Quiz 4

① a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \det I_4 = 1 \cdot 1 \cdot 1 \cdot 1 = 1.$

b) $\det A A^{-1} = \det I_4 = 1$

these are just numbers

c) $1 = \det A A^{-1} \overset{\textcircled{=}}{} \det A \cdot \det A^{-1} \longrightarrow \dfrac{\det A \cdot \det A^{-1}}{\det A} = \dfrac{1}{\det A}$

$\llcorner$ determinant property

$\rightarrow \det A^{-1} = \dfrac{1}{\det A}$

1st row expansion

② $\det A = 2\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} = 2(3-8) - 3(1-4)$

$= 2(-5) - 3(-3)$

$= -10 + 9 = -1 \neq 0 \rightarrow A$ is invertible

A^{-1}:

$\begin{bmatrix} 2 & 3 & 0 & | & 1 & 0 & 0 \\ 1 & 3 & 4 & | & 0 & 1 & 0 \\ 1 & 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & -1 & -2 & | & 1 & 0 & -2 \\ 0 & 1 & 3 & | & 0 & 1 & -1 \\ 1 & 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 & | & 1 & 1 & -3 \\ 0 & 1 & 3 & | & 0 & 1 & -1 \\ 1 & 0 & -5 & | & 0 & -2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 & | & 1 & 1 & -3 \\ 0 & 1 & 0 & | & -3 & -2 & 8 \\ 1 & 0 & 0 & | & 5 & 3 & -12 \end{bmatrix}$

$-R_3 + R_2 \rightarrow R_2$ $\qquad$ $R_2 + R_1 \rightarrow R_1$ $\qquad$ $-3R_1 + R_2 \rightarrow R_2$

$-2R_3 + R_1 \rightarrow R_1$ $\qquad$ $-2R_2 + R_3 \rightarrow R_3$ $\qquad$ $5R_1 + R_3 \rightarrow R_3$

$R_1 \leftrightarrow R_3$
$\rightarrow \begin{bmatrix} 1 & 0 & 0 & | & 5 & 3 & -12 \\ 0 & 1 & 0 & | & -3 & -2 & 8 \\ 0 & 0 & 1 & | & 1 & 1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} A^{-1} = \begin{bmatrix} 5 & 3 & -12 \\ -3 & -2 & 8 \\ 1 & 1 & -3 \end{bmatrix} \end{bmatrix}$

③ $\det B = 1\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} - 0 + 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

1st row expansion

$= 1(1-4) + 1(2-1)$

$= 1 \cdot (-3) + 1 \cdot 1$

$= -3 + 1 = -2$

Note $\det B^5 = (\det B)^5 \rightarrow \llbracket \det B \cdot \det B \cdot \det B \cdots$

Since $\det B = -2$

$\det B^5 = (-2)^5$

$= \boxed{-32}$