Quiz 6

$$A = \begin{array}{ccccc} \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 1 & 2 & 0 & 2 & 8 \\ 2 & 4 & -3 & 10 & 9 \\ 3 & 6 & 0 & 6 & 9 \end{bmatrix} \\ \vec{a}_1 \quad\quad \vec{a}_3 \quad\quad \vec{a}_5 \end{array} \leftarrow \text{pivots in column } 1, 3, 5$$

$-R_1 + R_2 \to R_2$
$-2R_1 + R_3 \to R_3$
$-3R_1 + R_4 \to R_4$

$$\to \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 0 & 0 & -3 & 6 & 0 \\ 0 & 0 & -9 & 18 & -7 \\ 0 & 0 & -9 & 18 & -15 \end{bmatrix} \xrightarrow[-3R_2 + R_4 \to R_4]{-3R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 0 & 0 & -3 & 6 & 0 \\ 0 & 0 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 & -15 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$-\frac{1}{3}R_2 \to R$

$-\frac{1}{7}R_3 \to R_3$
$\hookrightarrow -15R_3 + R_4 \to R_4$

① Basis for Col $A \to \mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 8 \\ 9 \\ 9 \end{bmatrix} \right\}$, Non-zero vector in Col $A \to \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$.

② There are 4 entries in each vector (4 rows), thus Col$A$ is a subspace of $\mathbb{R}^4$.

③ Matrix Reduction Cont'd:

augment (looking for Nul A)

$$\begin{bmatrix} 1 & 2 & 3 & -4 & 8 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[-8R_3 + R_1 \to R_1]{-3R_2 + R_1 \to R_1} \left[\begin{array}{ccccc|c} 1 & 2 & 0 & -10 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 + 2x_2 - 10x_4 = 0 \to x_1 = -2x_2 + 10x_4$
$x_3 + 2x_4 = 0 \to x_3 = -2x_4$
$x_5 = 0 \qquad x_5 = 0$

Considering $A\vec{x} = \vec{0} \to \vec{x} = \begin{bmatrix} -2x_2 + 10x_4 \\ x_2 \\ -2x_4 \\ x_4 \\ 0 \end{bmatrix} = x_2 \underbrace{\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\vec{u}} + x_4 \underbrace{\begin{bmatrix} 10 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}}_{\vec{v}}$

③ Basis for Nul $A \Rightarrow \mathcal{B}_2 = \{\vec{u}, \vec{v}\}$. Non-zero-vector in Nul$A$, let $x_2 = 1, x_4 = 0 \to \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

④ There are 5 entries in each vector (5 columns), thus Nul$A$ is a subspace of $\mathbb{R}^5$.