Math 30, Wednesday April 15, 2020

1pm class

Today: Please work on

- Practice Exam 3 (2020)
- Review Problems for Exam 3 (2019)
- Worksheets on typed notes in Canvas
- HW problems

Questions?

ask w/ voice or group chat.

Exam 3: same format as quizzes

posted at 6am
submit your work by 11:59pm

Friday

#1 on 2020 Review Problems.

I made up the word "rooverse."

Background: if f and g are inverses, then

$$g(f(x)) = x$$

and

$$f(g(y)) = y$$

Picture:

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & y \arrow[l, bend left, "g"] \end{tikzcd}$$

The "rooverse" problem: modify w/ a square root:

$$g(f(x)) = \sqrt{x} \text{ for all } x > 0.$$

Suppose f and g are such that

$$g(f(x)) = \sqrt{x} \text{ for all } x > 0.$$

Suppose $f(4) = 3$, $f'(4) = 5$.

<u>Q</u>: What is $g'(3)$?

Similar what did for inverses, differentiate both sides:

using chain rule

$$g'(f(x))\,f'(x) = \frac{1}{2}x^{-1/2} \left(= \frac{1}{2\sqrt{x}}\right)$$

$x=4$ $x=4$

Now plug in $x = 4$:

$$g'(3) \cdot 5 = \frac{1}{2 \cdot 2}$$

So

$$\boxed{g'(3) = \frac{1}{20}}$$

Very similar to how we found deriv.s of inverse functions:

$$e^{\ln x} = x \quad \text{for all } x>0$$

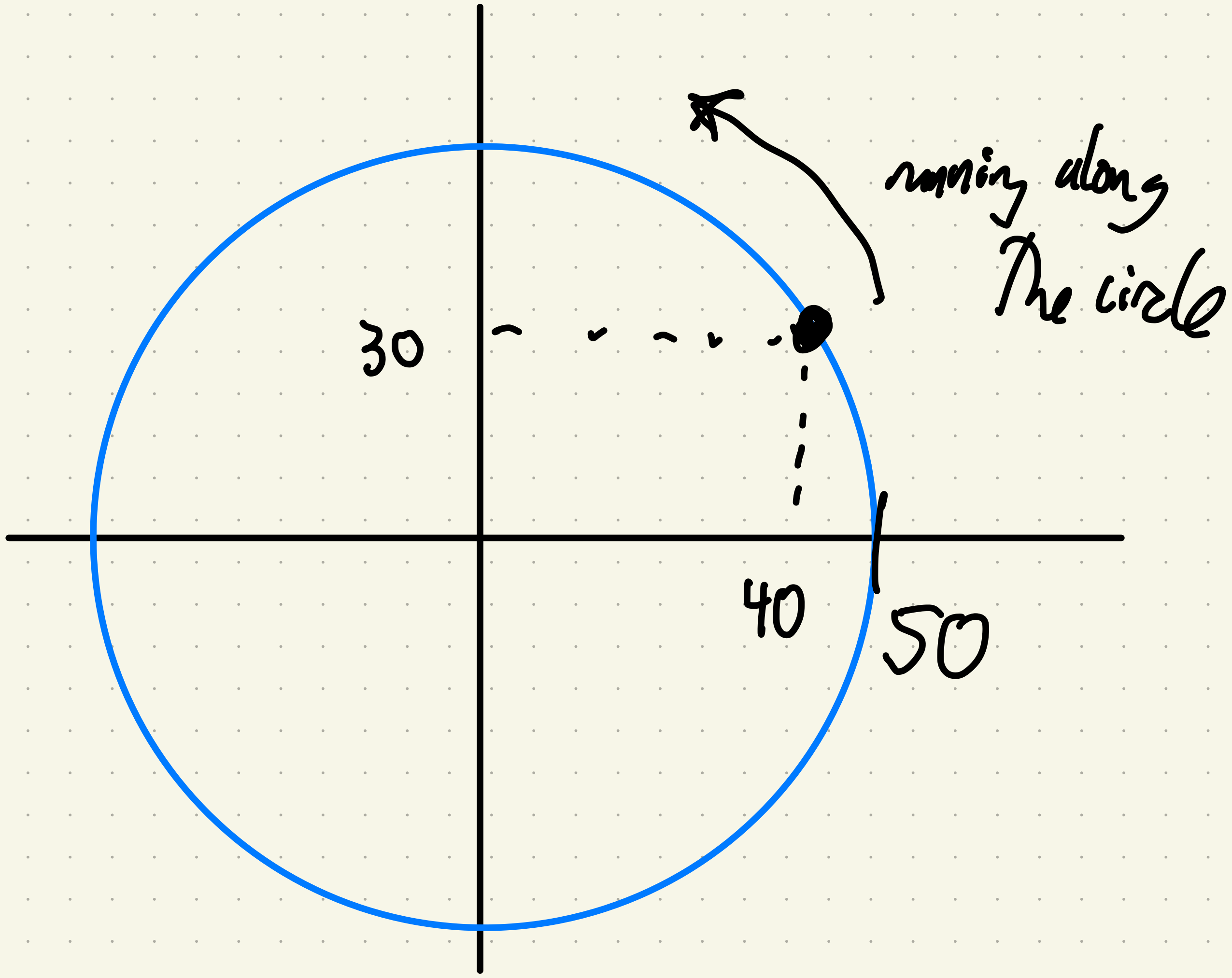
So differentiate:

$$\left(e^{\ln x}\right)\left(\frac{d}{dx}\ln x\right) = 1$$

using chain Rule

So $$\boxed{\frac{d}{dx}\ln x = \frac{1}{x}}$$

#2 on 2020 Review Problems:

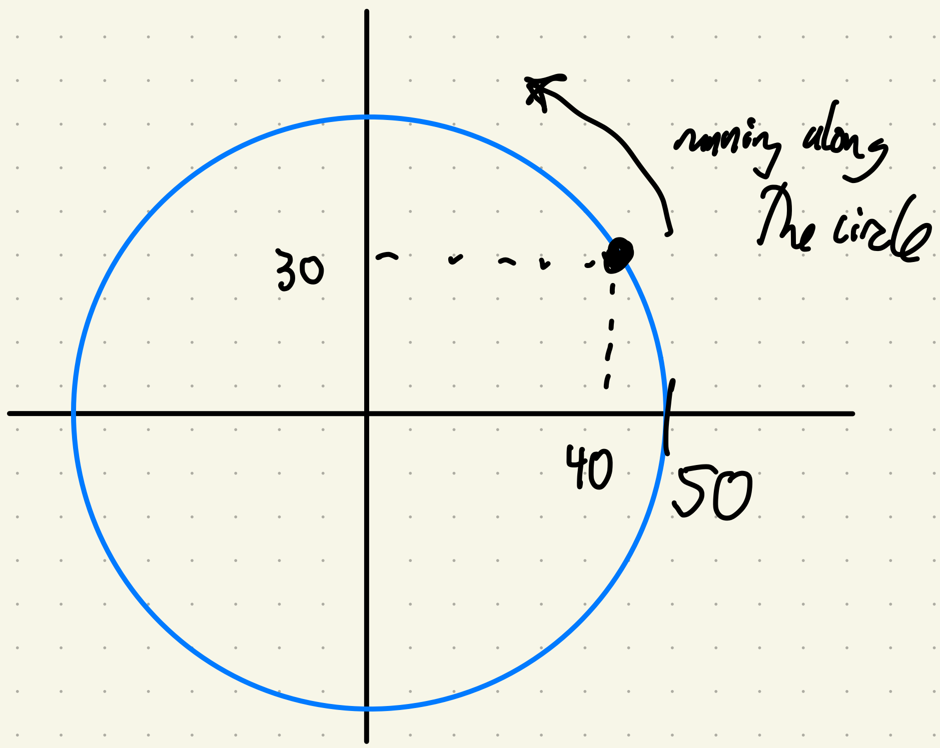


$$30^2 + 40^2 = 50^2$$

$$900 + 1600 = 2500 \checkmark \text{ ok}$$

Given: when she is at $(40,30)$ she has

$$x'(t) = -\frac{5}{4} \frac{\text{meters}}{\text{sec}}$$



when she is at $(40,30)$ she has

$$x'(t) = \frac{-5}{4}\ \frac{m}{sec.}$$

Q: Find $y'(t)$ at that moment.

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we can see $y'(t) > 0$ at that point from the picture.

Key Fact: $x(t)^2 + y(t)^2 = 2500$

b/c (x,y) is on the circle.

Diff. & use chain rule (or product rule).

The "creative" part of related rates problems:

Finding a function that relates all the variables.

Here, her position (x, y) is is on the circle $x^2 + y^2 = 2500$.

Q: on April 7/8 worksheet

#2. Sketch the graph of a function

with

$$\begin{cases} f'(0) = f'(2) = f'(4) = 0 \\ f'(x) > 0 \text{ if } x < 0 \\ \qquad \text{or } 2 < x < 4 \\ f'(x) < 0 \text{ if } 0 < x < 2 \\ \qquad \text{or } x > 4 \\ f''(x) > 0 \text{ if } 1 < x < 3 \\ f''(x) < 0 \text{ if } x < 1 \text{ or } x > 3. \end{cases}$$

There is more than one way to draw one...

#2. Sketch the graph of a function with

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tang. line is horiz.

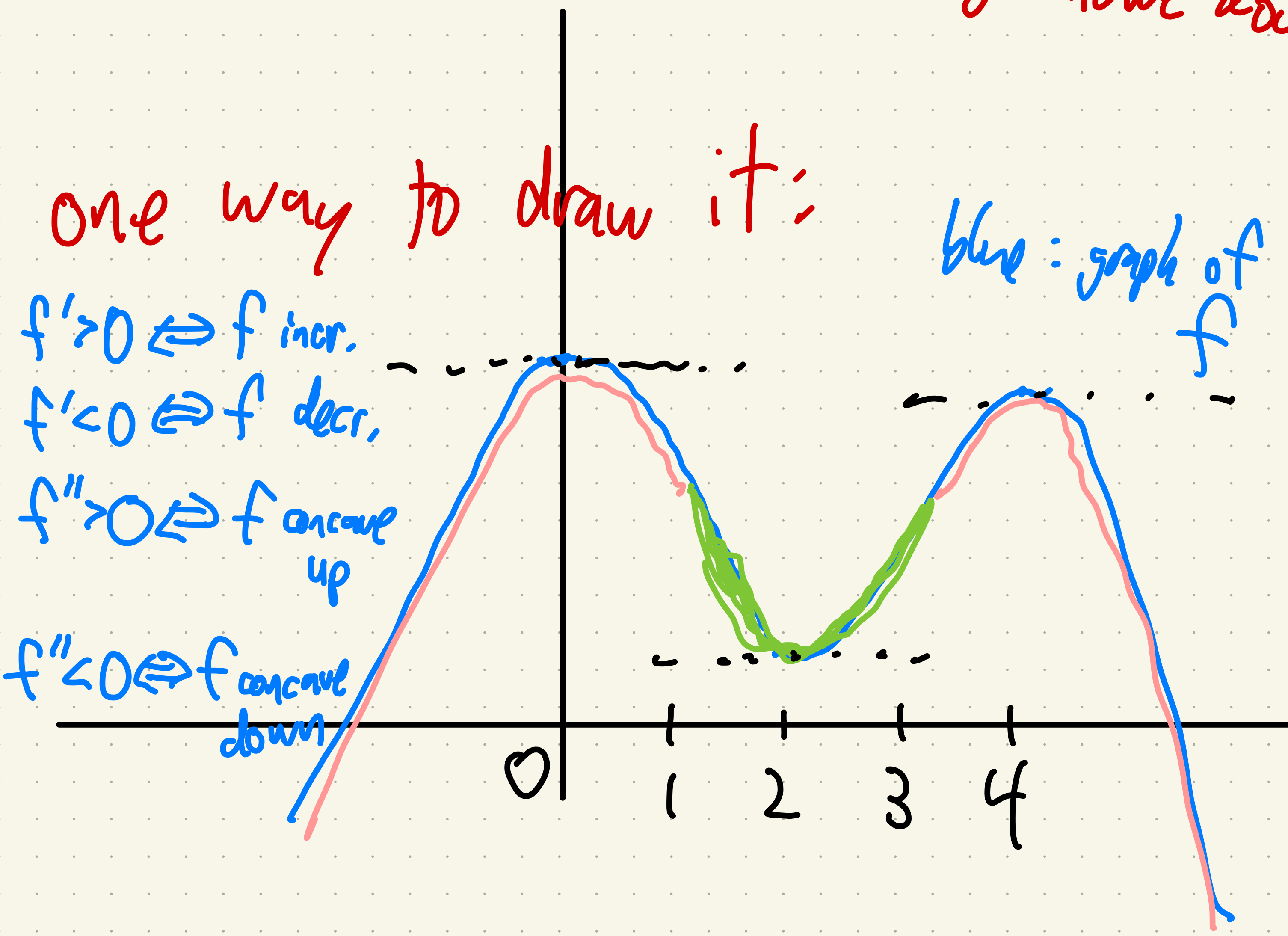
increasing

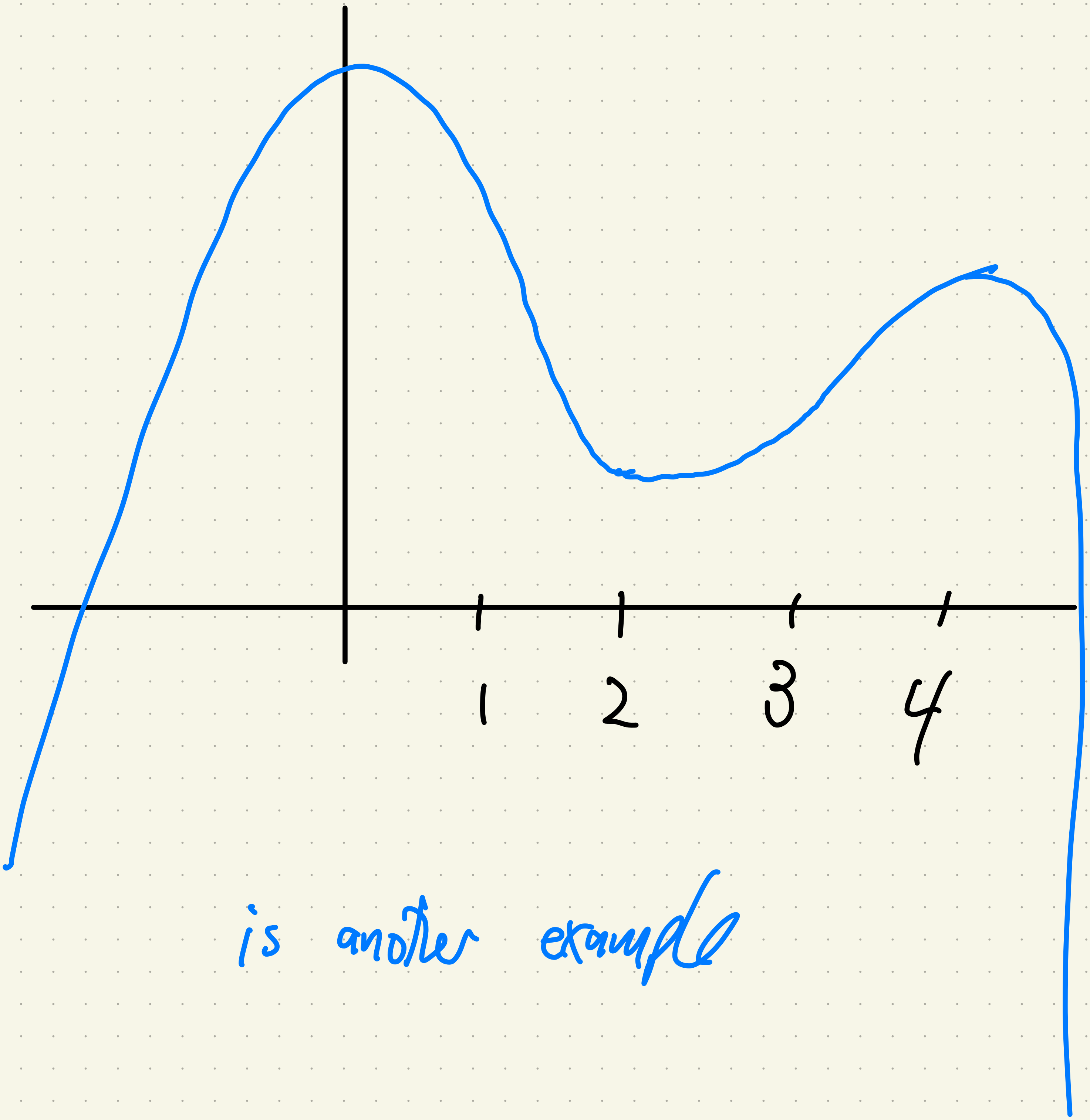
decreasing

concave up

concave down

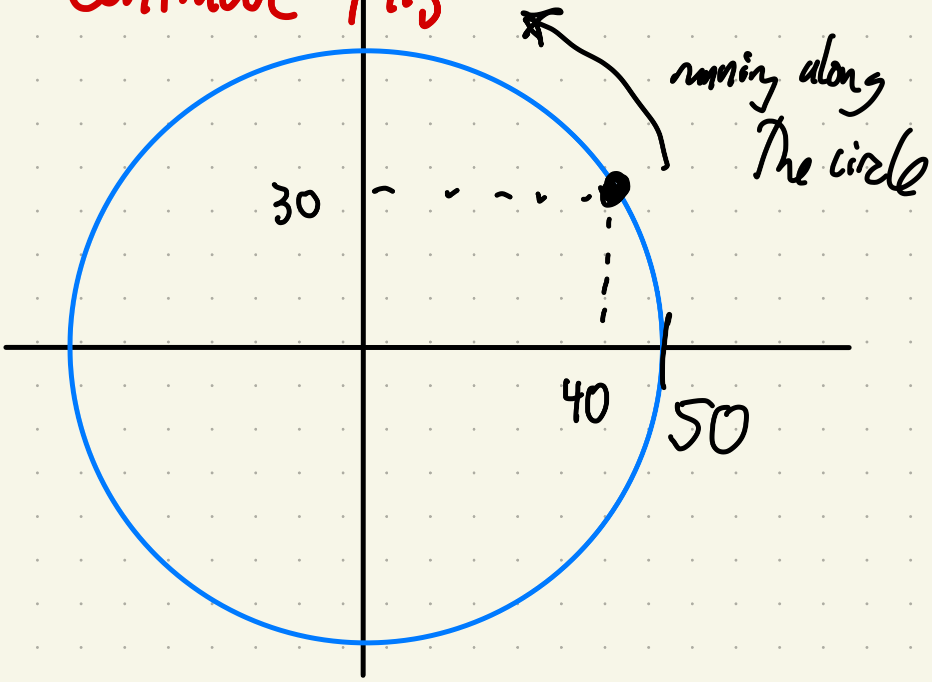
one way to draw it:





is another example

continue this:



when she is at $(40,30)$ she has

$$x'(t) = \frac{-5}{4}\ \frac{m}{sec}.$$

Q: Find $y'(t)$ at that moment.

$$x(t)^2 + y(t)^2 = 2500 \text{ for all time.}$$

b/c (x,y) is on the circle of radius 50.

diff. both sides:

b/c 2500 is constant

$$2x(t)x'(t) + 2y(t)y'(t) = 0$$

plug in $x(t)=40$, $y(t)=30$, $x'(t)=\frac{-5}{4}$.

to find $y'(t)$.

See you tomorrow!
(Thursday).