1. $n_1 = 60$ B.H
$n_2 = 30$ E.H
$n_3 = 10$ Dogs

a) 10 randomly chosen. Prob that 5 B.H, only 3 E.H, and 2 Dogs?

$60+30+10=100$

$$\boxed{\frac{\binom{60}{5}\binom{30}{3}\binom{10}{2}}{\binom{100}{10}}}$$

b) 2 chosen from 10 as Pres and Vice Pres. How many ways can this be done?

$10 * 9 = \boxed{90}$

2. Strikes occur → Poisson Process at a mean rate of 2 per hour

a) T = time until 3rd arrives. Identify Dist. of T and parameters, then find $P(T>3)$

$T \sim \Gamma(\alpha, \lambda)$ ⟸ Gamma Dist.

$T \sim \Gamma(3, 2)$ $\alpha = 3$, $\lambda = 2$ $P(T>3)$?

$$\int_3^\infty \frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)} \rightarrow \boxed{\int_3^\infty \frac{2^3 t^2 e^{-2t}}{2} dt}$$

$\Gamma(\alpha) = (\alpha-1)!$

$\Gamma(3) = 2! = 2$

Cont'd

cont'd

b) $T \to$ time until ship arrives identically dist, find $P(T>1)$. If 2 hrs since last ship what is prob that it will be more than 2 hrs until next?

$T \sim Exp(2) \quad P(T>1)?$

$P(T>1) = 1 - P(T \leq 1) = 1 - F(1)$

$1 - [1 - e^{-2(1)}] = 1 - [1 - e^{-2}] = e^{-2} = 0.1353$

$P(\overset{\text{next}}{> 2 \text{hrs}} \mid \overset{\text{1st}}{2 \text{hrs}})?$

cont'd

(cont'd*)

Q) 2 arrivals in 30 min. find Prob. random 0.26 and $P(X<2)$

$$Y \sim \text{Poisson}(\overset{2\text{ per hr}}{2} * \overset{1/2\text{ hr}}{0.5})$$

$$Y \sim \text{Poisson}(1) \quad P(X<2)?$$

$$P(X=0) \Rightarrow \frac{e^{-1} 1^0}{0!}$$

$$P(X=1) \Rightarrow \frac{e^{-1} 1^1}{1!} \rightarrow 0.36$$

$$P(X<2) = P(X=0) + P(X=1)$$
$$= 0.36 + 0.36$$
$$\rightarrow \boxed{0.7357}$$

3. R.V. $A \sim N(20,4)$, R.V. $B \sim N(30,9)$, R.V. $C \sim N(80,12)$

Assume independence

$$P(2C > 3A + 4B)?$$

$$P(2C - (3A+4B) > 0)$$

$$\sigma^2_{2C-(3A+4B)} = \sigma_C^2 + \sigma_A^2 + \sigma_B^2$$
$$= 2^2(12^2) + 3^2(4^2) + 4^2(9^2)$$
$$= 2016$$

$$P\left(\frac{2C - (3A+4B) - (-20)}{44.89} > \frac{0-(-20)}{44.89}\right) = P(Z > +0.445)$$

$$'' \sim N(-20, 2016)$$

$$1 - P(Z \leq 0.45) \rightarrow 0.6736$$

$$1 - 0.6736$$

$$\rightarrow \boxed{0.3264}$$

$$2(80) - (3(20) + 4(30))$$

$$\sqrt{2016} \approx 44.89$$

4. 10% excell   90% of excell get raise
75% Accept   60% of accept get raise
15% Unaccept   2% of unaccept get raise

If random employee chosen, what is odds they are accept?

$P(\text{Accept} \mid \text{raise})? \Rightarrow \frac{P(A \cap R)}{P(R)} \Rightarrow \frac{P(R \mid A)P(A)}{P(R)}$

$P(R) = 0.10(0.90) + 0.75(0.60) + 0.15(0.02)$
$= 0.09 + 0.45 + 0.003 = 0.543$

$\frac{(0.6)0.75}{0.543} = \boxed{0.8287}$

5. Ledger☆ contest of randomly chosen letters
a) 13 chosen at rand. $X \Rightarrow$ # of 13 names start w/(A–E)
Find $P(X<2)$
$X \sim Bin(13, 0.19)$ $\frac{5}{26}$
$P(X<2) = P(X=0) + P(X=1)$
$\frac{13!}{(13-0)!0!}(0.19)^0(1-0.19)^{13} + \frac{13!}{(13-1)!1!}(0.19)^1(1-0.19)^{12}$
0.06 0.19 $= \boxed{0.2549}$
b) Card chosen until 4th (A–E)
up to and including 4th, whose names → (A–E)
Find $P(X=10)$
$X \sim NB(r, p) \Rightarrow X \sim NB(4, \frac{5}{26})$
$P(X=10) = \binom{10-1}{4-1}\frac{5}{26}^4(1-\frac{5}{26})^{10-4} = \boxed{0.0319}$
$\binom{9}{3} = 84$

Cont'd ☆☆

Con'td A

5) Find P(A-G $\overset{\cup}{or}$ D-K) DEFGHIJK 8

P(A)+P(B) - P(A)P(B)

$\frac{7}{26}+\frac{8}{26}-\left(\frac{7}{26}\right)\left(\frac{8}{26}\right)=\boxed{0.4941}$

6. $f(x,y)=\begin{cases}\frac{2x+5y}{12} & 0\le x\le 1 \text{ and } 0\le y\le 2\\ 0 & \end{cases}$

otherwise

a) Find marginal densities of X and Y

$f_X=\frac{1}{12}\int_{y=0}^{2}(2x+5y)\,dy=\frac{1}{12}\left(2xy+\frac{5y^2}{2}\right)\Big|_{y=0}^{2}$

$\frac{1}{12}\left(2x(2)+\frac{5(2)^2}{2}\right)=\frac{1}{12}(4x+10)$

$\boxed{\begin{array}{l} f_X(x)=\frac{1}{12}(4x+10),\ 0\le x\le 1\\ f_Y(y)=\frac{1}{12}(1+5y),\ 0\le y\le 2\end{array}}$

$f_Y=\frac{1}{12}\int_{x=0}^{1}(2x+5y)\,dx=\frac{1}{12}\left(\frac{2x^2}{2}+5xy\right)\Big|_0^1$

$\frac{1}{12}(1+5y)$

~~$f_X(x)$~~

Con'td

Cont'd

b) Find Var(X) $\Rightarrow$ $Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = \int_0^1 x^2 \left[\frac{1}{12}(4x+10)\right] dx \quad \int_0^1 \frac{1}{12}(4x^3+10x^2) dx$

$\int_0^1 \left(\frac{x^3}{3} + \frac{5x^2}{6}\right) \Rightarrow \left(\frac{x^4}{12} + \frac{5x^3}{18}\right)\Big|_0^1 = \frac{1}{12} + \frac{5}{18} = 0.3611$

$\frac{10}{12} = \frac{5}{6}$

$E(X) = \int_0^1 x\left(\frac{1}{12}(4x+10)\right) dx \quad \int_0^1 \frac{1}{12}(4x^2+10x) dx$

$\int_0^1 \left(\frac{x^2}{3} + \frac{5x}{6}\right) \Rightarrow \frac{x^3}{9} + \frac{5x^2}{12}\Big|_0^1 = \frac{1}{9} + \frac{5}{12} = 0.5277$

$0.3611 - (0.5277)^2 = \boxed{0.0826}$

c) Write expression for Cov(X,Y)

Cov(X,Y) =

$\frac{1}{12}\int_0^2\int_0^1 xy\left(\frac{2xy+5y}{12}\right) dx\,dy -$

$\Rightarrow \left[\int_0^1 x\left(\frac{1}{12}(4x+10)\right) dx\right]\left[\int_0^2 y\left(\frac{1}{12}(1+5y)\right) dy\right]$

Cont'd 3A

Cont'd 3.8

use $\alpha = 0.10$

7. Random samp. of 20

$\mu = 37$
$s = 2.3$

Claim: mean < 38 Normal

a) State hypotheses

$\boxed{H_0: \mu = 38 \text{ vs } H_1: \mu < 38}$

~~reject~~

b) Complete test stat.

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{37 - 38}{2.3/\sqrt{20}} = \boxed{-1.944}$

c) Determine P-value

$t_{19}, 1.944$

$\cancel{P = P(Z < -1.944) = 0.0262}$ $\boxed{0.025 < P < 0.05}$

d) State conc in context.

We are between 0.025 and 0.05 confident that the

8. $\bar{x}_A = 1.875, s_A = 0.8345$ 40
$\bar{x}_B = 2.625, s_B = 0.9161$ 40

a) State hypotheses

$$\boxed{H_0: \mu_A = \mu_B \text{ vs. } H_1: \mu_A \neq \mu_B}$$

b) Compute test stat

$$z = \frac{(1.875 - 2.625) - 0}{\sqrt{\frac{0.8345^2}{40} + \frac{0.9161^2}{40}}} \approx \boxed{-3.837}$$

c) Compute P-value

$$P(Z < -3.8 \text{ or } Z > 3.8) = 2P(Z < -3.8)$$
$$= 2(0.0) \approx \boxed{0}$$

d) State conclusion

[illegible]

Cont'd ##

9. $n=500$ 　　 2.01.12

$\mu=5.4$

$\sigma=3.1$

a) Construct 95% C.I.

$1-0.95=0.05/2=0.025 \quad z_{0.025}\to 1.96$

$5.4 \pm 1.96\frac{3.1}{\sqrt{500}} \to \boxed{(5.12,\ 5.67)}$

b) Interpret C.I and explain.

We are 95% confident that the true mean is between 5.12 and 5.67. What we mean is that if we repeated this process a large number of times that 95% of the intervals would have the true mean.

c) Determine n s.t. ME .25, 95% C.I.

$1.96\frac{3.1}{\sqrt{n}} = 0.25$

$\frac{3.1}{\sqrt{n}} = \frac{0.25}{1.96}$

$0.12 \quad 0.13$

$\boxed{n=591}$

$\sqrt{n}=24.304 \to n=590.68\uparrow$

10. $n = 400$ $\alpha = 0.05$ conds sat!
$b = 40$

Claim: 10 or > 0.08

a) State hypothesis — right-tailed

$\boxed{H_0: \mu \leq 0.08 \text{ vs } H_1: \mu > 0.08}$

b) Compute test stat

$\hat{p} = \frac{40}{400} = 0.10$

$np_0 = 400(0.08) = 32$ $n(1-p_0) = 400(0.06) = 368$
$\geq 10? \checkmark$ $\geq 10? \checkmark$

$$z = \frac{0.10 - 0.08}{\sqrt{(0.08)(0.92)/400}} = 1.47 \quad \boxed{z = 1.47}$$

c) Compute p-value — $P < 0.05$? No $P < 0.01$

p-value area right of $1.47 = 1 - 0.9292$ $P < 0.11$
$= \boxed{0.0708}$

d) State concl in context.

Because p-value is greater than 0.05, we do not reject the null hypothesis and conclude that the mean is less than or equal to 0.08.