Prove $(A \cap B)^c = A^c \cup B^c$

- first prove $(A \cap B)^c \subseteq A^c \cup B^c$ — ①

$x \in (A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

- Now Prove $A^c \cup B^c \subseteq (A \cap B)^c$ — ②

$x \in A^c \cup B^c$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow x \in (A \cap B)^c$

Because we know $A = B$ only $A \subseteq B$ and $B \subseteq A$.

From ① and ②, we infer

$(A \cap B)^c = A^c \cup B^c$

Prove $(A \cup B)^c = A^c \cap B^c$

- first prove $(A \cup B)^c \subseteq A^c \cap B^c$ — ①

$x \in (A \cup B)^c$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

- Now prove $A^c \cap B^c \subseteq (A \cup B)^c$ — ②

$x \in A^c \cap B^c$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$

we know set $A =$ set B only if $A \subseteq B$ and $B \subseteq A$.

From ① and ②, we know that

$(A \cup B)^c = A^c \cap B^c$

Set difference law:

$$A - B = A \cap B^c$$

Proof: first prove $A - B \subseteq A \cap B^c$ — (1)

$x \in A, \; x \notin B$

$\Rightarrow x \in A, \; x \notin B^c$

$\Rightarrow x \in A \cap B^c$

— (2)

Now prove $A \cap B^c \subseteq A - B$

$x \in A \cap B^c$

$\Rightarrow x \in A, \; x \in B^c$

$\Rightarrow x \in A, \; x \notin B$

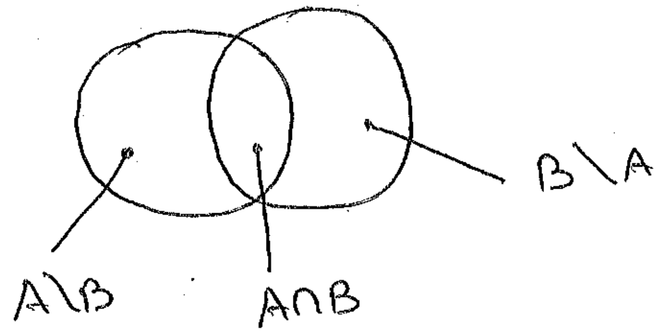
$= x \in A - B$

We know set A = set B if & only if $A \subseteq B$ and $B \subseteq A$.

From (1) and (2), we proved

$$A - B = A \cap B^c$$

Prove $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



$$n(A \setminus B) = n(A) - n(A \cap B) \quad -① $$
$$n(B \setminus A) = n(B) - n(A \cap B) \quad -② $$

We know from the diagram,

$$n(A \cup B) = n(A \setminus B) + n(A \cap B) + n(B \setminus A)$$

Apply ① and ② in the above equation,

$$n(A \cup B) = n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$$
$$= n(A) + n(B) - n(A \cap B).$$

. QED