

### MATH 30: LECTURE 55 REVIEW PROBLEMS

(1) Find the equation of the tangent line to the curve  $y = \frac{\sqrt{x}}{x^2 + 1}$  at the point  $(1, \frac{1}{2})$ .

(2) Let  $f(x) = \cos(x^4)$ .

(a) Define two functions  $g$  and  $h$  such that  $f(x) = h(g(x))$ .

(b) Calculate the derivatives of  $g$  and  $h$ .

(c) Calculate the derivative of  $f$ .

(3) Evaluate the limit, if it exists. State which limit laws you use.

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x} - \frac{1}{x^3 + x}}{x}.$$

(4) Does the limit  $\lim_{x \rightarrow 0} [\sin(x) \sin(\frac{1}{x})]$  exist? Explain why or why not.

(5) Fix  $c > 0$  and consider the curve  $y = e^{-cx^2}$ . At what point(s) does the tangent line to the curve have the steepest slope?

(6) Find the radius of a disc  $D$  with the property that:

the sum of (i) *the area of  $D$*  and (ii) *the reciprocal of the area of  $D$*  is as small as possible.

(7) A rollercoaster car moves from left to right along the curve  $y = \frac{1}{x^2 + 1}$  (in miles).

As it reaches the point  $(x, y) = (2, \frac{1}{5})$ , the  $y$ -coordinate is increasing at a rate of 30 miles per hour. How fast is the  $x$ -coordinate changing at that instant?

(8) Let  $f$  be continuous on  $[a, b]$ . Recall that the average of  $f$  on  $[a, x]$  is defined to be

$$f_{\text{avg}}(x) = \frac{1}{x - a} \int_a^x f(t) dt.$$

Find a formula for the rate of change of  $f_{\text{avg}}(x)$  with respect to  $x$ .