CSc 165
Computer Game Architecture

16 - Quaternions

Representing Orientation / Rotation

Angle-Axis

Euler Angles

- stored as homogenous 4x4 matrices
- simple to understand, easy to combine
- vulnerable to “gimbal lock”

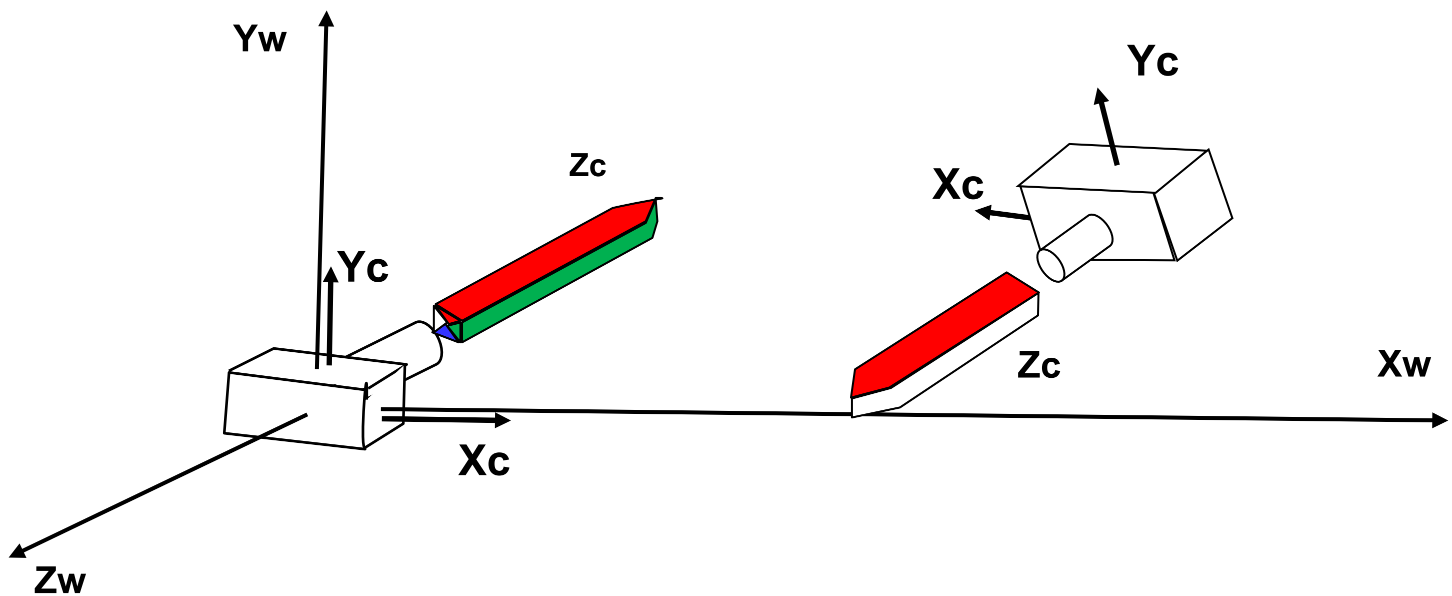
  https://www.youtube.com/watch?v=zc8b2Jo7mno

Quaternions

- stored as a 1x4 vector (more compact)
- complex to understand, but easy to combine
- scalar + 1x3 vector (imaginary component)
- not vulnerable to “gimbal lock”
- easier to “interpolate” for smooth rotations

Orientation as a *Rotation Vector*

- Consider the "look-at" vector (Zc)
  - Imagine Zc has "orientation" ***about its direction***
  - Then, ***camera orientation change*** can be considered as *"**transforming the Zc 'vector'** "*



<u>Quaternions</u>

A four-element object that represents a "<u>3D orientation</u>" **William Hamilton (1805-1865)**

$$q = (w, x, y, z) = (w, \vec{v}), \quad \text{where} \quad \vec{v} = [x\ y\ z]$$

- w represents "rotation angle"
- $\vec{v}$ represents "rotation axis"

But only if **`magnitude(q)`** = 1

- **`magnitude(q) = |q| = sqrt (`**$w^2 + x^2 + y^2 + z^2$**`)`**

Orientations as Quaternions

- **Any rotation "r" of amount α about an axis `v = [x y z]` can be represented as a quaternion**

$$q_r = \left(cos(\alpha/2), \quad sin(\alpha/2)\vec{v} \right)$$
$$= \left(cos(\alpha/2), \quad [x\, sin(\alpha/2), \quad y\, sin(\alpha/2), \quad z\, sin(\alpha/2)] \right)$$

- **Multiplying two (unit) quaternions is the same as concatenation of rotation transformations!**

$\mathbf{q_1 = (w_1, x_1, y_1, z_1)}$ *// an orientation*

$\mathbf{q_2 = (w_2, x_2, y_2, z_2)}$ *// another orientation*

$\mathbf{q_3 = q_2 * q_1}$ *// combined orientation*

Vectors as Quaternions

- Any vector v = [x y z] can be represented as a quaternion:

$$q_v = (0,[x\ y\ z]) = (0,\ x,\ y,\ z)$$

- Any "quaternion vector" q_v can be transformed by a "rotation quaternion" q_r

$$q_v' = q_r * q_v * \text{conjugate}(q_r)$$

Quaternion to Angle/Axis

- Given a quaternion

$$q = (w, [q_x, q_y, q_z])$$

- The corresponding angle/axis rotation is

$$\text{angle } \alpha = 2 * \arccos(w)$$
$$\text{xAxis} = q_x / \sin(\alpha/2)$$
$$\text{yAxis} = q_y / \sin(\alpha/2)$$
$$\text{zAxis} = q_z / \sin(\alpha/2)$$

Quaternion to Angle/Axis (cont.)

- Note the potential for *divide-by-zero*

$$\mathtt{xAxis} = q_x \;/\; \sin(\alpha/2)$$
$$\mathtt{yAxis} = q_y \;/\; \sin(\alpha/2)$$
$$\mathtt{zAxis} = q_z \;/\; \sin(\alpha/2)$$

- Occurs when $\alpha = 0$ (or 360)

- However, *if $\alpha = 0$*, axis doesn't matter

```
denom = sin(alpha/2);
if (abs(denom) < 0.0001 ) {
    denom = 1 ;
}
xAxis = qx / denom;
yAxis = qy / denom;
zAxis = qx / denom;
```

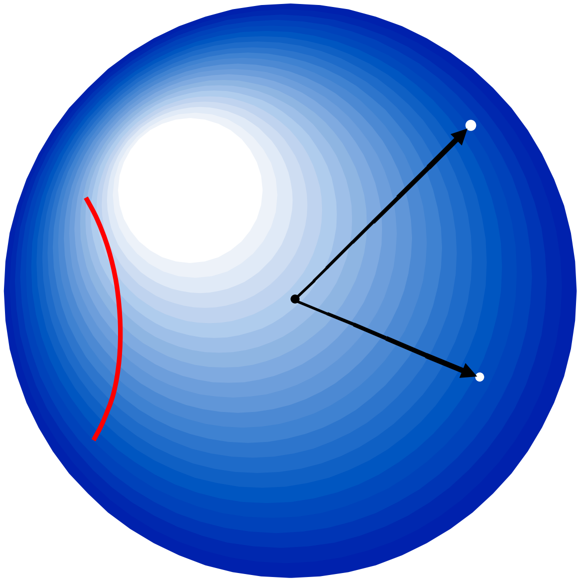
Orientation Interpolation

- Complicated when using Euler, UVN, or Angle/Axis
  - Many variables → many paths
  - How to choose one?

- Quaternions provide a simple, unique interpolation
  - Two approaches:
    - Linear ("*lerp*")
    - Spherical linear ("*slerp*")

Quaternion Interpolation

- Orientation quaternions represent radius vectors of a unit sphere
  - Actually, *infinitely many* unit spheres
- *Interpolation* = finding quaternions along the arc between surface points



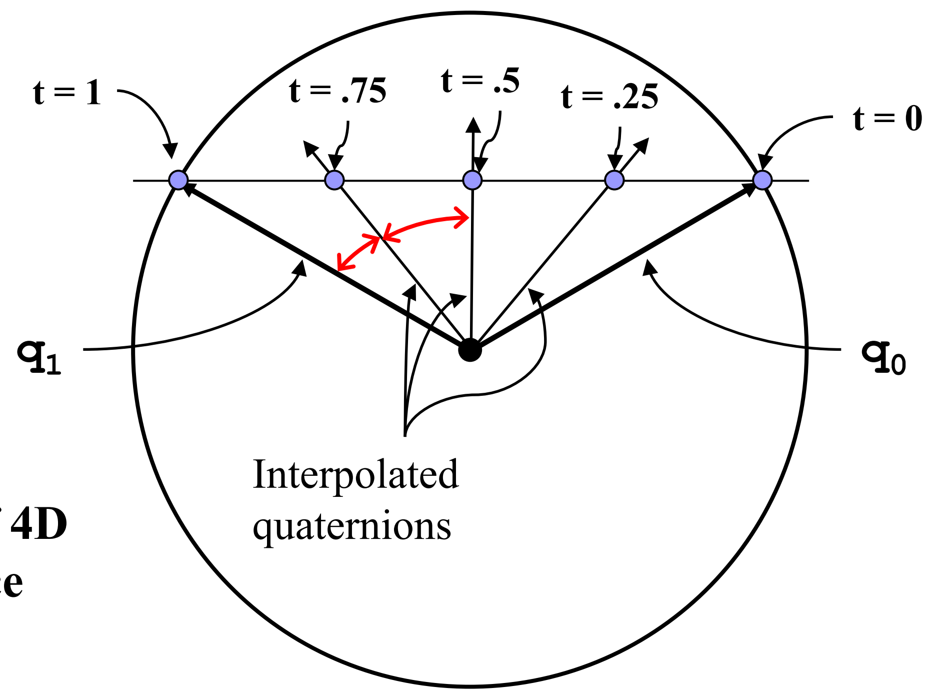
Linear Interpolation ("*lerp*")

- Given:
  - two quaternions $\mathbf{q_0}$ and $\mathbf{q_1}$
  - a parameter t $(0 \le t \le 1)$,

$$lerp(q_0, q_1, t) = (1 - t)q_0 + tq_1$$

Drawback of *lerp*

- Uniform parametric changes don't produce uniform angle changes



2D representation of 4D unit quaternion space

Spherical Linear Interpolation ("*slerp*")

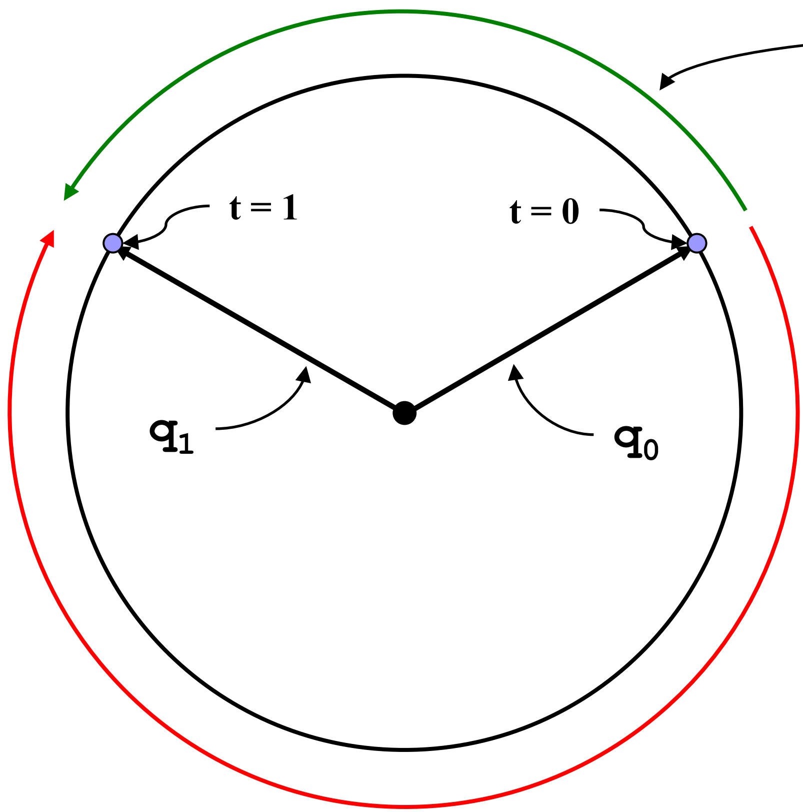
- Given:
  - two quaternions $\mathbf{q}_0$ and $\mathbf{q}_1$
  - a parameter t $(0 \le t \le 1)$
  - angle θ between $\mathbf{q}_0$ and $\mathbf{q}_1$ = *arccos*$(q_0 \bullet q_1)$

$$slerp(q_0, q_1, t) = \frac{q_0 \sin((1-t)\theta) \quad + \quad q_1 \sin(t\theta)}{\sin(\theta)} \qquad [1]$$

[1] Watt, Alan, 3D Computer Graphics, 3rd ed., p. 490-491

A Problem with *Slerp*

- There are always *two arcs around the sphere…*



Slerp will take this path when $(q_0 \bullet q_1) > 0$

Slerp will take this path when $(q_0 \bullet q_1) < 0$

Solution:

```
if ((q0•q1)<0)
{
  q1 = -q1;
}
```

Another Problem with *Slerp*

- Doesn't work well for very small angles
  - What happens in the limit – i.e. with the smallest possible angle ?

- Solution:

```
if ( abs(θ) < epsilon )
  use lerp()
else
  use slerp()
```

JOML Quaternion classes

- JOML includes classes for quaternions, both float and double types.
- They include functions for:
  - lerp
  - slerp
  - converting to and from Euler angles
  - applying quaternions as rotation transforms

  And many more!!