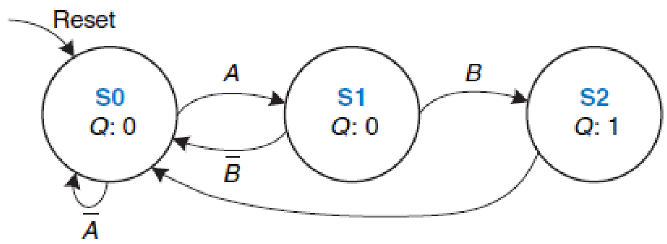
CSC 137 Cokgor Finite State Machines Exercises

(Exercises reference: David M. Harris, Sarah L. Harris, Digital Design and Computer Architecture, 2nd Edition, Elsevier, 2013, ISBN-13: 978-0-12-394424-5)

1) Consider the state transition diagram below and answer the questions. Assume Moore machine design.



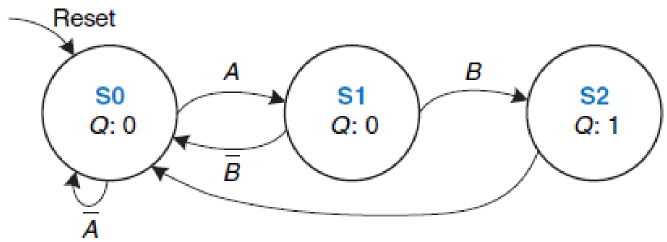
a) How many state variables are needed to represent all states?
b) Develop the state transition table.
c) Develop the output table.
d) Write the Boolean equations for the next state and output.
e) Draw the Moore machine for this FSM.

2) Gray codes have a useful property in that consecutive numbers differ in only a single bit position. Table below lists a 3-bit Gray code representing the numbers 0 to 7. Design a 3-bit modulo 8 Gray code counter FSM with no inputs and three outputs. (A modulo N counter counts from 0 to N − 1, then repeats.) When reset, the output should be 000. On each clock edge, the output should advance to the next Gray code. After reaching 100, it should repeat with 000.

Number	Gray code		
0	0	0	0
1	0	0	1
2	0	1	1
3	0	1	0
4	1	1	0
5	1	1	1
6	1	0	1
7	1	0	0

Solutions:

1) Consider the state transition diagram below and answer the questions. Assume Moore machine design.



a) How many state variables are needed to represent all states?

There are 3 states. Two state variables will be needed to encode three states. e.g.

State Variables		States
q1	q0	
0	0	S0
0	1	S1
1	0	S2

b) Develop the state transition table.

Present State, S	*Present State* q_1 q_0	Inputs, A B	Next State, S^+	*Next State* q_1^+ q_0^+
S0	0 0	0 X	S0	0 0
S0	0 0	1 X	S1	0 1
S1	0 1	X 0	S0	0 0
S1	0 1	X 1	S2	1 0
S2	1 0	X X	S0	0 0

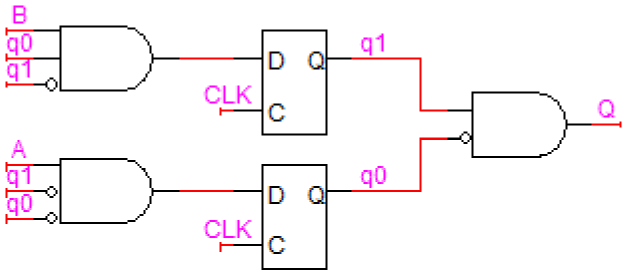
c) Develop the output table.

Present State, S	*Present State* q_1 q_0	Output, Q
S0	0 0	0
S1	0 1	0
S2	1 0	1

d) Write the Boolean equations for the next state and output.

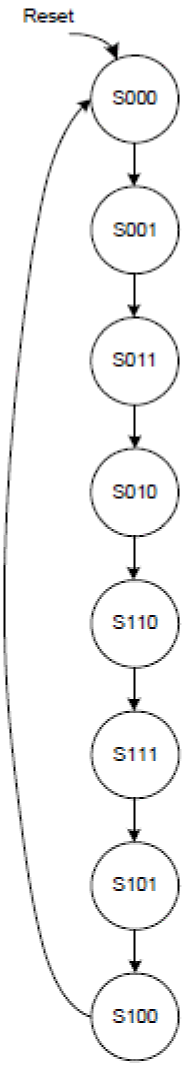
$q_1^+ = q_1' \, q_0 \, B$
$q_0^+ = q_1' \, q_0' \, A$
$Q = q_1 \, q_0'$

e) Draw the Moore machine for this FSM.



Reset has been omitted in this circuit.

2) State transition diagram.



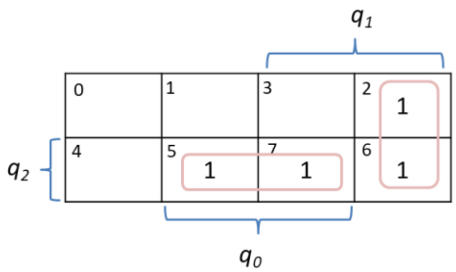
There are 8 states, hence $\log_2 2^3 = 3$ state variables.

State transition table:

Present State, S	*Present State* q_2	q_1	q_0	Next State, S^+	*Next State* q_2^+	q_1^+	q_0^+
S0	0	0	0	S0	0	0	1
S1	0	0	1	S2	0	1	1
S2	0	1	1	S3	0	1	0
S3	0	1	0	S4	1	1	0
S4	1	1	0	S5	1	1	1
S5	1	1	1	S6	1	0	1
S6	1	0	1	S7	1	0	0
S7	1	0	0	S0	0	0	0

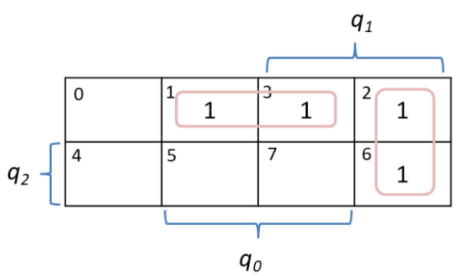
Next state Boolean equations:

$q_2^{+} = q_2'\, q_1\, q_0' + q_2\, q_1\, q_0' + q_2\, q_1\, q_0 + q_2\, q_1'\, q_0$



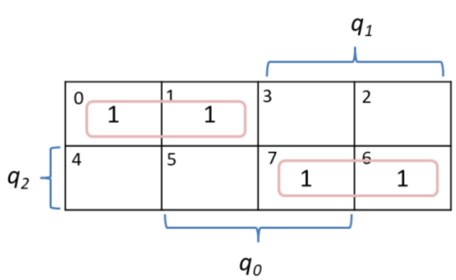
$q_2^{+} = q_2\, q_0 + q_1\, q_0'$

$q_1^{+} = q_2'\, q_1'\, q_0 + q_2'\, q_1\, q_0 + q_2'\, q_1\, q_0' + q_2\, q_1\, q_0'$



$q_1^{+} = q_2'\, q_0 + q_1\, q_0'$

$q_0^{+} = q_2'\, q_1'\, q_0' + q_2'\, q_1'\, q_0 + q_2\, q_1\, q_0' + q_2\, q_1\, q_0$



$q_0^{+} = q_2'\, q_1' + q_2\, q_1$

Output Equations:

Since this is a counter, the three state variables can represent the three outputs. Each output can be equal to a state variable.

$Q_2 = q_2$

$Q_1 = q_1$

$Q_0 = q_0$