Santiago Bermudez
4-5-21
STAT 50

Practice Midterm #2

1. $\mu = 15$ $X \sim N(15, 1.25)$
$\sigma = 1.25$

a) $P(13.5 \leq X \leq 15)$?

$\frac{13.5-15}{1.25} = -1.20 \rightarrow 0.1151$

$0.5 - 0.1151 = \boxed{0.3849}$

$\frac{15-15}{1.25} = 0 \Rightarrow 0.5$

b) $P(X \leq x) = 0.0287$. Find x.
$\rightarrow -1.91$

☆ Just work backwards on z-table. Don't overthink it!

$\boxed{x = -1.91}$

2. $1/9 = 0.\overline{11}$ tosses 18 independence
probability

$X =$ # among 18

a) Expected value? → For $X \sim B(n,p)$
$18 \cdot 0.\overline{11} = \boxed{2}$ ← $\mu_X = np$

b) $P(X \geq 1)$ $X \sim Bin(18, 0.\overline{11})$
$P(X=k) = \binom{n}{k} p^k q^{n-k}$

$1 - 0.12 = \boxed{0.88}$

$P(X \geq 1) \Rightarrow 1 - P(X<1) \Rightarrow 1 - P(X=0)$
$\binom{18}{0} (0.\overline{11})^0 (1-0.\overline{11})^{18} \approx 0.12$
1

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3. Life time of a chip is exponentially distributed with variance of 4,000,000 hrs

a) What proportion of chips will work for less than 5,000 days?

$T \sim Exp(5\times10^{-4}) \rightarrow E(x) = \frac{1}{\lambda} = \mu_x$

$V(x) = \frac{1}{\lambda^2} = \sigma_x^2$

$\lambda = 5\times10^{-4}$

$\cancel{\lambda^2}\ \frac{1}{\cancel{\lambda^2}} = 4{,}000{,}000\,\lambda^2$

$\sqrt{\frac{1}{4{,}000{,}000}} = \sqrt{\lambda^2}$

$Prob(T < 5{,}000) \rightarrow F(5{,}000) = 1 - e^{-5\times10^{-4}\cdot 5{,}000}$

$\rightarrow \boxed{0.9179}$

* Use $e^{\dots}$ if $T > 5{,}000$

b) A chip has worked for 1000 hrs, what is the prob that it will work for at least 6,000 hrs?

$P(T \geq \overset{t}{6000} \mid \overset{s}{T = 1000})$? memoryless property!!

$P(T > t+s \mid T > s) = P(T > t)$

If $T \sim Exp(\lambda)$ and t and s are positive nums

$P(T \geq \underbrace{5000}_{t} + \underbrace{1000}_{s} \mid T > \underbrace{1000}_{s}) = P(T > 5000)$

$1 - P(T \leq 5000)$

$1 - F(5000)$

* More like $1 - e^{\dots}$

$\cancel{1-}[1 - e^{-5\times10^{-4}(5000)}]$

$\boxed{0.9179}$

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4. Aircraft arrive at airport according to a Poisson process with mean rate of 8 per hour

$X \sim \text{Poisson}(8)$ 8 per hr → 4 per $\frac{hr}{2}$

a) T → waiting time until 4th aircraft arrives. Identify the distribution of T, including any parameters, and find $P(T \le 1)$.

Integrating ... of formula known as ...

$T \sim \Gamma(4, 8)$

$P(T \le 1) = F(x) = P(T \le x) = \begin{cases} 1 - \sum_{i=0}^{\alpha-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!} & x > 0 \\ 0 & x \le 0 \end{cases}$

$T \sim \Gamma(\alpha, \lambda)$

$1 - \sum_{i=0}^{3} e^{-8 \cdot 1} \frac{(8 \cdot 1)^i}{i!} = \sum_{i=0}^{3} e^{-8} \frac{(8)^i}{i!}$ $1 - 0.04 = \boxed{0.9576 \text{ and } T \sim \Gamma(4, 8)}$

$1 = e^{-8}\left(\overset{1}{\frac{8^0}{0!}} + \overset{8}{\frac{8^1}{1!}} + \overset{32}{\frac{8^2}{2!}} + \overset{85.3}{\frac{8^3}{3!}}\right) = 0.04$

b) Y → number that arrive in half hour period. Identify distribution of Y, including parameters, and find $P(Y \ge 3)$.

Poisson ↓ $1 \cdot 0.5$

$8 \cdot 0.5 = 4$ $Y \sim \text{Poisson}(4)$

$P(Y \ge 3)$ is 0.5ish?

$\frac{e^{-4} 4^y}{y!}$ ← distribution of Y

*See HW #10 for more details!

$P(Y \ge 3) = 1 - P(Y < 3)$
$= 1 - [P(Y=0) + P(Y=1) + P(Y=2)]$

$P(Y=0) = \frac{e^{-4} 4^0}{0!} = 0.0183$ $P(Y=1) = \frac{e^{-4} 4^1}{1!} = 0.0733$

$P(Y=2) = \frac{e^{-4} 4^2}{2!} = 0.146$ $1 - 0.2381 = \boxed{0.7619 \text{ and } Y \sim \text{Poisson}(4)}$

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5. mean tax paid = $2,000 & rand sample of 625
std dev = $500
$\bar{X}$ = mean tax paid of 625 forms

a) What will you use for the distribution of $\bar{X}$, including any parameters? Explain.

For the distribution of $\bar{X}$, I would use the Central Limit Theorem as the sample is 625 which is much larger than 30.
I would use the mean as it is and square the std dev and divide it by 625 for the parameters:
$\rightarrow \bar{X} \sim N(2{,}000, \frac{500^2}{625})$

b) Prob that mean tax paid > $1,980?

$P(\bar{X} > 1{,}980)$ $\quad \bar{X} \sim N(2{,}000, \frac{500^2}{625})$

$z = \frac{1{,}980 - 2{,}000}{\sqrt{400}} = \frac{-20}{20} = -1 \rightarrow 0.1587$

$1 - 0.1587 = \boxed{0.8413}$

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6. Process stopped when the fourth package is detected whose weight falls outside the specifications.
Packages have p of 0.01 of failing & resulting removal on p
weights are independent
$X \to$ # of ... until the process is stopped

a) Identify the distribution of X including parameters
$\boxed{X \sim NB(4, 0.01)}$ ← Same setting as geometric except trials continue until 4th success occurs.

b) Expected # until process is stopped?
$\mu_X = \frac{r}{p} = \frac{4}{0.01} = \boxed{400}$

7. 25% SC X_1 — 12 orders selected at random
30% CM X_2
15% SC + CM X_3
30% neither X_4

a) Find $P(X_1=3, X_2=2, X_3=1, X_4=6)$
$X_1, \dots X_4 \sim MN(n, p_1, \dots, p_k)$
$$= \boxed{\frac{12!}{3!\,2!\,1!\,6!}(0.25)^3(0.30)^2(0.15)^1(0.30)^6}$$

$\frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{3! \cdot 2 \cdot 1! \, 6!} = 55440$ Ok

$= \boxed{0.0085}$

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b) identify the distribution of X_1 and find $P(X_1=3)$

$\boxed{X_1 \sim Bin(12, 0.25),\ \binom{12}{3}(0.25)^3(0.75)^9}$

$P(X_1=3)=\binom{12}{3}(0.25)^3(1-0.25)^9$

$\frac{12\cdot11\cdot10\cdot\cancel{9!}}{\cancel{9!}\,3!}=220 \Rightarrow \boxed{0.2581}$

8. Lot of 100 ICs, 20 defective, 4 chosen at random
$X \to$ # of tested circuits that are defective
a) Identify the distribution of X, including any parameters, and find $P(X=2)$. No decimal answer required.

$X \sim H(\overset{N}{100}, \overset{R}{20}, \overset{n}{4})$ $P(X=x)=\binom{R}{x}\binom{N-R}{n-x}$

$\frac{80\cdot 79\cdot 78!}{78!\,2!}$ $\frac{80!}{2!}$ $\frac{80\cdot79\cdot78}{78\cdot2!}$ 3160

$\frac{\binom{20}{2}\binom{100-20}{4-2}}{\binom{100}{4}}$ $\frac{100\cdot99\cdot98\cdot97\cdot96!}{96!\,4!}$

$\frac{20\cdot19\cdot18!}{18!\,2!}$
190

$$\boxed{\frac{\binom{20}{2}\binom{80}{2}}{\binom{100}{4}} \text{ and } X_1 \sim H(100, 20, 4)}$$

b) If necessary (N.A.), estimate $P(X=2)$ using an appropriate method.

$X \sim Bin(4, 0.2)$

$P(X=2) \to \binom{4}{2}(0.2)^2(0.8)^2 = \boxed{0.1536}$

$\frac{4\cdot3\cdot\cancel{2!}}{2!\,\cancel{2!}}=\frac{12}{2}=6$

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Practice Midterm #2

9. P1 normally dist → mean = 20 in, std dev = 0.5 in

P2 normally dist → mean = 15 in, std dev = 0.4 in

overlap → normally dist → mean = 1 in, std dev = 0.1 in

*Assume independence between lengths and overlap

Prob that tot length after insertion is between 34.5 in and 35.1 in?

☆See HW #11

Mean tot length? = P1 + P2 − 2*overlap
20 + 15 − 2 = 33 in

Std dev tot length?
$\sigma = \sqrt{Var}$
$Var = V_1 + V_2 + V_o = (0.5)^2 + (0.4)^2 + 2^2(0.1)^2 = 0.45$

$\sqrt{0.42} = 0.64$ $\quad \frac{35.1-33}{0.67} = 3.13 \rightarrow 0.9991$
$P(34.5 \leq Y \leq 35.1)$

$\frac{34.5-33}{0.67} = 2.24 \rightarrow 0.9875$, $\quad 0.9991 - 0.9875$

$3.24 = 0.9994$

$\boxed{0.0116}$

$2.31 \rightarrow$

$z < \frac{35.1-34}{0.64} = 1.69 \rightarrow 0.9545$ $\quad z < \frac{34.5-34}{0.64} = 0.77 \rightarrow 0.7794$

$0.9545 - 0.7794 = \boxed{0.1751}$

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10. Resistors labeled 100Ω. Actual resistances are uniformly distributed on the interval $[96, 102]$.
→ prob distribution where all outcomes are equally likely

a) Find prob that resistance is between $97\ \Omega$ and $100\ \Omega$

$$\frac{100-97}{102-96}$$

mean → $E[R] = \frac{96+102}{2} = 99\ \Omega$

std dev → $\sqrt{V(R)} = \frac{(102-96)^2}{12} = 3$

$\sqrt{3} = 1.73\ \Omega$ $\quad 0.7190 - 0.1230 = \boxed{0.50}$

$Z_1 = \frac{100-99}{1.73} = 0.578 = 0.58 \rightarrow 0.7190$

$Z_2 = \frac{97-99}{1.73} = -1.155 = -1.16 \rightarrow 0.1230$

b) Assume independence. What is the probability that 2 of 7 resistances are between $97\ \Omega$ and $100\ \Omega$?

$P(97 \le x \le 100) = 0.5$

$P(Y=2) \rightarrow X \sim Bin(7, 0.50)$

$P(Y=2) = \binom{7}{2}(0.50)^2(1-0.50)^5 = \boxed{0.1641}$

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$\frac{7!}{5!2!} \rightarrow \frac{7\cdot 6\cdot 5!}{5!2!} = 21$

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11. It can be shown that if the random variable T represents the time to the αth event in a Poisson process with rate parameter λ, then T has a Gamma distribution with parameters α and λ. That is, $T \sim \Gamma(\alpha, \lambda)$. Assuming $T \sim \Gamma(2, 1)$ find $P(T \geq 2)$ in each of the following ways:

a) By integrating the gamma pdf from 0 to 2, $\Gamma(\alpha) = (\alpha - 1)!$, where α is an integer (see section 4.8).

$P(T \geq 2) = P(T < 2)$? $T \sim \Gamma(2, 1)$

$$\int_0^2 \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} dx \qquad \int u\,dv = uv - \int v\,du$$

$$1 - \int_0^2 \frac{(1)^2 x^1 e^{-1x}}{\Gamma(2)} dx = \int_0^2 x e^{-x} dx$$

$\Gamma(2) = 1$ $\qquad u = x \qquad v = -e^{-x}$

$\qquad du = 1\,dx \qquad dv = e^{-x}$

$$= \left[x \cdot -e^{-x} - \int -e^{-x} \cdot 1\,dx\right]_0^2 \qquad \int -e^{-x} = e^{-x}$$

$$= \left[-e^{-x}x - e^{-x}\right]_0^2$$

$$= (-e^{-2} \cdot 2 - e^{-2}) \cdot \overset{0-1}{(-e^0 \cdot 0 - e^0)}$$

$$= (-0.406)(-1) = 0.406$$

$$1 - 0.406 = \boxed{0.594}$$

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b) Using the formula $F(x) = P(T \le x) = 1 - \sum_{i=0}^{\alpha-1} e^{-\lambda x} \frac{(\lambda x)^i}{i!}$

where $T \sim \Gamma(\alpha, \lambda)$

$P(T \le 2)$

$T \sim \Gamma(2, 1)$ $x = 2$

$1 - \sum_{i=0}^{1} e^{-1 \cdot 2} \frac{(1 \cdot 2)^i}{i!} = \sum_{i=0}^{1} e^{-2} \frac{(2)^i}{i!}$

$e^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!}\right) = 0.406$

$1 - 0.406 = \boxed{0.594}$