

Bayes' Rule Problems:

- 1) An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident. Calculate the probability that the driver was age 16-20.

- 2) A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
1997	0.16	0.05
1998	0.18	0.02
1999	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident. Determine the probability that the model year of this automobile is 1997 .

- 3) The probability that a randomly chosen male has a circulation problem is 0.25 . Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?
- 4) A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.
- 5) An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

6) A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

7) Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

- (i) 10% of the emergency room patients were critical;
- (ii) 30% of the emergency room patients were serious;
- (iii) the rest of the emergency room patients were stable;
- (iv) 40% of the critical patients died;
- (v) 10% of the serious patients died; and
- (vi) 1% of the stable patients died.

Given that a patient survived, what is the probability that the patient was categorized as serious upon arrival?

8) An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year. A policyholder dies in the next year. What is the probability that the deceased policyholder was ultra-preferred?

9) You go to see the doctor about an ingrowing toenail. The doctor selects you at random to have a blood test for swine flu, which for the purposes of this exercise we will say is currently suspected to affect 1 in 10,000 people in Australia. The test is 99% accurate, in the sense that the probability of a false positive is 1%. The probability of a false negative is zero. You test positive. What is the new probability that you have swine flu?

Now imagine that you went to a friend's wedding in Mexico recently, and (for the purposes of this exercise) it is known that 1 in 200 people who visited Mexico recently come back with swine flu. Given the same test result as above, what should your revised estimate be for the probability you have the disease?

10) In one town, 8% of 18-29 year olds own a house, as do 21% of 30-50 year olds and 52% of those over 50. According to a recent census taken in the town, 27.0% of adults in the town are 18-29 years old, 38.7% are 30-50 years old, and 34.3% are over 50. What is the probability that a randomly selected adult owns a house? Round to the nearest thousandth when necessary.