$\underline{\text{Exam 1}}$

<u>Instructions:</u> Please answer the following legibly, logically, and **show all work** on a separate sheet of paper. No credit will be given for unjustified or unclear work. Please clearly label every problem and work them in order. When you are finished, please scan your work (or take pictures) and submit via the Exams link on Canvas.

1. Find the reduced echelon form of the following matrix: $\begin{bmatrix} 1 & 2 & 1 & 8 \\ 4 & 9 & 5 & 37 \\ 2 & 4 & 3 & 19 \end{bmatrix}$

2. Find a linear combination of 3 vectors that equals the following vector: $\begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$

   Please give the vectors and their coefficients.

3. Do the following vectors span $\mathbb{R}^3$? $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 10 \end{bmatrix}$ Justify your answer.

4. Find the vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$ given:

   $$A = \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$

5. Consider the following nonhomogeneous system: $\left[\begin{array}{ccc|c} 2 & 2 & 14 & -10 \\ 2 & 3 & 18 & -12 \\ 1 & 2 & 11 & -7 \end{array} \right]$

   (a) Find the solution for the system. Give the solution in parametric vector form.

   (b) What is the solution (in parametric vector form) of the homogeneous version of this system?

6. Find a value for h for which the following vectors are linearly *independent*. Justify your answer.

   $$\begin{bmatrix} 3 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}$$

7. Are the columns of A linearly independent? Why or why not?

   $$A = \begin{bmatrix} 3 & 1 & 3 \\ 7 & 4 & 2 \\ 9 & 5 & 3 \end{bmatrix}$$