STAT 50 HW #17 Sections 6.4 and 6.5

Section 6.4 #'s 3, 5, 8, 7, 11

3.

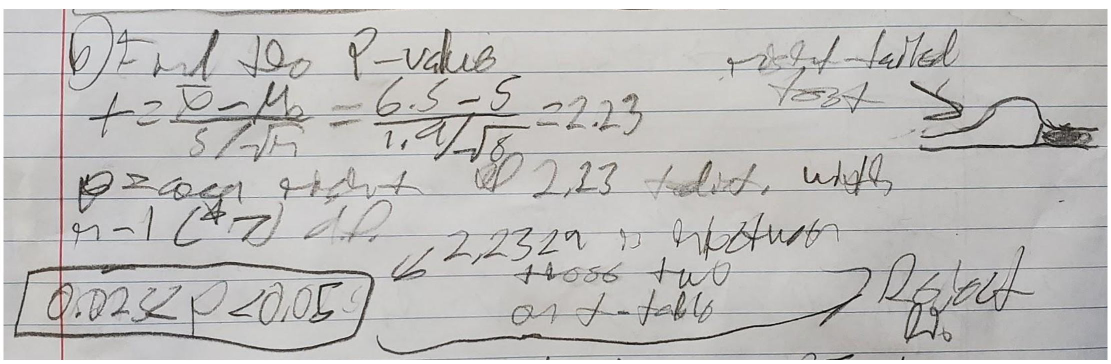
A new centrifugal pump is being considered for an application involving the pumping of ammonia. The specification is that the flow rate be more than 5 gallons per minute (gpm). In an initial study, eight runs were made. The average flow rate was 6.5 gpm and the standard deviation was 1.9 gpm. If the mean flow rate is found to meet the specification, the pump will be put into service.

a. **State the appropriate null and alternate hypotheses.**

$H_0: \mu \leq 5$vs $H_1: \mu > 5$

*Alternate hypothesis is sort of like what we are trying to prove. Null hypothesis is what we reject.

b. **Find the P-value.**



c. **Should the pump be put into service? Explain.**

Yes. As the p-value is less than 0.05, we would describe it as small and so we reject the null hypothesis and conclude that the mean flow rate is greater than 5 gpm.

5.

The article "Influence of Penetration Rate on Penetrometer Resistance" (G. Gagnon and J. Doubrough, Canadian Journal of Civil Engineering, 2011: 741–750) describes a study in which twenty 2-L specimens of water were drawn from a public works building in Bridgewater, Nova Scotia. The mean lead concentration was 6.7 μg/L with a standard deviation of 3.9 μg/L.

a. **The Health Canada guideline states that the concentration should be less than 10 <g/L. Can you conclude that the water in this system meets the guideline?**

5. $n = 20$

$\bar{x} = 6.7\ \mu g/L$

$s = 3.9\ \mu g/L$ $\qquad \mu$

a) Spec: $< 10\ \mu g/L$. Does the water meet this specification?

$H_0: \mu \geq 10\ \mu g/L$ vs $H_1: \mu < 10\ \mu g/L$

P-value:

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \dfrac{6.7 - 10}{3.9/\sqrt{20}} = -3.78 \rightarrow$ Look positive value!

$t_{19} = 3.78 \rightarrow 0.0005 < P < 0.001$

$P < 0.05? \rightarrow$ reject H_0 $\qquad$ P is less than 0.05

$\boxed{\text{Yes.}}$

b. **A stricter guideline is being considered, which would require the concentration to be less than 7.5 µg/L. Can you conclude that the water in this system meets this guideline?**

b) Spec: $< 7.5\ \mu g/L$. Does water meet this spec?

$H_0: \mu \geq 7.5\ \mu g/L$ vs $H_1: \mu < 7.5\ \mu g/L$

P-value:

$t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} \qquad \dfrac{6.7 - 7.5}{3.9/\sqrt{20}} = -0.91 \rightarrow 0.917$

$t_{19} = -0.91 \rightarrow 0.10 < P < 0.25$

↳ P is not less than 0.05

$P < 0.05? \rightarrow$ reject H_0

$\boxed{\text{No.}}$ ← we fail to reject H_0

8.

Environmental Protection Agency standards require that the amount of lead in drinking water be less than 15 ppb. Twelve samples of water from a particular source have the following concentrations, in ppb.

11.4	**13.9**	**11.2**	**14.5**	**15.2**	**8.1**

12.4	**8.6**	**10.5**	**17.1**	**9.8**	**15.9**

A hypothesis test will be performed to determine whether the water from this source meets the EPA standard.

a. **State the appropriate null and alternate hypotheses.**

$H_0: \mu \geq 15$ vs $H_1: \mu < 15$

b. **Compute the P-value.**

b) Find the P-value. σ unknown → Check 3.09

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{12.383 - 15}{2.9322/\sqrt{12}} = -3.09$ $\quad s = 2.9322$

$x_i - \bar{x}$ $\quad (x_i - \bar{x})^2$

$11.4 - 12.383 = -0.983^2 = 0.9669$

$12.4 - 12.383 = 0.016^2 = 2.77 \times 10^{-4}$

$13.9 - 12.383 = 1.516^2 = 2.3002$

$8.6 - 12.383 = -3.783^2 = 14.3136$

$11.2 - 12.383 = -1.183^2 = 1.4002$

$10.5 - 12.383 = -1.8833^2 = 3.5469$

$14.5 - 12.383 = 2.116^2 = 4.4802$ $\quad \Sigma = 94.5766$

$17.1 - 12.383 = 4.716^2 = 22.2469$

$15.2 - 12.383 = 2.816^2 = 7.9336$ $\quad \sqrt{\frac{94.58}{11}} = 2.9322$

$9.8 - 12.383 = -2.583^2 = 6.6736$

$8.1 - 12.383 = -4.283^2 = 18.3469$

$15.9 - 12.383 = 3.516^2 = 12.3669$

Cont'd $\quad P < 0.05?$

$t_{11} = -3.09 \rightarrow \boxed{0.005 < P < 0.01} \rightarrow P < 0.05$ ✓

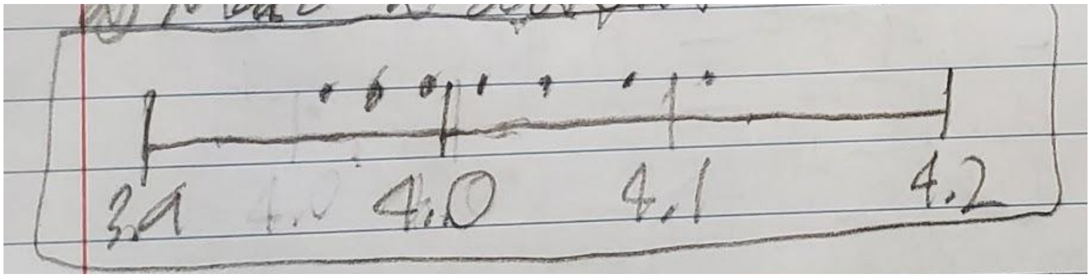
c. **Can you conclude that the water from this source meets the EPA standard? Explain.**

Yes. The P-value is less than 0.05, so we would describe it as small. We reject the null hypothesis and conclude that the water meets the EPA standard.

7.

Specifications call for the wall thickness of two-liter polycarbonate bottles to average 4.0 mils. A quality control engineer samples 7 two-liter polycarbonate bottles from a large batch and measures the wall thickness (in mils) in each. The results are: 3.999, 4.037, 4.116, 4.063, 3.969, 3.955, and 4.091. It is desired to test H0 : μ = 4.0 versus H1 : μ ≠ 4.0.

a. **Make a dotplot of the seven values.**



b. **Should a Student's t test be used to test H0? If so, perform the test. If not, explain why not.**

b) Should t-test be used to test H_0? If so perform the test. If not, explain.

Yes. → No outliers or skewness.

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.032 - 4}{0.06/\sqrt{7}} = 1.414$

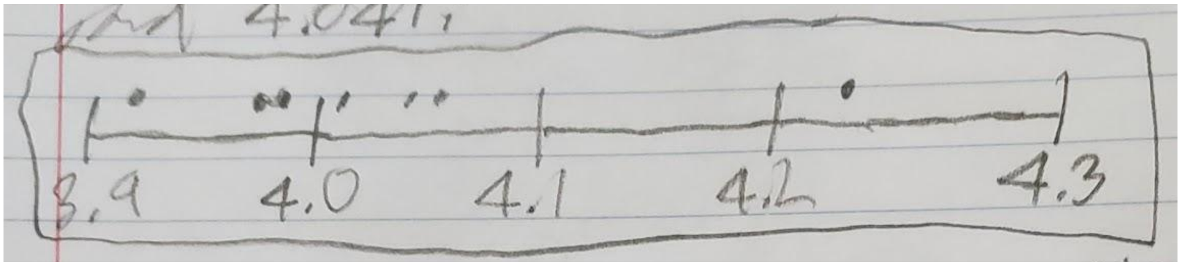
$P < 0.05$? No

$t_6 = 1.414 \rightarrow 0.10 < P < 0.25$ ← Two-tailed test so we double this!

$0.20 < P < 0.50$

$\boxed{\text{Yes. We do not reject the null hypothesis}}$

c. **Measurements are taken of the wall thicknesses of seven bottles of a different type. The measurements this time are: 4.004, 4.225, 3.924, 4.052, 3.975, 3.976, and 4.041. Make a dotplot of these values.**



d. **Should a Student's t test be used to test H0 : μ = 4.0 versus H1 : μ ≠ 4.0? If so, perform the test. If not, explain why not.**

No. There is an outlier in the dotplot so the population distribution is not approximately linear.

11.

Eight specimens of Incoloy 825 steel were immersed in acidified brine for four hours. The number of corrosive pits was counted on each sample. The results were

11 26 34 21 42 18 31 34

Can you conclude that the mean number of pits that develop under these conditions is less than 40?

11, 11, 26, 34, 21, 42, 18, 31, 34 $\bar{x} = 27.125$

$s = 10.09$

Can you conclude mean number of pits is less than 40?

$H_0: \mu \geq 40$ vs $H_1: \mu < 40$

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{27.125 - 40}{10.09/\sqrt{8}} = -3.608$

$P < 0.05$? Yes

$t_7 = -3.608 \Rightarrow 0.005 < P < 0.01$

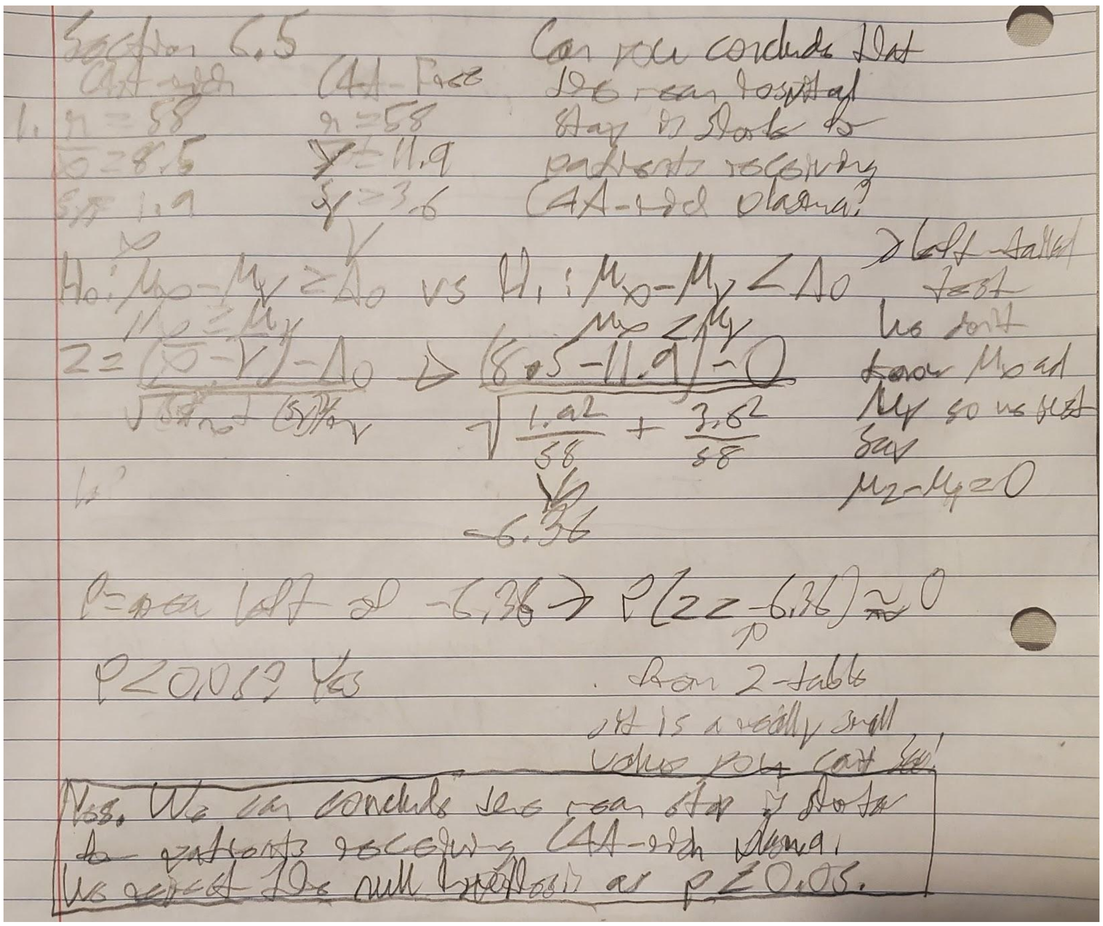
Yes, we can conclude the mean that develop is less than 40. We reject the null hypothesis as $P < 0.05$.

Section 6.5 #'s 1, 5, 7, 9, 10

1.

The article "Capillary Leak Syndrome in Children with C4A-Deficiency Undergoing Cardiac Surgery with Cardiopulmonary Bypass: A Double-Blind, Randomised Controlled

Study" (S. Zhang, S. Wang, et al., Lancet, 2005:556–562) presents the results of a study of the effectiveness of giving blood plasma containing complement component C4A to pediatric cardiopulmonary bypass patients. Of 58 patients receiving C4A-rich plasma, the average length of hospital stay was 8.5 days and the standard deviation was 1.9 days. Of 58 patients receiving C4A-free plasma, the average length of hospital stay was 11.9 days and the standard deviation was 3.6 days. Can you conclude that the mean hospital stay is shorter for patients receiving C4A-rich plasma?



Section 6.5

C4A-rich   C4A-Free

1. $n = 58$ $n = 58$

$\bar{x} = 8.5$ $\bar{y} = 11.9$

$s_x = 1.9$ $s_y = 3.6$

Can you conclude that the mean hospital stay is shorter for patients receiving C4A-rich plasma?

$H_0: \mu_x - \mu_y \geq \Delta_0$ vs $H_1: \mu_x - \mu_y < \Delta_0$ → left-tailed test

$\mu_x \geq \mu_y$ $\mu_x < \mu_y$

$z = \frac{(\bar{x} - \bar{y}) - \Delta_0}{\sqrt{s_x^2/n_x + s_y^2/n_y}} \to \frac{(8.5 - 11.9) - 0}{\sqrt{\frac{1.9^2}{58} + \frac{3.6^2}{58}}}$

$= -6.36$

We don't know μ_x and μ_y so we just say $\mu_x - \mu_y = 0$

P = area left of $-6.36 \to P(z \leq -6.36) \approx 0$

. from z-table

. It is a really small value you can see!

$P \leq 0.05$? Yes

Yes. We can conclude the mean stay is shorter for patients receiving C4A-rich plasma. We reject the null hypothesis as $p \leq 0.05$.

5.

Do you prefer taking tests on paper or online? A college instructor gave identical tests to two randomly sampled groups of 50 students. One group took the test on paper and the other took it online. Those who took the test on paper had an average score of 68.4 with a standard deviation of 12.1. Those who took the test online had an average score of 71.3 with a standard deviation of 14.2. Can you conclude that the mean scores differ between online and paper tests?

5. $n_X = 50$ $\quad n_Y = 50$ $\quad$ Can you conclude that the mean scores differ between online and paper tests?

$\bar{X} = 68.4$ $\quad \bar{Y} = 71.3$

$S_X = 12.1$ $\quad S_Y = 14.2$

Paper $\quad$ Online

$H_0: \mu_X = \mu_Y$ vs $H_1: \mu_X \neq \mu_Y$ → two-tailed test

$$z = \frac{(\bar{X}-\bar{Y})-\Delta_0}{\sqrt{S_X^2/n_X + S_Y^2/n_Y}} \rightarrow \frac{(68.4-71.3)-0}{\sqrt{\frac{12.1^2}{50}+\frac{14.2^2}{50}}} = -1.099$$

$P \rightarrow P(|Z| \geq |z| \mid H_0) = 2P(Z \geq 1.099 \mid H_0)$ Greater than 0.01, 0.05, and 0.1

$2(0.1357) = 0.2714$

Because the p-value is greater than any reasonable significance level α, we do not reject the null hypothesis and conclude that the mean scores do not differ.

7.

A statistics instructor who teaches a lecture section of 160 students wants to determine whether students have more difficulty with one-tailed hypothesis tests or with two-tailed hypothesis tests. On the next exam, 80 of the students, chosen at random, get a version of the exam with a 10-point question that requires a one-tailed test. The other 80 students get a question that is identical except that it requires a two-tailed test. The one-tailed students average 7.79 points, and their standard deviation is 1.06 points. The two-tailed students average 7.64 points, and their standard deviation is 1.31 points.

a. **Can you conclude that the mean score μ1 on one-tailed hypothesis test questions is higher than the mean score μ2 on two-tailed hypothesis test questions? State the appropriate null and alternate hypotheses, and then compute the P-value.**

one-tailed   two-tailed

7. $n_x = 80$ $n_y = 80$

$\bar{x} = 7.79$ $\bar{y} = 7.64$

$s_x = 1.06$ $s_y = 1.31$

a) Conclude that μ_1 mean score higher than μ_2? State appropriate hypotheses, then compute P-value.

$H_0: \mu_1 \le \mu_2$ vs $H_1: \mu_1 > \mu_2$ → right-tailed

$$z = \frac{(7.79 - 7.64) - 0}{\sqrt{\frac{1.06^2}{80} + \frac{1.31^2}{80}}} \approx 0.7961$$

$P = P(z > 0.7961) \approx 1 - P(z \le 0.80) = 1 - 0.7881$

$= 0.2119$

$P < 0.05$? No

No. We failed to reject the null hypothesis so we support the claim that the mean score on the one-tailed hypothesis test is higher.

b. **Can you conclude that the mean score µ1 on one-tailed hypothesis test questions differs from the mean score µ2 on two-tailed hypothesis test questions? State the appropriate null and alternate hypotheses, and then compute the P-value.**

b) Can you conclude mean score μ_1 differs from μ_2? State appropriate hypotheses and compute P-value.

$H_0: \mu_1 = \mu_2$ vs $H_1: \mu_1 \neq \mu_2 \rightarrow$ two-tailed

$$z = \frac{(7.79 - 7.64) - 0}{\sqrt{\frac{1.062}{80} + \frac{1.313}{80}}} \approx 0.7961$$

$P = P(Z < -0.80$ or $Z > 0.80) = 2P(Z < -0.80)$
$= 2(0.2119) = 0.4238$
$P < 0.05$? No

No. We failed to reject the null hypothesis so we cannot support the claim that the mean score on the two-tailed hypothesis test differs.

9.

Are low-fat diets or low-carb diets more effective for weight loss? This question was addressed in the article "Comparison of the Atkins, Zone, Ornish, and LEARN Diets for Change in Weight and Related Risk Factors Among Overweight Premenopausal Women: The A TO Z Weight Loss Study: A Randomized Trial" (C. Gardner, A. Kiazand, et al., Journal of the American Medical Association, 2007:969–977). A sample of 77 subjects went on a low-carbohydrate diet for six months. At the end of that time the sample mean weight loss was 4.7 kg with a sample standard deviation of 7.2 kg. A second sample of 79 subjects went on a low-fat diet. Their sample mean weight loss was 2.6 kg with a standard deviation of 5.9 kg.

a. **Can you conclude that the mean weight loss is greater for those on the lowcarbohydrate diet?**

a. low carb   low fat

$n_x = 77$ $n_y = 79$

$\bar{x} = 4.7$ $\bar{y} = 2.6$

$s_x = 7.2$ $s_y = 5.4$

a) Can you conclude mean weight loss greater for low carb?

$H_0: \mu_x \le \mu_y$ vs $H_1: \mu_x > \mu_y$ z right tailed

$$z = \frac{(4.7 - 2.6) - 0}{\sqrt{\frac{7.2^2}{77} + \frac{5.4^2}{79}}} \approx 1.964$$

$p = P(Z > 1.99) = 1 - P(Z \le 1.99) = 1 - 0.9767 \rightarrow 0.0233$

$p < 0.05$? Yes

$\boxed{\text{Yes. P is less than 0.05, so we reject the null hypothesis and conclude that the mean weight loss is greater for low carb diets.}}$

b. **Can you conclude that the mean weight loss on the low-carbohydrate diet is more than 1 kg greater than that of the low-fat diet?**

b) Can you conclude mean weight loss for low carb diet is more than 1kg greater?

claim: $\mu_1 - \mu_2 > 1$

$H_0: \mu_1 - \mu_2 \le 1$ vs $H_1: \mu_1 - \mu_2 > 1$ → right-tailed

$$z = \frac{(4.7 - 2.6) - 1}{\sqrt{\frac{7.2^2}{77} + \frac{5.9^2}{79}}} \approx 1.04$$

$P = P(z > 1.04) = 1 - P(z < 1.04) = 1 - 0.8508 = 0.1492$

$P < 0.05$? No.

$\boxed{\text{No. P is greater than 0.05 so we fail to reject the null hypothesis and cannot conclude the mean weight loss on low carb diets is more than 1kg greater.}}$

10.

In a certain supermarket, a sample of 60 customers who used a self-service checkout lane averaged 5.2 minutes of checkout time, with a standard deviation of 3.1 minutes. A sample of 72 customers who used a cashier averaged 6.1 minutes with a standard deviation of 2.8 minutes.

a. **Can you conclude that the mean checkout time is less for people who use the selfservice lane?**

10. self $\qquad$ cashier

$n_x = 60 \qquad n_y = 72$

$\bar{x} = 5.2 \qquad \bar{y} = 6.1$

$s_x = 3.1 \qquad s_y = 2.8$

a) Can you conclude mean time is less for people who use self-service?

$H_0: \mu_x \geq \mu_y$ vs $H_1: \mu_x < \mu_y \rightarrow$ left-tailed

$$z = \frac{(5.2-6.1)-0}{\sqrt{\frac{3.1^2}{60}+\frac{2.8^2}{72}}} \approx -1.735 \qquad p < 0.05 ? \text{ Yes}$$

$p = P(z < -1.74) = 0.0409$

$\boxed{\text{Yes. We can conclude } \cancel{\text{less}} \text{ mean time is less for people who use self-service. We reject null hypothesis as } p < 0.05.}$

b. **Can you conclude that if everyone used the selfservice lane, that the mean checkout time would decrease? Consider the number of items checked out when formulating your answer.**

No. If everyone used the self-service lane, there is no guarantee that the mean time will decrease as the customers that are now included could have more items. Obviously, the more items that are checked, the more time that will be required.