

Math 30 Exam 3
Friday, April 17, 2020

Professor Michael VanValkenburgh

Name: _____

Instructions: (Updated for Online Teaching.) Show all of your work, and clearly indicate your answers. You may use the class textbook, your lecture notes, and your homework, **but you may *not* get help from any other source.** This is for your long-term benefit.

Problem	Your score	Possible Points
1		10
2		10
3		10
4		10
5		10
6		10
Total		60

- 1a. We say the function g is the “natural logverse” of the function f if

$$g(f(x)) = \ln x \quad \text{for all } 0 < x < \pi.$$

Suppose $f(\pi/3) = \sqrt{3}$ and $f'(\pi/3) = 4$. Find the value of $g'(\sqrt{3})$. **Show all work.**

- b. Check that $g(x) = \ln(\tan^{-1} x)$ and $f(x) = \tan x$ satisfy the above conditions, and use these f and g to check your answer to part (a).

2. Evaluate the limit

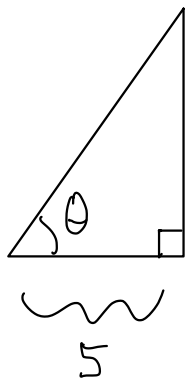
$$\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}.$$

- 3a. Use linear approximation to estimate $\sqrt{150}$ using “base point” $a = 144$.
- b. Now use linear approximation to estimate $\sqrt{150}$ using “base point” $a = 169$.
- c. Make a sketch illustrating your answers to (a) and (b).



4. Suppose you want to make a cylindrical pen for your cat to play in (with open top) and you want the volume to be 100 cubic feet. Suppose the material for the side costs \$3 per square foot, and the material for the bottom costs \$7 per square foot. What are the dimensions of the pen that *minimize* the cost of building it? (The bottom is a disc.)

5. For the function $f(x) = 3x^4 - 20x^3 + 36x^2$, determine
- intervals where f is increasing or decreasing,
 - local minima and maxima of f ,
 - intervals where f is concave up and concave down, and
 - the inflection points of f .
 - Also, sketch the graph of f .



6. Suppose we have a right triangle with base of constant length 5 inches. [See picture.] Suppose the angle θ increases at a constant rate of 3 radians per second. Find the rate of change of the area when $\theta = \frac{\pi}{3}$.