Least-Squares Problems

The Idea for this Section: Sometimes in applications you will have a system, $A\mathbf{x} = \mathbf{b}$, that is inconsistent, and thus has no solution (no $\mathbf{x}$ exists to make the system work). So the best thing to do at this point is to find an $\mathbf{x}$ that makes $A\mathbf{x}$ as close to $\mathbf{b}$ as possible. Meaning, we need to *approximate* $\mathbf{b}$ with $A\mathbf{x}$, which is, we want $||\mathbf{b} - A\mathbf{x}||$ to be as small as possible. The "general least-squares problem" is to find the $\mathbf{x}$ in question. It's called "least-squares" since $||\mathbf{b} - A\mathbf{x}||$ is the square root of a sum of squares.

Definition of Least Squares Solution:

Solution of the General Least-Squares Problem:

Theorem 28.1:

Example 28.2:

Theorem 28.3:

Example 28.4:

Example 28.5: