1 Applications of Derivatives

1. Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2.

2. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

3. Find the critical numbers of the function $g(y) = \frac{y-1}{y^2-y+1}$.

4. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a positive constant called the coefficient of friction and where $0 \leq \theta \leq \pi/2$. Show that F is minimized when $\tan\theta = \mu$.

5. Consider the function $f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$.
(a) Find the intervals on which f is increasing or decreasing.
(b) Find the local maximum and minimum values of f.
(c) Find the intervals of concavity and the inflection points.

6. Sketch a graph of the function $y = x\sqrt{2 - x^2}$.

7. A cone shaped paper drinking cup is to be made to hold 27 cm^3 of water. Find the height and radius of the cup that will use the smallest amount of paper.

2 Integration

1. Find the general antiderivative of $f(x) = x^{3.4} - 2x^{\sqrt{2}-1}$.

2. Find the function f if $f'''(x) = \cos x$ and $f(0) = 1, f'(0) = 2, f''(0) = 3$.

3. If $f(x) = x^2 - 4$, $0 \leq x \leq 3$, find the Riemann sum with $n = 6$, taking the sample points to be midpoints. What does the Riemann sum represent? Illustrate with a diagram.

4. Use part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

$$g(x) = \int_1^x \ln(1 + t^2)dt$$

5. Evaluate the integral $\int_{-1}^{2}(3u-2)(u+1)du$

6. Find the area of a shaded region that is bounded by the y-axis, the line $y=1$, and the curve $y=\sqrt[4]{x}$. Find the area by writing x as a function of y and integrating with respect to y.

7. Evaluate the indefinite integral.

$$\int \frac{dt}{\cos^2 t\sqrt{1-\tan t}}$$