Math 30, Wednesday April 22, 2020

1pm class

I'll have your exams graded by Friday.

Questions?

Exam will be out of 90, not 60.

<u>Mondy</u>: I gave a preview:

problems about tangents and derivatives are related to problems about areas and "integrals"

will define it tomorrow.

Now!

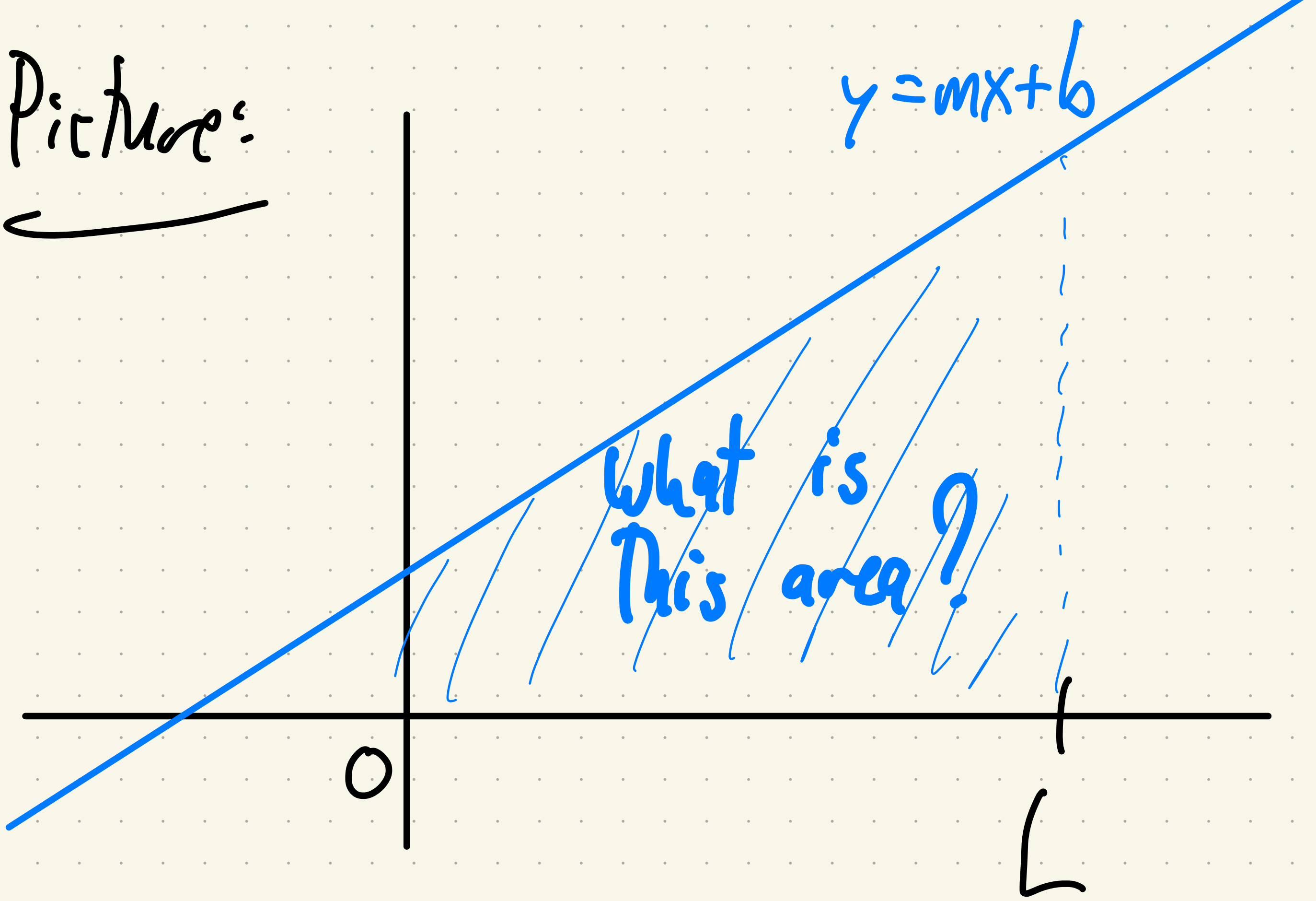
Example. Let $m>0$ and $b>0$ be fixed numbers.

Find the area under the line

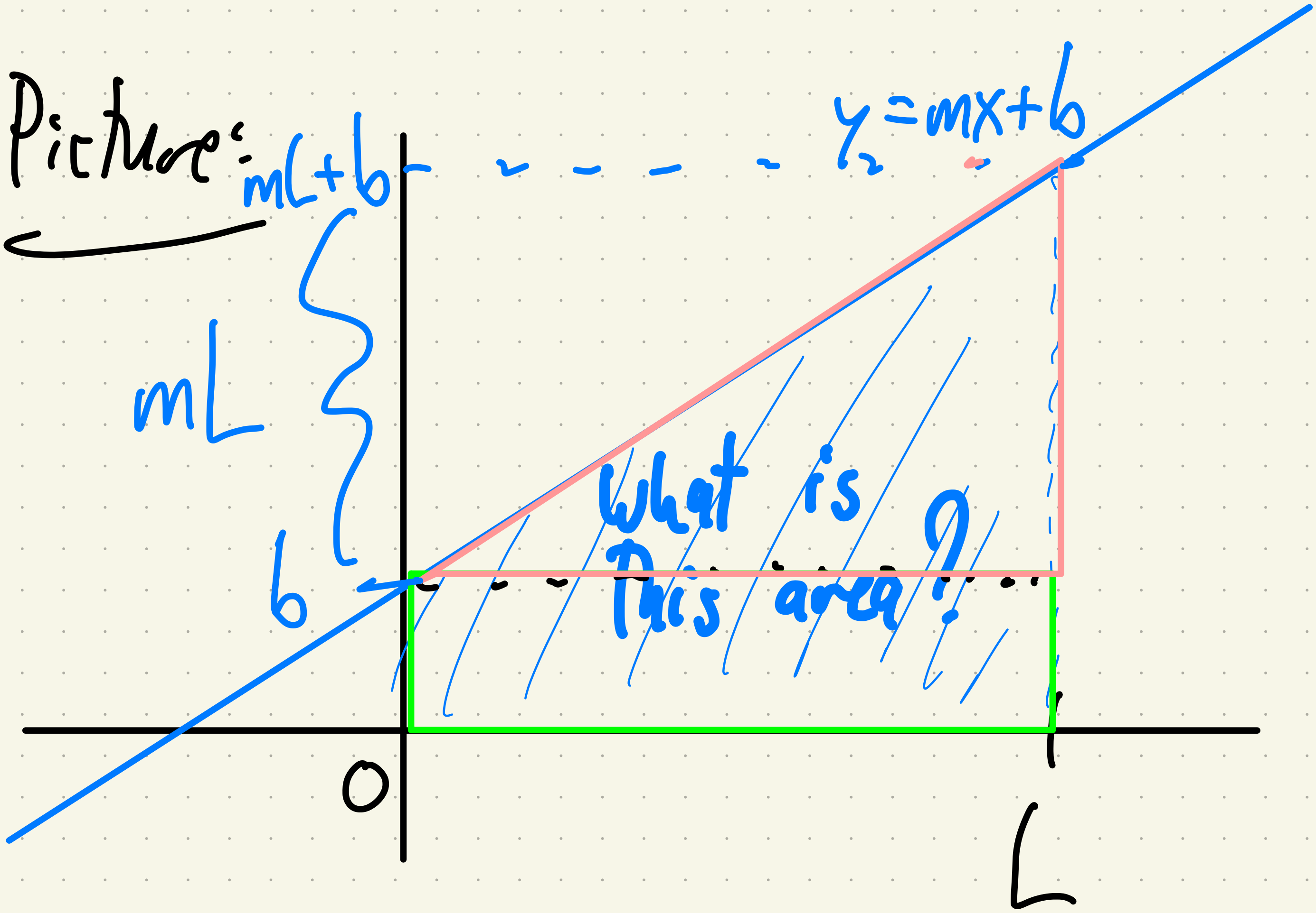
$$y = mx+b$$

from $x=0$ to $x=L$.

Picture:



Picture:



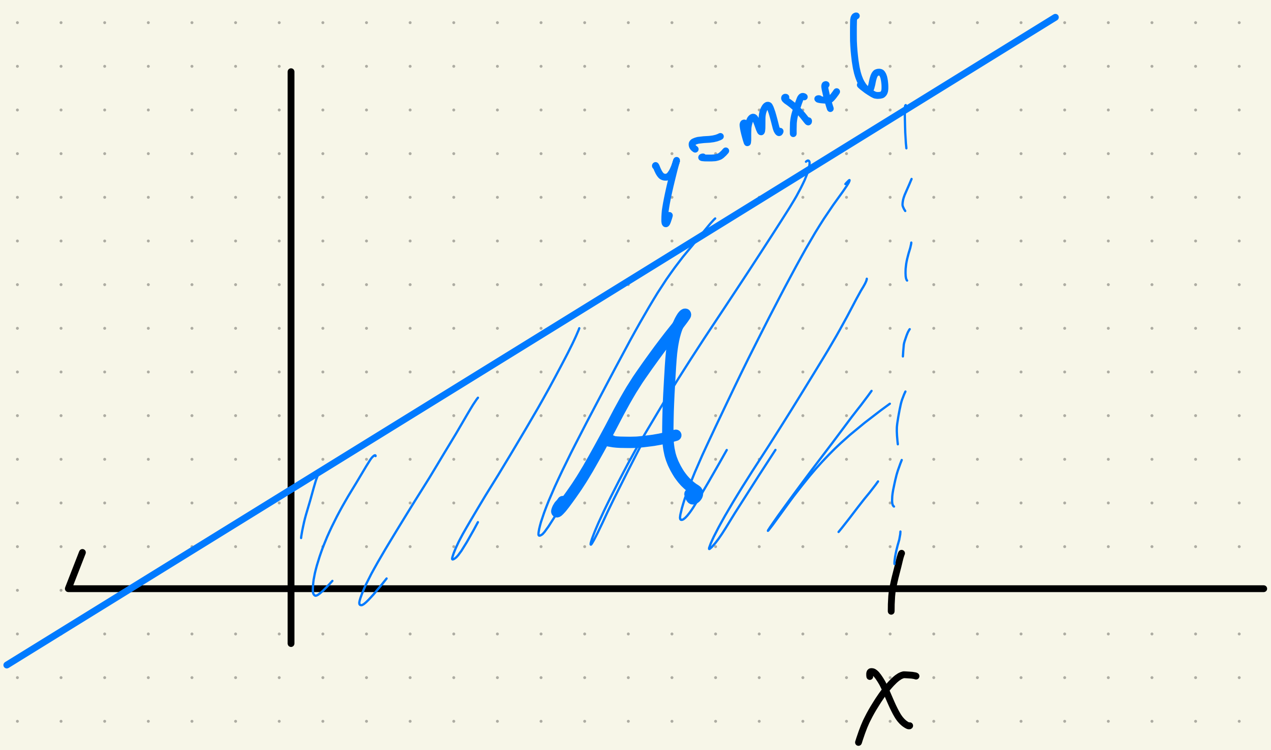
split into rectangle & triangle

so total area is:

$$\underbrace{bL}_{\text{area of rectangle}} + \frac{1}{2}\underbrace{L}_{\text{bas}}\underbrace{(mL)}_{\text{height}} = \frac{1}{2}mL^2 + bL$$

Area under the line $y = mx + b$
from 0 to x
$\leftarrow$ instead of L

is
$$A(x) = \frac{1}{2}mx^2 + bx$$

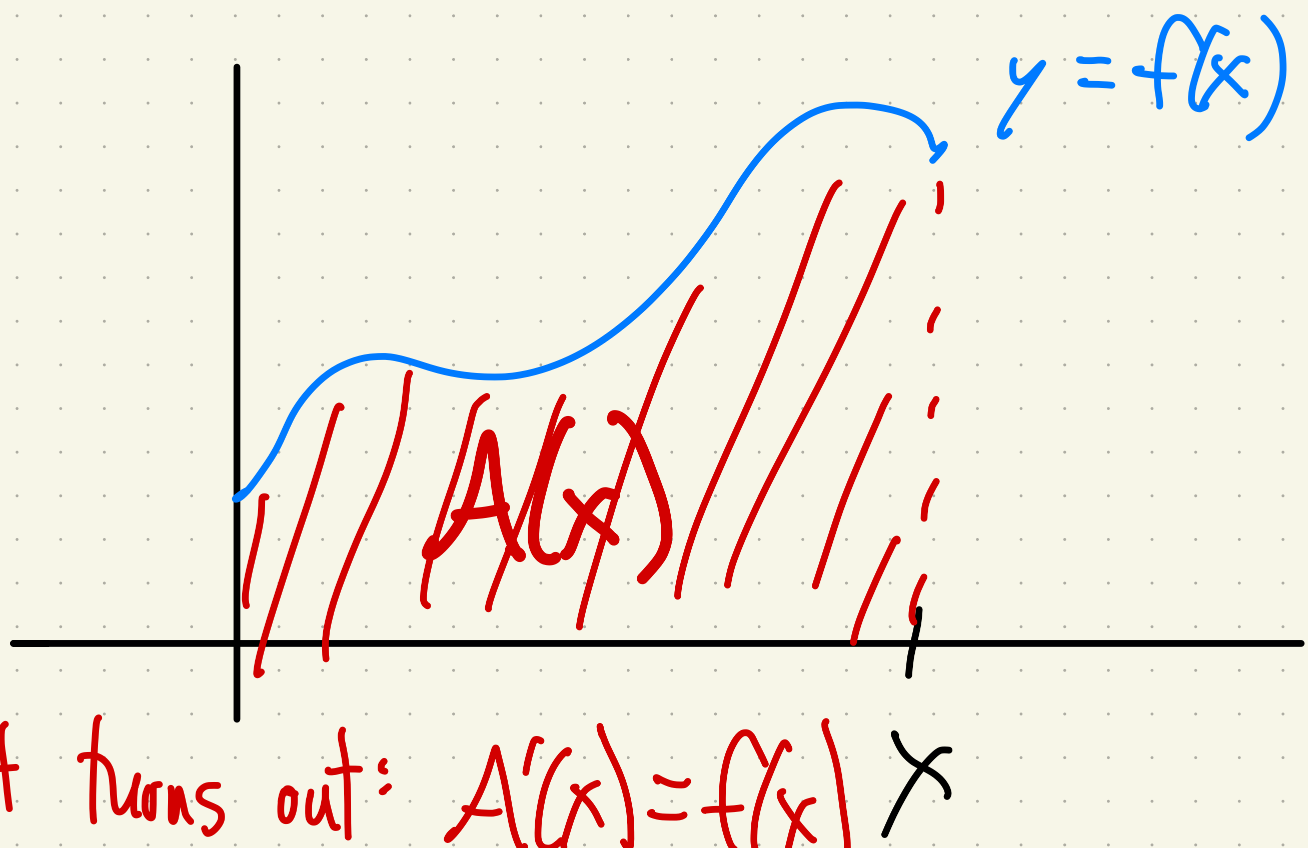


Note: $A(x) = \frac{1}{2}mx^2 + bx$
is an antiderivative of $f(x) = mx + b$

This works in general:

to find an antideriv. of a function $f(x)$, find the area under the curve $y = f(x)$

from 0 to x:

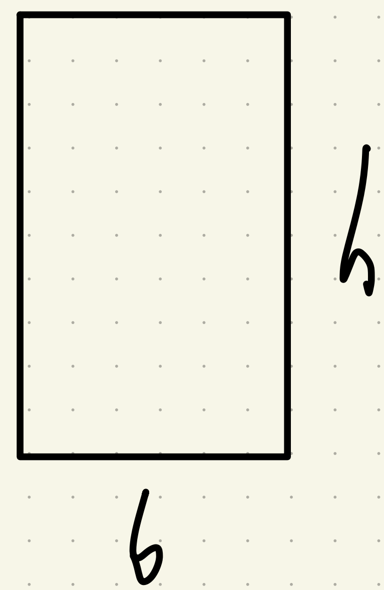


it turns out: $A'(x) = f(x)$

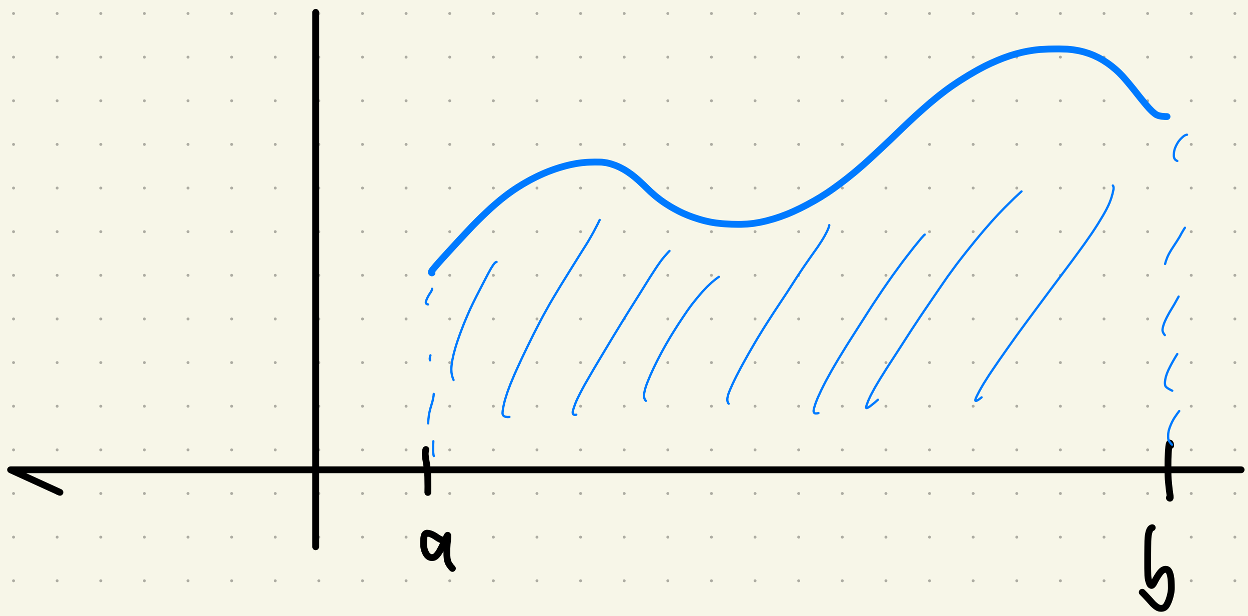
<u>Summary</u>: finding an antideriv.
is solved by finding an area!
Wow!

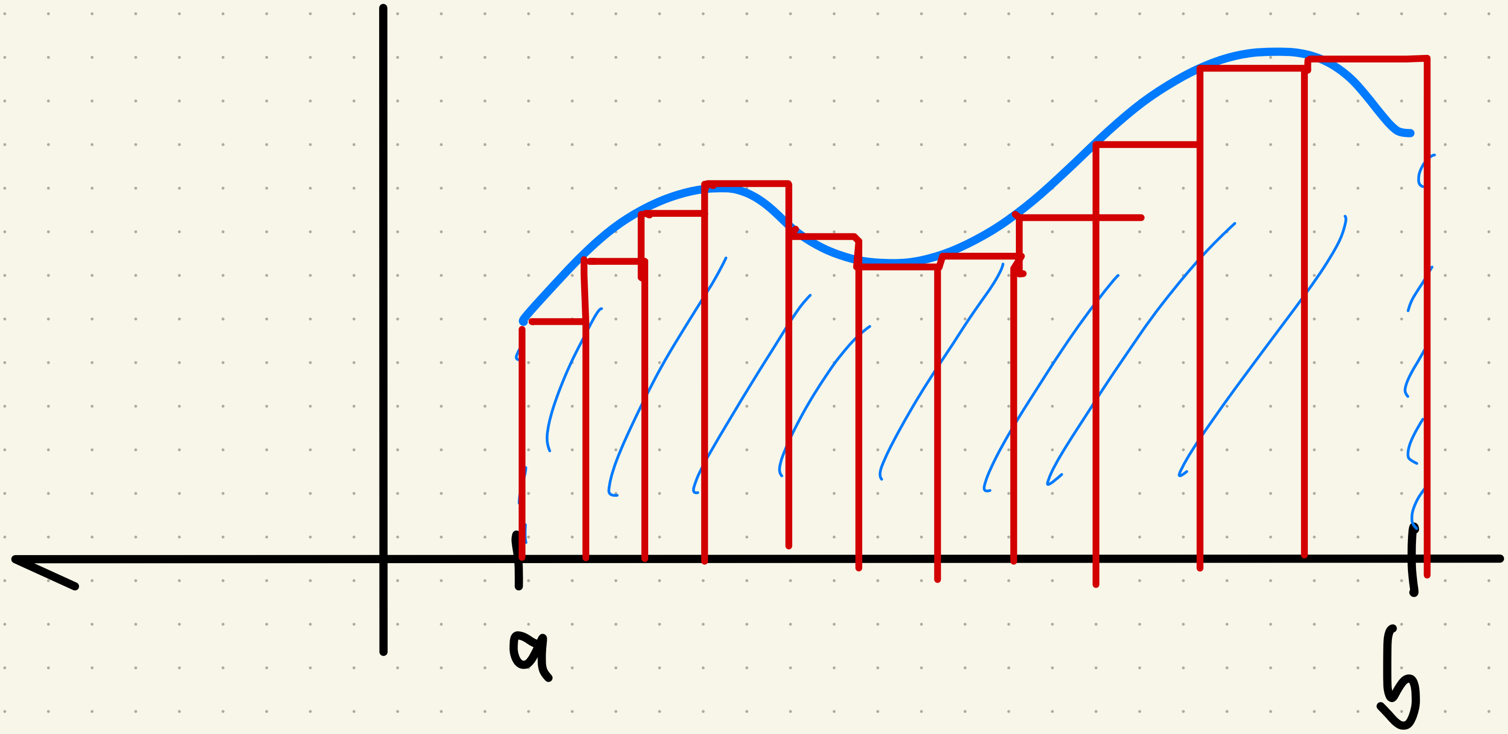
To fully explain this amazing fact,
we need to carefully study
what "area" <u>is</u>
and how to calculate it.

Starting from the fundamental fact that area of a rectangle is base x height ...



We'll see how to find the area under a curve:





Method: approximate using rectangles

something we understand well.

add up the areas of the rectangles to get an approximation.

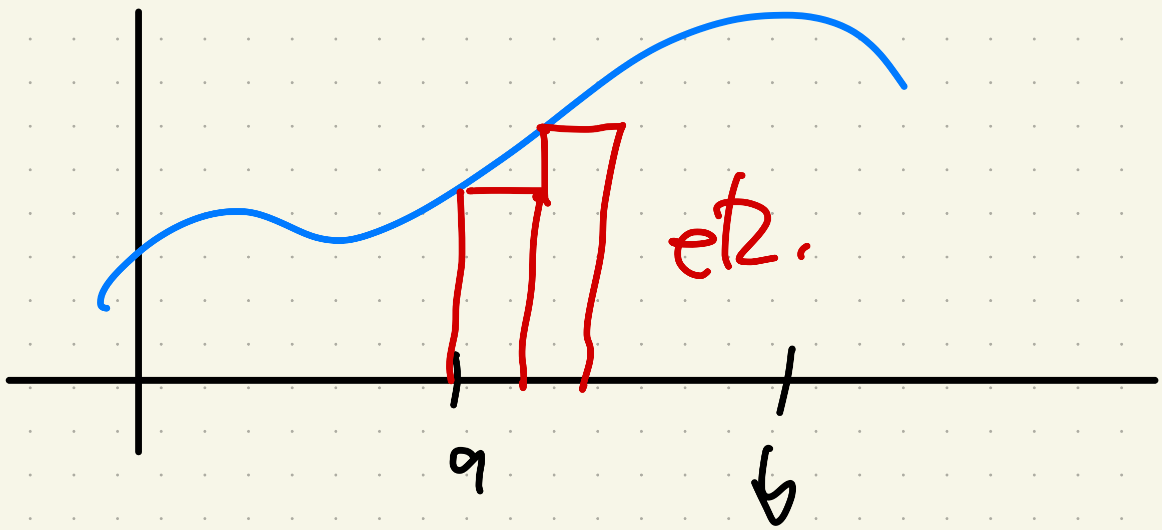
to get a better approximation, use more & more rectangles.

In the limit, we get the <u>exact</u> area under the curve.

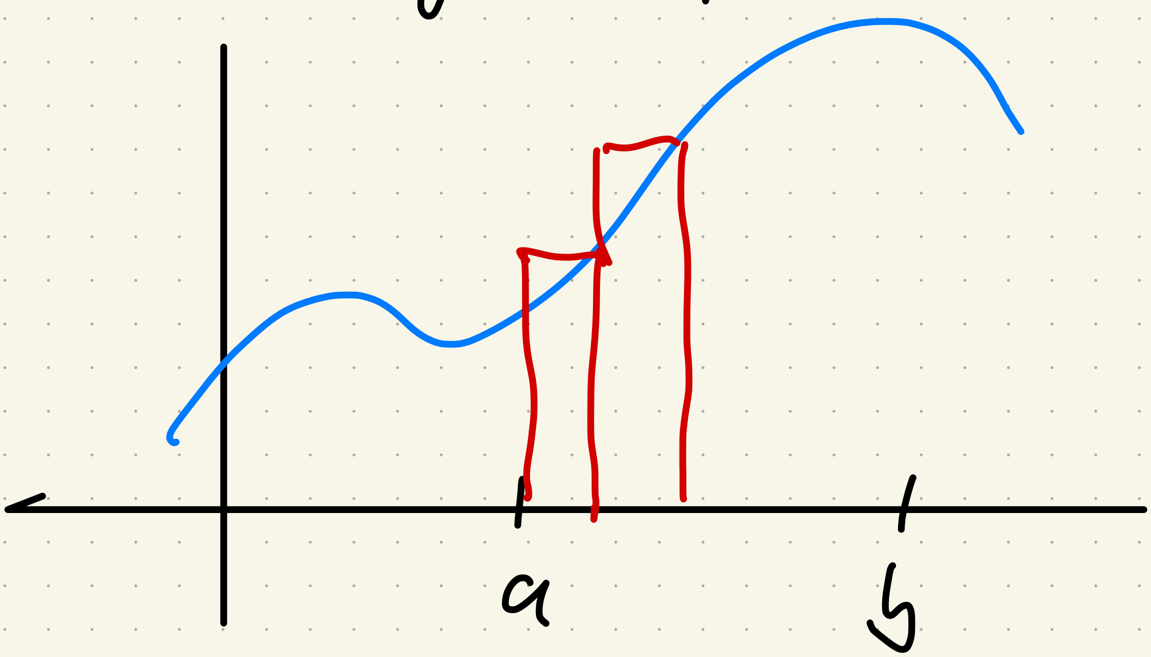
<u>Q</u>: what is the height of each rectangle?

<u>A</u>: more than one way to do it, but always get same limit in the end.

One way: use "left endpoints"



Can also use "right endpoints"



Can also use midpoints, etc.

Use n rectangles

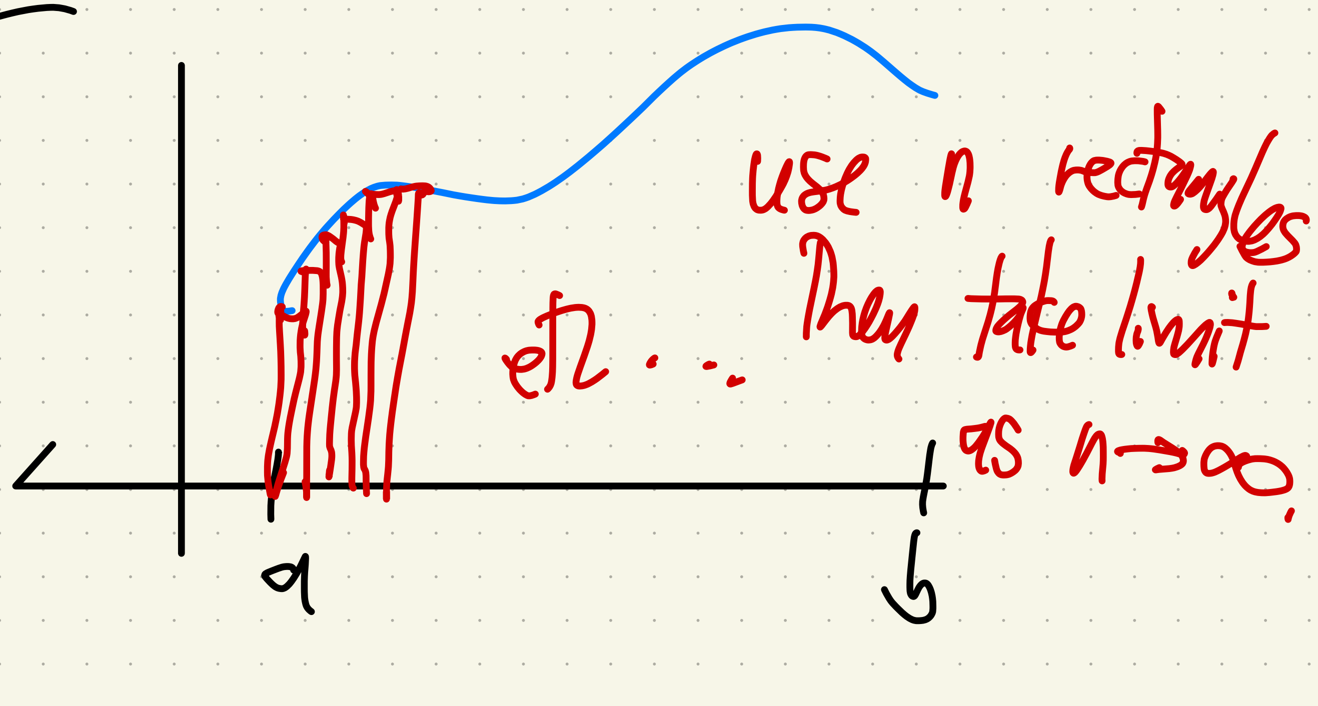
$(n = 10, 100, 1000, \ldots)$

Then take the limit as $n \to \infty$

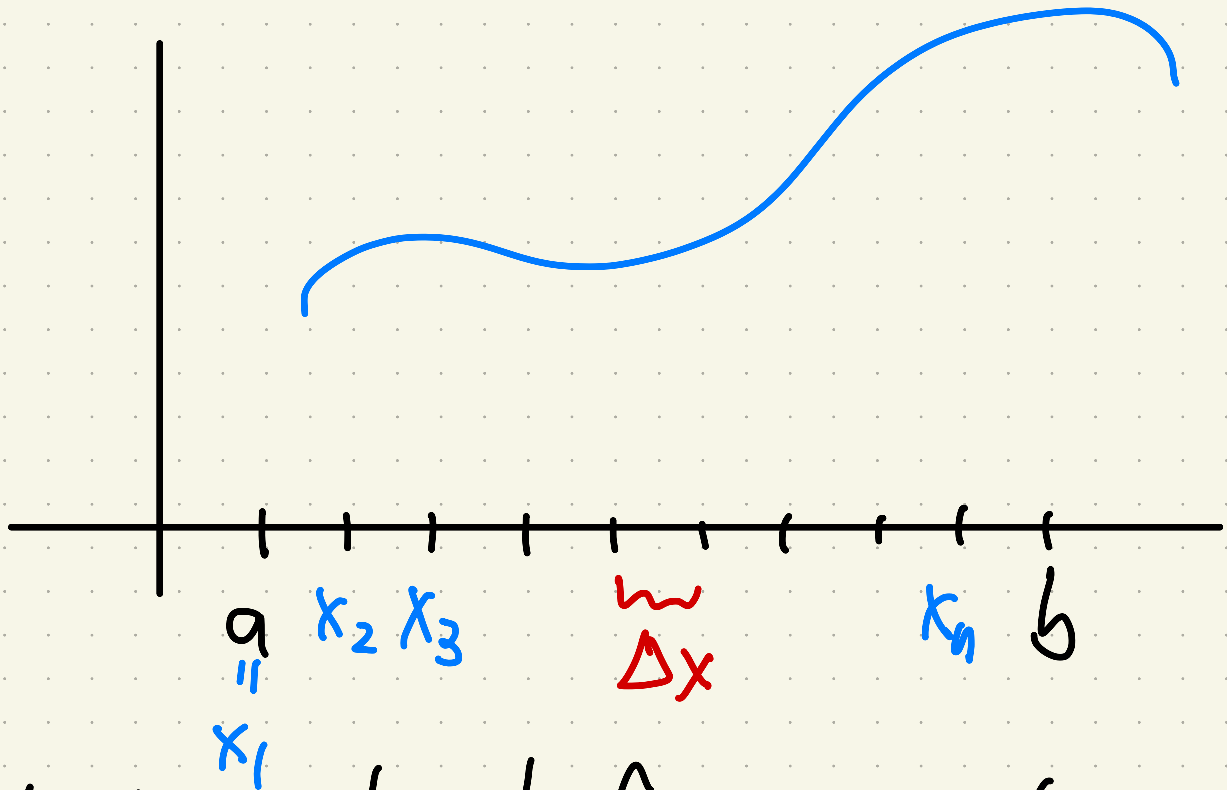
to get the exact area

under the curve.

Def$^{\underline{n}}$: The area A of the region S lying above the x-axis and below the graph of a positive continuous function f is defined to be the limit of the sums of the areas of the approximating rectangles.



Using math formulas:



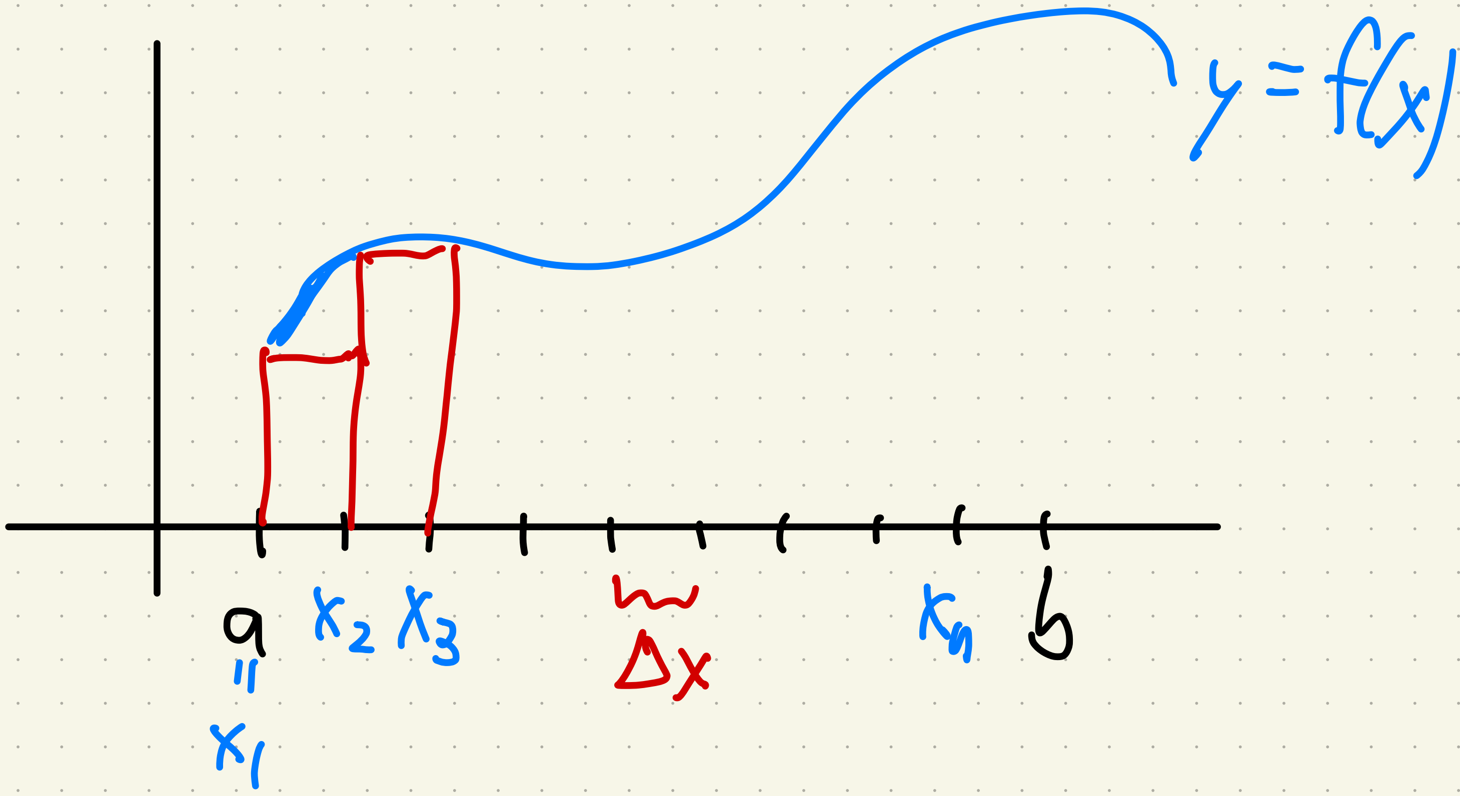
Divide the interval from a to b into n equal parts.

Each part has length $\frac{b-a}{n} = \Delta x$

give it a name.

Δ stands for "change"

so the symbol Δx means "change in x"

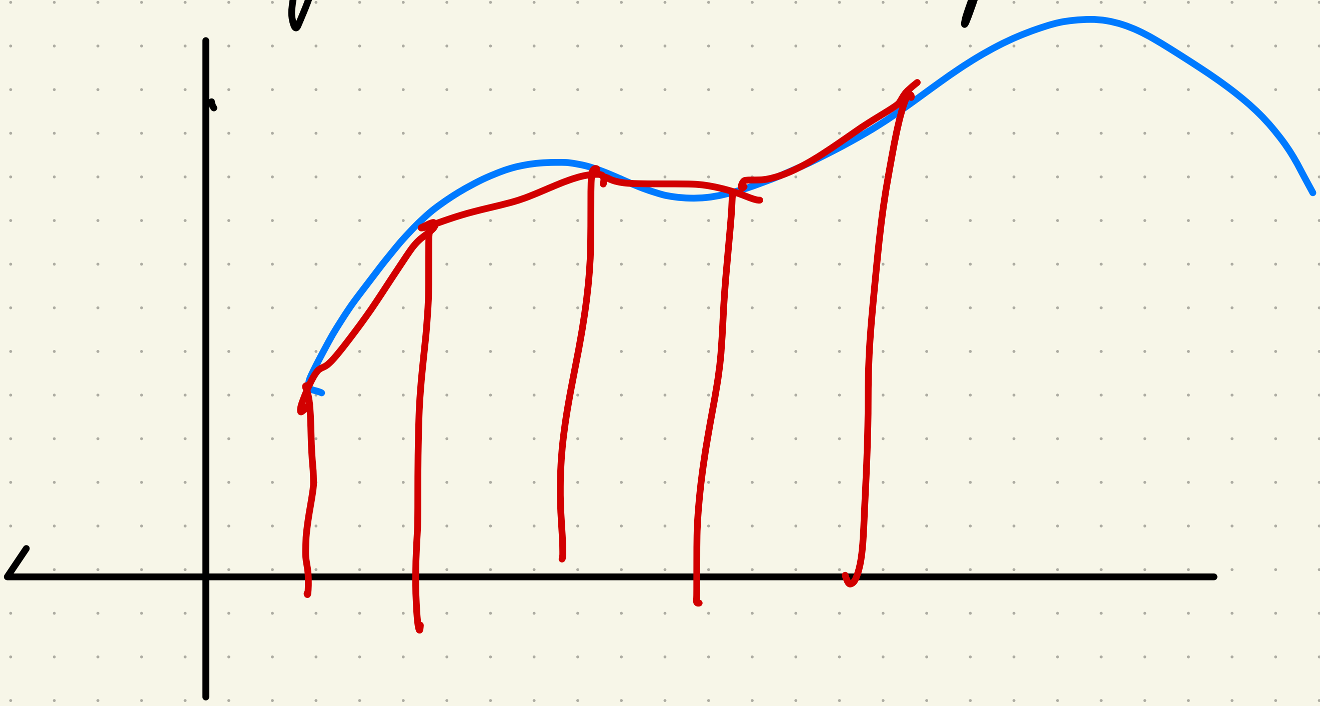


Use left endpoints (for example) to get the height of each rectangle.

$$\underbrace{f(x_1)\Delta x}_{\text{area of first rectangle}} + \underbrace{f(x_2)\Delta x}_{\text{area of second rectangle}} + \cdots + f(x_n)\Delta x$$

To answer D.D.'s Q:

To get a better approx, you might want to use trapezoids...



This subject belongs the theory of "Numerical Analysis"

Math 150

Summary: Using n rectangles and "left endpts",

the area under curve $y=f(x)$ is approximately

$$f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x.$$

The limit

$$\lim_{n\to\infty}\left(f(x_1)\Delta x + \cdots + f(x_n)\Delta x\right)$$

is the exact area.

We'll talk more tomorrow...

End with an application:

"area problems" turn up in surprising places.

Suppose your car's odometer is broken but speedometer still works.

Say you want to estimate how far you've driven.

Method: every 30 seconds $\left(\frac{1}{120} \text{ of an hour}\right)$

write down your speed.

Your speed is probably changing, but the distance traveled in the previous 30 seconds is approximately

$$\text{speed} \times \left(\frac{1}{120} \text{ hour}\right) = \text{dist.}$$

$$\frac{\text{miles}}{\text{hour}} \times \text{hours} = \text{miles}.$$

Say at time $t_k = \frac{1}{120} k$ hours

your speed is v_k.

Then distance travelled in those 30 sec.

is $\approx v_k \left(\underbrace{\frac{1}{120}} \right)$

$\frac{\text{miles}}{\text{hour}}$ hours $=$ miles.

an area of a rectangle in disguise!

total dist. traveled is

$$\approx v_1 \frac{1}{120} + v_2 \frac{1}{120} + \cdots + v_n \frac{1}{120}.$$

That's all for today —

see you tomorrow!