Math 30, Thursday 3/26/2020

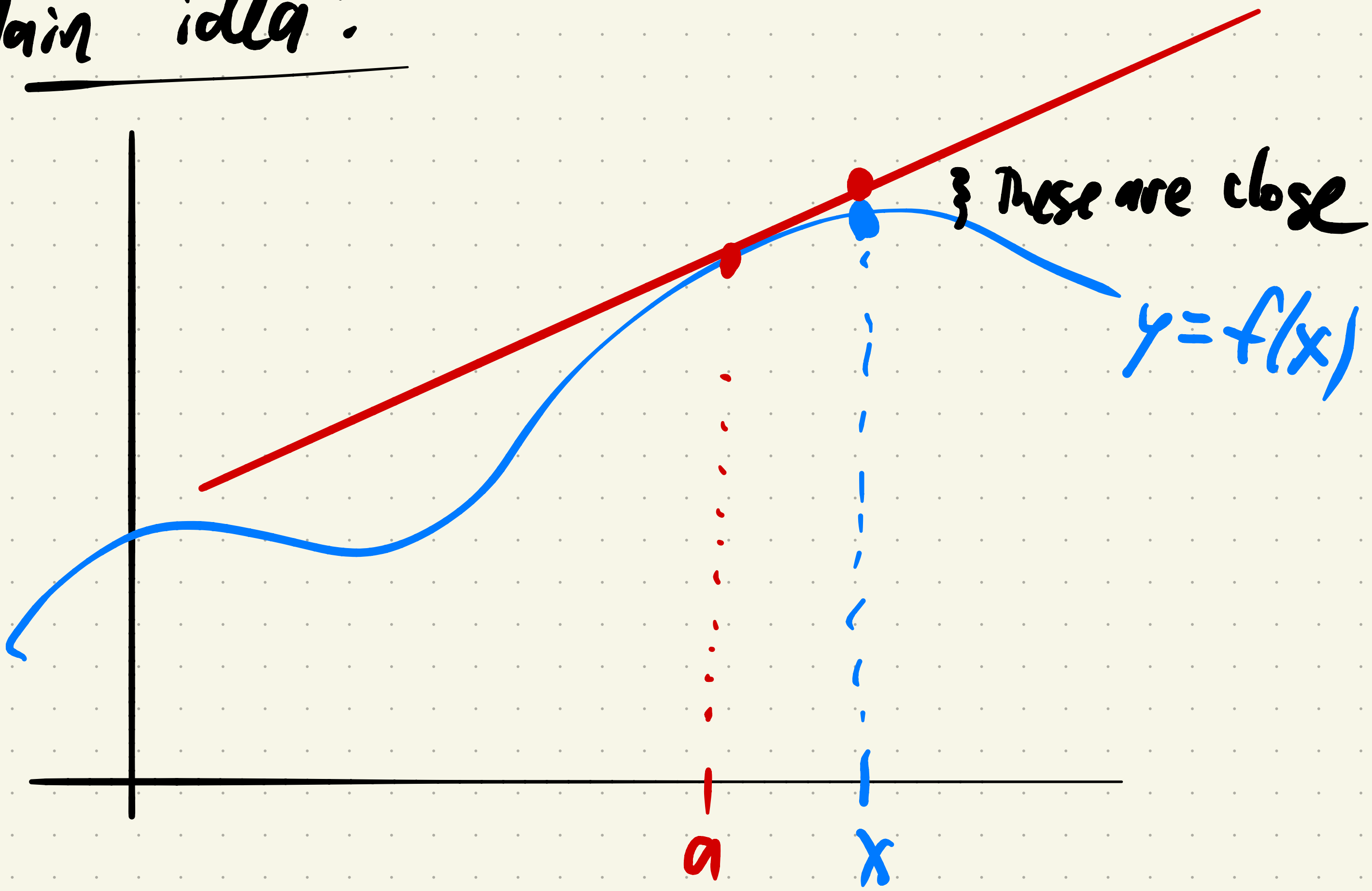
1pm class

Rolle's Theorem and

the Mean Value Theorem

Next quiz: Monday after Spring Break
on linear approx:

Main idea:



Q: what is $f(x)$?

Can approx like this: $f(a)$ might be easier to find, where a is near x, use tangent line.

Q's about closed interval method?

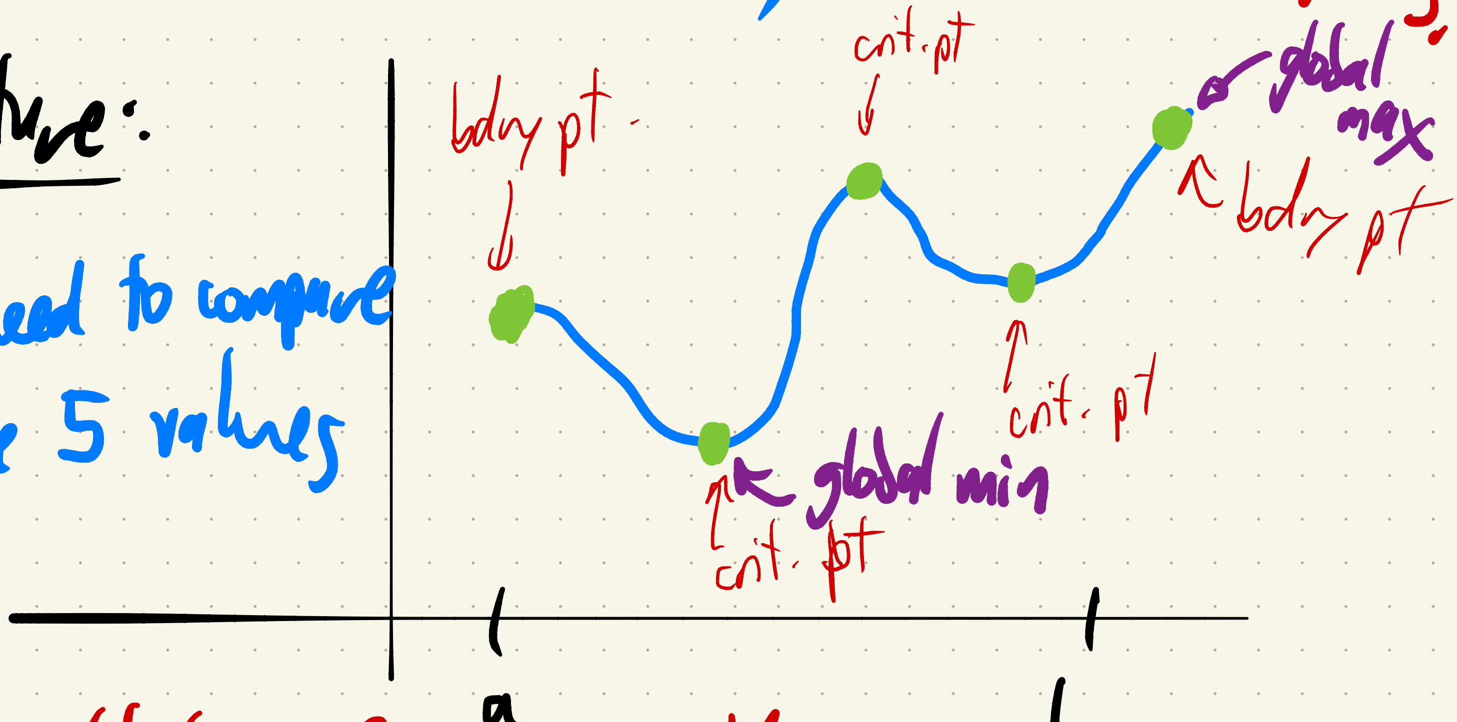
Idea: to find global max & min of f on a closed interval $[a,b]$,

The candidates are: The critical pts (where $f'=0$ or f' doesn't exist) and The bdry points.

(The only possible points where the global min & max can be)

Picture:

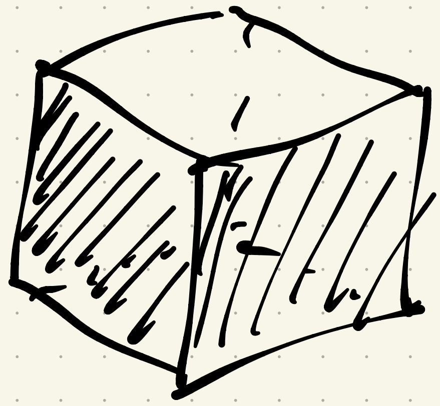
only need to compare those 5 values



The global min & max could only be at one of the green points

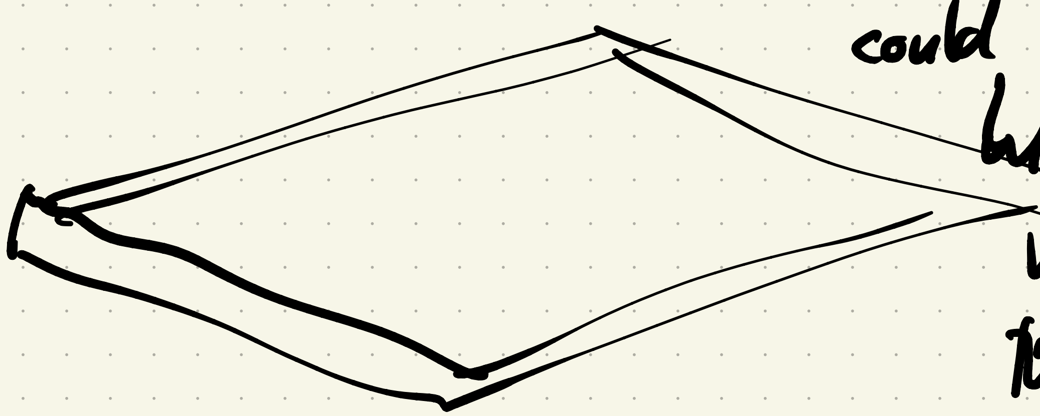
#5 on yesterday's lecture notes:

cardboard box w/ open top

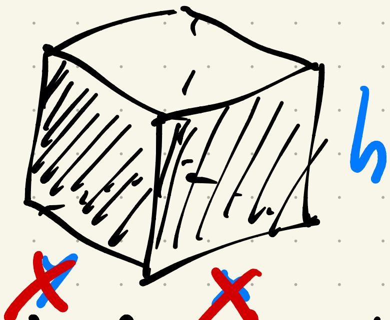


1200 cm^2 of cardboard is available.
(assume you don't need flaps —
just tape sides).

Q: Find largest possible vol. of box.



could make it short,
but then vol.
would be
too small



1200 cm^2 of cardboard is available.

oops – problem says bottom needs to be square

area of box?

use all cardboard

$$A = \underbrace{x^2}_{\text{area of base}} + \underbrace{4xh}_{\text{area of four sides}} \overset{\text{must be}}{=} 1200$$

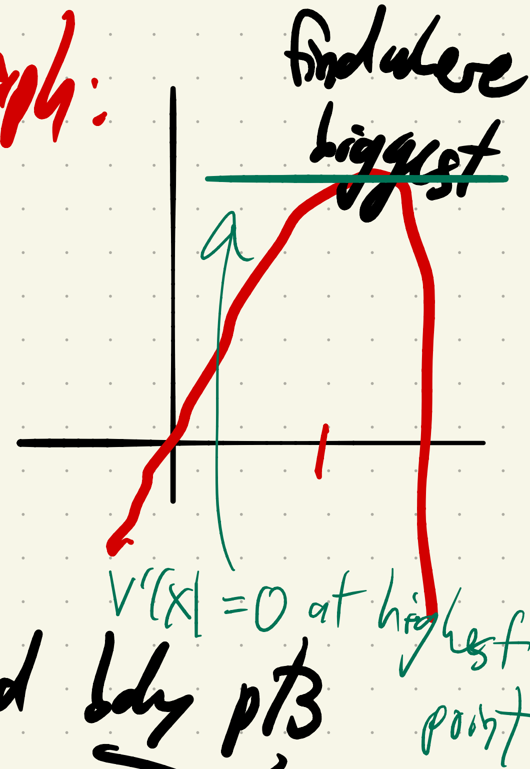
So $h = \dfrac{1200 - x^2}{4x}$

vol of box

$$V = \underbrace{x^2 h}_{\text{base} \times \text{height}} = x^2\left(\frac{1200-x^2}{4x}\right) = \frac{1}{4}\left(1200x - x^3\right)$$

Find the x that makes this biggest.

$V(x) = \frac{1}{4}(1200x - x^3)$ → graph:



vol. as a f= of x.

maximize this.

candidates: critical pts and bdy pts

where $V'(x) = 0$

can disregard b/c they give volume 0 (check)

$$V'(x) = \frac{1}{4}(1200 - 3x^2)$$

where is it zero?

$$3x^2 = 1200$$

$$x^2 = 400$$

this must be it...

$$x = \pm 20$$

$x = 20$ (positive lengths)

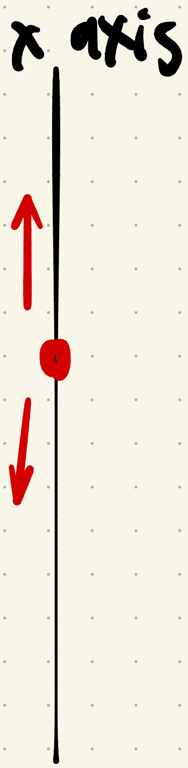
Now can figure out h

and figure out V (biggest possible volume)

Next topic: Rolle's Theorem.

Thought Experiment #1:

Imagine a particle is moving up & down in a straight line

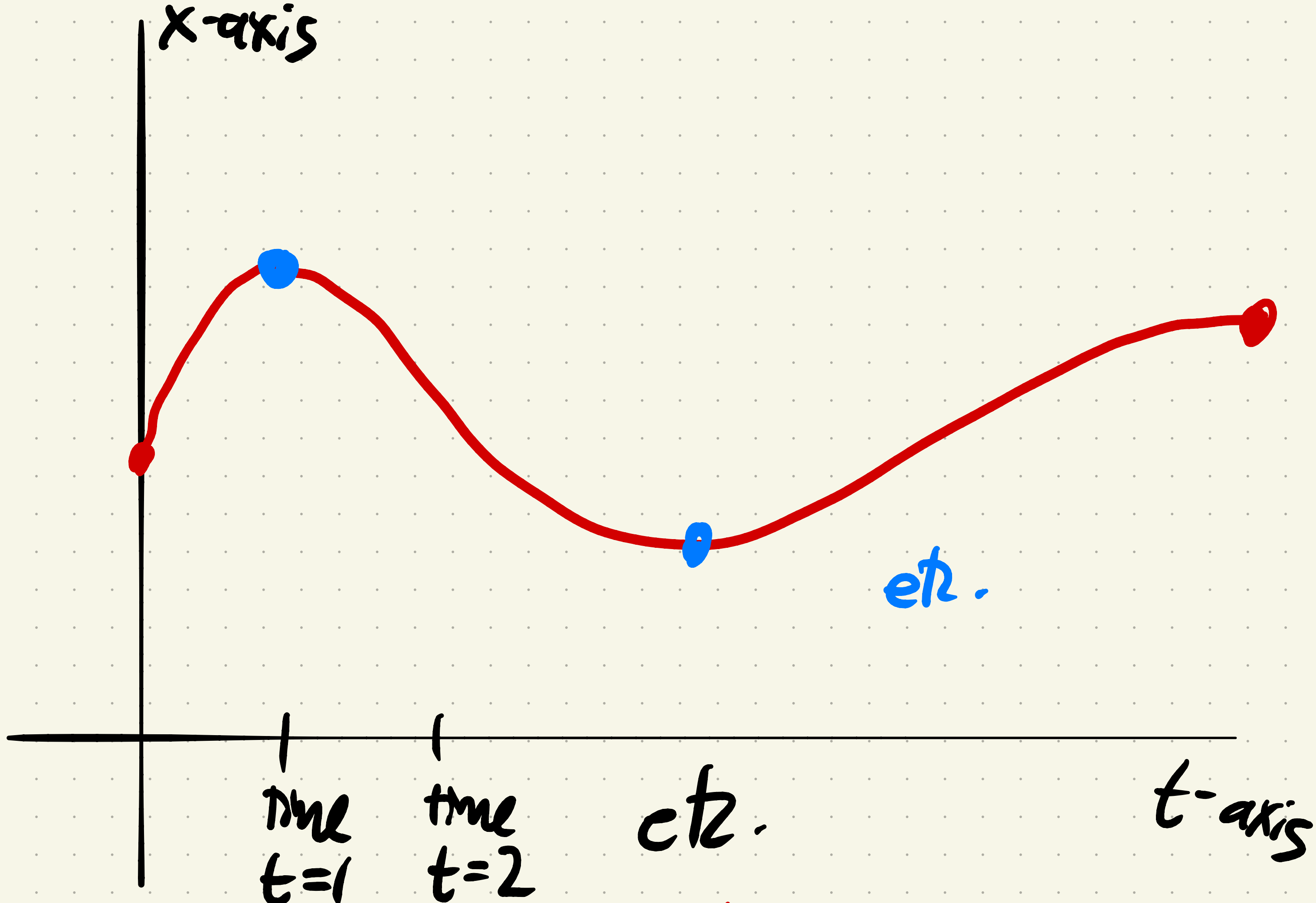


let $x(t)$ be the position on the line at time t?

Q: is its speed ever zero?

yes - whenever it changes direction.

Now draw time as the horizontal axis:



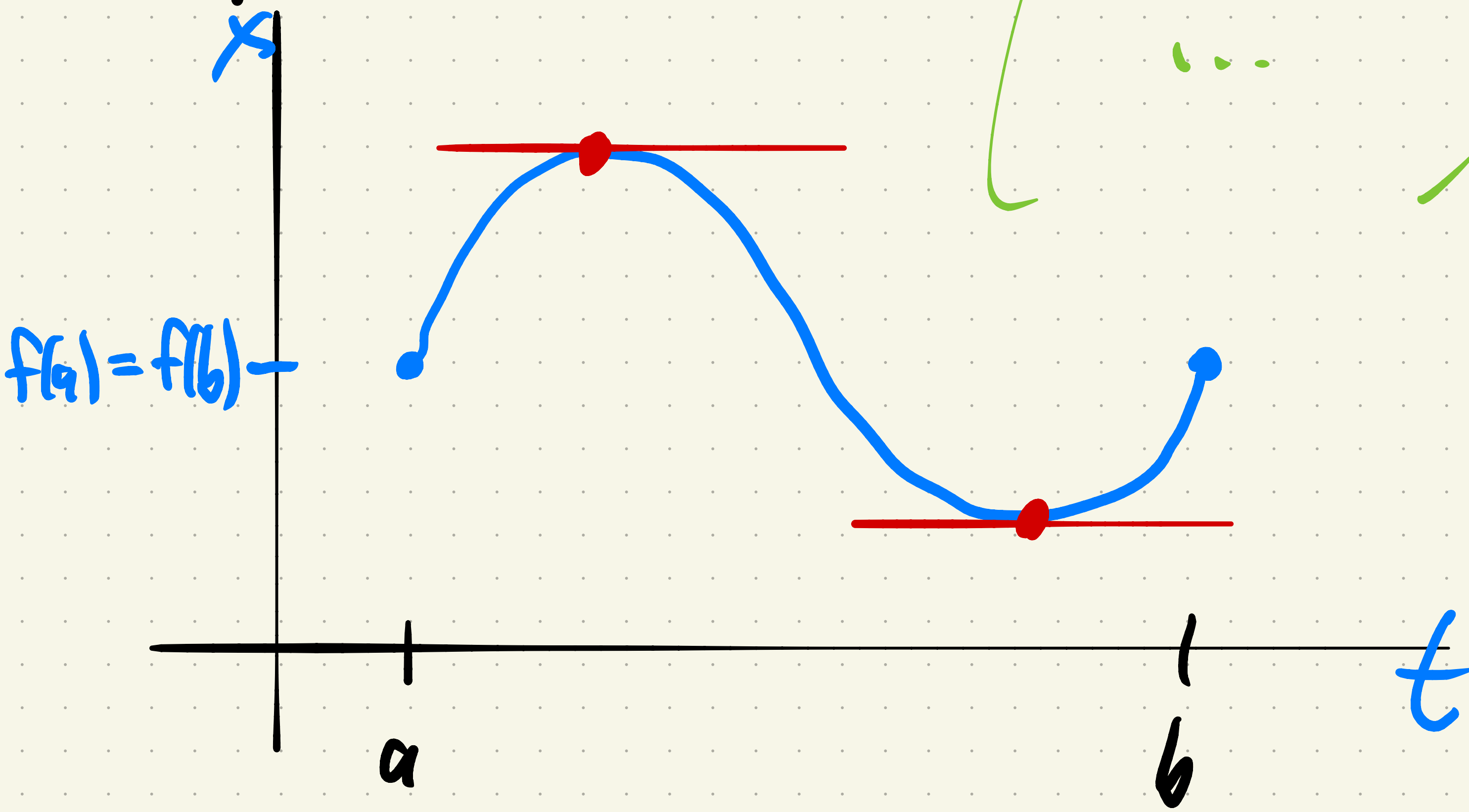
point traces out a curve

see where the speed is 0?

Rolle's Thm (see typed notes for precise statement).

Suppose $f(a) = f(b)$

Zoom crashed ...



Like for the particle in prev. example:
starts & ends on same pt on straight line
then at some point its speed is zero.

$$\frac{dx}{dt} = 0$$

Rolle's theorem says:
$f'(c) = 0$ somewhere.

It's useful!

Ex. Show that $1 + 2x + x^3 + 4x^5 = 0$

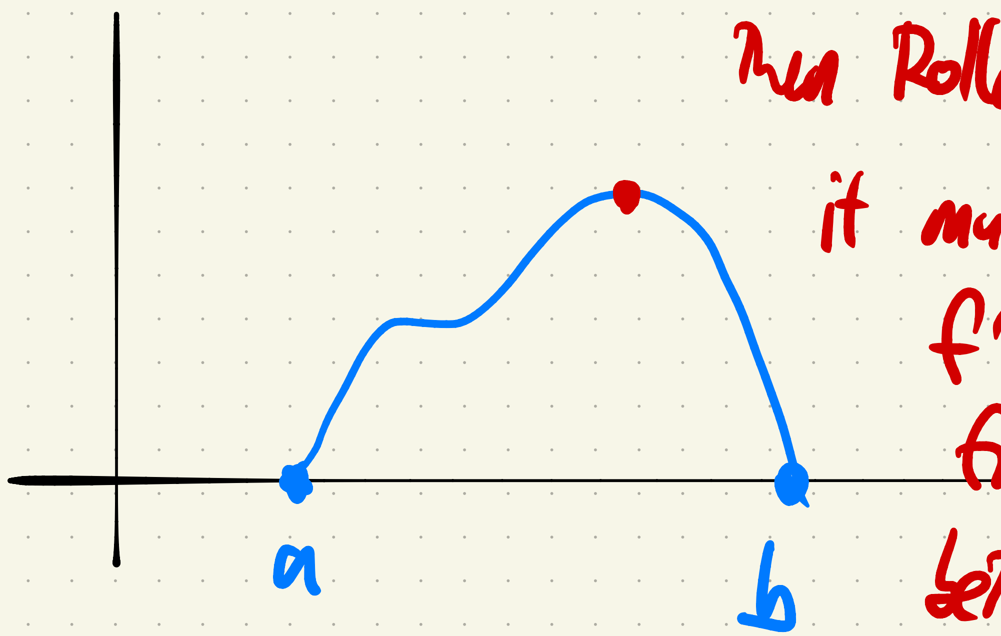
has exactly one solution

First, show it can't have two solutions.

let $f(x) = 1 + 2x + x^3 + 4x^5$.

Imagine $f(a) = f(b) = 0$

(two sol'ns a and b)



Then Rolle's Thm. says it must have $f'(c) = 0$ for some c between a and b.

$f(x) = 1 + 2x + x^3 + 4x^5$

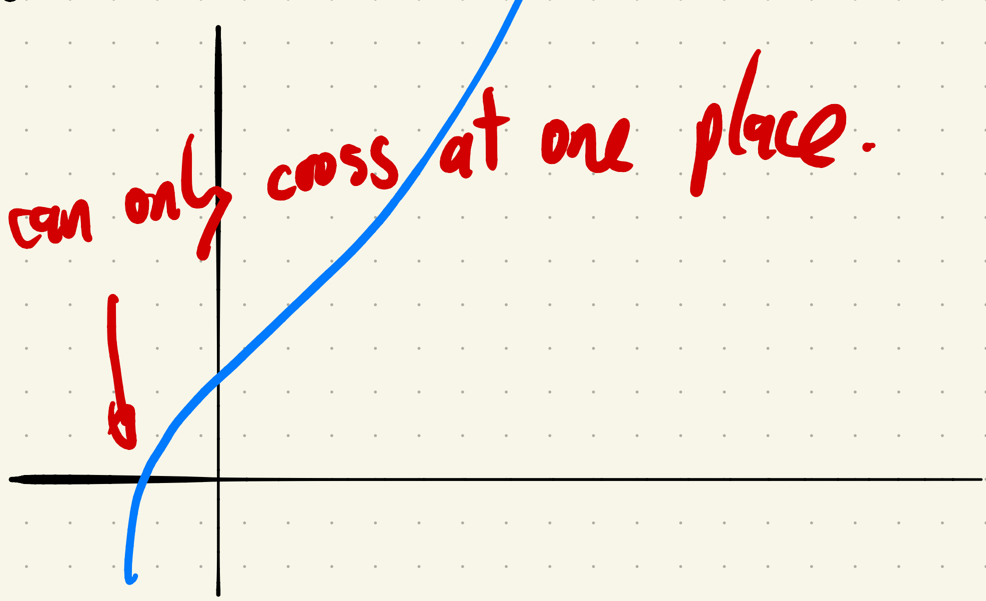
But... $f'(x) = 2 + 3x^2 + 20x^4$

$$\geq 2$$

So f' can never be zero!

So we can't have two solutions.

It looks more like:



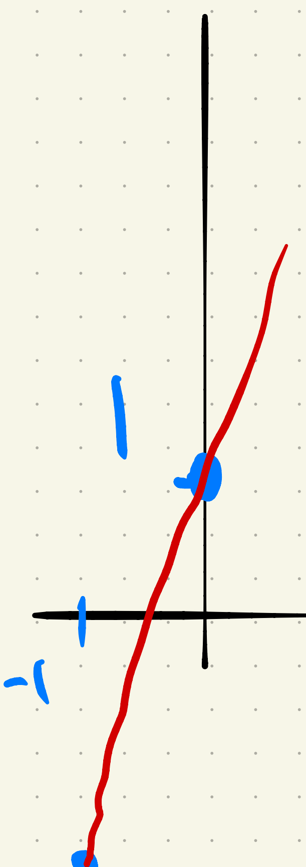
Technicality: we only showed it can have at most one solution.

How do we know it has a solution?

$$f(x) = 1 + 2x + x^3 + 4x^5.$$

Remember the Intermediate Value Theorem?

(a long time ago – lecture 7 or 8?).



Try some values: $f(0) = 1$

$$f(-1) = 1 - 2 - 1 - 4 = -6$$

Int. Value Theorem says the graph must cross the x-axis: $f(c) = 0$ somewhere.

So it has <u>one</u> solution $f(c)=0$

and not more than one solution.

<u>Next time:</u> Mean Value Theorem.

another word for "average"