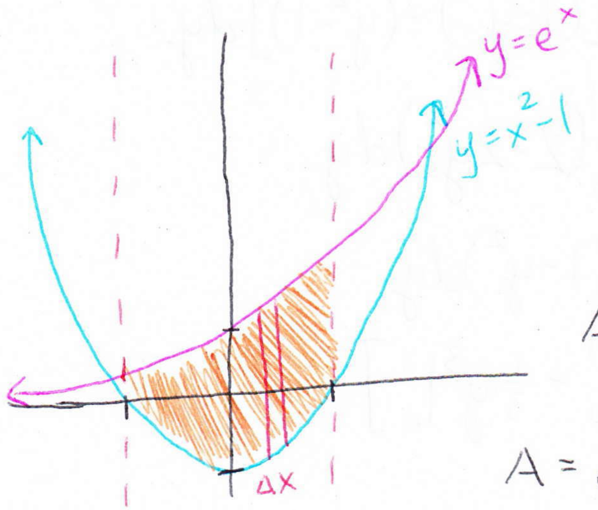
1 Areas Between Curves

1. Find the area of the region between the curves $y = e^x$ and $y = x^2 - 1$ that is bounded between $x = -1$ and $x = 1$. Make a sketch of the region that is being enclosed.



$$A = \int_a^b [f(x) - g(x)]\,dx$$

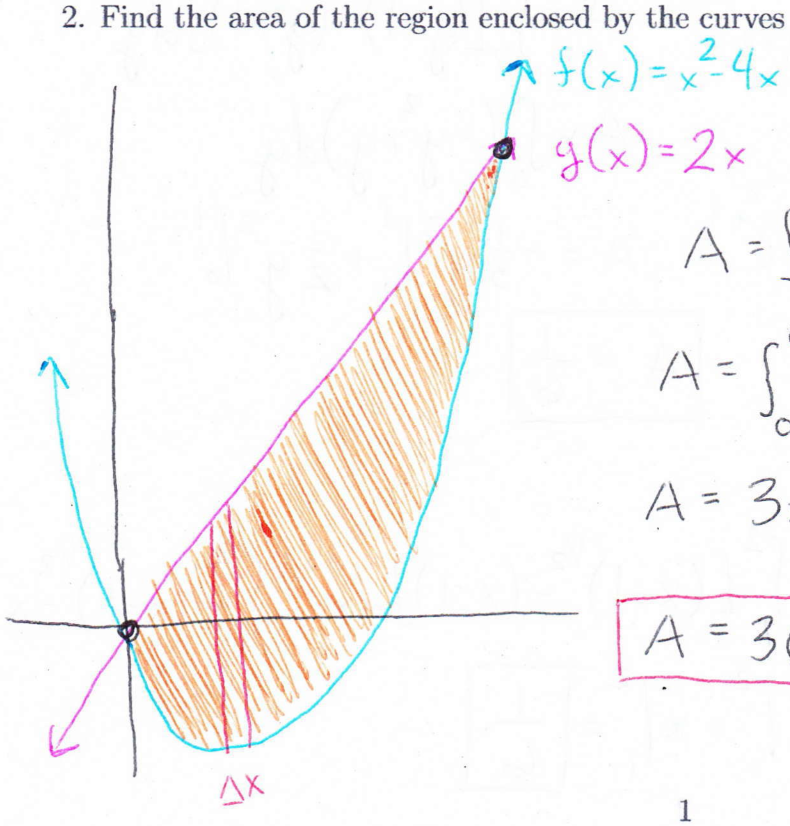
$$A = \int_{-1}^{1} [e^x - (x^2-1)]\,dx$$

$$A = \int_{-1}^{1} (e^x - x^2 + 1)\,dx$$

$$A = e^x\Big|_{-1}^{1} - \frac{1}{3}x^3\Big|_{-1}^{1} + x\Big|_{-1}^{1}$$

$$\boxed{A = e - \frac{1}{e} + \frac{4}{3}}$$

2. Find the area of the region enclosed by the curves $f(x) = x^2 - 4x$ and $g(x) = 2x$.



$$A = \int_a^b [f(x) - g(x)]\,dx$$

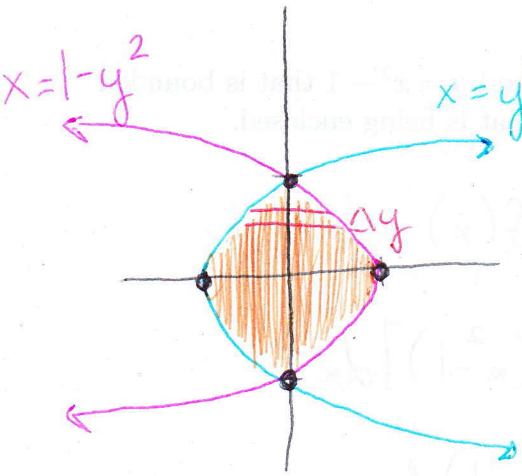
$$A = \int_0^6 [2x - (x^2 - 4x)]\,dx$$

$$A = \int_0^6 (6x - x^2)\,dx$$

$$A = 3x^2\Big|_0^6 - \frac{1}{3}x^3\Big|_0^6$$

$$\boxed{A = 36}$$

3. Find the area of the region enclosed by the curves $x = 1 - y^2$ and $x = y^2 - 1$.



$$A=\int_a^b [f(y)-g(y)]\,dy$$

$$A=\int_{-1}^{1} [(1-y^2)-(y^2-1)]\,dy$$

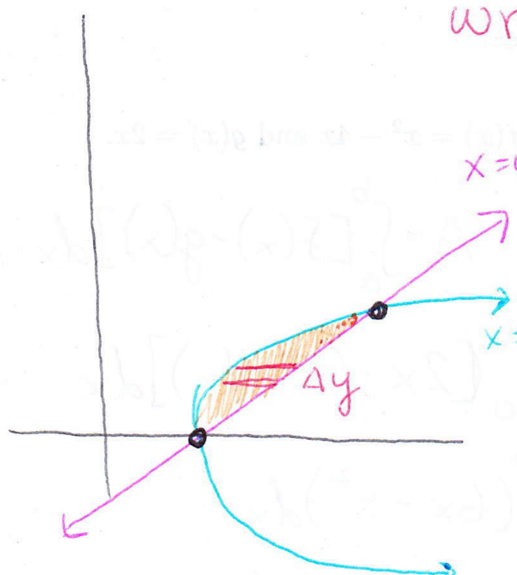
$$A=2\int_0^1 (2-2y^2)\,dy$$

$$A=4\int_0^1 (1-y^2)\,dy$$

$$A=4\left[\,y\Big|_0^1-\frac{1}{3}y^3\Big|_0^1\right]$$

$$\boxed{A=\frac{8}{3}}$$

4. Sketch the region enclosed by the curves $x - y = 1$ and $y^2 = x - 1$ and find the area.



wrt to y $\quad A=\int_a^b [f(y)-g(y)]\,dy$

$$A=\int_0^1 [(y+1)-(y^2+1)]\,dy$$

$$A=\int_0^1 (-y^2+y)\,dy$$

$$A=-\frac{1}{3}y^3\Big|_0^1+\frac{1}{2}y^2\Big|_0^1$$

$$\boxed{A=\frac{1}{6}}$$

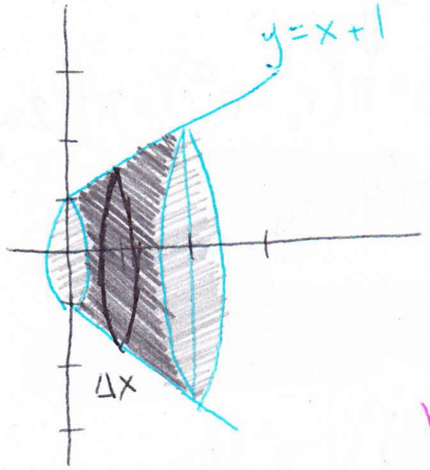
wrt to x

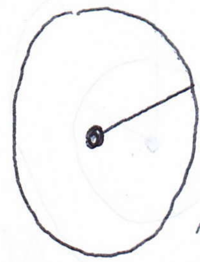
$$A=\int_a^b [f(x)-g(x)]\,dx=\int_1^2 [(x-1)^{1/2}-(x-1)]\,dx=\int_1^2 [(x-1)^{1/2}-x+1]\,dx$$

$$A=\frac{2}{3}(x-1)^{3/2}\Big|_1^2-\frac{1}{2}x^2\Big|_1^2+x\Big|_1^2=\boxed{\frac{1}{6}}$$

2 Volumes

1. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x + 1$ and $y = 0$ and $x = 0$ and $x = 2$ about the x-axis. Sketch the solid and a typical disc.





$r = x+1$

$A(x) = \pi r^2 = \pi(x+1)^2$

$A(x) = \pi(x^2+2x+1)$

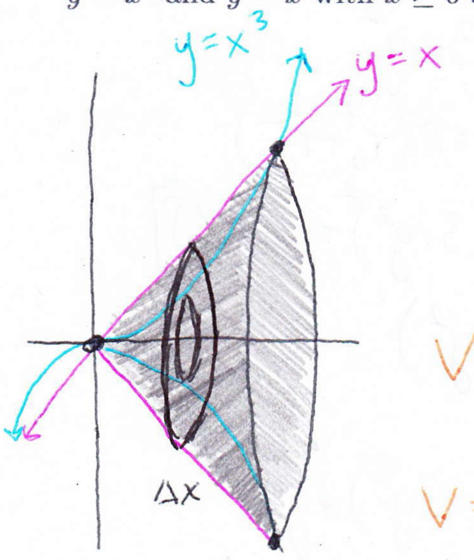
$V = \int_a^b A(x)\,dx = \int_0^2 \pi(x^2+2x+1)\,dx$

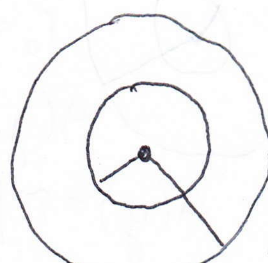
$V = \pi\int_0^2 (x^2+2x+1)\,dx$

$V = \pi\left(\frac{1}{3}x^3\Big|_0^2 + x^2\Big|_0^2 + x\Big|_0^2\right)$

$\boxed{V = \frac{26\pi}{3}}$

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^3$ and $y = x$ with $x \geq 0$ about the x-axis. Sketch the solid and a typical washer.





$r_i = x^3$

$r_o = x$

$A(x) = \pi(r_o^2 - r_i^2) = \pi(x^2 - x^6)$

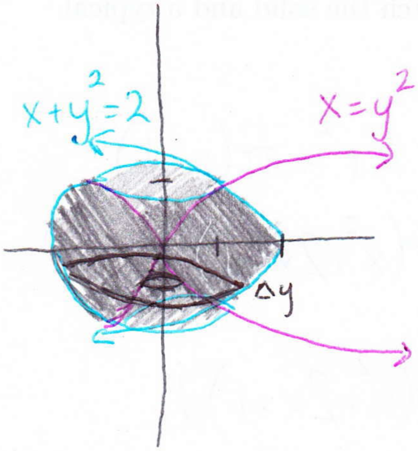
$V = \int_a^b A(x)\,dx = \int_0^1 \pi(x^2-x^6)\,dx$

$V = \pi\int_0^1 (x^2-x^6)\,dx$

$V = \pi\left(\frac{1}{3}x^3\Big|_0^1 - \frac{1}{7}x^7\Big|_0^1\right)$

$\boxed{V = \frac{4\pi}{21}}$

3. Find the volume of the solid obtained by rotating the region bounded by the curves $x + y^2 = 2$ and $x = y^2$ about the y-axis. Sketch the solid and a typical disc or washer.



$r_i = y^2$

$r_o = 2 - y^2$

$A(y) = \pi(r_o^2 - r_i^2) = \pi[(2-y^2)^2 - (y^2)^2]$

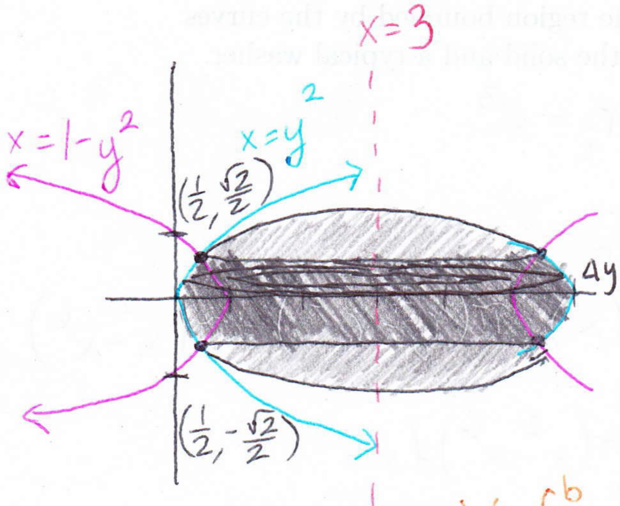
$A(y) = \pi(4 - 4y^2)$

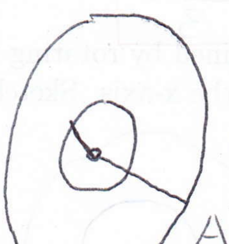
$V = \int_a^b A(y)\,dy = \int_{-1}^{1} \pi(4-4y^2)\,dy$

$V = \pi \int_{-1}^{1} (4-4y^2)\,dy = 4\pi \int_{-1}^{1} (1-y^2)\,dy$

$V = 4\pi\left(y\Big|_{-1}^{1} - \frac{1}{3}y^3\Big|_{-1}^{1}\right) = 4\pi\left(\frac{4}{3}\right) = \boxed{\frac{16\pi}{3}}$

4. Find the volume of the solid obtained by rotating the region bounded by the curves $x = y^2$ and $x = 1 - y^2$ about the line $x = 3$. Sketch the solid and a typical disc or washer.





$r_i = 3 - (1 - y^2) = 2 - y^2$

$r_o = 3 - y^2$

$A(y) = \pi(r_o^2 - r_i^2)$

$A(y) = \pi[(3-y^2)^2 - (2-y^2)^2]$

$A(y) = \pi(5 - 2y^2)$

$V = \int_a^b A(y)\,dy = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \pi(5-2y^2)\,dy$

$V = \pi \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} (5-2y^2)\,dy$

$V = \pi\left(5y\Big|_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} - \frac{2}{3}y^3\Big|_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}}\right) = \boxed{\frac{14\pi\sqrt{2}}{3}}$