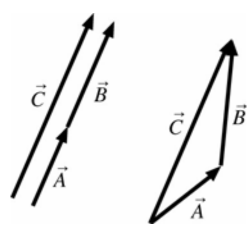
3

VECTORS AND COORDINATE SYSTEMS

Conceptual Questions

3.1. The magnitude of the displacement vector is the minimum distance traveled since the displacement is the vector sum of a number of individual movements. Thus, it is not possible for the magnitude of the displacement vector to be more than the distance traveled. If the individual movements are all in the same direction, the total displacement and the distance traveled are equal. However, it is possible that the total displacement is less than the distance traveled, if the individual movements are not in the same direction.

3.2. It is possible that $C = A + B$ only if $\vec{A}$ and $\vec{B}$ both point in the same direction as in the figure below. It is not possible that $C > A + B$ because, if $\vec{A}$ and $\vec{B}$ point in different directions, putting them tip to tail gives a resultant with a shorter length (see figure below).



3.3. It is possible that $C = 0$ if $\vec{A} = -\vec{B}.$ It is not possible for the length of a vector to be negative, so $C \geq 0.$ Even if $\vec{A}$ and $\vec{B}$ are parallel but in opposite directions, $\vec{C}$ will still have a length greater than or equal to zero.

3.4. No, it is not possible to add a scalar to a vector, since the scalar has no direction.

3.5. The zero vector $\vec{0}$ has zero length. It does not point in any direction.

3.6. A vector can have a component that is zero and still have nonzero length only if another component is nonzero. For example, consider the vector $\hat{i} = (1,\ 0),$ which points along the x-axis.

3.7. If one component of a vector is nonzero then it is not possible for the vector to have zero magnitude. The magnitude of the vector depends on the sum of the squares of the components, so any component signs do not matter.

3.8. No, it is not possible for two vectors with unequal magnitudes to add to zero. To add to zero, two vectors must be antiparallel and of the same length (magnitude).

3.9. **(a)** False, because the size of a vector is fixed. **(b)** False, because the direction of a vector in space is independent of any coordinate system. **(c)** True, because the orientation of the vector relative to the axes can be different.

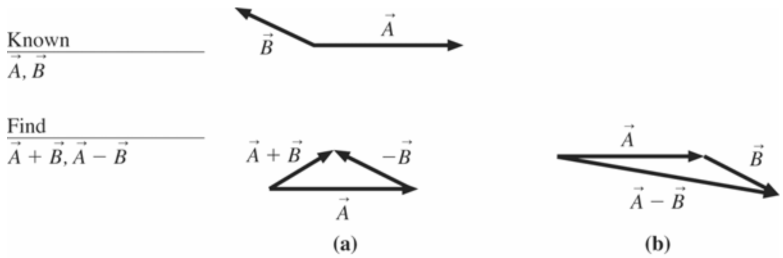
Exercises and Problems

Exercises

Section 3.1 Scalars and Vectors

Section 3.2 Using Vectors

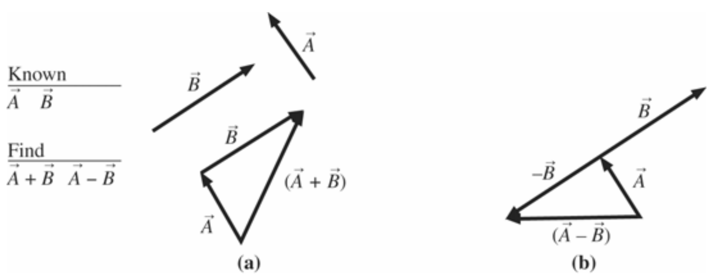
3.1. Visualize:



Solve: **(a)** To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and draw an arrow from the tail of vector $\vec{A}$ to the tip of vector $\vec{B}$.

(b) Since $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$, we place the tail of the vector $-\vec{B}$ on the tip of vector $\vec{A}$ and then draw an arrow from the tail of vector $\vec{A}$ to the tip of vector $-\vec{B}$.

3.2. Visualize:

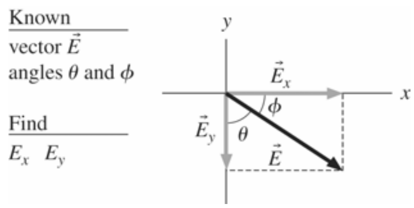


Solve: **(a)** To find $\vec{A}+\vec{B}$, we place the tail of vector $\vec{B}$ on the tip of vector $\vec{A}$ and then draw an arrow from vector $\vec{A}$'s tail to vector $\vec{B}$'s tip.

(b) To find $\vec{A}-\vec{B}$, we note that $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$. We place the tail of vector $-\vec{B}$ on the tip of vector $\vec{A}$ and then draw an arrow from vector $\vec{A}$'s tail to the tip of vector $-\vec{B}$.

Section 3.3 Coordinate Systems and Vector Components

3.3. Visualize:



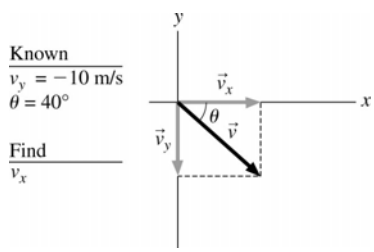
Solve: Vector $\vec{E}$ points to the right and down, so the components E_x and E_y are positive and negative, respectively, according to the Tactics Box 3.1.

(a) $E_x = E\sin\theta$ and $E_y = -E\cos\theta$.

(b) $E_x = E\cos\varphi$ and $E_y = -E\sin\varphi$.

Assess: Note that the role of sine and cosine are reversed because we are using a different angle. θ and ϕ are complementary angles.

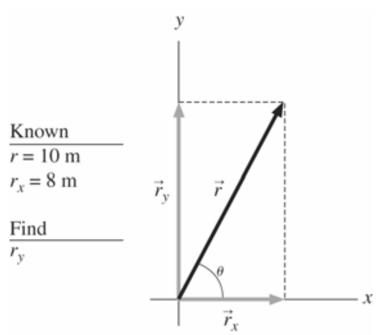
3.4. Visualize: The figure shows the components v_x and v_y, and the angle θ.



Solve: We have $v_y = -v\sin\theta$ where we have manually inserted the minus sign because $\vec{v}_y$ points in the negative-y direction. The x-component is $v_x = v\cos\theta$. Taking the ratio v_x/v_y and solving for v_x gives $v_x = -v_y(\tan\theta)^{-1} = -(-10\text{ m/s})(\tan 40°)^{-1} = 12\text{ m/s}$.

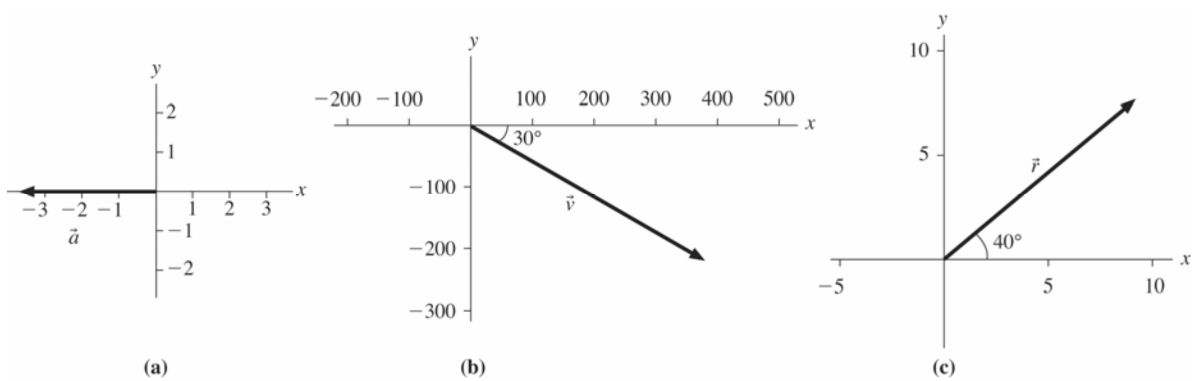
Assess: The x-component is positive since the position vector is in the fourth quadrant.

3.5. Visualize: The position vector $\vec{r}$ whose magnitude r is 10 m has an x-component of 6 m. It makes an angle θ with the $+x$-axis in the first quadrant.



Solve: Using trigonometry, $r_x = r\cos\theta$, or $6\text{ m} = (10\text{ m})\cos\theta$. This gives $\theta = 53.1°$. Thus the y-component of the position vector $\vec{r}$ is $r_y = r\sin\theta = (10\text{ m})\sin(53.1°) = 8\text{ m}$.

3.6. Visualize:

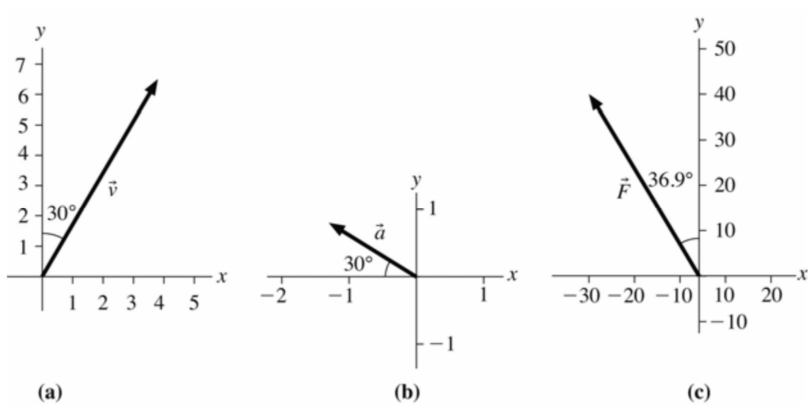


Solve: **(a)** $a_x = -3.5\ \text{m/s}^2;\ \ a_y = 0\ \text{m/s}^2$

(b) $v_x = (440\ \text{m/s})(\cos 30°) = 380\ \text{m/s};\ \ v_y = -(440\ \text{m/s})(\sin 30°) = -220\ \text{m/s}$

(c) $r_x = (12\ \text{m})(\cos 40°) = 9.2\ \text{m};\ \ r_y = (12\ \text{m})(\sin 40°) = 7.7\ \text{m}$

3.7. Visualize:

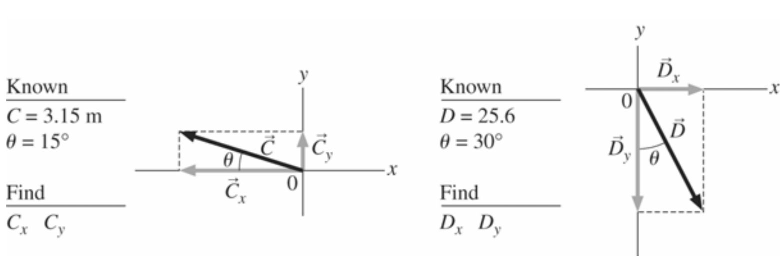


Solve: **(a)** $v_x = (7.5\ \text{m/s})(\sin 30°) = 3.8\ \text{m/s};\ \ v_y = (7.5\ \text{m/s})(\cos 30°) = 6.5\ \text{m/s}$

(b) $a_x = -(1.5\ \text{m/s}^2)(\cos 30°) = -1.3\ \text{m/s}^2;\ \ a_y = (1.5\ \text{m/s}^2)(\sin 30°) = 0.80\ \text{m/s}^2$

(c) $F_x = -(50.0\ \text{N})(\sin 36.9°) = -30\ \text{N};\ \ F_y = (50.0\ \text{N})(\cos 36.9°) = 40\ \text{N}$

3.8. Visualize:

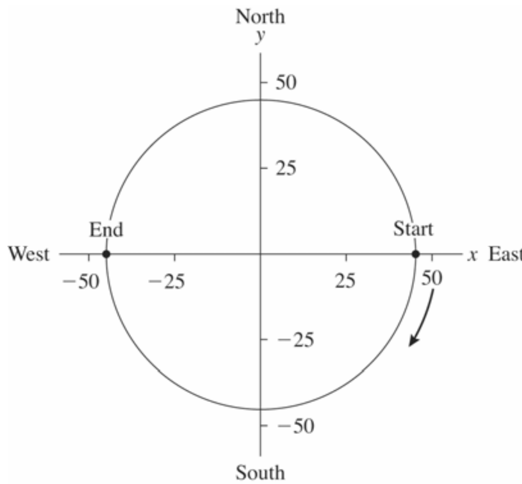


The components of the vector $\vec{C}$ and $\vec{D}$, and the angles θ are shown.

Solve: For $\vec{C}$ we have $C_x = -(3.15\text{ m})\cos(15°) = -3.04\text{ m}$ and $C_y = (3.15\text{ m})\sin(15°) = 0.815\text{ m}$. For $\vec{D}$ we have $D_x = (25.6\text{ m})\sin(30°) = 12.8\text{ m}$ and $D_y = -(25.67\text{ m})\cos(30°) = -22.2\text{ m}$.

Assess: The components of the vectors $\vec{C}$ and $\vec{D}$ have the same units as the vectors themselves. Note the minus signs we have manually inserted, as per the rules of Tactics Box 3.1.

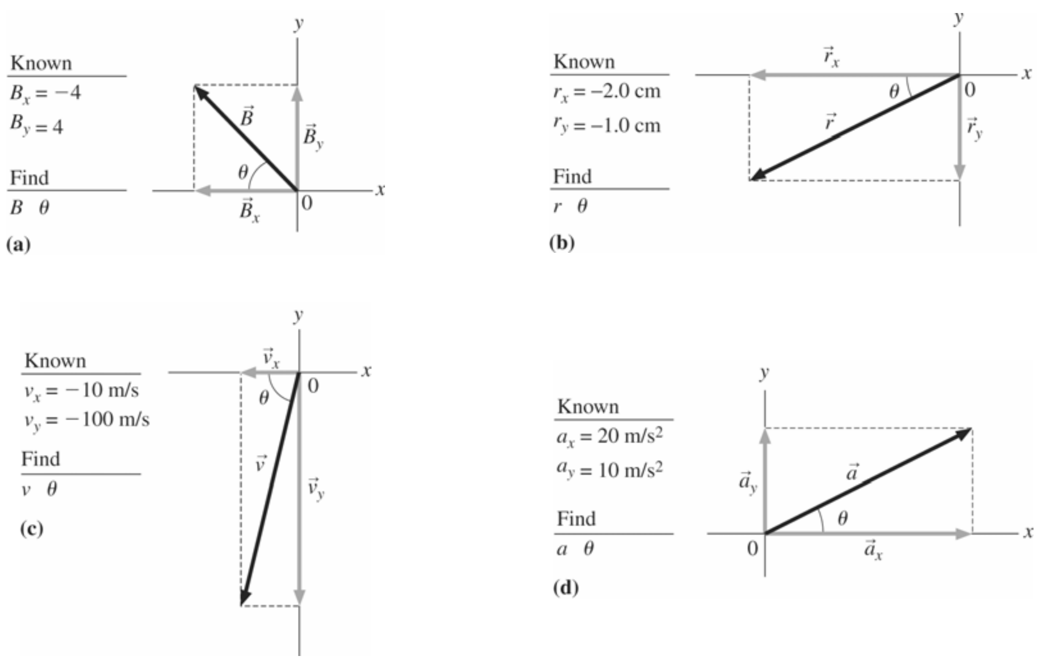
3.9. Visualize:



Solve: The runner ends up at the point $(x, y) = (-50\text{ m}, 0\text{ m})$ after 2.5 times around (which is the same as 0.5 times around). The displacement from the starting point to the ending point is 100 m, west.
Assess: The position is only 50 m west of the origin, but the displacement goes from the first position to the last.

Section 3.4 Unit Vectors and Vector Algebra

3.10. Visualize:



Solve: (a) Using the formulas for the magnitude and direction of a vector, we have:

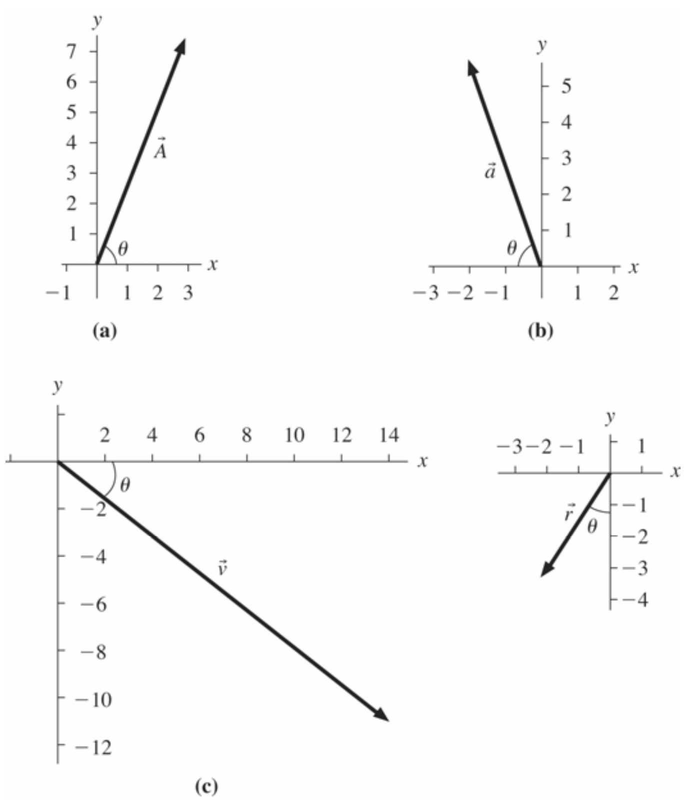
$$B = \sqrt{(-4)^2 + (4)^2} = 5.7, \quad \theta = \tan^{-1}\left(\frac{4}{4}\right) = 45°$$

(b) $r = \sqrt{(-2.0\ \text{cm})^2 + (-1.0\ \text{cm})^2} = 2.2\ \text{cm}, \quad \theta = \tan^{-1}\left(\frac{1.0}{2.0}\right) = 27°$

(c) $v = \sqrt{(-10\ \text{m/s})^2 + (-100\ \text{m/s})^2} = 100\ \text{m/s}, \quad \theta = \tan^{-1}\left(\frac{100}{10}\right) = 84°$

(d) $a = \sqrt{(10\ \text{m/s}^2)^2 + (20\ \text{m/s}^2)^2} = 22\ \text{m/s}^2, \quad \theta = \tan^{-1}\left(\frac{10}{20}\right) = 27°$

3.11. Visualize:



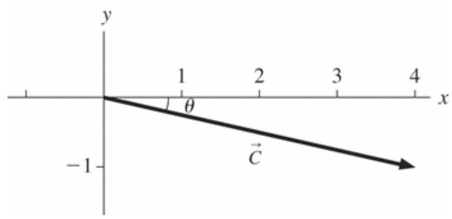
Solve: (a) $A = \sqrt{(3.0)^2 + (7.0)^2} = 7.6, \quad \theta = 67°$

(b) $a = \sqrt{(-2.0\ \text{m/s}^2)^2 + (4.5\ \text{m/s}^2)^2} = 4.9\ \text{m/s}^2, \quad \theta = 66°$

(c) $v = \sqrt{(14\ \text{m/s})^2 + (-11\ \text{m/s})^2} = 18\ \text{m/s}, \quad \theta = 38°$

(d) $r = \sqrt{(-2.2\ \text{m})^2 + (-3.3\ \text{m})^2} = 4.0\ \text{m}, \quad \theta = 34°$

3.12. Visualize:

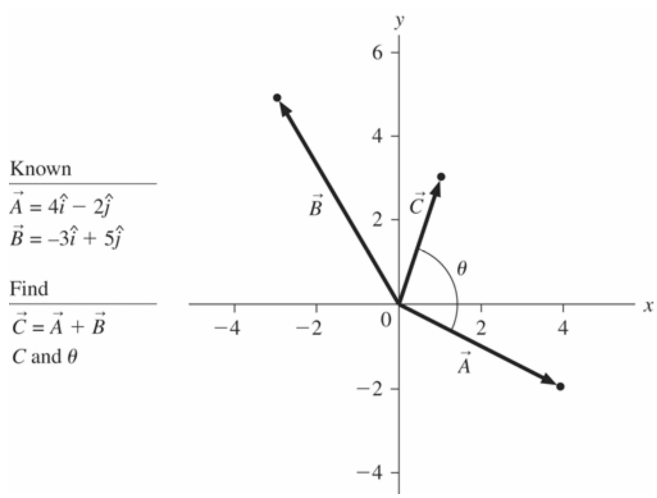


Solve: (a) $\vec{C} = (2\hat{i} + 3\hat{j}) + (2\hat{i} - 4\hat{j}) = (4\hat{i} - \hat{j})$

(b) Vector $\vec{C}$ is shown in the figure above.

(c) $C = \sqrt{(4.0)^2 + (-1.0)^2} = 4.1 \quad \theta = 14°$ below the $+x$-axis

3.13. Visualize: The vectors $\vec{A}$, $\vec{B}$, and $\vec{C} = \vec{A} + \vec{B}$ are shown.



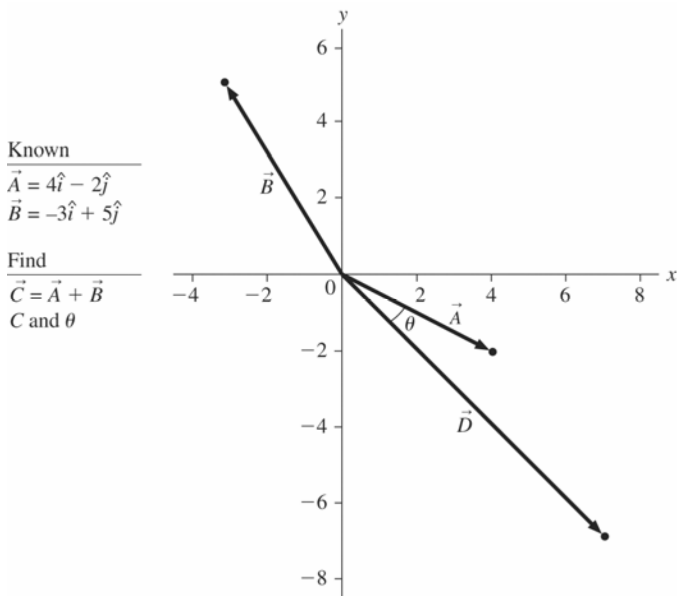
Solve: (a) We have $\vec{A} = 4\hat{i} - 2\hat{j}$ and $\vec{B} = -3\hat{i} + 5\hat{j}$. Thus, $\vec{C} = \vec{A} + \vec{B} = (4\hat{i} - 2\hat{j}) + (-3\hat{i} + 5\hat{j}) = 1\hat{i} + 3\hat{j}$.

(b) Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are shown in the figure above.

(c) Since $\vec{C} = 1\hat{i} + 3\hat{j} = C_x\hat{i} + C_y\hat{j}$, $C_x = 1$, and $C_y = 3$. Therefore, the magnitude and direction of $\vec{C}$ are $C = \sqrt{(1)^2 + (3)^2} = \sqrt{10} = 3.2$ and $\theta = \tan^{-1}(C_y/C_x) = \tan^{-1}(3/1) = 72°$, respectively.

Assess: The vector $\vec{C}$ is to the right and up, thus implying that both the x and y components are positive. Also $\theta > 45°$ since $|C_y| > |C_x|$.

3.14. Visualize:



Solve: (a) We have $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $-\vec{B} = 3\hat{\imath} - 5\hat{\jmath}$. Thus, $\vec{D} = \vec{A} + (-\vec{B}) = (4+3)\hat{\imath} + (-2-5)\hat{\jmath} = 7\hat{\imath} - 7\hat{\jmath}$.

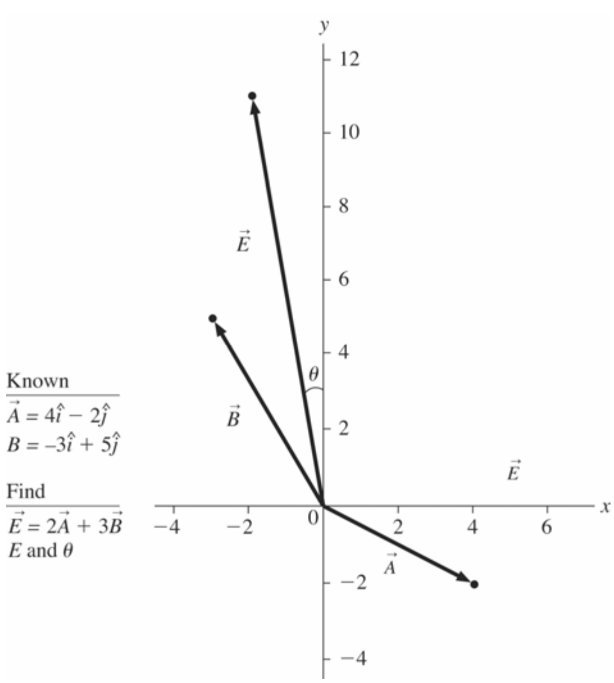
(b) Vectors $\vec{A}$, $\vec{B}$, and $\vec{D}$ are shown in the figure above.

(c) Since $\vec{D} = 7\hat{\imath} - 7\hat{\jmath} = D_x\hat{\imath} + D_y\hat{\jmath}$, $D_x = 7$ and $D_y = -7$. Therefore, the magnitude and direction of $\vec{D}$ are

$$D = \sqrt{(7)^2 + (-7)^2} = 7\sqrt{2} = 9.9 \qquad \theta = \tan^{-1}\left(\left|D_y\right|/D_x\right) = \tan^{-1}(7/7) = 45°$$

Assess: Since $\left|D_y\right| = \left|D_x\right|$, the angle $\theta = 45°$, as expected.

3.15. Visualize:



Solve: (a) We have $\vec{A} = 4\hat{i} - 2\hat{j}$ and $\vec{B} = -3\hat{i} + 5\hat{j}$. This means $2\vec{A} = 8\hat{i} - 4\hat{j}$ and $3\vec{B} = -9\hat{i} + 15\hat{j}$. Thus, $\vec{E} = 2\vec{A} + 3\vec{B} = [8 + (-9)]\hat{i} + [(-4) + 15]\hat{j} = -1\hat{i} + 11\hat{j}$.

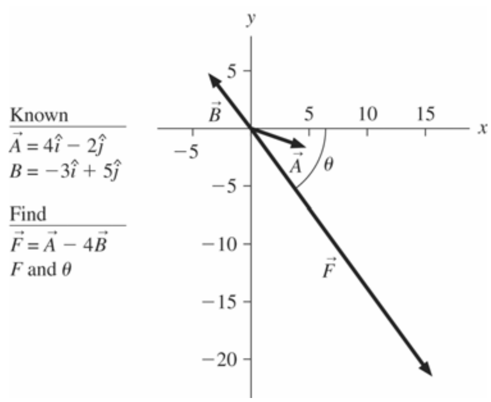
(b) Vectors $\vec{A}$, $\vec{B}$, and $\vec{E}$ are shown in the figure above.

(c) From the $\vec{E}$ vector, $E_x = -1$ and $E_y = 11$. Therefore, the magnitude and direction of $\vec{E}$ are

$$E = \sqrt{(-1)^2 + (11)^2} = \sqrt{122} = 11,\ \ \theta = \tan^{-1}(E_x/E_y) = \tan^{-1}(-1/11) = 5.2°$$

So $\vec{E}$ is 5.2° counterclockwise from the +*y*-axis.

3.16. Visualize:



Solve: (a) We have $\vec{A} = 4\hat{i} - 2\hat{j}$ and $\vec{B} = -3\hat{i} + 5\hat{j}$. This means $4\vec{B} = -12\hat{i} + 20\hat{j}$. Hence, $\vec{F} = \vec{A} - 4\vec{B} = [4 - (-12)]\hat{i} + [-2 - 20]\hat{j} = 16\hat{i} - 22\hat{j} = F_x\hat{i} + F_y\hat{j}$, so $F_x = 16$ and $F_y = -22$.

(b) The vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$ are shown in the above figure.

(c) The magnitude and direction of $\vec{F}$ are

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(16)^2 + (-22)^2} = 27$$

$$\theta = \tan^{-1}\left(|F_y|/F_x\right) = \tan^{-1}(22/16) = 54°$$

Assess: $F_y > F_x$ implies $\theta > 45°$, which is consistent with the figure.

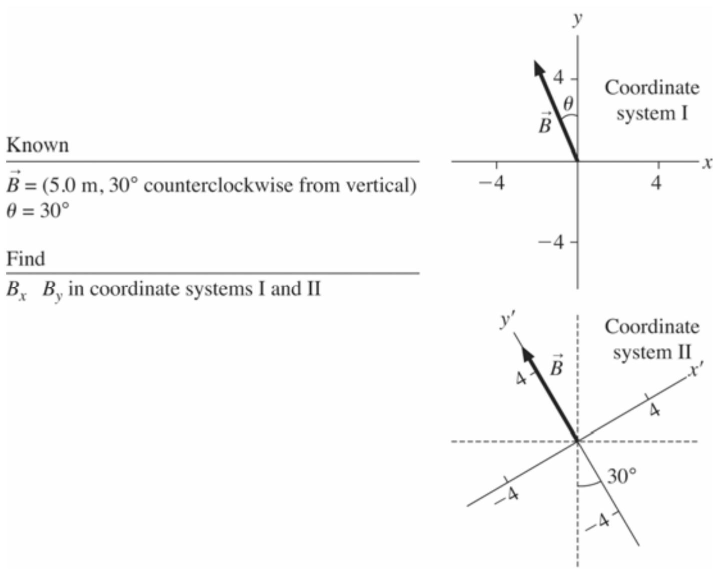
3.17. Solve: We have $\vec{E} = E_x\hat{i} + E_y\hat{j} = 2\hat{i} + 3\hat{j}$, which means $E_x = 2$ and $E_y = 3$. Also, $\vec{F} = F_x\hat{i} + F_y\hat{j} = 2\hat{i} - 2\hat{j}$, which means $F_x = 2$ and $F_y = -2$.

(a) The magnitude of $\vec{E}$ is given by $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(2)^2 + (3)^2} = 3.6$ and the magnitude of $\vec{F}$ is given by $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2)^2 + (2)^2} = 2.8$.

(b) Since $\vec{E} + \vec{F} = 4\hat{i} + 1\hat{j}$, the magnitude of $\vec{E} + \vec{F}$ is $\sqrt{(4)^2 + (1)^2} = 4.1$.

(c) Since $-\vec{E} - 2\vec{F} = -(2\hat{i} + 3\hat{j}) - 2(2\hat{i} - 2\hat{j}) = -6\hat{i} + 1\hat{j}$, the magnitude of $-\vec{E} - 2\vec{F}$ is $\sqrt{(-6)^2 + (1)^2} = 6.1$.

3.18. Visualize:



Solve: In coordinate system I, the vector $\vec{B}$ makes an angle of 30° counterclockwise from vertical, so it has an angle of $\theta = 60°$ with the negative *x*-axis. Since $\vec{B}$ points to the left and up, it has a negative *x*-component and a positive *y*-component. Thus, $B_x = -(5.0 \text{ m})\cos(60°) = -2.5 \text{ m}$ and $B_y = +(5.0 \text{ m})\cos(60°) = 4.3 \text{ m}$. Thus, $\vec{B} = -(4.3 \text{ m})\hat{i} + (2.5 \text{ m})\hat{j}$.

In coordinate system II, the vector $\vec{B}$ is along the +*y*-axis. This means the *x*-component is zero. Thus, $B_x = 0.0$, and $B_y = 5.0$ m. Thus $\vec{B} = (0.0 \text{ m})\hat{i} + (5.0 \text{ m})\hat{j}$.

3.19. Visualize: Refer to Figure EX3.19 in your textbook. The velocity vector $\vec{v}$ points south and makes an angle of 30° with the –*y*-axis. The vector $\vec{v}$ points to the left and down, implying that both v_x and v_y are negative.

Solve: We have $v_x = -v\sin(30°) = -(100 \text{ m/s})\sin(30°) = -50 \text{ m/s}$ and

$v_y = -v\cos(30°) = -(100 \text{ m/s})\cos(30°) = -87 \text{ m/s}$.

Assess: Notice that v_x and v_y have the same units as $\vec{v}$.

3.20. Visualize: Refer to Figure EX3.20 in your textbook.

Solve: (a) We are given that $\vec{A} + \vec{B} + \vec{C} = 1\hat{j}$ with $\vec{A} = 3\hat{i}$, and $\vec{C} = -4\hat{j}$. This means $\vec{A} + \vec{C} = 3\hat{i} - 4\hat{j}$. Thus, $\vec{B} = (1\hat{j}) - (\vec{A} + \vec{C}) = (1\hat{j}) - (3\hat{i} - 4\hat{j}) = -3\hat{i} + 5\hat{j}$.

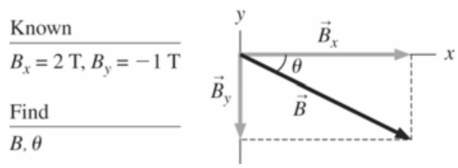
(b) We have $\vec{B} = B_x\hat{i} + B_y\hat{j}$ with $B_x = -3$ and $B_y = 5$. Hence, $B = \sqrt{(-3)^2 + (5)^2} = 5.8$

$$\theta = \tan^{-1}\frac{B_y}{B_x} = \tan^{-1}\left(\frac{5}{-3}\right) = -59°$$

Since $\vec{B}$ has a negative *x*-component and a positive *y*-component, the vector $\vec{B}$ is in the second quadrant and the angle θ made by $\vec{B}$ is 59 degrees above the –*x*-axis or 31 degrees left of + *y*-axis.

Assess: Since $|B_y| < |B_x|$, $\theta < 45°$ as obtained above.

3.21. Visualize:



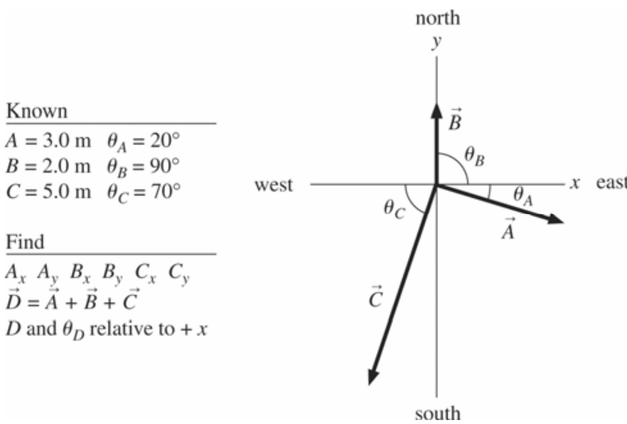
Solve: The magnitude of the vector is $B = \sqrt{B_x^2 + B_y^2} = \sqrt{(2.0\text{ T})^2 + (-1.0\text{ T})^2} = \sqrt{5.0}\text{ T} = 2.2\text{ T}$. The angle θ is

$$\theta = \tan^{-1}\frac{|B_y|}{B_x} = \tan^{-1}\left(\frac{1.0\text{ T}}{2.0\text{ T}}\right) = 27°$$

Assess: Since $|B_y| < |B_x|$, the angle θ made with the +*x*-axis is less than $45°$. $\theta = 45°$ for $|B_y| = |B_x|$.

Problems

3.22. Visualize: (a)



Solve: (b) The components of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are
$A_x = (3.0\text{ m})\cos(20°) = 2.8$ m and $A_y = -(3.0\text{ m})\sin(20°) = -1.0$ m; $B_x = 0$ m and $B_y = 2.0$ m;
$C_x = -(5.0\text{ m})\cos(70°) = -1.7$ m and $C_y = -(5.0\text{ m})\sin(70°) = -4.7$ m. This means the vectors can be written as

$$\vec{A} = (2.8\hat{i} + 1.0\hat{j})\text{ m}, \quad \vec{B} = (2.0\hat{j})\text{ m}, \quad \vec{C} = (-1.7\hat{i} - 4.7\hat{j})\text{ m}$$

(c) We have $\vec{D} = \vec{A} + \vec{B} + \vec{C} = (1.1\text{ m})\hat{i} - (3.7\text{ m})\hat{j}$. This means

$$D = \sqrt{(1.1\text{ m})^2 + (3.7\text{ m})^2} = 3.9\text{ m} \qquad \theta = \tan^{-1}(3.9/1.09) = 74°$$

The direction of $\vec{D}$ is south of east, 74° below the +*x*-axis.

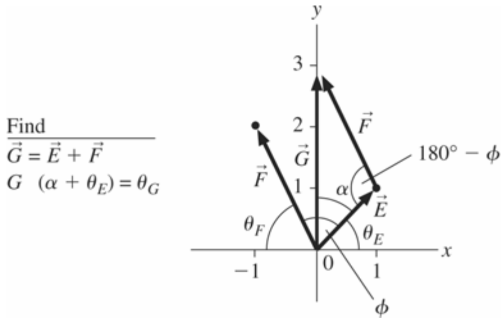
3.23. Solve: We have $\vec{r} = (5.0\hat{i} + 4.0\hat{j})t^2$ m. This means that $\vec{r}$ does not change the ratio of its components as t increases; that is, the direction of $\vec{r}$ is constant. The magnitude of $\vec{r}$ is given by $r = \sqrt{(5.0t^2)^2 + (4.0t^2)^2}\text{m} = 6.40t^2$ m.

(a) The particle's distance from the origin at $t = 0$ s, $t = 2$ s, and $t = 5$ s is 0 m, 26 m, and 160 m.

(b) The particle's velocity is $\vec{v}(t) = \dfrac{d\vec{r}}{dt} = (5.0\hat{i} + 4.0\hat{j})\dfrac{dt^2}{dt}\text{m/s} = (5.0\hat{i} + 4.0\hat{j})2t\text{ m/s} = (10\hat{i} + 8.0\hat{j})t\text{ m/s}$.

(c) The magnitude of the particle's velocity is given by $v = \sqrt{(10t)^2 + (8.0t)^2} = 13t$ m/s. The particle's speed at $t = 0$ s, $t = 2$ s, and $t = 5$ s is 0 m/s, 26 m/s, and 64 m/s.

3.24. Visualize:



Solve: (a) $\theta_E = \tan^{-1}\left(\frac{1}{1}\right) = 45°$, $\theta_F = \tan^{-1}\left(\frac{2}{1}\right) = 63°$. Thus $\phi = 180° - \theta_E - \theta_F = 72°$

(b) From the figure, $E = \sqrt{2}$ and $F = \sqrt{5}$. Using

$$G^2 = E^2 + F^2 - 2EF\cos\varphi = (\sqrt{2})^2 + (\sqrt{5})^2 - 2(\sqrt{2})(\sqrt{5})\cos(180° - 72°)$$
$$G = 3$$

Furthermore, using $\frac{\sin\alpha}{\sqrt{5}} = \frac{\sin(180° - 72°)}{2.975} \Rightarrow \alpha = 45°$

Since $\theta_E = 45°$, the angle made by the vector $\vec{G}$ with the +*x*-axis is $\theta_G = (\alpha + \theta_E) = 45° + 45° = 90°$.

(c) We have

$$E_x = +1.0, \quad \text{and} \quad E_y = +1.0$$
$$F_x = -1.0, \quad \text{and} \quad F_y = +2.0$$
$$G_x = 0.0, \quad \text{and} \quad G_y = 3.0$$
$$G = \sqrt{(0.0)^2 + (3.0)^2} = 3.0, \quad \text{and} \quad \theta = \tan^{-1}\frac{|G_y|}{|G_x|} = \tan^{-1}\left(\frac{3.0}{0.0}\right) = 90°$$

That is, the vector $\vec{G}$ makes an angle of 90° with the *x*-axis.

Assess: The graphical solution and the vector solution give the same answer within the given significance of figures.

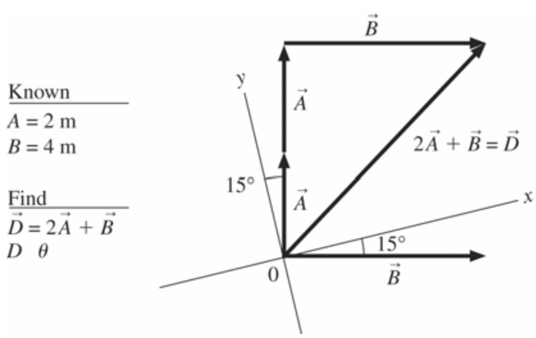
3.25. Visualize: Refer to Figure P3.25 in your textbook.

Solve: From the rules of trigonometry, we have $A_x = (4\text{ m})\cos(60°) = 2.0\text{ m}$ and $A_y = (4\text{ m})\sin(60°) = 3.5\text{ m}$.

Also, $B_x = -(3\text{ m})\cos(20°) = -2.8$ and $B_y = +(3\text{ m})\sin(20°) = 1.0\text{ m}$. Since $\vec{A} + \vec{B} + \vec{C} = \vec{0}$,

$\vec{C} = -\vec{A} - \vec{B} = (-\vec{A}) + (-\vec{B}) = (-2.0\hat{i} - 3.5\hat{j}) + (+2.8\hat{i} - 1.0\hat{j}) = 0.8\hat{i} - 4.5\hat{j}$.

3.26. Visualize:



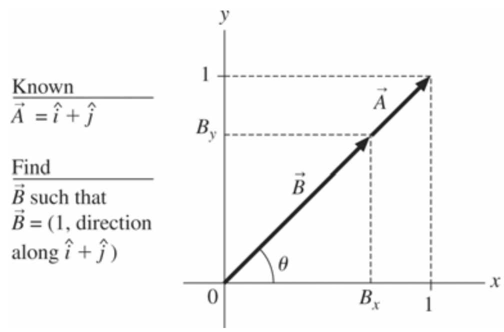
Solve: In the tilted coordinate system, the vectors $\vec{A}$ and $\vec{B}$ are expressed as:

$$\vec{A}=[2\sin(15°)\text{ m}]\hat{i}+[2\cos(15°)\text{ m}]\hat{j} \text{ and } \vec{B}=[4\cos(15°)\text{ m}]\hat{i}-[4\sin(15°)\text{ m}]\hat{j}.$$

Therefore, $\vec{D}=2\vec{A}+\vec{B}=(4\text{ m})[\sin(15°)+\cos(15°)]\hat{i}+(4\text{ m})[\cos(15°)-\sin(15°)]\hat{j}=(4.9\text{ m})\hat{i}+(2.9\text{ m})\hat{j}.$

Assess: The magnitude of this vector is $D=\sqrt{(4.9\text{ m})^2+(2.9\text{ m})^2}=5.7\text{ m},$ and it makes an angle of $\theta=\tan^{-1}(2.9\text{ m}/4.9\text{ m})=31°$ with the +*x*-axis. The resultant vector can be obtained graphically by using the rule of tail-to-tip addition.

3.27. Visualize:



The magnitude of the unknown vector is 1 and its direction is along $\hat{i}+\hat{j}$. Let $\vec{A}=\hat{i}+\hat{j}$ as shown in the diagram. That is, $\vec{A}=1\hat{i}+1\hat{j}$ and the *x*- and *y*-components of $\vec{A}$ are both unity. Since $\theta=\tan^{-1}(A_y/A_x)=45°,$ the unknown vector must make an angle of 45° with the +*x*-axis and have unit magnitude.

Solve: Let the unknown vector be $\vec{B}=B_x\hat{i}+B_y\hat{j}$ where

$$B_x=B\cos(45°)=\frac{1}{\sqrt{2}}B \quad \text{and} \quad B_y=B\sin(45°)=\frac{1}{\sqrt{2}}B$$

We want the magnitude of $\vec{B}$ to be 1, so we have

$$B=\sqrt{B_x^2+B_y^2}=1\Rightarrow\sqrt{\left(\frac{1}{\sqrt{2}}B\right)^2+\left(\frac{1}{\sqrt{2}}B\right)^2}=1\Rightarrow\sqrt{B^2}=1\Rightarrow B=1$$

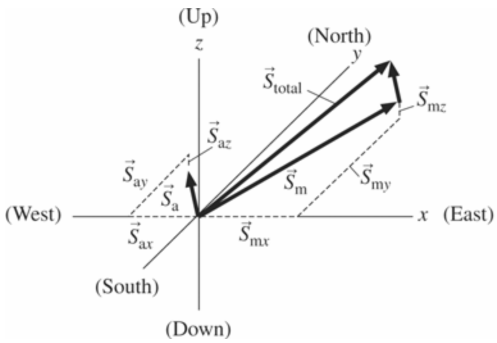
Thus,

$$B_x=B_y=\frac{1}{\sqrt{2}}$$

Finally,

$$\vec{B}=B_x\hat{i}+B_y\hat{j}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$$

3.28. Visualize: The coordinate system (*x*, *y*, *z*) is shown here; +*x* denotes east, +*y* denotes north, and +*z* denotes upward vertical. The vectors $\vec{S}_{\text{morning}}$ (shortened to $\vec{S}_{\text{m}}$), $\vec{S}_{\text{afternoon}}$ (shortened to $\vec{S}_{\text{a}}$), and the total displacement vector $\vec{S}_{\text{total}}=\vec{S}_{\text{a}}+\vec{S}_{\text{m}}$ are also shown.



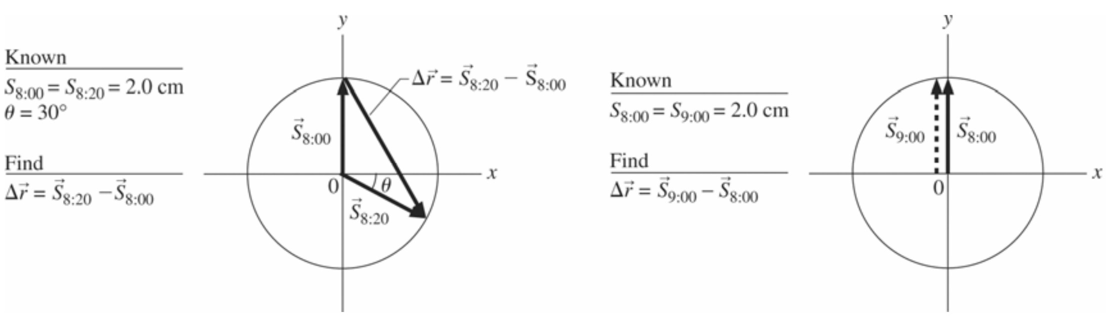
Solve: $\vec{S}_{\text{m}} = (2000\hat{i} + 3000\hat{j} + 200\hat{k})$ m, and $\vec{S}_{\text{a}} = (-1500\hat{i} + 2000\hat{j} - 300\hat{k})$ m. The total displacement is the sum of the individual displacements.

(a) The sum of the *z*-components of the afternoon and morning displacements is $S_{\text{az}} + S_{\text{mz}} = -300 \text{ m} + 200 \text{ m} = -100 \text{ m};$ that is, 100 m lower.

(b) $\vec{S}_{\text{total}} = \vec{S}_{\text{a}} + \vec{S}_{\text{m}} = (500\hat{i} + 5000\hat{j} - 100\hat{k})$ m; that is, (500 m east) + (5000 m north) – (100 m vertical). The magnitude of your total displacement is

$$S_{\text{total}} = \sqrt{(500)^2 + (5000)^2 + (-100)^2} \text{ m} = 5.0 \text{ km}$$

3.29. Visualize:



Only the minute hand is shown in the figure.

Solve: (a) We have $\vec{S}_{8:00} = (2.0 \text{ cm})\hat{j}$ and $\vec{S}_{8:20} = (2.0 \text{ cm})\cos(30°)\hat{i} - (2.0 \text{ cm})\sin(30°)\hat{j}$. The displacement vector is

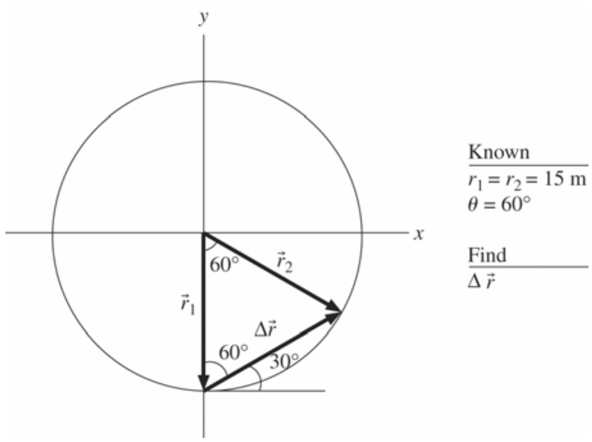
$$\begin{aligned}\Delta\vec{r} &= \vec{S}_{8:20} - \vec{S}_{8:00} \\ &= (2.0 \text{ cm})[\cos 30°\hat{i} - (\sin 30° + 1)\hat{j}] \\ &= (2.0 \text{ cm})(0.87\hat{i} - 1.50\hat{j}) \\ &= (1.7 \text{ cm})\hat{i} - (3.0 \text{ cm})\hat{j}\end{aligned}$$

(b) We have $\vec{S}_{8:00} = (2.0 \text{ cm})\hat{j}$ and $\vec{S}_{9:00} = (2.0 \text{ cm})\hat{j}$. The displacement vector is $\Delta\vec{r} = \vec{S}_{9:00} - \vec{S}_{8:00} = 0.0$ cm.

Assess: The displacement vector in part (a) has a positive *x*-component and a negative *y*-component. The vector thus is to the right and points down, in quadrant IV. This is where the vector drawn from the tip of the 8:00 a.m. arm to the tip of the 8:20 a.m. arm will point.

3.30. Model: Model Betty as a particle. Set the origin of the coordinate system at the center of the wheel.

Visualize:



Solve:

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = [((15\text{ m})\sin 60°)\hat{i} + ((15\text{ m})\cos 60°)\hat{j}] - [(0.0\text{ m})\hat{i} - (15\text{ m})\hat{j}]$$
$$= [(13\text{ m})\hat{i} - (7.5\text{ m})\hat{j}] - [(0.0\text{ m})\hat{i} - (15\text{ m})\hat{j}] = (13\text{ m})\hat{i} + (7.5\text{ m})\hat{j}$$

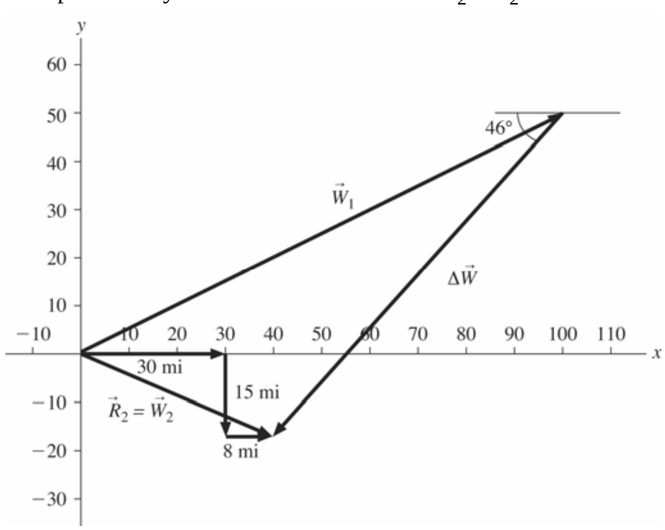
Now find the magnitude and direction.

$$\left|\Delta\vec{r}\right| = \sqrt{(13\text{ m})^2 + (7.5\text{ m})^2} = 15\text{ m} \qquad \theta = \tan^{-1}\left(\frac{7.5\text{ m}}{13\text{ m}}\right) = 30° \text{ above } +x\text{-axis}$$

Assess: The above method is the right way to solve the problem for an arbitrary angle. But in this particular case it is easy to see that the triangle is equilateral, so the sides are all the same length and all the interior angles are $60°$. Simple analysis of the resulting right angle gives $\theta = 30°$ above $+x$-axis.

3.31. Model: Model Ward as a particle. Set the origin of the coordinate system at Ruth's initial location.

Visualize: When Ward picks up Ruth they are at the same location: $\vec{W}_2 = \vec{R}_2$.



Solve: $\vec{R}_2 = ((30\text{ mi})\hat{i} + (0.0\text{ mi})\hat{j}) + ((0.0\text{ mi})\hat{i} - (15\text{ mi})\hat{j}) + ((8\text{ mi})\hat{i} + (0.0\text{ mi})\hat{j}) = (38\text{ mi})\hat{i} - (15\text{ mi})\hat{j}$

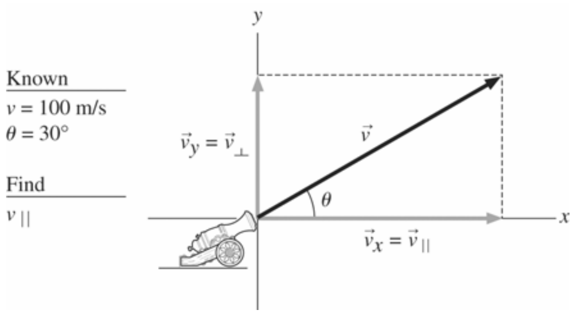
$$\Delta\vec{W} = \vec{W}_2 - \vec{W}_1 = ((38\text{ mi})\hat{i} - (15\text{ mi})\hat{j}) - ((100\text{ mi})\hat{i} + (50\text{ mi})\hat{j}) = -(62\text{ mi})\hat{i} - (65\text{ mi})\hat{j}$$

Now find the magnitude and direction.

$$\left|\Delta\vec{W}\right| = \sqrt{(-62\text{ mi})^2 + (-65\text{ mi})^2} = 90\text{ mi} \qquad \theta = \tan^{-1}\left(\frac{-65\text{ mi}}{-62\text{ mi}}\right) = 46°\text{ south of west}$$

Assess: The length of $\Delta\vec{W}$ and the angle in the scale diagram measure about what we calculated.

3.32. Visualize:

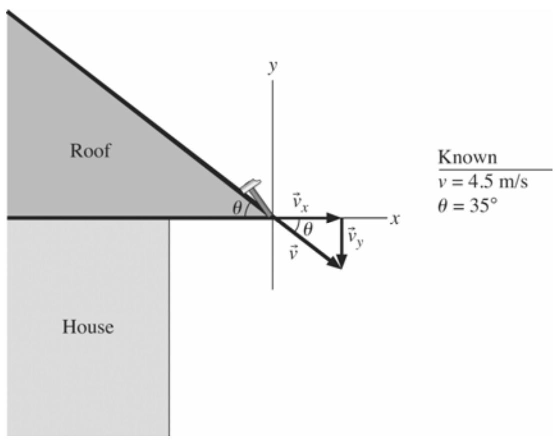


Solve: We have $\vec{v} = v_x\hat{i} + v_y\hat{j} = v_{||}\hat{i} + v_{\perp}\hat{j} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$. Thus, $v_{||} = v\cos\theta = (100\text{ m/s})\cos(30°) = 87\text{ m/s}$.

Assess: For the angle of 30°, 87 m/s for the horizontal component seems reasonable.

3.33. Model: Model the hammer as a particle.

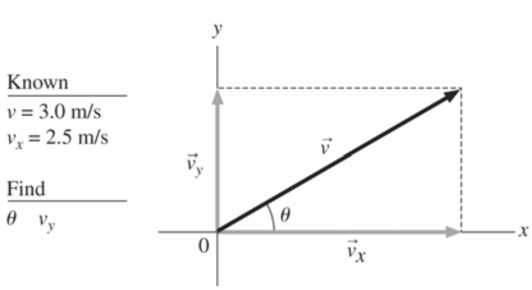
Visualize: Put the origin at the point the hammer leaves the roof.



Solve: $v_x = (4.5\text{ m/s})\cos 35° = 3.7\text{ m/s} \qquad v_y = (4.5\text{ m/s})\sin 35° = 2.6\text{ m/s}$

Assess: These seem like reasonable answers for the velocity given.

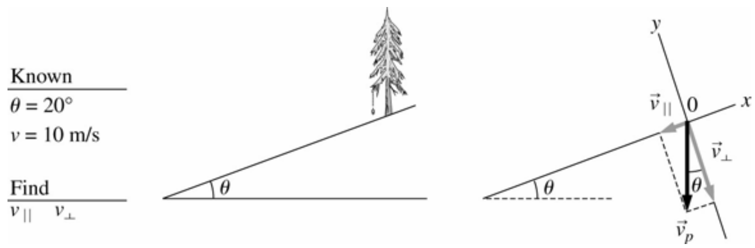
3.34. Visualize:



Solve: **(a)** Since $v_x = v\cos\theta$, we have $2.5\text{ m/s} = (3.0\text{ m/s})\cos\theta \Rightarrow \theta = \cos^{-1}\left(\dfrac{2.5\text{ m/s}}{3.0\text{ m/s}}\right) = 34°.$

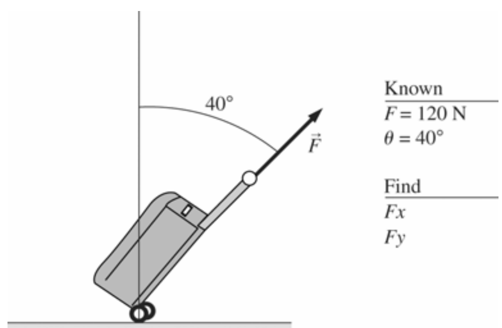
(b) The vertical component is $v_y = v\sin\theta = (3.0\text{ m/s})\sin(34°) = 1.7\text{ m/s}.$

3.35. Visualize:



The coordinate system used here is tilted so that the *x*-axis is along the slope.
Solve: The component of the velocity parallel to the *x*-axis is $v_{||} = -v\cos(70°) = -v\sin(20°) = -(10\text{ m/s})(0.34) =$ -3.4 m/s. This is the speed down the slope. The component of the velocity perpendicular to the slope is $v_\perp = -v\sin(70°) = -v\cos(20°) = -(10\text{ m/s})(0.94) = -9.4\text{ m/s}$. This is the speed toward the ground.
Assess: A final speed of approximately 10 m/s implies a fall time of approximately 1 second under free fall. Note that $g = -9.8\text{ m/s}^2$. This time is reasonable for a drop of approximately 5 m, or 16 feet.

3.36. Visualize:

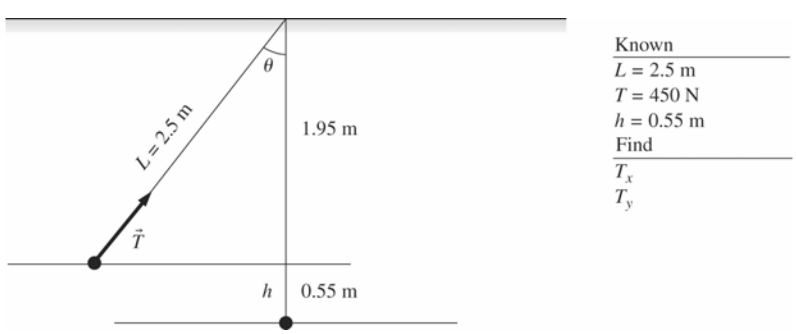


Solve:

$$F_x = (120\text{ N})\sin 40° = 77\text{ N} \qquad F_y = (120\text{ N})\cos 40° = 92\text{ N}$$

Assess: Kami would not have to pull as hard if the angle from the vertical were larger, but it can become awkward.

3.37. Model: Model Dee as a particle.
Visualize:



Solve: First find the angle from the vertical.

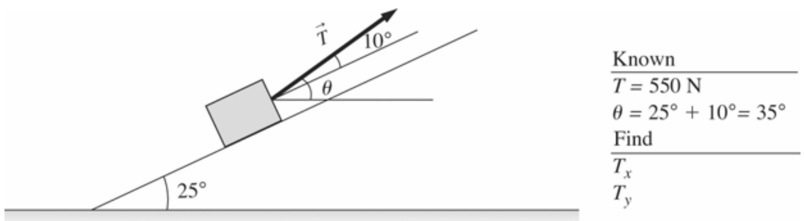
$$(2.5\text{ m})\cos\theta = (2.5\text{ m} - 0.55\text{ m}) = 1.95\text{ m} \Rightarrow \theta = \cos^{-1}\left(\frac{1.95\text{ m}}{2.5\text{ m}}\right) = 38.7°$$

$$T_x = (450\text{ N})\sin 38.7° = 280\text{ N} \qquad T_y = (450\text{ N})\cos 38.7° = 350\text{ N}$$

Assess: As the angle gets bigger the vertical component of the tension decreases.

3.38. Model: Model the crate as a particle.

Visualize:



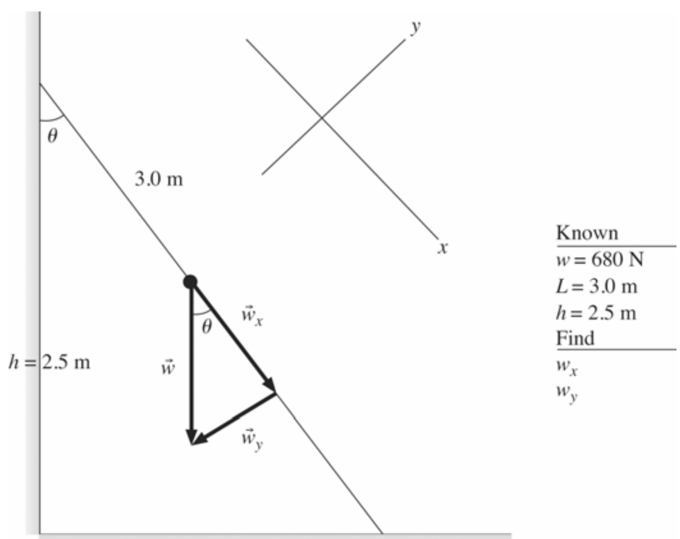
Solve: First find the angle from the horizontal: $\theta = 25° + 10° = 35°$.

$$T_x = (550\text{ N})\cos 35° = 450\text{ N} \qquad T_y = (550\text{ N})\sin 35° = 310\text{ N}$$

Assess: The horizontal component of the tension would decrease if the angle of the ramp decreases or if the angle of the rope from the ramp decreases.

3.39. Model: Model Tom as a particle.

Visualize:



Solve: First find the angle of the ladder from the vertical: $\theta = \cos^{-1}\left(\frac{2.5\text{ m}}{3.0\text{ m}}\right) = 33.56°$.

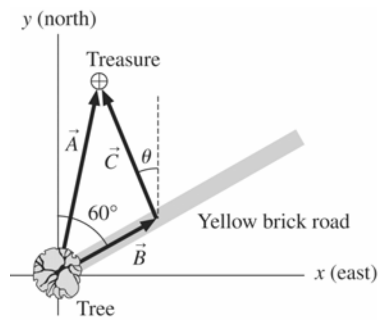
$$w_x = (680\text{ N})\cos 33.56° = 570\text{ N} \qquad w_y = (680\text{ N})\sin(-33.56°) = -380\text{ N}$$

Assess: The figure could have been drawn differently to give negative values for both components; the magnitudes would be the same, however.

3.40. Visualize: Establish a coordinate system with origin at the tree and with the *x*-axis pointing east. Let $\vec{A}$ be a displacement vector directly from the tree to the treasure. Vector $\vec{A}$ is $\vec{A} = (100\hat{i} + 500\hat{j})$ paces.

This describes the displacement you would undergo by walking north 500 paces, then east 100 paces. Instead, you follow the road for 300 paces and undergo displacement

$$\vec{B} = [300\sin(60°)\hat{i} + 300\cos(60°)\hat{j}] \text{ paces} = (260\hat{i} + 150\hat{j}) \text{ paces}$$



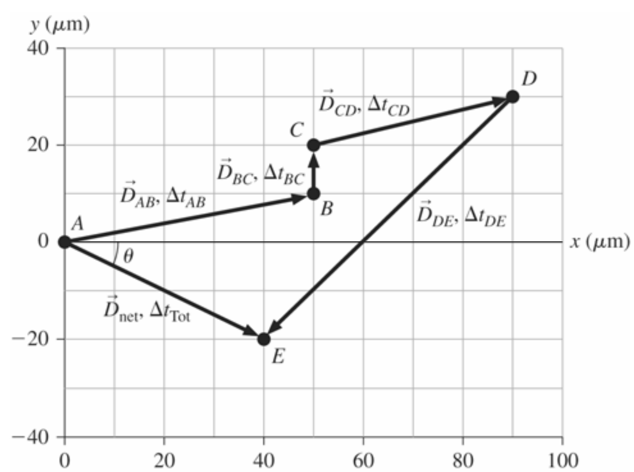
Solve: Now let $\vec{C}$ be the displacement vector from your position to the treasure. From the figure $\vec{A} = \vec{B} + \vec{C}$. So the displacement you need to reach the treasure is $\vec{C} = \vec{A} - \vec{B} = (-160\hat{i} + 350\hat{j})$ paces.

If θ is the angle measured between $\vec{C}$ and the y-axis,

$$\theta = \tan^{-1}\left(\frac{160}{350}\right) = 25°$$

You should head 25° west of north. You need to walk distance $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-160)^2 + (350)^2}$ paces $= 385$ paces to get to the treasure.

3.41. Visualize: The average velocity is the net displacement $\vec{D}_{\text{net}}$ divided by the total time, which are marked on the graph. We also mark on the graph of the bacterium's individual displacements and the time for each.



Solve: The magnitude of the net displacement is found with Pythagorean rule, taking the values from the graph. We have $D_{\text{net}} = \sqrt{D_{\text{net},\,x}^2 + D_{\text{net},\,y}^2} = \sqrt{(40\ \mu\text{m})^2 + (-20\ \mu\text{m})^2} = 45\ \mu\text{m}$. The direction of this displacement is

$$\theta = \tan^{-1}\left(\frac{|D_{\text{net},y}|}{D_{\text{net},x}}\right) = \tan^{-1}\left(\frac{20\ \mu\text{m}}{40\ \mu\text{m}}\right) = 27°$$

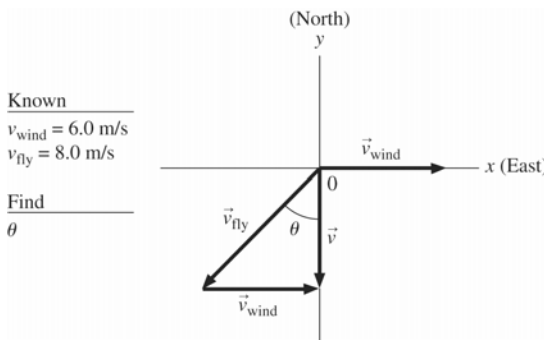
The total time for the displacement is the sum of the individual times, which may be found by dividing each individual distance by the bacterium's constant speed of 20 μm/s. This gives

$$\Delta t_{AB} = D_{AB}/(20\ \mu\text{m}/s) = \sqrt{(50\ \mu\text{m})^2 + (10\ \mu\text{m})^2}/(20\ \mu\text{m}/s) = (51.0\ \mu\text{m})/(20\ \mu\text{m}/s) = 2.55\ \text{s}$$
$$\Delta t_{BC} = D_{BC}/(20\ \mu\text{m}/s) = (10\ \mu\text{m})/(20\ \mu\text{m}/s) = 0.50\ \text{s}$$
$$\Delta t_{CD} = D_{CD}/(20\ \mu\text{m}/s) = \sqrt{(40\ \mu\text{m})^2 + (10\ \mu\text{m})^2}/(20\ \mu\text{m}/s) = (41.0\ \mu\text{m})/(20\ \mu\text{m}/s) = 2.06\ \text{s}$$
$$\Delta t_{DE} = D_{DE}/(20\ \mu\text{m}/s) = \sqrt{(-50\ \mu\text{m})^2 + (-50\ \mu\text{m})^2}/(20\ \mu\text{m}/s) = (70.7\ \mu\text{m})/(20\ \mu\text{m}/s) = 3.54\ \text{s}$$

The total time is therefore $\Delta t_{\text{Tot}} = 2.55\,\text{s} + 0.50\,\text{s} + 2.06\,\text{s} + 3.54\,\text{s} = 8.65\,\text{s}$ and the magnitude of the bacterium's net velocity is

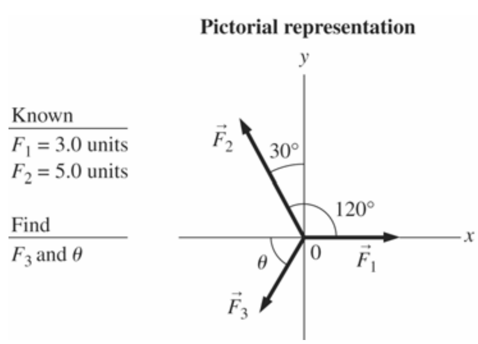
$$v_{\text{net}} = \frac{D_{\text{net}}}{\Delta t_{\text{Tot}}} = \frac{45\ \mu\text{m}}{8.65\ \text{s}} = 5.2\ \mu\text{m}/s$$

3.42. Visualize:



Solve: The resulting velocity is given by $\vec{v} = \vec{v}_{\text{fly}} + \vec{v}_{\text{wind}}$, where $\vec{v}_{\text{wind}} = (6.0\ \text{m/s})\hat{i}$ and $\vec{v}_{\text{fly}} = -v\sin\theta\,\hat{i} - v\cos\theta\,\hat{j}$. Substituting the known values we get $\vec{v} = -(8.0\ \text{m/s})\sin\theta\,\hat{i} - (8.0\ \text{m / s})\cos\theta\,\hat{j} + (6.0\ \text{m/s})\,\hat{i}$. We need to have $v_x = 0$. This means $0 = -(8.0\ \text{m/s})\sin\theta + (6.0\ \text{m/s})$, so $\sin\theta = \frac{6}{8}$ or $\theta = 49°$. Thus the ducks should head 49° west of south.

3.43. Model: We will treat the knot in the rope as a particle in static equilibrium.
Visualize:



Solve: Expressing the vectors in component form, we have $\vec{F}_1 = 3.0\hat{i}$ and $\vec{F}_2 = -5.0\sin(30°)\hat{i} + 5.0\cos(30°)\hat{j}$. Since we must have $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$ for the know to remain stationary, we can write $\vec{F}_3 = -\vec{F}_1 - \vec{F}_2 = -0.50\,\hat{i} - 4.33\hat{j}$. The magnitude of $\vec{F}_3$ is given by $F_3 = \sqrt{(-0.50)^2 + (-4.33)^2} = 4.4\,\text{units}$ The angle between $\vec{F}_3$ and the negative x-axis is $\theta = \tan^{-1}(4.33/0.50) = 83°$ below the negative x-axis.

Assess: The resultant vector has both components negative, and is therefore in quadrant III. Its magnitude and direction are reasonable. Note the minus sign that we have manually inserted with the force $\vec{F}_2$.

3.44. Visualize: $\vec{F}_3$ and $\vec{F}_4$ are along the axes. The vertical components must add to zero. We treat the horizontal and vertical components separately.

Solve: First find the component vectors of $\vec{F}_2$.

$$\vec{F}_1 = (0.0\text{ N})\hat{i} - (5.0\text{ N})\hat{j} \qquad \vec{F}_2 = (6.0\text{ N})\cos 20°\hat{i} + (6.0\text{ N})\sin 20°\hat{j} = (5.6\text{ N})\hat{i} + (2.1\text{ N})\hat{j}$$

There is no $\hat{i}$ component of $\vec{F}_3$.

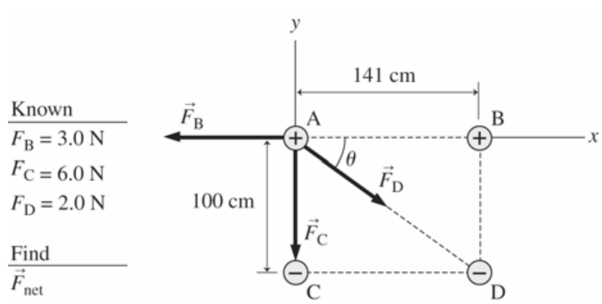
$$\vec{F}_3 + (\vec{F}_1)_y + (\vec{F}_2)_y = (0.0\text{ N})\hat{j} \Rightarrow \vec{F}_3 = (5.0\text{ N})\hat{j} - (2.1\text{ N})\hat{j} = (2.9\text{ N})\hat{j}$$

There is no $\hat{j}$ component of $\vec{F}_4$.

$$\vec{F}_4 + (\vec{F}_2)_x = (4.0\text{ N})\hat{i} \Rightarrow \vec{F}_4 = (4.0\text{ N})\hat{i} - (5.6\text{ N})\hat{i} = (-1.6\text{ N})\hat{i}$$

Assess: The figure could have been drawn differently to give negative values for both components; the magnitudes would be the same, however.

3.44. Visualize:



Solve: Using trigonometry to calculate θ, we get $\theta = \tan^{-1}(100\text{ cm}/141\text{ cm}) = 35.3°$. Expressing the three forces component form gives $\vec{F}_B = -(3.0\text{ N})\hat{i}$, $\vec{F}_C = -(6.0\text{ N})\hat{j}$, and $\vec{F}_D = +(2.0\text{ N})\cos(35.3°)\hat{i} - (2.0\text{ N})\ \sin(35.3°)\hat{j} =$ $(1.63\text{ N})\hat{i} - (1.16\text{ N})\hat{j}$. The total force is $\vec{F}_{net} = \vec{F}_B + \vec{F}_C + \vec{F}_D = -1.37\text{ N}\,\hat{i} - 7.2\text{ N}\,\hat{j}$. The magnitude of $\vec{F}_{net}$ is $F_{net} = \sqrt{(1.37\text{ N})^2 + (7.2\text{ N})^2} = 7.3\text{ N}$.

$$\theta_{net} = \tan^{-1}\frac{|(F_{net})_y|}{|(F_{net})_x|} = \tan^{-1}\left(\frac{7.2\text{ N}}{1.37\text{ N}}\right) = 79°$$

$\vec{F}_{net} = (7.3\text{ N},\ 79°$ below the negative x-axis in quadrant III).