STAT 50 HW #15 Sections 5.4 and 5.2

Section 5.4 #'s 1, 3, 5, 9, 11

1.

To study the effect of curing temperature on shear strength of a certain rubber compound, 80 specimens were cured at 150°C and 95 were cured at 130°C. The specimens cured at 150°C had an average shear strength of 620 psi, with a standard deviation of 20 psi. Those cured at 130°C had an average shear strength of 750 psi, with a standard deviation of 30 psi. Find a 95% confidence interval for the difference between the mean shear strengths of specimens cured at the two temperatures.

Section 5.4

1. 180 cured at 150°C mean str = 620 psi, $\sigma = 20$ psi
95 cured at 130°C mean str = 750 psi, $\sigma = 30$ psi
Find a 95% C.I. for the difference between the mean shear strengths.

$X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$

then, $X - Y \sim N(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2)$

$\bar{X} - \bar{Y} \pm z_{\alpha/2} \sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}$ $1 - 0.95 = \frac{0.05}{2} = 0.025 \to 1.96 \leftarrow z_{0.025}$

$750 - 620 \pm 1.96 \sqrt{\frac{30^2}{95} + \frac{20^2}{80}}$

$\sqrt{\frac{900}{95} + \frac{400}{80}}$ $\boxed{(122.54, 137.46)}$

$130 \pm 1.96 \cdot 3.8 \to 130 \pm 7.45$

3.

The article "Effects of Diets with Whole Plant-Origin Proteins Added with Different Ratios of Taurine:Methionine on the Growth, Macrophage Activity and Antioxidant Capacity of Rainbow Trout (Oncorhynchus mykiss) Fingerlings" (O. Hernandez, L. Hernandez, et al., Veterinary and Animal Science, 2017:4-9) reports that a sample of 210 juvenile rainbow trout fed a diet fortified with equal amounts of the amino acids taurine and methionine for a period of 70 days had a mean weight gain of 313 percent with a standard deviation of 25, while 210 fish fed with a control diet had a mean weight gain of 233 percent with a

standard deviation of 19. Units are percent. Find a 99% confidence interval for the difference in weight gain on the two diets.

3. 210 → treat $\bar{x}$ gain: 313% σ_x: 25%

210 → control $\bar{y}$ gain: 233% σ_y: 19%

Find 99% C.I. for difference

$1 - 0.99 = \frac{0.01}{2} = 0.005 \rightarrow Z_{0.005} \rightarrow 2.57$

$313 - 233 \pm 2.57 \sqrt{\frac{25^2}{210} + \frac{19^2}{210}}$

$2.57 \sqrt{\frac{625}{210} + \frac{361}{210}}$

$80 \pm 5.56 \rightarrow \boxed{(74.43, 85.57)}$

5.

The article "Automatic Filtering of Outliers in RR Intervals Before Analysis of Heart Rate Variability in Holter Recordings: a Comparison with Carefully Edited Data" (M. Karlsson, et al., Biomedical Engineering Online, 2012) reports measurements of the total power, on the log scale, of the heart rate variability, in the frequency range 0.003 to 0.4 Hz, for a group of 40 patients aged 25–49 years and for a group of 43 patients aged 50–75 years. The mean for the patients aged 25–49 years was 3.64 with a standard deviation of 0.23, and the mean for the patients aged 50–75 years was 3.40 with a standard deviation of 0.28. Find a 95% confidence interval for the difference in mean power between the two age groups.

5\. 40 25-49 yrs $\bar{x} = 3.64$ $\sigma_x = 0.23$

43 50-75 yrs $\bar{y} = 3.40$ $\sigma_y = 0.28$

Find 95% C.I. for difference

$$3.64 - 3.40 \pm 1.96\sqrt{\frac{0.23^2}{40} + \frac{0.28^2}{43}}$$

$$0.24 \pm 1.96(0.0561) \rightarrow \boxed{(0.1301, 0.3499)}$$

9.

In a study to compare two different corrosion inhibitors, specimens of stainless steel were immersed for four hours in a solution containing sulfuric acid and a corrosion inhibitor. Forty-seven specimens in the presence of inhibitor A had a mean weight loss of 242 mg and a standard deviation of 20 mg, and 42 specimens in the presence of inhibitor B had a mean weight loss of 220 mg and a standard deviation of 31 mg. Find a 95% confidence interval for the difference in mean weight loss between the two inhibitors.

9\. 47 A $\bar{x} = 242$ mg $\sigma_x = 20$ mg

42 B $\bar{y} = 220$ mg $\sigma_y = 31$ mg

Find 95% C.I. for difference

$$242 - 220 \pm 1.96\sqrt{\frac{20^2}{47} + \frac{31^2}{42}}$$

$$22 \pm 10.98 \rightarrow \boxed{(11.02, 32.98)}$$

11.

In a study of the effect of cooling rate on the hardness of welded joints, 50 welds cooled at a rate of 10°C/s had an average Rockwell (B) hardness of 91.1 and a standard deviation of 6.23, and 40 welds cooled at a rate of 30°C/s had an average hardness of 90.7 and a standard deviation of 4.34.

a. **Find a 95% confidence interval for the difference in hardness between welds cooled at the different rates.**

11. 80 10°C/s $\bar{X} = 91.1$ $\sigma_X = 6.23$
40 30°C/s $\bar{Y} = 90.7$ $\sigma_Y = 4.34$
a) Find 95% C.I. of difference.

$$91.1 - 90.7 \pm 1.96\sqrt{\frac{6.23^2}{80} + \frac{4.34^2}{40}}$$

$$0.4 \pm 1.96(1.1162)$$

$$0.4 \pm 2.188 \rightarrow \boxed{(-1.789, 2.589)}$$

b. **Someone says that the cooling rate has no effect on the hardness. Do these data contradict this claim? Explain.**

No. Since 0 is in the confidence interval, it would indicate that there is no difference in the means. The data would support this claim as there does not appear to be an effect in this case.

Section 5.2 #'s 1, 5, 7, 13, 15

1.

In a simple random sample of 70 automobiles registered in a certain state, 28 of them were found to have emission levels that exceed a state standard.

a. **What proportion of the automobiles in the sample had emission levels that exceed the standard?**

1. SRS of 70
28 exceed
a) Proportion of automobiles in the sample had emission levels that exceed the standard.

$$28/70 = \boxed{0.4}$$

b. **Find a 95% confidence interval for the proportion of automobiles in the state whose emission levels exceed the standard.**

b) Find 95% C.I. for proportion of [illegible] vehicles

$1-0.95=\frac{0.05}{2}=0.025 \rightarrow z_{0.025} \rightarrow 1.96$

$\tilde{n}=70+4=74 \quad \tilde{p}=\frac{28+2}{74}=0.405$

$x=28$

$\tilde{p} \pm z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} \quad \boxed{(0.294, 0.517)}$

$0.405 \pm 1.96\sqrt{\frac{0.405(0.595)}{74}}$

c. **Find a 98% confidence interval for the proportion of automobiles whose emission levels exceed the standard.**

c) Find 98% C.I.

$1-0.98=\frac{0.02}{2}=0.01 \rightarrow z_{0.01} \rightarrow 0.99 \rightarrow 2.33$

$0.405 \pm 2.33\sqrt{\frac{0.405(0.599)}{74}}$

$\boxed{(0.272, 0.538)}$

d. **How many automobiles must be sampled to specify the proportion that exceed the standard to within ±0.10 with 95% confidence?**

d) How many automobiles must be sampled to specify the proportion that exceed the standard to within ± 0.10 with 95% confidence

$1.96\sqrt{\frac{0.405(0.594)}{n+4}}=0.10 \quad 1.96\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}}$

$n=\left(\frac{1.96\sqrt{0.405(0.594)}}{0.10}\right)^2-4 \rightarrow 88.6 \rightarrow \boxed{89}$

e. **How many automobiles must be sampled to specify the proportion that exceed the standard to within ±0.10 with 98% confidence?**

e) " to within ± 0.10 with 98% confidence

$$2.33\sqrt{\frac{0.405(0.594)}{n+4}} = 0.10 \qquad \boxed{127}$$

$$n = \left(\frac{2.33\sqrt{0.405(0.594)}}{0.10}\right)^2 - 4 \to 126.8$$

f. **Someone claims that less than half of the automobiles in the state exceed the standard. With what level of confidence can this statement be made?**

f) Someone claims that less than half of the automobiles in the state exceed the standard. With what level of confidence can this statement be made?

claim $p < 0.5$

$$0.5 = \tilde{p} - z_\alpha\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} = 0.405 - z_\alpha\sqrt{\frac{(0.405)(0.594)}{74}}$$

$$z_\alpha = \frac{0.5-0.405}{\sqrt{\frac{(0.405)(0.594)}{74}}} = 1.66$$

$$P(z<1.66) = 0.9515 \to \boxed{95.15\%}$$

5.

High levels of blood sugar are associated with an increased risk of type 2 diabetes. A level higher than normal is referred to as "impaired fasting glucose." The article "Association of Low-Moderate Arsenic Exposure and Arsenic Metabolism with Incident Diabetes and Insulin Resistance in the Strong Heart Family Study" (M. Grau-Perez, C. Kuo, et al., Environmental Health Perspectives, 2017, online) reports a study in which 47 of 155 people with impaired fasting glucose had type 2 diabetes. Consider this to be a simple random sample

a. **Find a 95% confidence interval for the proportion of people with impaired fasting glucose who have type 2 diabetes.**

5. $n = 165$ SRS $\frac{47}{165} = 0.303$

$x = 47$

a) Find 95% C.I.

$1 - 0.95 = \frac{0.05}{2} = 0.025 \rightarrow z_{0.025} \rightarrow 1.96$

$\tilde{n} = n + 4 = 159 \quad \tilde{p} = \frac{47+2}{159} = 0.308$

$x = 47$

$0.308 \pm 1.96\sqrt{\frac{0.308(0.692)}{159}}$

$\boxed{(0.236, 0.380)}$

b. **Find a 99% confidence interval for the proportion of people with impaired fasting glucose who have type 2 diabetes.**

b) Find 99% C.I.

$0.308 \pm 2.57\sqrt{\frac{0.308(0.692)}{159}}$

$\boxed{(0.214, 0.402)}$

c. **A doctor claims that less than 35% of people with impaired fasting glucose have type 2 diabetes. With what level of confidence can this claim be made?**

c) Claim $p < 0.35$, level of confidence?

$0.35 = \tilde{p} - z_\alpha\sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} = 0.308 - z_\alpha\sqrt{\frac{0.308(0.692)}{159}}$

$z_\alpha = \frac{0.35 - 0.308}{\sqrt{\frac{0.308(0.692)}{159}}} = 1.14 \quad P(Z < 1.14)$

0.8729

$\boxed{87.29\%}$

7.

Refer to Exercise 2. Find a 98% upper confidence bound for the proportion of residences that reduced their water consumption.

2. During a recent drought, a water utility in a certain town sampled 100 residential water bills and found that 73 of the residences had reduced their water consumption over that of the previous year.

7. Refer to exercise 2. Find 98% UCB for proportion of residences that reduced their water consumption.

$n = 100$ $\quad \tilde{n} = n + 4 = 104$ $\quad \tilde{p} = \frac{73+2}{104} = 0.72$

$x = 73$

$1 - \alpha = 0.98$ $\quad z_{\alpha/2} = z_{0.02} \to -2.05 \to 2.05$

from z-table

MOE:

$E = z_{\alpha/2} \cdot \sqrt{\frac{\tilde{p}(1-\tilde{p})}{\tilde{n}}} = 2.05 \cdot \sqrt{\frac{0.7212(0.2788)}{104}} = 0.09$

upper bound of C.I. is:

$\tilde{p} + E = 0.7212 + 0.09 = \boxed{0.8113}$

13.

A sociologist is interested in surveying workers in computer-related jobs to estimate the proportion of such workers who have changed jobs within the past year.

a. In the absence of preliminary data, how large a sample must be taken to ensure that a 95% confidence interval will specify the proportion to within ±0.05?

13. # to sample for 95% C.I. within ± 0.05?

① *No preliminary data → let's use $\tilde{p} = 0.5$ as it will result in largest possible sample size

$n = \left(\frac{z_{\alpha/2} \sqrt{\tilde{p}(1-\tilde{p})}}{E} \right)^2 - 4$

$\left(\frac{1.96 \cdot \sqrt{0.5(1-0.5)}}{0.05} \right)^2 - 4 = 380.16$

$\boxed{381}$

b. In a sample of 100 workers, 20 of them had changed jobs within the past year. Find a 95% confidence interval for the proportion of workers who have changed jobs within the past year.

b) $n=100$, $x=20$ Find 95% C.I.

$\tilde{n} = n+4 = 104$ $\tilde{p} = \frac{20+2}{104} \approx 0.211$

$0.211 \pm 1.96\sqrt{\frac{0.211(0.788)}{104}}$

$\boxed{(0.1330, 0.2900)}$

c. **Based on the data in part (b), estimate the sample size needed so that the 95% confidence interval will specify the proportion to within ±0.05.**

c) Based on B estim., the sample size needed so that 95% C.I. will specify the proportion to within ± 0.05.

MOE:

$E = z_{\alpha/2} \cdot \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}} \Rightarrow n = \left(\frac{z_{\alpha/2} \cdot \sqrt{\tilde{p}(1-\tilde{p})}}{E}\right)^2 - 4$

$= \left(\frac{1.96 \cdot \sqrt{0.2115(1-0.2115)}}{0.05}\right)^2 - 4 \approx 252.29$

$\boxed{253}$

15.

The article "A Music Key Detection Method Based on Pitch Class Distribution Theory" (J. Sun, H. Li, and L. Ma, International Journal of Knowledge-based and Intelligent Engineering Systems, 2011:165–175) describes a method of analyzing digital music files to determine the key in which the music is written. In a sample of 335 classical music selections, the key was identified correctly in 293 of them.

a. **Find a 90% confidence interval for the proportion of pieces for which the key will be correctly identified.**

15. $n = 335$

$x = 293$ $\quad \frac{293}{335} = 0.87$

a) Find 90% C.I.

$1 - 0.9 = 0.1$

$\frac{0.1}{2} = 0.05 \rightarrow z_{0.05} \rightarrow 1.645$

$\tilde{n} = n + 4 = 339$ $\quad \tilde{p} = \frac{293+2}{339} = 0.87$

$x = 293$

$0.87 \pm 1.645\sqrt{\frac{0.87(0.129)}{339}}$

$\boxed{(0.84, 0.90)}$

b. **How many music pieces should be sampled to specify the proportion to within ±0.025 with 90% confidence?**

b) How many to sample → ± 0.025 with 90% confidence

$1.645\sqrt{\frac{0.87(0.129)}{n+4}} = 0.025$

$n = \left(\frac{1.645\sqrt{0.87(0.129)}}{0.025}\right)^2 - 4 = 485.02 \rightarrow \boxed{486}$

c. **Another method of key detection is to be tested. At this point, there is no estimate of the proportion of the time this method will be identified correctly. Find a conservative estimate of the sample size needed so that the proportion will be specified to within ±0.03 with 90% confidence.**

c) No est. of proportion of time, Find a conservative estimate of the sample size needed so that the proportion will be specified to within ±0.03 with 90% confidence

$E = 0.03$ $\quad C = 0.90$

MOE!

$E = z_{\alpha/2}\sqrt{\frac{\tilde{p}(1-\tilde{p})}{n+4}}$

$n = \left(\frac{1.645\sqrt{0.5(1-0.5)}}{0.03}\right)^2 - 4$

$747.67 \rightarrow \boxed{748}$