MATH 30, 3/24-25/2020: THE CLOSED INTERVAL METHOD

Last time: Critical points and Fermat's theorem. If you want to find local max and min, the candidates are the critical points (you can ignore all the other points).

The Closed Interval Method. To find the *global* max and min values of f on $[a, b]$:

(1) Find the values of f at the critical points of f in (a, b).
(2) Find $f(a)$ and $f(b)$.
(3) Compare all these values, and take the biggest and smallest values of f.

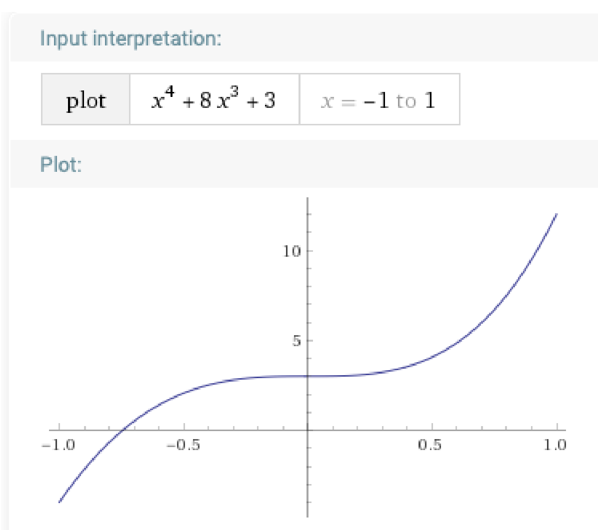
Example. Find the global max and min of the function $f(x) = x^4 + 8x^3 + 3$ on the interval $[-1, 1]$.

Solution. First, let's find the critical points: $f'(x) = 4x^3 + 24x^2 = 4x^2(x + 6)$. So the critical points are $x = 0$ and ... **not** $x = -6$ only because that point is not in the interval $[-1, 1]$.
Now compare the values of the function f at $x = -1, 0, 1$:

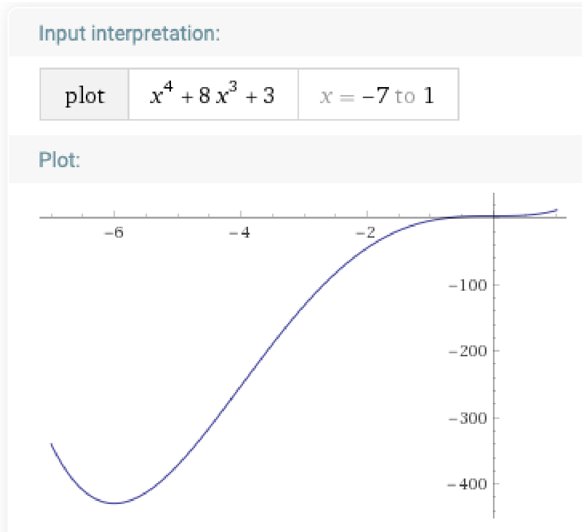
$$\begin{aligned} f(-1) &= -4 \\ f(0) &= 3 \\ f(1) &= 12. \end{aligned}$$

So we see that the global maximum is $f(1) = 12$, and the global minimum is $f(-1) = -4$.

For fun, let's look at the graph of the function on the given interval $[-1, 1]$:



Also for fun (even though I wasn't asking about this), let's look at the graph on the interval $[-7, 1]$:



Now it's your turn! Here are some "optimization and pessimization" problems: (You will probably need scratch paper...)

1. Find the point on the line $y = 2x - 3$ that is closest to the origin. Make a sketch.

2. Prove that among all rectangles of a given perimeter (say they all have perimeter P), the square has the largest area.

3. Find the global max and min of the function $f(x) = x^5 + x + 1$ on the interval $[-1, 1]$.

4. Find the global max and min of the function $f(x) = x^3 - x^2 - 8x + 1$ on the interval $[-2, 2]$.

5. If 1200 cm^2 of cardboard is available to make a box with a square base and an open top, find the largest possible volume of the box.