

2. $A = \{10,11,12,13\}$

$B = \{12,13,14\}$

List all the partitions(subsets) of $A \cap B$

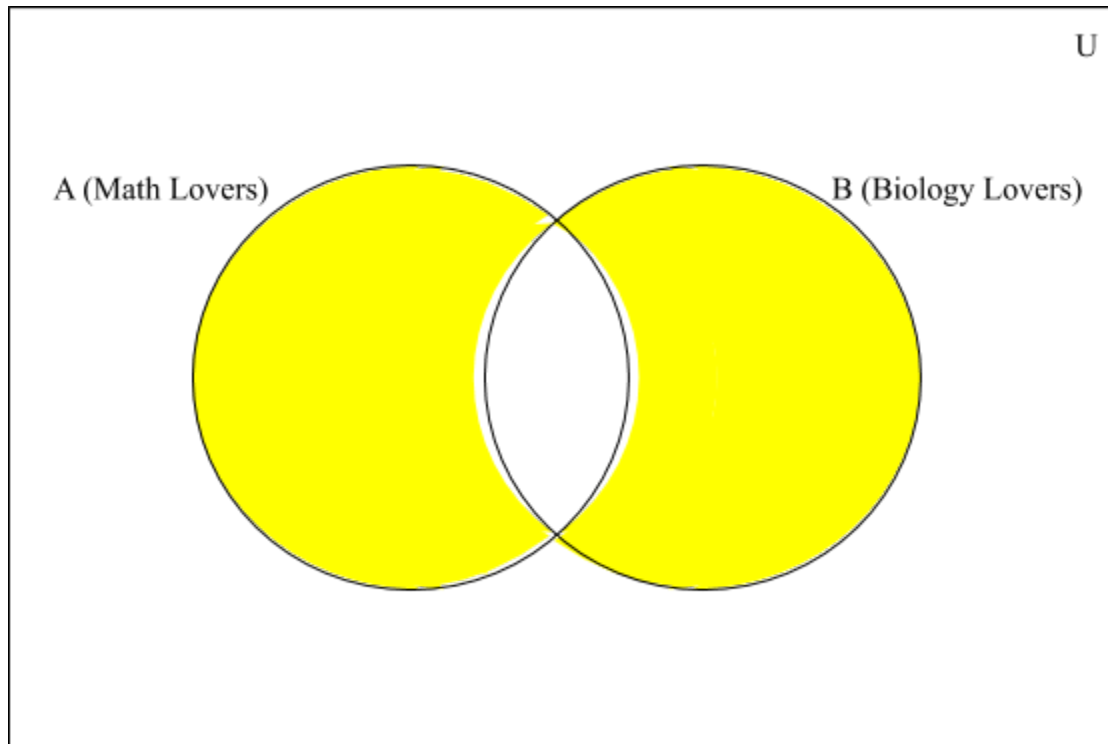
- $\{12,13\}$
- $\{12\}$
- $\{13\}$
- $\{\emptyset\}$ (This is an empty set)

8. $A = \{7,8,9\}$

$B = \{8,9,10,11\}$

What is $A \oplus B$?

$A \oplus B$ is the symmetric difference of sets A and B. It refers to elements that belong to A and B but not both. For example, if there was a group of people who loved math and a group of people who loved biology, as well as people who loved both, the symmetric difference would be the people who love math and the people who love biology, but not the people who love both. You could also think of it as the relative difference between the union of sets A and B, and the intersection of sets A and B $\{(A \cup B) \setminus (A \cap B)\}$. This can be thought of as the combination of all the students who only love math, who only love biology, and who love both, minus the students that love both math and biology. Another way to think of it is to see it as the union between the relative difference of sets A and B, and the relative difference of sets B and A $\{(A \setminus B) \cup (B \setminus A)\}$. This can be thought of as the combination of students who love math but not biology with the students who love biology but not math. Below is a visual representation:



Also prove $A \cap B \subseteq A$, $A \subseteq A \cup B$, $A \cap B \subseteq B$, $B \subseteq A \cup B$.

Assuming:

$$A = \{7, 8, 9\}$$

$$B = \{8, 9, 10, 11\}$$

$$A \cap B \subseteq A$$

$A \cap B \subseteq A$ would be true because the intersection of sets A and B are 8 and 9 ($A \cap B = \{8, 9\}$), and $A \cap B$ is a subset of set A ($A = \{7, 8, 9\}$). What this means is that the elements held in the intersection of set A and B are contained in set A.

$$A \subseteq A \cup B$$

$A \subseteq A \cup B$ would be true because the union of sets A and B are greater than just set A itself. $A \cup B = \{7, 8, 9, 10, 11\}$, whereas $A = \{7, 8, 9\}$. A can be considered a subset of the union of sets A and B as all of the elements contained in sets A are contained in the union of sets A and B.

$$A \cap B \subseteq B$$

$A \cap B \subseteq B$ would be true because (like in one of the previous examples) the intersection of sets A and B are 8 and 9 ($A \cap B = \{8,9\}$), and $A \cap B$ is a subset of set B ($B = \{8,9,10,11\}$).

What this means is that the elements held in the intersection of set A and B are contained in set B.

$$B \subseteq A \cup B$$

$B \subseteq A \cup B$ would be true because the union of sets A and B are greater than just set B itself.

$A \cup B = \{7,8,9,10,11\}$, whereas $B = \{8,9,10,11\}$. B can be considered a subset of the union of sets A and B as all of the elements contained in set B are contained in the union of sets A and B.

What can you deduce from this?

If we combine the properties above, we get that:

$$A \cap B \subseteq A \subseteq A \cup B$$

$$A \cap B \subseteq B \subseteq A \cup B$$

This gives us the self-evident principle that the intersection of sets A and B are not only subsets of set A and set B, but subsets of the union of set A and B as well.