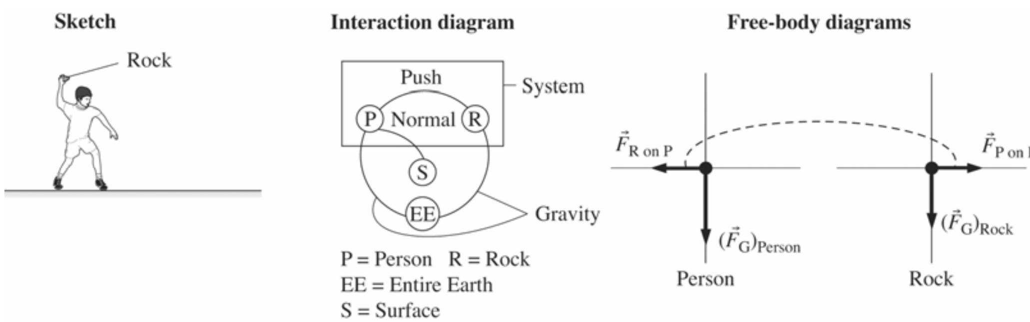
7

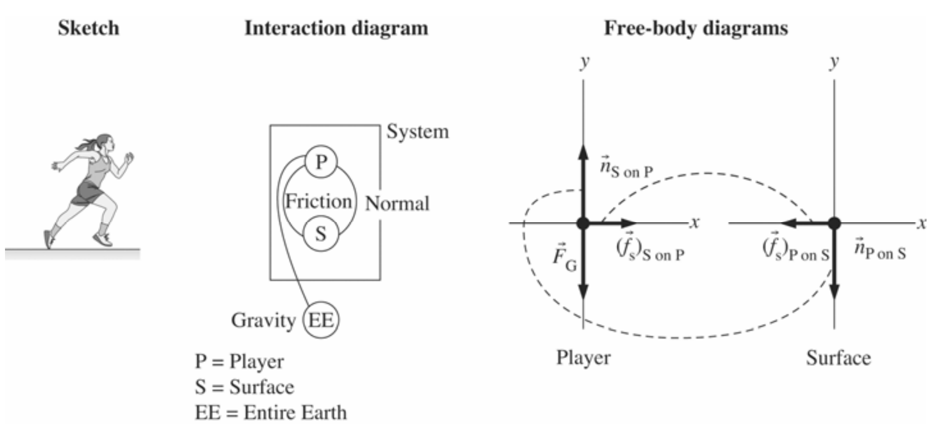
NEWTON'S THIRD LAW

Conceptual Questions

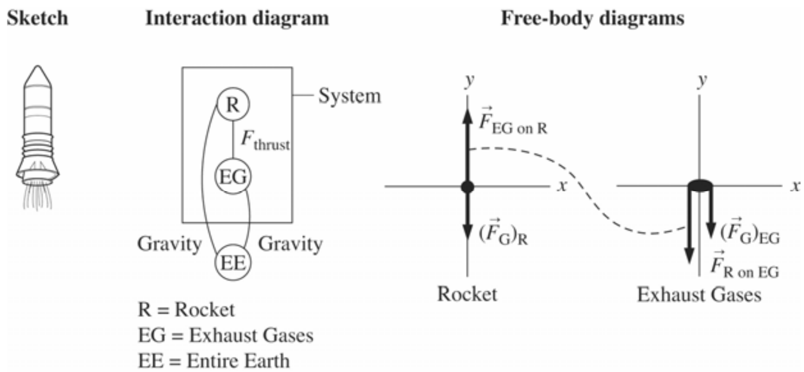
7.1. If you were to throw the rocks in the opposite direction you wanted to go, you would be pushed by the rocks in the right direction. Throwing the rocks requires a force to accelerate them (Newton's second law). So you exert a force on the rock in one direction and the rock exerts an equal force on you in the opposite direction (Newton's third law). This force will cause you to slide along the ice in the opposite direction that you threw the rock. Note that you will move most efficiently when you use a horizontal force, which means that you throw the rock horizontally.



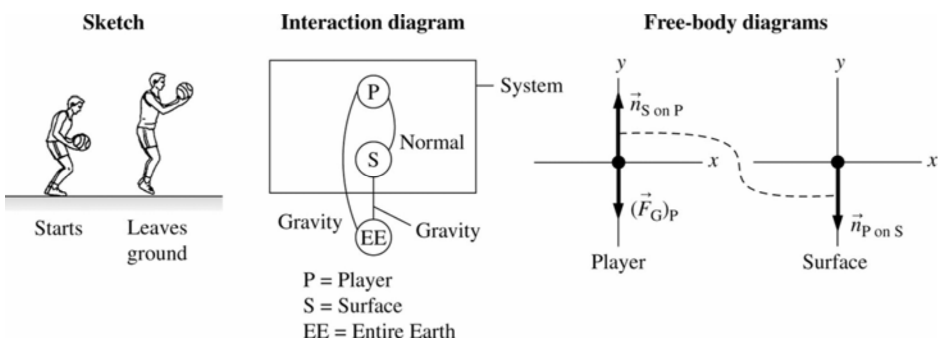
7.2. The sprinter pushes backward on the ground, which pushes back (forward) on her. This is the only horizontal force on the sprinter, so she accelerates forward.



7.3. The rocket forces the exhaust gases down, and the hot gases push up on the rocket. The two forces are a Newton's third-law pair. The rocket accelerates upward because the force of the exhaust gases on the rocket is greater than the force of gravity.



7.4. The player pushes down on the floor, which pushes back up on him. The player accelerates upward because the force of his push is greater than the force of gravity.



7.5. Newton's third law tells us that the force of the mosquito on the car has the same magnitude as the force of the car on the mosquito.

7.6. The mosquito has a much smaller mass than the car, so the magnitude of the interaction force between the car and mosquito, although equal on each, causes the mosquito to have a much larger acceleration. In fact, the acceleration is usually fatal to the mosquito.

7.7. Newton's third law tells us that the magnitudes of the forces are equal. The acceleration of the truck and car are determined by the *net* force on each.

7.8. The force of the wagon on the girl acts on the girl, whereas the force of the girl on the wagon acts on the wagon. The wagon's motion is determined by the net force acting on it, so if the girl pulls hard enough to overcome any other opposing forces acting *on the wagon*, the wagon will move forward. So try saying, "But, my dear, the *net* force on the wagon determines if it will move forward. The forces you mention act on different objects, and so cannot cancel."

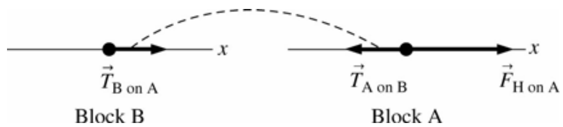
7.9. The *net* force on each team determines that team's motion. The net horizontal force on each team is the difference between the rope's pull and friction with the ground. So the team that wins the tug-of-war is not the team that pulls harder, but the team that is best able to keep from sliding along the ground.

7.10. This technique will not work because the magnet is part of the cart, not external to it. The forces between the magnet and cart have the same magnitude but act in the opposite directions. Therefore, although the two objects may accelerate toward each other, the cart-magnet system as a whole will not move. (Actually, it would be more precise to say that the center of mass of the cart-magnet system does not move.)

7.11. The scale reads 5 kg. The left-hand mass performs a function no different than the ceiling would if the rope were attached to the ceiling (i.e., both pull upward with the force required to suspend 5 kg). The force of gravity acting on the right-hand mass provides the downward force on the spring scale that the spring scale converts to mass in its display.

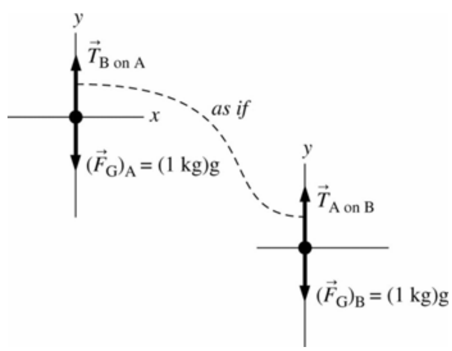
7.12. The scale reads 5 kg. The left-hand mass performs a function no different than the wall would if the rope were attached to the wall (i.e., both pull leftward with the force required to suspend 5 kg). The right-hand mass provides the rightward force on the spring scale that the spring scale converts to mass and displays.

7.13.



The figure shows the horizontal forces on blocks B and A using the massless-string approximation in the absence of friction. The hand must accelerate both blocks A and B, so more force is required to accelerate the greater mass. Thus the force of the string on B is smaller than the force of the hand on A.

7.14.



The pulley will not rotate. As shown in the free-body diagrams above, the force of gravity pulls down equally on both blocks so the tension forces, which act as if they were a Newton third-law pair, pull up equally on each with the same magnitude force as the force of gravity. The net force on each block is therefore zero, so they do not move and the pulley does not rotate.

7.15. Block A's acceleration is greater in case b. In case a, the hanging 10 N must accelerate both the mass of A and its own mass, leading to a smaller acceleration than case b, where the entire 10 N force accelerates the mass of block A.

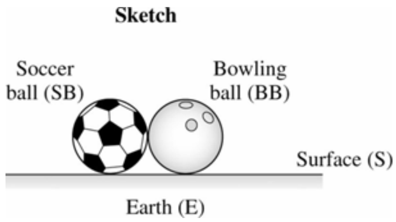
Case a	Case b
$10\text{ N} = (M_A + M_{10\text{ N}})a$ $a = \dfrac{10\text{ N}}{(M_A + M_{10\text{ N}})}$	$10\text{ N} = M_A a$ $a = \dfrac{10\text{ N}}{M_A}$

Exercises and Problems

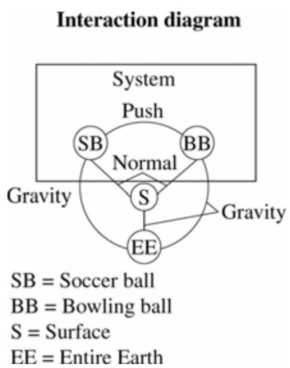
Exercises

Section 7.2 Analyzing Interacting Objects

7.1. Visualize:

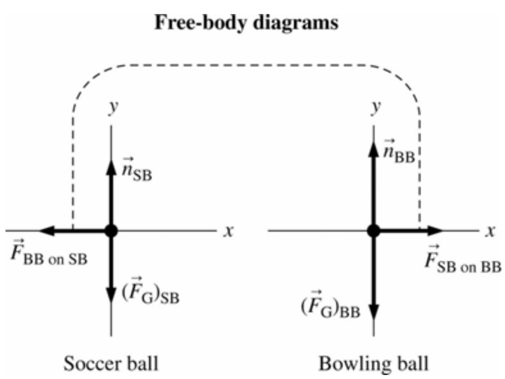


Solve: (a) Both the bowling ball and the soccer ball have a normal force from the surface and gravitational force on them. The interaction forces between the two are equal and opposite.



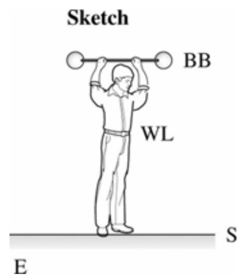
(b) The system consists of the soccer ball and bowling ball, as indicated in the figure.

(c)

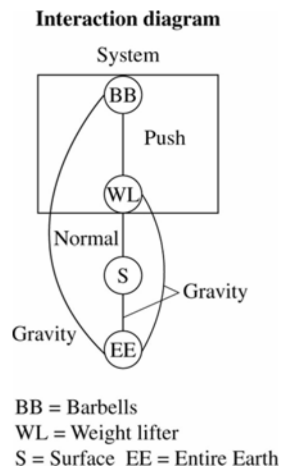


Assess: Even though the soccer ball bounces back more than the bowling ball, the forces that each exerts on the other are part of an action/reaction pair, and therefore have equal magnitudes. Each ball's acceleration due to the forces on it is determined by Newton's second law, $a = F_{net}/m$, which depends on the mass. Since the masses of the balls are different, their accelerations are different.

7.2. Visualize:

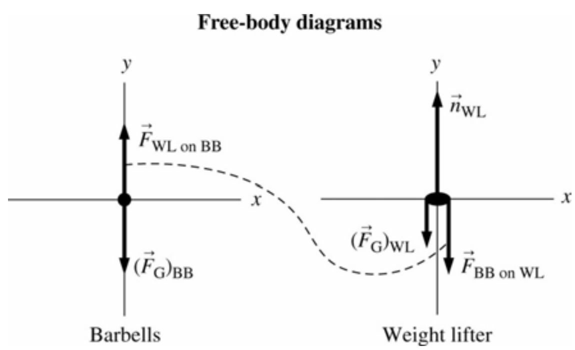


Solve: (a) The weight lifter is holding the barbell in dynamic equilibrium as he stands up, so the net force on the barbell and on the weight lifter must be zero. The barbells have an upward contact force from the weight lifter and the gravitational force downward. The weight lifter has a downward contact force from the barbells and an upward one from the surface. Gravity also acts on the weight lifter.

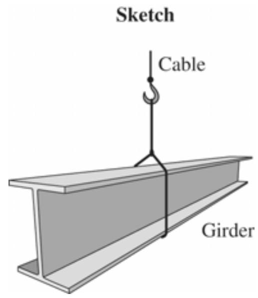


(b) The system is the weight lifter and barbell, as indicated in the figure.

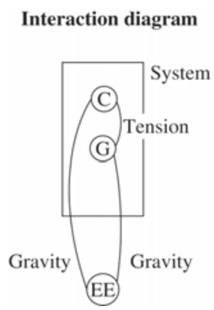
(c)



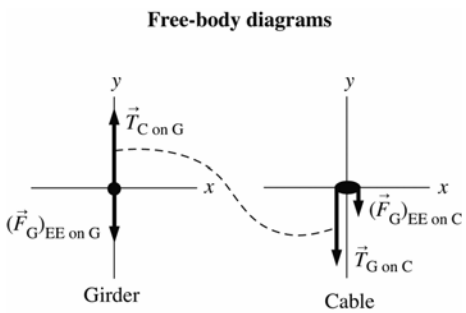
7.3. Visualize:



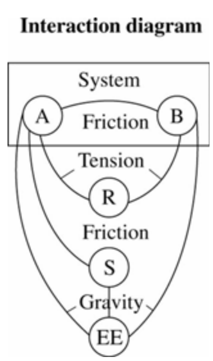
Solve: (a) Both the cable and girder have gravitational forces on them from the Entire Earth. The interaction forces (tension) between the two are equal and opposite.



(b) The system consists of the cable and girder, as indicated in the figure.
(c)

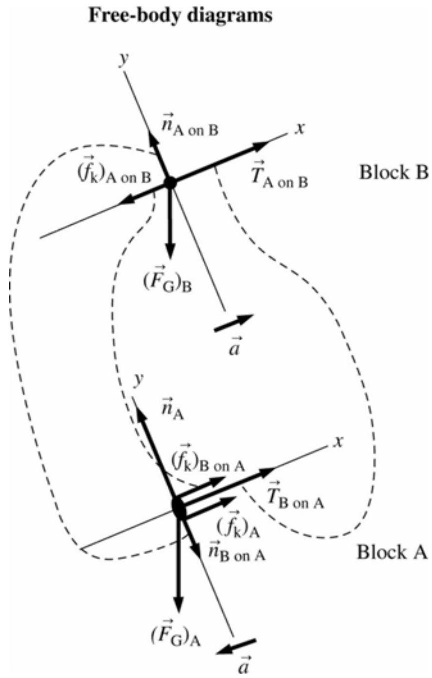


7.4. Visualize: Please refer to Figure EX7.4.
Solve: (a) Gravity acts on both blocks. Block A is in contact with the floor and experiences a normal force and friction. The string tension is the same on both blocks since the rope and pulley are massless and the pulley is frictionless. There are two third-law pair of forces at the surface where the two blocks touch. block B pushes against block A with a normal force, while block A pushes back against block B. There is also friction between the two blocks at the surface.



(b) A string that will not stretch constrains the two blocks to accelerate at the same rate but in opposite directions. Block A accelerates down the incline with an acceleration equal in magnitude to the acceleration of block B up the incline. The system consists of the two blocks, as indicated in the previous figure.

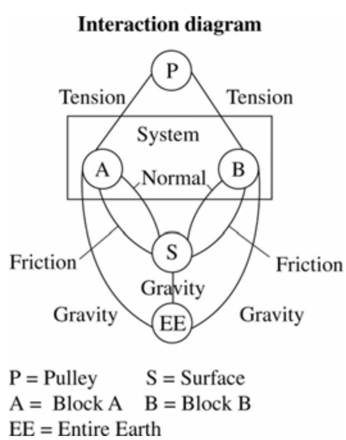
(c)



Assess: The inclined coordinate system allows the acceleration *a* to be purely along the *x*-axis. This is convenient because it simplifyies the mathematical expression of Newton's second law.

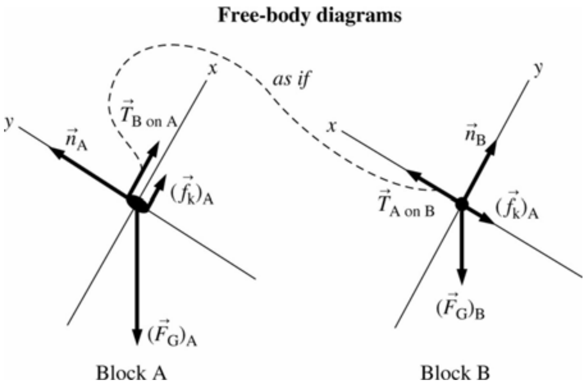
7.5. Visualize: Please refer to Figure EX7.5.

Solve: (a) For each block, there is a gravitational force due to the earth, a normal force and kinetic friction due to the surface, and a tension force due to the rope.



(b) The tension in the massless ropes over the frictionless pulley is the same on both blocks. Block A accelerates down the incline with the same magnitude acceleration that block B has up the incline. The system consists of the two blocks, as indicated in the figure.

(c)

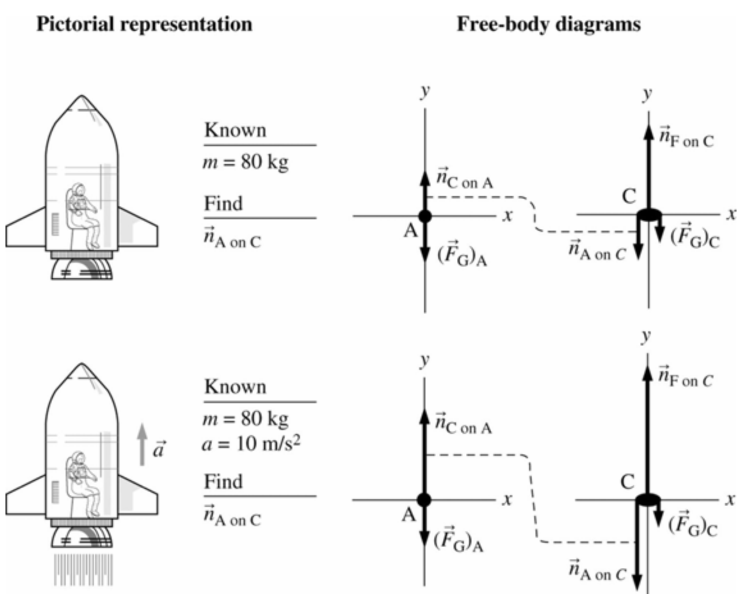


Assess: The inclined coordinate systems allow the acceleration *a* to be purely along the *x*-axis. This is convenient since then one component of *a* is zero, simplifying the mathematical expression of Newton's second law.

Section 7.3 Newton's Third Law

7.6. Model: We will model the astronaut and the chair as particles. The astronaut and the chair will be denoted by A and C, respectively, and they are separate systems. The launch pad is a part of the environment.

Visualize:



Solve: **(a)** Newton's second law for the astronaut is

$$\Sigma(F_{\text{on A}})_y = n_{\text{C on A}} - (F_G)_A = m_A a_A = 0 \text{ N} \quad \Rightarrow \quad n_{\text{C on A}} = (F_G)_A = m_A g$$

By Newton's third law, the astronaut's force on the chair is

$$n_{\text{A on C}} = n_{\text{C on A}} = m_A g = (80 \text{ kg})(9.8 \text{ m/s}^2) = 7.8\times10^2 \text{ N}$$

(b) Newton's second law for the astronaut is:

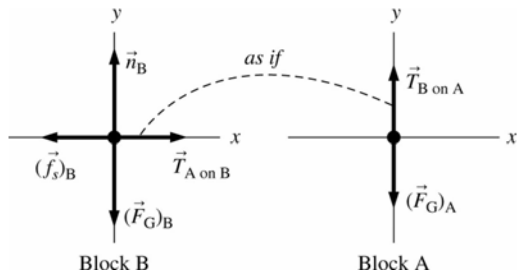
$$\Sigma(F_{\text{on A}})_y = n_{\text{C on A}} - (F_G)_A = m_A a_A \quad \Rightarrow \quad n_{\text{C on A}} = (F_G)_A + m_A a_A = m_A(g + a_A)$$

By Newton's third law, the astronaut's force on the chair is

$$n_{\text{A on C}} = n_{\text{C on A}} = m_A(g + a_A) = (80 \text{ kg})(9.8 \text{ m/s}^2 + 10 \text{ m/s}^2) = 1.6\times10^3 \text{ N}$$

Assess: This is a reasonable value because the astronaut's acceleration is greater than *g*.

7.7. Visualize: Please refer to Figure EX7.7.



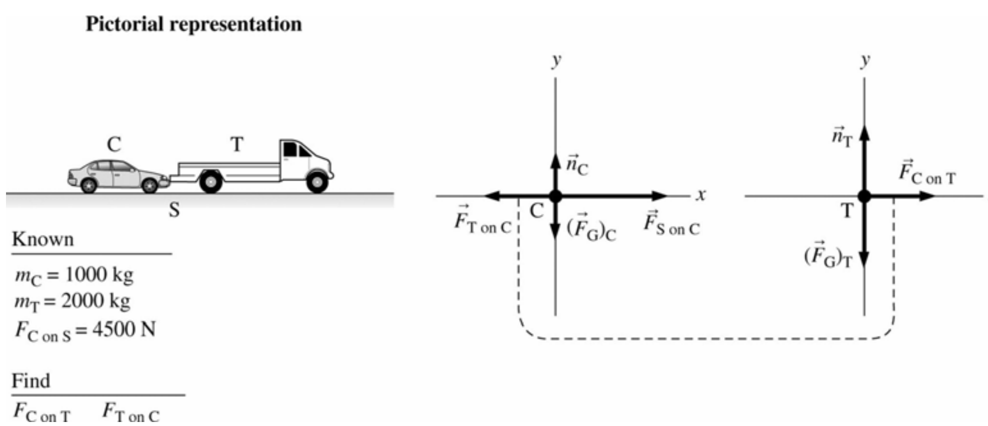
Solve: Since the ropes are massless we can treat the tension force they transmit as a Newton's third law force pair on the blocks. The connection shown in Figure EX7.7 has the same effect as a frictionless pulley on these massless ropes. The blocks are in equilibrium as the mass of A is increased until block B slides, which occurs when the static friction on B is at its maximum value. Applying Newton's first law to the vertical forces on block B gives $n_B = (F_G)_B = m_B g$. The static friction force on B is thus

$$(f_s)_B = \mu_s n_B = \mu_s m_B g.$$

Applying Newton's first law to the horizontal forces on B gives $(f_s)_B = T_{A\text{ on }B}$, and the same analysis of the vertical forces on A gives $T_{B\text{ on }A} = (F_G)_A = m_A g$. Since $T_{A\text{ on }B} = T_{B\text{ on }A}$, we have $(f_s)_B = m_A g$, so

$$\mu_s m_B g = m_A g \quad \Rightarrow \quad m_A = \mu_s m_B = (0.60)(20\text{ kg}) = 12\text{ kg}$$

7.8. Model: Model the car and the truck as particles denoted by the symbols C and T, respectively. Denote the surface of the ground by the symbol S.
Visualize:



Solve: (a) The *x*-component of Newton's second law for the car gives

$$\sum(F_{\text{on }C})_x = F_{S\text{ on }C} - F_{T\text{ on }C} = m_C a_C$$

The *x*-component of Newton's second law for the truck gives

$$\sum(F_{\text{on }T})_x = F_{C\text{ on }T} = m_T a_T$$

Using $a_C = a_T \equiv a$ and $F_{T\text{ on }C} = F_{C\text{ on }T}$, we get

$$(F_{C\text{ on }S} - F_{C\text{ on }T})\left(\frac{1}{m_C}\right) = a \quad\text{and}\quad (F_{C\text{ on }T})\left(\frac{1}{m_T}\right) = a$$

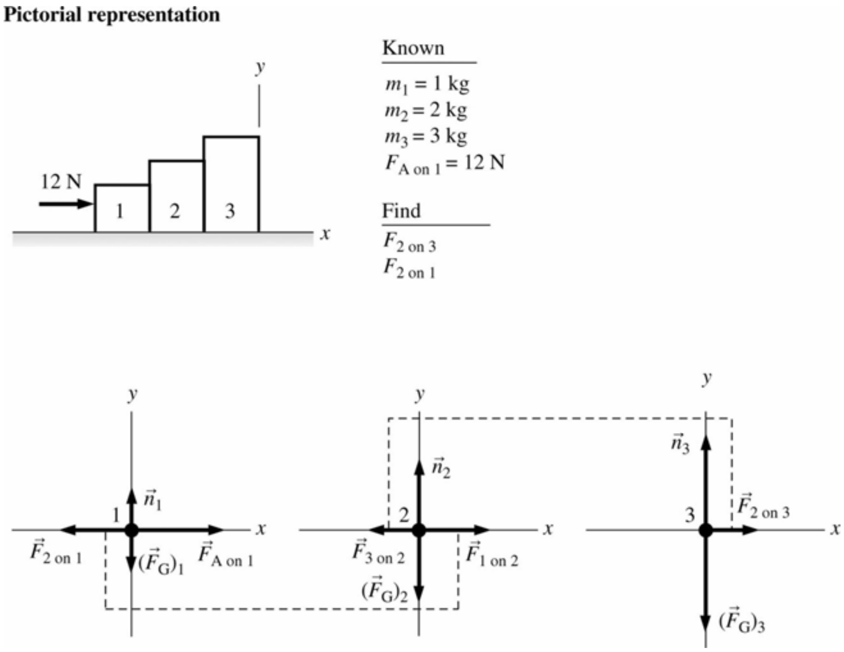
Combining these two equations,

$$(F_{\text{C on S}} - F_{\text{C on T}})\left(\frac{1}{m_{\text{C}}}\right) = (F_{\text{C on T}})\left(\frac{1}{m_{\text{T}}}\right) \quad \Rightarrow \quad F_{\text{C on T}}\left(\frac{1}{m_{\text{C}}} + \frac{1}{m_{\text{T}}}\right) = (F_{\text{C on S}})\left(\frac{1}{m_{\text{C}}}\right)$$

$$F_{\text{C on T}} = (F_{\text{C on S}})\left(\frac{m_{\text{T}}}{m_{\text{C}} + m_{\text{T}}}\right) = (4500\text{ N})\left(\frac{2000\text{ kg}}{1000\text{ kg} + 2000\text{ kg}}\right) = 3000\text{ N}$$

(b) Due to Newton's third law, $F_{\text{T on C}} = 3000\text{ N}$.

7.9. Model: The blocks are to be modeled as particles and denoted as 1, 2, and 3. The surface is frictionless and along with the earth it is a part of the environment. The three blocks are our three systems of interest.
Visualize:



The force applied on block 1 is $F_{\text{A on 1}} = 12\text{ N}$. The acceleration for all the blocks is the same and is denoted by *a*.
Solve: (a) Newton's second law for the three blocks along the *x*-direction is

$$\Sigma(F_{\text{on 1}})_x = F_{\text{A on 1}} - F_{\text{2 on 1}} = m_1 a,\ \ \Sigma(F_{\text{on 2}})_x = F_{\text{1 on 2}} - F_{\text{3 on 2}} = m_2 a,\ \ \Sigma(F_{\text{on 3}})_x = F_{\text{2 on 3}} = m_3 a$$

Summing these three equations and using Newton's third law $(F_{\text{2 on 1}} = F_{\text{1 on 2}}$ and $F_{\text{3 on 2}} = F_{\text{2 on 3}})$, we get

$$F_{\text{A on 1}} = (m_1 + m_2 + m_3)a \quad \Rightarrow \quad (12\text{ N}) = (1\text{ kg} + 2\text{ kg} + 3\text{ kg})a \quad \Rightarrow \quad a = 2\text{ m/s}^2$$

Using this value of *a*, the force equation for block 3 gives

$$F_{\text{2 on 3}} = m_3 a = (3\text{ kg})(2\text{ m/s}^2) = 6\text{ N}$$

(b) Substituting into the force equation on block 1,

$$12\text{ N} - F_{\text{2 on 1}} = (1\text{ kg})(2\text{ m/s}^2) \quad \Rightarrow \quad F_{\text{2 on 1}} = 10\text{ N}$$

Assess: Because all three blocks are pushed forward by a force of 12 N, the value of 10 N for the force that the 2 kg block exerts on the 1 kg block seems reasonable.

7.10. Model: The asteroid and earth are to be modeled as particles. No air resistance. The system is the earth and asteroid.
Visualize: The acceleration of the asteroid right before it hits the earth is 9.8 m/s^2, as indicated by Galileo's law of falling bodies.

Solve: The force applied by the earth on the asteroid is $F = m_a a = (3000 \text{ kg})(9.8 \text{ m/s}^2) = 29400 \text{ N}$.

The acceleration of the earth is

$$a = \frac{F}{m_e} = \frac{29400 \text{ N}}{5.98\times10^{24} \text{ kg}} = 4.9\times10^{-21} \text{ m/s}^2$$

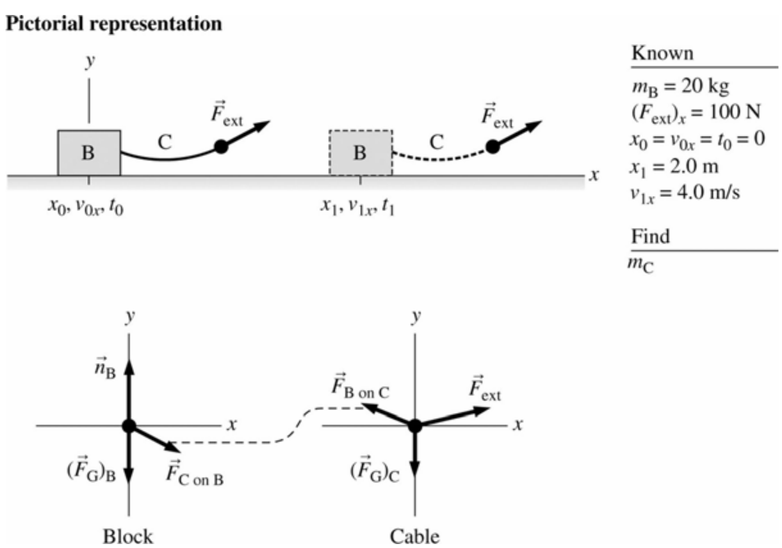
Assess: This acceleration is immeasurably small. The acceleration of the earth would be tiny in this situation, but the energy released in the actual collision would be great.

7.11. Model: The sprinter is a particle.
Visualize: Both the second and third laws will come into play here.
Solve: **(a)** The force applied by the ground is a static friction force because there is no relative motion between the two surfaces.

(b) $f_s = F = ma = m\frac{\Delta v}{\Delta t} = (55 \text{ kg})\left(\frac{2.0 \text{ m/s}}{0.25 \text{ s}}\right) = 440 \text{ N}$

Assess: This is a moderate amount of force.

7.12. Model: We treat the two objects of interest, the block (B) and steel cable (C), like particles. The motion of these objects is governed by the constant-acceleration kinematic equations. The horizontal component of the external force is 100 N.
Visualize:



Solve: Using $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$, we find

$$(4.0 \text{ m/s})^2 = 0 \text{ m}^2/\text{s}^2 + 2a_x(2.0 \text{ m}) \Rightarrow a_x = 4.0 \text{ m/s}^2$$

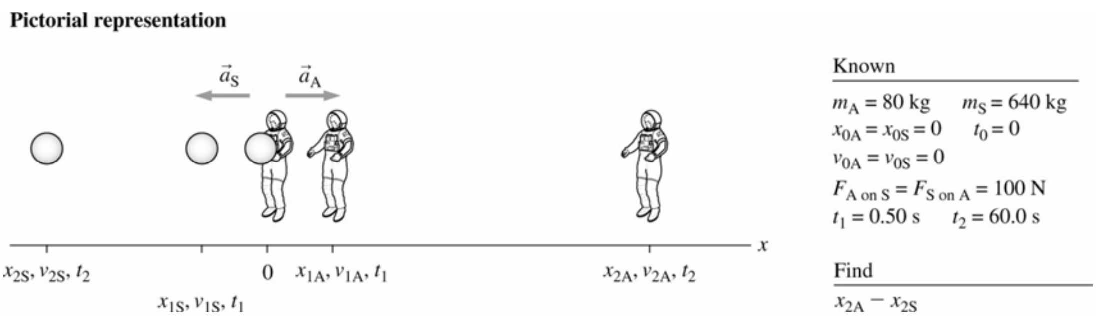
From the free-body diagram on the block:

$$\Sigma(F_{\text{on B}})_x = (F_{\text{C on B}})_x = m_B a_x \quad \Rightarrow \quad (F_{\text{C on B}})_x = (20 \text{ kg})(4.0 \text{ m/s}^2) = 80 \text{ N}$$

Also, according to Newton's third law $(F_{\text{B on C}})_x = (F_{\text{C on B}})_x = 80\text{ N}$. Applying Newton's second law to the cable gives

$$\Sigma(F_{\text{on C}})_x = (F_{\text{ext}})_x - (F_{\text{B on C}})_x = m_{\text{C}}a_x \quad\Rightarrow\quad 100\text{ N} - 80\text{ N} = m_{\text{C}}(4.0\text{ m/s}^2) \quad\Rightarrow\quad m_{\text{C}} = 5.0\text{ kg}$$

7.13. Model: The astronaut and the satellite, the two objects in our system, will be treated as particles.
Visualize:



Solve: The astronaut and the satellite accelerate in opposite directions for 0.50 s. The force on the satellite and the force on the astronaut are an action/reaction pair, so both have a magnitude of 100 N. Newton's second law for the satellite along the *x*-direction gives

$$\Sigma(F_{\text{on S}})_x = F_{\text{A on S}} = m_{\text{S}}a_{\text{S}} \quad\Rightarrow\quad a_{\text{S}} = \frac{F_{\text{A on S}}}{m_{\text{S}}} = \frac{-(100\text{ N})}{640\text{ kg}} = -0.156\text{ m/s}^2$$

Newton's second law for the astronaut along the *x*-direction is

$$\Sigma(F_{\text{on A}})_x = F_{\text{S on A}} = m_{\text{A}}a_{\text{A}} \quad\Rightarrow\quad a_{\text{A}} = \frac{F_{\text{S on A}}}{m_{\text{A}}} = \frac{F_{\text{A on S}}}{m_{\text{A}}} = \frac{100\text{ N}}{80\text{ kg}} = 1.25\text{ m/s}^2$$

Let us first calculate the positions and velocities of the astronaut and the satellite at $t_1 = 0.50\text{ s}$ under the accelerations a_{A} and a_{S}:

$$x_{1\text{A}} = x_{0\text{A}} + v_{0\text{A}}(t_1 - t_0) + \tfrac{1}{2}a_{\text{A}}(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(1.25\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s})^2 = 0.156\text{ m}$$

$$x_{1\text{S}} = x_{0\text{S}} + v_{0\text{S}}(t_1 - t_0) + \tfrac{1}{2}a_{\text{S}}(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-0.156\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s})^2 = -0.020\text{ m}$$

$$v_{1\text{A}} = v_{0\text{A}} + a_{\text{A}}(t_1 - t_0) = 0\text{ m/s} + (1.25\text{ m/s}^2)(0.50\text{ s} - 0.00\text{ s}) = 0.625\text{ m/s}$$

$$v_{1\text{S}} = v_{0\text{S}} + a_{\text{S}}(t_1 - t_0) = 0\text{ m/s} + (-0.156\text{ m/s}^2)(0.5\text{ s} - 0.00\text{ s}) = -0.078\text{ m/s}$$

With $x_{1\text{A}}$ and $x_{1\text{S}}$ as initial positions, $v_{1\text{A}}$ and $v_{1\text{S}}$ as initial velocities, and zero accelerations, we can now obtain the new positions at $(t_2 - t_1) = 59.5\text{ s}$:

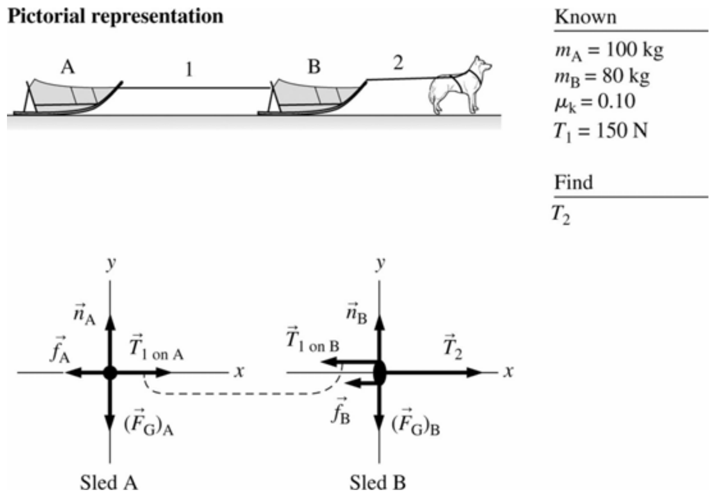
$$x_{2\text{A}} = x_{1\text{A}} + v_{1\text{A}}(t_2 - t_1) = 0.156\text{ m} + (0.625\text{ m/s})(59.5\text{ s}) = 37.34\text{ m}$$

$$x_{2\text{S}} = x_{1\text{S}} + v_{1\text{S}}(t_2 - t_1) = -0.02\text{ m} + (-0.078\text{ m/s})(59.5\text{ s}) = -4.66\text{ m}$$

Thus the astronaut and the satellite are $x_{2\text{A}} - x_{2\text{S}} = (37.34\text{ m}) - (-4.66\text{ m}) = 42\text{ m}$ apart.

7.14. Model: Sled A, sled B, and the dog (D) are treated like particles in the model. The rope is considered to be massless.

Visualize:



Solve: The acceleration constraint is $(a_A)_x = (a_B)_x = a_x$. Newton's second law applied to sled A gives

$$\Sigma(\vec{F}_{\text{on A}})_y = n_A - (F_G)_A = 0 \text{ N} \quad \Rightarrow \quad n_A = (F_G)_A = m_A g$$

$$\Sigma(\vec{F}_{\text{on A}})_x = T_{1\text{ on A}} - f_A = m_A a_x$$

Using $f_A = \mu_k n_A$, the x-equation yields

$$T_{1\text{ on A}} - \mu_k n_A = m_A a_x \quad \Rightarrow \quad 150 \text{ N} - (0.10)(100 \text{ kg})(9.8 \text{ m/s}^2) = (100 \text{ kg})a_x \quad \Rightarrow \quad a_x = 0.52 \text{ m/s}^2$$

Newton's second law applied to sled B gives

$$\Sigma(\vec{F}_{\text{on B}})_y = n_B - (F_G)_B = 0 \text{ N} \quad \Rightarrow \quad n_B = (F_G)_B = m_B g$$

$$\Sigma(\vec{F}_{\text{on B}})_x = T_2 - T_{1\text{ on B}} - f_B = m_B a_x$$

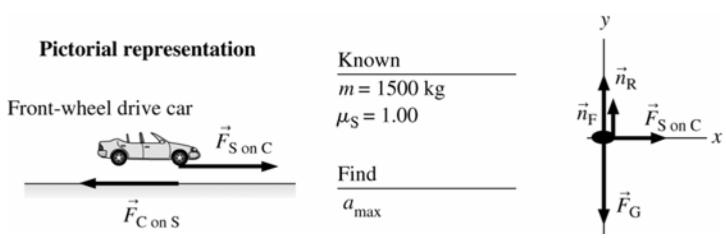
$T_{1\text{ on B}}$ and $T_{1\text{ on A}}$ act as if they are an action/reaction pair, so $T_{1\text{ on B}} = 150$ N. Using $f_B = \mu_k n_B = (0.10)(80 \text{ kg})(9.8 \text{ m/s}^2) = 78.4$ N, we find

$$T_2 - 150 \text{ N} - 78.4 \text{ N} = (80 \text{ kg})(0.52 \text{ m/s}^2) \quad \Rightarrow \quad T_2 = 270 \text{ N}$$

Thus the tension $T_2 = 2.7 \times 10^2$ N.

7.15. Model: For car tires on dry concrete, the coefficient μ_s of static friction is typically about 1.00 (see Table 6.1).

Visualize: The car and the ground are denoted by C and S, respectively.



Solve: The car presses down against the ground with both the drive wheels (assumed to be the front wheels F, although this is not critical) and the nondrive wheels. For this car, two-thirds of the gravitational force rests on the front wheels. Physically, force $\vec{F}_{\text{S on C}}$ is a static friction force, so its maximum value is $(\vec{F}_{\text{S on C}})_{\max} = (f_s)_{\max} = \mu_s n.$ The maximum acceleration of the car on the ground (or concrete surface) occurs when the static friction reaches this maximum possible value:

$$(F_{\text{S on C}})_{\max} = (f_s)_{\max} = \mu_s n_F = \mu_s (F_G)_F = \mu_s (\tfrac{2}{3}mg) = (1.00)(\tfrac{2}{3})(1500\text{ kg})(9.8\text{ m/s}^2) = 9800\text{ N}$$

Use this force in Newton's second law to find the acceleration:

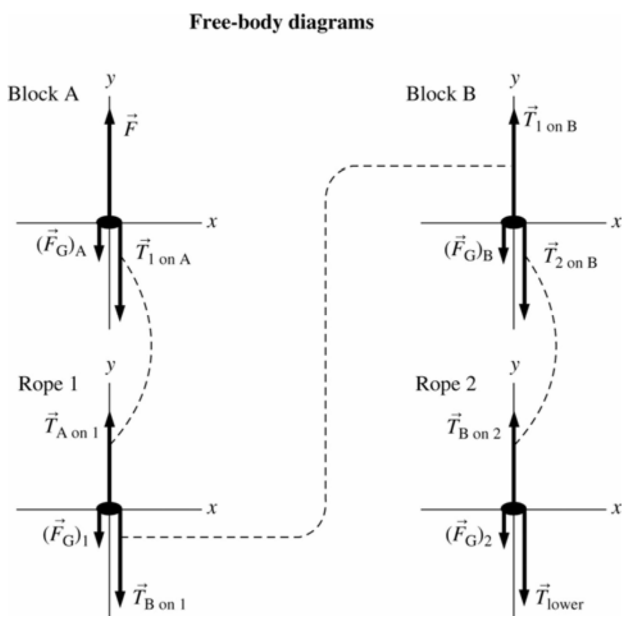
$$a_{\max} = \frac{(F_{\text{S on C}})_{\max}}{m} = \frac{9800\text{ N}}{1500\text{ kg}} = 6.5\text{ m/s}^2$$

Assess: The greater the fraction of weight on the drive wheels, the larger the maximum acceleration.

Section 7.4 Ropes and Pulleys

7.16. Model: The two ropes and the two blocks (A and B) will be treated as particles.

Visualize:



Solve: (a) The two blocks and two ropes form a combined system of total mass $M = 2.5$ kg. This combined system is accelerating upward at $a = 3.0\text{ m/s}^2$ under the influence of a force F and the gravitational force $-Mg\,\hat{j}$. Newton's second law applied to the combined system gives

$$(F_{\text{net}})_y = F - Mg = Ma \quad \Rightarrow \quad F = M(a+g) = (2.5\text{ kg})(3.0\text{ m/s}^2 + 9.8\text{ m/s}^2) = 32\text{ N}$$

(b) The ropes are *not* massless. We must consider both the blocks and the ropes as systems. The force F acts only on block A because it does not contact the other objects. We can proceed to apply the y-component of Newton's second law to each system, starting at the top. Each object accelerates upward at $a = 3.0\text{ m/s}^2$. For block A,

$$(F_{\text{net on A}})_y = F - m_A g - T_{\text{1 on A}} = m_A a \quad \Rightarrow \quad T_{\text{1 on A}} = F - m_A(a+g) = 19\text{ N}$$

(c) Applying Newton's second law to rope 1 gives

$$(F_{\text{net on 1}})_y = T_{\text{A on 1}} - m_1 g - T_{\text{B on 1}} = m_1 a$$

where $\vec{T}_{\text{A on 1}}$ and $\vec{T}_{\text{1 on A}}$ are an action/reaction pair. But, because the rope has mass, the two tension forces $\vec{T}_{\text{A on 1}}$ and $\vec{T}_{\text{B on 1}}$ are *not* the same. The tension at the lower end of rope 1, where it connects to B, is

$$T_{\text{B on 1}} = T_{\text{A on 1}} - m_1(a+g) = 16\text{ N}$$

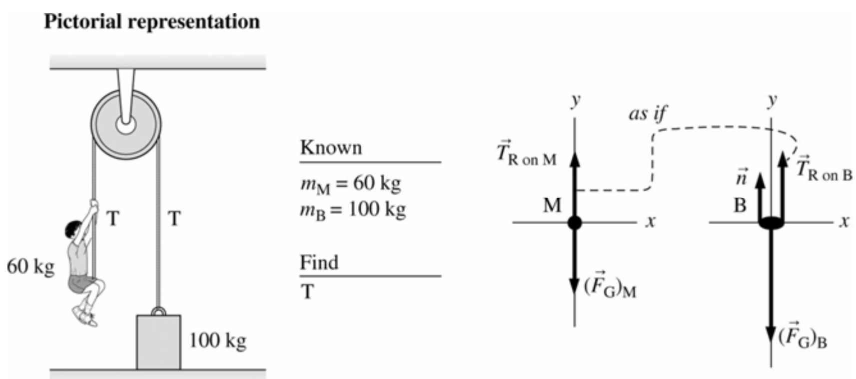
(d) We can continue to repeat this procedure, noting from Newton's third law that

$$T_{1\text{ on B}} = T_{\text{B on }1} \text{ and } T_{2\text{ on B}} = T_{\text{B on }2}$$

Newton's second law applied to block B is

$$(F_{\text{net on B}})_y = T_{1\text{ on B}} - m_{\text{B}}g - T_{2\text{ on B}} = m_{\text{B}}a \quad \Rightarrow \quad T_{2\text{ on B}} = T_{1\text{ on B}} - m_{\text{B}}(a+g) = 3.2\text{ N}$$

7.17. Model: The man (M) and the block (B) are interacting with each other through a rope. We will assume the pulley to be frictionless, which implies that the tension in the rope is the same on both sides of the pulley. The system is the man and the block.
Visualize:

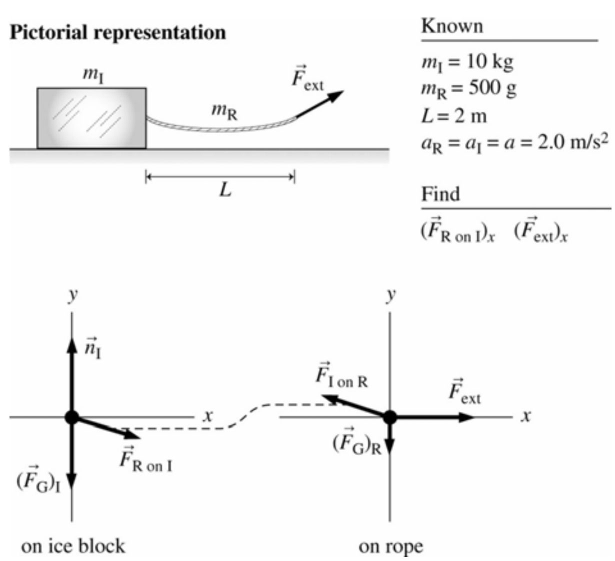


Solve: Clearly the entire system remains in equilibrium since $m_{\text{B}} > m_{\text{M}}$. The block would move downward but it is already on the ground. From the free-body diagrams, we can write Newton's second law in the vertical direction as

$$\sum(F_{\text{on M}})_y = T_{\text{R on M}} - (F_{\text{G}})_{\text{M}} = 0\text{ N} \quad \Rightarrow \quad T_{\text{R on M}} = (F_{\text{G}})_{\text{M}} = (60\text{ kg})(9.8\text{ m/s}^2) = 590\text{ N}$$

Since the tension is the same on both sides, $T_{\text{B on R}} = T_{\text{M on R}} = T = 590$ N.

7.18. Model: The block of ice (I) is a particle and so is the rope (R) because it is not massless. We must therefore consider both the block of ice and the rope as objects in the system.
Visualize:



Solve: **(a)** The force $\vec{F}_{\text{ext}}$ acts only on the rope. Since the rope and the ice block move together, they have the same acceleration. Also because the rope has mass, F_{ext} on the front end of the rope is not the same as $F_{\text{I on R}}$ that acts on the rear end of the rope. Applying Newton's second law along the *x*-axis to the ice block and the rope gives

$$\sum(F_{\text{on I}})_x = (F_{\text{R on I}})_x = m_{\text{I}}a = (10\text{ kg})(2.0\text{ m/s}^2) = 20\text{ N}$$

(b) Applying Newton's second law to the rope gives

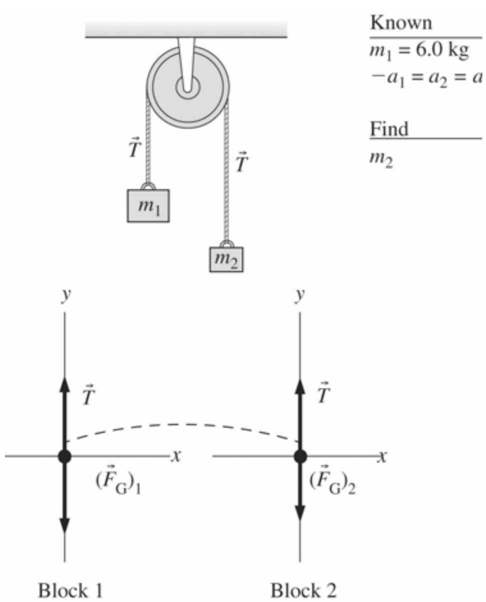
$$\sum(F_{\text{on R}})_x = (F_{\text{ext}})_x - (F_{\text{I on R}})_x = m_{\text{R}}a \quad\Rightarrow\quad (F_{\text{ext}})_x = (F_{\text{R on I}})_x + m_{\text{R}}a = 20\text{ N} + (0.500\text{ kg})(2.0\text{ m/s}^2) = 21\text{ N}$$

7.19. Model: The crates are particles, the rope is massless, and the pulley is massless and frictionless.
Visualize: Because "the tension in a massless string remains constant as it passes over a massless, frictionless pulley" the tension will be the same everywhere in the rope.
Solve: The tension in the vertical part of the rope right above the crate is given by the second law.

$$\Sigma F = T - mg = 0 \Rightarrow T = mg = (25\text{ kg})(9.8\text{ m/s}^2) = 250\text{ N}$$

Since the tension is the same everywhere along the rope the woman must pull with a force of 250 N to keep a constant speed.
Assess: This is within the capabilities of most people.

7.20. Model: The blocks are particles, the rope is massless, and the pulley is massless and frictionless.
Visualize: Because "the tension in a massless string remains constant as it passes over a massless, frictionless pulley" the tension will be the same everywhere in the rope. There is an acceleration constraint: $-a_1 = a_2 = a$.



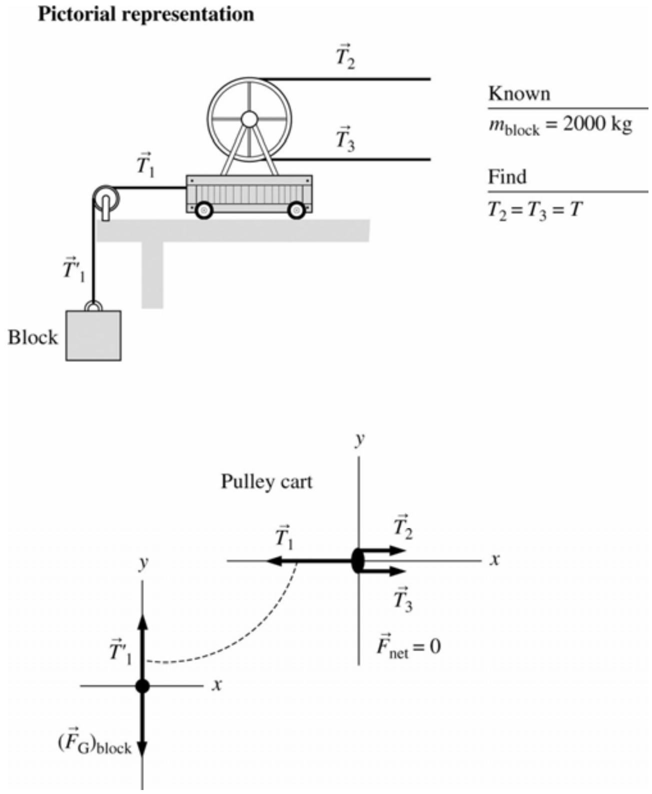
Solve: First find the tension in the rope above the 6.0 kg mass. Then use that tension to find the mass of the other block.

$$\Sigma F_1 = T - m_1 g = -m_1 a \Rightarrow T = m_1(g - a) = m_1\left(g - \tfrac{3}{4}g\right) = (6.0\text{ kg})\left(\tfrac{1}{4}g\right)$$

$$\Sigma F_2 = T - m_2 g = m_2 a \Rightarrow (6.0\text{ kg})\left(\tfrac{1}{4}g\right) - m_2 g = m_2\left(\tfrac{3}{4}g\right) \Rightarrow m_2 = \frac{(6.0\text{ kg})\left(\tfrac{1}{4}g\right)}{g + \tfrac{3}{4}g} = \tfrac{6}{7}\text{ kg} = 0.86\text{ kg}$$

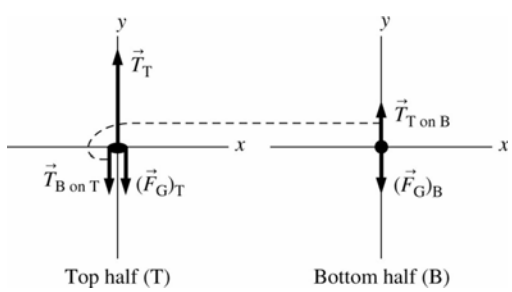
Since the tension is the same everywhere along the rope the woman must pull with a force of 250 N to keep a constant speed.
Assess: This can be checked by considering the two blocks as one compound object with a net force of $(m_1 - m_2)g$ and a total mass of $m_1 + m_2$.

7.21. Model: The hanging block and the rail car are objects in the systems.
Visualize:



Solve: The mass of the rope is very small in comparison to the 2000-kg block, so we assume a massless rope. In this case, the forces $\vec{T}_1$ and $\vec{T}_1'$ act as if they are an action/reaction pair. The hanging block is in static equilibrium, with $\vec{F}_{net} = 0$ N, so $T_1' = m_{block}g = (2000\text{ kg})(9.8\text{ m/s}^2) = 19{,}600$ N. The rail car with the pulley is also in static equilibrium, so $T_2 + T_3 - T_1 = 0$ N. Notice how the tension force in the cable pulls both the top and bottom of the pulley to the right. Now, $T_1 = T_1' = 19{,}600$ N by Newton's third law. Also, the cable tension is $T_2 = T_3 = T$. Thus, $T = \frac{1}{2}T_1' = 9800$ N.

7.22. Visualize:



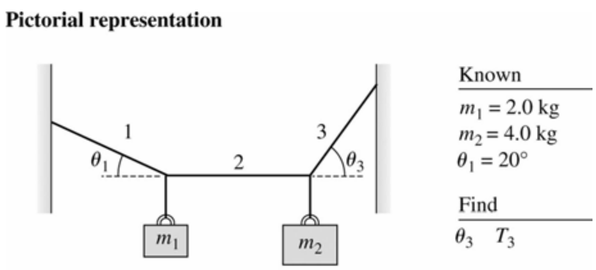
Solve: The rope is treated as two 1.0-kg interacting objects. At the midpoint of the rope, the rope has a tension $T_{B\text{ on }T} = T_{T\text{ on }B} \equiv T$. Apply Newton's first law to the bottom half of the rope to find T.

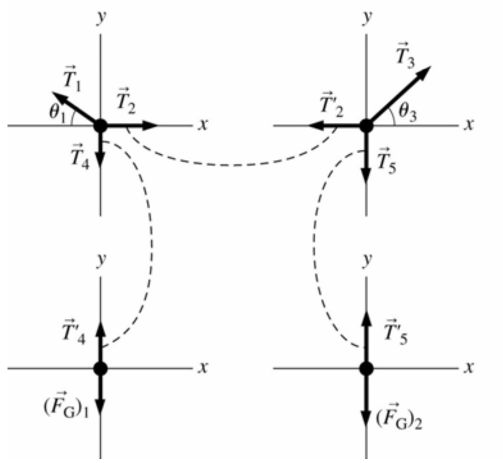
$$(F_{net})_y = 0 = T - (F_G)_B \quad \Rightarrow \quad T = m_B g = (1.0\text{ kg})(9.8\text{ m/s}^2) = 9.8\text{ N}$$

Assess: 9.8 N is half the gravitational force on the whole rope. This is reasonable since the top half is holding up the bottom half of the rope against gravity.

7.23. Model: The cat and dog are modeled as two blocks in the pictorial representation below. The rope is assumed to be massless. The two points (knots) where the blocks are attached to the rope and the two hanging blocks form a system. These four objects are treated at particles, form the system, and are in static equilibrium.

Visualize:





Solve: (a) We consider both the two hanging blocks *and* the two knots. The blocks are in static equilibrium with $\vec{F}_{net} = 0$ N. Note that there are three action/reaction pairs. For Block 1 and Block 2, $\vec{F}_{net} = 0$ N and we have

$$T_4' = (F_G)_1 = m_1 g, \quad T_5' = (F_G)_2 = m_2 g$$

By Newton's third law:

$$T_4 = T_4' = m_1 g, \quad T_5 = T_5' = m_2 g$$

The knots are also in equilibrium. Newton's law applied to the left knot is

$$(F_{net})_x = T_2 - T_1 \cos\theta_1 = 0 \text{ N} \quad \Rightarrow \quad (F_{net})_y = T_1 \sin\theta_1 - T_4 = T_1 \sin\theta_1 - m_1 g = 0 \text{ N}$$

The y-equation gives $T_1 = m_1 g / \sin\theta_1$. Substitute this into the x-equation to find

$$T_2 = \frac{m_1 g \cos\theta_1}{\sin\theta_1} = \frac{m_1 g}{\tan\theta_1}$$

Newton's law applied to the right knot is

$$(F_{net})_x = T_3 \cos\theta_3 - T_2' = 0 \text{ N} \quad \Rightarrow \quad (F_{net})_y = T_3 \sin\theta_3 - T_5 = T_3 \sin\theta_3 - m_2 g = 0 \text{ N}$$

These can be combined just like the equations for the left knot to give

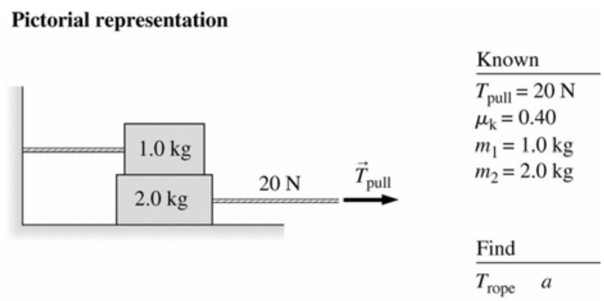
$$T_2' = \frac{m_2 g \cos\theta_3}{\sin\theta_3} = \frac{m_2 g}{\tan\theta_3}$$

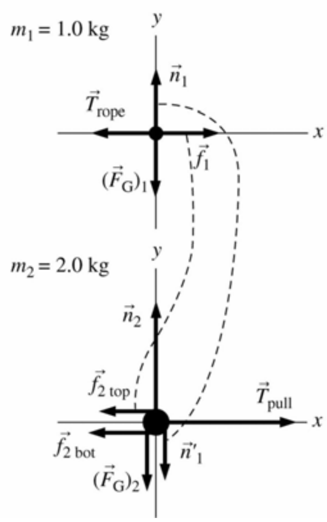
But the forces $\vec{T}_2$ and $\vec{T}_2'$ are an action/reaction pair, so $T_2 = T_2'$. Therefore,

$$\frac{m_1 g}{\tan\theta_1} = \frac{m_2 g}{\tan\theta_3} \quad \Rightarrow \quad \tan\theta_3 = \frac{m_2}{m_1}\tan\theta_1 \quad \Rightarrow \quad \theta_3 = \tan^{-1}[2\tan(20°)] = 36°$$

We can now use the y-equation for the right knot to find $T_3 = m_2 g/\sin\theta_3 = 67\ \text{N}$.

7.24. Model: The two blocks (1 and 2) form the system of interest and will be treated as particles. The ropes are assumed to be massless, and the model of kinetic friction will be used.
Visualize:





Solve: (a) The separate free-body diagrams for the two blocks show that there are two action/reaction pairs. Notice how block 1 both pushes down on block 2 (force $\vec{n}_1'$) and exerts a retarding friction force $\vec{f}_{2\ \text{top}}$ on the top surface of block 2. Block 1 is in static equilibrium ($a_1 = 0\ \text{m/s}^2$) but block 2 is accelerating to the right. Newton's second law for block 1 is

$$(F_{\text{net on 1}})_x = f_1 - T_{\text{rope}} = 0\ \text{N} \quad \Rightarrow \quad T_{\text{rope}} = f_1$$
$$(F_{\text{net on 1}})_y = n_1 - m_1 g = 0\ \text{N} \quad \Rightarrow \quad n_1 = m_1 g$$

Although block 1 is stationary, there is a *kinetic* force of friction because there is motion between blocks 1 and 2. The friction model means $f_1 = \mu_k n_1 = \mu_k m_1 g$. Substitute this result into the x-equation to get the tension in the rope:

$$T_{\text{rope}} = f_1 = \mu_k m_1 g = (0.40)(1.0\ \text{kg})(9.8\ \text{m/s}^2) = 3.9\ \text{N}$$

(b) Newton's second law for block 2 is

$$a_x \equiv a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - f_{2\text{ top}} - f_{2\text{ bot}}}{m_2}$$

$$a_y = 0 \text{ m/s}^2 = \frac{(F_{\text{net on 2}})_y}{m_2} = \frac{n_2 - n_1' - m_2 g}{m_2}$$

Forces $\vec{n}_1$ and $\vec{n}_1'$ are an action/reaction pair, so $n_1' = n_1 = m_1 g$. Substituting into the y-equation gives $n_2 = (m_1 + m_2)g$. This is not surprising because the combined weight of both objects presses down on the surface. The kinetic friction on the bottom surface of block 2 is then

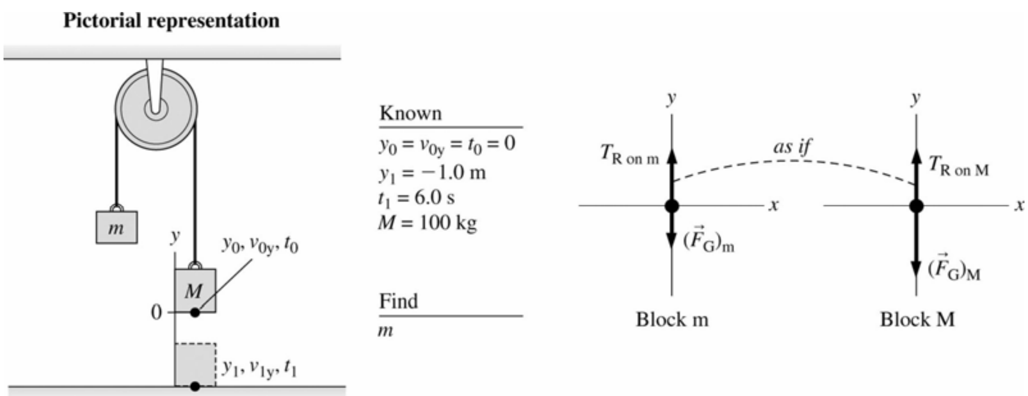
$$f_{2\text{ bot}} = \mu_\text{k} n_2 = \mu_\text{k}(m_1 + m_2)g$$

The forces $\vec{f}_1$ and $\vec{f}_{2\text{ top}}$ are an action/reaction pair, so $f_{2\text{ bot}} = f_1 = \mu_\text{k} m_1 g$. Inserting these friction results into the x-equation gives

$$a = \frac{(F_{\text{net on 2}})_x}{m_2} = \frac{T_{\text{pull}} - \mu_\text{k} m_1 g - \mu_\text{k}(m_1 + m_2)g}{m_2}$$

$$= \frac{20 \text{ N} - (0.40)(1.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.40)(1.0 \text{ kg} + 2.0 \text{ kg})(9.8 \text{ m/s}^2)}{2.0 \text{ kg}} = 2.2 \text{ m/s}^2$$

7.25. Model: The masses m and M are to be treated in the particle model. We will also assume a massless rope and frictionless pulley, and use the constant-acceleration kinematic equations for m and M.
Visualize:



Solve: Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_\text{M}(t_1 - t_0)^2$,

$$(-1.0 \text{ m}) = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}a_\text{M}(6.0 \text{ s} - 0 \text{ s})^2 \quad \Rightarrow \quad a_\text{M} = -0.0556 \text{ m/s}^2$$

Newton's second law for m and M gives

$$\sum(F_{\text{on m}})_y = T_{\text{R on m}} - (F_\text{G})_\text{m} = ma_\text{m} \quad \sum(F_{\text{on M}})_y = T_{\text{R on M}} - (F_\text{G})_\text{M} = Ma_\text{M}$$

The acceleration constraint is $a_m = -a_M$. Also, the tensions are an pseudo-action/reaction pair, so $T_{\text{R on m}} = T_{\text{R on M}}$. With these, the second-law equations become

$$T_{\text{R on M}} - Mg = Ma_\text{M}$$
$$T_{\text{R on M}} - mg = -ma_\text{M}$$

Subtracting the second from the first gives

$$-Mg + mg = Ma_M + ma_M$$

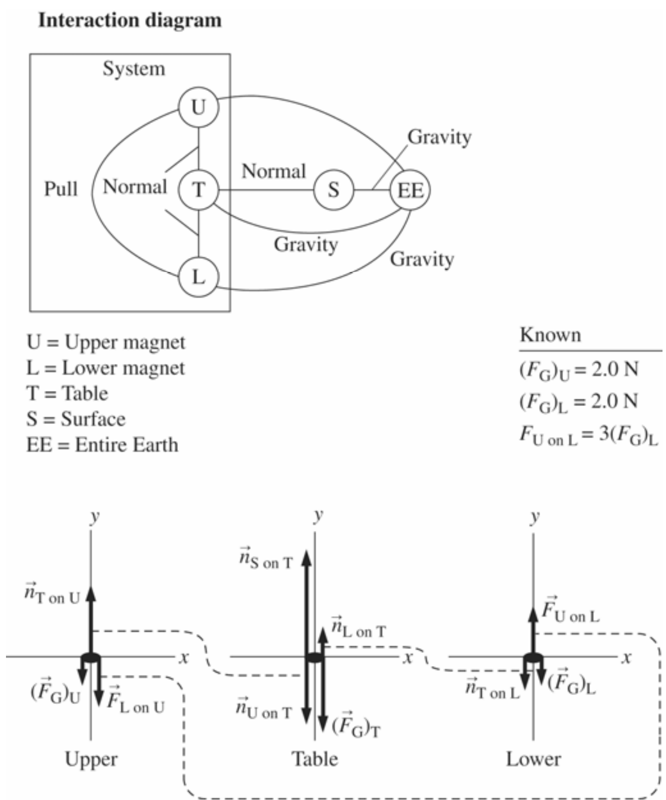
$$m = M\left[\frac{g + a_M}{g - a_M}\right]$$

$$= (100\text{ kg})\left[\frac{9.8\text{ m/s}^2 - 0.556\text{ m/s}^2}{9.8\text{ m/s}^2 + 0.556\text{ m/s}^2}\right] = 99\text{ kg}$$

Assess: Note that $a_m = -a_M = 0.0556\text{ m/s}^2$. For such a small acceleration, the 1% mass difference seems reasonable.

Problems

7.26. (a) Visualize: The upper magnet is labeled U and the lower magnet L. Each magnet exerts a long-range magnetic force on the other. Each magnet and the table exert a contact force (normal force) on each other. In addition, the table experiences a normal force due to the surface.

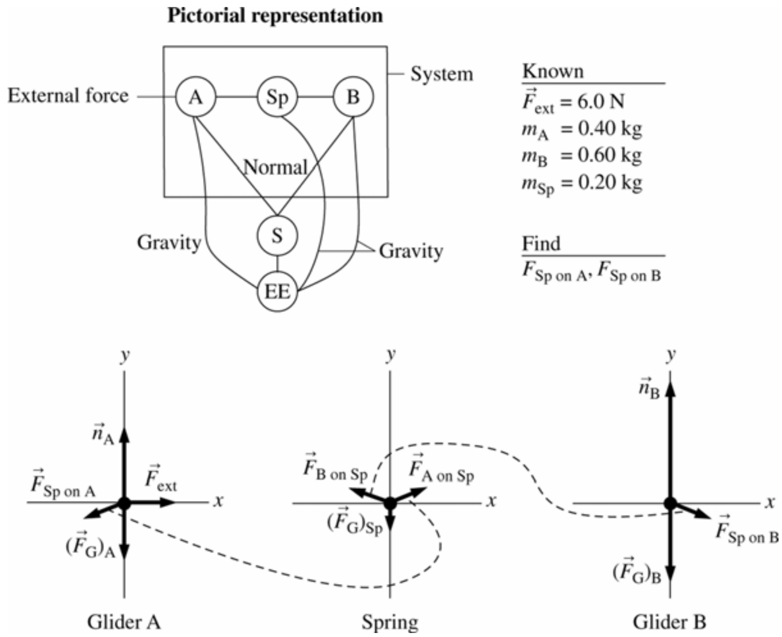


(b) Solve: Each object is in static equilibrium with $F_{net} = 0$. Examine the lower magnet. The upward force is from the upper magnet; the downward forces are gravity and the normal force from the table. With $F_{L\text{ on }U} = 6.0\text{ N}$, then by the third law $F_{U\text{ on }L} = 6.0\text{ N}$. Since the gravitational force is 2.0 N equilibrium requires $n_{T\text{ on }L} = -4.0\text{ N}$ and therefore by the third law $n_{L\text{ on }T} = 4.0\text{ N}$.

Assess: Each object is in equilibrium. The forces are all in the proper range.

7.27. Model: The gliders and spring form the system and are modeled as particles. Because the spring is compressed, it may be modeled as a rigid rod, so the three objects are constrained to have the same acceleration.

Visualize:



Solve: By Newton's third law, we know the action/reaction forces are of equal magnitude but point in the opposite direction, so they cancel when considering the entire system. Using Newton's second law in the x-direction, the acceleration of the system is

$$\sum(F)_x = F_{\text{ext}} = (m_A + m_{Sp} + m_B)a \quad \Rightarrow \quad a = \frac{F_{\text{ext}}}{m_A + m_{Sp} + m_B} = \frac{6.0\text{ N}}{0.40\text{ kg} + 0.20\text{ kg} + 0.60\text{ kg}} = 5.0\text{ m/s}^2$$

Applying Newton's second law to glider A gives

$$\sum(F)_x = F_{\text{ext}} - (\vec{F}_{\text{Sp on A}})_x = m_A a \quad \Rightarrow \quad (\vec{F}_{\text{Sp on A}})_x = F_{\text{ext}} - m_A a = 6.0\text{ N} - (0.40\text{ kg})(5.0\text{ m/s}^2) = 4.0\text{ N}$$

Applying Newton's second law to glider B gives

$$\sum(F)_x = (\vec{F}_{\text{Sp on B}})_x = m_B a = (0.60\text{ kg})(5.0\text{ m/s}^2) = 3.0\text{ N}$$

Applying Newton's second law to the spring in the y-direction, and using the fact that $(\vec{F}_{\text{Sp on A}})_y = (\vec{F}_{\text{Sp on B}})_y$ by symmetry, we find

$$\sum(F)_y = (\vec{F}_{\text{Sp on A}})_y + (\vec{F}_{\text{Sp on B}})_y - (F_G)_{Sp} = 0 \quad \Rightarrow \quad 2(\vec{F}_{\text{Sp on B}})_y = m_{Sp}(9.8\text{ m/s}^2)$$
$$(\vec{F}_{\text{Sp on A}})_y = (\vec{F}_{\text{Sp on B}})_y = \tfrac{1}{2}(0.20\text{ kg})(9.8\text{ m/s}^2) = 0.98\text{ N}$$

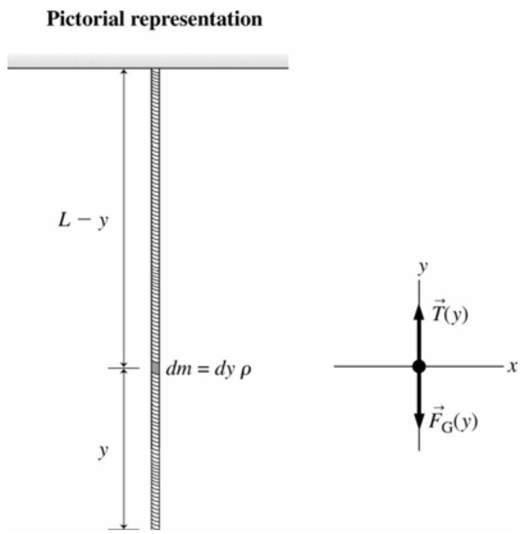
Adding the x- and y-components in quadrature gives the total force exerted by the spring on each block:

$$\text{glider A:} \quad F_{\text{Sp on A}} = \sqrt{(4.0\text{ N})^2 + (0.98\text{ N})^2} = 4.1\text{ N}$$
$$\text{glider B:} \quad F_{\text{Sp on B}} = \sqrt{(3.0\text{ N})^2 + (0.98\text{ N})^2} = 3.2\text{ N}$$

Assess: The result seems reasonable because more force is exerted on glider A by the spring than on glider B, as expected. The force exerted on glider A by the spring is, by Newton's third law, the force that must accelerate the spring + glider B, whereas the force exerted by the spring on glider B only has to accelerate glider B

7.28. Model: Consider an element of the rope $dm = \rho dy$, where $\rho = m/L$ is the mass density of the rope. Model this element as a particle.

Visualize:



Solve: The rope is stationary, so Newton's second law applied to the particle gives

$$\sum(F_{dm})_y = T(y) - F_G(y) = 0 \quad \Rightarrow \quad T(y) = F_G(y)$$

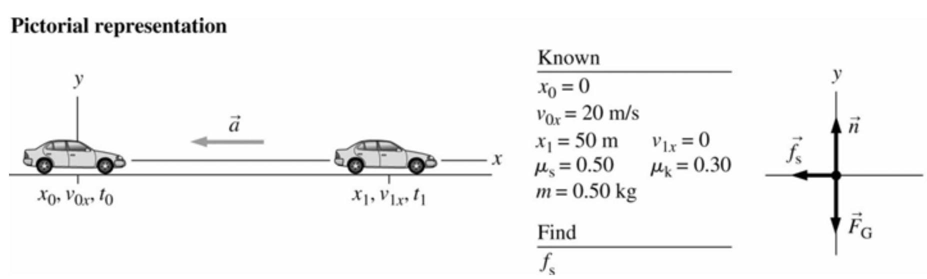
The force $F_G(y)$ due to gravity is the weight of the rope below the point y, which is

$$F_G(y) = y\rho g = y(m/L)g$$

Inserting this into the expression above gives the tension: $T(y) = ymg/L$.

Assess: The result seems reasonable because $T(y) = 0$ at the bottom of the rope ($y = 0$) and $T(y) = mg$ at the top of the rope ($y = L$).

7.29. Model: The coffee mug (M) is the only object in the system, and it will be treated as a particle. The model of friction and the constant-acceleration kinematic equations will also be used.
Visualize:



Solve: The mug and the car have the same velocity. If the mug does not slip, their accelerations will also be the same. Using $v_{1x}^2 = v_{0x}^2 + 2a_x(x_1 - x_0)$, we get

$$0 \text{ m}^2/\text{s}^2 = (20 \text{ m/s})^2 + 2a_x(50 \text{ m}) \Rightarrow a_x = -4.0 \text{ m/s}^2$$

The static force needed to stop the mug is

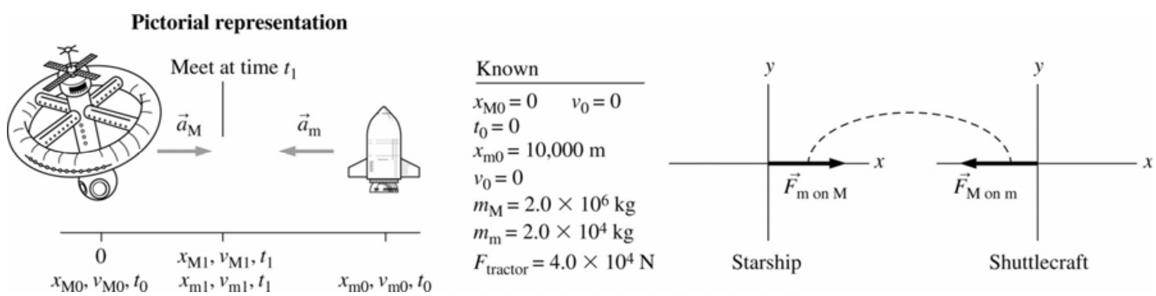
$$(F_{\text{net}})_x = -f_s = ma_x = (0.5 \text{ kg})(-4.0 \text{ m/s}^2) = -2.0 \text{ N} \Rightarrow f_s = 2.0 \text{ N}$$

The maximum force of static friction is

$$(f_s)_{max} = \mu_s n = \mu_s F_G = \mu_s mg = (0.50)(0.50\text{ kg})(9.8\text{ m/s}^2) = 2.5\text{ N}$$

Since $(f_s)_{max} < (f_s)_{max}$, the mug does not slide.

7.30. Model: The starship and the shuttlecraft will be denoted as M and m, respectively, and both will be treated as particles. We will also use the constant-acceleration kinematic equations.
Visualize:



Solve: (a) The tractor beam is some kind of long-range force $\vec{F}_{M\text{ on }m}$. Regardless of what kind of force it is, by Newton's third law there *must* be a reaction force $\vec{F}_{m\text{ on }M}$ on the starship. As a result, both the shuttlecraft *and* the starship move toward each other (rather than the starship remaining at rest as it pulls the shuttlecraft in). However, the very different masses of the two crafts means that the distances they each move will also be very different. The pictorial representation shows that they meet at time t_1 when $x_{M1} = x_{m1}$. There's only one force on each craft, so Newton's second law is very simple. Furthermore, because the forces are an action/reaction pair,

$$F_{M\text{ on }m} = F_{m\text{ on }M} = F_{\text{tractor beam}} = 4.0\times10^4\text{ N}$$

The accelerations of the two craft are

$$a_M = \frac{F_{m\text{ on }M}}{M} = \frac{4.0\times10^4\text{ N}}{2.0\times10^6\text{ kg}} = 0.020\text{ m/s}^2 \text{ and } a_m = \frac{\vec{F}_{M\text{ on }m}}{m} = \frac{-4.0\times10^4\text{ N}}{2.0\times10^4\text{ kg}} = -2.0\text{ m/s}^2$$

Acceleration a_m is negative because the force and acceleration vectors point in the negative *x*-direction. Now we have a constant-acceleration problem in kinematics. At a later time t_1 the positions of the crafts are

$$x_{M1} = x_{M0} + v_{M0}(t_1 - t_0) + \tfrac{1}{2}a_M(t_1 - t_0)^2 = \tfrac{1}{2}a_M t_1^2$$
$$x_{m1} = x_{m0} + v_{m0}(t_1 - t_0) + \tfrac{1}{2}a_m(t_1 - t_0)^2 = x_{m0} + \tfrac{1}{2}a_m t_1^2$$

The craft meet when $x_{M1} = x_{m1}$, so

$$\tfrac{1}{2}a_M t_1^2 = x_{m0} + \tfrac{1}{2}a_m t_1^2 \quad\Rightarrow\quad t_1 = \sqrt{\frac{2x_{m0}}{a_M - a_m}} = \sqrt{\frac{2x_{m0}}{a_M + |a_m|}} = \sqrt{\frac{2(10{,}000\text{ m})}{2.02\text{ m/s}^2}} = 99.5\text{ s}$$

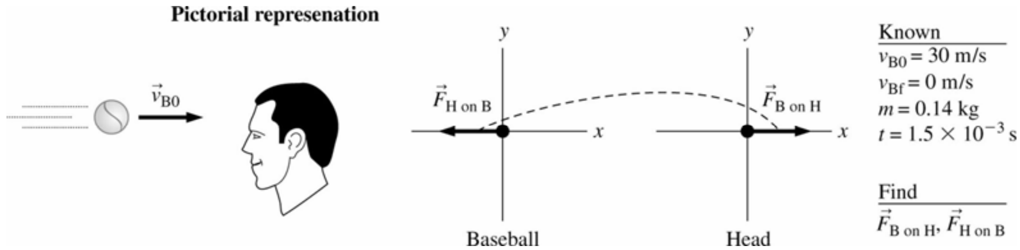
Knowing t_1, we can now find the starship's position as it meets the shuttlecraft:

$$x_{M1} = \tfrac{1}{2}a_M t_1^2 = 99\text{ m}$$

The starship moves 99 m as it pulls in the shuttlecraft from 10 km away.

7.31. Model: We shall only consider horizontal forces. The head and the baseball are the two objects in our system and are treated as particles. We will also use the constant-acceleration kinematic equations.

Visualize:



Solve: **(a)** The ball experiences an average acceleration of

$$a_B = \frac{v_{B1} - v_{B0}}{t} = \frac{-30\text{ m/s}}{1.5\times10^{-3}\text{ s}} = -20{,}000\text{ m/s}^2$$

Insert this into Newton's second law to find the force on the baseball:

$$F_{\text{H on B}} = m_B a_B = (0.14\text{ kg})\left|-20{,}000\text{ m/s}^2\right| = 2800\text{ N}$$

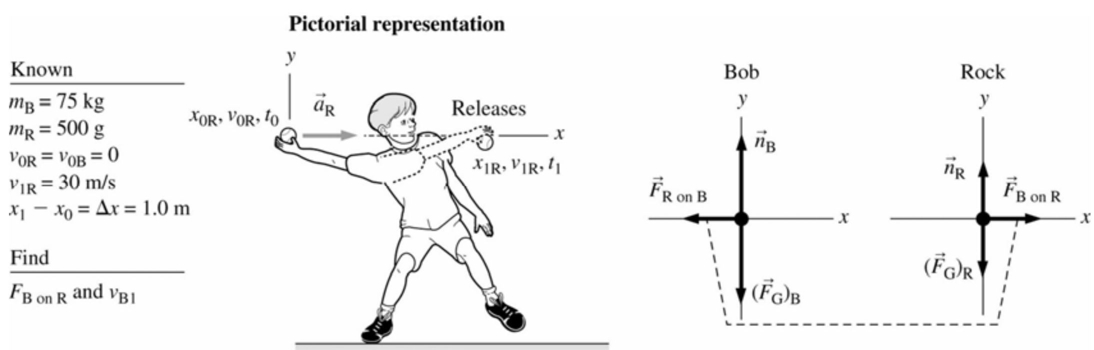
(b) By Newton's third law, the magnitude of the force exerted by the ball on the head is the same as that exerted by the head on the ball. Thus, $F_{\text{B on H}} = 2800$ N.

(c) Because 2800 N < 6000 N, the ball will not fracture your forehead, but will fracture your cheekbone because 2800 N > 1300 N.

Assess: A 90 mph fastball travels at $(90\text{ mph})(1609.3\text{ m/mile})(1\text{ h}/3600\text{ s}) = 40\text{ m/s}$, so it will not fracture your forehead, but it will fracture your cheekbone. This explains why baseball helmets protect the cheekbone.

7.32. Model: The rock (R) and Bob (b) are the two objects in our system, and will be treated as particles. We will also use the constant-acceleration kinematic equations.

Visualize:



Solve: **(a)** Bob exerts a forward force $\vec{F}_{\text{B on R}}$ on the rock to accelerate it forward. The rock's acceleration is calculated as follows:

$$v_{1R}^2 = v_{0R}^2 + 2a_{0R}\Delta x \quad\Rightarrow\quad a_R = \frac{v_{1R}^2}{2\Delta x} = \frac{(30\text{ m/s})^2}{2(1.0\text{ m})} = 450\text{ m/s}^2$$

The force is calculated from Newton's second law:

$$F_{\text{B on R}} = m_R a_R = (0.500\text{ kg})(450\text{ m/s}^2) = 225\text{ N}$$

Bob exerts a force of 2.3×10^2 N on the rock.

(b) Because Bob pushes on the rock, the rock pushes back on Bob with a force $\vec{F}_{\text{R on B}}$. Forces $\vec{F}_{\text{R on B}}$ and $\vec{F}_{\text{B on R}}$ are an action/reaction pair, so $F_{\text{R on B}} = F_{\text{B on R}} = 225\ \text{N}$. The force causes Bob to accelerate backward with an acceleration of

$$a_{\text{B}} = \frac{(F_{\text{net on B}})_x}{m_{\text{B}}} = -\frac{F_{\text{R on B}}}{m_{\text{B}}} = -\frac{225\ \text{N}}{75\ \text{kg}} = -3.0\ \text{m/s}^2$$

This is a rather large acceleration, but it lasts only until Bob releases the rock. We can determine the time interval by returning to the kinematics of the rock:

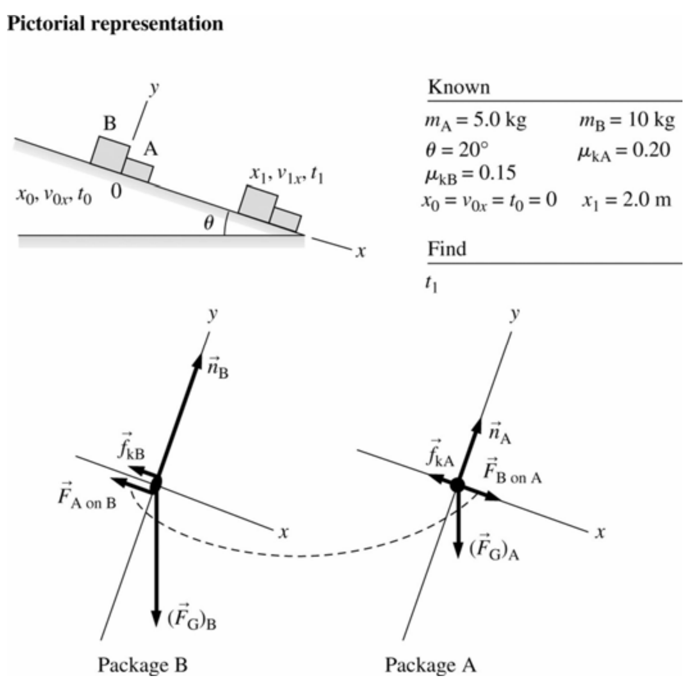
$$v_{1\text{R}} = v_{0\text{R}} + a_{\text{R}}\Delta t = a_{\text{R}}\Delta t \quad \Rightarrow \quad \Delta t = \frac{v_{1\text{R}}}{a_{\text{R}}} = 0.0667\ \text{s}$$

At the end of this interval, Bob's velocity is

$$v_{1\text{B}} = v_{0\text{B}} + a_{\text{B}}\Delta t = a_{\text{B}}\Delta t = -0.20\ \text{m/s}$$

Thus his recoil speed is 0.20 m/s.

7.33. Model: Assume package A and package B are particles. Use the model of kinetic friction and the constant-acceleration kinematic equations.
Visualize:



Solve: Package B has a smaller coefficient of friction, so its acceleration down the ramp is greater than that of package A. It will therefore push against package A and, by Newton's third law, package A will push back on B. The acceleration constraint is $a_{\text{A}} = a_{\text{B}} \equiv a$.

Newton's second law applied to each package gives

$$\sum(F_{\text{on A}})_x = F_{\text{B on A}} + (F_{\text{G}})_{\text{A}} \sin\theta - f_{\text{kA}} = m_{\text{A}}a$$
$$F_{\text{B on A}} + m_{\text{A}}g\sin\theta - \mu_{\text{kA}}(m_{\text{A}}g\cos\theta) = m_{\text{A}}a$$

$$\sum(F_{\text{on B}})_x = -F_{\text{A on B}} - f_{\text{kB}} + (F_{\text{G}})_{\text{B}} \sin\theta = m_{\text{B}}a$$
$$-F_{\text{A on B}} - \mu_{\text{kB}}(m_{\text{B}}g\cos\theta) + m_{\text{B}}g\sin\theta = m_{\text{B}}a$$

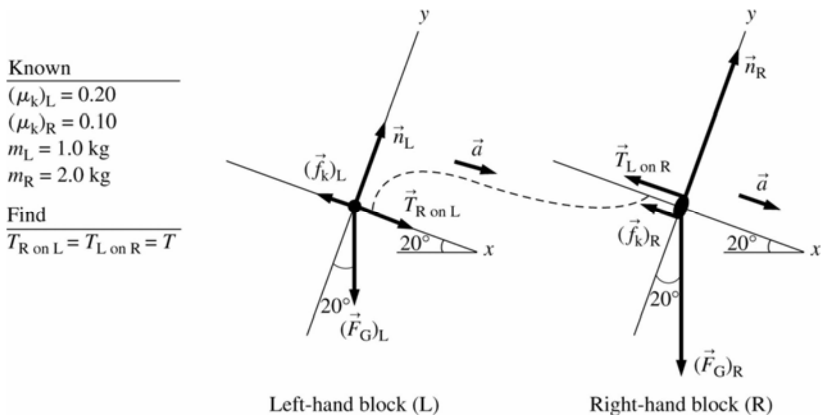
where we have used $n_A = m_A \cos\theta g$ and $n_B = m_B \cos\theta g$. Adding the two force equations, and using $F_{\text{A on B}} = F_{\text{B on A}}$ because they are an action/reaction pair, we get

$$a = g\sin\theta - \frac{(\mu_{kA}m_A + \mu_{kB}m_B)(g\cos\theta)}{m_A + m_B} = \frac{[(020)(5.0\text{ kg}) + (0.15)(10\text{ kg})](9.8\text{ m/s}^2)\cos(20°)}{5.0\text{ kg} + 10\text{ kg}} = 1.82\text{ m/s}^2$$

Finally, using $x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2$, we find

$$2.0\text{ m} = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(1.82\text{ m/s}^2)(t_1 - 0\text{ s})^2 \quad \Rightarrow \quad t_1 = \sqrt{2(2.0\text{ m})/(1.82\text{ m/s}^2)} = 1.5\text{ s}$$

7.34. Model: The two blocks form a system of interacting objects. We shall treat them as particles.
Visualize: Please refer to Figure P7.34.



Solve: It is possible that the left-hand block (block L) is accelerating down the slope faster than the right-hand block (block R), causing the string to be slack (zero tension). If that were the case, we would get a zero or negative answer for the tension in the string. Newton's first law applied in the *y*-direction on block L yields

$$(\Sigma F_L)_y = 0 = n_L - (F_G)_L\cos(20°) \quad \Rightarrow \quad n_L = m_L g\cos(20°)$$

Therefore

$$(f_k)_L = (\mu_k)_L m_L g\cos(20°) = (0.20)(1.0\text{ kg})(9.80\text{ m/s}^2)\cos(20°) = 1.84\text{ N}$$

A similar analysis of the forces in the *y*-direction on block R gives $(f_k)_R = 1.84$ N as well. Using Newton's second law in the *x*-direction for block L gives

$$(\Sigma F_L)_x = m_L a = T_{\text{R on L}} - (f_k)_L + (F_G)_L\sin(20°) \quad \Rightarrow \quad m_L a = T_{\text{R on L}} - 1.84\text{ N} + m_L g\sin(20°)$$

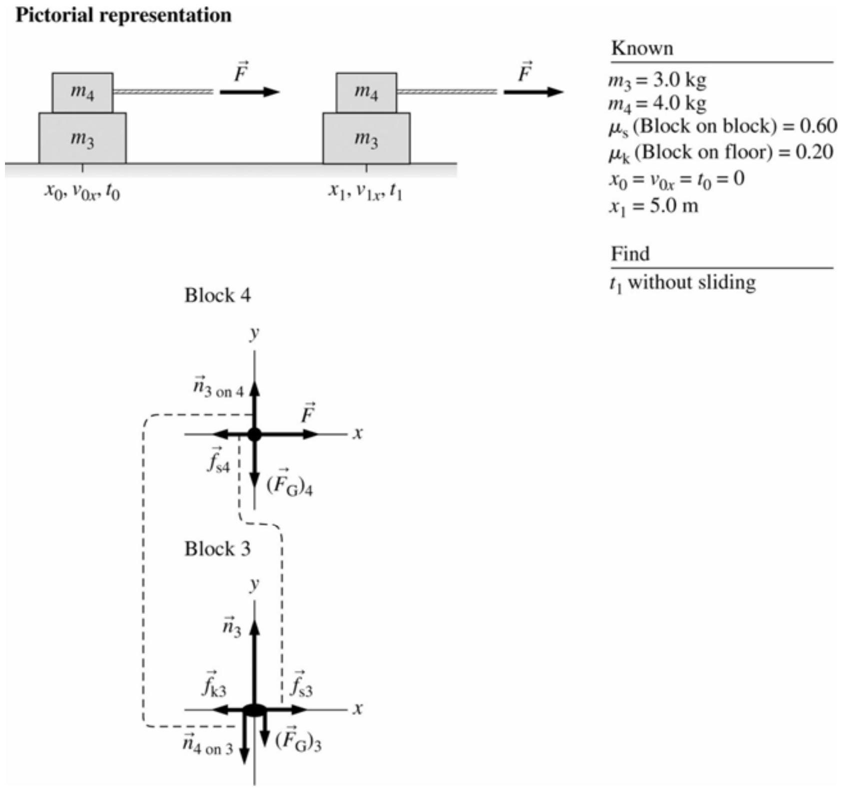
For block R,

$$(\Sigma F_R)_x = m_R a = (F_G)_R\sin(20°) - 1.84\text{ N} - T_{\text{L on R}} \quad \Rightarrow \quad m_R a = m_R g\sin(20°) - 1.84\text{ N} - T_{\text{L on R}}$$

Solving these two equations in the two unknowns a and $T_{\text{L on R}} = T_{\text{R on L}} \equiv T$, we obtain $a = 2.12\text{ m/s}^2$ and $T = 0.61$ N.
Assess: The tension in the string is positive, and is about 1/3 of the kinetic friction force on each of the blocks, which is reasonable.

7.35. Model: The 3-kg and 4-kg blocks constitute the system and are to be treated as particles. The models of kinetic and static friction and the constant-acceleration kinematic equations will be used.

Visualize:



Solve: The minimum time will be achieved when static friction is at its maximum possible value. Newton's second law for the 4-kg block is

$$\Sigma(F_{\text{on }4})_y = n_{3\text{ on }4} - (F_G)_4 = 0 \text{ N} \quad\Rightarrow\quad n_{3\text{ on }4} = (F_G)_4 = m_4 g = (4.0 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$$

$$f_{s4} = (f_s)_{\max} = \mu_s n_{3\text{ on }4} = (0.60)(39.2 \text{ N}) = 23.5 \text{ N}$$

Newton's second law for the 3-kg block is

$$\Sigma(F_{\text{on }3})_y = n_3 - n_{4\text{ on }3} - (F_G)_3 = 0 \text{ N} \quad\Rightarrow\quad n_3 = n_{4\text{ on }3} + (F_G)_3 = 39.2 \text{ N} + (3.0 \text{ kg})(9.8 \text{ m/s}^2) = 68.6 \text{ N}$$

Friction forces f and f_{s4} are an action/reaction pair. Thus

$$\Sigma(F_{\text{on }3})_x = f_{s3} - f_{k3} = m_3 a_3 \quad\Rightarrow\quad f_{s4} - \mu_k n_3 = m_3 a_3 \quad\Rightarrow\quad 23.5 \text{ N} - (0.20)(68.6 \text{ N}) = (3.0 \text{ kg})a_3$$

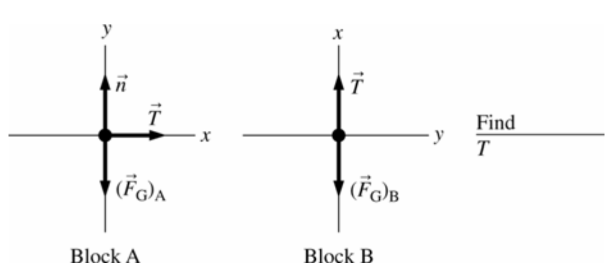
$$a_3 = 3.27 \text{ m/s}^2$$

Since block 3 does not slip, this is also the acceleration of block 4. The time is calculated as follows:

$$x_1 - x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a(t_1 - t_0)^2 \quad\Rightarrow\quad 5.0 \text{ m} = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(3.27 \text{ m/s}^2)(t_1 - 0 \text{ s})^2 \quad\Rightarrow\quad t_1 = 1.8 \text{ s}$$

7.36. Model: Blocks A and B make up the system of interest and will be treated as particles. There is no friction anywhere.

Visualize:



Notice that the coordinate system of for block B is rotated so that the motion in the positive x-direction is consistent between the two free-body diagrams.

Solve: The blocks are constrained to have the same magnitude acceleration. Applying Newton's second law to block B gives

$$\sum(F)_y = -T + (F_G)_B = ma \quad \Rightarrow \quad T - mg = -ma$$

Applying Newton's second law in both the x- and y-directions to the block A gives

$$\sum(F)_y = n - (F_G)_A = 0 \quad \Rightarrow \quad n = Mg$$
$$\sum(F)_x = T = Ma \quad \Rightarrow \quad T = Ma$$

Using the first equation to eliminate the acceleration a gives the tension:

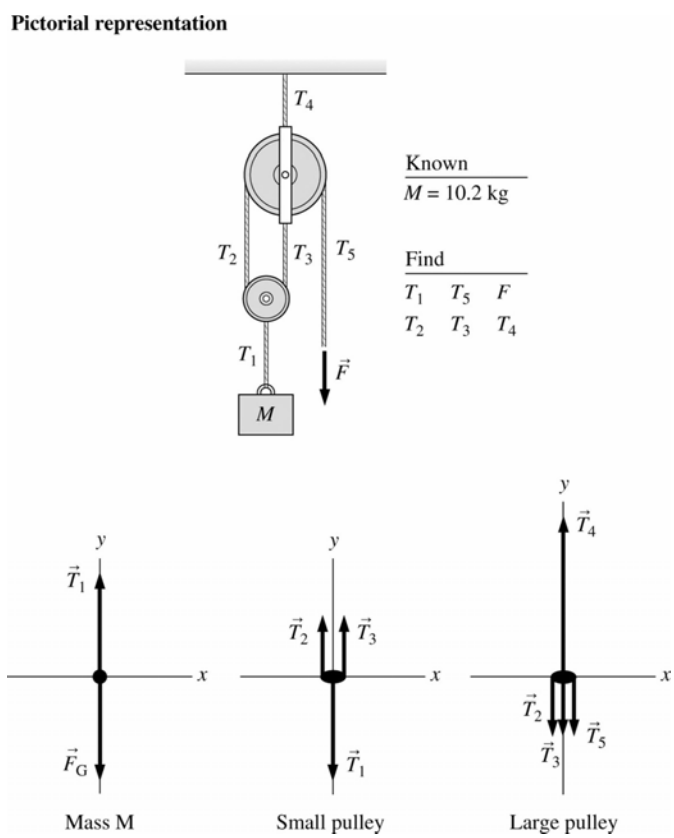
$$T = Ma = M(g - T/m) \quad \Rightarrow \quad T = \frac{mMg}{m+M}$$

Assess: The result is positive, as it should be for our choice of coordinate system. Consider $m = 0$. In this case, $T = 0$, as expected. For $m >> M$, the tension is independent of the mass of the hanging block because its acceleration will be g, as we can see by solving for the acceleration:

$$a = -\frac{T}{m} + g = g - \frac{Mg}{m+M} \to g \text{ for } m >> M$$

7.37. Model: Use the particle model for the block of mass M and the two massless pulleys. Additionally, the rope is massless and the pulleys are frictionless. The block is kept in place by an applied force $\vec{F}$.

Visualize:



Solve: Since there is no friction on the pulleys, $T_2 = T_3 = T_5$. Newton's second law for mass M gives

$$T_1 - F_G = 0 \text{ N} \quad \Rightarrow \quad T_1 = Mg = (10.2 \text{ kg})(9.8 \text{ m/s}^2) = 100 \text{ N}$$

Newton's second law for the small pulley is

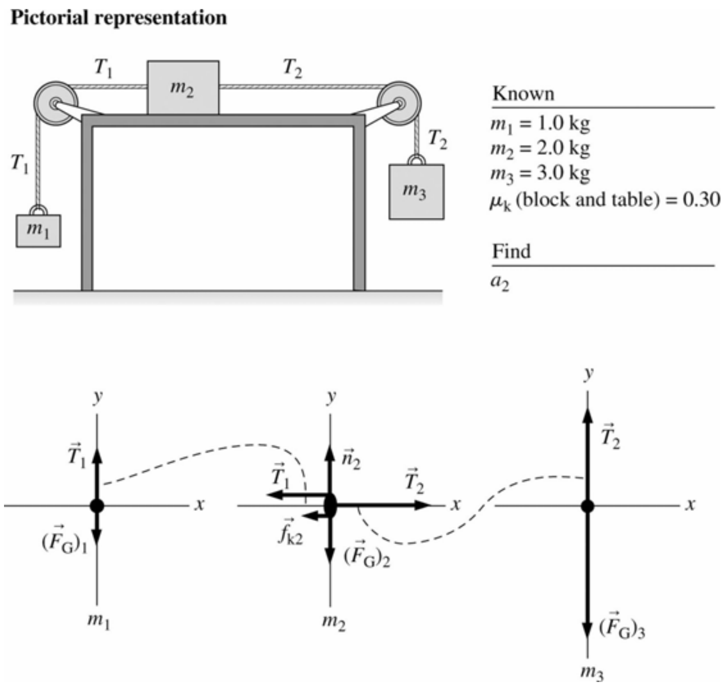
$$T_2 + T_3 - T_1 = 0 \text{ N} \quad \Rightarrow \quad T_2 = T_3 = \frac{T_1}{2} = 50 \text{ N} = T_5 = F$$

Newton's second law for the large pulley is

$$T_4 - T_2 - T_3 - T_5 = 0 \text{ N} \quad \Rightarrow \quad T_4 = T_2 + T_3 + T_5 = 150 \text{ N}$$

7.38. Model: Assume the particle model for $m_1, m_2,$ and $m_3,$ and the model of kinetic friction. Assume the ropes to be massless, and the pulleys to be frictionless and massless.

Visualize:



Solve: Newton's second law for m_1 gives $T_1 - (F_G)_1 = m_1 a_1$. Newton's second law for m_2 gives

$$\Sigma(F_{\text{on } m_2})_y = n_2 - (F_G)_2 = 0 \text{ N} \quad \Rightarrow \quad n_2 = m_2 g$$

$$\Sigma(F_{\text{on } m_2})_x = T_2 - f_{k2} - T = m_2 a_2 \quad \Rightarrow \quad T_2 - \mu_k n_2 - T_1 = m_2 a_2$$

Newton's second law for m_3 gives $T_2 - (F_G)_3 = m_3 a_3$. Since $m_1, m_2,$ and m_3 move together, $a_1 = a_2 = -a_3 \equiv a$. The equations for the three masses thus become

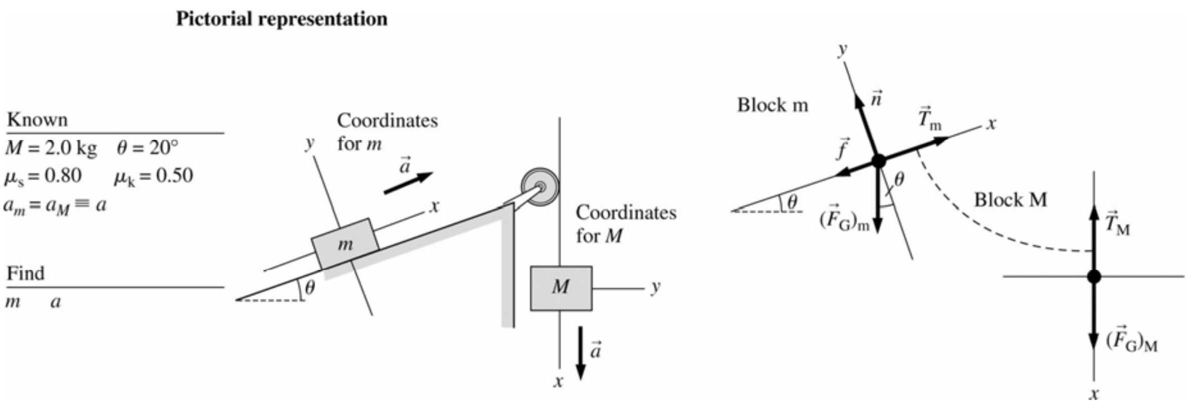
$$T_1 - (F_G)_1 = m_1 a \quad T_2 - \mu_k n_2 - T_1 = m_2 a \quad T_2 - (F_G)_3 = -m_3 a$$

Subtracting the third equation from the sum of the first two equations yields:

$$-(F_G)_1 - \mu_k n_2 + (F_G)_3 = -m_1 g - \mu_k m_2 g + m_3 g = (m_1 + m_2 + m_3)a$$

$$a = \frac{-m_1 g - \mu_k m_2 g + m_3 g}{(m_1 + m_2 + m_3)} = \frac{-1.0 \text{ kg} - (0.30)(2.0 \text{ kg}) + 3.0 \text{ kg}}{1.0 \text{ kg} + 2.0 \text{ kg} + 3.0 \text{ kg}}(9.8 \text{ m/s}^2) = 2.3 \text{ m/s}^2$$

7.39. Model: Assume the particle model for the two blocks, and the model of kinetic and static friction.
Visualize:



Solve: (a) If the mass *m* is too small, the hanging 2.0 kg mass will pull it up the slope. We want to find the smallest mass that will stick as a result of friction. The smallest mass will be the one for which the force of static friction is at its maximum possible value: $f_s = (f_s)_{max} = \mu_s n.$ As long as the mass *m* is stuck, both blocks are at rest with $\vec{F}_{net} = 0$ N. In this situation, Newton's second law for the hanging mass *M* gives

$$(F_{net})_x = -T_M + Mg = 0 \text{ N} \quad \Rightarrow \quad T_M = Mg = (2.0 \text{ kg})(9.8 \text{ m/s}^2) = 19.6 \text{ N}$$

For the smaller mass *m*,

$$(F_{net})_x = T_m - f_s - mg\sin\theta = 0 \text{ N} \quad (F_{net})_y = n - mg\cos\theta \quad \Rightarrow \quad n = mg\cos\theta$$

For a massless string and frictionless pulley, forces $\vec{T}_m$ and $\vec{T}_M$ act as if they are an action/reaction pair. Thus $T_m = T_M$. Mass *m* is a minimum when $f_s = (f_s)_{max} = \mu_s n = \mu_s mg\cos\theta.$ Substituting these expressions into the *x*-equation for m gives

$$T_M - \mu_s mg\cos\theta - mg\sin\theta = 0 \text{ N}$$

$$m = \frac{T_M}{(\mu_s\cos\theta + \sin\theta)g} = \frac{19.6 \text{ N}}{[(0.80)\cos(20°) + \sin(20°)](9.8 \text{ m/s}^2)} = 1.83 \text{ kg}$$

or 1.8 kg to two significant figures.
(b) Because $\mu_k < \mu_S$ the 1.8 kg block will begin to slide up the ramp and the 2.0 kg mass will begin to fall if the block is nudged ever so slightly. In this case, the net force and the acceleration are *not* zero. Notice how, in the pictorial representation, we chose different coordinate systems for the two masses. The magnitudes of the accelerations are the same because the blocks are tied together. Thus, the acceleration constraint is $a_m = a_M \equiv a$, where *a* will have a positive value. Newton's second law for block M gives

$$(F_{net})_x = -T + Mg = Ma_M = Ma$$

For block m we have

$$(F_{net})_x = T - f_k - mg\sin\theta = T - \mu_k mg\cos\theta - mg\sin\theta = ma_m = ma$$

In writing these equations, we used Newton's third law to obtain $T_m = T_M = T$. Also, notice that the *x*-equation and the friction model for block m don't change, except for μ_s becoming μ_k, so we already know the expression for f_k from part (a). Notice that the tension in the string is *not* the gravitational force *Mg*. We have two equations with the two unknowns *T* and *a*:

$$Mg - T = Ma \quad T - (\mu_k\cos\theta + \sin\theta)mg = ma$$

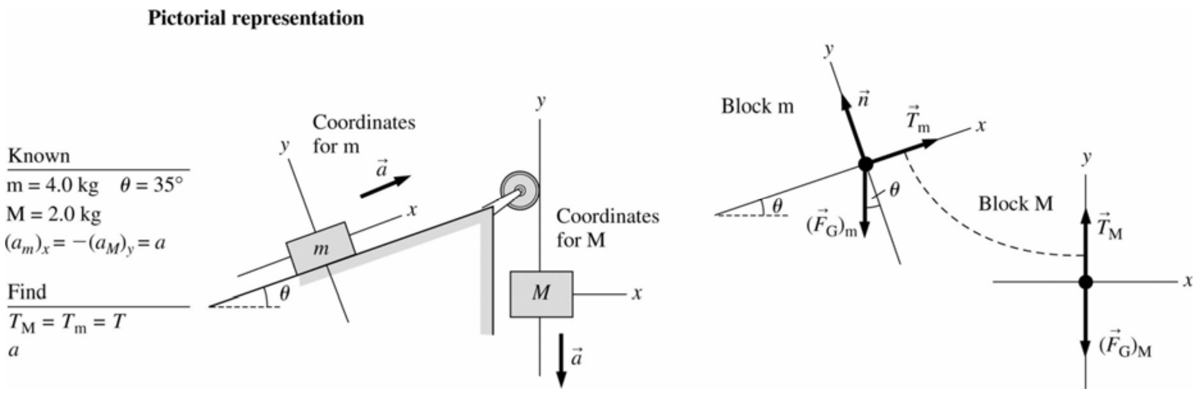
Adding the two equations to eliminate T gives

$$Mg-(\mu_k\cos\theta+\sin\theta)mg=Ma+ma$$

$$a=g\frac{M-(\mu_k\cos\theta+\sin\theta)m}{M+m}$$

$$=(9.8\text{ m/s}^2)\frac{2.0\text{ kg}-[(0.50)\cos(20°)+\sin(20°)](1.83\text{ kg})}{2.0\text{ kg}+1.83\text{ kg}}1.3\text{ m/s}^2$$

7.40. Model: Assume the particle model for the two blocks and use the friction model.
Visualize:



Solve: (a) The slope is frictionless, so the blocks stay in place *only* if held. Once m is released, the blocks will move one way or the other. As long as m is held, the blocks are in static equilibrium with $\vec{F}_{\text{net}}=0$ N. In this case, Newton's second law for the hanging block M is

$$(F_{\text{net on M}})_y=T_M-Mg=0\text{ N}\quad\Rightarrow\quad T_M=Mg=19.6\text{ N}$$

Because the string is massless and the pulley is frictionless, $T_M=T_m=T=20$ N (to two significant figures).

(b) The free-body diagram shows box m after it is released. Whether it moves up or down the slope depends on whether the acceleration a is positive or negative. The acceleration constraint is $(a_m)_x-(a_M)_y\equiv a$ Newton's second law for each system gives

$$(F_{\text{net on m}})_x=T-mg\sin\theta=m(a_m)_x=ma\quad(F_{\text{net on M}})_y=T-Mg=M(a_M)_y=-Ma$$

We have two equations in two unknowns. Subtract the second from the first to eliminate T:

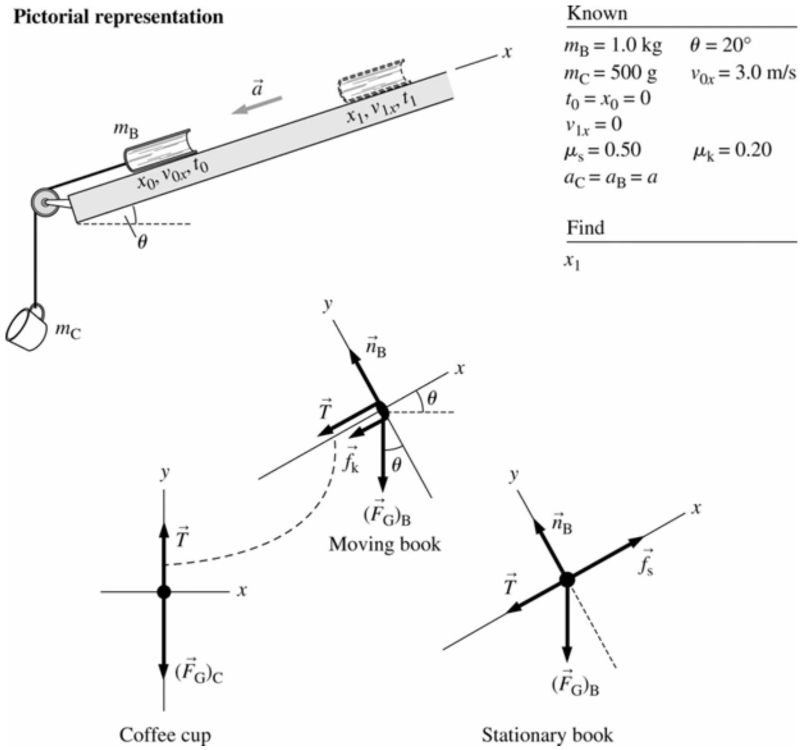
$$-mg\sin\theta+Mg=(m+M)a\quad\Rightarrow\quad a=\frac{M-m\sin\theta}{M+m}g=\frac{2.0\text{ kg}-(4.0\text{ kg})\sin(35°)}{2.0\text{ kg}+4.0\text{ kg}}=-0.48\text{ m/s}^2$$

Since $a<0$ m/s^2, the box accelerates *down* the slope.

(c) It is now straightforward to compute $T=Mg-Ma=21$ N. Notice how the tension is *larger* than when the blocks were motionless.

7.41. Model: Use the particle model for the book (B) and the coffee cup (C), the models of kinetic and static friction, and the constant-acceleration kinematic equations.

Visualize:



Solve: (a) Using $v_{1x}^2 = v_{0x}^2 + 2a(x_1 - x_0)$, we find

$$0\text{ m}^2/\text{s}^2 = (3.0\text{ m/s})^2 + 2a(x_1) \quad \Rightarrow \quad ax_1 = -4.5\text{ m}^2/\text{s}^2$$

To find x_1, we must first find *a*. Newton's second law applied to the book and the coffee cup gives

$$\sum(F_{\text{on B}})_y = n_B - (F_G)_B\cos(20°) = 0\text{ N} \quad \Rightarrow \quad n_B = (1.0\text{ kg})(9.8\text{ m/s}^2)\cos(20°) = 9.21\text{ N}$$

$$\sum(F_{\text{on B}})_x = -T - f_k - (F_G)_B\sin(20°) = m_B a_B \quad \sum(F_{\text{on C}})_y = T - (F_G)_C = m_C a_C$$

The last two equations can be rewritten, using $a_C = a_B = a$, as

$$-T - \mu_k n_B - m_B g\sin(20°) = m_B a \quad T - m_C g = m_C a$$

Adding the two equations gives

$$a(m_C + m_B) = -g[m_C + m_B\sin(20°)] - \mu_k(9.21\text{ N})$$

$$(1.5\text{ kg})a = -(9.8\text{ m/s}^2)[0.500\text{ kg} + (1.0\text{ kg})\sin 20°] - (0.20)(9.21\text{ N}) \quad \Rightarrow \quad a = -6.73\text{ m/s}^2$$

Using this value for *a*, we can now find x_1 as follows:

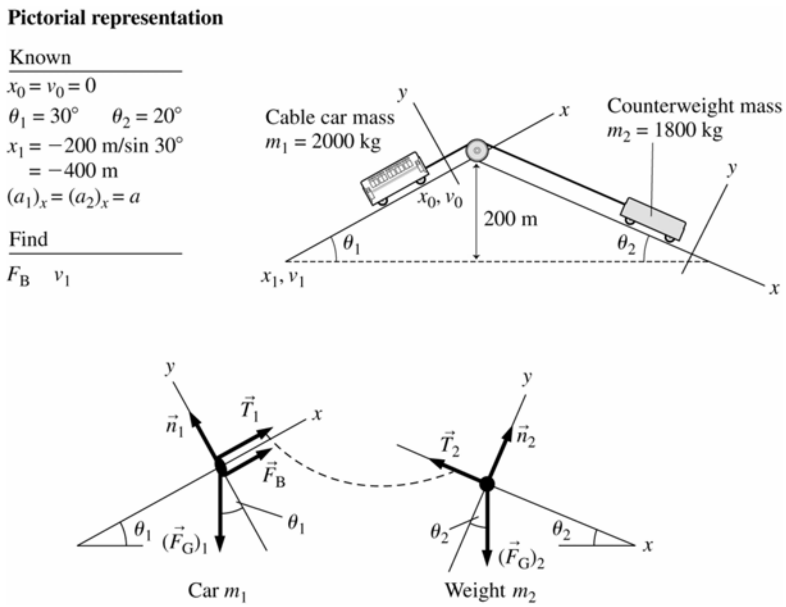
$$x_1 = \frac{-4.5\text{ m}^2/\text{s}^2}{a} = \frac{-4.5\text{ m}^2/\text{s}^2}{-6.73\text{ m/s}^2} = 0.67\text{ m}$$

(b) The maximum static friction force is $(f_s)_{\text{max}} = \mu_s n_B = (0.50)(9.21\text{ N}) = 4.60\text{ N}$. We'll see if the force f_s needed to keep the book in place is larger or smaller than $(f_s)_{\text{max}}$. When the cup is at rest, the string tension is $T = m_C g$. Newton's first law for the book is

$$\sum(F_{\text{on B}})_x = f_s - T - w_B\sin(20°) = f_s - m_C g - m_B g\sin(20°) = 0$$
$$f_s = (M_C + M_B\sin 20°)g = 8.25\text{ N}$$

Because $f_s > (f_s)_{\text{max}}$, the book slides back down.

7.42. Model: Use the particle model for the cable car and the counterweight. Assume a massless cable.
Visualize:



Solve: (a) Notice the separate coordinate systems for the cable car (object 1) and the counterweight (object 2). Forces $\vec{T}_1$ and $\vec{T}_2$ act as if they are an action/reaction pair. The braking force $\vec{F}_B$ works with the cable tension $\vec{T}_1$ to allow the cable car to descend at a constant speed. Constant speed means dynamic equilibrium, so $\vec{F}_{net} = 0$ N for both systems. Newton's second law applied to the cable car gives

$$(F_{\text{net on 1}})_x = T_1 + F_B - m_1 g \sin\theta_1 = 0 \text{ N} \quad (F_{\text{net on 1}})_y = n_1 - m_1 g \cos\theta_1 = 0 \text{ N}$$

Newton's second law applied to the counterweight gives

$$(F_{\text{net on 2}})_x = m_2 g \sin\theta_2 - T_2 = 0 \text{ N} \quad (F_{\text{net on 2}})_y = n_2 - m_2 g \cos\theta_2 = 0 \text{ N}$$

From the x-equation for the counterweight, $T_2 = m_2 g \sin\theta_2$. Because we can neglect the pulley's friction and the cable is assumed to be massless, $T_1 = T_2$. Thus the x-equation for the cable car then becomes

$$F_B = m_1 g \sin\theta_1 - T_1 = m_1 g \sin\theta_1 - m_2 g \sin\theta_2 = 3770 \text{ N} = 3.8 \text{ kN}$$

(b) If the brakes fail, then $F_B = 0$ N. The car will accelerate down the hill on one side while the counterweight accelerates up the hill on the other side. Both will have *negative* accelerations because of the direction of the acceleration vectors. The constraint is $a_{1x} = a_{2x} = a$, where a will have a negative value. Using $T_1 = T_2 = T$, the two x-equations are

$$(F_{\text{net on 1}})_x = T - m_1 g \sin\theta_1 = m_1 a_{1x} = m_1 a \quad (F_{\text{net on 2}})_x = m_2 g \sin\theta_2 - T = m_2 a_{2x} = m_2 a$$

Note that the y-equations aren't needed in this problem. Add the two equations to eliminate T:

$$-m_1 g \sin\theta_1 + m_2 g \sin\theta \; = (m_1 + m_2)a \quad \Rightarrow \quad a = -\frac{m_1 \sin\theta_1 - m_2 \sin\theta_2}{m_1 + m_2} g = -0.991 \text{ m/s}^2$$

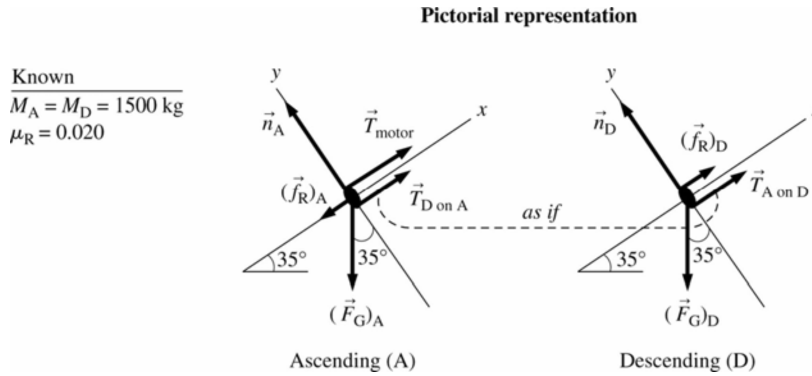
Now we have a problem in kinematics. The speed at the bottom is calculated as follows:

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) = 2ax_1 \quad \Rightarrow \quad v_1 = \sqrt{2ax_1} = \sqrt{2(-0.991 \text{ m/s}^2)(-400 \text{ m})} = 28 \text{ m/s}$$

Assess: A speed of approximately 60 mph as the cable car travels a distance of 2000 m along a frictionless slope of 30° is reasonable.

7.43. Model: Assume the cable mass is negligible compared to the car mass and that the pulley is frictionless. Use the particle model for the two cars.

Visualize: Please refer to Figure P7.43.



Solve: (a) The cars are moving at constant speed, so they are in dynamic equilibrium. Consider the descending car D. We can find the rolling friction force on car D, and then find the cable tension by applying Newton's first law. In the y-direction for car D,

$$(F_{net})_y = 0 = n_D - (F_G)_D \cos(35°)$$
$$n_D = m_D g \cos(35°)$$

So the rolling friction force on car D is

$$(f_R)_D = \mu_R n_D = \mu_R m_D g \cos(35°)$$

Applying Newton's first law to car D in the x-direction gives

$$(F_{net})_x = T_{A\text{ on }D} + (f_R)_D - (F_G)_D \sin(35°) = 0$$

Thus,

$$\begin{aligned} T_{A\text{ on }D} &= m_D g \sin(35°) - \mu_R m_D g \cos(35°) \\ &= (1500\text{ kg})(9.80\text{ m/s}^2)[\sin(35°) - (0.020)\cos(35°)] \\ &= 8.2 \times 10^3\text{ N} \end{aligned}$$

(b) Similarly, we find that for car A, $(f_R)_A = \mu_R m_A g \cos(35°)$. In the x-direction for car A,

$$(F_{net})_x = T_{motor} + T_{D\text{ on }A} - (f_R)_A - (F_G)_A \sin(35°) = 0$$
$$T_{motor} = m_A g \sin(35°) + \mu_R m_A g \cos(35°) - m_D g \sin(35°) + \mu_R m_D g \cos(35°)$$

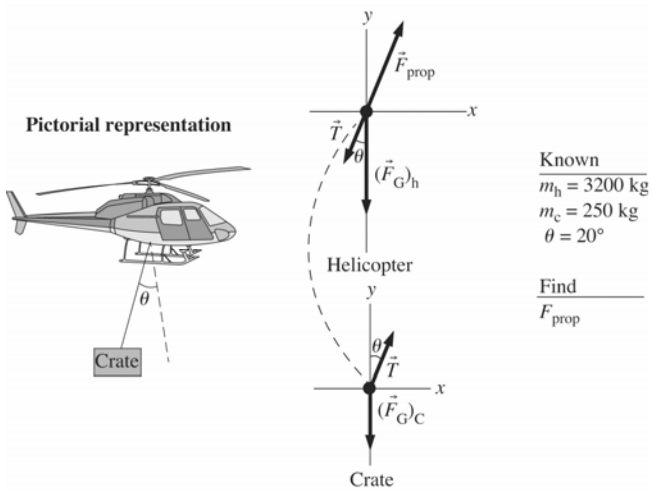
Here, we have used $T_{A\text{ on }D} = T_{D\text{ on }A}$. If we also use $\mathrm{m_A} = \mathrm{m_D}$, then

$$T_{motor} = 2\mu_R m_A g \cos(35°) = 4.8 \times 10^2\text{ N}.$$

Assess: Careful examination of the free-body diagrams for cars D and A yields the observation that $T_{motor} = 2(F_R)_A$ in order for the cars to be in dynamic equilibrium. It is a tribute to the design that the motor must only provide such a small force compared to the tension in the cable connecting the two cars.

7.44. Model: The helicopter and crate are particles. They are accelerating in the x-direction and are not in equilibrium. Ignore air resistance. The cable is massless.

Visualize:



Solve: First apply the second law to the crate.

$$\Sigma F_x = T\sin\theta = m_c a_x$$
$$\Sigma F_y = T\cos\theta - m_c g = 0 \Rightarrow T\cos\theta = m_c g$$

Divide the two equations to get $\tan\theta = a_x/g \Rightarrow a_x = g\tan\theta$. Now apply the second law to the helicopter and insert what we learned above.

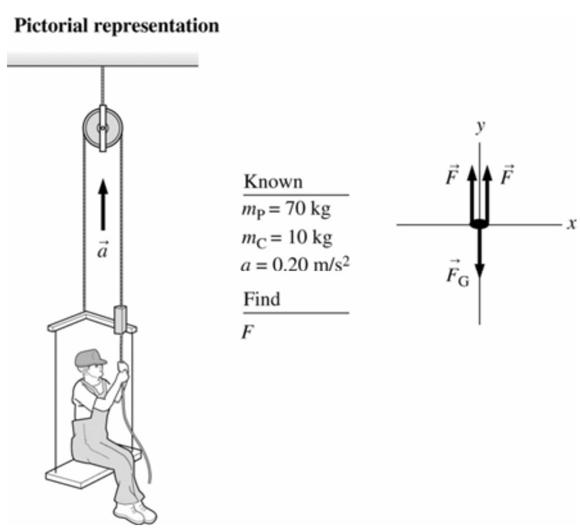
$$\Sigma F_x = (F_{\text{prop}})_x - T\sin\theta = m_h a_x \Rightarrow (F_{\text{prop}})_x = m_c a_x + m_h a_x = (m_c + m_h)g\tan\theta$$
$$\Sigma F_y = (F_{\text{prop}})_y - T\cos\theta - m_h g = 0 \Rightarrow (F_{\text{prop}})_y = m_c g + m_h g = (m_c + m_h)g$$

Plug in the values given.

$$\vec{F}_{\text{prop}} = ((m_c + m_h)g\tan\theta)\hat{i} + ((m_c + m_h)g)\hat{j}$$
$$= ((3450\text{ kg})(9.8\text{ m/s}^2)\tan 20°)\hat{i} + ((3450\text{ kg})(9.8\text{ m/s}^2))\hat{j}$$
$$= (12\text{ kN})\hat{i} + (34\text{ kN})\hat{j}$$

Assess: This answer is in the same range as other helicopter problems.

7.45. Model: The painter and the chair are treated as a single object and represented as a particle. We assume that the rope is massless and that the pulley is massless and frictionless.
Visualize:



Solve: If the painter pulls down on the rope with force F, Newton's third law requires the rope to pull up on the painter with force F. This is just the tension in the rope. With our model of the rope and pulley, the same tension force F also pulls up on the painter's chair. Newton's second law for $(\text{painter} + \text{chair})$ gives

$$2F - F_G = (m_P + m_C)a$$
$$F = \left(\tfrac{1}{2}\right)[(m_P + m_C)a + (m_P + m_C)g] = \tfrac{1}{2}(m_P + m_C)(a+g)$$
$$= \left(\tfrac{1}{2}\right)(70\text{ kg} + 10\text{ kg})(0.20\text{ m/s}^2 + 9.8\text{ m/s}^2) = 4.0\times10^2\text{ N}$$

Assess: A force of 400 N, which is approximately one-half the total gravitational force, is reasonable since the upward acceleration is small.

7.46. Model: The rope is not massless.

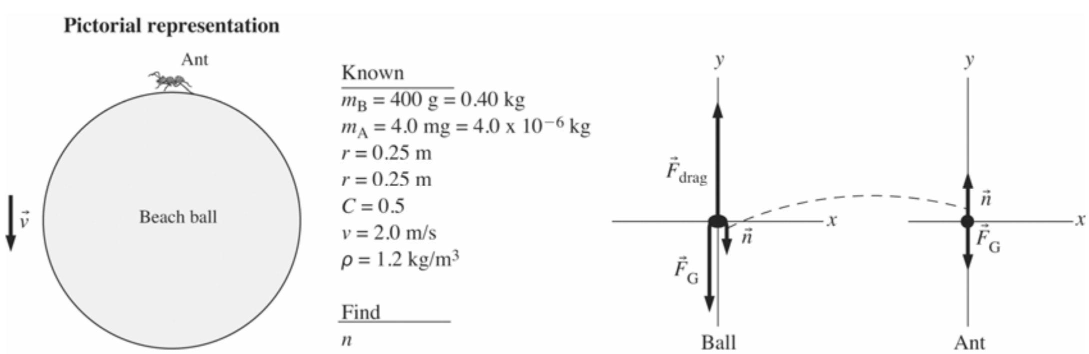
Visualize: Because the weight of the rope already creates $(1.0\text{ kg})(9.8\text{ m/s}^2) = 9.8\text{ N}$ of tension in the rope, the tension due to the robot mouse cannot exceed $40\text{ N} - 9.8\text{ N} = 30.2\text{ N}$.

Solve: Apply the second law to the mouse.

$$\Sigma F_y = T - m_m g = m_m a \Rightarrow a = \frac{T - m_m g}{m_m} = \frac{30.2\text{ N} - (2.0\text{ kg})(9.8\text{ m/s}^2)}{2.0\text{ kg}} = 5.3\text{ m/s}^2$$

The direction is up, because if the robot accelerates down that creates less tension in the rope, not more.
Assess: This seems like a very powerful robot.

7.47. Model: The air resistance on the ball is not negligible, but it is on the ant (the ball shields the ant).
Visualize: The normal force of the ball on the ant is equal in magnitude to the normal force of the ant on the ball.



Solve: Apply the second law to both objects; the n is the same magnitude in each equation since the normal forces are an action/reaction pair.

$$\text{ball: } \Sigma F_y = F_{drag} - n - m_b g = m_b a$$
$$\text{ant: } \Sigma F_y = n - m_a g = m_a a$$

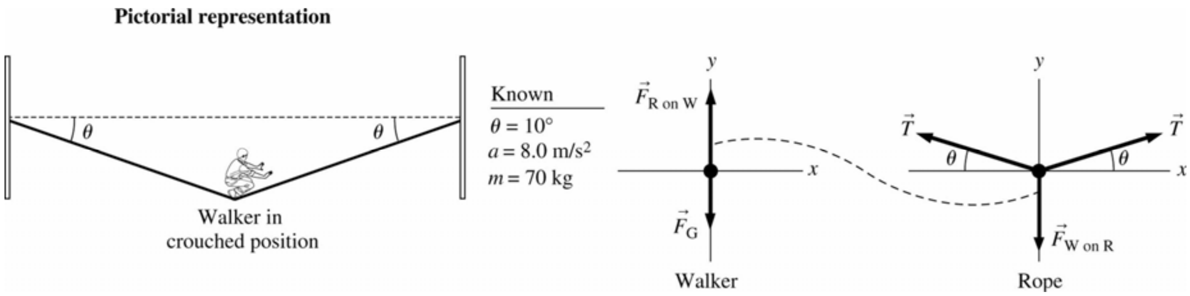
We have two equations in two unknowns; one approach to solve the system is to divide the two equations (eliminating a) and solve for n.

$$\frac{F_{drag} - n - m_b g}{n - m_a g} = \frac{m_b}{m_a} \quad \Rightarrow \quad n = \frac{m_a}{m_a + m_b}F_{drag}$$
$$= \frac{4.0\times10^{-6}\text{ kg}}{4.0\times10^{-6}\text{ kg} + 0.40\text{ kg}}\left(\frac{1}{2}(0.5)(1.2\text{ kg/m}^3)\pi(0.25\text{ m})^2(2.0\text{ m/s})^2\right) = 2.4\times10^{-6}\text{ N}$$

Assess: The normal force is very small, as one would expect for an ant; it is even smaller than the gravitational force on the ant because the ball-ant system is accelerating downward.

7.48. Model: Use the particle model for the tightrope walker and the rope. The rope is assumed to be massless, so the tension in the rope is uniform.

Visualize:



Solve: Newton's second law applied to the tightrope walker gives

$$F_{\text{R on W}} - F_{\text{G}} = ma \quad \Rightarrow \quad F_{\text{R on W}} = m(a+g) = (70 \text{ kg})(8.0 \text{ m/s}^2 + 9.8 \text{ m/s}^2) = 1.25 \times 10^3 \text{ N}$$

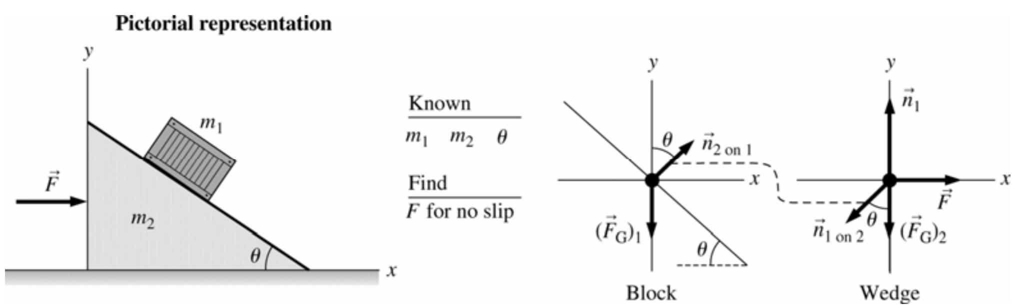
Newton's second law applied to the rope gives

$$\Sigma(F_{\text{on R}})_y = T\sin\theta + T\sin\theta - F_{\text{W on R}} = 0 \text{ N} \quad \Rightarrow \quad T = \frac{F_{\text{W on R}}}{2\sin(10°)} = \frac{F_{\text{R on W}}}{2\sin(10°)} = \frac{1.25\times 10^3 \text{ N}}{2\sin(10°)} = 3.6\times 10^3 \text{ N}$$

We used $F_{\text{W on R}} = F_{\text{R on W}}$ because they are an action/reaction pair.

7.49. Model: Use the particle model for the wedge and the block.

Visualize:



The block will not slip relative to the wedge if they both have the same horizontal acceleration a. Note that $n_{1 \text{ on } 2}$ and $n_{2 \text{ on } 1}$ form a third-law pair, so $n_{1 \text{ on } 2} = n_{2 \text{ on } 1}$.

Solve: Newton's second law applied to block m_1 in the y-direction gives

$$\Sigma(F_{\text{on 1}})_y = n_{2 \text{ on } 1}\cos\theta - (F_G)_1 = 0 \text{ N} \quad \Rightarrow \quad n_{2 \text{ on } 1} = \frac{m_1 g}{\cos\theta}$$

Combining this equation with the x-component of Newton's second law yields:

$$\Sigma(F_{\text{on 1}})_x = n_{2 \text{ on } 1}\sin\theta = m_1 a \quad \Rightarrow \quad a = \frac{n_{2 \text{ on } 1}\sin\theta}{m_1} = g\tan\theta$$

Newton's second law applied to the wedge-block system gives

$$F = m_1 a + m_2 a = (m_1 + m_2)a = (m_1 + m_2)g\tan\theta$$

7.50. Model: No air resistance in space.

Visualize: The small pellet of fuel accelerates because of a force on it, but it also exerts a force forward on the rocket. $dm/dt = 5.0$ kg/s.

Solve: **(a)** Use the second law. The force on the pellet is given by its mass Δm multiplied by its acceleration. If we assume the pellet starts at rest with respect to the rocket, then $\Delta v = v_{ex} - 0 = v_{ex}$.

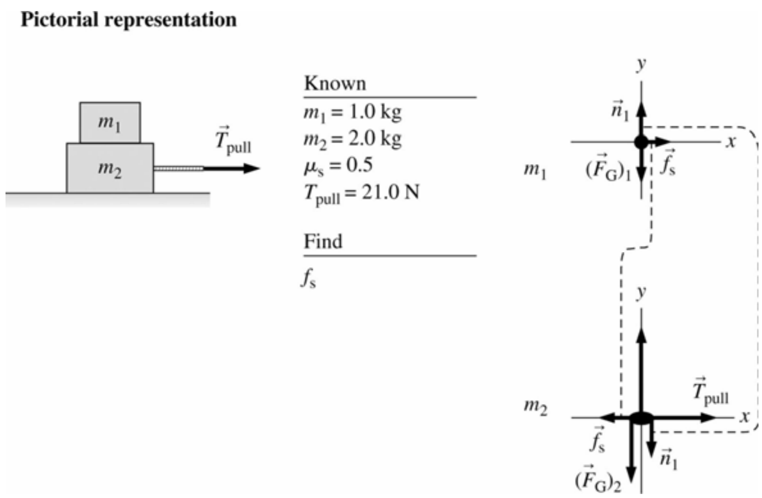
$$F = (\Delta m)a = (\Delta m)\frac{\Delta v}{\Delta t} = \Delta v \frac{\Delta m}{\Delta t} = v_{ex}\frac{\Delta m}{\Delta t}$$

That is the magnitude of the force on the pellet and also the magnitude of the force of the pellet on the rocket.
(b) In the limit as Δm and Δt approach zero $\Delta m/\Delta t$ becomes the derivative dm/dt.

$$F_{thrust} = v_{ex}\frac{\Delta m}{\Delta t} = v_{ex}\frac{dm}{dt} = (4.0\text{ km/s})(5.0\text{ kg/s}) = 20\text{ kN}$$

Assess: This is a very small thrust force for a rocket motor. Note that the units work out correctly.

7.51. A 1.0 kg wood block is placed on top of a 2.0 kg wood block. A horizontal rope pulls the 2.0 kg block across a frictionless floor with a force of 21.0 N. Does the 1.0 kg block on top slide?
Visualize:



Solve: The 1.0 kg block is accelerated by static friction. It moves smoothly with the lower block if $f_s < (f_s)_{max}$. It slides if the force that would be needed to keep it in place exceeds $(f_s)_{max}$. Begin by assuming that the blocks move together with a common acceleration a. Newton's second law gives

$$\text{Top block:} \quad \sum(F_{on\ 1})_x = f_s = m_1 a$$
$$\text{Bottom block:} \quad \sum(F_{on\ 2})_x = T_{pull} - f_s = m_2 a$$

Adding these two equations gives $T_{pull} = (m_1 + m_2)a$, or $a = (21.0\text{ N})/(1.0\text{ kg} + 2.0\text{ kg}) = 7.0\text{ m/s}^2$. The static friction force needed to accelerate the top block at 7.0 m/s^2 is

$$f_s m_1 a = (1.0\text{ kg})(7.0\text{ m/s}^2) = 7.0\text{ N}$$

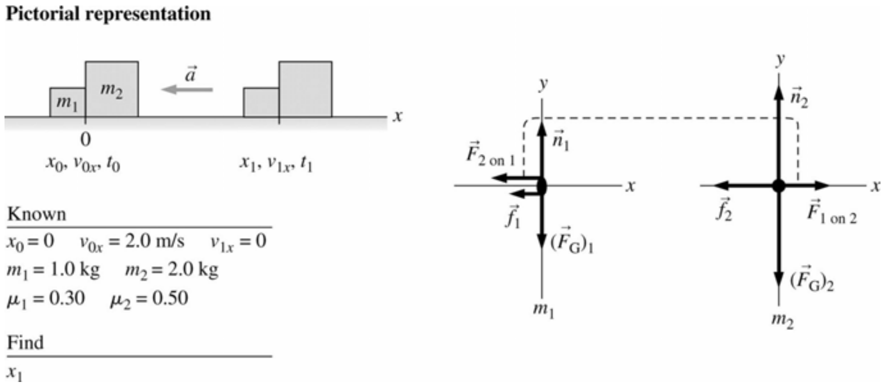
To find the maximum possible static friction force $(f_s)_{max} = \mu_s n_1$, the y-equation of Newton's second law for the top block shows that $n_1 = m_1 g$. Thus

$$(f_s)_{max} = \mu_s m_1 g = (0.50)(1.0\text{ kg})(9.8\text{ m/s}^2) = 4.9\text{ N}$$

Because $7.0\text{ N} > 4.9\text{ N}$, static friction is *not* sufficient to accelerate the top block, so it slides.

7.52. A 1.0 kg wood block is placed behind a 2.0 kg wood block on a horizontal table. The coefficients of kinetic friction with the table are 0.30 for the 1.0 kg block and 0.50 for the 2.0 kg block. The 1.0 kg block is pushed forward, against the 2.0 kg block, and released with a speed of 2.0 m/s. How far do the blocks travel before stopping?

Visualize:



Solve: The 2.0 kg block in front has a larger coefficient of friction. Thus the 1.0 kg block pushes against the rear of the 2.0 kg block and, in reaction, the 2.0 kg block pushes back against the 1.0 kg block. There's no vertical acceleration, so $n_1 = m_1 g$ and $n_2 = m_2 g$, leading to $f_1 = \mu_1 m_1 g$ and $f_2 = \mu_2 m_2 g$. Applying Newton's second law along the *x*-axis gives

$$\text{1 kg block:}\quad \Sigma(F_{\text{on 1}})_x = -F_{\text{2 on 1}} - f_1 = -F_{\text{2 on 1}} - \mu_1 m_1 g = m_1 a$$
$$\text{2 kg block:}\quad \Sigma(F_{\text{on 2}})_x = F_{\text{1 on 2}} - f_2 = F_{\text{2 on 1}} - \mu_2 m_2 g = m_2 a$$

where we used $a_1 = a_2 = a$. Also, $F_{\text{1 on 2}} = F_{\text{2 on 1}}$ because they are a third-law action/reaction pair. Adding these two equations gives

$$-(\mu_1 m_1 + \mu_2 m_2)\, g = (m_1 + m_2) a$$
$$a = -\frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} g = -\frac{(0.30)(1.0\text{ kg}) + (0.50)(2.0\text{ kg})}{1.0\text{ kg} + 2.0\text{ kg}}(9.8\text{ m/s}^2) = -4.25\text{ m/s}^2$$

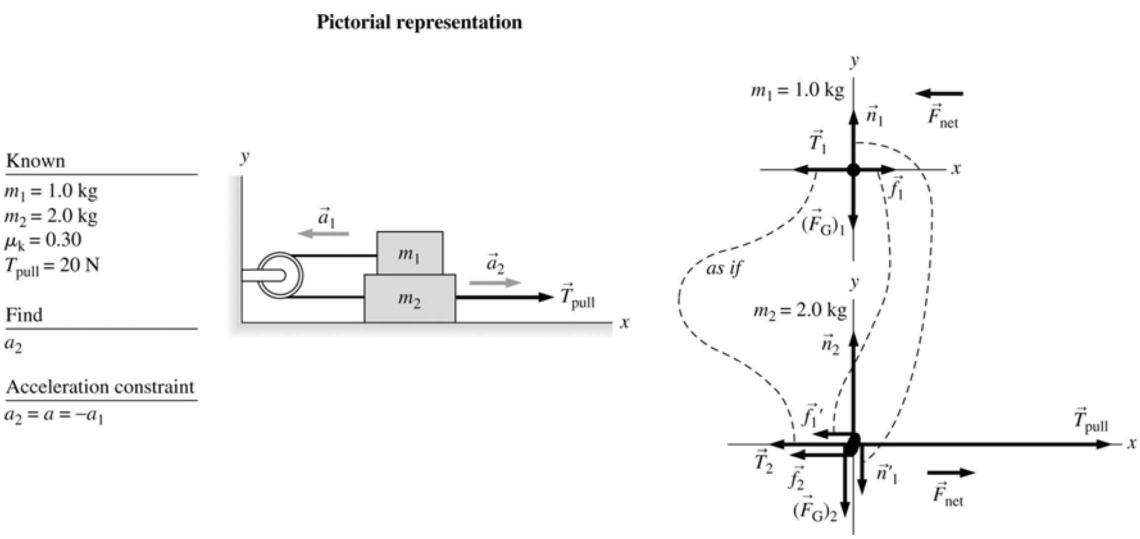
We can now use constant-acceleration kinematics to find

$$v_{1x}^2 = 0 = v_{0x}^2 + 2a(x_1 - x_0) \quad\Rightarrow\quad x_1 = -\frac{v_{0x}^2}{2a} = -\frac{(2.0\text{ m/s})^2}{2(-4.25\text{ m/s}^2)} = 0.47\text{ m}$$

Challenge Problems

7.53. Model: Blocks 1 and 2 make up the system of interest and will be treated as particles. Assume a massless rope and frictionless pulley.

Visualize:



Solve: The blocks accelerate with the same magnitude but in opposite directions. Thus the acceleration constraint is $a_2 = a = -a_1$, where a will have a positive value. There are two real action/reaction pairs. The two tension forces will act as if they are action/reaction pairs because we are assuming a massless rope and a frictionless pulley. Make sure you understand why the friction forces point in the directions shown in the free-body diagrams, especially force $\vec{f}_1'$ exerted on block 2 by block 1. We have quite a few pieces of information to include. First, Newton's second law applied to blocks 1 and 2 gives

$$(\vec{F}_{\text{net on 1}})_x = f_1 - T_1 = \mu_k n_1 - T_1 = m_1 a_1 = -m_1 a$$
$$(F_{\text{net on 1}})_y = n_1 - m_1 g = 0\text{ N} \quad\Rightarrow\quad n_1 = m_1 g$$
$$(F_{\text{net on 2}})_x = T_{\text{pull}} - f_1' - f_2 - T_2 = T_{\text{pull}} - f_1' - \mu_k n_2 - T_2 = m_2 a_2 = m_2 a$$
$$(F_{\text{net on 2}})_y = n_2 - n_1' - m_2 g = 0\text{ N} \quad\Rightarrow\quad n_2 = n_1' + m_2 g$$

We've already used the kinetic friction model in both x-equations. Next, Newton's third law gives

$$n_1' = n_1 = m_1 g \qquad f_1' = f_1 = \mu_k n_1 = \mu_k m_1 g \qquad T_1 = T_2 = T$$

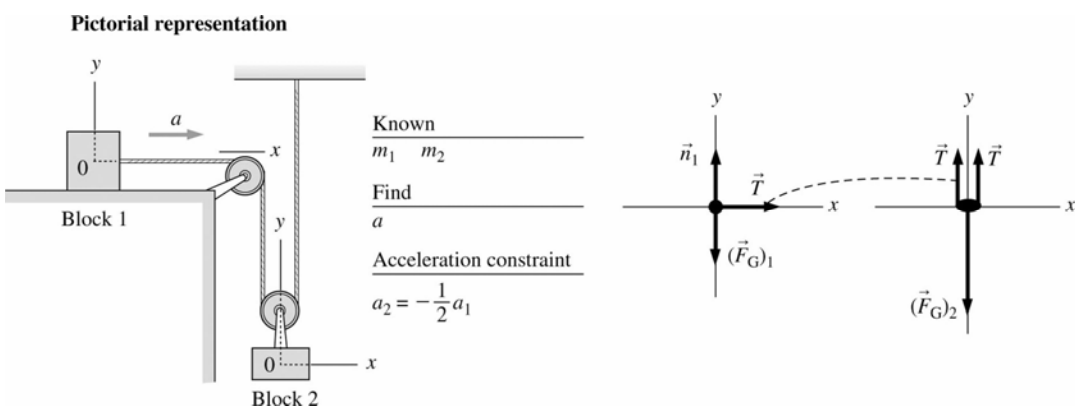
Knowing n_1', we can now use the y-equation of block 2 to find n_2. Substitute all these pieces into the two x-equations, and we end up with two equations with two unknowns:

$$\mu_k m_1 g - T = -m_1 a \qquad T_{\text{pull}} - T - \mu_k m_1 g - \mu_k (m_1 + m_2) g = m_2 a$$

Subtract the first equation from the second to get

$$T_{\text{pull}} - \mu_k (3m_1 + m_2) g = (m_1 + m_2) a$$
$$a = \frac{T_{\text{pull}} - \mu_k (3m_1 + m_2) g}{m_1 + m_2} = \frac{20\text{ N} - (0.30)[3(1.0\text{ kg}) + 2.0\text{ kg}](9.8\text{ m/s}^2)}{1.0\text{ kg} + 2.0\text{ kg}} = 1.8\text{ m/s}^2$$

7.54. Model: Use the particle model for the two blocks. Assume a massless rope and massless, frictionless pulleys.
Visualize:



Note that for every meter block 1 moves forward, one meter is provided to block 2. So each rope on m_2 has to be lengthened by one-half meter. Thus, the acceleration constraint is $a_2 = -\frac{1}{2}a_1$.

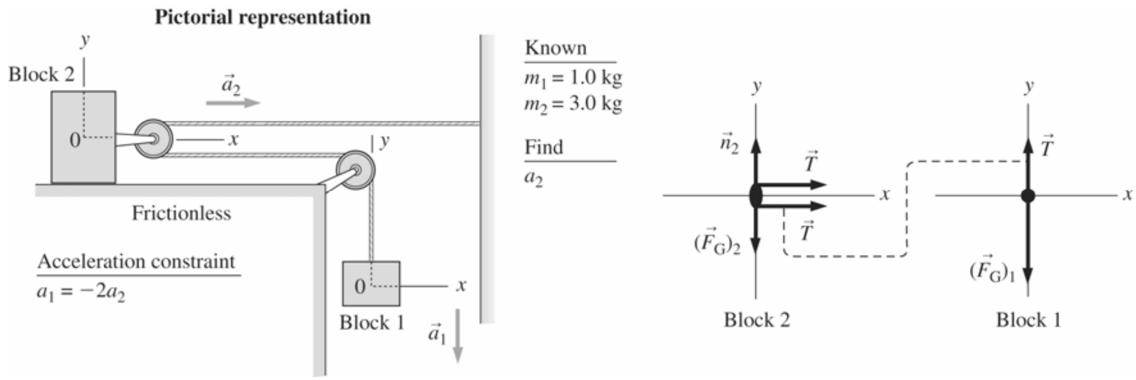
Solve: Newton's second law applied to block 1 gives $T = m_1 a_1$. Newton's second law applied to block 2 gives $2T - (F_G)_2 = m_2 a_2$. Combining these two equations gives

$$2(m_1 a_1) - m_2 g = m_2\left(-\tfrac{1}{2}a_1\right) \quad\Rightarrow\quad a_1(4m_1 + m_2) = 2m_2 g \quad\Rightarrow\quad a_1 = \frac{2m_2 g}{4m_1 + m_2}$$

where we have used $a_2 = -\frac{1}{2}a_1$.

Assess: If $m_1 = 0$ kg, then $a_2 = -g$. This is what is expected for a freely falling object.

7.55. Model: Use the particle model for the two blocks. Assume a massless rope and massless, frictionless pulleys.
Visualize:



For every one meter that the 1.0-kg block goes down, each rope on the 3.0-kg block will be shortened by one-half meter. Thus, the acceleration constraint is $a_1 = -2a_2$.
Solve: Newton's second law applied to the two blocks gives

$$2T = m_2 a_2 \quad T - (F_G)_1 = m_1 a_1$$

Since $a_1 = -2a_2$, the above equations become

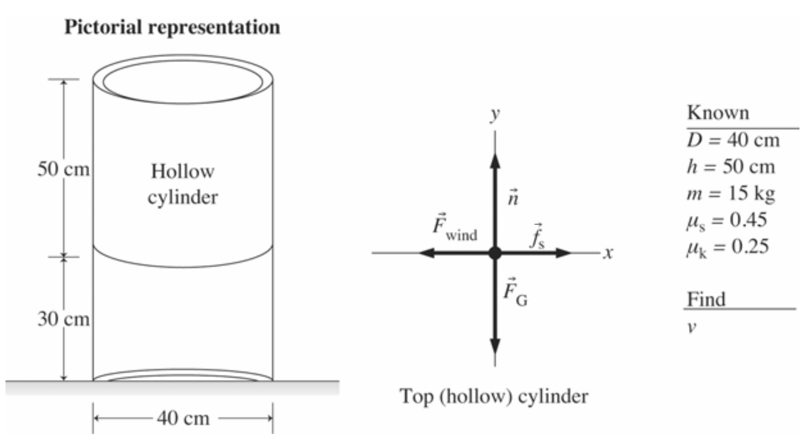
$$2T = m_2 a_2 \quad T - m_1 g = m_1(-2a_2)$$

$$m_2 \frac{a_2}{2} + m_1(2a_2) = m_1 g$$

$$a_2 = \frac{2m_1 g}{m_2 + 4m_1} = \frac{2(1.0\text{ kg})(9.8\text{ m/s}^2)}{(3.0\text{ kg} + 4.0\text{ kg})} = 2.8\text{ m/s}^2$$

Assess: If $m_1 = 0$ kg, then $a_2 = 0$ m/s^2, which is expected.

7.56. Model: This appears to be a hard problem until we shift reference frames and realize the answer to the given problem is the same as if the cylinders were stationary and the wind is blowing against them. The force of the wind on the stationary cylinder would be the same as the drag force if the cylinder were moving through still air, and the computed speed of the wind would be the same as the desired speed of the cylinders through still air.
Visualize: The two cylinders do not move with respect to each other so we use the coefficient of static friction rather than the coefficient of kinetic friction. We use $(f_s)_{max} = \mu_s n$. We draw the following figure with the idea that the cylinders are at rest and the wind is blowing against them. We then only need a free-body diagram of the top cylinder.



Solve: Use the second law on the upper cylinder, where v in the drag force is the speed of the wind right when the top cylinder begins to slide off of the bottom one.

$$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$$

$$\Sigma F_x = (f_s)_{max} - F_{drag} = 0 \Rightarrow \mu_s n = F_{drag} = \tfrac{1}{2} C\rho A v^2$$

Solve for v.

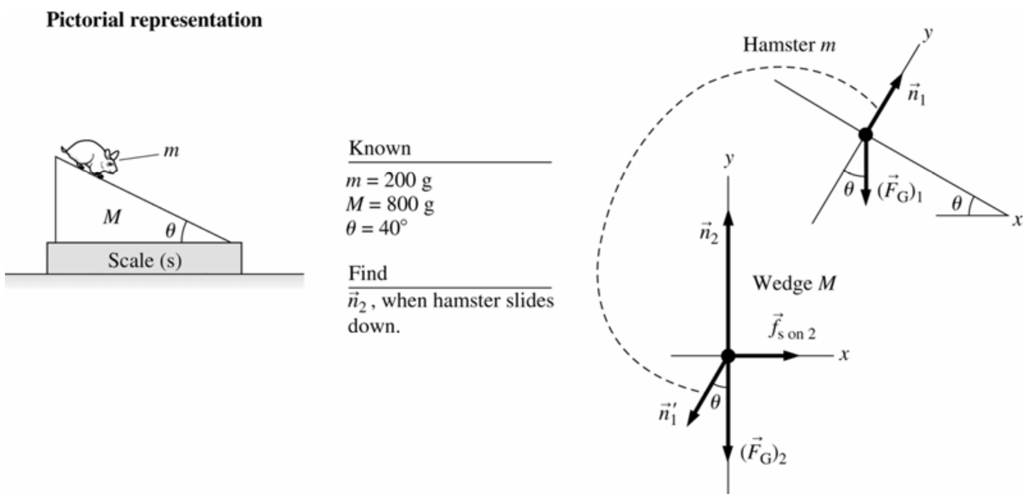
$$v = \sqrt{\frac{2\mu_s mg}{C\rho A}} = \sqrt{\frac{2(0.45)(15\text{ kg})(9.8\text{ m/s}^2)}{(1.1)(1.2\text{ kg/m}^3)(0.40\text{ m})(0.50\text{ m})}} = 22\text{ m/s}$$

This is the speed wind would have to make the top cylinder slid off a stationary lower cylinder; it is also the speed the two cylinders would have together if moving across frictionless ice through still air at which the top cylinder would begin to slide off.

Assess: The free-body diagram for the lower cylinder would have five forces on it, so we're glad we didn't need it.

7.57. Model: The hamster of mass m and the wedge with mass M will be treated as objects 1 and 2, respectively. They will be modeled as particles.

Visualize:



The scale is denoted by the letter s.

Solve: The reading of the scale is the magnitude of the force $\vec{n}_2$ that the scale exerts upward. Because of the shape of the wedge, it is not clear whether the scale has to exert a horizontal friction force $\vec{f}_{\text{s on 2}}$ to prevent horizontal motion. We've included one just in case. The hamster is accelerating down the wedge and the total mass is $m + M$. System 1 (hamster) has an acceleration that is along the x-axis, so $a_{1y} = 0\text{ m/s}^2$. The hamster's y-equation is

$$(F_{\text{net on 1}})_y = n_1 - mg\cos\theta = 0\text{ N} \quad \Rightarrow \quad n_1 = mg\cos\theta$$

We have $n_1' = n_1 = mg\cos\theta,$ so the y-equation for the wedge (with $a_{2y} = 0\text{ m/s}^2$) is

$$(F_{\text{net on 2}})_y = n_2 - n_1'\cos\theta - Mg = n_2 - mg\cos^2\theta - Mg = 0\text{ N}$$

$$n_2 = mg\cos^2\theta + Mg = (M + m\cos^2\theta)g = 9.0\text{ N}$$

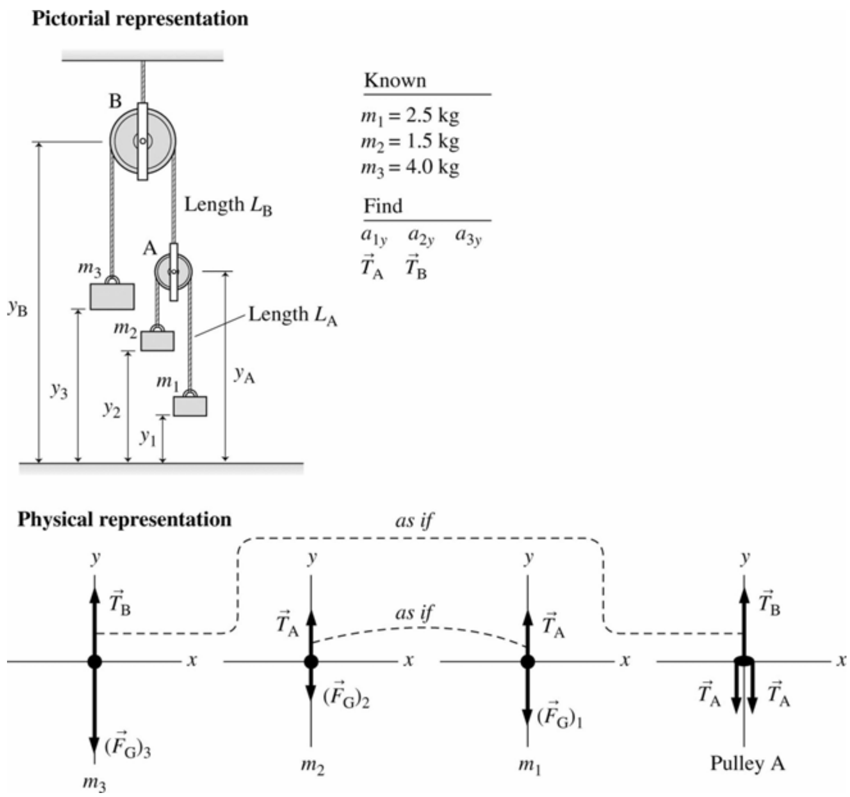
The scale reading in grams will be

$$n_2/\text{g} = M + m\cos^2\theta = 800\text{ g} + 200\text{ g}\cos^2 40° = 920\text{ g}$$

Assess: If the face of the wedge is vertical, then the hamster is simply in free fall and can have no effect on the scale (at least until impact!). So for $\theta = 90°$ we expect the scale to record Mg only, and that is indeed what the expression for n_2 gives.

7.58. Model: The hanging masses $m_1, m_2,$ and m_3 are modeled as particles. Pulleys A and B are massless and frictionless. The strings are massless.

Visualize:



Solve: (a) The length of the string over pulley B is constant. Therefore,

$$(y_B - y_3) + (y_B - y_A) = L_B \quad \Rightarrow \quad y_A = 2y_B - y_3 - L_B$$

The length of the string over pulley A is constant. Thus,

$$(y_A - y_2) + (y_A - y_1) = L_A = 2y_A - y_1 - y_2$$
$$2(2y_B - y_3 - L_B) - y_1 - y_2 = L_A \quad \Rightarrow \quad 2y_3 + y_2 + y_1 = \text{ constant}$$

This constraint implies that

$$2\frac{dy_3}{dt} + \frac{dy_2}{dt} + \frac{dy_1}{dt} = 0 \text{ m/s} = 2v_{3y} + v_{2y} + v_{1y}$$

Also by differentiation, $2a_{3y} + a_{2y} + a_{1y} = 0 \text{ m/s}^2$.

(b) Applying Newton's second law to the masses $m_3, m_2, m_1,$ and pulley A gives

$$T_B - m_3 g = m_3 a_{3y} \quad T_A - m_2 g = m_2 a_{2y} \quad T_A - m_1 g = m_1 a_{1y} \quad T_B - 2T_A = 0 \text{ N}$$

The pulley equation is zero because the pulley is massless. These four equations plus the acceleration constraint consitute five equations with five unknowns (two tensions and three accelerations). To solve for T_A, multiply the m_3 equation by 2, substitute $2T_B = 4T_A$, then divide each of the mass equations by the mass. This gives the three equations

$$4T_A / m_3 - 2g = 2a_{3y}$$
$$T_A / m_2 - g = a_{2y}$$
$$T_A / m_1 - g = a_{1y}$$

If these three equations are added, the right side adds to zero because of the acceleration constraint. Thus

$$(4/m_3 + 1/m_2 + 1/m_2)T_A - 4g = 0 \quad \Rightarrow \quad T_A = \frac{4g}{(4/m_3 + 1/m_2 + 1/m_2)}$$

(c) Using numerical values, we find $T_A = 18.97\ \text{N}$. Then

$$a_{1y} = T_A/m_1 - g = -2.2\ \text{m/s}^2$$
$$a_{2y} = T_A/m_2 - g = 2.9\ \text{m/s}^2$$
$$a_{3y} = 2T_A/m_3 - g = -0.32\ \text{m/s}^2$$

(d) $m_3 = m_1 + m_2$, so it appears at first as if m_3 should hang in equilibrium. For this to be so, tension T_B would need to equal $m_3 g$. However, T_B is not $(m_1 + m_2)g$ because masses m_1 and m_2 are accelerating rather than hanging at rest. Consequently, tension T_B is not able to balance the weight of m_3.