

## CSC 28 Fall 2020

### Relations (Total 150 points)

**1.1)** From the given ordered pairs (3, 5); (8, 4); (6, 6); (9, 11); (6, 3); (3, 0); (1, 2) find the following relations. Also, find the **domain and range** of each relation.

- (a) Is greater than (2 points)
- (b) Is less than (2 points)
- (c) Is less than, by at least two (2 points)
- (d) Is equal to (2 points)

**1.2)**

(a) Determine the domain and range of the relation R defined by

$$R = \{x + 2, x - 3\}; x \in \{6, 7, 8, 9\} \quad (\text{domain and range: 4 points}) \text{ and}$$

(b) draw the arrow diagram for relation R. (4 points)

(c) What is the range of the relation R if the co-domain includes additional values 7, 12? (1 points)

**2)** Draw the **arrow diagram** and the **matrix representation** for the following two relations:

(a) Define the set  $A = \{r, o, t, p, c\}$  and  $B = \{\text{discrete, math, position, computer, science}\}$ . Define the relation  $R \subseteq A \times B$  such that (letter, word) is in the relation if that letter doesn't occur somewhere in the word. Be careful - recheck your answer again to see if you by mistake reversed the definition for any pair (if you mistakenly used 'does occur' for 'doesn't occur'). Even if you know the answer psychological distraction can play a role. It can happen to me too!

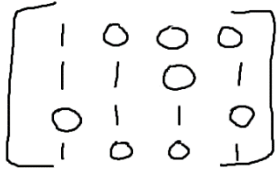
(Arrow Diagram 5 points + Matrix representation 5 points)

(b) The domain for the relation M is the set  $\{2, 3, 4, 12, 16, 27, 32, 48\}$ . For  $x, y$  in the domain,  $xMy$  if there is a positive integer  $n$  such that  $x \text{ power } n = y$ . Be careful - include only (x, y) pairs (not (x, n) or n, y) pairs). Even if you know the answer, psychological distraction can play a role. It can happen to too!

(Arrow Diagram 5 points + Matrix representation 5 points)

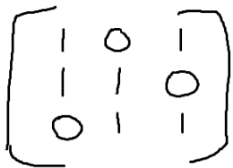
**3)** Give the arrow diagram for each of the two matrices, then express the relation as a set of ordered pairs (The rows and columns are numbered 1 through 3 or 4. In other words, the domain and co-domain contain the row and column numbers.)

(a)



Arrow Diagram (8 points) + Relation as ordered pairs (8 points) + correct domain and co-domain (1 point)

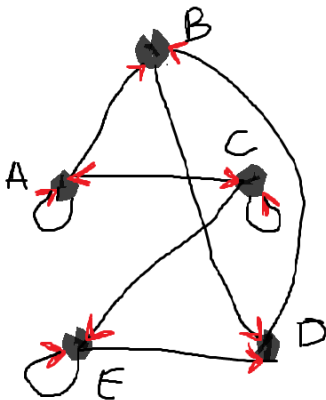
(b)



Arrow Diagram (6 points) + Relation as ordered pairs (6 points) + correct domain and co-domain (1 point)

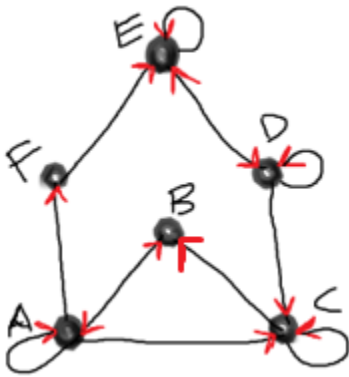
**4)** Give the matrix representation for the relation depicted in each arrow diagram. Then express the relation as a set of ordered pairs. Assume the domain and co-domain for each graph contain the letters in the corresponding graph.

(a)



(matrix representation 10 points) + (relation as ordered pairs 10 points)

(b)



(matrix representation 10 points) + (relation as ordered pairs 10 points)

5) What are the types of Relations, Explain each with **one example** on your own.

(3 \* 2 points)

6) True/ False (To get full marks explain the answer in one or two sentences) (5 \* 1 points)

(a) If R is the set of all females in a family, then the relation “is sister of” is reflexive over R.

(b) If R is the set of all females in a family, then the relation “is sister of” is not symmetric over R.

(c) If R is the set of mothers and S is the set of children in a family then a relation F on  $R \times S$  is a symmetric relation.

(d) “Is the same height as” is a reflexive relation.

(e) “Is descendent of” is a transitive relation.

7) Let  $A = \{5, 6, 7, 8\}$ , Let  $R: A \rightarrow A$  and  $S: A \rightarrow A$ . Let  $R = \{(5, 6) (6, 8) (7, 6) (8, 7)\}$  and  $S = \{(5, 5) (6, 8) (7, 5) (8, 8)\}$ . Find the answers for the following and draw the arrow diagram for each result.

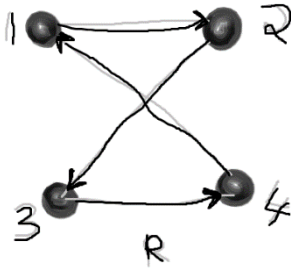
(a)  $R \circ S$  (5 points)

(b)  $S \circ R$  (5 points)

8) Below is the arrow diagram (graph) for relation R with the domain  $\{1, 2, 3, 4\}$ .

Define a new relation A to be  $R \circ R$

(a) Express relation A as a set of related pairs. (5 points)



(b) Draw the arrow diagram for  $R \circ A$ . (5 points)

9) What are the three types of closures in relation and **explain with an example.**

(3 \* 2 points)

10) Given  $R = \{w, x, y, z\}$  and relation  $A = \{(w, w) (x, y) (w, z)\}$ .

(a) Is relation A reflexive closure? if not what are the sets should be added to make it a proper reflexive closure? (2 points)

Explain in detail and then draw arrow diagram for the result.

(b) Is relation A symmetric closure? if not what are the sets should be added to make it a proper symmetric closure? Explain in detail and then draw arrow diagram for the result. (2 points)

(c) Is relation A transitive closure? if not what are the sets should be added to make it a proper transitive closure? Explain in detail and then draw arrow diagram for the result. (2 points)