Midterm #1 Solutions

1. a) $P(X=8) = \frac{1}{16}$, $P(X=4) = \frac{1}{8}$

or $P(X=8) = .15$, $P(X=4) = .30$

$E(x) = 5$ or $E(x) = 5.7$

Some got $E(x) = 7$ but $P(x=4)$, $P(x=8)$ were reversed

b) $E(x^2) = 1(.1) + 16(\frac{1}{8}) + 36(.15) + 64(\frac{1}{16}) + 100(.3)$

$= 41.5$, or other

$Var(x) = E(x^2) - (E(x))^2$

Answers vary

2. a)

X_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$
30	9.125	83.2656
16	-4.875	23.7656
22	1.125	1.2656
23	2.125	4.5156
20	-.875	.7656
24	3.125	9.7656
19	-1.875	3.5156
13	-7.875	62.0156
167		188.8748

$\bar{X} = \frac{167}{8} = 20.875$

$S_x^2 = \frac{188.8748}{7} = 26.982$

2. b) $\bar{Y} = 3\bar{x} - 5 = 57.625$

$S_Y^2 = 3^2 S_x^2 = 9(26.982) = 242.838$

3. a) $P(V \cap W) = P(V) + P(W) - P(V \cup W)$

$= .1 + .05 - .145 = .005$

b) $P(V) - P(V \cap W) = .1 - .005 = .995$

c) Yes, $P(V \cap W) = .005$

$P(V)P(W) = .1(.05) = .005$

4. a) $.25(25+1) = 6.5$, $Q1 = \dfrac{71+72}{2} = 71.5$

$.75(25+1) = 19.5$, $Q3 = \dfrac{86+88}{2} = 87$

b) $IQR = 87 - 71.5 = 15.5$ Middle 50% of data

c) $.5(25+1) = 13$, $Q2 = 78$

$Min = 60$
$Q1 = 71.5$
$Q2 = 78$
$Q3 = 87$
$Max = 98$

5. a) $\int_0^1 (cy^2 + y)\,dy = \frac{cy^3}{3} + \frac{y^2}{2}\Big|_0^1 = \frac{c}{3} + \frac{1}{2} = 1, \ c = \frac{3}{2}$

b) $F(y) = \int_0^y \left(\frac{3}{2}x^2 + x\right)dx = \frac{x^3}{2} + \frac{x^2}{2}\Big|_0^y = \frac{y^3 + y^2}{2}, \ 0 \leq y \leq 1$
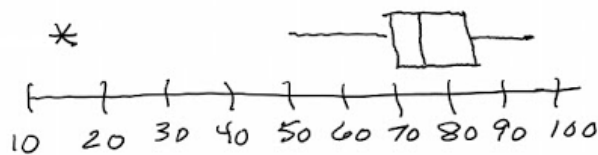
$$F(y) = \begin{cases} 0 & y < 0 \\ \dfrac{y^3 + y^2}{2} & 0 \leq y \leq 1 \\ 1 & y > 1 \end{cases}$$

c) $P(Y \leq .5) = F(.5) = \frac{.5^3 + .5^2}{2} = .1875$

d) $87 + 1.5(15.5) = 118.25$, None

$71.5 - 1.5(15.5) = 48.25$, 16 is outlier

e)

6. $P(+|D) = .85$ $P(+|D^c) = .03$
   $P(D) = .002$
   $P(D^c) = .998$

$$P(D|+) = \frac{P(D \cap +)}{P(+)} = \frac{P(+|D)P(D)}{P(+)}$$

$$= \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|D^c)P(D^c)}$$

$$= \frac{.85(.002)}{.85(.002) + .03(.998)} = .054$$

7. $$\frac{\binom{12}{3}\binom{8}{2}}{\binom{20}{5}}$$

8. $R_1 = 2.6X_1$
   $R_2 = 2.9X_2$
   $E(R_1 + R_2) = E(R_1) + E(R_2) = 2.6(1500) + 2.9(300) = 4770$
   $V(R_1 + R_2) = V(R_1) + V(R_2) = 2.6^2(180^2) + 2.9^2(40^2) = 232480$
   $E(R_1 - R_2) = E(R_1) - E(R_2) = 2.6(1500) - 2.9(300) = 3030$
   $V(R_1 - R_2) = V(R_1) + V(R_2) = 232480$