STAT 50 HW #9

Section 4.4 #'s 1, 5, 6, 9, 10, 11, 13, 15

1.
Twenty-five automobiles have been brought in for service. Fifteen of them need tuneups and ten of them need new brakes. Nine cars are chosen at random to be worked on. What is the probability that three of them need new brakes?

P(X = 3)?
Let X ~ Bin(9, 0.4).

P(X = 3) = $\binom{9}{3}(0.4)^3(1-0.4)^6$
$\binom{9}{3}$ or ${}_9C_3 = \frac{9!}{(9-3)!3!} = 84$
84((0.4)^3)((0.6)^6) = 0.2508

<u>P(X = 3) = 0.2508</u>

5.
Refer to Exercise 4. Let Y denote the number of days up to and including the third day on which a red light is encountered.

> **4.** A traffic light at a certain intersection is green 50% of the time, yellow 10% of the time, and red 40% of the time. A car approaches this intersection once each day. Let X represent the number of days that pass up to and including the first time the car encounters a red light. Assume that each day represents an independent trial.

a. **Find P(Y = 7).**

5. a) $G = 0.5$ $\quad$ X = # of days of red light

$Y = 0.1$ $\quad$, assume independence

$R = 0.4$

Y = # of days up to and including 3rd day on which a red light is encountered

$P(Y = 7)$?

using $P(X=x) = \binom{x-1}{r-1}(p)^r(1-p)^{x-r}$

$\frac{6\cdot5\cdot4!}{2!\,4!} = 15$ $\qquad$ $\binom{7-1}{3-1}(0.4)^3(0.6)^4$

$\binom{6}{2}(0.4)^3(0.6)^4$

$\boxed{0.1244}$

b. Find μ_y.

b) μ_Y $\quad$ $Y \sim NB(r, p) = NB(3, 0.4)$

$\mu_Y = E(Y_1) + E(Y_2) + E(Y_3) = r \cdot \frac{1}{p} = 3 \cdot \frac{1}{0.4} = \boxed{7.5}$

c. Find σ_Y^2.

g Find σ_V^2

$\sigma_V^2 = \sigma_{Y_1}^2 + \sigma_{Y_2}^2 + \sigma_{Y_3}^2 = \frac{r(1-p)}{p^2} = \frac{3(1-0.4)}{0.4^2}$

$= \boxed{11.25}$

6.

Refer to Exercise 4. What is the probability that in a sequence of 10 days, four green lights, one yellow light, and five red lights are encountered?

4. A traffic light at a certain intersection is green 50% of the time, yellow 10% of the time, and red 40% of the time. A car approaches this intersection once each day. Let X represent the number of days that pass up to and including the first time the car encounters a red light. Assume that each day represents an independent trial.

$P(G=4, Y=1, R=5)$

$Y_1, \dots, Y_k \sim MN(n, p_1, \dots, p_k)$

$= \frac{10!}{4!1!5!}(0.5)^4(0.1)^1(0.4)^5 = \boxed{0.08064}$

$\frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5! \, 1!} = 1260$

9.

A system is tested for faults once per hour. If there is no fault, none will be detected. If there is a fault, the probability is 0.8 that it will be detected. The tests are independent of one another.

a. If there is a fault, what is the probability that it will be detected in 3 hours or less?

a) $p = 0.8$ $X \sim Geom(0.8) \Rightarrow P(X=1) + P(X=2) + P(X=3)$

$P(X \leq 3) = P(n=1, n=2, n=3)$

$= (0.8)^1(0.2)^0 + (0.8)(0.2)^1 + (0.8)(0.2)^2$

$= \boxed{0.992}$

b. **Given that a fault has gone undetected for 2 hours, what is the probability that it will be detected in the next hour?**

b) Prob that detected in next hour, given that fault was undetected for 2 hours

$P(n=3 \mid n>2) = \frac{P(n=3) \cap P(n>2)}{P(n>2)} = \frac{P(n=3)}{P(n>2)}$

$p = 0.8$

$P(n=3) = p(1-p)^{n-1} = 0.8(0.2)^2 = 0.032$

$P(n>2) = 1 - P(n \le 2) = 1 - [0.8(0.2)^{(2-1)} + 0.8(2)^{(1-1)}]$

$= 0.04$

$P(n=3 \mid n>2) = \frac{0.032}{0.04} = \boxed{0.8}$

c. **What is the mean number of tests that must be conducted in order to detect a fault?**

c) $E(n) = \frac{r}{p} = \frac{1}{p} = \frac{1}{0.8} = \boxed{1.25}$

Because $X \sim \text{Geom}(0.8)$

10.

A computer program has a bug that causes it to fail once in every thousand runs, on average. In an effort to find the bug, independent runs of the program will be made until the program has failed five times.

a. **What is the mean number of runs required?**

a) $P(\frac{1}{1000})$ Assuming Runs to [illegible]

$E(X) = \frac{r}{p} = \frac{5}{0.001} = \boxed{5000}$

b. **What is the standard deviation of the number of runs required?**

b) Std dev? $X \sim \text{Geom}(p)$

$$\sigma_X^2 = \frac{r(1-p)}{p^2} = \frac{5(1-0.001)}{0.001^2} = 4995000$$

$$\sigma_X = \sqrt{4995000} = \boxed{2234.94}$$

11.

In a lot of 15 truss rods, 12 meet a tensile strength specification. Four rods are chosen at random to be tested. Let X be the number of tested rods that meet the specification.

a. **Find P(X = 3).**

11. 15 truss rods — $X \sim \text{Bin}(n, p)$

12 meet spec. — $X \sim \text{Bin}(4, 0.8)$

4 chosen to test

X = # of tested rods that meet spec.

a) $P(X=3)$? — $\frac{12}{15} = 0.8$

$$\binom{4}{3}(0.8)^3(0.2)^1 = \boxed{0.4096}$$

$$\frac{4 \cdot \cancel{3!}}{\cancel{3!}} = 4$$

b. **Find µX.**

b) μ_X?

$$\mu_X = n \cdot p = 4 \cdot 0.8 = \boxed{3.2}$$

c. **Find σ X.**

c) 15.2

$V_x = 4(0.8)(0.2) = 0.64$

σ^2_x $\qquad \sigma_x = \sqrt{0.64} = \boxed{0.8}$

13.

Ten items are to be sampled from a lot of 60. If more than one is defective, the lot will be rejected. Find the probability that the lot will be rejected in each of the following cases.

a. The number of defective items in the lot is 5.

lot of 60 $\qquad X \sim H(60, 5, 10)$

10 chosen

>1 defective means rejected lot

a) $N = 60$ $\qquad P(X>1) = 1 - P(X \le 1) = 1 - P(X=0) - P(X=1)$

$R = 5$

$n = 10$

$$\frac{\binom{5}{0}\binom{60-5}{10-0}}{\binom{60}{10}} = \frac{1\left(\frac{55!}{10!45!}\right)}{\frac{60!}{10!50!}} = 0.387$$

cont'd

cont'd

$$\frac{\binom{5}{1}\binom{60-5}{10-1}}{\binom{60}{10}} = \frac{5\left(\frac{55!}{9!46!}\right)}{\frac{60!}{10!50!}} = 0.4216$$

$1 - (0.387 + 0.4216) = \boxed{0.1904}$

b. The number of defective items in the lot is 10.

(b) $1 - P(X=0) - P(X=1)$

$$\frac{\binom{10}{0}\binom{60-10}{10-0}}{\binom{60}{10}} = \frac{1\left(\frac{50!}{10!\,40!}\right)}{\frac{60!}{10!\,50!}} = 0.1362$$

$$\frac{\binom{10}{1}\binom{60-10}{10-1}}{\binom{60}{10}} = \frac{10\left(\frac{50!}{9!\,41!}\right)}{\frac{60!}{10!\,50!}} = 0.3323$$

$$1-(0.1362-0.3323) = \boxed{0.5314}$$

c. **The number of defective items in the lot is 20.**

(c) $1 - P(X=0) - P(X=1)$

$$\frac{\binom{20}{0}\binom{60-20}{10-0}}{\binom{60}{10}} = \frac{1\left(\frac{40!}{10!\,30!}\right)}{\frac{60!}{10!\,50!}} = 0.011$$

$$\frac{\binom{20}{1}\binom{60-20}{10-1}}{\binom{60}{10}} = \frac{20\left(\frac{40!}{9!\,31!}\right)}{\frac{60!}{10!\,50!}} = 0.0725$$

$$1-(0.011+0.0725) = \boxed{0.9162}$$

15.

At a certain fast-food restaurant, 25% of drink orders are for a small drink, 35% for a medium, and 40% for a large. A random sample of 20 orders is selected for audit.

a. **What is the probability that the numbers of orders for small, medium, and large drinks are 5, 7, and 8, respectively?**

a) $P(S=5, M=7, L=8)$?

$$\frac{20!}{5!\,7!\,8!}(0.25)^5(0.35)^7(0.40)^8 = \boxed{0.0411}$$

b. What is the probability that more than 10 orders are for large drinks?

b) $P(L>10)$? $X \sim Bin(20, 0.40)$

$$P(L>10) = 1 - P(L \leq 10) = 1 - [P(0) \ldots + P(10)]$$

cont'd

20, .4

00003

$$P(0) = \binom{20}{0}(0.4)^0(0.6)^{20} \approx 0.00003$$

$$P(1) = \binom{20}{1}(0.4)^1(0.6)^{19} \approx 0.0004$$

$$P(2) = \binom{20}{2}(0.4)^2(0.6)^{18} \approx 0.0031$$

$$P(3) = \binom{20}{3}(0.4)^3(0.6)^{17} \approx 0.0123$$

$$P(4) = \binom{20}{4}(0.4)^4(0.6)^{16} \approx 0.0349$$

$$P(5) = \binom{20}{5}(0.4)^5(0.6)^{15} \approx 0.0746$$

$$P(6) = \binom{20}{6}(0.4)^6(0.6)^{14} \approx 0.1244$$

$$P(7) = \binom{20}{7}(0.4)^7(0.6)^{13} \approx 0.1659$$

$$P(8) = \binom{20}{8}(0.4)^8(0.6)^{12} \approx 0.1797$$

$$P(9) = \binom{20}{9}(0.4)^9(0.6)^{11} \approx 0.1597$$

$$P(10) = \binom{20}{10}(0.4)^{10}(0.6)^{10} \approx 0.1171$$

$$1 - (0.8725) \approx \boxed{0.1275}$$