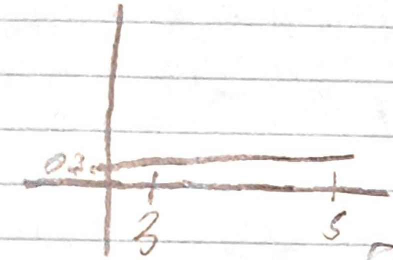
Santiago Barudos
11-11-22
Stat 1...

Module 3 HW

1. P.D.F.:

$$f(x) = \begin{cases} 0.075x + 0.2 & 3 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

a) Graph and verify that area is 1!



$\int_3^5 0.075x + 0.2\,dx$

$0.075\frac{x^2}{2}\Big|_3^5 + 0.2x\Big|_3^5$

$0.075\frac{(5)^2}{2} = 0.9375$ $\quad 0.9375 - 0.3375 = 0.6$

$0.075\frac{(3)^2}{2} = 0.3375$ $\quad 0.6 + 0.4 = \boxed{1}$

$0.2(5) = 1$ $\quad 1 - 0.6 = 0.4$

$0.2(3) = 0.6$

b) $P(x \le 4)$?

$= \int_3^4 0.075x + 0.2\,dx$ $\quad 0.075\frac{x^2}{2}\Big|_3^4 + 0.2x\Big|_3^4$

$0.075\frac{(4)^2}{2} = 0.6$ $\quad 0.6 - 0.3375 = 0.2625$

$0.075\frac{(3)^2}{2} = 0.3375$ $\quad 0.2625 + 0.2 = 0.4625$

$0.2(4) = 0.8$ $\quad 0.8 - 0.6 = 0.2$

$0.2(3) = 0.6$

$\boxed{\text{0.4625, this probability is the same as } P(x<4).}$

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d) $P(3.5 \le X \le 4.5)$ and $P(4.5 < X)$

$= \int_{3.5}^{4.5} 0.075x + 0.2\,dx \quad 0.075\frac{x^2}{2}\Big|_{3.5}^{4.5} + 0.2x\Big|_{3.5}^{4.5}$

$0.075\frac{(4.5)^2}{2} = 0.759375 \quad 0.759375 - 0.459375 = 0.3$

$0.075\frac{(3.5)^2}{2} = 0.459375 \quad 0.3 + 0.2 = \boxed{0.5 \text{ and } 0.278125}$

$0.2(4.5) = 0.9 \quad 0.9 - 0.7 = 0.2$

$0.2(3.5) = 0.7$

$\int_{4.5}^{5} 0.075x + 0.2\,dx \quad 0.075\frac{x^2}{2}\Big|_{4.5}^{5} + 0.2x\Big|_{4.5}^{5}$

$0.075\frac{(5)^2}{2} = 0.9375 \quad 0.9375 - 0.759375 = 0.178125$

$0.075\frac{(4.5)^2}{2} = 0.759375 \quad 0.178125 + 0.1 = 0.278125$

$0.2(5) = 1 \quad 1 - 0.9 = 0.1$

$0.2(4.5) = 0.9$

2. PDF:

$f(x) = \begin{cases} 0.15e^{-0.15(x-0.5)} & x \ge 0.5 \\ 0 & \text{otherwise} \end{cases}$

a) $P(x \le 6)$?

$\int_{0.5}^{6} 0.15e^{-0.15(x-0.5)} \Rightarrow 0.15\int_{0.5}^{6} e^{-0.15(x-0.5)}$

$e^u du \quad u = -0.15x + 0.075$

$du = -0.15dx \Rightarrow dx = \frac{du}{-0.15}$

$-\frac{1}{0.15} e^u du \Rightarrow \cancel{0.15} \cdot -\frac{1}{\cancel{0.15}} e^{-0.15(x-0.5)}$

$-e^{-0.15(6-0.5)} + e^{-0.15(0.5-0.5)} = \boxed{0.5617}$ cont'd

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b) $P(X \geq 6)? \rightarrow 1 - P(X < 6)$

$= \int_{0.5}^{6} 0.15 e^{-0.15(x-0.5)} \rightarrow 0.15 \int_{0.5}^{6} e^{-0.15(x-0.5)}$

$-e^{-0.15(x-0.5)} \Big|_{0.5}^{6}$ $\quad 1 - 0.56 = \boxed{0.4382}$

$-e^{-0.15(6-0.5)} - (-e^{-0.15(0.5-0.5)}) = 0.56$

c) $P(5 < X < 6)?$

$= \int_{5}^{6} 0.15 e^{-0.15(x-0.5)} \rightarrow 0.15 \int_{5}^{6} e^{-0.15(x-0.5)}$

$-e^{-0.15(x-0.5)} \Big|_{5}^{6}$

$-e^{-0.15(6-0.5)} - (-e^{-0.15(5-0.5)}) = \boxed{0.07}$

3. CDF:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{4}, & 0 \leq x < 2 \\ 1, & 2 \leq x \end{cases}$$

a) $P(X \leq 1)?$

$\rightarrow F(1)$ so, $\frac{(1)^2}{4} = \frac{1}{4} = \boxed{0.25}$

b) $P(0.5 \leq X \leq 1)$

$\rightarrow F(1) - F(0.5)$

$\frac{(0.5)^2}{4} = 0.0625 \quad 0.25 - 0.0625 = \boxed{0.1875}$

c) $P(X > 1.5) = 1 - P(X \leq 1.5)$

$1 - F(1.5) \quad 1 - 0.5625 = \boxed{0.4375}$

$\frac{(1.5)^2}{4} = 0.5625$

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c) Median $\tilde{\mu}$ [solve $0.5 = F(\tilde{\mu})$]

$F(\tilde{\mu}) = 0.5$

$\frac{(\tilde{\mu})^2}{4} = 0.5$

$(\tilde{\mu})^2 = 2$

$\tilde{\mu} = \sqrt{2}$ $\boxed{\tilde{\mu} = 1.414}$

e) $F'(x)$ for density function $f(x)$

$f(x) = \frac{d}{dx} F(x)$ $\boxed{f(x) = \frac{x}{2}}$ $\rightarrow f(x) = \begin{cases} \frac{x}{2} & 0 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$

$= \frac{d}{dx} \frac{(x)^2}{4}$

$= \frac{2x}{4} = \frac{x}{2}$

f) $E(x)$?

$\int_0^2 x f(x)\,dx = \int_0^2 x \cdot \frac{x}{2}\,dx = \frac{1}{2}\int_0^2 x^2\,dx$

$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 \rightarrow \frac{1}{2}\frac{(2)^3}{3} = 1.33$ $\boxed{E(x) = 1.33}$

g) $V(x)$ and σ_x

$E(x^2) = \int_0^2 x^2 f(x)\,dx = \int_0^2 x^2 \frac{x}{2}\,dx$

$= \frac{1}{2}\int_0^2 x^3\,dx = \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2 = \frac{1}{2}\frac{(2)^4}{4} = 2$

$E(x^2) = 2$ $V(x) = E(x^2) - (E(x))^2$

$= 2 - (1.33)^2$

$V(x) = 0.22$

$\boxed{\begin{array}{l} V(x) = 0.22 \\ \sigma_x = 0.4714 \end{array}}$

$\sigma_x = \sqrt{V(x)} = \sqrt{0.22} = 0.4714$

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Module 3 HW

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b) $h(x) = x^2$ when hashing to X, compute $E[h(X)]$

$E[h(x)] = \int h(x) f(x)\,dx$

$= \int_0^2 x^2 \frac{x}{2}\,dx$ $\boxed{E[h(x)] = 2}$

$= \frac{1}{2}\int_0^2 x^3\,dx$

$= \frac{1}{2}\left[\frac{x^4}{4}\right]\Big|_0^2$ $\frac{1}{2}\left(\frac{(2)^4}{4}\right) = 2$

4. $f(x) = \begin{cases} \frac{k}{x^4} & x > 1 \\ 0 & x \le 1 \end{cases}$ PDF

a) Find k for which $f(x)$ is a pdf

$\int f(x)\,dx = 1$

$\int_0^1 0\,dx + \int_1^{\infty} \frac{k}{x^4}\,dx = 1$

$k\int_1^{\infty} \frac{1}{x^4}\,dx = 1$ $k\left[\frac{x^{-3}}{-3}\right]\Big|_1^{\infty} = 1$

x^{-4} $k\left[\frac{1}{-3x^3}\right]\Big|_1^{\infty} = 1$

$k\left[\frac{1}{-3\infty^3} - \frac{1}{-3(1)^3}\right] = 1$

$k\left[0 + \frac{1}{3}\right] = 1$

$\frac{k}{3} = 1$ $\boxed{k = 3}$

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b) CDF?

$$f(x) = \begin{cases} \frac{3}{x^4} & x > 1 \\ 0 & x \le 1 \end{cases}$$

$$\int_1^n \frac{3}{x^4} \quad 3x^{-4} \quad \frac{3x^{-3}}{-3} \quad -x^{-3}\Big|_1^n$$

$$-n^{-3} - (-1^{-3}) = -n^{-3} - (-1)$$

$$-n^{-3} + 1 \rightarrow -\frac{1}{n^3} + 1$$

$$\boxed{F(x) = \begin{cases} -\frac{1}{x^3} + 1 & x > 1 \\ 0 & x \le 1 \end{cases}}$$

c) Use CDF for $P(x > 2)$ and $P(2 < x < 3)$

$$F(2) = 1 - \frac{1}{(2)^3} = 1 - \frac{1}{8} = 0.875$$

$$1 - 0.875 = \boxed{0.125}$$

$$F(3) = 1 - \frac{1}{(3)^3} = 1 - \frac{1}{27} = 0.9629$$

$$0.9629 - 0.875 = 0.0879$$

$$\boxed{\begin{aligned} P(x > 2) &= 0.125 \\ P(2 < x < 3) &= 0.0879 \end{aligned}}$$

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Module 3 HW

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A) Mean and SD of hardware?

$\mu_x = \int_{-\infty}^{\infty} x f(x) dx$

PDF:

$$f(x) = \begin{cases} \frac{3}{x^4} & x > 1 \\ 0 & x \leq 1 \end{cases}$$

$= \int_1^{\infty} \frac{3x}{x^4} dx$

$x^{-3} \rightarrow \frac{x^{-2}}{-2}$

$\int_1^{\infty} \frac{3}{x^3}$

$\frac{3}{-2} x^{-2} \Big|_1^{\infty}$

$-\frac{3}{2}\left[\frac{1}{x^2} - \frac{1}{1^2}\right]$

$[0 - 1]$

$-\frac{3}{2}(-1) = \frac{3}{2}$

$\mu_x = 3/2$

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$\sigma_x^2 = Var(x) = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)\,dx$

$= \int_1^{\infty} (x - 3/2)^2 \frac{3}{x^4}\,dx$

$= \int_1^{\infty} \left(x^2 - \frac{3}{2}x + \frac{9}{4}\right) \frac{3}{x^4}\,dx$

$= \int_1^{\infty} \left(\frac{3}{x^2} - \frac{9}{2x^3} + \frac{27}{4x^4}\right) dx$

$3\left[\frac{x^{-1}}{-1}\right]\Big|_1^{\infty} - \frac{9}{2}\left[\frac{x^{-2}}{-2}\right]\Big|_1^{\infty} + \frac{27}{4}\left[\frac{x^{-3}}{-3}\right]\Big|_1^{\infty}$

$-3(0-1) + \frac{9}{4}(0-1) - \frac{27}{12}(0-1)$

$3 - \frac{9}{4} + \frac{27}{12} = 3$

$\sigma_x = \sqrt{3}$

$= 1.73$

$\boxed{\mu_x = 3/2 \quad \sigma_x = 1.73}$

c) Prob that reading within 1 SD of mean?

$P(\mu_x - \sigma_x < X < \mu_x + \sigma_x)$?

$= P(3/2 - 1.73 < X < 3/2 + 1.73)$

$= P(-0.23 < X < 3.23)$

$= \int_1^{3.23} \frac{3}{x^4}\,dx$

$3(x^{-4})\,dx$

$\frac{3}{-3}(x^{-3})\Big|_1^{3.23}$

$-1[0.029 - 1] = \boxed{0.97}$

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Module 3 HW

5. X has uniform dist on $[A, B]$

a) Expression for (100p)th percentile?

$F(x) = \frac{x-A}{B-A} = p$ $\quad x = \boxed{p(B-A)+A}$

$\cancel{(B-A)\frac{x-A}{B-A}} = p(B-A) \rightarrow x - A + A = p(B-A) + A$

b) $E(X)$, $V(X)$, and σ_X?

$\frac{B^2-A^2}{B-A}$ $\quad B-A$

$E(X) = \int_A^B x \frac{1}{B-A} dx \rightarrow \frac{x^2}{2} \frac{1}{B-A}\Big|_A^B \rightarrow \frac{1}{2}\frac{1}{B-A}(B^2-A^2)$ $\quad \frac{B+A}{2}$

$\frac{1}{2}\frac{(B^2-A^2)}{B-A} = \frac{(B+A)(B-A)}{2(B-A)}$

$E(X) = \frac{A+B}{2}$

$E(X^2) = \int_A^B x^2 \frac{1}{B-A} dx \rightarrow \frac{x^3}{3}\frac{1}{B-A}\Big|_A^B \rightarrow \frac{1}{3}\frac{1}{B-A}(B^3-A^3)$

$\frac{\cancel{(B-A)}(B^2+BA+A^2)}{3\cancel{(B-A)}} \leftarrow \frac{1}{3}\frac{(B^3-A^3)}{B-A}$

$\rightarrow E(X^2) = \frac{B^2+BA+A^2}{3}$

$$\boxed{E(X) = \frac{A+B}{2} \quad E(X^2) = \frac{A^2+AB+B^2}{3} \quad \sigma_X = \frac{(B-A)}{\sqrt{12}}}$$

$V(X) = \left(\frac{A^2+AB+B^2}{3}\right) - \left(\frac{A+B}{2}\right)^2$ $\quad \sigma_X = \frac{B-A}{\sqrt{12}}$ $\quad \frac{(B-A)^2}{12}$

$4 \cdot \left(\frac{A^2+AB+B^2}{3}\right) - \left(\frac{A^2+AB+B^2}{4}\right) \cdot 3$

$\frac{4(A^2+AB+B^2)}{12} - \frac{3(A^2+AB+B^2)}{12} = \frac{A^2-2AB+B^2}{12}$

$(B-A)^2$

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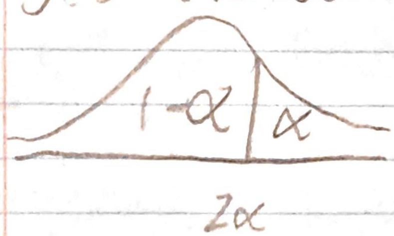
5) For n, a positive integer, compute E(X^n)

$E(X^n) = \int_A^B x^n \frac{1}{B-A}\,dx = \frac{x^{n+1}}{n+1}\left(\frac{1}{B-A}\right)\Big|_A^B$

$\boxed{\frac{B^{n+1} - A^{n+1}}{(n+1)(B-A)}}$

6. Find Z_α for the following

a) $\alpha = 0.0055$



$C = 1 - \alpha$ from Z-table

$= 1 - 0.0055$

$C = 0.9945 \rightarrow 2.54$

$\boxed{Z_\alpha = 2.54}$

b) $\alpha = 0.09$ $C = 1 - 0.09$

$C = 0.91 \rightarrow \boxed{Z_\alpha = 1.34}$

c) $\alpha = 0.663$ $C = 1 - 0.663$

$= 0.337 \rightarrow \boxed{Z_\alpha = -0.43}$

7. $X \sim N(\mu, \sigma^2)$

$X \sim N(8.46, 0.913^2)$

$\sigma = 0.913$

a) $P(X \geq 10)$? $P\left(\frac{X-\mu}{\sigma} \geq \frac{10-8.46}{0.913}\right)$

$P(Z \geq 1.686)$ $1 - 0.9545 = \boxed{0.0455}$

0.9545

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Module 8 HW

cont #1

b) $P(x>13)$? $P\left(z > \frac{13-8.46}{0.913}\right)$

$z > 7.163$

$\downarrow$

0.999

$\boxed{P(x>13) \approx 0}$ $1-0.999 \approx 0$

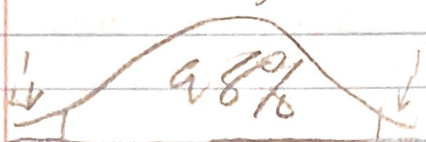
c) $P(8<x<10)$? $P\left(z < \frac{10-8.46}{0.913}\right) - P\left(z < \frac{8-8.46}{0.913}\right)$

$P(z<1.69) - P(z<-0.503)$

$0.9545 - 0.3085 = 0.646$

$\boxed{P(8<x<10) \approx 0.65}$

d) Value c such that 98% of all x in interval $[8.46-c, 8.46+c]$?



$8.46-c \rightarrow$ 1st percentile $\rightarrow -2.33$

$8.46+c \rightarrow$ 99th percentile $\rightarrow 2.33$

$8.46-c = \mu + (-2.33)\sigma = 8.46 - 2.33(0.913)$

$\downarrow$

$\boxed{c = 2.127}$ $c = 2.33(0.913) = 2.127$

e) 4 x chosen, prob that $1 \; x > 10$?

$P(x>10) = 0.0455$

$P(\text{At least } 1 \; x) = 1 - P(\text{no } x) = 1 - P(x)^4$

$= 1-(1-0.0455)^4$

$= 1-0.83 = \boxed{0.169}$

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Stat 11ba

6. $X \sim N(104, 5^2)$
$\sigma = 5$
a) $P(X \leq 105)$? $P(X < 105)$? $P(60 \leq 105)$?
$P\left(Z \leq \frac{105-104}{5}\right) \to P(Z \leq 0.2) \to \boxed{0.5793}$

b) Prob that X differs from mean by more than 1 SD?
Prob depend on μ and σ?
$P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)$
$= 0.8413 - 0.1587$
$= 0.6826$
$\boxed{\text{0.6826. No, the probability does not depend on the values of } \mu \text{ and } \sigma!}$

c) How would you characterize most extreme 0.1% of X?

$\alpha = 0.001$
$\downarrow$
$Z \approx \pm 3.09$
$Z = \frac{X - \mu}{\sigma}$

$X = \mu + Z\sigma$
$= 104 + 3.09 \cdot 2$
$= 110.18$

$X = \mu - Z\sigma$
$= 104 - 3.09 \cdot 2$
$= 97.82$

$\boxed{\text{The most extreme 0.1\% of X values are less than 97.82 and greater than 110.18!}}$

9. $X \sim N(200, 30^2)$ Damage at $X < 100$
$\sigma = 30$ Prob of damage of at least 1 of 5 [illegible]?
$P(X < 100)? \to P\left(Z < \frac{100-200}{30}\right) \to P(Z < -3.33) \to 0.0004$
$1 - 0.0004 = 0.9996$
$1 - P(\text{No } X) \to 1 - P(X)^5$
$= 1 - (0.9996)^5$
$= 1 - 0.998 = \boxed{0.002}$

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Module 3 HW

10. $X \sim N(\mu, \sigma)$ Prob. that X is...

a) within 1.5 SDs of mean?

$P(\mu - 1.5\sigma \le X \le \mu + 1.5\sigma)$
$= P(-1.5 \le Z \le 1.5)$
$= \Phi(1.5) - \Phi(-1.5)$
$= 0.9332 - 0.0668$
$= \boxed{0.8664}$

b) Farther than 2.5 SDs from mean?

$P(X < \mu - 2.5\sigma$ or $X > \mu + 2.5\sigma)$
$= 1 - P(\mu - 2.5\sigma \le X \le \mu + 2.5\sigma)$
$= 1 - P(-2.5 \le Z \le 2.5)$
$= 1 - [\Phi(2.5) - \Phi(-2.5)]$
$= 1 - [0.9938 - 0.0062]$
$= 1 - 0.9876 = \boxed{0.0124}$

c) Between 1 and 2 SDs from mean?

$P(\mu - 2\sigma \le X \le \mu - 2\sigma$ or $\mu - \sigma \le X \le \mu + \sigma)$
$= P(\mu - 2\sigma \le X \le \mu + 2\sigma) - P(\mu - \sigma \le X \le \mu + \sigma)$
$= P(-2 \le Z \le 2) - P(-1 \le Z \le 1)$
$= [\Phi(2) - \Phi(-2)] - [\Phi(1) - \Phi(-1)]$
$= (0.9772 - 0.0228) - (0.8413 - 0.1587)$
$= 0.9544 - 0.6826$
$= \boxed{0.2718}$

11. $X \sim N(12, 3.5^2)$ Value c such that 99% of all X are at least 1 lb under sigadogs weight?

$\sigma = 3.5$

$P(X > x) = 1 - P(X < x) = 1 - 0.99 = 0.01$ from z-table

$\downarrow$

-2.33

$z = \frac{x - \mu}{\sigma}$ $3.5 \cdot -2.33 = \frac{x - 12}{\cancel{3.5}}$ 3.5

$-8.155 = x - 12$
$+12$ $+12$

$\boxed{x = 3.845}$

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Stat 1Ba

12. 0.75 wear seatbelt... sample of 800 ... that...

a) Between 360 and 400 (inclusive) wear a belt?

$P(360 \le X \le 400)$?

$X \sim B(800, 0.75)$ $\quad E(X) = n \cdot p = 800 \cdot 0.75 = 375$

σ^2 $\quad V(X) = n \cdot p \cdot q = 800 \cdot 0.75 \cdot 0.25 = 93.75$

$\sqrt{93.75} = 9.68$

$$P\left[\frac{360-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{400-\mu}{\sigma}\right]$$

$$P\left[\frac{360-375}{9.68} < \frac{X-375}{9.68} < \frac{400-375}{9.68}\right]$$

$P(-1.55 < Z < 2.58)$

$P(Z < 2.58) - (1 - P(Z < -1.55))$

$0.9951 - (1 - 0.0606)$

$= \boxed{0.0557}$

b) < 400 wear a belt?

$P(X < 400)$?

$$P\left(Z < \frac{400-375}{9.68}\right) = P(Z < 2.58)$$

$\boxed{0.9951}$