Math 30, Friday May 8, 2020

Review

Final Exam is on Monday May 11

6am – 11:59pm.

Questions?

but it should take

~ 2 hrs.

Maybe 10 or so problems...

Santiago's Question: #4 on "Integration Worksheet"

Mary L.

4. Find an expression of the form $\sum_{j=1}^{n} f(x_j)\Delta x$

expressing the area under $y = \sin x$ from

$x = 0$ to $x = \pi$

w/ 100 rectangles of equal width

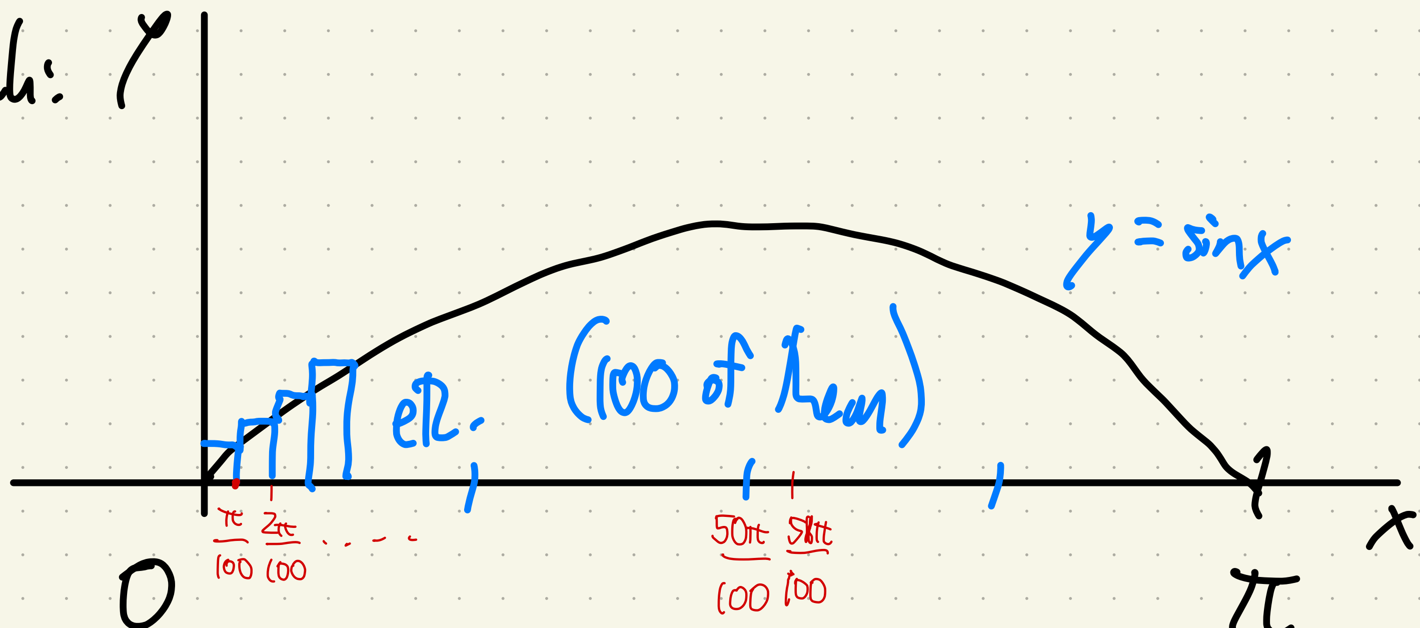
and "right endpts"

4. Find an expression of the form $\sum_{j=1}^{n} \underbrace{f(x_j)\Delta x}_{\text{area of } j^{th} \text{ rectangle.}}$

expressing the area under $y = \sin x$ from $x = 0$ to $x = \pi$

w/ 100 rectangles of equal width and "right endpts"

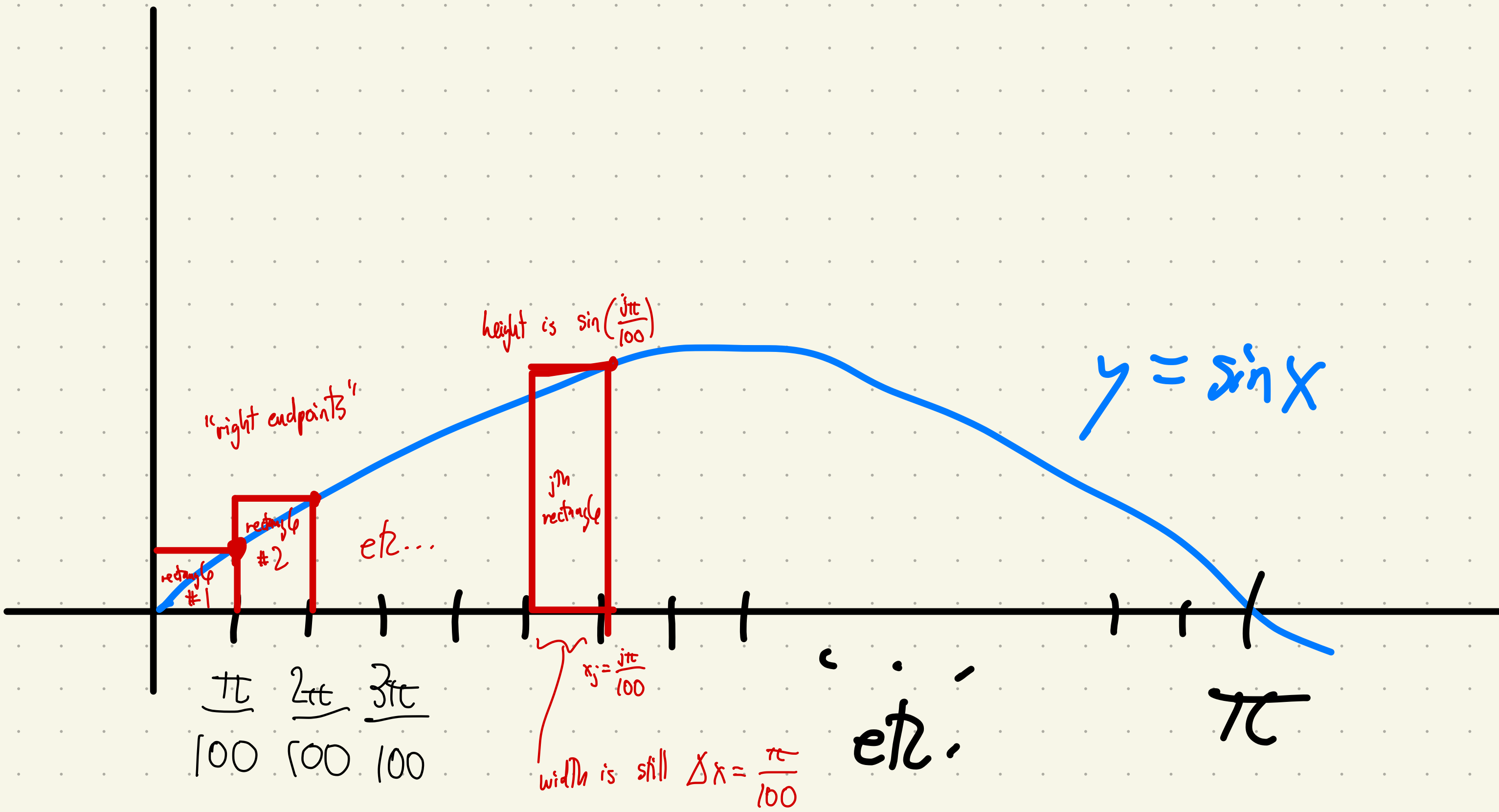
Sketch:



each rectangle has width $\frac{\pi}{100} = \Delta x$

height of j^{th} rectangle is $\sin\left(\frac{j\pi}{100}\right)$

$$\boxed{\text{Final answer} = \sum_{j=1}^{100} \sin\left(\frac{j\pi}{100}\right) \cdot \frac{\pi}{100}}$$



I want 100 rectangles of equal width.

1. First split the interval $[0, \pi]$ into 100 equal parts

each has width $\Delta x = \frac{\pi}{100}$

in general, to split $[a, b]$ into n equal parts

each has width $\frac{b-a}{n} = \Delta x$.

#2. on same worksheet from May 1:

Find the avg. value of $f(x) = \cos x$ over the interval $[0, \frac{\pi}{2}]$.

& sketch.

recall: $\text{avg} = \dfrac{\text{Area}}{\text{length of interval}} =$ height of rectangle w/ same area

$$\text{Avg} = \underbrace{\frac{1}{\left(\frac{\pi}{2}\right)}}_{\text{divided by width}} \underbrace{\int_0^{\pi/2} \cos x \, dx}_{\text{area}}$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \cos x \, dx = \ldots$$

$$\text{Avg} = \frac{1}{\left(\frac{\pi}{2}\right)} \int_0^{\pi/2} \cos x \, dx$$

The avg. is the height of the rectangle w/ the same area

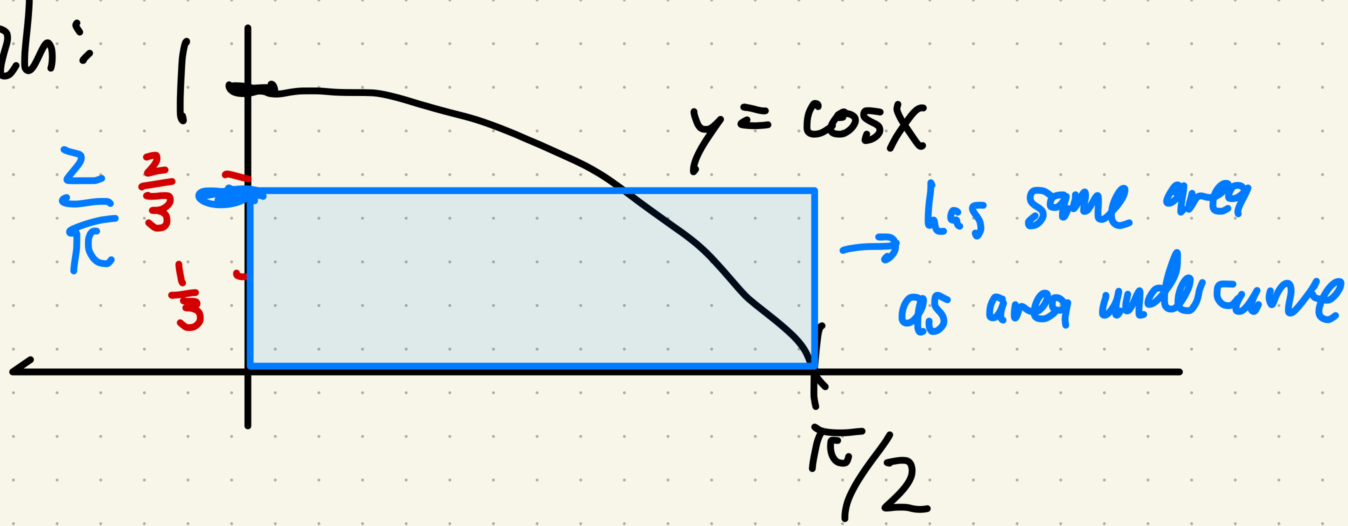
$$= \frac{2}{\pi} \int_0^{\pi/2} \cos x \, dx = \frac{2}{\pi} \Big[\sin x \Big]_0^{\frac{\pi}{2}}$$

$$\frac{1}{\left(\frac{\pi}{2}\right)} = \frac{2}{\pi}$$

$$= \frac{2}{\pi} [1 - 0]$$

$$= \boxed{\frac{2}{\pi}} \approx \frac{2}{3}$$

Sketch:



Q: #9 on Practice Final.

#9a. Find the derivative of $f(x) = x \ln x$.

Product Rule:

$$f'(x) = 1 \cdot \ln x + x\left(\frac{1}{x}\right)$$

$$= \ln x + 1.$$

This almost works, just need to remove the "+1"

#9b. Now evaluate $\int \ln x \, dx$.

can you find an antiderivative?

Thanks to part (a),

try $g(x) = x \ln x - x$

Check: $g'(x) = \ln x + 1 - 1 = \ln x$ ☺

So $\boxed{\int \ln x \, dx = x \ln x - x + C}$

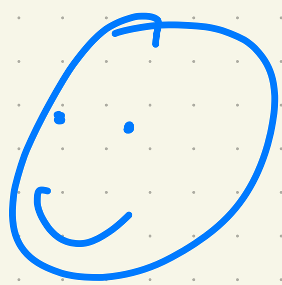
remember: can check by differentiating:

$$\frac{d}{dx}\left(x \ln x - x + C\right)$$

↓ product Rule

$$= \ln x + 1 - 1 + 0$$

$$= \ln x$$



Santiago's Q:

May 4.

#2 on Substitution Worksheet

#1 helps:

#1. $g(x) = e^{x^2}$. $g'(x) = ?$

Chain Rule: $g'(x) = e^{x^2} \cdot 2x$

$= 2xe^{x^2}$

#2. Use this to evaluate $\int xe^{x^2}\,dx$

can you find an antiderivative?

Use $G(x) = \frac{1}{2}e^{x^2}$

Final answer:

$$\boxed{\int x e^{x^2}\,dx = \frac{1}{2}e^{x^2} + C}$$

As always, check by differentiating:

$$\frac{d}{dx}\left(\frac{1}{2}e^{x^2} + C\right)$$

Chain Rule

$$= \frac{1}{2}e^{x^2}\cdot 2x + 0$$

$$= x e^{x^2}$$

Can also do it as a substitution:

$u = x^2 \qquad du = 2x\,dx \qquad \frac{1}{2}du = x\,dx$ etc.

Other Questions?

Q: "Lecture 54 Worksheet"

#3. Define $f(x) = \int_{-1}^{x} t^2 \sin t \, dt$.

Find the critical numbers of f on the interval $(-1, 4)$.

In general, for any function f, find where

$$f'(x) = 0$$

F.T. of Calc. Part I

Here $f'(x) = x^2 \sin x$) This is zero when $x = 0$, $x = \pi$

That's all on the interval $(-1, 4)$.

Plug these into f to find the values:

$$f(0) = \int_{-1}^{0} t^2 \sin t \, dt = ?$$

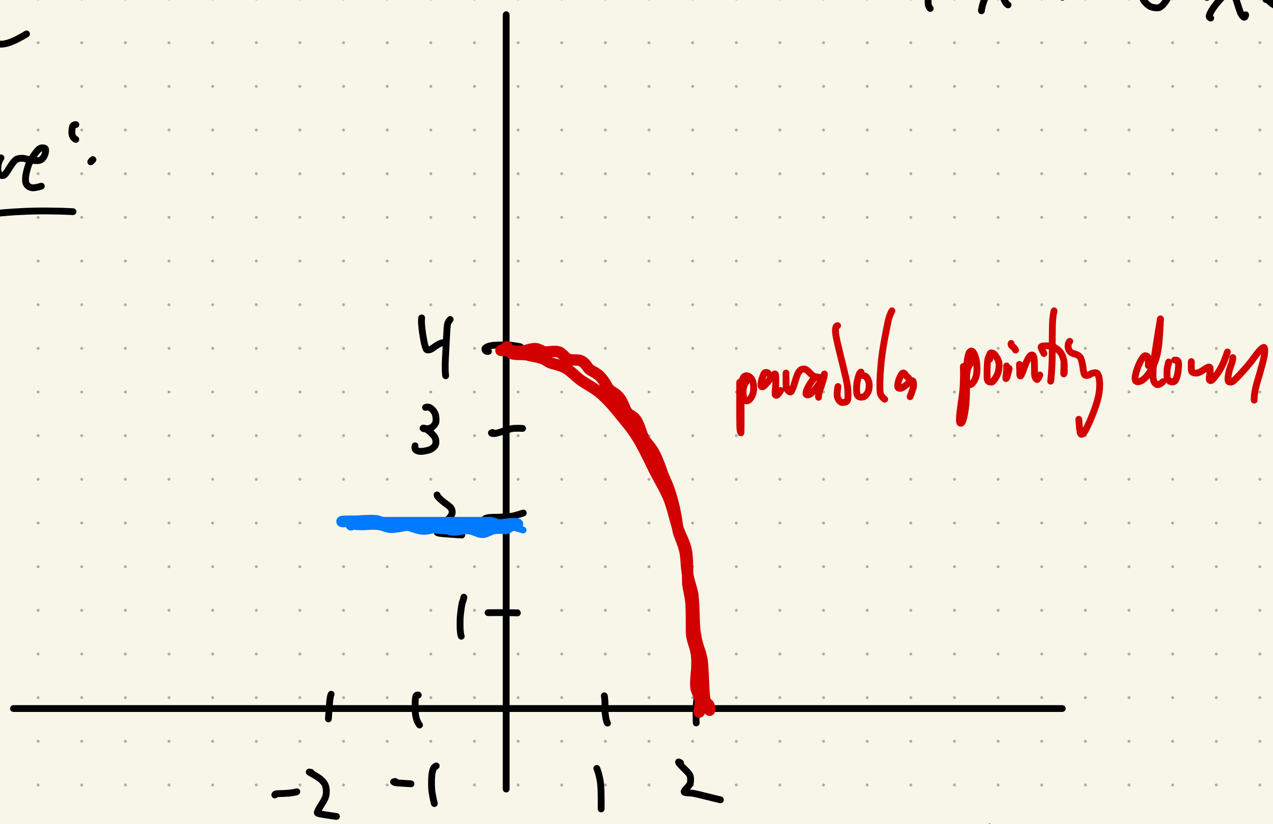
$$f(\pi) = \int_{-1}^{\pi} t^2 \sin t \, dt = ?$$

nevermind, I just want the x-values $x = 0, \pi$.

#4. on "Lecture 54 Worksheet":

$\int_{-2}^{2} f(x)\,dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \le x \le 0 \\ 4-x^2 & \text{if } 0 < x \le 2. \end{cases}$

Picture:



So $\int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$ do the parts separately

$= 4 + \int_0^2 (4-x^2)\,dx$ etc...

now use F.T. of Calc. Part II...

Thanks!

Have fun Studying!