Hw 5

(1) a) $2\cdot\begin{vmatrix}1 & 2\\ 4 & -1\end{vmatrix} - (-4)\begin{vmatrix}3 & 2\\ 1 & -1\end{vmatrix} + 3\cdot\begin{vmatrix}3 & 1\\ 1 & 4\end{vmatrix} \rightarrow$

$\rightarrow 2\cdot(-1-8)+4\cdot(-3-2)+3(12-1) = 2\cdot(-9)+4\cdot(-5)+3(11)$

$= -18-20+33 = 33-38 = -5$

b) $5\cdot\begin{vmatrix}3 & -5\\ -4 & 7\end{vmatrix} - (-2)\begin{vmatrix}0 & -5\\ 2 & 7\end{vmatrix} + 4\begin{vmatrix}0 & 3\\ 2 & -4\end{vmatrix}$

$\rightarrow 5\cdot(21-20)+2\cdot(0-(-10))+4(0-6) = 5\cdot 1+2\cdot 10+4(-6)$

$= 5+20-24 = 25-24 = 1$

(2) a) 3rd Row $+2\begin{vmatrix}0 & 0 & 5\\ 7 & 2 & -5\\ 3 & 1 & 8\end{vmatrix} \rightarrow$ across 1st row $\rightarrow 2\cdot 5\cdot\begin{vmatrix}7 & 2\\ 3 & 1\end{vmatrix} = 10\cdot(7-6) = 10\cdot 1 = 10$

b) 1st Row $+4\begin{vmatrix}-1 & 0 & 0\\ 6 & 3 & 0\\ -8 & 4 & -3\end{vmatrix} \rightarrow$ down 3rd column $\rightarrow 4\cdot(-3)\cdot\begin{vmatrix}-1 & 0\\ 6 & 3\end{vmatrix} = 4\cdot(-3)(-3-0)$

$= -12\cdot -3 = 36$

(3) a) $\begin{vmatrix}1 & 5 & 3\\ 3 & -3 & 3\\ 2 & 13 & -7\end{vmatrix} \rightarrow \begin{vmatrix}1 & 5 & 3\\ 0 & -18 & -6\\ 0 & 3 & -13\end{vmatrix} \rightarrow 6\cdot\begin{vmatrix}1 & 5 & 3\\ 0 & -3 & -1\\ 0 & 3 & -13\end{vmatrix} \rightarrow 6\cdot\begin{vmatrix}1 & 5 & 3\\ 0 & -3 & -1\\ 0 & 0 & -14\end{vmatrix} \rightarrow 6\cdot[1\cdot(-3)\cdot(-14)]$

$\rightarrow 6\cdot(42) = 252$

b) $\begin{vmatrix}1 & 3 & 3 & -4\\ 0 & 1 & 2 & -5\\ 2 & 5 & 4 & -3\\ -3 & -7 & -5 & 2\end{vmatrix} \rightarrow \begin{vmatrix}1 & 3 & 3 & -4\\ 0 & 1 & 2 & -5\\ 0 & -1 & -2 & 5\\ 0 & 2 & 4 & -10\end{vmatrix} \rightarrow \begin{vmatrix}1 & 3 & 3 & -4\\ 0 & 1 & 2 & -5\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{vmatrix} \rightarrow 1\cdot 1\cdot 0\cdot 0 = 0$

(4) a) $\begin{vmatrix} 4 & -7 & -3 \\ 6 & 0 & -5 \\ -7 & 2 & 6 \end{vmatrix}$ → across 2^{nd} row: $-6\begin{vmatrix} -7 & -3 \\ 2 & 6 \end{vmatrix} - (-5)\begin{vmatrix} 4 & -7 \\ -7 & 2 \end{vmatrix}$

→ $-6(-42-(-6)) + 5(8-49)$

→ $-6(-36) + 5(-41) = 216 - 205 = 11 \neq 0$

→ thus these vectors are L.I.

b) $\begin{vmatrix} 7 & -8 & 7 \\ -4 & 5 & 0 \\ -6 & 7 & -5 \end{vmatrix}$ → down 3^{rd} column $7 \cdot \begin{vmatrix} -4 & 5 \\ -6 & 7 \end{vmatrix} - 5\begin{vmatrix} 7 & -8 \\ -4 & 5 \end{vmatrix}$

→ $7 \cdot (-28-(-30)) - 5 \cdot (35-(32))$

→ $7 \cdot 2 - 5 \cdot 3 = 14 - 15 = -1 \neq 0$

→ these vectors are L.I.

(5) $\det A = -1$, $\det B = 2$

a) $\det AB = \det A \cdot \det B = (-1) \cdot 2$ -2

c) $\det 2A = \det 2I_4 \cdot \det A$
$= 2^4 \cdot -1 = -16$

b) $\det B^5 = (\det B)^5 = 2^5 = 32$

d) Note: $\det A^T = \det A$ → $\det A^T A = \det A^T \cdot \det A = (-1)(-1) = 1$

e) Watch this! $\det B^{-1} A B = \det B^{-1} \cdot \det A \cdot \det B$ → these are just #'s

$= \det B^{-1} \cdot \det B \cdot \det A$

$\det B^{-1} \det B = \det B^{-1} \cdot B$ ← $= \det B^{-1} \cdot B \cdot \det A$

$= \det I$

$= 1$

$= \det I_4 \cdot (-1)$

$= 1 \cdot -1 = -1$

Scanned with CamScanner

⑥ a) $A = \begin{bmatrix} 3 & -2 \\ -5 & 6 \end{bmatrix} \rightarrow \det A = 18 - 10 = \underline{8}$ and $\vec{b} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$

✓ $A_1(\vec{b}) = \begin{bmatrix} 7 & -2 \\ -5 & 6 \end{bmatrix} \rightarrow \det A_1(\vec{b}) = 42 - 10 = 32$

✓ $A_2(\vec{b}) = \begin{bmatrix} 3 & 7 \\ -5 & -5 \end{bmatrix} \rightarrow \det A_2(\vec{b}) = -15 - (-35) = 20$

$\rightarrow \left(x_1 = \frac{32}{8} = 4, \; x_2 = \frac{20}{8} = \frac{5}{2} \right) \leftarrow$

b) $A = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix} \rightarrow \det A = (-1)(-1) \cdot \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} + (-1)(2) \cdot \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$

(second row)

$= 1(3-1) - 2 \cdot (2-3) = 2 + 2 = \underline{4}$

✓ $A_1(\vec{b}) = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 0 & 2 \\ -2 & 1 & 3 \end{bmatrix} \rightarrow \det A_1(\vec{b}) = (-1)(1) \cdot \begin{vmatrix} 2 & 2 \\ -2 & 3 \end{vmatrix} + (-1)(1) \begin{vmatrix} 4 & 1 \\ 2 & 2 \end{vmatrix}$

(2^{nd} column)

$= -1 \cdot (6 - (-4)) - 1 \cdot (8 - 2)$

$= -1 \cdot (10) - 6 = \underline{-16}$

✓ $A_2(\vec{b}) = \begin{bmatrix} 2 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & -2 & 3 \end{bmatrix} \rightarrow \det A_2(\vec{b}) = 2 \cdot \begin{vmatrix} 2 & 2 \\ -2 & 3 \end{vmatrix} + (-1)(4) \cdot \begin{vmatrix} -1 & 2 \\ 3 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}$

(1^{st} row)

$= 2 \cdot 10 - 4 \cdot (-3 - 6) + 1 \cdot (2 - 6)$

$= 20 + 36 - 4 = 56 - 4 = 52$

✓ $A_3(\vec{b}) = \begin{bmatrix} 2 & 1 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & -2 \end{bmatrix} \rightarrow \det A_3(\vec{b}) = (+1) \cdot \begin{vmatrix} 1 & 4 \\ 1 & -2 \end{vmatrix} + (-1)(2) \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$

(2^{nd} row)

$= +1 \cdot (-2 - 4) - 2(2 - 3)$

$= -6 + 2 = -4$

✓ $x_1 = \frac{-16}{4} = -4, \; x_2 = \frac{52}{4} = 13, \; x_3 = \frac{-4}{4} = -1 \rightarrow$

$x_1 = -4$
$x_2 = 13$
$x_3 = -1$

(7) a) $C_{11} = (-1)^2 \begin{vmatrix} -2 & 1 \\ 1 & 0 \end{vmatrix} = 1\cdot(-1) = \underline{-1}$

$C_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix} = -1\cdot(0-0) = \underline{0}$

$C_{13} = (-1)^4 \cdot \begin{vmatrix} 2 & -2 \\ 0 & 1 \end{vmatrix} = 1\cdot(2-0) = \underline{2}$

$C_{21} = (-1)^3 \cdot \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} = -1\cdot(0-3) = \underline{3}$

$C_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} = 1\cdot(0-0) = \underline{0}$

$C_{23} = (-1)^5 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = (-1)\cdot(1-0) = \underline{-1}$

$C_{31} = (-1)^4 \cdot \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = 1\cdot(1-(-6)) = 1\cdot 7 = \underline{7}$

$C_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = (-1)\cdot(1-6) = (-1)\cdot(-5) = \underline{5}$

$C_{33} = (-1)^6 \cdot \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} = (1)\cdot(-2-2) = 1\cdot(-4) = \underline{-4}$

Cofactor Matrix: $\begin{bmatrix} -1 & 0 & 2 \\ 3 & 0 & -1 \\ 7 & 5 & -4 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} -1 & 3 & 7 \\ 0 & 0 & 5 \\ 2 & -1 & -4 \end{bmatrix} = adj(A)$

$\det A = 0 + 1\cdot(C_{32}) + 0 = 1\cdot 5 = \underline{5} \rightarrow A^{-1} = \frac{1}{\det A}\cdot adj(A) = \left(\frac{1}{5}\right)\cdot \begin{bmatrix} -1 & 3 & 7 \\ 0 & 0 & 5 \\ 2 & -1 & -4 \end{bmatrix} = \begin{bmatrix} -1/5 & 3/5 & 7/5 \\ 0 & 0 & 1 \\ 2/5 & -1/5 & -4/5 \end{bmatrix}$

(3rd row)

b) $C_{11} = (-1)^2 \cdot \begin{vmatrix} -3 & 1 \\ 0 & 3 \end{vmatrix} = 1\cdot(-9-0) = \underline{-9}$

$C_{12} = (-1)^3 \cdot \begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = (-1)\cdot(0-0) = \underline{0}$

$C_{13} = (-1)^4 \cdot \begin{vmatrix} 0 & -3 \\ 0 & 0 \end{vmatrix} = (1)\cdot(0-0) = \underline{0}$

$C_{21} = (-1)^3 \cdot \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} = (-1)(6-0) = \underline{-6}$

$C_{22} = (-1)^4 \cdot \begin{vmatrix} 1 & 4 \\ 0 & 3 \end{vmatrix} = 1\cdot(3-0) = \underline{3}$

$C_{23} = (-1)^5 \cdot \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = (-1)\cdot(0-0) = \underline{0}$

$C_{31} = (-1)^4 \cdot \begin{vmatrix} 2 & 4 \\ -3 & 1 \end{vmatrix} = 1\cdot(2-(-12)) = \underline{14}$

$C_{32} = (-1)^5 \cdot \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -1\cdot(1-0) = \underline{-1}$

$C_{33} = (-1)^6 \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = 1\cdot(-3-0) = \underline{-3}$

→ Cofactor matrix:

$\begin{bmatrix} -9 & 0 & 0 \\ -6 & 3 & 0 \\ 14 & -1 & -3 \end{bmatrix}$

Adjugate:

$\begin{bmatrix} -9 & -6 & 14 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{bmatrix}$

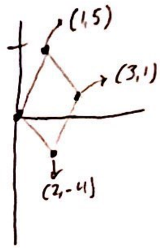
$\det A = 1\cdot C_{11} = \underline{-9}$

(1st column)

$A^{-1} = \frac{1}{\det A}\cdot adj(A) = \left(-\frac{1}{9}\right)\begin{bmatrix} -9 & -6 & 14 \\ 0 & 3 & -1 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2/3 & -14/9 \\ 0 & -1/3 & 1/9 \\ 0 & 0 & 1/3 \end{bmatrix}$

(8) $(-1,0), (0,5), (1,-4), (2,1) \rightarrow (0,0), (1,5), (2,-4), (3,1)$

$(+1,+0)$ $(+1\ +0)$ $(+1\ +0)$ $(+1\ +0)$



Area $= \left|\begin{vmatrix}1 & 2\\ 5 & -4\end{vmatrix}\right| = |(-4-10)| = |-14| = 14$

(9) Area $= \left|\begin{vmatrix}1 & -2 & -1\\ 4 & -5 & 2\\ 0 & 2 & -1\end{vmatrix}\right| =$ (down 1st column) $\left|1\cdot\begin{vmatrix}-5 & 2\\ 2 & -1\end{vmatrix} + (-1)(4)\begin{vmatrix}-2 & -1\\ 2 & -1\end{vmatrix} + 0\right| = \left|1\cdot(5-4) - 4(2-(-2))\right| = |1-16| = |-15| = 15$

(10) a) let $\vec{u} = \begin{bmatrix}x\\ y\end{bmatrix}$ where $xy \geq 0$ and

i) let $c=0$ then $c\cdot\vec{u} = 0\cdot\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$ and $0\cdot 0 \geq 0 \rightarrow c\vec{u}$ is in W

ii) Let $c>0$ then $c\cdot\vec{u} = c\cdot\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}cx\\ cy\end{bmatrix}$ then $(cx)(cy) \geq 0$ since $c^2 > 0$ and $xy \geq 0$ $\rightarrow c\vec{u}$ is in W

iii) let $c<0$ then $c\cdot\vec{u} = c\cdot\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}cx\\ cy\end{bmatrix}$ then $(cx)(cy) \geq 0$ since $c^2 > 0$ and $xy \geq 0$ $\rightarrow c\vec{u}$ is in W

$\exists\, c^2\cdot(xy) \geq 0$

b) Let $\vec{u} = \begin{bmatrix}1\\ 2\end{bmatrix}$ and let $\vec{v} = \begin{bmatrix}-2\\ -1\end{bmatrix}$ then $\vec{u}$ is in W since $(1)(2) = 2 \geq 0$ and $\vec{v}$ is in W since $(-1)(-2) = 2 \geq 0$

but $\vec{u}+\vec{v} = \begin{bmatrix}1\\ 2\end{bmatrix} + \begin{bmatrix}-2\\ -1\end{bmatrix} = \begin{bmatrix}-1\\ 1\end{bmatrix}$ and $(-1)(1) = -1 < 0$, thus, $\vec{u}+\vec{v}$ is not in W.

(11)

a) Note that since 'a' is in $\mathbb{R}$, then this set is Span$\{t^2\}$, and since this set can be described as a spanning set then all polynomials of the form $\vec{p}(t) = at^2$ is a subspace of $\mathbb{P}_m$ $m \geq 2$.

b) Note that the zero vector has the form $0 + 0t + 0t^2$, and even if $a=0$ then $\vec{p}(t) = 0 + t^2 = t^2$ and the coefficient of t^2 will always be '1', so the zero vector is not in the set of all polynomials of the form $\vec{p}(t) = a + t^2$. Not a subspace.