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Assignment 2 Solution

(Note this assignment is useful in a number of computer science related problems and primarily useful when you do algorithm analysis)

NOTE in exam follow this advice:

(a) it is not necessary to fully unfold and calculate the final number value of the answer. For example if you have C(5, 3) then unfold it to 5! / (3! * (5-3)!)

That is enough. It is not necessary to solve the above and conclude the answer is 10.

(b) Remember that **AND** becomes multiplication (*) and **OR** becomes Addition (+).

1) A Student wants some books from the library. He has to choose from 4 different comic books, 5 different fiction books, and 4 different medical books:

a) In how many ways can a student choose one of each kind of book? **Explain your answer.**

Product rule principle: there are 4 ways to choose comic books and 5 ways to choose fiction books and 4 ways to choose medical books.

4 * 5 * 4 = 80 ways to choose one of each kind of book.

b) In how many ways can a student just choose one of the books?

Explain your answer.

Sum rule principle applies here: there are 13 total books to choose from.

4 + 5 + 4 = 13 ways to choose one of the books.

2) There are 7 red and 9 black marbles. Each marble is unique. There should be 10 of the marbles placed in a box. How many different combinations can there be in one box if:

a) There must be equal numbers of both colors of marbles.

Equal number: Implies: 5 red and 5 black marbles:

Combination of 5 red marbles (out of 7) **and** 5 black marbles (out of 9).

C(7,5) * C(9,5) = (7! / ((7-5)! * 5!) * (9! / ((9-5)! * 5!) = 21 * 126 = 2646

b) There must be at least 6 red marbles in a box.

10 marbles can be obtained as follows:

Combination of:

- 6 red marbles and 4 black marbles **OR**
- 7 red marbles and 4 black marbles.

C(7,6) * C(9,4) **+** C(7,7) * C(9,3)

= (7! / ((7-6)! * 6!) * (9! / ((9-4)! * 4!) + 1 * (9! / ((9-3)! * 4!) = 7 * 126 + 1 * 21 = 903

c) All the red marbles should be used.

Combination of all 7 red marbles (out of 7) **and** (10-7 = 3) black marbles (out of 9).

C(7,7) * C(9,3) = 1 * (9! / ((9-3)!3!) = 84

d) All the black marbles should be used.

Combination of 9 black marbles (out of 9) **and** (10-9 = 1) red marble (out of 7):

C(9,9) * C(7,1) = 1 * (7! / ((7-1)! * 1!) = 7

3) There are 5 buses between Sac State and 65th Street, and 4 train lines between 65th Street and Folsom. Find the number of ways that a man can travel by bus:

(a) from Sac State to Folsom by way of 65th Street; 5 * 4 = 20.
(SacState to 65^{th} St) **and** (65^{th} St to Folsom)

Product rule principle applies, there are 5 buses **and** then 4 trains.

(b) roundtrip from Sac State to Folsom by the way of 65th St;
Combinations from:
(SacState to Folsom) **and** (Folsom to SacState) =
((SacState to 65th st) **and** (65th St to Folsom)) **and**
((Folsom to 65th St) **and** (65th St to SacState)) =

(5 * 4) * (4 * 5) = 400.

Product rule principle, there are 5 buses then 4 trains then return trip 4 trains then 5 buses.

(c) roundtrip from Sac State to Folsom by way of 65th Street but without using a transportation mode more than once (Do not use the same bus or train again).

(d)
5 ***** 4 ***** 3 ***** 4 = 240.

Product rule principle, there are 5 buses then 4 trains then 3 trains (since cannot repeat a train 4-1), then 4 buses (since cannot repeat a bus 5-1).
Hence 5 buses **and** 4 trains **and** 3 trains **and** 4 buses.

4) a. How many distinguishable ways can the letters of the word MISSISSIPPI be arranged in order?

Permutations with indistinguishable objects:

1 M, 4 I, 4 S, 2 P and 11 total letters:

11! / (1! * 4! * 4! * 2!) = 34650

b. How many distinguishable orderings of the letters of MISSISSIPPI begin with M and end with I?

Remove M and one I from the calculation:

3 I, 4 S, 2 P and 11-2 = 9 total letters so:

9! / (3! * 4! * 2!) = 1260

5) A team is selected with 12 players including the captain.

a) How many different combinations of 3 can be chosen?

C(12,3) = 12! / ((12-3)! * 3!) = 220

b) How many of these combinations include the captain?

From 12 remove captain to select from since Captain is mandatorily in total hence total is 11. Out of 3 slots, one is already selected for Captain hence we have only two slots.

If we assume the captain is in all combinations, then we have 11 players and 2 slots so:

C(11,2) = 11!/(11-2)! * 2! = 55

c) How many do not include the captain?

C(12, 3) – C(11, 2) =

220 total combinations with or without captain minus 55 with captain so:

220 - 55 = 165 which do not include the captain.

6) A computer programming team of 5 should be formed from 9 employees. Two of the employees are managers. However, to avoid dispute problems, the 2 managers cannot both be chosen. Find the number of teams that can be formed?

Two managers cannot both be chosen implies:

We can have one manager **OR** we can have zero managers.

Number of Combinations with one manager **+** Number of combinations with no manager.

(i) If team includes a manager:

Out of 7 employee who are not managers choose 4 employees **and** out of 2 managers choose 1 manager:

C(2,1) ***** C(7,4) = (2!/(2-1)! * 1!) * (7!/(7-4)! * 4!) = 2 * 35 = 70

(ii) If team does not include a manager:

Out of 7 employees who are not managers, choose 5 employees **and** out of 2 managers choose 0 managers.

C(7, 5) * C(2, 0) = (7! / (5! * (7-5)!)) * (2! / (0! * (2-0)!) = 6*7/2 * 1 = 21

Summing solutions from (i) and (ii):

Number of Combinations with one manager + Number of combinations with no manager.

= 70 + 21 = 91.

7) A photo has to be captured with 8 different celebrities. There are also some chairs available. So, they have the option to either sit on the chair or stand while taking pictures. How many different photos are possible?

Assume there are enough chairs for all of the people to sit simultaneously.

Celebrities can permutate in any combination **and** either each one sits or stands.

= Number of permutations ***** Number of combinations of Sitting or standing per permutation.

(The celebrities can permutate: 8! permutations) *

(Each of 8 celebrities has 2 options to sit or stand, 2^8 = 256)

= 8! * 256 total combinations.

8) I have to create one computer password. A password is of length 5 characters, first two of which are distinct numbers, next character can be any upper-case letter, and the remaining 2 characters can be any digit or letter (upper- or lower-case). How many combinations are allowed? Note: Repetition of characters are not allowed.

(one digit is chosen from 10 = 10) * (one digit is chosen from remaining 9 = 9) * (number of upper case characters = 26) * (total digits + total upper case + total lower case – previous 3) * (total from previous 59 -1)

10 * 9 * 26 * 59 * 58 = 8007480

Product rule principle, and no character can be used twice. 10 digits * (10-1) digits * 26 letters * (62 - 3) characters * (62 - 4) characters.

9) You have 3 red pens and 7 blue pens. If you line up all the 10 pens one pen per day for 10 days, where the pens are indistinguishable by color, how many weeks (plus days) will it take to complete all combinations?

Permutations with indistinguishable objects:

10! / (3! * 7!) = 120 ways to order pens.

120 ways * 10 days per way = 1200 days.

Number of Weeks = 1200 / 7 days per week = 171 weeks and 3 days

10) There are 4 entry and exit points (A single point can be used either for entry or exit). In how many ways can a person enter and leave a space if he or she has to use different points?

4 * 3 = 12

Product rule principle, 4 choices to enter, and 3 choices to exit.

(since cannot repeat-use the same entry point to exit).

What if the person can use the same points?

4 * 4 = 16

Product rule principle, 4 choices to enter, and 4 choices to exit.