Chapter 6-8 HW - **Prasad Prabhu**

1. Represent the number 263.3 in 32-bit floating point representation
    a. **0 10000111 00000111010011001100110**

① 263.3

263 in binary is

100000111

0.3 × 2 = 0.6 | 0
0.6 × 2 = 1.2 | 1
0.2 × 2 = 0.4 | 0
0.4 × 2 = 0.8 | 0
0.8 × 2 = 1.6 | 1
0.6 × 2 = 1.2

= 100000111.01001

Mantisa = $1.00000111\,01001\ldots \times 2^8$

Exp 8+127= 135 = 10000111   Sign= 0

0 10000111 00000111010011001100110

2. Represent the number -17.625 in 32-bit floating point representation
    a. **1 10000011 00011010000000000000000**

② 17=10001

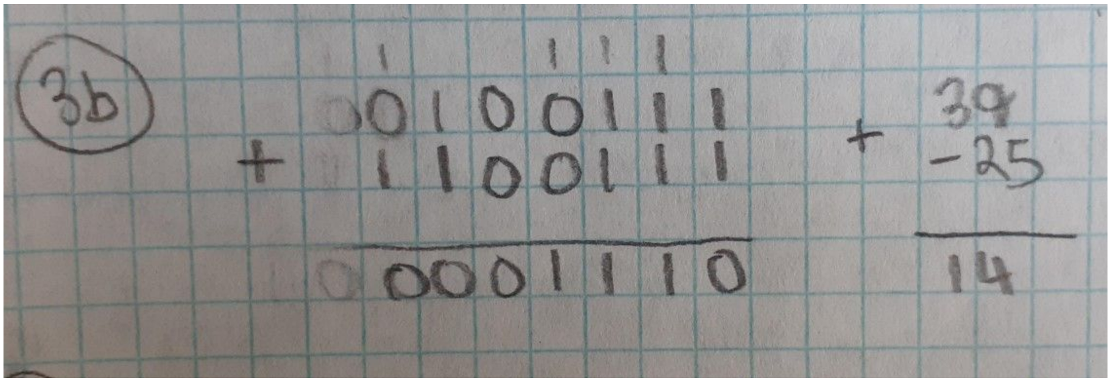
0.625 · 2 = 1.25 | 1
0.25 × 2 = 0.5 | 0
0.5 × 2 = 1 | 1

10001.101   Mantisa: 1.0001101×2^4

Exp: 4+127 = 10000011   Sign=1

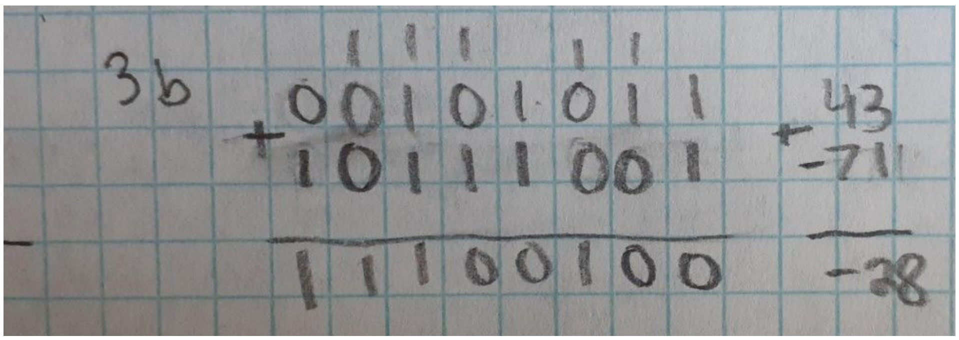
1 10000011 00011010000000000000000

3.
    a. Using the 2's complement method, express the following negative numbers in binary -7, -12
        i. -7: 7 in binary is 0111. The 1's complement becomes 1000. Adding 1 to make it a 2's complement becomes 1001. **1001**.
        ii. -12: 12 in binary is 0 1100. The 1's complement becomes 1 0011. Adding 1 to make it a 2's complement becomes 1 0100. **1 0100.**
    b. Using the 2's complement method, find the value of the following
        i. 39 + (-25). 39= 010 0111. -25= 110 0111. Using 7 bit binary system
            1. **000 1110**



ii. 43 - (+71) . 43=0010 1011. -71= 1011 1001

  1. **1110 0100**



4. Simplify the following Boolean Expression using Boolean laws:
   a. ABC+A’+AB’C
      i. C(AB+AB’)+A’ by Distributive Law of C
      ii. C(A(B+B’))+A’ by Distributive Law of A
      iii. C(A(1))+A’ by Complementary Law B+B’ =1
      iv. (CA)+A’ by Law of Intersection A*1=A
      v. **A’+C** by Law of Common Identities A+(A’B)= A+B
   b. A’B’C’+A’B’C+A’C’
      i. A’(B’C’+B’C+C’) by Distributive Law A’
      ii. A’(B’(C’+C)+C’) by Distributive Law B’
      iii. A’(B’(1)+C’) by Complementary Law C+C’ =1
      iv. **A’(B’+C’)** by Law of Intersection B’*1=B’
   c. (AB’(C+BD)+A’B’)C
      i. C(AB’(C+BD)+A’B’) by Commutative Law
      ii. C((AB’C)+(AB’BD)+A’B’) by Distributive Law
      iii. C((AB’C)+(A(0)D)+A’B’) by Complementary Law B’ * B = 0
      iv. C((AB’C)+(0)+A’B’) by Law of Intersection twice A*0=0
      v. C((AB’C)+A’B’) by Law of Union A+0=A
      vi. C(B’((AC)+A’)) by Distributive Law B’
      vii. C(B’(A’+C)) by Law of Common Identities A+(A’B)= A+B
      viii. B’C(A’+C) rewritten
      ix. **B’C** by Absorption Law
5. Given the Boolean function

a. Obtain the truth table of the function . AB’C+A’B’C+D’AB+DA’B+DAB

A	B	C	D	AB’C	A’B’C	D’AB	DA’B	DAB	AB’C+A’B’C+D’AB+DA’B+DAB
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	**1**	0	0	0	**1**
0	0	1	1	0	**1**	0	0	0	**1**
0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	**1**	0	**1**
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	**1**	0	**1**
1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0
1	0	1	0	**1**	0	0	0	0	**1**
1	0	1	1	**1**	0	0	0	0	**1**
1	1	0	0	0	0	**1**	0	0	**1**
1	1	0	1	0	0	0	0	**1**	**1**
1	1	1	0	0	0	**1**	0	0	**1**
1	1	1	1	0	0	0	0	**1**	**1**

b. Simplify the function to a minimum number of laterals using Boolean algebra.

i. AB’C+A’B’C+D’AB+DA’B+DAB
ii. B’C(A+A’)+B(D’A+DA’+DA) by Distributive Law
iii. B’C(1)+B(D’A+DA’+DA) by Complementary Law A+A’=1
iv. B’C + B(D’A+DA’+DA) by Law of Intersection A*1=A
v. B’C + B(A(D’+D)+DA’) by Commutative Law and Distributive Law
vi. B’C + B(A(1)+DA’) by Complementary Law A+A’=1
vii. B’C + B(A+DA’) by Law of Intersection A*1=A
viii. B’C + B(A+D) by Law of Common Identities
ix. B’C+BA+BD

c. Obtain the truth table of the simplified function

A	B	C	D	B'C	BA	BD	B'C+BA+BD
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	**1**	0	0	**1**
0	0	1	1	**1**	0	0	**1**
0	1	0	0	0	0	0	0
0	1	0	1	0	0	**1**	**1**
0	1	1	0	0	0	0	0
0	1	1	1	0	0	**1**	**1**
1	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0
1	0	1	0	**1**	0	0	**1**
1	0	1	1	**1**	0	0	**1**
1	1	0	0	0	**1**	0	**1**
1	1	0	1	0	**1**	**1**	**1**
1	1	1	0	0	**1**	0	**1**
1	1	1	1	0	**1**	**1**	**1**

6. For the following function prove F+F'=1 and F*F'=0
    a. F=DA+BC
        i. F+F'=1. F+F' = DA+BC + (DA+BC)'. Applying DeMorgan's Laws twice gives us DA+BC+ (DA)'(BC)' = DA+BC+(D'+A')(B'+C').

A	B	C	D	DA	BC	DA+BC	D'+A'	B'+C'	(D'+A')(B'+C')	DA+BC+(D'+A')(B'+C')
0	0	0	0	0	0	0	1	1	1	1
0	0	0	1	0	0	0	1	1	1	1
0	0	1	0	0	0	0	1	1	1	1
0	0	1	1	0	0	0	1	1	1	1

0	1	0	0	0	0	0	1	1	1	1
0	1	0	1	0	0	0	1	1	1	1
0	1	1	0	0	1	1	1	0	0	1
0	1	1	1	0	1	1	1	0	0	1
1	0	0	0	0	0	0	1	1	1	1
1	0	0	1	1	0	1	0	1	0	1
1	0	1	0	0	0	0	1	1	1	1
1	0	1	1	1	0	1	0	1	0	1
1	1	0	0	0	0	0	1	1	1	1
1	1	0	1	1	0	1	0	1	0	1
1	1	1	0	0	1	1	1	0	0	1
1	1	1	1	1	1	1	0	0	0	1

This truth table proves that F+F'=1 because DA+BC+(D'+A')(B'+C') is 1 for all combinations.

ii. F*F'=0. F*F' = (DA+BC) * (DA+BC)'. Applying DeMorgan's Laws twice gives us DA+BC * ((DA)'(BC)') = (DA+BC) * ((D'+A')(B'+C')).

A	B	C	D	DA	BC	DA+BC	D'+A'	B'+C'	(D'+A')(B'+C')	DA+BC * (D'+A')(B'+C')
0	0	0	0	0	0	0	1	1	1	0
0	0	0	1	0	0	0	1	1	1	0
0	0	1	0	0	0	0	1	1	1	0
0	0	1	1	0	0	0	1	1	1	0
0	1	0	0	0	0	0	1	1	1	0
0	1	0	1	0	0	0	1	1	1	0
0	1	1	0	0	1	1	1	0	0	0
0	1	1	1	0	1	1	1	0	0	0
1	0	0	0	0	0	0	1	1	1	0

1	0	0	1	1	0	1	0	1	0	0
1	0	1	0	0	0	0	1	1	1	0
1	0	1	1	1	0	1	0	1	0	0
1	1	0	0	0	0	0	1	1	1	0
1	1	0	1	1	0	1	0	1	0	0
1	1	1	0	0	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0	0

This truth table proves that F*F’=0 because DA+BC * (D’+A’)(B’+C’) is 0 for all combinations.

b. F=A+BC
   i. F+F’=1. F+F’ = (A+BC)+(A+BC)’. Applying DeMorgan’s Laws twice gives us (A+BC)+(A’ * (BC)’) = (A+BC)+(A’ * (B’+C’)).

A	B	C	BC	B’+C’	A+BC	A’ * (B’+C’)	(A+BC)+(A’ * (B’+C’))
0	0	0	0	1	0	1	1
0	0	1	0	1	0	1	1
0	1	0	0	1	0	1	1
0	1	1	1	0	1	0	1
1	0	0	0	1	1	0	1
1	0	1	0	1	1	0	1
1	1	0	0	1	1	0	1
1	1	1	1	0	1	0	1

This truth table proves that F+F’=1 because (A+BC)+(A’ * (B’+C’)) is 1 for all combinations

   ii. F*F’=0. F*F’ = (A+BC) * (A+BC)’. Applying DeMorgan’s Laws twice gives us (A+BC) * (A’ * (BC)’) = (A+BC) * (A’ * (B’+C’)).

A	B	C	BC	B’+C’	A+BC	A’ * (B’+C’)	(A+BC) * (A’ * (B’+C’))
0	0	0	0	1	0	1	0
0	0	1	0	1	0	1	0

0	1	0	0	1	0	1	0
0	1	1	1	0	1	0	0
1	0	0	0	1	1	0	0
1	0	1	0	1	1	0	0
1	1	0	0	1	1	0	0
1	1	1	1	0	1	0	0

This truth table proves that F*F'=0 because (A+BC) * (A' * (B'+C')) is 0 for all combinations.

7. Demonstrate by means of truth tables the validity of the following identities
    a. De-Morgan's theorem for three variables: (A+B+C)' = A'B'C' and (ABC)'=A'+B'+C'
        i. (A+B+C)' = A'B'C'.

A	B	C	A+B+C	(A+B+C)'	A'B'	A'B'C'
0	0	0	0	**1**	1	**1**
0	0	1	1	**0**	1	**0**
0	1	0	1	**0**	0	**0**
0	1	1	1	**0**	0	**0**
1	0	0	1	**0**	0	**0**
1	0	1	1	**0**	0	**0**
1	1	0	1	**0**	0	**0**
1	1	1	1	**0**	0	**0**

Therefore, the truth table proves that (A+B+C)' = A'B'C'

        ii. (ABC)'=A'+B'+C'.

A	B	C	ABC	(ABC)'	A'+B'	A'+B'+C'
0	0	0	0	**1**	1	**1**
0	0	1	0	**1**	1	**1**
0	1	0	0	**1**	1	**1**

0	1	1	0	**1**	1	**1**
1	0	0	0	**1**	1	**1**
1	0	1	0	**1**	1	**1**
1	1	0	0	**1**	0	**1**
1	1	1	1	**0**	0	**0**

Therefore, the truth table proves that (ABC)’=A’+B’+C’

b. The distributive law: A+BC = (A+B) (A+C).

A	B	C	BC	A+BC	A+B	A+C	(A+B)*(A+C)
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	1	**0**
0	1	0	0	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	1	1	1	**1**	1	1	**1**

Therefore, the truth table proves that A+BC = (A+B) (A+C)

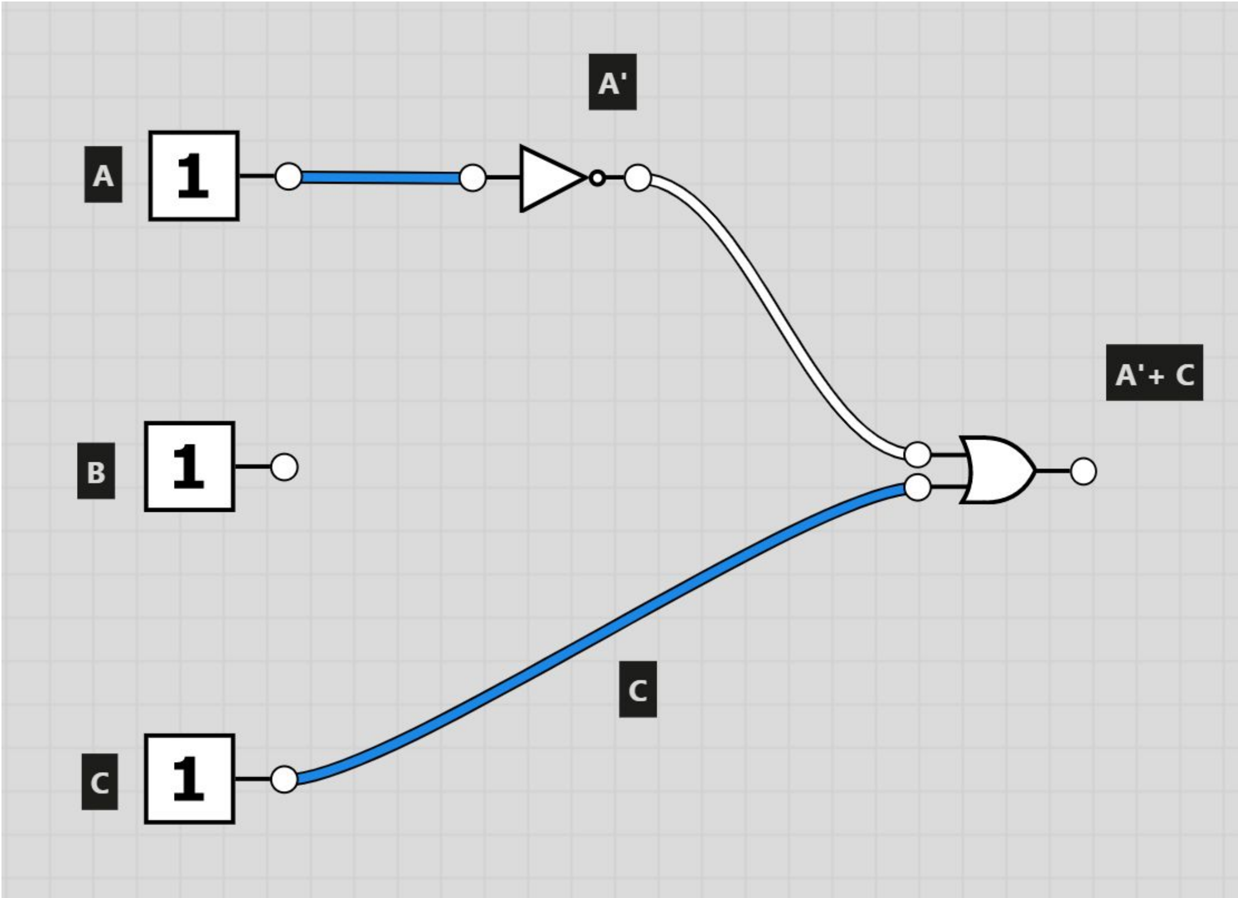
8. Reduce the following Boolean expressions to the indicated number of literals
   a. R’T’ + RST + RT’ to 3 literals.
      i. R’T’ + RST + RT’
      ii. R’T’ + R(ST+T’) by Distributive Law
      iii. R’T’ + R(T’+S) by Law of Common Identities A’+(AB)=A’+B
      iv. R’T’ + RT’ + RS by Distributive Law
      v. T’(R’+R) + RS by Distributive Law
      vi. T’(1) + RS by Complementary Law A’+A=1
      vii. **T’ + RS** by Law of Intersection A*1=A
   b. (R’S’ + T)’ + T + RS + UT to 3 literals
      i. (R’S’ + T)’ + T + RS + UT
      ii. ((R’S’)’ * T’) + T + RS + UT by DeMorgan’s Law
      iii. ((R+S)* T’) + T + RS + UT by DeMorgan’s Law

iv. ((R+S)* T') + T + RS + UT by Associative Law
v. ((R+S)* T') + RS +T+ UT by Commutative Law A+B= B+A
vi. ((R+S)* T') + RS +T by Absorption Law A*(A+B)=A
vii. T+(T'*(R+S)) + RS by Commutative and AssociativeLaw
viii. T+(R+S) + RS by Law of Common Identities A+(A'B) = A+B
ix. **T+R+S** by Associative and Absorption Law

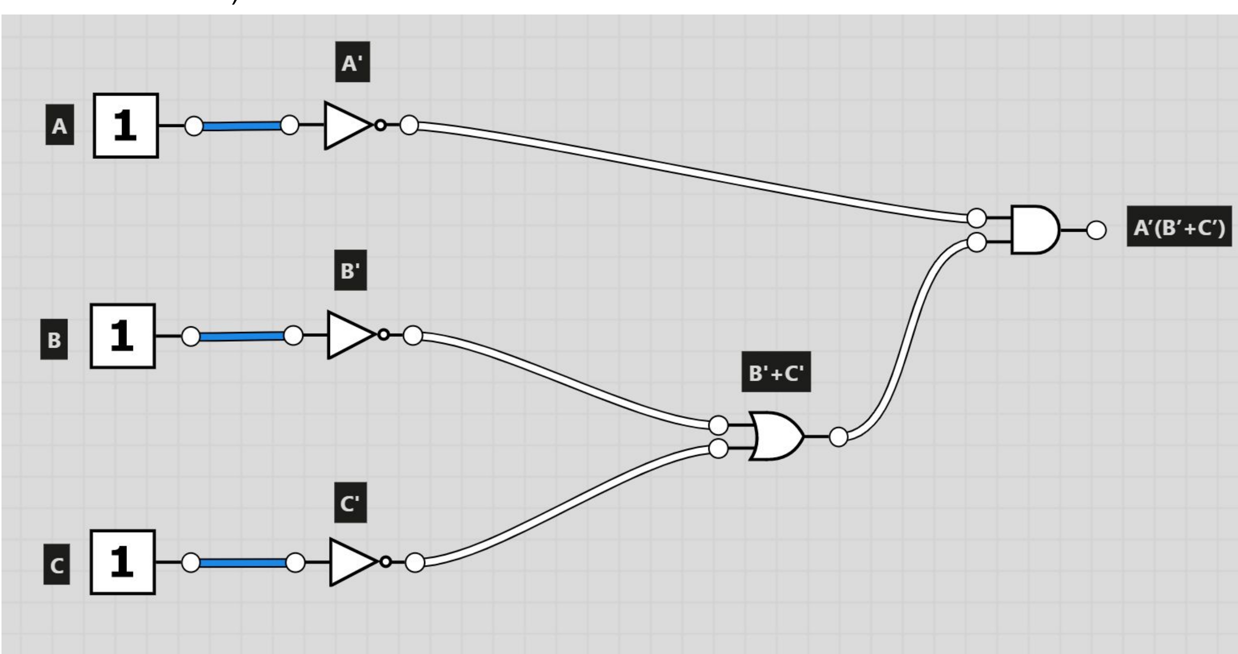
c. R'S(U' + T'U) + S(R + R'TU) to 1 literal
   i. R'S(U' + UT') + S(R + R'TU) by Associative Law A*(B*C) = A*B*C
   ii. R'S(U' + T') + S(R + TU) by Law of Common Identities twice A+(A'B) = A+B
   iii. R'SU' + R'ST' + SR + STU by Distributive Law Twice
   iv. S(R'U' + R'T'+R+TU) by Distributive Law
   v. S(R+R'(U' + T')+TU) by Associative and Distributive Law
   vi. S(R+(U' + T')+TU) by Law of Common Identities A+(A'B) = A+B
   vii. S(R+U' + T'+TU) by Associative Law A*(B*C) = A*B*C
   viii. S(R+U' +T'+U) by Law of Common Identities A+(A'B) = A+B
   ix. S(R +T'+U+U') by Commutative Law A+B= B+A
   x. S(R +T'+1) by Complementary Law A'+A=1
   xi. S(1) by Law of Union Twice A+1=1
   xii. **S** by Law of Intersection

d. (R' + T) (R' + T') (R + S + T'U) to four literals
   i. R'+(T*T') (R + S + T'U) by Distribution Law
   ii. R'+(0) (R + S + T'U) by Complementary Law
   iii. R' (R + S + T'U) by Law of Union
   iv. R'R + R'S + R'T'U by Distributive Law
   v. 0 + R'S + R'T'U by Complementary Law A*A'=0
   vi. R'S + R'T'U by Law of Union A+0=A
   vii. (R'S+R')(R'S+T')(R'S+U) by Distributive Law
   viii. (R')(R'S+T')(R'S+U) by Absorption Law
   ix. (R')(T'+R')*(T'+S)(R'S+U) by Distributive Law
   x. R'(T'+S)(R'S+U) by Absorption Law
   xi. R'(T'+S)(U+R')(U+S) by Distributive Law
   xii. R'(R'+U)(T'+S)(U+S) by Associative Law
   xiii. R'(T'+S)(U+S) by Absorption Law
   xiv. R'(S+T')(S+U) by Commutative Law
   xv. **R'(S+(T'U))** by Distributive Law

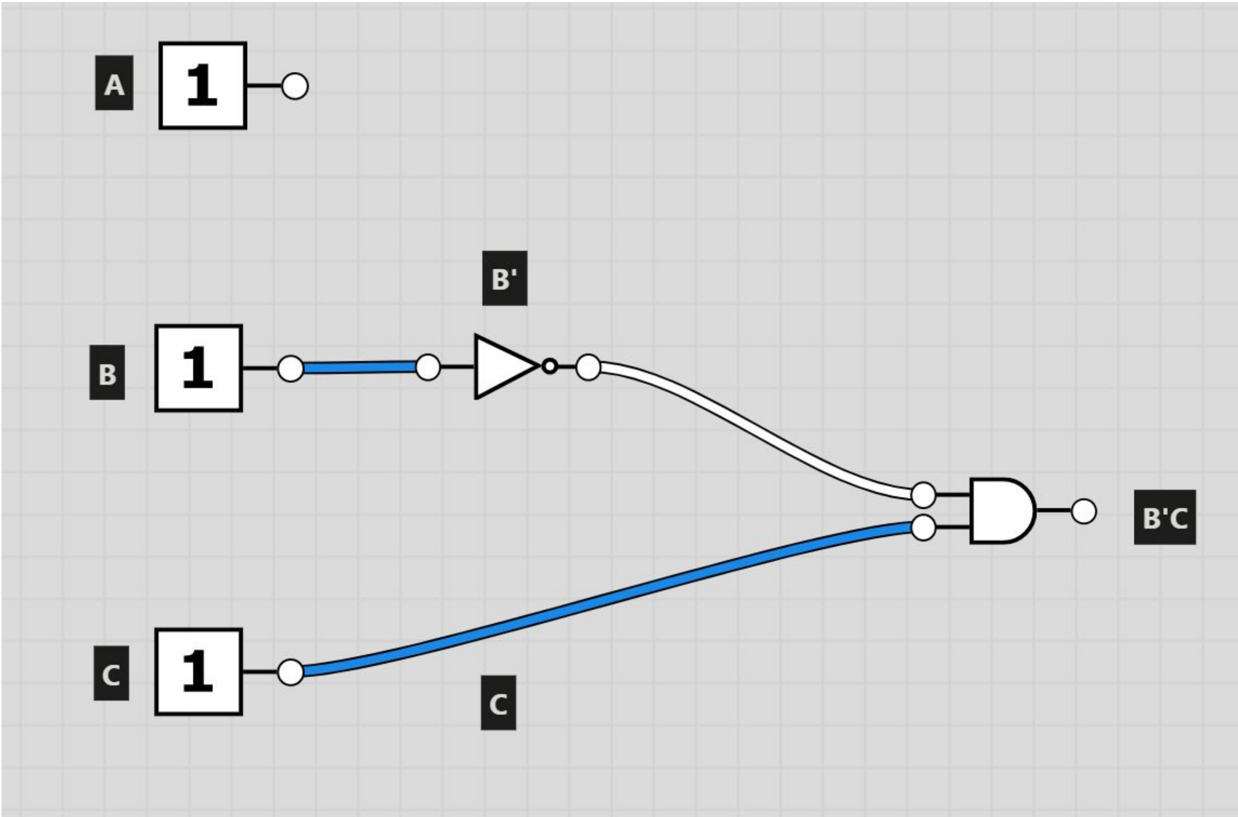
9. Using only minimum gates, draw a logic gate diagram for the following expressions:
   a. ABC+A'+AB'C (This statement is simplified to <u>A'+C</u> as proven in question #4)



b. A’B’C’+A’B’C+A’C’ (This statement is simplified to A’(B’+C’) as proven in question #4)



c. (AB’(C+BD)+A’B’)C   (This statement is simplified to B’C as proven in question #4)



10. Simply the following functions using 3 variable maps:

The following tables and maps used for question #10 and #11 are

x	y	z	f
0	0	0	A
0	0	1	B
0	1	0	C
0	1	1	D
1	0	0	E
1	0	1	F

1	1	0	G
1	1	1	H

The corresponding map is

	y'z'	y'z	yz	yz'
x'	A	B	D	C
x	E	F	H	G

a. F(x,y,z) = Σ(0,2,6,7)
    i. 0=000=A. 2=010=C. 6 = 110= G. 7 = 111=H

	y'z'	y'z	yz	yz'
x'	1	0	0	1
x	0	0	1	1

    ii. X'y'z'+x'yz' + xyz+xyz' = X'z'(y'+y) + xy(z+z')
    iii. f= x'z'+xy

b. F(x,y,z) = Σ(0,2,3,4,6)
    i. 0=000=A 2=010=C 3=011=D 4=100=E 6=110=G

	y'z'	y'z	yz	yz'
x'	1	0	1	1
x	1	0	0	1

    ii. AECG= z'. DC= x'yz+x'yz' = x'y(z+z')=x'y
    iii. f= z' + x'y

c. F(x,y,z) = Σ(0,1,2,3,7)
    i. 0=000=A 1=001=B 2=010=C 3=011=D 7 = 111=H

	y'z'	y'z	yz	yz'
x'	1	1	1	1
x	0	0	1	0

    ii. ABDC=x' DH= x'yz+xyz = yz(x+x')=yz
    iii. f= x'+yz

d. F(x,y,z) = Σ(3,5,6,7)
    i. 3=011=D 5=101=F 6=110=G 7 = 111=H

	y’z’	y’z	yz	yz’
x’	0	0	1	0
x	0	1	1	1

ii. DH= x’yz+zyz= yz. FH=xy’z+xyz= xz GH=xyz+xyz’=xy

iii. f= yz + xz + xy

11. Simplify the following Boolean expressions, using three-variable maps

a. F(x,y,z) = xy + x’y’z’ + x’yz’

	y’z’	y’z	yz	yz’
x’	1	0	0	1
x	0	0	1	1

i. AC= x’y’z’+x’yz’= x’z’ HG= xyz+xyz’= xy

ii. f= x’z’ + xy

b. F(x,y,z) = x’y’ + yz + x’yz’

	y’z’	y’z	yz	yz’
x’	1	1	1	1
x	0	0	1	0

i. ABDC= x’ DH= x’yz+xyz = yz

ii. f= x’ + yz

c. F(x,y,z) = x’y + yz’ + y’z’

	y’z’	y’z	yz	yz’
x’	1	0	1	1
x	1	0	0	1

i. AECG= z’ DC= x’yz+x’yz’ = x’y

ii. f= z’ + x’y

d. F(x,y,z) = xyz + x’y’z + xy’z’

	y’z’	y’z	yz	yz’
x’	0	1	0	0
x	1	0	1	0

i. **f= xyz + x’y’z + xy’z’**. The function is already simplified

12. Prove by induction the recursive formula for the Fibonacci numbers:

F1 = 1
F2 = F1
F3 = F1+F2
F4 = F2+F3
F5 = F3+F4

a. Define the two Recursive Formula Rules, with the basic rule and the recursive rule.
    i. Basic Rule Formula: F(1) = 1
    ii. Recursive Rule: The recursive definition is defined by the equation F(n)= F(n-2) + F(n-1).
    iii. The closed form equation for F(n):

$F_n = (x - y)/ \sqrt{5}$. Where $x = ((1 + \sqrt{5}) / 2)^n$ *and* $y = ((1 - \sqrt{5}) / 2)^n$

b. Then, using the below information, validate the formula for Fn:
    i. Base Case: P(1) is true since = f((1)+2) – 1 = f(3)-1 = 2-1 = 1, which is the correct value for the sum of the first n terms.
    ii. Inductive Step:

Inductive Step: Assume the statement P(k) is true for some arbitrary positive integer k, k ∈ N; this means that f(k+2)-1 = f1+f2+f3+...fn

f1+f2+...fk+f(k+1) = f(k+2) - 1 + f(k+1) = f(k+1) + f(k+2) - 1 = f(k+3) - 1

Therefore the claim holds and proof by induction is complete