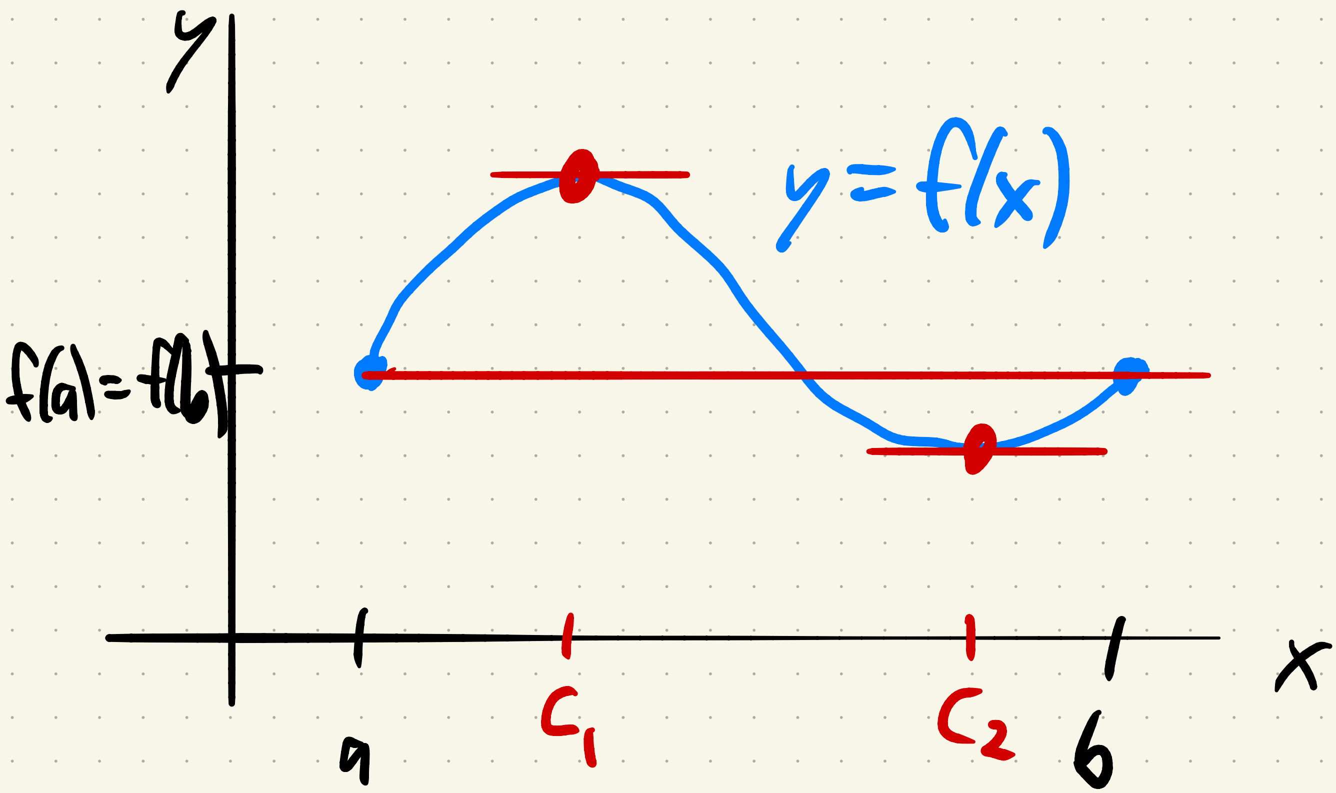
Math 30, Friday 3/27/2020

1pm class

The Mean Value Theorem and

increasing/decreasing functions

Rolle's Theorem:



It says: There is at least one point c between a and b where

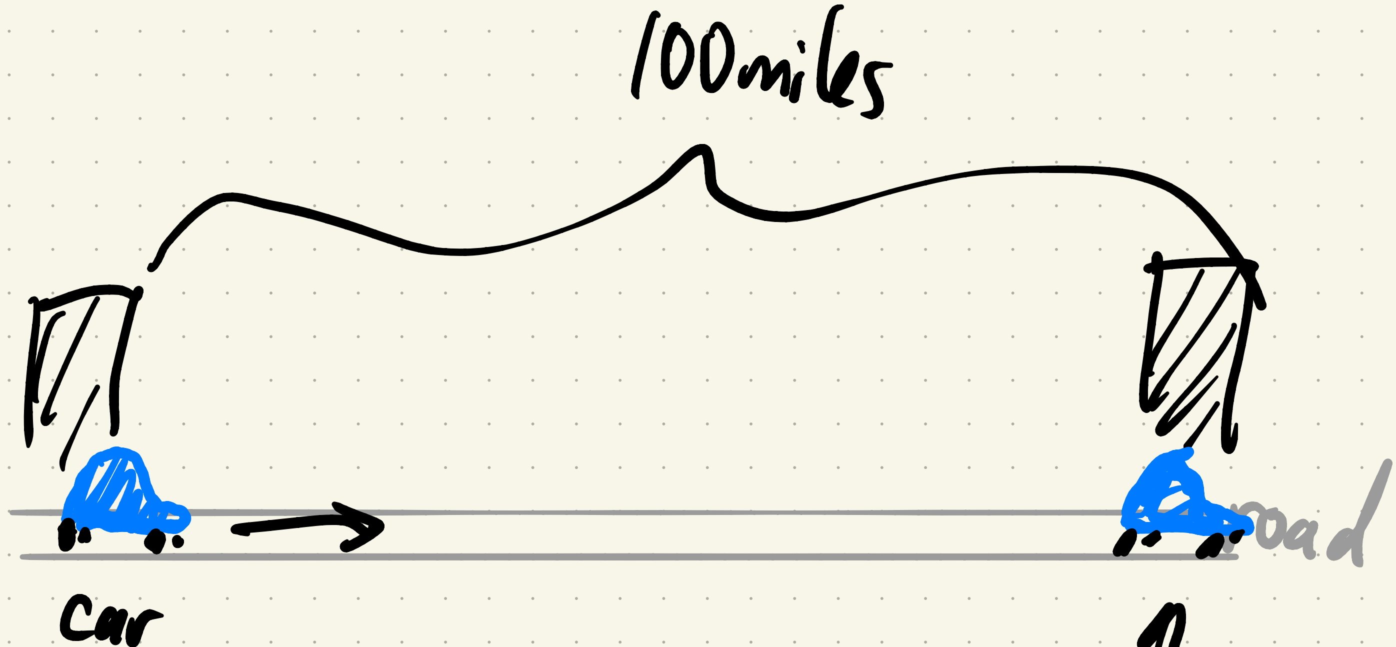
$$f'(c) = 0$$

related result:

The Mean Value Theorem

$\underbrace{\text{Mean}}_{\text{"average"}}$

"Thought experiment #2"

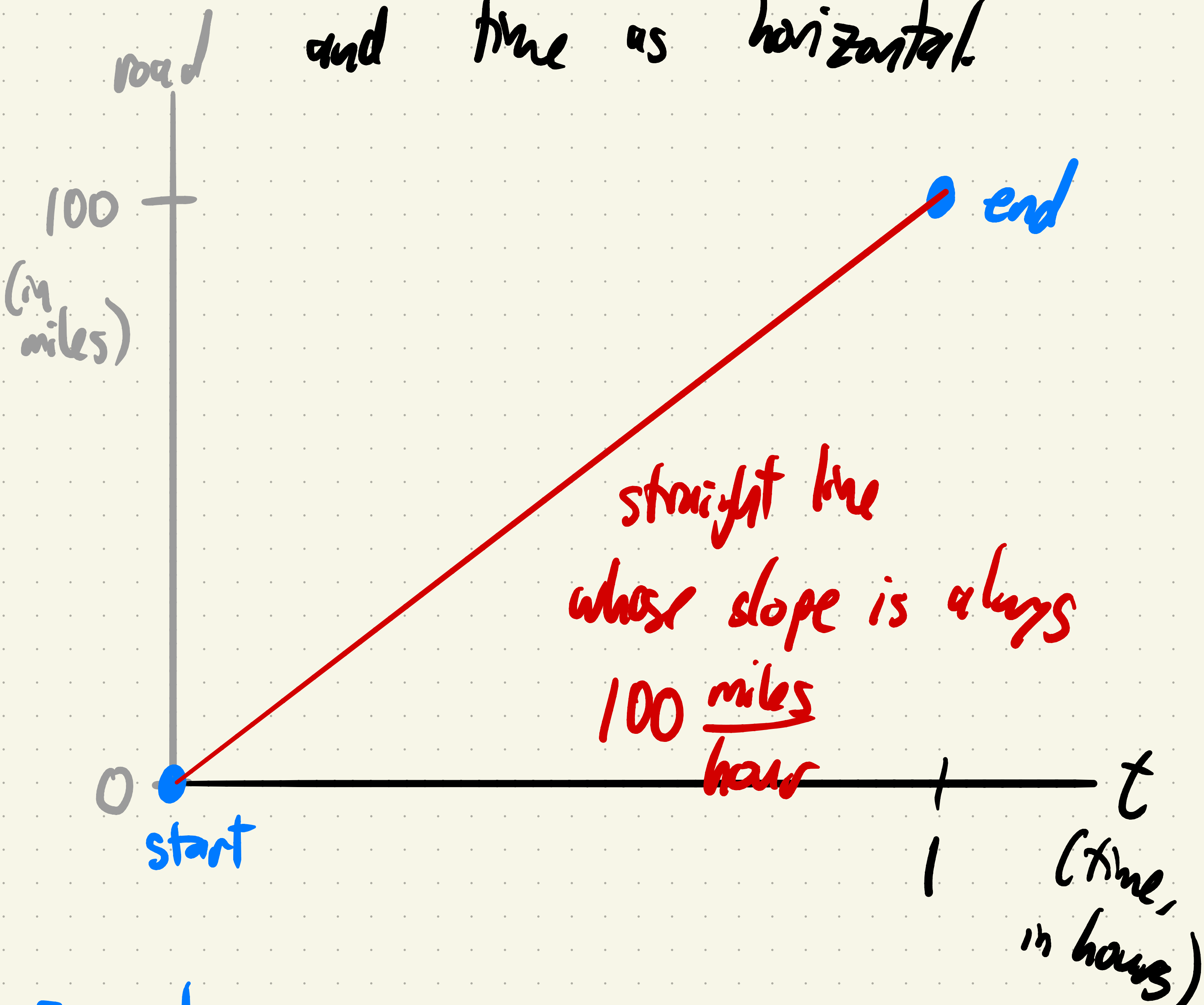


at time $t=0$ a tollbooth observes the car

one hour later ($t=1$ hour) another tollbooth observes the car.

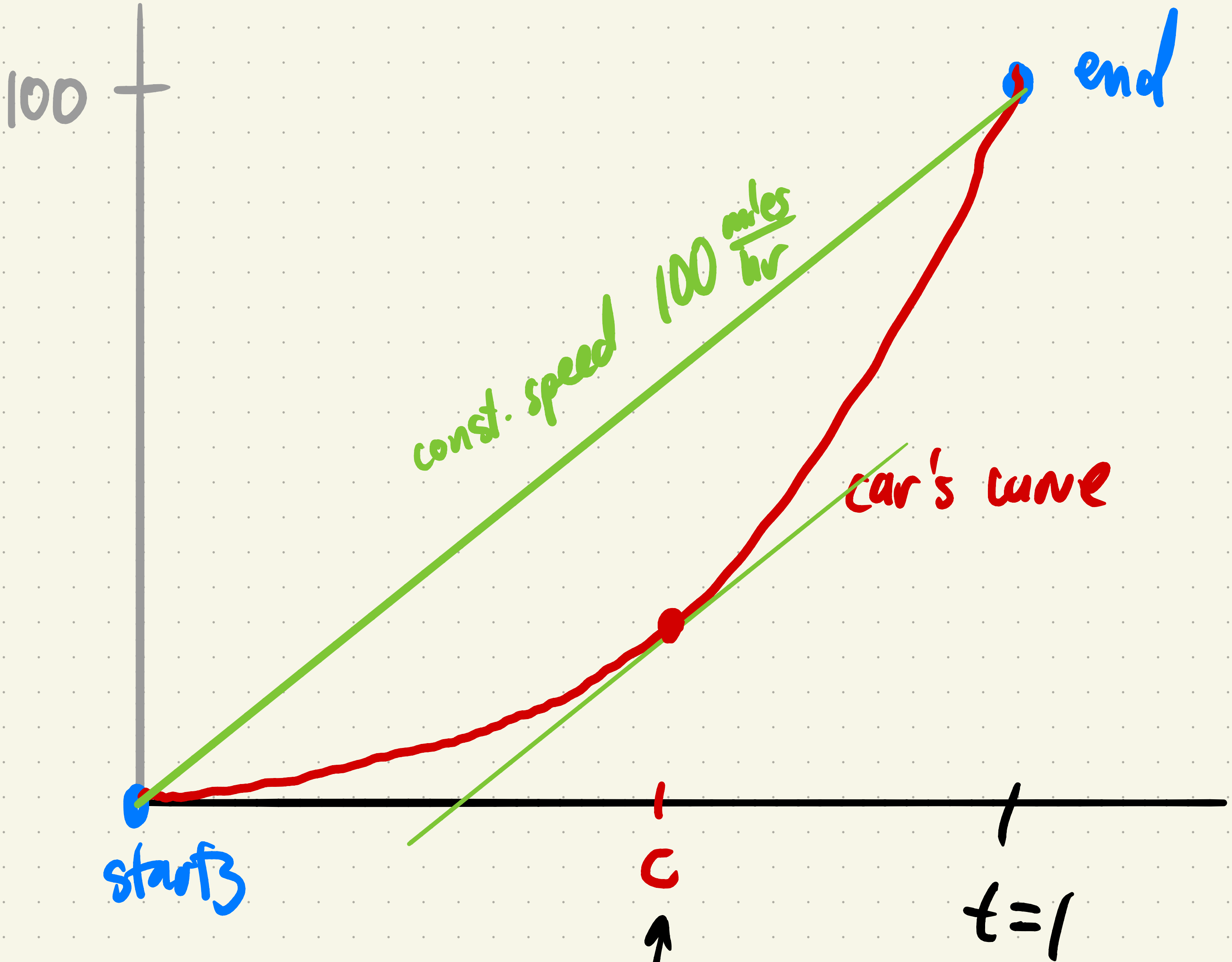
Fact: at some time the car was driving exactly 100 mph (give you a ticket!)

Now let's draw the position as vertical and time as horizontal.



Special cases:

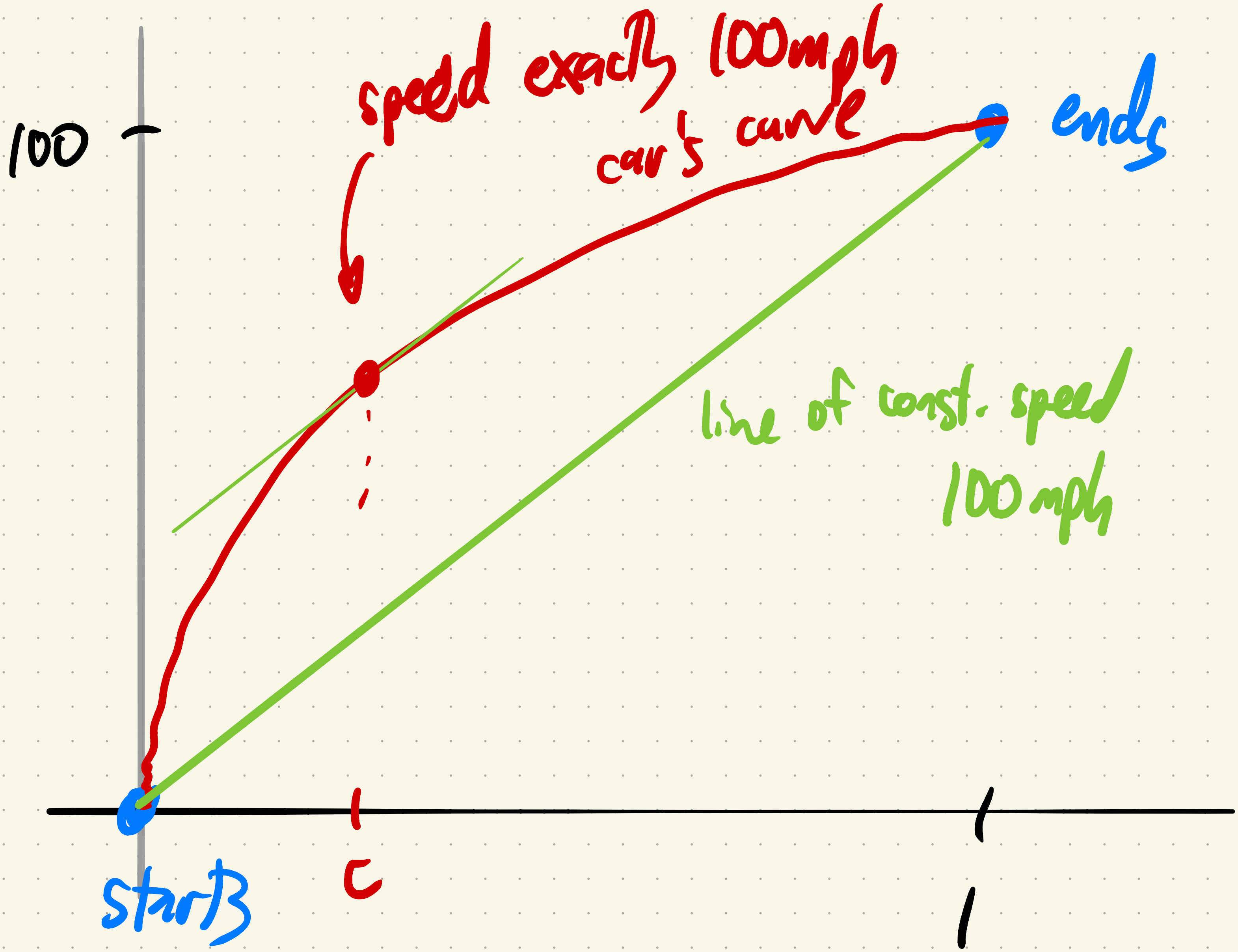
suppose car is going constant speed.



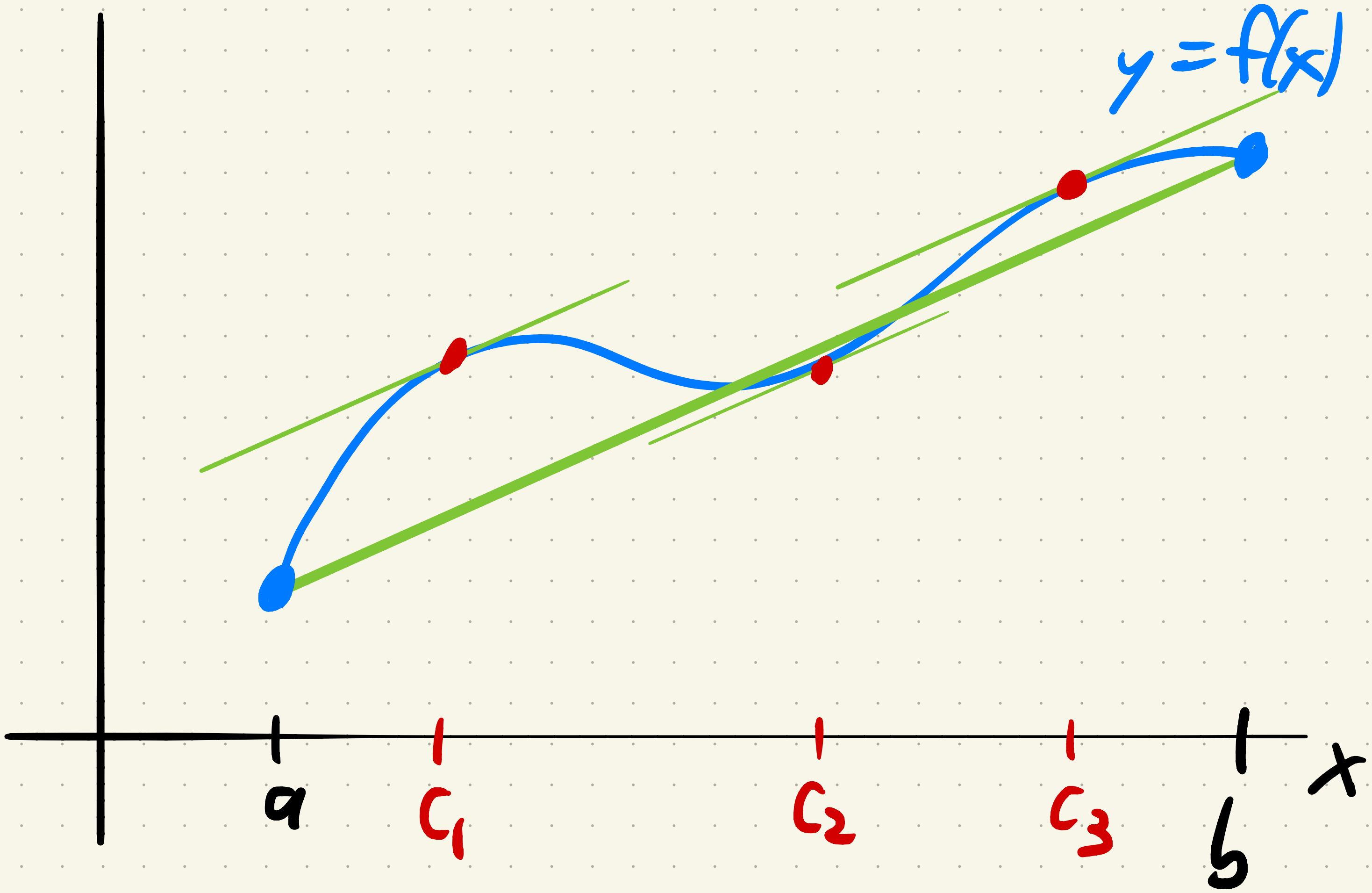
speed is exactly 100mph here

another special case:

car starts slow – behind schedule, so need to speed up (accelerate)



sp. case: starts fast, then ahead of schedule
so it must slow down



MVT: There is at least one point c where

$$\underbrace{f'(c)}_{\text{slope of tang. line}} = \underbrace{\frac{f(b) - f(a)}{b-a}}_{\text{slope b/w endpts.}}$$

definition of slope (not a derivative)

make sense?

The MVT says c exists

but it doesn't tell you

where it is.

(it's more of a theoretical result.)

an "existance result"

Application: The most important differential eq$^{\underline{n}}$:

Find all functions $f(x)$ such that $f'(x) = 0$ for all x.

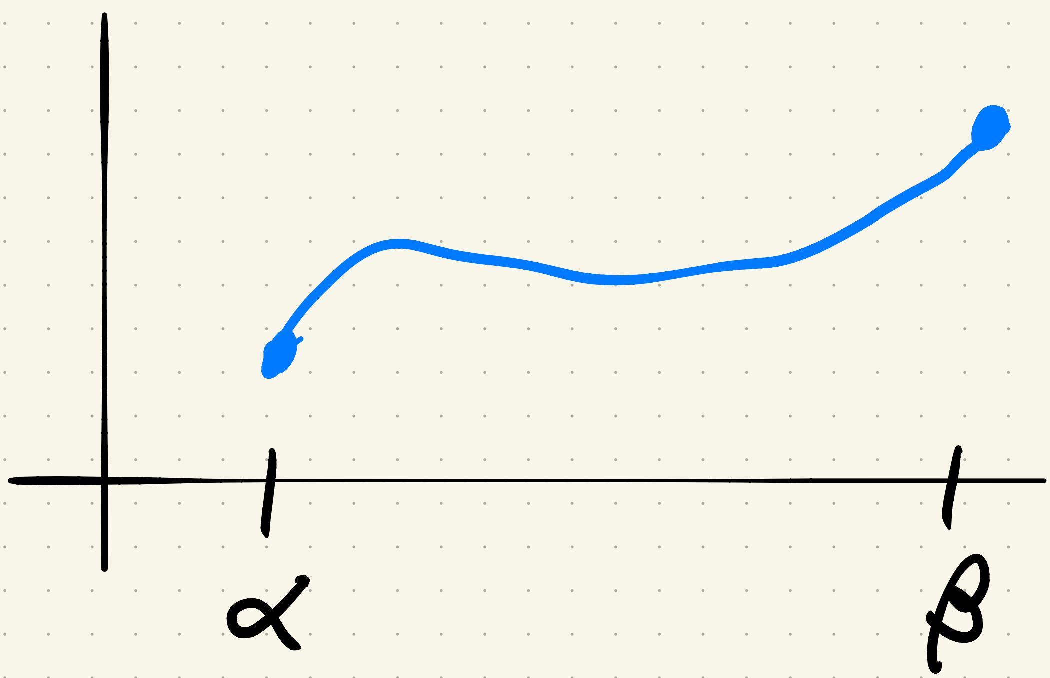
Example of a f$^{\underline{n}}$ like this?

a constant function!

Q: is that the only kind whose deriv. is always zero?

Suppose f is <u>not</u> constant:

So $f(\alpha) \neq f(\beta)$ for some α and β.



MVT says: at some point c,

$$f'(c) = \frac{f(\beta) - f(\alpha)}{\beta - \alpha} \neq 0$$

This shows: if $f'(x) = 0$ for all x, then f can only be a const. function.

Next topic: increasing / decreasing functions.

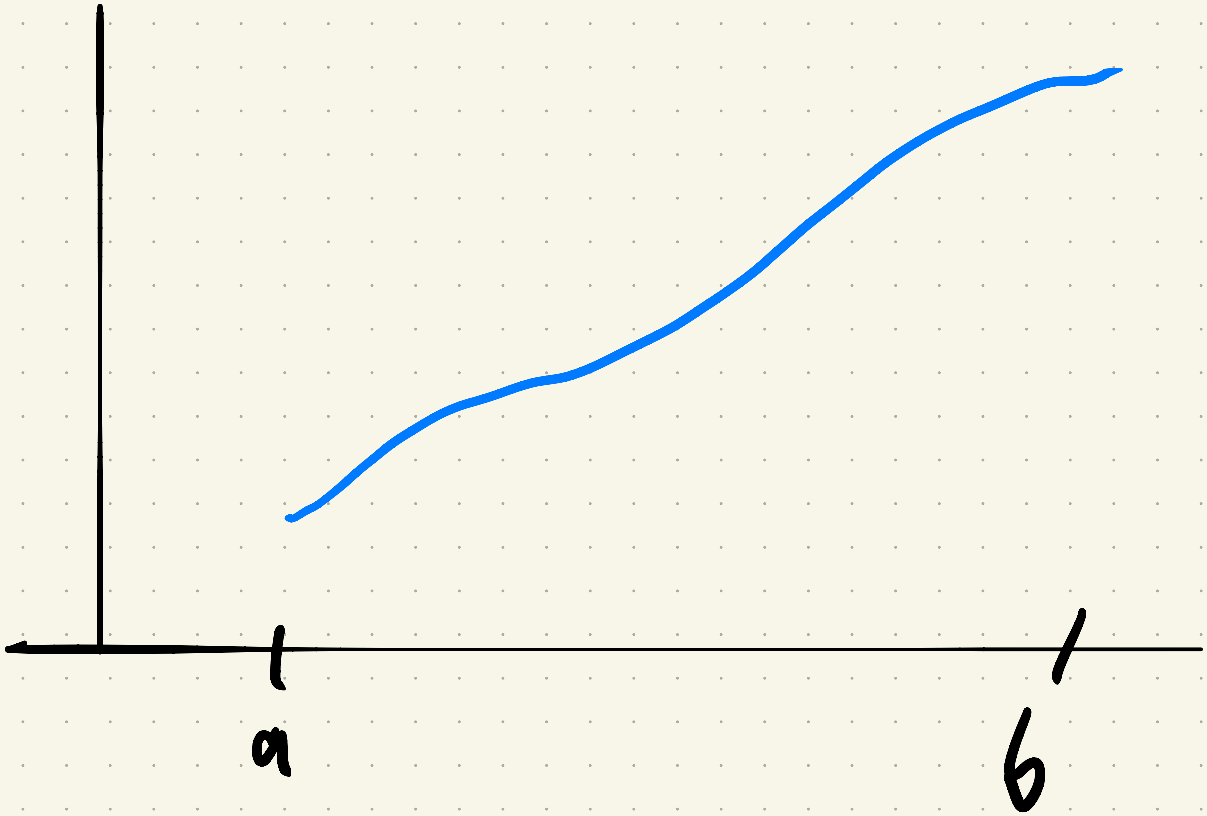
This relates calculus to geometry.

Q: What does $f'(x)$ tell you about the shape of the curve $y=f(x)$?

I could prove this using the MVT, but (in interest of saving time) I'll just say:

If $f'(x)>0$ for all x in the interval (a,b)

Then f is increasing in (a,b).



draw a curve whose slope is always positive (>0).

The function is necessarily increasing.

Similarly, if $f'(x)<0$ for all x in (a,b), The function is decreasing on (a,b).

This helps us draw graphs of functions.

Ex. Sketch the graph of

$$f(x) = 3x^4 - 8x^3 + 6x^2$$

To see where incr / decr. :

$$f'(x) = 12x^3 - 24x^2 + 12x$$

$$= 12x(x^2 - 2x + 1)$$

$$= 12x(x-1)^2$$

$$f(x) = 3x^4 - 8x^3 + 6x^2$$

$$f'(x) = 12x(x-1)^2$$

$f(x) = 3x^4 - 8x^3 + 6x^2$

$f'(x) = 12x(x-1)^2$

Then when $x < 0$, $f'(x) < 0$

so f is decreasing

on $(-\infty, 0)$.

when $0 < x < 1$, $f'(x) > 0$

so f is increasing on $(0,1)$.

$f'(1) = 0$ so tang. line is horizontal line

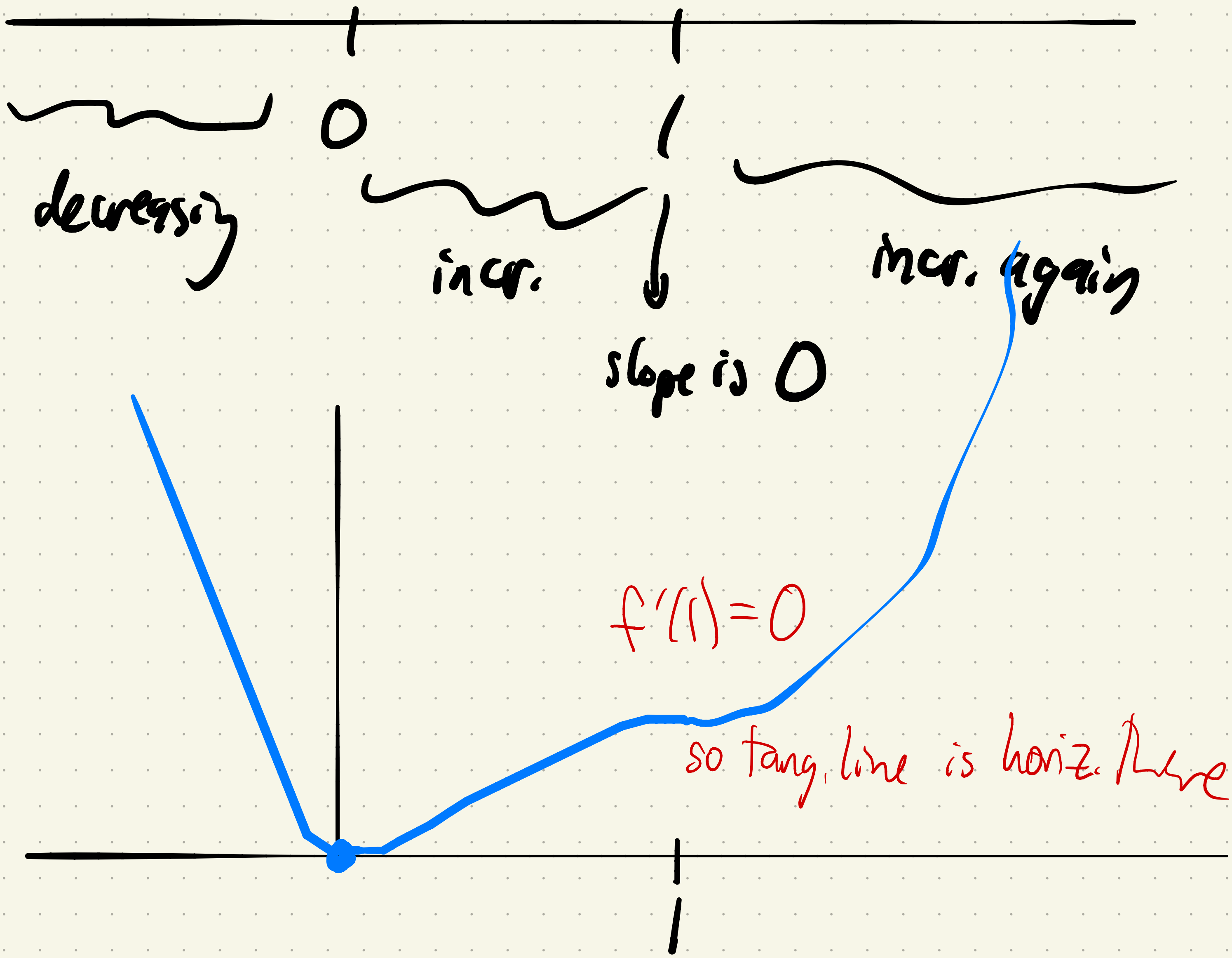
when $x > 1$, $f'(x) > 0$ so

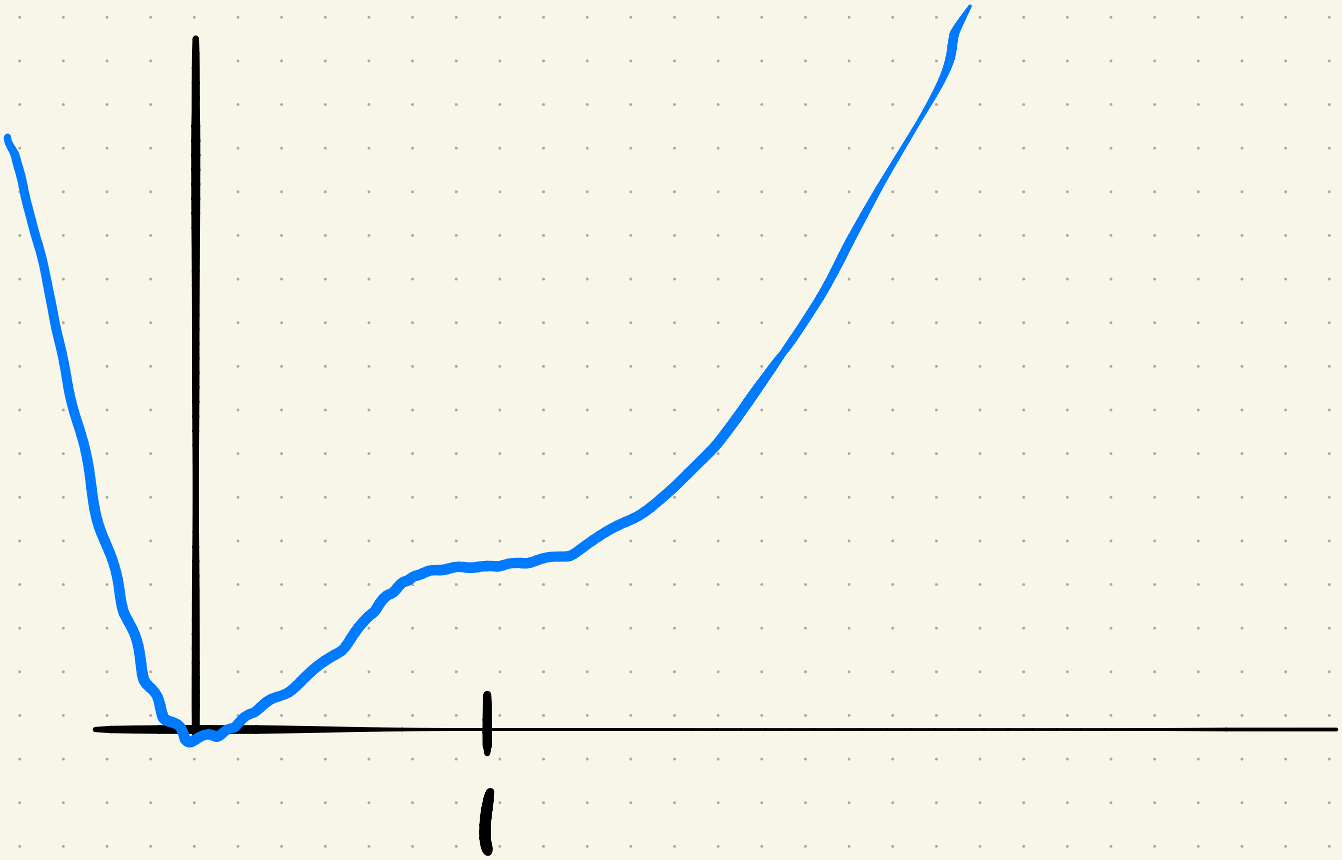
f is increasing on $(1, \infty)$.

$f(x) = 3x^4 - 8x^3 + 6x^2$ $\qquad f(0) = 0$

$f'(x) = 12x(x-1)^2$

Summary:





Compare w/ Wolfram Alpha to see how good we did!

Quiz on Monday after Spring Break: Linear Approx.

We'll postpone Exam #3 (I'll email you soon)

After Sp. Break:

Monday: Lin. Approx

Wed: Related Rates

Fri: Min/Max.

a pdf posted on Canvas

It'll look like an in-class quiz
but you'll have more time
& need to upload your work
to Canvas