0.30 0.15

1.

x	1	4	6	8	10
$P(x=x)$	0.10		0.15		0.30

a) Given $P(x=4) = 2P(x=8)$ and $E(x) = 5$, find $P(x=4)$ and $P(x=8)$

$0.10 + 0.15 + 0.30 = 0.55$
$1 - 0.55 = 0.45$

$x + 2x = 0.45$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{0.45}{3} = 0.15$

$1(0.10) + 4(\quad) + 6(0.15) + 8(\quad) + 10(0.30) = 5$
0.10 0.9 3

0.10 $4x + 8x = 1$
$\frac{12x}{12} = \frac{1}{12}$

$1 + 3 = 5$
4

$$\boxed{\begin{array}{l} P(x=4) = 0.1\overline{66} \\ P(x=8) = 0.08\overline{3} \end{array}}$$

b) $V(x) = 1^2(0.10) + 4^2(0.16) + 6^2(0.15) + 8^2(0.08\overline{3}) + 10^2(0.3)$
$0.10 + 16(0.16) + 36(0.15) + 64(0.08\overline{3} + 100(0.3)$
$= 43.47 - 5^2$

$$\boxed{V(x) = 18.\overline{47}86}$$

2.

30	16	22	23
20	24	19	13

a) $\frac{30+20+16+24+22+19+23+13}{8} = 20.875$

Variance

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
30	$30 - 20.875 = 9.125$	83.265
20	$20 - 20.875 = -0.875$	0.765
16	$16 - 20.875 = -4.875$	23.765
24	$24 - 20.875 = 3.125$	9.765
22	$22 - 20.875 = 1.125$	1.265
19	$19 - 20.875 = -1.875$	3.515
23	$23 - 20.875 = 2.125$	4.515
13	$13 - 20.875 = -7.875$	62.015

$$\Sigma = \frac{188.875}{7}$$

$$\boxed{\begin{array}{l} \text{Mean} = 20.875 \\ \text{Variance} = 26.98 \end{array}}$$

b) $Y = 3X - 5$

$\bar{Y} = 3\bar{X} - 5 = 3(20.875) - 5 = 57.625$

$S_Y^2 = 3^2 S_X^2 \rightarrow 9(26.98) = 242.834$

$$\boxed{\begin{array}{l} \bar{Y} = 57.625 \\ S_Y^2 = 242.839 \end{array}}$$

3. V = CPU virus

W = Computer worm

a)

$P(V) = 0.1$ $P(W) = 0.05$ $P(V \text{ or } W) = 0.145$

a) Find $P(V \text{ and } W)$ $P(V \wedge W)$ 0.005 overlap

$P(V \wedge W) = 0.1 + 0.05 = 0.15$

$0.15 - 0.145 = 0.05$

$\boxed{P(V \wedge W) = 0.005}$

b) Find $P(V \wedge W^c)$

$W^c = 1 - 0.05 = 0.95$

$= 0.1 \times 0.95 = 0.095$

$\boxed{P(V \wedge W^c) = 0.095}$

c) V and W independent?

$\boxed{\text{No because } P(V \wedge W) \neq P(V \wedge W^c)}$

4.

a) Find Q_1 and Q_3

1 6
2
3
4
5 0
6 2 6 8
7 1 2 2 5 5 5 7 8 8 8 8 8
8 3 6 8 8 8
9 2 2 8

~~16~~ ~~50~~ ~~62~~ ~~66~~ ~~68~~
~~71~~ ~~72~~ ~~72~~ ~~75~~ ~~75~~ 10
~~75~~ ~~77~~ ~~78~~ ~~78~~ ~~78~~ 15
~~78~~ ~~78~~ ~~85~~ ~~86~~ ~~88~~
~~88~~ ~~88~~ ~~92~~ ~~92~~ ~~98~~

$Q_1 = 0.25(n+1)$
$0.25(25+1)$
$0.25(26) = 6.5$

$Q_3 = 0.75(n+1)$
$0.75(25+1)$
$0.75(26) = 19.5$

$\frac{71+72}{2} = 71.5$

$\frac{86+88}{2} = 87$

$$\boxed{\begin{array}{l} Q_1 = 71.5 \\ Q_3 = 87 \end{array}}$$

b) Determine and interpret IQR

$87 - 71.5 = 15.5$

The IQR is 15.5, The middle half (50%) of the data has a range of 15.5 points between 71.5 and 87.

5. $f(x) = \begin{cases} cx^2+x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$ $\frac{1}{x^2} \Rightarrow \frac{-2}{x^3}$

a) Find c

$cx^2 + x = 1 - x$ $\boxed{c = \frac{1-x}{x^2}}$ $x^{-2} \Rightarrow$

$-x$

$\frac{\cancel{cx^2}}{\cancel{x^2}} = \frac{1-x}{x^2}$

b) Find E(Y) $x^{-3} \Rightarrow \frac{x^{-2}}{-2}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$\int_0^1 x\left(\frac{1-x}{x^2} x^2 + x\right) dx$

$\int_0^1 \frac{x-x^2}{x^3}(x^3+x^2)$ $\frac{f}{g}$ $\frac{gf'-fg'}{g^2}$

$\frac{x^{-2}}{-2}(x^2-x^3)\; x^4+x^3 \Big|_0^1$ $\frac{1}{2}, 2$

~~$x - x^2(x^3+x^2)$~~

$\cancel{x^4} + x^3 - x^5 \cancel{-x^4}$ $\int_0^1 \frac{x^3-x^5}{x^3}$

$\frac{-2}{x^2}\left(\frac{x^4}{4} - \frac{x^6}{6}\right)\Big|_0^1$

$-\frac{2}{2}\left(\frac{1}{4} - \frac{1}{6}\right) - 0$ $\boxed{E(Y) = 0.0416}$

c) Five number summary

Min: 16
Q1: 71.5
Q2: 78
Q3: 87
Max: 98

0.5(25+1)
0.5(26) = 13

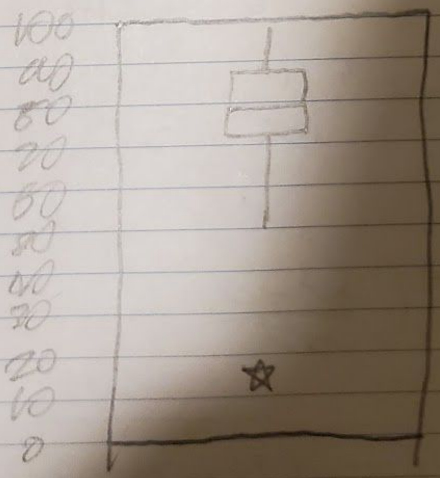
d) Identify outliers

15.5×1.5 = 23.25

71.5 - 23.25 = 48.25
87 + 23.25 = 110.25

We have an outlier below the data at 16 points.

e) boxplot



e) P(less than half ton)

$x < 0.5$

$\int_0^{0.5} 4\left(\frac{x^3}{x^3} - \frac{x^5}{x^5}\right) dx$ $\quad \frac{x^{-3-1}}{-3-1} = \frac{x^{-4}}{-4}$

$-\frac{2}{x^2}$ $\quad \frac{4}{x^4}\left(\frac{x^4}{4} - \frac{x^6}{6}\right)\Big|_0^{0.5}$

$-2 \cdot x^{-2}$

$4x$ $\quad 64\left(\frac{0.0625}{4} - \frac{0.015625}{6}\right)$

$x^{-3} \Rightarrow \frac{-3x^{-2-1}}{-2-1}$

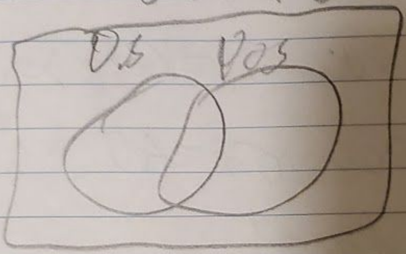
$\boxed{\text{P(less than half ton} = 0.8\overline{3}}$

6. $P(Ds) = 0.02$ Cost = $800

$P(Ds \mid Pos)$? $P(Pos \mid Ds) = 0.85$

$P(Pos \mid No\ Ds) = 0.03$

$0.85 + 0.03 = 0.88$



$Pos \mid Ds = 0.85$

$Pos \mid No\ Ds = 0.03$

$$P(Ds \mid Pos) = \frac{P(P \mid B)P(A)}{P(P \mid D)P(D) + P(P \mid D^c) + P(D^c)}$$

$$\frac{0.85 \times 0.02}{0.85 \times 0.02 + 0.03 + 0.13}$$

$$\boxed{P(Ds \mid Pos) = 0.066}$$

7. 20

EL 12 ink 5 dozen

$P(X=3)$
$\uparrow$
MK

$\boxed{\binom{12}{3}\binom{8}{2}}$

6. $2.60 reg $\mu_1 = 1500$ $\sigma_1 = 160$ X_1 2

$2.40 prem $\mu_2 = 300$ $\sigma_2 = 40$ X_2 =

$R_1 =$ independent

$R_2 =$

a) Find $E(R_1 + R_2)$ and $Var(R_1 + R_2)$

$= 2.60(1500) + 2.40(300)$

$= 4770$

$\sigma_R = \sqrt{2.60^2(160^2) + 2.40^2(40^2)}$ std = $\sqrt{var}$

$= \sqrt{232480} = 482^2$

$$\boxed{\begin{array}{l} E(R_1+R_2) = 4770 \\ Var(R_1+R_2) = 232480 \end{array}}$$

b) $E(R_1 - R_2)$ and $Var(R_1 - R_2)$

$E(R_1 - R_2) = 3900 - 870 = 3030$

$$\boxed{\begin{aligned} E(R_1 - R_2) &= 3030 \\ Var(R_1 - R_2) &= 232480 \end{aligned}}$$