Math 100 - Assignment 1 (56) pts

1) a) (15pts)

$$\left[\begin{array}{ccc|c} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} & '' & & \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -2 \end{array}\right] \rightarrow \text{no solution}$$

1st: $R_1 \leftrightarrow R_2$

2nd: $-2R_1 + R_3 \rightarrow R_3$

$R_2 + R_3 \rightarrow R_3$

$0 = -2 \rightarrow$ no.

b)

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right] \rightarrow \left[\begin{array}{cccc} & '' & & \\ & '' & & \\ 0 & 0 & 5 & -5 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$, $-2R_1 + R_3 \rightarrow R_3$

$-2R_2 + R_3 \rightarrow R_3$

$\frac{1}{5}R_3 \rightarrow R_3$

(left) (side)

$x_1 = 5,\ x_2 = 3,\ x_3 = -1$

$-5R_3 + R_2 \rightarrow R_2$

$3R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right] \rightarrow$$

c)

$$\left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} & '' & & \\ & '' & & \\ 0 & 0 & -5 & -10 \end{array}\right] \rightarrow -\frac{1}{5}R_3 \rightarrow R_3$$

$\frac{1}{2}R_1 \rightarrow R_1$

$-3R_1 + R_3 \rightarrow R_3$

$-6R_2 + R_3 \rightarrow R_3$

$-2R_3 + R_2 \rightarrow R_2$

$3R_3 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & +2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & +2 \end{array}\right]$$

$x_1 = +2$

$x_2 = -1$

$x_3 = +2$

2) 3pts

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

3) a) REF b) Echelon form

2pts

4) 5pts

$$\left[\begin{array}{cccc} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cccc} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & -3 & -12 & -18 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow$$

$-2R_1 + R_2 \rightarrow R_2$

$-4R_1 + R_3 \rightarrow R_3$

$R_2 \leftrightarrow R_3$, $-\frac{1}{3}R_3 \rightarrow R_3$, $-\frac{1}{3}R_2 \rightarrow R_2$

$-4R_3 + R_2 \rightarrow R_2$

$-4R_3 + R_1 \rightarrow R_1$

$(\ -4\ -8)$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$-2R_2 + R_1 \rightarrow R_1$

25

(5) $\frac{31}{}$

a) 5pt

$$\begin{bmatrix} 1 & -1 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix} \xrightarrow{2R_1+R_2\to R_2} \begin{bmatrix} 1 & -1 & -1 & 4 \\ 0 & 2 & -7 & 14 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2\to R_2} \begin{bmatrix} 1 & -1 & -1 & 4 \\ 0 & 1 & -\frac{7}{2} & 7 \end{bmatrix}$$

$-\frac{2}{2}$

$$\xrightarrow{R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{9}{2} & 11 \\ 0 & 1 & -\frac{7}{2} & 7 \end{array}\right] \to \begin{matrix} x_1 - \frac{9}{2}x_3 = 11 \\ x_2 - \frac{7}{2}x_3 = 7 \end{matrix} \to \begin{bmatrix} x_1 = 11 + \frac{9}{2}x_3 \\ x_2 = 7 + \frac{7}{2}x_3 \end{bmatrix}$$

x_3 is free

b)

$$\left[\begin{array}{ccc|c} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{array}\right] \xrightarrow[\frac{1}{2}R_3\to R_3]{\frac{1}{3}R_2\to R_2} \left[\begin{array}{ccc|c} 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \end{array}\right] \xrightarrow[R_1+R_3\to R_3]{R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to 0=0 ✓$$

5pt

$3x_1 - 2x_2 + 4x_3 = 0 \to \left[x_1 = \frac{2}{3}x_2 - \frac{4}{3}x_3\right]$

x_2, x_3 are free

13pt

(6) a) T

2 b) False, you Do need same number of rows, but they

1 c) T

2 d) F, if equiv. then can make the other via row ops see c.

1 e) T

2 f) F → Red. ech. Matrix are unique

2 g) F → need [0 0 0 0 5] different

2 h) F → each choice gives ~~the~~ a solution → infinite

3pt.

(7) No, will have a free variable → infinite sol's

3 8) There are no free variables, pivots yield a leading 1, every variable is accounted f

2 9) (Varies) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & \pi \end{array}\right]$