2. Let H and K be subspaces of a vector space V. The **intersection** of H and K, denoted by $H \cap K$, is the set of \mathbf{v} in V such that \mathbf{v} belongs to both H and K. Show that $H \cap K$ is a subspace of V.

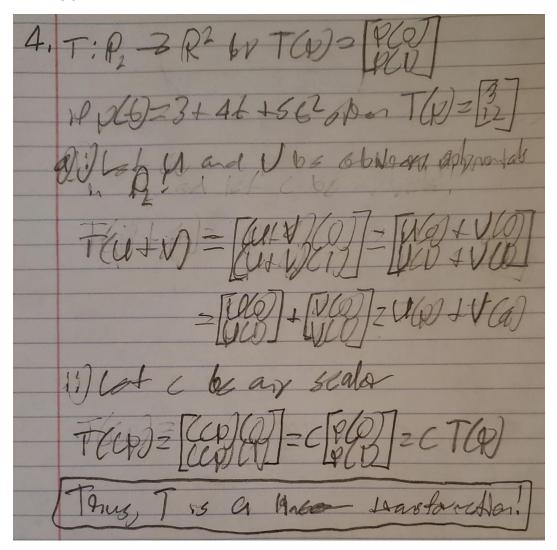
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*My work above is messy, but I was able to prove that $H \cap K$ is a subspace of V! We know that H and K both have the zero vector of V if they are subspaces of V. As such, the zero vector would

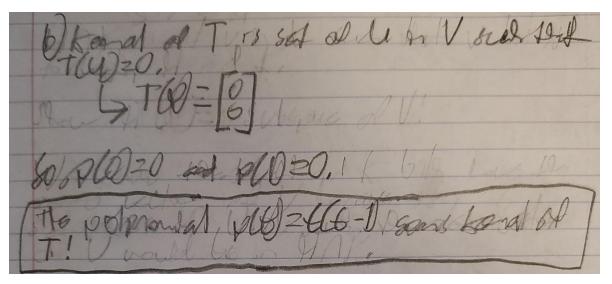
be in $H \cap K$. Next, we had to show that u + v is in $H \cap K$. We know that u and v are in H, so u + v would be in H since H is a subspace. We also know that u and v are in H, so u + v would be in H since H is a subspace. This gives us that u + v is in $H \cap K$! Last, we had to show that for any scalar v and for u in $H \cap K$, that v is in v in

4. Define
$$T: \mathbb{P}_2 \to \mathbb{R}^2$$
 by $T(\mathbf{p}) = \begin{bmatrix} \mathbf{p}(0) \\ \mathbf{p}(1) \end{bmatrix}$.
For example if $\mathbf{p}(t) = 3 + 4t + 5t^2$ then $T(\mathbf{p}) = \begin{bmatrix} 3 \\ 12 \end{bmatrix}$.

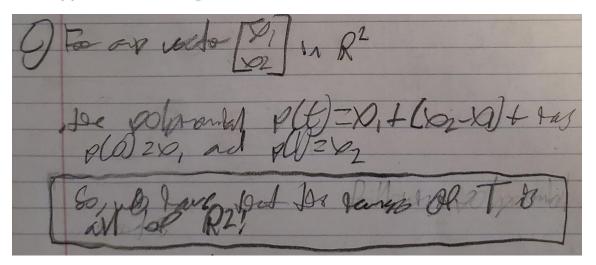
(a) Show that T is a linear transformation.



(b) Find a polynomial in the kernel of T.

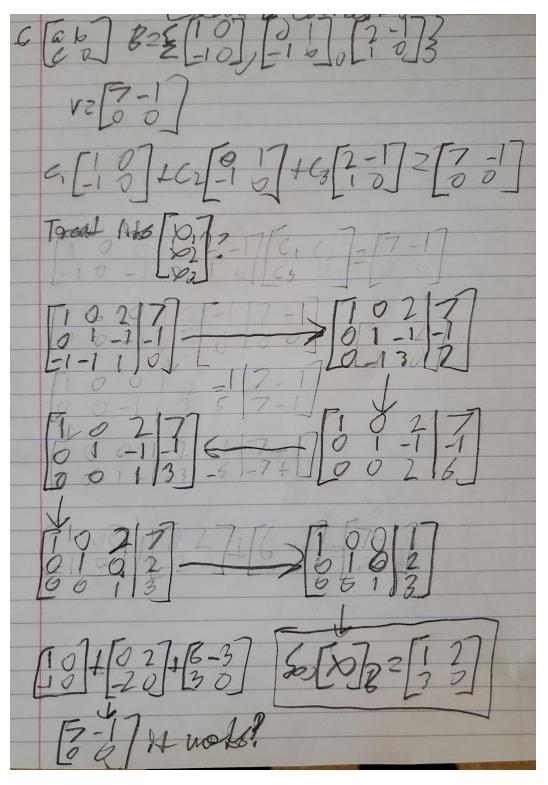


(c) What is the range of T?

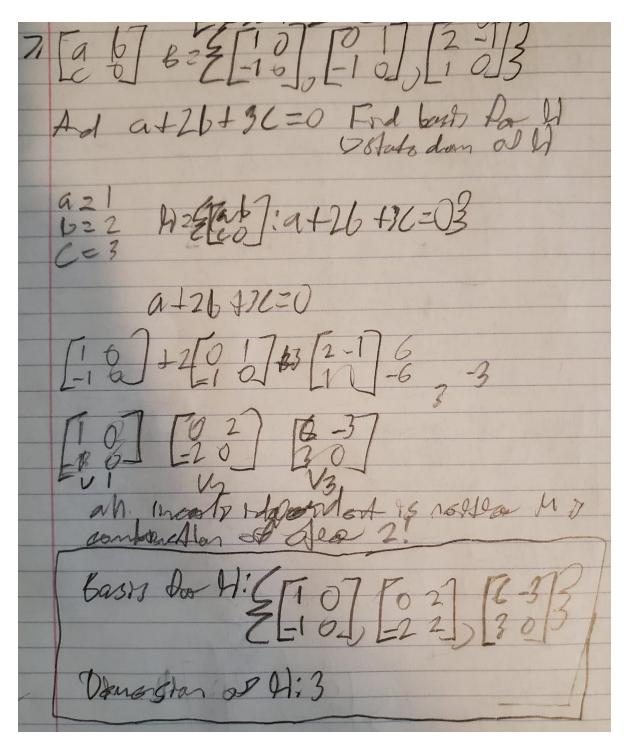


6. Let H be a subspace of $M_{2\times 2}$ whose vectors are of the form $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix}$. Then, $\mathcal{B} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$ is a basis for H.

Find the coordinate vector of $\mathbf{v} = \begin{bmatrix} 7 & -1 \\ 0 & 0 \end{bmatrix}$ according to the basis, \mathcal{B} .

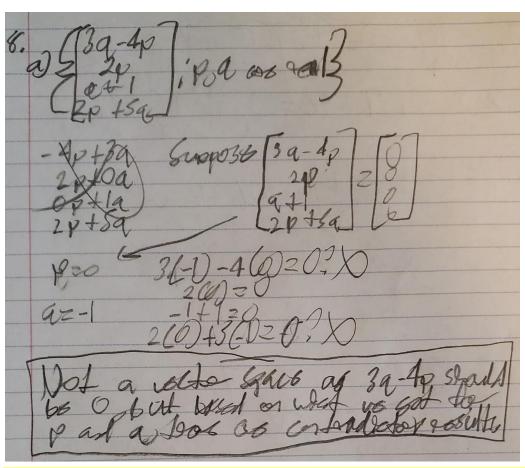


7. Let H be the subspace of $M_{2\times 2}$ in Question 6 with the added restriction that a+2b+3c=0. Find a basis for H, and state the dimension of H.



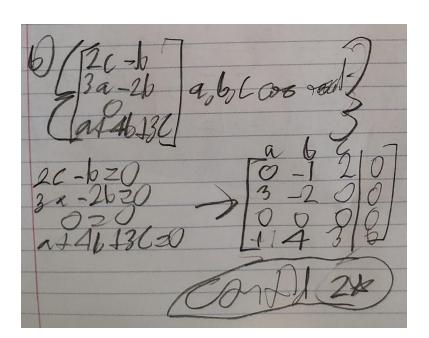
8. Let *H* be the set of all vectors of the forms given below and either find a basis for the vector space, or give an example to show that it is not a vector space.

(a)
$$\left\{ \begin{bmatrix} 3q - 4p \\ 2p \\ q + 1 \\ 2p + 5q \end{bmatrix} : p, q \text{ are real} \right\}$$



*Not a vector space as 3q - 4p should be 0, but based on what we got for p and q, there are contradictory results.

(b)
$$\left\{ \begin{bmatrix} 2c - b \\ 3a - 2b \\ 0 \\ a + 4b + 3c \end{bmatrix} a, b, c \text{ are real} \right\}$$



Cantil In Lawrence
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*Not a vector space as the only solution is trivial and a zero vector!