Math 30, Thursday 4/16/2020

1pm class

Exam 3 is tomorrow (Friday).

- 2019 review prob.s
- 2020 practice exam
- worksheets → in lecture notes
- book problems

Questions?

No Zoom Meeting tomorrow — just work on your exam.

§4.5 #224. $f(x) = x^2 - 6x$

Find: (a) where it's incr or decr.

(b) local max/min

(c) where it's concave up/down

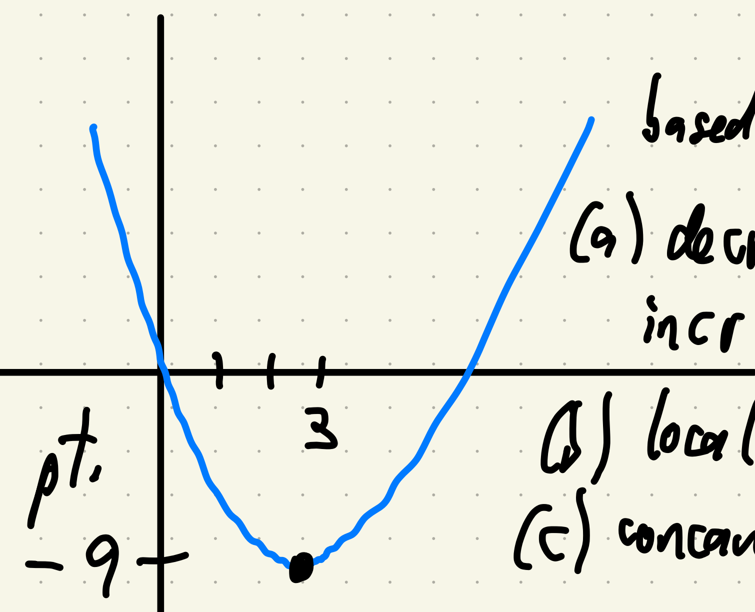
(d) inflection pts

This is a little "easier" b/c its a parabola:

$$f(x) = (x-3)^2 - 9 \quad \text{(check)}$$

"complete the square"

so the graph is:
(not to scale)



based on graph:

(a) decr: $x<3$

incr: $x>3$

(b) local min at 3

(c) concave up everywhere

(d) no inflection pt.

Check using calculus: $f(x) = x^2 - 6x$

<u>(a) incr/decr.?</u>

$$f'(x) = 2x - 6$$

$f' < 0$ when $x < 3$

so f is decr. for $x < 3$

$f' > 0$ when $x > 3$

so f is incr. for $x > 3$.

(b) so local min at $x = 3$

(c) $f''(x) = 2 > 0$

concave up everywhere

(d) There are no inflection pts.

look good?

In other problems it may be hard to graph, so need to rely on calculus...

More Questions?

L'Hôpital's rule is for limits where indeterminate forms like

$$\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty \text{ show up.}$$

If you get

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \text{“}\frac{0}{0}\text{”}$$

<u>not</u> the answer.

$$\dots = \lim_{x\to 0} \frac{f'(x)}{g'(x)}$$

§4.6 #274. Find horiz & vert. asymptote

$$f(x) = \frac{x^2+3}{x^2+1}.$$

✓ For any x, This is a finite number, so There is no vertical asymptote.

what about horiz. asymptotes?

$$\lim_{x \to \infty} f(x) = ?$$

$$f(x) = \frac{x^2+3}{x^2+1}.$$

$$\lim_{x\to\infty} f(x) = ?$$

one way: $$f(x) = \frac{x^2\left(1+\frac{3}{x^2}\right)}{x^2\left(1+\frac{1}{x^2}\right)}$$

$$= \frac{1+\frac{3}{x^2}}{1+\frac{1}{x^2}}$$

So $$\lim_{x\to\infty} f(x) = 1$$

and $$\lim_{x\to-\infty} f(x) = 1$$

$$f(x) = \frac{x^2+3}{x^2+1}$$

another way: (overkill) L'Hôpital Rule.

$$\lim_{x\to\infty} \frac{x^2+3}{x^2+1}$$ looks like $"\frac{\infty}{\infty}"$

need to do more work.

L'Hôp

$$= \lim_{x\to\infty} \frac{2x}{2x} = \boxed{1}$$

Other Questions?

L'Hôpital works for

$$\lim_{x \to a} \frac{f(x)}{g(x)} \quad \text{where} \quad f(a) = 0 \text{ and } g(a) = 0$$

and for

$$\lim_{x \to \infty} \frac{f(x)}{g(x)}$$

where $\lim_{x \to \infty} f(x) = \infty$

and $\lim_{x \to \infty} g(x) = \infty$

Time now to work on review problems.
Please let me know if you have Q's.

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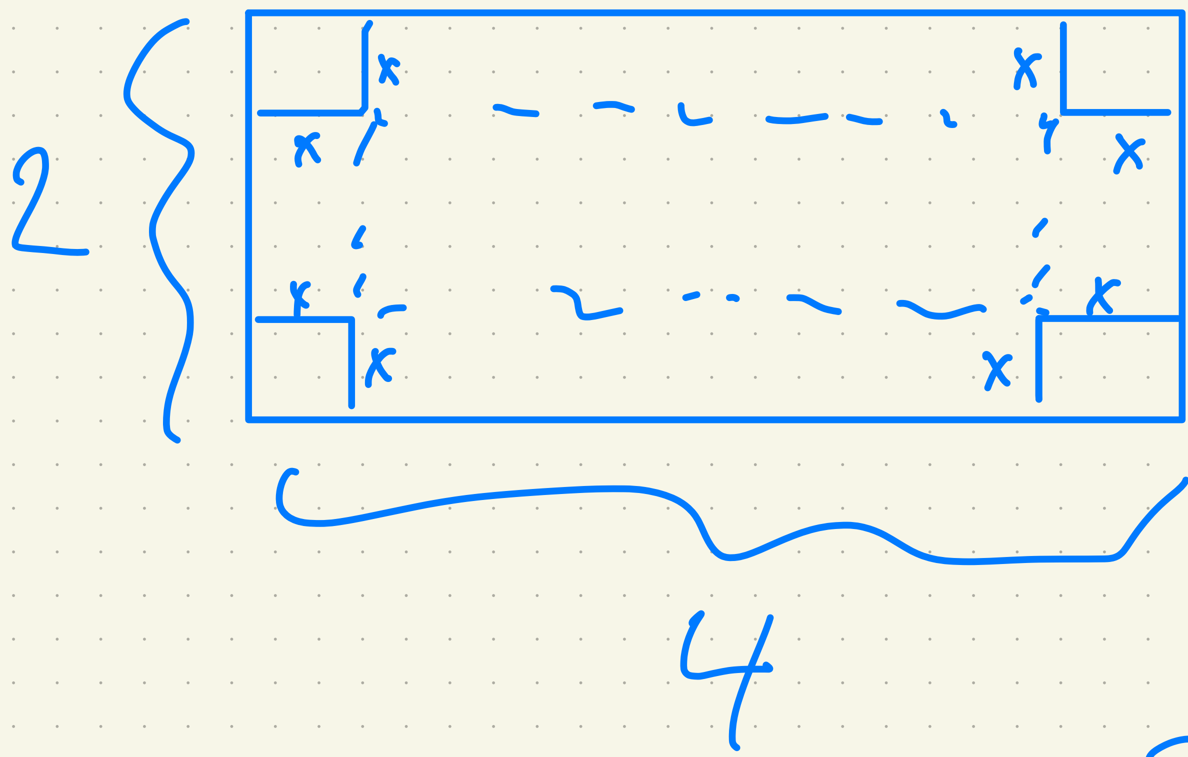
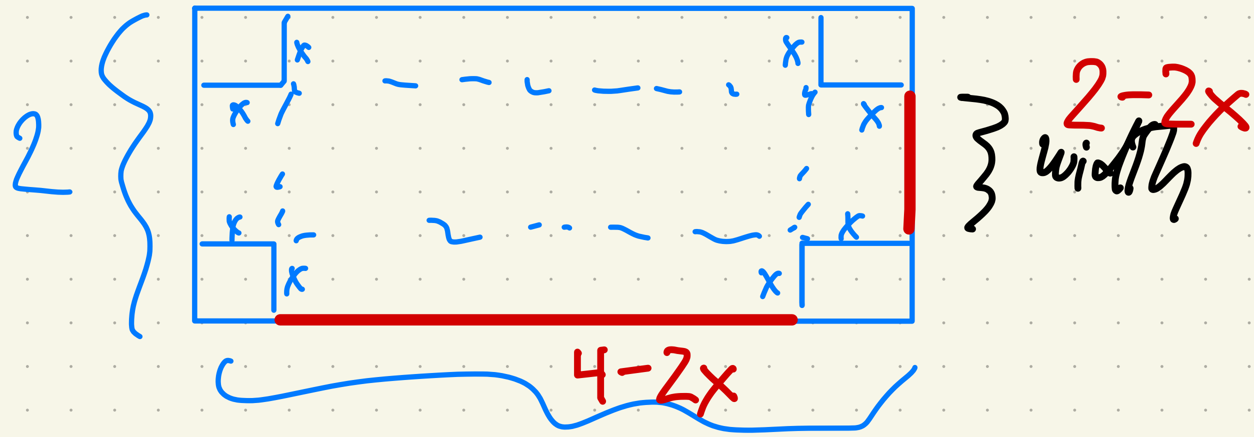
§4.7 #316. make a box w/
dimensions 2 meters by 4 meters



How to get biggest volume?



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ℓ length

height $= x$

note: $0 < x < 1$ physically speaking

Vol. of box $= \ell w h$

$= (4-2x)(2-2x)x$

Find x to make this as big as possible.

$= 2(2-x)2(1-x)x$

$= 4(2-2x-x+x^2)x$

$$V(x) = 4(2-2x-x+x^2)x$$

$$= 4x(2-3x+x^2)$$

$$= 4x^3-12x^2+8x$$

find x That maximizes This.

quad. formula for $ax^2+bx+c=0$

roots: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

crit pts:

$$V'(x) = 12x^2-24x+8$$

$$= 4(3x^2-6x+2)$$

roots: $x = \dfrac{6 \pm \sqrt{36-24}}{6}$ etc.

find crit. pt in The interval $(0, 1)$.

$$x = \frac{6 \pm \sqrt{36-24}}{6}$$

Note: $\frac{6+\sqrt{12}}{6} > 1$ (outside interval)

so only look at the crit. pt.

$$x = \frac{6-\sqrt{12}}{6} = 1 - \frac{\sqrt{3}}{3}$$

is it a local min or max?

two ways: (1) incr/decr.

(2) concavity.

at this point $V'' < 0$

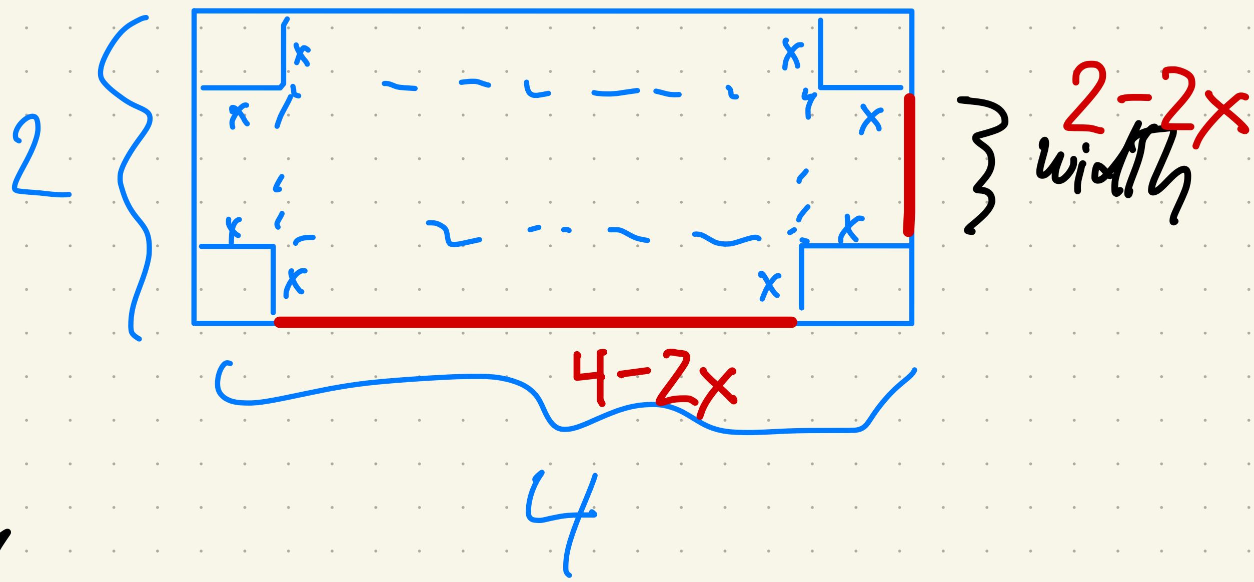
$V'(x) = 12x^2 - 24x + 8$

$V''(x) = 24x - 24$

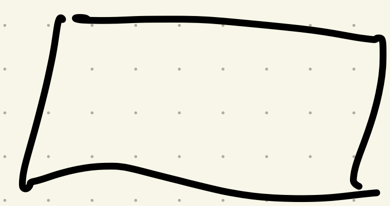
so this is a local max

$V(x) = 2(2-x)2(1-x)x$

$= 4x(2-x)(1-x)$

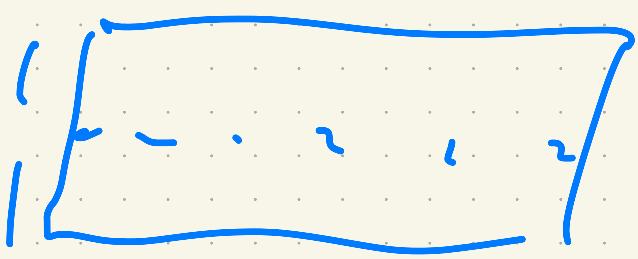


note: if $x=0$: $h=0$



$V=0$ flat piece of cardboard

if $x=1$:



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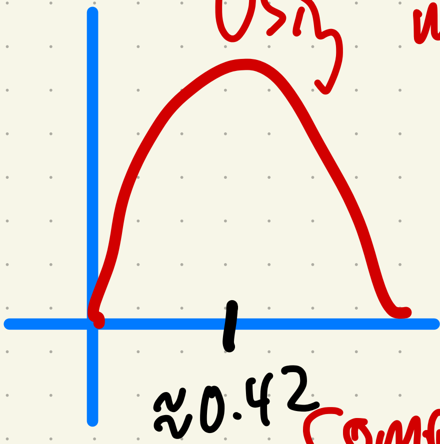
- does not fold into a box.

$V=0$.

so max V occurs for $0<x<1$

Using mathematica:



max V happens at ≈ 0.42

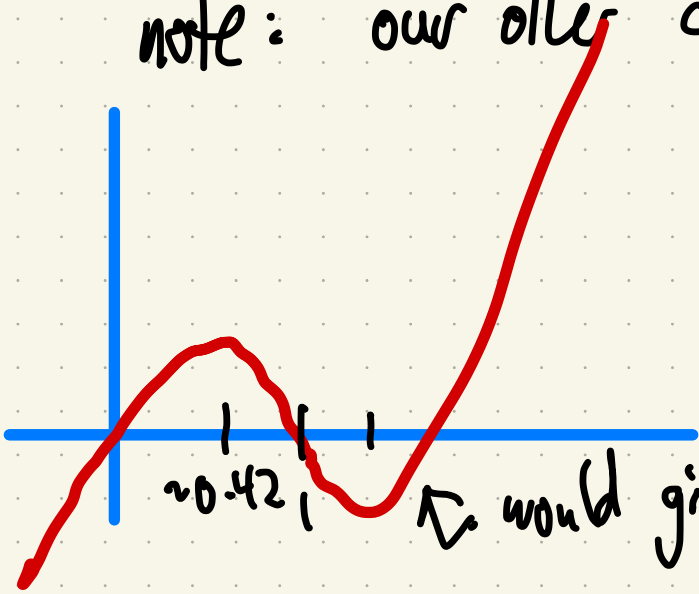
Compare w/ our answer:

$$x = 1 - \frac{1}{\sqrt{3}} \approx 0.423$$

Wow!

Calculus works!

note: our other critical pt. is not physically valid for our box.



← would give a <u>negative</u> volume.

Last Q's?

Have fun studying!