

Problem 1	/25
Problem 2	/25
Problem 3	/25
Problem 4	/25
<hr/>	
Total	/75

Best 3 of 4 problems are used

READ the instructions!

Writing on any portion of this exam indicates your acceptance of the rules.

Instructions and Rules:

1. You must answer **3 of the 4 problems**. If you have time to do all 4, your three highest scores will be used in your total score.
2. You may use the given equation sheet but all other materials are forbidden
3. You must show your work to receive full credit.
4. Be sure to put you name at the top.
5. Write all answers on this test. If you need additional space, you can continue working on the back, but be sure it's clear that you are doing so. Write "see reverse for additional work"
6. Remember to include units.
7. This test contains multi-part problems where several of the later parts depend on answers to the previous parts. If you are unable to solve one part of the problem, **do not give up on the rest of it.** Either clearly state a guess for the unknown answer you need and proceed to solve the rest of the problem using that guess OR solve the subsequent parts using symbols only like m, k, v, etc. and state the correct units for the final answer.
8. **Use at least 3 significant figures for each answer**
9. You may use a scientific calculator or graphing calculator but you may not have any unauthorized material on them (e.g. notes, equations). I reserve the right to inspect you calculator.
10. **Any phone I see in use without prior clearance from me will mean failure of the exam.**
11. Turn off or silence your phone during the exam!
12. If wandering eyes become a problem, I may ask you to change seats. I may move the offender or the victim, so please don't panic if I ask you to move seats
13. If there is something unclear on the exam, please come up and ask me about it. If it is a reasonable concern, I will announce it to the class.

Section 1: Problem Solving (75 Points)

Do at least 3 of the 4 problems

Problem 1 (25 points)

A 1500kg car approaches an intersection traveling at $v = +20\text{m/s}$. The light at the intersection turns yellow when the car reaches point A (see the diagram).

- a) If the driver brakes at point A so that a constant deceleration of 5.25 m/s^2 is applied, how far from A will the car stop? [8 points]

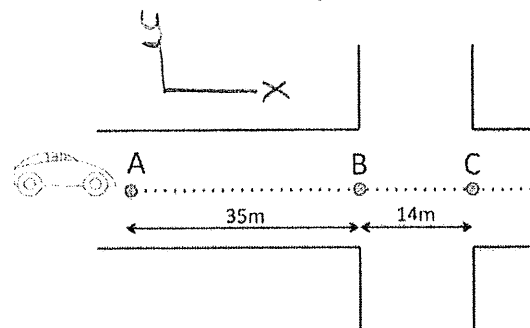
$$a = -5.25\text{ m/s}^2 \quad x_0 = 0$$

$$v_0 = +20\text{ m/s} \quad x = ?$$

$$v = 0 \text{ after } a \text{ is applied}$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$0^2 = 20^2 + 2(-5.25\text{ m/s}^2)(x) \rightarrow x = \frac{400}{2(5.25)}\text{ m} = \boxed{38.1\text{ m}}$$



- b) How long in seconds does it take the car to stop? [4 points]

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - 0} \quad \text{if } t_i = 0$$

$$t_f = \frac{0 - 20\text{ m/s}}{5.25\text{ m/s}^2} = 3.81\text{ s}$$

- c) The yellow light is known to last 2.0s. If the driver decides to increase speed to clear the intersection, find the distance the car will move (from point A) before the light turns red. Assume the car accelerates at a constant rate from 20m/s to 30m/s in 5 seconds. [8 points]

$$a_{\text{ave}} = \frac{\Delta v}{\Delta t} = \frac{(30 - 20)\text{ m/s}}{5\text{ s}} = +2\text{ m/s}^2$$

$$x(t) = \frac{1}{2}at^2 + v_0t + x_0$$

$$x_0 = 0$$

$$v_0 = 20\text{ m/s}$$

$$t = 2\text{ s}$$

$$x(t) = \frac{1}{2}(2\text{ m/s}^2)(2\text{ s})^2 + (20\text{ m/s})(2\text{ s}) + 0$$

$$x(t) = 44\text{ m}$$

- d) On the basis of your results from a) and c): (choose 1 and explain) [5 points]

○ The driver should brake since

○ The driver should try to clear the intersection since

○ Neither choice is advisable since

in either case, the car stops in the intersection!

$x = 35\text{ m} \rightarrow x = 44\text{ m}$ is in the intersection

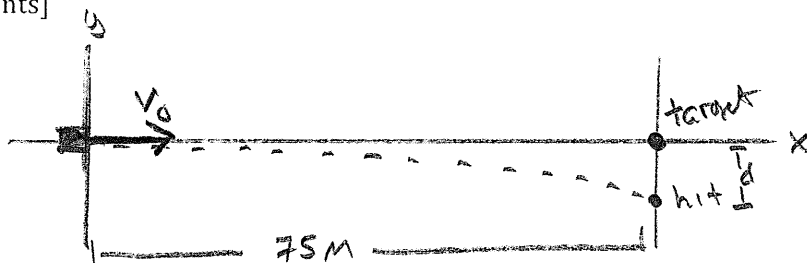
Section 1: Problem Solving (75 Points)

Do at least 3 of the 4 problems

Problem 2 (25 points)

A gun is aimed horizontally with its barrel pointing directly at a black dot painted on a wall 75m away. The initial speed of the bullet, v_0 , as it leaves the barrel is unknown.

- a) Sketch the situation. Label the x and y-axes, the distances involved, the initial velocity of the bullet and the target. Sketch the trajectory of the bullet and label approximately where the bullet hits the wall. [7 points]



- b) If the bullet hits the wall 20cm below the target, what was its initial speed, v_0 ? [7 points]

constant acc. of $-g\hat{y}$

$$y(t) = -\frac{1}{2}gt^2 + v_{0y}t + y_0$$

$$y_0 = 0$$

$$v_{0y} = 0$$

at t_f , $y = 20\text{cm} = .2\text{m}$

so $.2\text{m} = -\frac{1}{2}gt_f^2$ or $t_f = \sqrt{\frac{(-2)(.2)}{9.8}} = \boxed{.202\text{s}}$

$$x(t) = v_0t + x_0$$

$$x_0 = 0$$

$$x(t_f) = 75\text{m}$$

$$75\text{m} = v_0(.202\text{s})$$

or $\boxed{v_0 = 371.2\text{m/s}}$

- c) At what angle (relative to the horizontal) should the gun be aimed so that the bullet hits the target? Sketch the trajectory. [7 points]

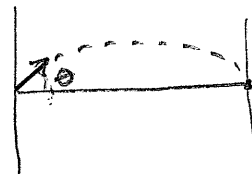
to hit the target, its horizontal range must be 75m

$$R = \frac{v_0^2}{g} \sin 2\theta$$

$$\sin 2\theta = \frac{(75\text{m})(9.8\text{m/s}^2)}{(371.2\text{m/s})^2} = 0.005334$$

$$\theta = \frac{1}{2} \sin^{-1}(0.005334)$$

$$\theta = \frac{1}{2}(.3056) = \boxed{0.153^\circ} \text{ or } 0.00267 \text{ radians}$$



- d) In which case (firing horizontal or firing at an angle) does the bullet reach the wall in less time and why? [4 points]

In both cases, $v_0 = 371.2\text{m/s}$

but for (B) $v_{0x} = 371.2\text{m/s}$ and for (C) $v_{0x} = (371.2\text{m/s})(\cos .153^\circ)$
 $= 371.199\text{m/s}$

so distance is covered faster in (b) than in (c)

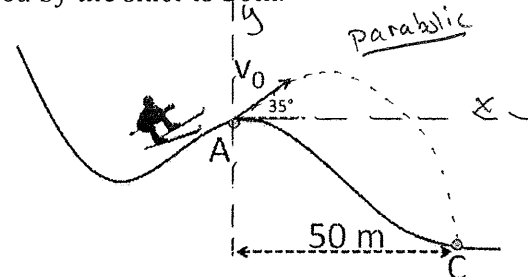
Section 1: Problem Solving (75 Points)

Do at least 3 of the 4 problems

Problem 3 (25 points)

In a ski jump competition, a skier takes off from point A of the diagram with a speed of 20 m/s at an angle of 35° above the horizontal and lands at point C. The total horizontal distance traveled by the skier is 50m.

- a) Setup an appropriate coordinate system, draw and label the axes on the diagram and sketch the path of the skier. [2 points]



- b) Using the information given, what are the equations describing the position and velocity of the skier as a function of time: $x(t)$, $y(t)$, $v_x(t)$, $v_y(t)$? [6 points]

$$\begin{aligned} v_0 &= 20 \text{ m/s} & v_{0x} &= 20 \text{ m/s} \cos 35^\circ = 16.4 \text{ m/s} \\ x_0 &= 0 & v_{0y} &= 20 \text{ m/s} \sin 35^\circ = 11.5 \text{ m/s} \\ y_0 &= 0 \\ a_x &= 0 \\ a_y &= -g \end{aligned}$$

$$\begin{aligned} x(t) &= (16.4 \text{ m/s})t \\ v_x(t) &= (16.4 \text{ m/s}) \\ y(t) &= -\frac{1}{2}(9.8 \text{ m/s}^2)t^2 + (11.5 \text{ m/s})t \\ v_y(t) &= -9.8 \text{ m/s}^2 t + 11.5 \text{ m/s} \end{aligned}$$

- c) What is the vertical distance from the takeoff point (A) to the landing point (C)? [6 points]

$$v_x = 16.4 \text{ m/s} = \frac{\Delta x}{\Delta t}$$

$$\Delta x = 50 \text{ m} = 16.4 \text{ m/s} \Delta t$$

$$\text{or } \Delta t = \frac{50 \text{ m}}{16.4 \text{ m/s}} = \boxed{3.05 \text{ s}}$$

$$\begin{aligned} y(3.05 \text{ s}) &= (-4.9 \text{ m/s}^2)(3.05 \text{ s})^2 + (11.5 \text{ m/s})(3.05 \text{ s}) \\ &= -10.5 \text{ m} \end{aligned}$$

$$\text{so } \Delta y = \underbrace{-10.5 \text{ m}}_{(\text{final})} - \underbrace{0}_{(\text{initial})} = \boxed{-10.5 \text{ m}}$$

- d) How high above point A does the skier rise in its trajectory? [6 points]

$$\text{at } y_{\text{max}}, v_y = 0$$

$$t_{\text{max}} = \frac{11.5 \text{ m/s}}{9.8 \text{ m/s}^2} = 1.17 \text{ s}$$

$$y(1.17 \text{ s}) = -\frac{1}{2}(9.8 \text{ m/s}^2)(1.17 \text{ s})^2 + (11.5 \text{ m/s})(1.17 \text{ s})$$

$$y(1.17 \text{ s}) = \boxed{6.75 \text{ m} = y_{\text{max}}}$$

- e) What is the magnitude of the skier's velocity at point C? [5 points]

$$\begin{aligned} v_y(3.05 \text{ s}) &= (-9.8 \text{ m/s}^2)(3.05 \text{ s}) + 11.5 \text{ m/s} \\ &= 18.39 \text{ m/s} \end{aligned}$$

$$v_x(3.05 \text{ s}) = 16.5 \text{ m/s}$$

$$\begin{aligned} \text{so } |v| &= \sqrt{(18.39 \text{ m/s})^2 + (16.4 \text{ m/s})^2} \\ |v| &= \boxed{24.64 \text{ m/s}} \end{aligned}$$

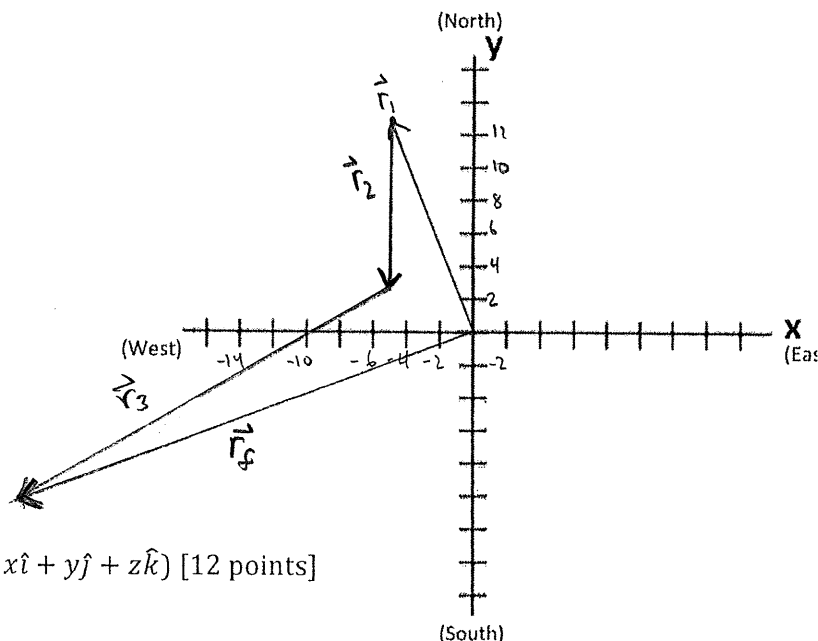
Section 1: Problem Solving (75 Points)

Do at least 3 of the 4 problems

Problem 4 (25 points)

A group of students decide to go sailing for the weekend to relax after their exams. They depart from San Francisco and sail 14.0km in a direction 20° West of North (\vec{r}_1). They then travel south for 10.0km (\vec{r}_2) before sailing 25.0km in a direction 60° west of south (\vec{r}_3). At this point, they encounter a storm and their mast is broken which leaves them stationary and stranded.

- a) Sketch their path, labeling each displacement vector (\vec{r}_1 , \vec{r}_2 , \vec{r}_3). [5 points]



- b) Write each vector in component form (i.e. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$) [12 points]

$$\vec{r}_1 = 14 \text{ km } 20^\circ \text{ West of N } (\theta_1 = 110^\circ)$$

$$\vec{r}_2 = 10 \text{ km } \theta_2 = 270^\circ \text{ (South)}$$

$$\vec{r}_3 = 25 \text{ km } 60^\circ \text{ West of S } (\theta_3 = 210^\circ)$$

$$\vec{r}_1 = -4.79 \text{ km } \hat{i} + 13.2 \text{ km } \hat{j}$$

$$\vec{r}_2 = 0 \text{ km } \hat{i} - 10.0 \text{ km } \hat{j}$$

$$\vec{r}_3 = -21.7 \text{ km } \hat{i} - 12.5 \text{ km } \hat{j}$$

- c) A rescue ship is sent to their location to help. How far and in which direction must the boat travel to reach the students? Give your direction as an angle from the +x direction (east). [6 points]

$$\begin{aligned} \vec{r}_f &= \vec{r}_1 + \vec{r}_2 + \vec{r}_3 \\ &= -26.49 \text{ km } \hat{i} - 9.3 \text{ km } \hat{j} \end{aligned}$$

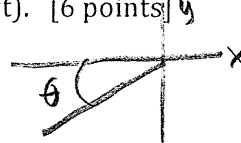
$$\begin{aligned} |\vec{r}_f| &= \sqrt{(26.49)^2 + (9.3)^2} \\ &= \boxed{28.1 \text{ km}} \end{aligned}$$

$$\tan \theta = \frac{9.3}{26.49}$$

$$\theta = 19.3^\circ$$

from +x axis

$$\theta_f = 19.3^\circ + 180^\circ = \boxed{199.3^\circ}$$



- d) Draw the final displacement vector on your sketch and label the angle. [2 points]