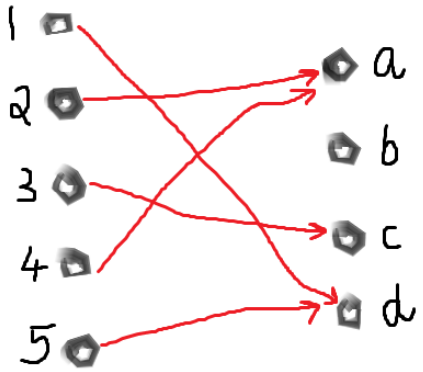
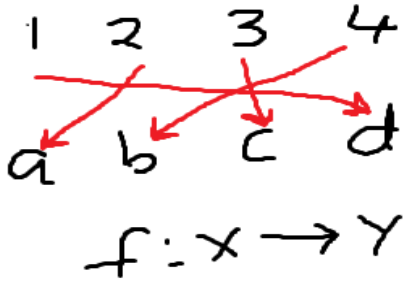
Functions

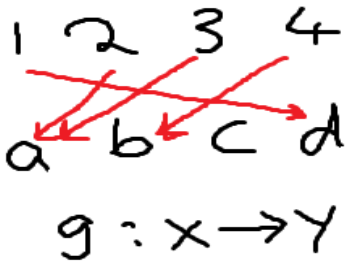
1) Define what is a function with example? The drawing below shows the arrow diagram for a function f. Give answers for what is domain, codomain, and range of function f.

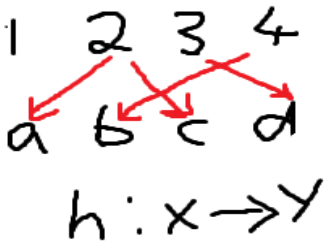


2) (a) Which of the following diagrams represent a function? Explain in detail.

Let X= {1, 2, 3, 4} and Y= {a, b, c, d}.







(b) Are the three expressions given below well-defined functions from R to R? Explain your answer in detail.

$$f(x) = \frac{1}{x-2}$$

$$g(x) = \sqrt{x^2+2}$$

$$h(x) = \pm\sqrt{x^2+5}$$

3) (a) Consider the function f: A→A Given by f (0) = 0 and f (a+1) = f(a)+2a+1. **Find f (6).**

(b) Give recursive definitions for the functions described below.

(i) f: B→B gives the number of butterflies in your terrarium 'b' years after you built it, assuming you started with 3 butterflies and the number of butterflies doubles each year.

(ii) g: B→B gives the number of Punches you do 'b' days after you started your Punching challenge, assuming you could do 7 Punches on day zero and you can do 2 more Punches each day.

4) (i) The following functions have **{a, b, c, d, e}** as both their domain and codomain. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

(a) $f = \begin{pmatrix} a & b & c & d & e \\ e & e & e & e & e \end{pmatrix}$

(b) $f = \begin{pmatrix} a & b & c & d & e \\ b & c & a & e & d \end{pmatrix}$

(Hint: $\begin{array}{c|cccc} x & 0 & 1 & 2 & 3 & 4 \\ \hline f(x) & 3 & 3 & 2 & 4 & 1 \end{array} \Rightarrow f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 3 & 2 & 4 & 1 \end{pmatrix}$.)

(ii) The following functions have **{1, 2, 3, 4, 5}** as both their domain and codomain. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

(c) $f(x) = 6 - x$

(d) $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ (x+1)/2 & \text{if } x \text{ is odd} \end{cases}$

5) Use the definition of the functions **f** below to **Compute f (2.2), f (2.9), f (2.5), f (2), and f (3).**

(a) $f(x) = \left\lceil \frac{x}{2} + 2x + 1 \right\rceil$

(b) $f(x) \left\lfloor x + 1.5 \right\rfloor$

6) (a) Define five types of functions with examples.

(b) match the following relative sizes of the domain (D) and target (T) of functions:

1) $|D| \geq |T|$ (i) bijection

2) $|D| \leq |T|$ and $|D| \geq |T|$ (ii) one-to-one
$|D| = |T|$

3) $|D| \leq |T|$ (iii) onto

(c) At the end of the semester, professor assigns letter grades to each of his students. Is this a function? If so, what sets make up the domain and codomain, and is the function injective, surjective, bijective, or neither?

7) (a) Given g = {(4, x), (5, y), (6, w)}, a function from X = {4, 5, 6} to Y = {w, x, y, z} and

f = {(w, b), (x, b), (y, d), (z, a)}, a function from Y to Z = {a, b, c, d}.Write as a set of ordered pairs and draw the arrow diagram of (f o g).

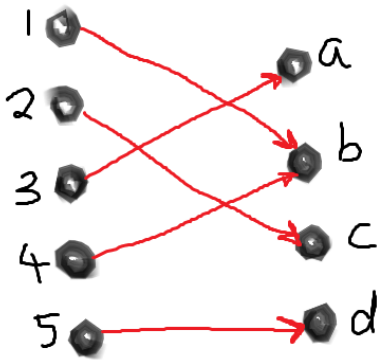
(b) Let *f* and *g* be functions from the positive real numbers to the positive real numbers defined by the equations given below (observe the brackets carefully). Find the compositions of (f o g), (g o f),

(f o f) and (g o g).

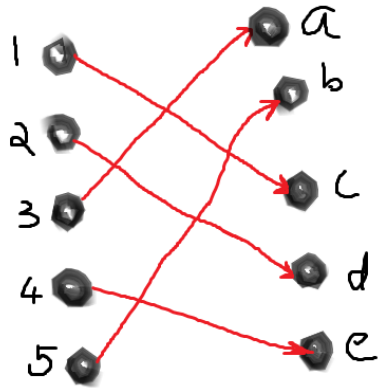
$$f(x) = \lfloor 3x \rfloor, \; g(x) = x^2$$

8) Each of the arrow diagrams below define a function f. For each arrow diagram, indicate whether f^-1 (f inverse) is well-defined. If f^-1 is not well-defined, indicate why. If f^-1 is well-defined, give an arrow diagram showing f^-1.

(a)



(b)



9) Answer the following questions using logarithms**:**

(a) $6^x = 45$, find x (approx)

(b) $\log_5 10 + \log_5 6 = ?$

(c) $\log_2 45 - \log_2 9 = ?$

(d) $\log_9 16 \Rightarrow \boxed{?} \log_9 4$

fill the box

(e) $\log_7 10 + \log_7 8 - \log_7 4 = ?$

10) (a) Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(i) s: Z → Z, where s(x) = x^3.
h: Z → Z, where h(x) = |x|^3.

(ii) s: Z → Z, where s(x) = x^4.
h: Z → Z, where h(x) = |-x|^4

(b) Express the range of function g.

Let A = {2, 3, 4, 5, 6}.
g: A → Z such that g(x) = 2x + x^2 - 1.