# Quiz 3

(1) An $n \times n$ matrix $A$ is invertible if there exists an $n \times n$ matrix $C$ such that $CA = I_n$ and $AC = I_n$

(2)
$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1 + R_3 \to R_3]{3R_1 + R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right] \to$$

$$\xrightarrow{3R_2 + R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \xrightarrow[\substack{R_3 + R_2 \to R_2 \\ \frac{1}{2}R_3 \to R_3}]{R_3 + R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right] \to \left[ A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix} \right]$$

(3) a) If $A$ is $n \times n$ and invertible then it is row equivalent to the $n \times n$ identity matrix, $I_n$.

b) Consider $A\vec{x} = 0$, which we solve by row reducing $[A | \vec{0}]$. Since $A$ is row equivalent to $I_n$ then $[A | \vec{0}] \sim [I_n | \vec{0}]$ and this suggests that $\vec{x} = 0$ is the one and only solution, which shows that the columns of $A$ are linearly independent.