Math 30 Final Exam
Monday, May 11, 2020

Professor Michael VanValkenburgh

Name: _____

Instructions: (Updated for Online Teaching.) Show all your work, and clearly indicate your answers. You may use the class textbook, your lecture notes, and your homework, **but you may *not* get help from any other source.** This is for your long-term benefit.

Also, by submitting your work you are agreeing to NOT discuss the contents of this exam with anyone until your grade is posted.

Problem	Your score	Possible Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		90

1. Consider the function $f(x) = 3x^2 - 2x$.

   (a) Use the definition of the derivative (as a limit) to find $f'(1)$.

   (b) Find the equation of the tangent line to the graph of f at $x_0 = 1$.

   (c) Find where the line in (b) intersects the x-axis.

   (d) Make a sketch illustrating your answer to part (c).

2. Consider the function $h(x) = \sqrt{1 + \sqrt{x}}$.

   (a) Write h as a composition of simpler functions $g(u)$ and $f(x)$.

   (b) Find $g'(u)$ and $f'(x)$.

   (c) Find $h'(x)$.

3. Consider the function $f(x) = 3x^3 + x^2 + 1$.

   (a) Find the intervals on which f is increasing and decreasing.

   (b) Find those x where $f(x)$ has a local maximum or local minimum.

   (c) Find the inflection points.

   (d) Find where the graph of f is concave up and concave down.

   (e) Sketch the graph of f.

4. The curve given by $x^4 + y^4 = 17$ is a "squarish circle." Find the equation of the tangent line to the curve at $(x, y) = (1, -2)$.

5. Suppose a cylinder has constant volume 100 cubic inches. Suppose its radius is decreasing at a constant rate of 1/3 inch per second. What is the rate of increase of the length when the radius is 1/2 inch? (I am imagining a baker rolling out a long tube of dough.)

6. Write the integral $\int_0^1 4x^3\,dx$ as a limit of sums of areas of rectangles with equal width and using "right endpoints." Make a sketch illustrating your answer.

7. Evaluate the definite integral $\displaystyle\int_1^2 \frac{e^x}{1+e^{2x}}\,dx$ using u-substitution.

8. Find the average value of the function $f(x) = x^{2/3}$ over the interval $[1, 8]$. Illustrate your answer with a sketch.

9. Consider the function $f(x) = x \tan^{-1} x$. ($\tan^{-1} x$ is another name for $\arctan x$.)

   (a) Find $f'(x)$.

   (b) Use part (a) to evaluate the indefinite integral $\int \tan^{-1} x \, dx$.