Math 100 **Assignment 1** Fall 21

Instructions: Please answer the following legibly, logically, and **show all work**. No credit will be given for unjustified or unclear work. Please clearly label every problem and work them in order. When you are finished, please scan your work (or take pictures) and submit via the Assignments link on Canvas if you are an online student. Otherwise, you can turn in a separate sheet of paper in-person.

1. Use augmented matrices and elementary row operations to solve the following systems. Please use row notation to show your steps.

   For example: $-3R_1 + R_2 \to R_2$.

   (a)

   $$\begin{aligned} x_2 + 5x_3 &= -4 \\ x_1 + 4x_2 + 3x_3 &= -2 \\ 2x_1 + 7x_2 + x_3 &= -2 \end{aligned}$$

   (b)

   $$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

   (c)

   $$\begin{aligned} 2x_1 - 6x_3 &= -8 \\ x_2 + 2x_3 &= 3 \\ 3x_1 + 6x_2 - 2x_3 &= -4 \end{aligned}$$

2. Construct an augmented matrix for a linear system whose solution set is $x_1 = 3$, $x_2 = -2$, and $x_3 = -1$.

3. Determine which matrices are in reduced echelon form and which are only in echelon form (you don't need to care if the system is consistent or not):

   (a) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

4. Row reduce the following matrix into reduced echelon form:
   *(Use row notation to show your steps)*

   $$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$$

5. Find the general solution for the augmented matrices:
*(Use row notation to show your steps)*

(a) $\begin{bmatrix} 1 & -1 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$

6. For the following statements determine whether they or true or false. If false, then explain why it is false.
   (a) Every elementary row operation is reversible.
   (b) Two matrices are row equivalent if they have the same number of rows.
   (c) Elementary row operations on an augmented matrix never change the solution set of the associated linear system.
   (d) Two equivalent linear systems can have different solution sets.
   (e) A consistent system of linear equations has one or more solutions.
   (f) In some cases, a matrix may be row reduced to more than one reduced echelon matrix if using different sequences of row operations.
   (g) If one row in an echelon form of an augmented matrix reads

   $$\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$$

   then the associated matrix is inconsistent.
   (h) If a system has free variables, then there is only one unique solution.

7. A system of linear equations with fewer equations than unknowns (variables) is sometimes called a *underdetermined system* Can such a system have a unique solution. Explain why or why not.

8. Suppose the coefficient matrix of a linear system of four equations in four variables has a pivot in each column. Explain why the system has a unique solution.

9. Give an example of an *inconsistent* underdetermined system of two equations in three variables.