Chapter 1: Concepts of Motion

displacement:
$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

Average speed:
$$s_{ave} = \frac{\text{total distance}}{\Delta t} = \frac{d}{\Delta t}$$

Average velocity:
$$\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t}$$

Average Acceleration:
$$\vec{a}_{ave} = \frac{\Delta \vec{v}}{\Delta t}$$

Chapter 2: Kinematics in one dimension

displacement:
$$\Delta x = x_f - x_i$$

$$x_f = x_i + \int_{t_i}^{t_f} v_x dx$$

Average speed:
$$s_{ave} = \frac{\text{total distance}}{\Delta t}$$

Instantaneous velocity:
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous acceleration:
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

For Constant Acceleration

constant acceleration:
$$a = a_{ave} = \frac{v_f - v_i}{\Delta t}$$

velocity (with a and t):
$$v(t) = v_0 + at$$

position (with a and t):
$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

velocity (with a and x):
$$v^2 = v_0^2 + 2a(x - x_0)$$

position (with v and t):
$$x(t) = \frac{1}{2}(v_0 + v)t + x_0$$

average velocity:
$$v_{ave} = \frac{1}{2}(v_0 + v)$$

average velocity:
$$v_{ave} = v_0 + \frac{1}{2}at^2$$

free fall acceleration:
$$a_{freefall} = -g$$

Chapter 3: Vectors and Coordinate Systems

Unit Vectors: have magnitude (length) = 1

x-direction,
$$(i)$$

y-direction,
$$(j)$$

z-direction,
$$(k)$$

Vector equation:
$$\vec{s} = \vec{a} + \vec{b}$$

Vector commutative law:
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

Vector associative law:
$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

Vector subtraction:
$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$

Vector components:
$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$a_x = a \cos \theta$$
 and $a_y = a \sin \theta$

$$a = \sqrt{a_x^2 + a_y^2}$$
 and $\tan \theta = \frac{\mathrm{a_y}}{\mathrm{a_x}}$

Vector addition:
$$\vec{r} = \vec{a} + \vec{b}$$

$$\vec{r} = r_x \hat{i} + r_y \hat{j} + r_z \hat{k}$$

$$r_x = a_x + b_x$$

$$r_y = a_y + b_y$$

$$r_z = a_z + b_z$$

vector by scalar multip.: $2\vec{a} = 2a_x\hat{i} + 2a_u\hat{j} + 2a_z\hat{k}$

Scalar Product:
$$\vec{a} \cdot \vec{b} = ab \cos \phi$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

Vector Product:
$$\vec{c} = \vec{a} \times \vec{b}$$

$$|c| = ab \sin \phi$$

direction from Right-hand rule

$$\vec{c} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j}$$

$$+ (a_x b_y - a_y b_x)\hat{k}$$

$$\vec{a}\times\vec{b}=-(\vec{b}\times\vec{a})$$

Chapter 4: Kinematics in two dimensions

 $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ Position:

 $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$ Displacement:

 $\Delta \vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

 $\vec{v}_{ave} = \frac{\Delta \vec{r}}{\Delta t}$ $\vec{v} = \frac{d\vec{r}}{dt}$ Average Velocity:

Instantaneous Velocity:

 $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$

 $v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$

 $\vec{a}_{ave} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$ Average acceleration:

Instantaneous acceleration:

 $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

 $a_x = \frac{dv_x}{dt}, a_y = \frac{dv_y}{dt}, a_z = \frac{dv_z}{dt}$

Projectile Motion

 $v_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ Initial velocity:

 $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$

 $x(t) = v_{0x}t + x_0$ Horizonatal Motion:

 $y(t) = -\frac{1}{2}gt^2 + v_{0y}t + y_0$ Vertical Motion:

 $v_{u} = v_{0u}t - gt$

 $v_y^2 = v_{0y}^2 - 2g(y - y_0)$

 $y = (\tan \theta_0)\mathbf{x} - \frac{\mathbf{g}\mathbf{x}^2}{2(\mathbf{v}_0 \cos \theta_0)^2}$ Equation of the path (trajectory):

 $R = \frac{v_0^2}{\hat{z}} \sin 2\theta_0$ Horizontal Range:

Relative Motion: Labels are for a point P in reference frames A,B

1-dimension: $x_{PA} = x_{PB} + x_{BA}$

 $v_{PA} = v_{PB} + v_{BA}$

 $a_{PA} = a_{PB}$

2-dimensions: $\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$

 $\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

 $\vec{a}_{PA} = \vec{a}_{PB}$

Uniform Circular Motion

 $\theta(radians) = \frac{\text{arclength}}{\text{radius}} = \frac{s}{r}$ angular position:

 $\theta_f = \theta_i + \omega \Delta t$

 $s = r\theta$ arclength:

 $\omega_{ave} = \frac{\Delta\theta}{\Delta t}$ $\omega = \frac{d\theta}{\Delta t}$ angular velocity:

$$\begin{split} &\omega = \frac{dt}{dt} \\ &\vec{a} = \frac{v^2}{r}, \text{towardscenterofcircle} \end{split}$$
centripedal acceleration:

 $T = \frac{2\pi r}{r}$ (Uniform Circular Motion) Period of revolution:

Units

Length: 1 m = 3.28 ft

1 mi = 1.61 km

1 ft = 12 in

 $1 \min = 60 s$ Time:

 $60 \min = 1 \text{ hr}$

24 hr = 1 day

365 days = 1 year

1 m/s = 3.28 ft/sSpeed:

1 km/hr = 0.621 mi/hr

SI Prefixes		
Exponent	Prefix	SI Symbol
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	cento	c
10^{-1}	deci	d
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^9	giga	G
10^{12}	tera	\mid T

Chapter 5: Force and Motion-I

Newton's 1st law: if $\vec{F}_{net} = 0$, then $\vec{a} = 0$

Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$

 $F_{net,x} = ma_x$

 $F_{net,y} = ma_y$

 $F_{net,z} = ma_z$

Superposition of Forces: (Net force) $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$

Gravitational Force: $F_g = mg$ (magnitude)

Weight: W = mg

Newton's 3rd law: (for two objects, B & C) $\vec{F}_{BC} = -\vec{F}_{CB}$

Chapter 6,7,8: Forces

Static Friction: $f_{s,max} = \mu_s F_N$

Kinetic Friction: $f_k = \mu_k F_N$

Drag Force: $D = \frac{1}{2}C\rho Av^2$

Terminal speed: $v_t = \sqrt{\frac{2F_g}{C\rho A}}$

Centripetal acceleration: $a = \frac{v_2}{R}$

Centripetal Force: $F = m \frac{v_2}{R}$

Constants

Gravitational Acceleration: $g = 9.8 \frac{m}{s^2}$

Atomic Mass Units: $1 \text{ u} = 1.66 \times 10^{-27} \text{kg}$

Chapter 9: Kinetic Energy and Work

Kinetic Energy: $K = \frac{1}{2}mv^2$

Work from a constant force: $W = Fd \cos \phi$

 $W = \vec{F} \cdot \vec{d}$

Work-Kinetic Energy Theorem: $\Delta K = K_f - Ki = W$

Work done by the gravitational force: $W_g = mgd \cos \phi$

Work down lifting and lowering object: $\Delta K = K_f - Ki = W_a + W_g$

Hooke's Law (spring force): $\vec{F}_s = -k\vec{d}$

1-D: $F_x = -kx$

Work done by a spring force: $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Work done by a variable force (1-D): $W = \int_{x_i}^{x_f} F(x) dx$

Work done by a variable force (3-D): $W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$

Average power: $P_{ave} = \frac{W}{\Delta t}$

Instantaneous power: $P = \frac{\mathrm{d}W}{\mathrm{d}t}$

Instantaneous power: $P = \vec{F} \cdot \vec{v}$

Chapter 10: Potential Energy and Conservation of Energy

Potential Energy: $\Delta U = -W$

> $\Delta U = -\int_{x_i}^{x_f} F(x) dx$ 1-D:

 $\Delta U = mg(y_f - y_i)$ Gravitational Pot. Energy (change):

U(y) = mgyGravitational Pot. Energy:

 $U(x) = \frac{1}{2}kx^2$ Elastic Potential Energy, (spring):

 $E_{mech} = K + U$ Mechanical Energy:

 $K_2 + U_2 = K_1 + U_1$ Cons. of Mechanical Energy:

 $\Delta E_{mech} = \Delta K + \Delta U = 0$

 $F(x) = -\frac{dU(x)}{dx}$ Force from Pot. Energy curve, 1-D:

 $K(x) = E_{mech} - U(x)$ Kinetic energy from Pot. Energy curve, 1-D:

 $W = \Delta E_{mech}$ Work done on System, no friction:

Increase in thermal Energy by sliding: $\Delta E_{th} = f_k d$

Work done on System, with friction: $W = \Delta E_{mech} + \Delta E_{th}$

 $W = \Delta E = \Delta E_{mech} + \Delta E_{th} + \Delta E_{int}$ Total Energy change of a system:

 $\Delta E_{mech} + \Delta E_{th} + \Delta E_{int} = 0$ Total Energy change of an isolated system:

 $P_{ave} = \frac{\Delta E}{\Delta t}$ $P = \frac{dE}{\Delta t}$ Average Power:

Instantaneous Power:

Chapter 11: Impulse and Momentum

 $F_{net,x} = Ma_{com,x}$ $F_{net,y} = Ma_{com,y}$ $F_{net,z} = Ma_{com,z}$

 $\vec{p} = m\vec{v}$ Linear momentum of a particle:

 $\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\vec{a}$ Newton's 2nd, Force and Momentum (particle):

 $\vec{P}_s y stem = \sum_{i=1}^n m_i \vec{v}_i$ Linear momentum of a system of particles:

 $\vec{F}_{net} = \frac{d\vec{P}_{system}}{dt}$ Newton's 2nd, Force and Momentum (system):

 $\vec{J} = \int_{t}^{t_f} \vec{F}(t) dt$ Impulse (collision):

 $\Delta \vec{p} = \vec{J}$ Linear momentum-impulse theorem:

 $\vec{P}_{sustem} = constant$ linear momentum, closed, isolated system:

 $\vec{P}_i = \vec{P}_f$ Cons. of linear momentum, system:

Cons. of linear momentum, two body, closed system: $\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$

 $V = \frac{m_1}{m_1 + m_2} v_{1,i}$ Completely inelastic collision $(v_{2,i} = 0)$, 1-D:

 $v_{1,f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1,i} + \frac{2m_2}{m_1 + m_2} v_{2,i}$ Elastic Collision, 1-D final velocities:

 $v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2,i}$

 $\vec{P}_{1,i} + \vec{P}_{2,i} = \vec{P}_{1,f} + \vec{P}_{2,f}$ Collision in 2-D: momentum:

 $K_{1,i} + K_{2,i} = K_{1,f} + K_{2,f}$ Collision in 2-D (elastic only):

Geometry and other useful equations

Surface Area:

square: $l \times w$

circle: πr

sphere: $4\pi r^2$

cylinder: $2\pi r^2 + 2\pi rh$

Circumference of a circle: $C = 2\pi r$

Area of a circle: $A_{circle} = \pi r^2$

Density: $\rho = \frac{m}{V}$

Quadratic formula: $(ax^2 + bx + c = 0)$

 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Angular velocity, ω : $\omega = \frac{v}{r}$

Volume:

Cube: $l \times w \times h$

Sphere: $\frac{4}{3}\pi r$

Cylinder: $\pi r^2 h$