MATH 30, 3/20/2020: LINEAR APPROXIMATION

We will start with a **related rates** example, for review:

1. Read the problem carefully.
2. Draw a picture if possible.
3. Introduce notation.
4. Express the given information mathematically.
5. Write an equation that relates the various quantities.
6. Use the Chain Rule.
7. Substitute into the resulting equation and solve for the related rate.

Example. Water is being poured into an inverted cone (vertex down) of radius 4 inches and height 10 inches at a rate of 3 cubic inches per second. Find the rate at which the water level is rising when the depth of the water over the vertex is 6 inches.

Next topic: Another cool, smart math shortcut.

Main idea: If a function f is differentiable at $x = a$, then the tangent line is the *best linear approximation* near that point. [Sketch.] Zoom in! The curve looks flat to a bug, just like how the Earth looks flat to us on the surface. The tangent line really is a good linear approximation (the best!).

This will look really cool on my tablet!

Remember: the tangent line through $(a, f(a))$ has slope $f'(a)$, so its equation is

$$y = f(a) + f'(a)(x - a).$$

(It's a function of x!)

Check: It has the right slope ✓ It goes through $(a, f(a))$ ✓

The tangent line gives a good approximation, so

$$f(x) \approx f(a) + f'(a)(x - a)$$

for x close to a.

Example. What is the decimal expansion for the side length of a square of area 125? [Sketch.]

Solution. Here the relevant function is $f(x) = \sqrt{x}$. We know $\sqrt{121} = 11$, and 121 is pretty close to 125, so use $a = 121$ as a "base point."

Using the above formula, we have

$$f(125) \approx f(121) + f'(121)(125 - 121).$$

That is,

$$\sqrt{125} \approx 11 + \frac{4}{2 \cdot 11} = 11 + \frac{2}{11}.$$

Using long division, we can find $\frac{2}{11} = 0.1818\ldots$, so

$$\sqrt{125} \approx 11.18.$$

Now compare this with your calculator:

$$\sqrt{125} = 11.18033\ldots$$

We were pretty close!

For review, see the "pop quiz" on differentiation of inverse functions and derivatives of exponentials and logarithms.