1. $\begin{bmatrix} 1 & 2 & 1 & 8 \\ 4 & 9 & 5 & 37 \\ 2 & 4 & 3 & 19 \end{bmatrix} \xrightarrow{R_3 - 2 \cdot R_1 \to R_3} \begin{bmatrix} 1 & 2 & 1 & 8 \\ 4 & 9 & 5 & 37 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ $4 \times 8 = 3$

$\xrightarrow{R_2 - 4R_1 \to R_2} \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_1 - 2 \cdot R_2 \to R_1} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

$\xrightarrow{R_1 + R_3 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 - R_3 \to R_2} \boxed{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}}$

2. $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$

$\boxed{x_1 \underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{a_1} \quad x_2 \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{a_2} \quad x_3 \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{a_3}}$

cons [illegible] mat
for linear sys
[illegible] solution

3. $\begin{bmatrix}1\\2\\1\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}4\\5\\10\end{bmatrix}$ Span $\mathbb{R}^3$?

$$\begin{bmatrix}1 & 2 & 4 & b_1\\ 2 & 3 & 5 & b_2\\ 1 & 4 & 10 & b_3\end{bmatrix} \xrightarrow{R_2 - 2\cdot R_1 \to R_2} \begin{bmatrix}1 & 2 & 4 & b_1\\ 0 & -1 & -3 & b_2-2b_1\\ 1 & 4 & 10 & b_3\end{bmatrix}$$

$$\xrightarrow{R_3 - R_1 \to R_3} \begin{bmatrix}1 & 2 & 4 & b_1\\ 0 & -1 & -3 & b_2-2b_1\\ 0 & 2 & 6 & b_3-b_1\end{bmatrix}$$

$$\xrightarrow{R_3 + 2\cdot R_1} \begin{bmatrix}1 & 2 & 4 & b_1\\ 0 & -1 & -3 & b_2-2b_1\\ 0 & 0 & 0 & b_3-b_1+2(b_2-2b_1)\end{bmatrix}$$

$b_3 - b_1 + 2(b_2 - 2b_1)$

$\downarrow$

$b_3 - b_1 + 2b_2 - 4b_1$

$\downarrow$

$b_3 - 5b_1 + 2b_2 = 0$

No. The vectors do not span R_3 as it has a row of the form:
$[0 \ldots 0 \; b]$
where the system will be inconsistent if
$b_3 - 5b_1 + 2b_2 \neq 0$!

4.

$$\begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & -1 & 1 \\ -3 & -4 & 2 & 2 \\ 5 & 2 & 3 & -3 \end{bmatrix} \text{ RREF}$$

$\downarrow R_2 + 3 \cdot R_1 \to R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 5 & 2 & 3 & -3 \end{bmatrix} \xrightarrow{R_3 - 5 \cdot R_1 \to R_3} \begin{array}{cccc} -10 & 5 & -5 & \\ \end{array} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 0 & -8 & 8 & -8 \end{bmatrix}$$

$\downarrow \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & -1/2 & 5/2 \\ 0 & -8 & 8 & -8 \end{bmatrix} \xrightarrow{R_1 - 2 \cdot R_2 \to R_1} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & -1/2 & 5/2 \\ 0 & -8 & 8 & -8 \end{bmatrix}$$

$-\frac{1}{2} \cdot 8 = -4$

$\downarrow R_3 + 8 \cdot R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & -1/2 & 5/2 \\ 0 & 0 & 4 & 12 \end{bmatrix} \xrightarrow{\frac{1}{4} \cdot R_3} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & -1/2 & 5/2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$\frac{5}{2} \cdot 8 = 20$

4   6

2.5

10   8

20 . 0

$20 - 8 \to 12$

$\downarrow R_2 + \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$5/2 + 3/2 \to 8/2 = 4$

$$\boxed{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix}}$$

5.

RREF

(a)

$$\left[\begin{array}{ccc|c} 2 & 2 & 14 & -10 \\ 2 & 3 & 18 & -12 \\ 1 & 2 & 11 & -7 \end{array}\right] \xrightarrow{\frac{1}{2}\cdot R_1} \begin{bmatrix} 1 & 1 & 7 & -5 \\ 2 & 3 & 18 & -12 \\ 1 & 2 & 11 & -7 \end{bmatrix}$$

$$\downarrow R_3 - R_1 \to R_3$$

$$\begin{bmatrix} 1 & 1 & 7 & -5 \\ 0 & 1 & 4 & -2 \\ 0 & 1 & 4 & -2 \end{bmatrix} \xleftarrow{R_2 - 2\cdot R_1 \to R_2} \begin{bmatrix} 1 & 1 & 7 & -5 \\ 2 & 3 & 18 & -12 \\ 0 & 1 & 4 & -2 \end{bmatrix}$$

$-7+5$

$\downarrow$

-2

$$\downarrow R_3 - R_2 \to R_3$$

$-12+10=-2$

$$\begin{bmatrix} 1 & 1 & 7 & -5 \\ 0 & 1 & 4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - R_2 \to R_1} \begin{bmatrix} 1 & 0 & 3 & -3 \\ 0 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$-5+2=-3$

$x_1 + 3x_3 = -3$
$x_2 + 4x_3 = 2$
x_3 is free

$x_1 = -3 - 3x_3$
$x_2 = 2 - 4x_3$
x_3 is free

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}}$$

5.

(b)

$$\left[\begin{array}{ccc|c} 2 & 2 & 14 & 0 \\ 2 & 3 & 16 & 0 \\ 1 & 2 & 11 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cccc} 1 & 1 & 7 & 0 \\ 2 & 3 & 16 & 0 \\ 1 & 2 & 11 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_1 \to R_3} \left[\begin{array}{cccc} 1 & 1 & 7 & 0 \\ 2 & 3 & 16 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right] \xrightarrow{R_2 - 2\cdot R_1 \to R_2} \left[\begin{array}{cccc} 1 & 1 & 7 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2 \to R_3} \left[\begin{array}{cccc} 1 & 1 & 7 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc} 1 & 0 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 3x_3 = 0$
$x_2 + 4x_3 = 0$
x_3 is free

$x_1 = -3x_3$
$x_2 = -4x_3$
~~x_3 is free~~

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}}$$

6. $\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$

$v_1 \quad v_2 \quad v_3$

$$\left[\begin{array}{ccc|c} 3 & -6 & 9 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{array}\right]$$

$-12 \quad 16$

$$\xrightarrow{\frac{1}{3}\cdot R_1} \begin{bmatrix} 1 & -2 & 3 & 0 \\ -6 & 4 & h & 0 \\ 1 & -3 & 3 & 0 \end{bmatrix} \xrightarrow{R_2+6\cdot R_1 \to R_2} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -8 & h+18 & 0 \\ 1 & -3 & 3 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3-R_1 \to R_3} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -8 & h+18 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -8 & h+18 & 0 \end{bmatrix}$$

$$\xrightarrow{-1\cdot R_2} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -8 & h+18 & 0 \end{bmatrix} \xrightarrow{R_3+8\cdot R_2} \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & h+18 & 0 \end{bmatrix}$$

$h+18=0$
$h=-18$

Vectors are linearly independent when $h \neq -18$! This is because we get a trivial sol when $h \neq -18$ and a free variable when $h+18=0$.

7. $\begin{bmatrix} 3 & 1 & 3 \\ 7 & 4 & 2 \\ 9 & 5 & 3 \end{bmatrix}$ L.I.?

REF

$12/3 - 7/3 = 5/3$

$7/3$

$$\xrightarrow{\frac{1}{3} \cdot R_1} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 7 & 4 & 2 & 0 \\ 9 & 5 & 3 & 0 \end{array}\right] \xrightarrow{R_2 - 7R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 5/3 & -5 & 0 \\ 9 & 5 & 3 & 0 \end{array}\right]$$

$9/3 = 3$

$$\xrightarrow{R_3 - 9 \cdot R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 5/3 & -5 & 0 \\ 0 & 2 & -6 & 0 \end{array}\right] \xrightarrow{\frac{1}{2} \cdot R_3} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 5/3 & -5 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{3 \cdot R_2} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 5 & -15 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow{\frac{1}{5} \cdot R_2} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1/3 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Free vars!

So, has nontrivial sol.

The columns of A are linearly dependent. There are free variables, which means the homogeneous system has non-trivial solutions. As such, it is linearly dependent.