STAT 50 HW #7

Section 2.6 #'s 1, 7, 9, 16, 17, 18, 19 (a and b only for # 19), 31

1.
In a certain community, levels of air pollution may exceed federal standards for ozone or for particulate matter on some days. For a particular summer season, let X be the number of days on which the ozone standard is exceeded and let Y be the number of days on which the particulate matter standard is exceeded. Assume that the joint probability mass function of X and Y is given in the following table:

	y		
x	**0**	**1**	**2**
0	0.10	0.11	0.05
1	0.17	0.23	0.08
2	0.06	0.14	0.06

a. **Find P(X = 1and Y = 0).**

P(x = 1 and Y = 0) = 0.17

b. **Find P(X ≥ 1and Y < 2).**

P(X ≥ 1and Y < 2) = 0.17 + 0.06 + 0.23 + 0.14 = 0.6

c. **Find P(X < 1).**

P(X < 1) = 0.10 + 0.11 + 0.05 = 0.26

d. **Find P(Y ≥ 1).**

P(Y ≥ 1) = 0.11 + 0.23 + 0.14 + 0.05 + 0.08 + 0.06 = 0.67

e. **Find the probability that the standard for ozone is exceeded at least once.**

P(X ≥ **1**) = 0.17 + 0.23 + 0.08 + 0.06 + 0.14 + 0.06 = 0.74

f. **Find the probability that the standard for particulate matter is never exceeded.**

P(Y = 0) = 0.10 + 0.17 + 0.06 = 0.33

g. **Find the probability that neither standard is ever exceeded.**

P(X = 0 and Y = 0) = 0.10

7.

Refer to Exercise 4. Assume that the cost of repairing an assembly whose clearance is too little is $2, and the cost of repairing an assembly whose clearance is too much is $3.

4. In a piston assembly, the specifications for the clearance between piston rings and the cylinder wall are very tight. In a lot of assemblies, let X be the number with too little clearance and let Y be the number with too much clearance. The joint probability mass function of X and Y is given in the table below:

	y			
x	**0**	**1**	**2**	**3**
0	0.15	0.12	0.11	0.10
1	0.09	0.07	0.05	0.04
2	0.06	0.05	0.04	0.02
3	0.04	0.03	0.02	0.01

a. **Express the total cost of repairs in terms of X and Y.**

2X + 3Y

b. **Find the mean of the total cost of repairs.**

$\mu_X = 1(0.25) + 2(0.17) + 3(0.1) = 0.89$
$\mu_Y = 1(0.27) + 2(0.22) + 3(0.17) = 1.22$
$\mu_{2X} + \mu_{2Y} = 2\,\mu_X + 3\,\mu_Y = 2(0.89) + 3(1.22) = \$5.44$

c. **Find the standard deviation of the total cost of repairs.**

$\mu_X = 1(0.25) + 2(0.17) + 3(0.1) = 0.89$
$E(x^2) = 1^2(0.25) + 2^2(0.17) + 3^2(0.1) = 1.83$
$\sigma_x^2 = 1.83 - (0.89^2) = 1.0379$

$\mu_Y = 1(0.27) + 2(0.22) + 3(0.17) = 1.22$
$E(y^2) = 1^2(0.27) + 2^2(0.22) + 3^2(0.17) = 2.68$
$\sigma_y^2 = 2.68 - (1.22^2) = 1.1916$

E(X,Y) = (0)(0)(0.15) +(0)(1)(0.12) +(0)(2)(0.11) +(0)(3)(0.10)
+ (1)(0)(0.09) +(1)(1)(0.07) +(1)(2)(0.05) +(1)(3)(0.04)
+ (2)(0)(0.06) +(2)(1)(0.05) +(2)(2)(0.04) +(2)(3)(0.02)
+ (3)(0)(0.04) +(3)(1)(0.03) +(3)(2)(0.02) +(4)(3)(0.01)
= 0.97

$Cov(X,Y)$ = E(X,Y) - E(X)E(Y)
= 0.97 - (0.89*1.22)
= 0.97 - (1.0858) = -0.1158

$= 2^2\sigma_x^2 + 3^2\sigma_y^2 + 2(2)(3)Cov(X,Y)$
$= \sqrt{2^2(1.0379) + 3^2(1.1916) - 2(2)(3)Cov(-0.1158)}$
= $3.67

9.
The General Social Survey asked a sample of adults how many siblings (brothers and sisters) they had (X) and also how many children they had (Y). We show results for those who had no more than 4 children and no more than 4 siblings. Assume that the joint probability mass function is given in the following contingency table.

	y				
x	**0**	**1**	**2**	**3**	**4**
0	0.03	0.01	0.02	0.01	0.01
1	0.09	0.05	0.08	0.03	0.01
2	0.09	0.05	0.07	0.04	0.02
3	0.06	0.04	0.07	0.04	0.02
4	0.04	0.03	0.04	0.03	0.02

a. **Find the marginal probability mass function of X.**

*This can be found by summing the rows of the joint probability mass function.
$P_x(0) = 0.08, P_x(1) = 0.26, P_x(2) = 0.27, P_x(3) = 0.23, P_x(4) = 0.16$

b. **Find the marginal probability mass function of Y.**

*This can be found by summing the columns of the joint probability mass function.
$P_y(0) = 0.31, P_y(1) = 0.18, P_y(2) = 0.28, P_y(3) = 0.15, P_y(4) = 0.08$

c. **Are X and Y independent? Explain.**

$P_{xy}(0,0) = 0.03$
$P_x(0) * P_y(0)$ =0.08*0.31 = 0.0248
$No,\ because\ P_{xy}(0,0) \neq P_x(0) * P_y(0)$

d. **Find μX and μY.**

μX = 0($P_x(0)$) + 1($P_x(1)$) + 2($P_x(2)$) + 3($P_x(3)$) + 4($P_x(4)$)
= 0(0.08) + 1(0.26) + 2(0.27) + 3(0.23) + 4(0.16)

$\mu X = 2.13$

$\mu Y = 0(P_y(0)) + 1(P_y(1)) + 2(P_y(2)) + 3(P_y(3)) + 4(P_y(4))$
$= 0(0.31) + 1(0.18) + 2(0.28) + 3(0.15) + 4(0.08)$
$\mu Y = 1.51$

e. Find σX and σY.

$\sigma_x^2 = 0^2(P_x(0)) + 1^2(P_x(1)) + 2^2(P_x(2)) + 3^2(P_x(3)) + 4^2(P_x(4))$ - μX^2
= 0^2(0.08) + 1^2(0.26) + 2^2(0.27) + 3^2(0.23) + 4^2(0.16) - (2.13)^2
= 5.97 - 4.5369 = 1.4331
$\sigma_x = \sqrt{1.4331} = 1.1971$

$\sigma_y^2 = 0^2(P_y(0)) + 1^2(P_y(1)) + 2^2(P_y(2)) + 3^2(P_y(3)) + 4^2(P_y(4))$ - μY^2
= 0^2(0.31) + 1^2(0.18) + 2^2(0.28) + 3^2(0.15) + 4^2(0.08) - (1.51)^2
= 3.93 - 2.2801 = 1.6499
$\sigma_y = \sqrt{1.6499} = 1.2845$

f. Find Cov(X, Y).

Cov(X,Y) = $\mu_{xy} - \mu_x\mu_y$
μ_{xy} = (0)(0)(0.03) +(0)(1)(0.01) +(0)(2)(0.02) +(0)(3)(0.01) +(0)(4)(0.01)
+ (1)(0)(0.09) +(1)(1)(0.05) +(1)(2)(0.08) +(1)(3)(0.03) +(1)(4)(0.01)
+ (2)(0)(0.09) +(2)(1)(0.05) +(2)(2)(0.07) +(2)(3)(0.04) +(2)(4)(0.02)
+ (3)(0)(0.06) +(3)(1)(0.04) +(3)(2)(0.07) +(3)(3)(0.04) +(3)(4)(0.02)
+ (4)(0)(0.04) +(4)(1)(0.03) +(4)(2)(0.04) +(4)(3)(0.03) +(4)(4)(0.02)
= 3.38

$Cov(X, Y)$ = E(X,Y) - E(X)E(Y)
= 3.38 - (2.13*1.51)
= 3.38 - (3.2163) = 0.1637

g. Find ρ(X, Y).

$$\rho(X, Y) = \frac{Cov(X,Y)}{\sigma_x\sigma_y} = \frac{0.1637}{(1.1971)(1.2845)} = 0.1065$$

16.
For continuous random variables X and Y with joint probability density function

$$f(x) = \begin{cases} xe^{-(x+xy)} & x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

a. Find P(X > 1 and Y > 1).

$$f(x) = \begin{cases} xe^{-(x+xy)} & x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

a)

$$\int_1^{\infty}\left(\int_1^{\infty} xe^{-(x+xy)}\,dy\right)dx \qquad (fg)' = f'g + gf'$$

$$x\int_1^{\infty} e^{-(x+xy)}\,dy \qquad u = -(x+xy), \quad du = -x\,dy, \quad dy = \frac{du}{-x}$$

$$\frac{d}{dy}(x + xy) \to -(0 + x) = -x$$

$$x\int \frac{e^u}{-x}\,du \to -e^u$$

$$\int_1^{\infty}\left(-e^{-(x+xy)}\Big|_1^{\infty}\right)dx$$

$$(-e^{-\infty}) - (-e^{-2x}) \to 0$$

$$\int_1^{\infty} e^{-2x}\,dx \to e^u\frac{du}{-2} \qquad u = -2x, \quad du = -2\,dx, \quad dx = \frac{du}{-2}$$

$$\frac{e^{-2x}}{-2}\Big|_1^{\infty}$$

$$\left(\frac{e^{-2\infty}}{-2}\right) - \left(\frac{e^{-2}}{-2}\right)$$

$$0 + \frac{e^{-2}}{2} = \boxed{0.06766}$$

b. Find the marginal probability density functions fX (x) and fY (y).

b) Find the marginal pdf functions $f_X(x)$ and $f_Y(y)$

$f_X(x) = \int_0^{\infty} f(x,y)\,dy = \int_0^{\infty} \left(x e^{-(x+xy)}\right) dy$

$x \int_0^{\infty} \left(e^{-(x+xy)}\right) dy$

$u = -x + xy$

$du = -x\,dy$

$dy = \frac{du}{-x}$

$x \int \frac{e^{u}}{-x}\,du$

$-e^{u} \Rightarrow -e^{-(x+xy)} \Big|_0^{\infty}$

$\left(-e^{-x-\infty}\right) - \left(-e^{-x}\right)$

$\cancel{e^{-\infty}} + e^{-x}$

$-e^{-\infty}$

0

$f_X(x) = e^{-x}$

$f_Y(y) = \int_0^{\infty} f(x,y)\,dx = \int_0^{\infty} \left(x e^{-(x+xy)}\right) dx \qquad (fg)' = f'g + gf'$

$x \int_0^{\infty} e^{u} \qquad u = -(x+xy)$

$du =$

$-\left(1 + y + x\frac{d}{dx}(y)\right)$

$x \frac{e^{-(x+xy)}}{-\left(1+y+x\frac{d}{dx}(y)\right)} \Big|_0^{\infty}$

$-x)^{\infty}$

$-0)^{\infty}$

$0 - \left(\frac{e^{-0-0y}}{-\left(1+y\frac{d}{dx}(y)\right)}\right) \Rightarrow \frac{1}{(1+y)y'}$

$y' + yy'$

$y =$

$\frac{0}{1}$

$-x - xy$

$-1 - y - x\frac{d}{dx}(y)$

$-1 - y - \frac{d}{dx}y$

$-1 - y - y'$

$$\boxed{\begin{aligned} f_X(x) &= e^{-x} \\ f_Y(y) &= \frac{1}{1+y^2} \end{aligned}}$$

c. **Are X and Y independent? Explain.**

No, because f(x,y) $\neq f_x(x) * f_y(y)$

17.
Refer to Example 2.54.

Example 2.54

Assume that for a certain type of washer, both the thickness and the hole diameter vary from item to item. Let X denote the thickness in millimeters and let Y denote the hole diameter in millimeters, for a randomly chosen washer. Assume that the joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{6}(x+y) & \text{if } 1 \le x \le 2 \text{ and } 4 \le y \le 5 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that a randomly chosen washer has a thickness between 1.0 and 1.5 mm, and a hole diameter between 4.5 and 5 mm.

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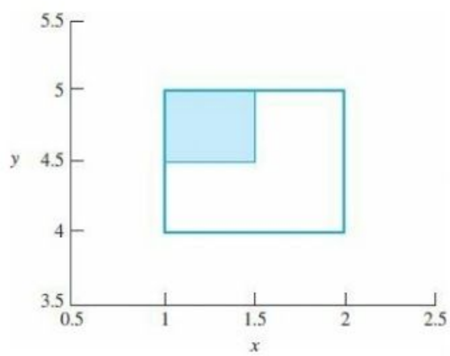
Solution

We need to find $P(1 \le X \le 1.5$ and $4.5 \le Y \le 5)$. The large rectangle in the figure indicates the region where the joint density is positive. The shaded rectangle indicates the region where $1 \le x \le 1.5$ and $4.5 \le y \le 5$, over which the joint density is to be integrated.



We integrate the joint probability density function over the indicated region:

$$\begin{aligned} P(1 \le X \le 1.5 \text{ and } 4.5 \le Y \le 5) &= \int_1^{1.5} \int_{4.5}^{5} \frac{1}{6}(x+y)\,dy\,dx \\ &= \int_1^{1.5} \left\{ \frac{xy}{6} + \frac{y^2}{12} \bigg|_{y=4.5}^{y=5} \right\} dx \\ &= \int_1^{1.5} \left(\frac{x}{12} + \frac{19}{48} \right) dx \\ &= \frac{1}{4} \end{aligned}$$

Note that if a joint probability density function is integrated over the entire plane, that is, if the limits are $-\infty$ to ∞ for both x and y, we obtain the probability that both X and Y take values between $-\infty$ and ∞, which is equal to 1.

a. Find Cov(X,Y).

16 $f(x,y) = \begin{cases} \frac{1}{6}(x+y) & \text{if } 1 \le x \le 2 \text{ and } 4 \le y \le 5 \\ 0 & \text{otherwise} \end{cases}$

a) Find Cov(X,Y)

$\mu_{xy} = \int_1^2 \int_4^5 \frac{1}{6} x y (x+y) \, dy \, dx$

$= \int_4^5 \frac{1}{6} (x^2 y + x y^2) \, dy$

$\frac{1}{6} \left(\frac{x^2 y^2}{2} + \frac{x y^3}{3} \right) \Big|_4^5$

$\frac{1}{6} \left(\left(x^2 \frac{\overset{25}{5^2}}{2} + x \frac{\overset{125}{5^3}}{3} \right) - \left(x^2 \frac{\overset{16}{4^2}}{2} + x \frac{\overset{64}{4^3}}{3} \right) \right)$

$\frac{1}{6} \left(\left(\frac{25x^2}{2} + \frac{125x}{3} \right) - \left(\frac{16x^2}{2} + \frac{64x}{3} \right) \right)$

$\int_1^2 \frac{1}{6} \left(\frac{9x^2}{2} + \frac{61x}{3} \right)$ Cont'd

$\frac{1}{6} \int_1^2 \left(\frac{9x^2}{2} + \frac{61x}{2} \right) \to \frac{1}{6} \left(\frac{9x^3}{3} + \frac{61x^2}{2} \right) \Big|_1^2$ $= \frac{41}{6}$

$\frac{1}{6} \left(\left(\frac{9}{2} \frac{2^3}{3} + \frac{61}{2} \frac{2^2}{2} \right) - \left(\frac{9}{2} \frac{1^3}{3} + \frac{61}{2} \frac{1^2}{2} \right) \right)$

$\frac{1}{6} \left(\left(\frac{72}{6} + \frac{244}{4} \right) - \left(\frac{9}{6} + \frac{61}{4} \right) \right)$

$\frac{1}{6} \left(\frac{63}{6} + \frac{183}{4} \right) = \frac{1}{6} \left(\frac{126}{12} + \frac{549}{12} \right) = \frac{675}{12} \cdot \frac{1}{6} =$

cont'd

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_4^5 \frac{1}{6}(x+y)\,dy$$
$$= \frac{1}{6}\left(x + \frac{9}{2}\right) \text{ for } 1 \le x \le 2$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_1^2 \frac{1}{6}(x+y)\,dx$$
$$= \frac{1}{6}\left(y + \frac{3}{2}\right) \text{ for } 4 \le y \le 5$$

$$\mu_x = \int_1^2 \frac{1}{6} x\left(x + \frac{9}{2}\right)dx = \int_1^2 \frac{1}{6}\left(x^2 + \frac{9}{2}x\right)dx = \frac{1}{6}\left(\frac{x^3}{3} + \frac{9x^2}{4}\right)\Big|_1^2 = \frac{109}{72}$$

$$\mu_Y = \int_4^5 \frac{1}{6} y\left(y + \frac{3}{2}\right) = \int_4^5 \frac{1}{6}\left(y^2 + \frac{3}{2}y\right) = \frac{1}{6}\left(\frac{y^3}{3} + \frac{3y^2}{4}\right)\Big|_4^5 = \frac{325}{72}$$

$$\text{Cov}(X,Y) = \mu_{XY} - \mu_X\mu_Y = \frac{41}{6} - \left(\frac{109}{72}\right)\left(\frac{325}{72}\right)$$

$$\boxed{\text{Cov}(X,Y) = -0.000193}$$

b. Find ρX,Y.

b) Find $\rho(X,Y)$

$$\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\sigma_X^2 = \int_1^2 \frac{1}{6} x^2\left(x + \frac{9}{2}\right)dx - \mu_x^2$$

$$= \frac{1}{6}\left(\frac{x^4}{4} + \frac{3x^3}{2}\right)\Big|_1^2 - \left(\frac{109}{72}\right)^2 = 0.08319$$

cont'd

$$\sigma_V^2 = \int_4^5 \frac{1}{6} v^2\left(v+\frac{3}{2}\right) dv - \mu_V^2$$

$$= \frac{1}{6}\left(\frac{v^4}{4}+\frac{v^3}{2}\right)\Big|_4^5 - \left(\frac{325}{72}\right)^2 = 0.08314$$

$$\frac{-0.000193}{(\sqrt{0.08314})(\sqrt{0.08314})} = -0.00232$$

$$\boxed{\rho(X,Y) = -0.00232}$$

18.

A production facility contains two machines that are used to rework items that are initially defective. Let X be the number of hours that the first machine is in use, and let Y be the number of hours that the second machine is in use, on a randomly chosen day. Assume that X and Y have joint probability density function given by

$$f(x) = \begin{cases} \frac{3}{2}(x^2+y^2) & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. **What is the probability that both machines are in operation for more than half an hour?**

18. $f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & 0<x<1 \text{ and } 0<y<1 \\ 0 & \text{Otherwise} \end{cases}$

a) $P(X>0.5 \text{ and } Y>0.5)$

$$\int_{0.5}^{1}\left(\int_{0.5}^{1} \frac{3(x^2+y^2)}{2}\,dy\right)dx$$

$$\frac{3}{2}\int_{0.5}^{1}(x^2+y^2)\,dy = \frac{3}{2}\left(x^2y+\frac{y^3}{3}\right)\Big|_{0.5}^{1}$$

$$\frac{3}{2}\left(\left(x^2+\frac{1}{3}\right)-\left(\frac{x^2}{2}+\frac{0.5^3}{3}\right)\right) \qquad \frac{21}{48}$$

$$\frac{3}{2}\left(\frac{x^2}{2}+\frac{0.875}{3}\right) = \frac{3x^2}{4}+\frac{0.875}{2}$$

$$\int_{0.5}^{1}\left(\frac{3x^2}{4}+\frac{21}{48}\right)dx = \left(\frac{3x^3}{12}+\frac{21x}{48}\right)\Big|_{0.5}^{1}$$

$$\left(\frac{1}{4}+\frac{21}{48}\right)-\left(\frac{.375}{12}+\frac{10.5}{48}\right) = \boxed{\frac{21}{48}}$$

b. Find the marginal probability density functions fX (x) and fY (y).

b) Find marginal pdf functions $f_X(x)$ and $f_Y(y)$

$$f_X(x) = \int_0^1 \frac{3(x^2+y^2)}{2}\,dy$$

$$\frac{3}{2}\int_0^1 (x^2+y^2)\,dy = \frac{3}{2}\left(x^2 y + \frac{y^3}{3}\right)\Big|_0^1$$

$$f_X(x) = \frac{3}{2}x^2 + \frac{1}{2} \qquad \frac{3}{2}\left(x^2+\frac{1}{3}\right) - (0)$$

$$f_Y(y) = \int_0^1 \frac{3(x^2+y^2)}{2}\,dx$$

$$\frac{3}{2}\int_0^1 (x^2+y^2)\,dx = \frac{3}{2}\left(\frac{x^3}{3}+y^2 x\right)\Big|_0^1$$

$$f_Y(y) = \frac{1}{2} + \frac{3y^2}{2} \qquad \frac{3}{2}\left(\frac{1}{3}+y^2\right) - (0)$$

$$\boxed{\begin{aligned} f_X(x) &= \frac{3}{2}x^2 + \frac{1}{2} \\ f_Y(y) &= \frac{1}{2} + \frac{3y^2}{2} \end{aligned}}$$

c. Are X and Y independent? Explain.

(3) X and Y independent?

$f_{X,Y}(x,y) = f_X(x) f_Y(y)$?

$$\frac{3}{2}(x^2+y^2) \neq \left(\frac{3}{2}x^2+\frac{1}{2}\right)\left(\frac{1}{2}+\frac{3y^2}{2}\right)$$

$\boxed{\text{No, because } f_{X,Y}(x,y) \neq f_X(x) * f_Y(y)}$

19.

Refer to Exercise 18.

a. **Find Cov(X, Y).**

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2) & 0<x<1 \text{ and } 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find $Cov(X,Y)$

$$Cov(X,Y) = \mu_{XY} - \mu_X \mu_Y$$

$$\mu_{XY} = \int_0^1 \left(\int_0^1 xy \, \frac{3(x^2+y^2)}{2} \, dx \right) dy$$

$$\frac{3}{2}\int_0^1 \cancel{xy(x^2+y^2)} \, dx$$

$$\int_0^1 (x^3 y + x y^3) = \left(\frac{x^4}{4} y + \frac{x^2}{2} y^3 \right) \Big|_0^1$$

$$\int_0^1 \frac{3}{2}\left(\frac{1}{4} y + \frac{1}{2} y^3 \right) dy$$

$$\frac{3}{2}\left(\frac{1}{4} \frac{y^2}{2} + \frac{1}{2} \frac{y^4}{4} \right) \Big|_0^1$$

$$\frac{3}{2}\left(\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4} \right)$$

$$\frac{1}{8} + \frac{1}{8} \qquad \frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8}$$

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$$\mu_X = \int_0^1 x(f_X(x)) \, dx = \int_0^1 x\left(\frac{3x^2}{2} + \frac{1}{2} \right) dx$$

$$\mu_Y = \int_0^1 y(f_Y(y)) \, dy = \int_0^1 y\left(\frac{1}{2} + \frac{3y^2}{2} \right) dy = \frac{5}{8}$$

$$Cov(X,Y) = \frac{3}{8} - \frac{5}{8}^2 = -\frac{1}{64} \qquad \boxed{Cov(X,Y) = -\frac{1}{64}}$$

b. Find ρX,Y .

b) Find $\rho(X,Y)$

$$\rho(X,Y) = \frac{\operatorname{Cov}(X,Y)}{\sigma_X \sigma_Y}$$

$$\sigma_X^2 = \int_0^1 x^2\left(\frac{3x^2}{2}+\frac{1}{2}\right)dx - \mu_X^2$$

$$= \int_0^1 \left(\frac{3x^4}{2}+\frac{1}{2}x^2\right)dx - \mu_X^2$$

$$= \left(\frac{3}{2}\frac{x^5}{5} - \frac{1}{2}\frac{x^3}{3}\right)\Big|_0^1 = \left(\frac{3x^5}{10} - \frac{x^3}{6}\right)\Big|_0^1 - \left(\frac{5}{8}\right)^2 = \frac{73}{960}$$

$$\sigma_Y^2 = \int_0^1 y^2\left(\frac{1}{2}+\frac{3y^2}{2}\right)dy - \mu_Y^2$$

$$= \int_0^1 \left(\frac{1}{2}y^2 + \frac{3y^4}{2}\right)dy = \left(\frac{1}{2}\frac{y^3}{3} + \frac{3}{2}\frac{y^5}{5}\right)\Big|_0^1 - \left(\frac{5}{8}\right)^2 = \frac{73}{960}$$

$$\rho(X,Y) = \frac{-\frac{1}{64}}{\left(\sqrt{\frac{73}{960}}\right)\left(\sqrt{\frac{73}{960}}\right)} = \boxed{-0.2055}$$

31.

Refer to Exercise 30.

30. The oxygen equivalence number of a weld is a number that can be used to predict properties such as hardness, strength, and ductility. The article "Advances in Oxygen Equivalence Equations for Predicting the Properties of Titanium Welds" (D. Harwig, W. Ittiwattana, and H. Castner, *The Welding Journal*, 2001:126s–136s) presents several equations for computing the oxygen equivalence number of a weld. An equation designed to predict the strength of a weld is $X = 1.12C + 2.69N + O - 0.21\,Fe$, where X is the oxygen equivalence, and C, N, O, and Fe are the amounts of carbon, nitrogen, oxygen, and iron, respectively, in

weight percent, in the weld. Suppose that for welds of a certain type, $\mu_C = 0.0247$, $\mu_N = 0.0255$, $\mu_O = 0.1668$, $\mu_{Fe} = 0.0597$, $\sigma_C = 0.0131$, $\sigma_N = 0.0194$, $\sigma_O = 0.0340$, and $\sigma_{Fe} = 0.0413$. Furthermore assume that correlations are given by $\rho_{C,N} = -0.44$, $\rho_{C,O} = 0.58$, $\rho_{C,Fe} = 0.39$, $\rho_{N,O} = -0.32$, $\rho_{N,Fe} = 0.09$, and $\rho_{O,Fe} = -0.35$.

An equation to predict the ductility of a titanium weld is Y = 7.84C + 11.44N + O − 1.58Fe, where Y is the oxygen equivalence used to predict ductility, and C, N, O, and Fe are the amounts of carbon, nitrogen, oxygen, and iron, respectively, in weight percent, in the weld.

Using the means, standard deviations, and correlations presented in Exercise 30, find μY and σY.

$Y = 7.84C + 11.44N + O - 1.58Fe$

Find μ_Y

$\mu_Y = \mu(7.84C + 11.44N + O - 1.56Fe)$

$= 7.64\mu_C + 11.44\mu_N + \mu_O - 1.58\mu_{Fe}$

$= 7.84(0.0247) + 11.46(0.0285) + 0.1668 - 1.58(0.0347)$

$= 0.5578$ $\quad \boxed{\mu_Y = 0.5578}$

Find σ_Y

$\sigma_Y^2 = \sigma^2(7.84C + 11.44N + O - 1.58Fe)$

$= (7.84^2\sigma_C^2) + (11.44^2\sigma_N^2) + \sigma_O^2 + 1.58^2\sigma_{Fe}^2$

$+ 2(7.84)(11.44)\,Cov(C,N) + 2(7.84)\,Cov(C,O)$
$- 2(7.84)(1.58)\,Cov(C,Fe) + 2(11.44)\,Cov(N,O)$
$- 2(11.44)(1.58)\,Cov(N,Fe) - 2(1.58)\,Cov(O,Fe)$

$= 7.84^2(0.0131)^2 + 11.44^2(0.0194)^2 + 0.0340^2 + 1.58^2(0.0413)^2$
$+ 2(7.84)(11.44)(-0.0001118) + 2(7.84)(0.0002583)$
$- 2(7.84)(1.58)(0.0002110) + 2(11.44)(-0.0002111)$
$- 2(11.44)(1.58)(0.0000721) - 2(1.58)(0.0004415)$
$= 0.038100$

$= \sqrt{0.038100} = 0.1952$

$\boxed{\mu_Y = 0.5578,\ \sigma_Y = 0.1952}$