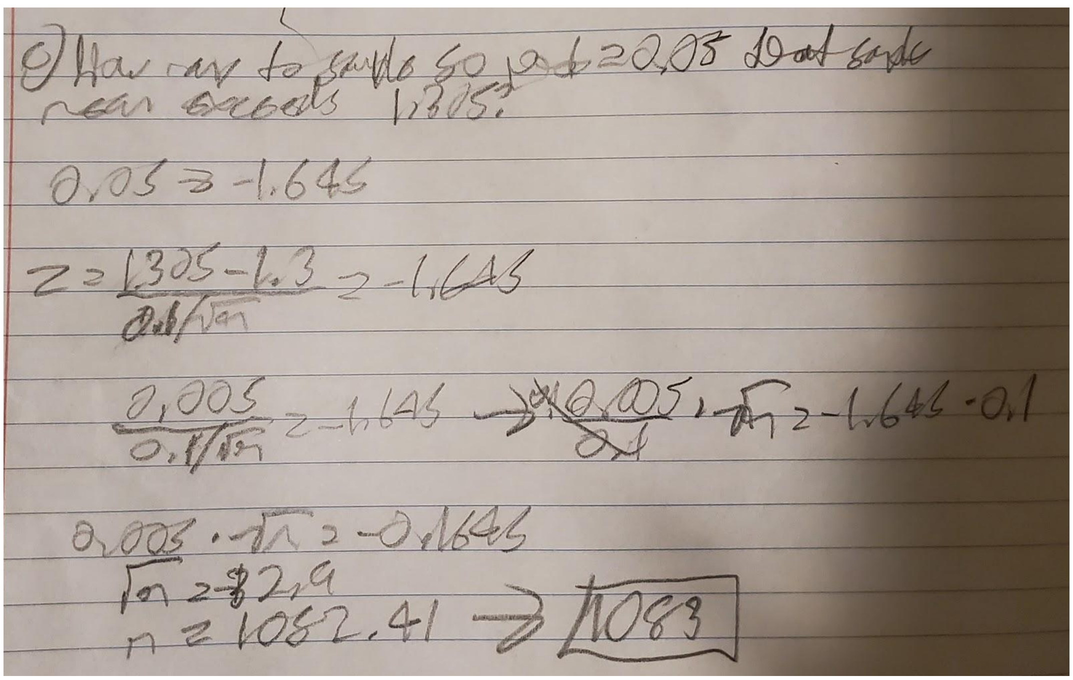
$Z = \frac{1.305 - 1.3}{\sqrt{0.00005}} = 0.71 \to 0.7611$

$1 - 0.7611 = \boxed{0.2389}$

b. **Find the 25th percentile of the sample mean.**



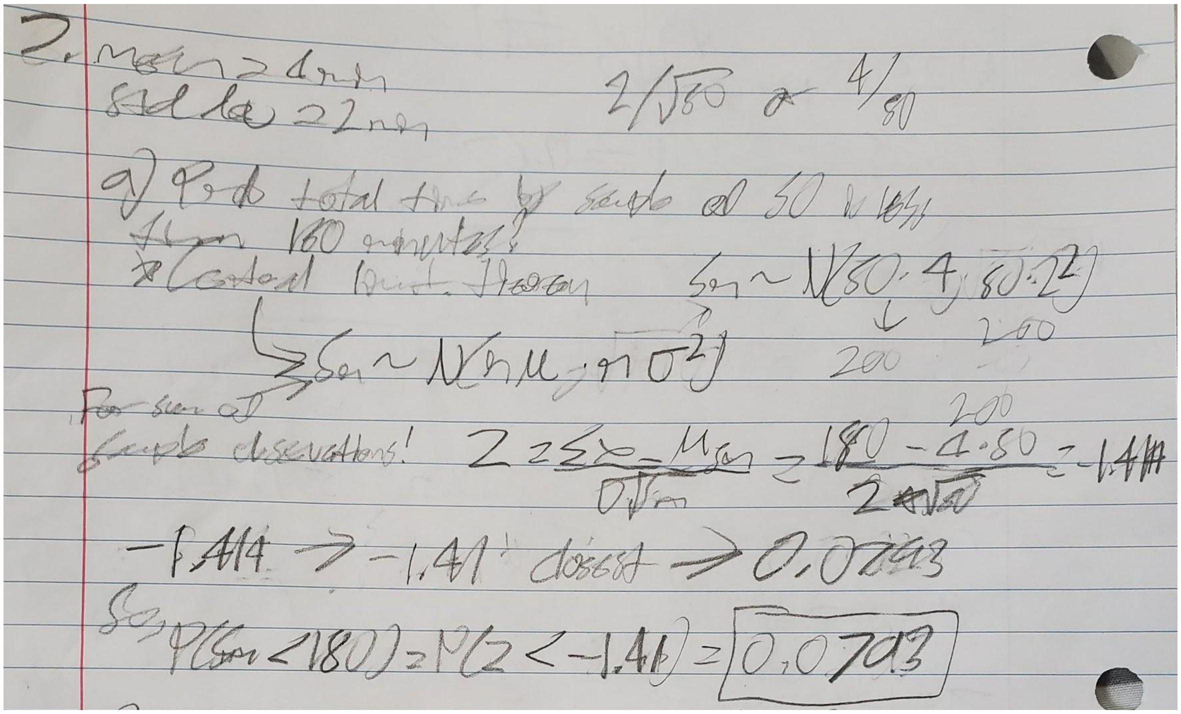
c. **How many wafers must be sampled so that the probability is 0.05 that the sample mean exceeds 1.305?**



7.

The time spent by a customer at a checkout counter has mean 4 minutes and standard deviation 2 minutes.

a. **What is the probability that the total time taken by a random sample of 50 customers is less than 180 minutes?**



2. Mean = 4 min
Std dev = 2 min $\quad 2/\sqrt{50}$ or $4/50$

a) Prob total time by sample of 50 is less than 180 minutes?

* Central Limit Theorem $\quad S_n \sim N(50\cdot 4, 50\cdot 2^2)$ → 200, 200

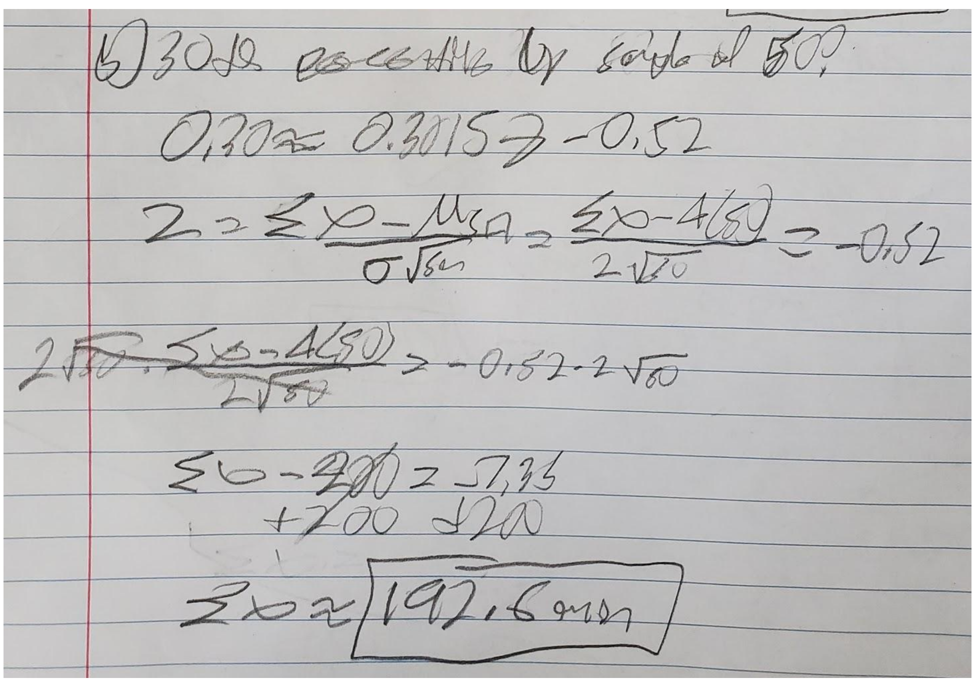
$\sum S_n \sim N(n\mu, n\sigma^2)$

For sum of sample observations! $\quad Z = \frac{\sum x - \mu_{sn}}{\sigma\sqrt{n}} = \frac{180 - 4\cdot 50}{2\sqrt{50}} = -1.414$

$-1.414 \rightarrow -1.41$ closest $\rightarrow 0.0793$

So, $P(S_n < 180) = P(Z < -1.41) = \boxed{0.0793}$

b. **Find the 30th percentile of the total time taken by 50 customers.**



b) 30th percentile by sample of 50?

$0.30 \approx 0.3015 \rightarrow -0.52$

$Z = \frac{\sum x - \mu_{sn}}{\sigma\sqrt{sn}} = \frac{\sum x - 4(50)}{2\sqrt{50}} = -0.52$

$2\sqrt{50}\cdot\frac{\sum x - 4(50)}{2\sqrt{50}} = -0.52\cdot 2\sqrt{50}$

$\sum x - 200 = -7.35$
$\quad +200 \quad +200$

$\sum x \approx \boxed{192.6 \text{ min}}$

8.

The time taken by an automobile mechanic to complete an oil change is random with mean 29.5 minutes and standard deviation 3 minutes.

a. **What is the probability that 50 oil changes take more than 1500 minutes?**

8. mean = 29.5 min random
std dev = 3 min

a) Prob that 50 takes more than 1500 min?

normal
$S_n \sim N(n\mu, n\sigma^2) \Rightarrow S_n \sim N(50 \cdot 29.5, 50 \cdot 3^2)$
1475 450

So, $P(S_n > 1500) = 1 - P(S_n \leq 1500)$

$= 1 - P\left(\frac{S_n - n\mu}{\sqrt{n}\,\sigma} \leq \frac{1500 - 50 \cdot 29.5}{\sqrt{50} \cdot 3}\right)$

$= 1 - P(Z \leq 1.1785)$

1.18 → 0.8810

$= 1 - 0.8810 = \boxed{0.119}$

b. **What is the probability that a mechanic can complete 80 or more oil changes in 40 hours?**

b) Prob that 80 or more changes done in 40 hrs?
n = 80
$40 \times 60 = 2400$ min
$P(S_n < 2400) = P\left(\frac{S_n - n\mu}{\sqrt{n}\,\sigma} \leq \frac{2400 - 80 \cdot 29.5}{\sqrt{80} \cdot 3}\right)$

$= P(Z < 1.4907)$

1.49 → $\boxed{0.9319}$

c. **The mechanic wants to reduce the mean time per oil change so that the probability is 0.99 that 80 or more oil changes can be completed in 40 hours. What does the mean time need to be? Assume the standard deviation remains 3 minutes.**

c) What should mean be reduced to so that prob is 0.99 that 80 or more changes can be completed in 40 hours? Std dev still 3 min

mean = ?

std dev = 3 min

$$P(S_n < 2400) = P\left(\frac{S_n - n\mu}{\sqrt{n}\sigma} < \frac{2400 - 80\cdot\mu}{\sqrt{80}\cdot 3}\right)$$

$$= P\left(Z < \frac{2400 - 80\cdot\mu}{\sqrt{80}\cdot 3}\right)$$

$$= 0.99 \rightarrow 2.33$$

$$\sqrt{80}\cdot 3 \quad \frac{2400 - 80\cdot\mu}{\cancel{\sqrt{80}\cdot 3}} = 2.33\sqrt{80}\cdot 3$$

$$2400 - 80\cdot\mu = 62.52$$

$$-2400 \qquad -2400$$

$$\frac{-80\cdot\mu}{-80} = \frac{-2337.47}{-80} \qquad \mu = \boxed{29.22 \text{ min}}$$

18.

The manufacture of a certain part requires two different machine operations. The time on machine 1 has mean 0.5 hours and standard deviation 0.4 hours. The time on machine 2 has mean 0.6 hours and standard deviation 0.5 hours. The times needed on the machines are independent. Suppose that 100 parts are manufactured.

a. **What is the probability that the total time used by machine 1 is greater than 55 hours?**

HW #13

18. m1:
mean = 0.5 hrs
Std dev = 0.4 hrs

m2:
mean = 0.6 hrs
Std dev = 0.5 hrs

time independent for both machines
100 parts manufactured

a) Prob that tot time used by m1 > 55 hrs?

$S_n \sim N(n\mu, n\sigma^2) \rightarrow S_n \sim N(100 \cdot 0.5, 100 \cdot 0.4^2)$

$50, 16$

$Z = \frac{\sum x - \mu_{S_n}}{\sigma_{S_n}} = \frac{55 - \overset{50}{100(0.5)}}{\sqrt{100}(0.4)} = 1.25$

$P(S_n > 55) = P(Z > 1.25) = 1 - P(Z < 1.25)$ $\boxed{0.1056}$

$= 1 - 0.8944 = 0.1056$

b. What is the probability that the total time used by machine 2 is less than 55 hours?

b) Prob that tot time by m2 < 55 hrs?

$S_n \sim N(\underset{60}{100 \cdot 0.6}, \underset{25}{100 \cdot 0.5^2})$

$Z = \frac{\sum x - \mu_{S_n}}{\sigma_{S_n}} = \frac{55 - \overset{60}{100(0.6)}}{\sqrt{100}(0.5)} = -1$

$P(S_n < 55) = P(Z < -1) = \boxed{0.1587}$

c. What is the probability that the total time used by both machines together is greater than 115 hours?

(cont'd)

together

c) Prob that time used by both radios is greater than 115 hrs?

$\mu = \mu_1 + \mu_2 = 0.5 + 0.6 = 1.1$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{0.4^2 + 0.5^2} \approx 0.6403$

$S_n \sim N(n\mu, n\sigma^2) = S_n \sim N(100(1.1), 100 \cdot 0.64^2)$

$z = \dfrac{\sum x - \mu_{S_n}}{\sigma_{S_n}} = \dfrac{115.5 - 100(1.1)}{\sqrt{100}\,(0.6403)} \approx 0.7804 \rightarrow 0.7823$

$P(S_n > 115) = P(Z > 0.78) = 1 - P(Z < 0.78)$ from Z-table

$= 1 - 0.7823 = \boxed{0.2177}$

d. What is the probability that the total time used by machine 1 is greater than the total time used by machine 2?

d) Prob that tot time used by m1 is greater than tot time used by m2?

$\mu = \mu_1 - \mu_2 = 0.5 - 0.6 = -0.1$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} = \sqrt{0.4^2 + 0.5^2} \approx 0.6403$

$S_n \sim N(n\mu, n\sigma^2) = S_n \sim N(100(-0.1), 100 \cdot 0.64^2)$

$z = \dfrac{\sum x - \mu_{S_n}}{\sigma_{S_n}} = \dfrac{0 - 100(0.1)}{\sqrt{100}\,(0.6403)} \approx 1.56$

$P(S_1 > S_2) = P(S_1 - S_2 > 0) = P(Z > 1.56) = 1 - P(Z < 1.56)$

$1 - 0.9406 = \boxed{0.0594}$ $\quad 0.9406$