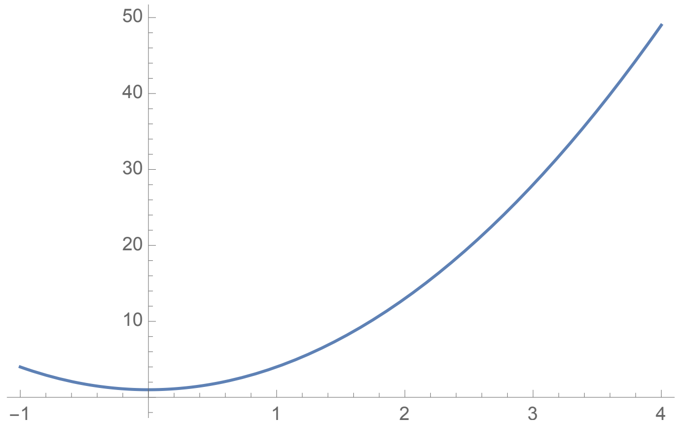
MATH 30, SPRING 2020: PRACTICE FINAL

1. Consider the following graph of $f(x) = 3x^2 + 1$.

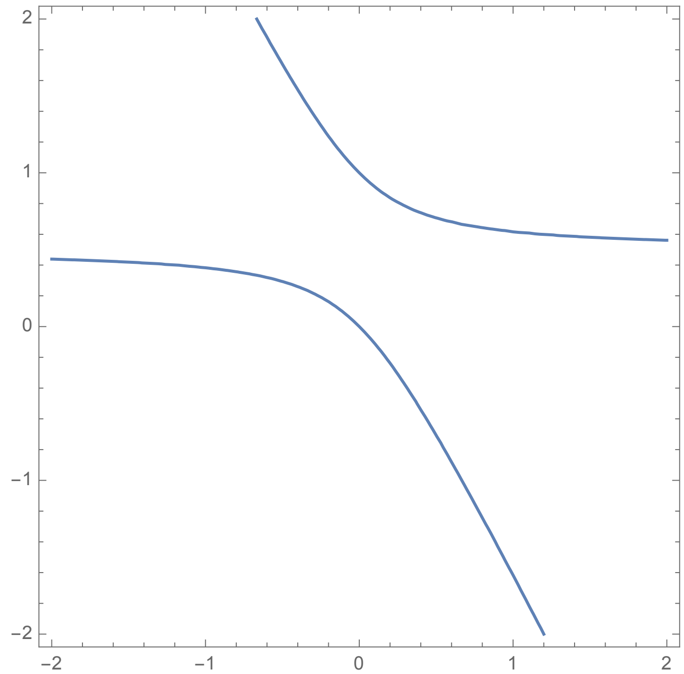


   (a) Find the slope of the line connecting the point $(1, f(1))$ to the point $(3, f(3))$.
   (b) Given any $h \neq 1$, find the slope of the line connecting the point $(1, f(1))$ to the point $(h, f(h))$.
   (c) In part (b), what happens as h approaches 1?
   (d) Sketch the lines in parts (a), (b), and (c).

2. Find the derivative of the function $f(x) = \sqrt{2e^{3x} + \sin x}$.

3. Consider the function $f(x) = -x^3 + 3x^2 - 1$.
   (a) Find the intervals on which f is increasing or decreasing.
   (b) Find those x for which $f(x)$ is a local maximum or local minimum.
   (c) Find the intervals of concavity and the inflection points.
   (d) Sketch the graph of f.

4. Say hello to Gerbi, whose equation is $2xy + y^2 = x + y$.
Find the equation of the tangent line at the point $(x, y) = (0, 1)$ and plot it.



Gotta Diff 'Em All!

5. Find the point on the parabola $y = 2x^2$ closest to the point $(9, 0)$. Illustrate your answer with a sketch.
[Hint: The distance from (a, b) to (c, d) is $\sqrt{(c-a)^2 + (d-b)^2}$, and this is smallest exactly when $(c-a)^2 + (d-b)^2$ is smallest.]

6a. Find an expression of the form $\sum_{j=1}^{n} f(x_j)\,\Delta x$ that represents the approximate area under the curve $y = \sin(x)$ between $x = 0$ and $x = \pi$, using 4 rectangles of equal width and using "right endpoints." Make a sketch illustrating your answer.

b. What do you get when you use n rectangles of equal width, and let $n \to \infty$?

7. Evaluate the definite integral $\int_1^2 \left(x^2 + \frac{1}{x} + \frac{1}{x^2}\right) dx$.
Leave your answer as a sum of simple numbers.

8a. Use the substitution $u = 2x^2$ to evaluate the indefinite integral

$$\int x\cos(2x^2)\,dx.$$

b. Check your answer to part (a) by differentiating.

9a. Find the derivative of $f(x) = x\ln x$.

b. Use your answer to part (a) to evaluate the indefinite integral $\int \ln x\,dx$. Check your answer by differentiating.