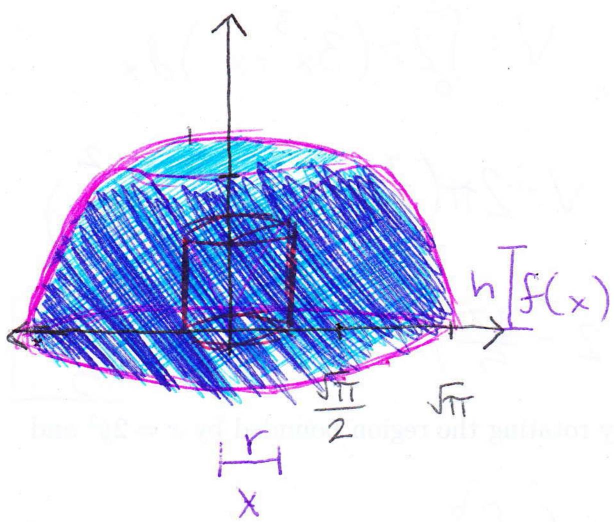
1 Shell Method

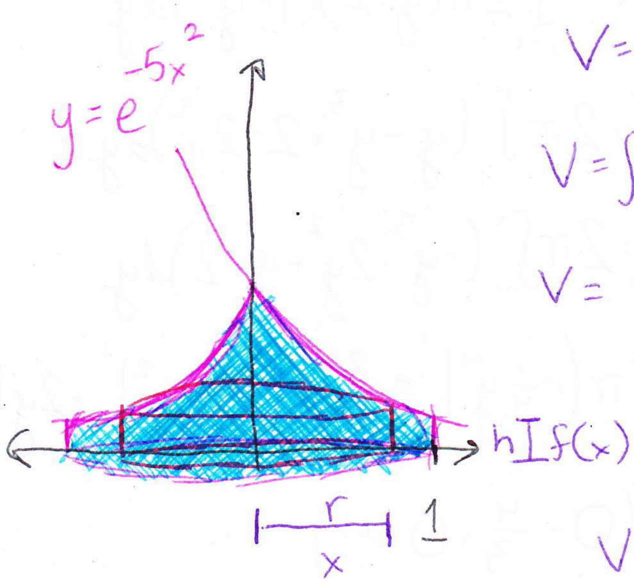
1. Let S be the solid obtained by rotating the region bounded by $y = \sin(x^2)$ and above the x-axis and rotated about the y-axis. Sketch the solid S and a typical cylindrical shell. Find the circumference and height of the shell and set up an integral. You do not need to solve the integral.



$V = \int_a^b 2\pi r h w$

$$\boxed{V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx}$$

2. Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by $y = e^{-5x^2}$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.



$V = \int_a^b 2\pi r \cdot h \cdot w$

$V = \int_0^1 2\pi x e^{-5x^2}\,dx$

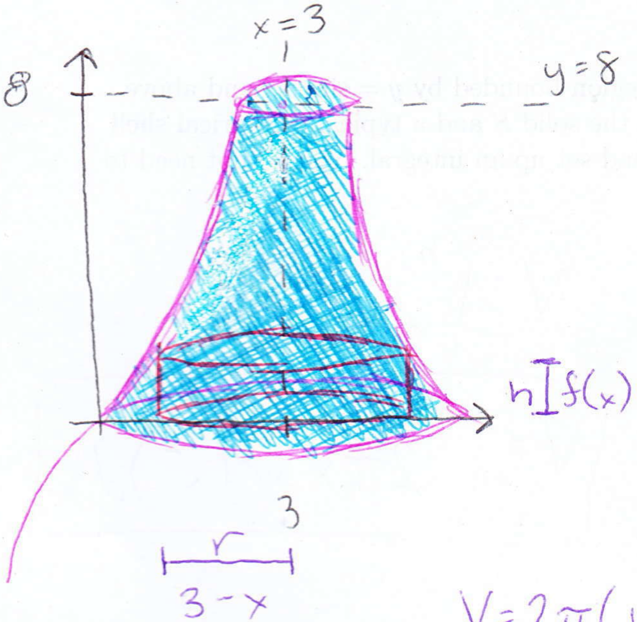
$V = -\frac{1}{10} \cdot 2\pi \int_0^1 -10x e^{-5x^2}\,dx$

$V = -\frac{\pi}{5}\left(e^{-5x^2}\Big|_0^1\right)$

$V = -\frac{\pi}{5}\left(\frac{1}{e^5} - 1\right)$

$$\boxed{V = \frac{\pi}{5}\left(1 - \frac{1}{e^5}\right) \approx 0.624}$$

3. Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by $f(x) = x^3, y = 8$, and $x = 0$ about the line $x = 3$.



$$V = \int_a^b 2\pi r \cdot h \cdot w$$

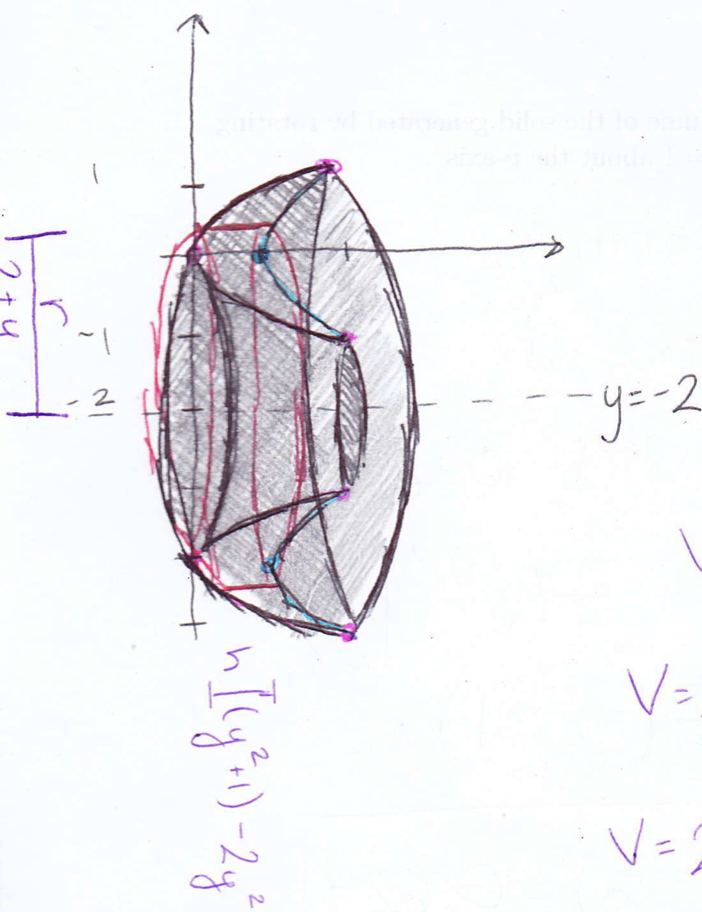
$$V = \int_0^2 2\pi(3-x)x^3\,dx$$

$$V = \int_0^2 2\pi(3x^3 - x^4)\,dx$$

$$V = 2\pi\left(\frac{3}{4}x^4\Big|_0^2 - \frac{1}{5}x^5\Big|_0^2\right)$$

$$V = 2\pi\left(12 - \frac{32}{5}\right) = 2\pi\left(\frac{28}{5}\right) = \boxed{\frac{56\pi}{5}}$$

4. Find the volume of the solid generated by rotating the region bounded by $x = 2y^2$ and $x = y^2 + 1$ about the line $y = -2$.



$$V = \int_a^b 2\pi r \cdot h \cdot w$$

$$V = \int_{-1}^1 2\pi(y+2)(1-y^2)\,dy$$

$$V = 2\pi\int_{-1}^1 (y - y^3 + 2 - 2y^2)\,dy$$

$$V = 2\pi\int_{-1}^1 (-y^3 - 2y^2 + y + 2)\,dy$$

$$V = 2\pi\left(-\frac{1}{4}y^4\Big|_{-1}^1 - \frac{2}{3}y^3\Big|_{-1}^1 + \frac{1}{2}y^2\Big|_{-1}^1 + 2y\Big|_{-1}^1\right)$$

$$V = 2\pi\left(0 - \frac{4}{3} + 0 + 4\right)$$

$$V = 2\pi\left(\frac{8}{3}\right) = \boxed{\frac{16\pi}{3}}$$

2 Work

1. A 360 pound gorilla climbs a tree to a height of 20 feet. Find the work done if the gorilla reaches that height in 10 seconds.

$$W = F \cdot d = 360(20) = \boxed{7200 \text{ ft-lb}}$$

2. How much work is done when a hoist lifts a 200 kg rock to a height of 3 m?

$$W = F \cdot d = (m \cdot a) \cdot d = 200(9.81)(3) = \boxed{5886 \text{ J}}$$

3. A spring has a natural length of 40 cm. If a 60 N force is required to keep the spring compressed 10 cm, how much work is done during this compression? How much work is required to compress the spring to a length of 25 cm?

$10 \text{ cm} = 0.1 \text{ m}$ If $F = 60 = f(x)$,

$25 \text{ cm} = 0.25 \text{ m}$ then $60 = k \cdot x \Rightarrow 60 = k(0.1) \Rightarrow k = 600 \text{ N/m}$

and $f(x) = 600x$.

Hence $W = \int_a^b f(x)\,dx = \int_0^{0.1} 600x\,dx = 300x^2 \Big|_0^{0.1} = \boxed{3 \text{ J}}$

and for 25 cm, $W = \int_0^{0.25} 600x\,dx = 300x^2 \Big|_0^{0.25} = \boxed{18.75 \text{ J}}$

4. A 50 ft long heavy rope weighing 0.5 lb/ft hangs over the edge of a building that is 120 ft high. How much work is done to pull the rope to the top of the building?

The force function $f(x) = 0.5x$ or $f(x) = \frac{1}{2}x$ and the rope needs to be pulled up 50ft.

$$W = \int_a^b f(x)\,dx = \int_0^{50} \frac{1}{2}x\,dx = \frac{1}{4}x^2 \Big|_0^{50} = \boxed{625 \text{ ft-lb}}$$

5. Find the average value of the function $f(x) = e^{\sin t}\cos t$ on the interval $[0, \pi/2]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} e^{\sin t}\cos t\,dt$$

Let $u = \sin t$
$du = \cos t\,dt$

$$f_{ave} = \frac{2}{\pi}\int_0^1 e^u\,du = \frac{2}{\pi}e^{\sin t}\Big|_0^{\pi/2} = \frac{2}{\pi}(e-1)$$

$$\boxed{f_{ave} = \frac{2(e-1)}{\pi}}$$

6. The linear density in a rod that is 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

$$\rho_{ave} = \frac{1}{b-a}\int_a^b \rho(x)\,dx = \frac{1}{8-0}\int_0^8 \frac{12}{\sqrt{x+1}}\,dx$$

$$\rho_{ave} = \frac{1}{8}\int_0^8 12(x+1)^{-1/2}\,dx = \frac{3}{2}\int_0^8 (x+1)^{-1/2}\,dx$$

$$\rho_{ave} = \frac{3}{2}\left(2\sqrt{x+1}\Big|_0^8\right) = 3(3-1) = \boxed{6 \text{ kg/m}}$$