STAT 50 HW #5
Section 2.4
2.
Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

a. Find $P(X \leq 2)$.

$P(X \leq 2) = P(0) + P(1) + P(2)$
$P(X \leq 2) = 0.4 + 0.3 + 0.15 = 0.85$

b. Find $P(X > 1)$.

$P(X > 1) = P(2) + P(3) + P(4)$
$P(X > 1) = 0.15 + 0.10 + 0.05 = 0.30$

c. Find μ_x.

$\mu_x = \sum xP(x) = 0(0.4) + 1(0.3) + 2(0.15) + 3(0.10) + 4(0.05)$
$\mu_x = 1.1$

d. Find σ_x^2.

*This is the 2nd method. Just for reference.

$\sigma_x^2 = \sum_x x^2 P(x) - (\mu_x)^2 =$ E(x^2) - (E(x))^2
$\sigma_x^2 = 0^2(0.4) + 1^2(0.3) + 2^2(0.15) + 3^2(0.10) + 4^2(0.05) - 1.1^2$
$\sigma_x^2 = 2.6 - 1.21$
$\sigma_x^2 = 1.39$

3.
A chemical supply company ships a certain solvent in 10-gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume X has the following probability mass function:

x	1	2	3	4	5
$p(x)$	0.4	0.2	0.2	0.1	0.0

a. Find the mean number of drums ordered.

*There was a mistake with the data in this problem, as addressed in class. I will be solving #3 with the correct data.

$\mu_x = \sum xP(x) = 1(0.4) + 2(0.2) + 3(0.2) + 4(0.1) + 5(0.1)$

$\mu_x = 0.4 + 0.4 + 0.6 + 0.4 + 0.5 = 2.3$

b. Find the variance of the number of drums ordered.

*This is the 1st method. Just for reference.

$\sigma_x^2 = \sum_x (x - \mu_x)^2$ P(x = x) = (1 - 2.3)^2 * 0.4 + (2 - 2.3)^2 * 0.2 + (3 - 2.3)^2 * 0.2 + (4 - 2.3)^2 * 0.1 + (5 - 2.3)^2 * 0.1

$\sigma_x^2 = 1.81$

c. Find the standard deviation of the number of drums ordered.

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{1.81} = 1.345$

d. Let Y be the number of gallons ordered. Find the probability mass function of Y.

*You can ignore the highlighted lines below. It's just for my reference.
The pmf of Y is p(Y) = P(Y=Y).
The pmf is sometimes called the probability distribution.

10 gallons = 1 drum

y	10	20	30	40	50
p(y)	0.4	0.2	0.2	0.1	0.1

e. Find the mean number of gallons ordered.

$\mu_x = \sum xP(x) = 10(0.4) + 20(0.2) + 30(0.2) + 40(0.1) + 50(0.1)$

$\mu_x = 4 + 4 + 6 + 4 + 5 = 23$

Or

*10 gallons go into 1 drum, so you could multiply the previous mean by 10

$\mu_x = 2.3 * 10 = 23$

f. Find the variance of the number of gallons ordered.

$\sigma_x^2 = \sum_x x^2 P(x) - (\mu_x)^2 = E(x^2) - (E(x))^2$

$\sigma_x^2 = 10^2(0.4) + 20^2(0.2) + 30^2(0.2) + 40^2(0.1) + 50^2(0.1) - 23^2$

$\sigma_x^2 = 710 - 529$

$\sigma_x^2 = 181$

g. **Find the standard deviation of the number of gallons ordered.**

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{181} = 13.45$

7.

A computer sends a packet of information along a channel and waits for a return signal acknowledging that the packet has been received. If no acknowledgement is received within a certain time, the packet is re-sent. Let X represent the number of times the packet is sent. Assume that the probability mass function of X is given by

$$p(x) = \begin{cases} cx & \text{for } x = 1, 2, 3, 4, \text{ or } 5 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

a. **Find the value of the constant c so that p(x) is a probability mass function.**

$1c + 2c + 3c + 4c + 5c = 1$

$15c = 1$

$c = \frac{1}{15}$

b. **Find P(X = 2).**

$P(x=2) = 2(\frac{1}{15}) = 2/15$

c. **Find the mean number of times the packet is sent.**

$E(x) = \mu_x = 1(1/15 * 1) + 2(1/15 * 2) + 3(1/15 * 3) + 4(1/15 * 4) + 5(1/15 * 5)$

$E(x) = \mu_x = 1/15 + 2(2/15) + 3(3/15) + 4(4/15) + 5(5/15)$

$E(x) = \mu_x = 1/15 + 4/15 + 9/15 + 16/15 + 25/15$

$E(x) = \mu_x = 55/15 = 11/3$

d. **Find the variance of the number of times the packet is sent.**

$\sigma_x^2 = \sum_x x^2 P(x)$ - $(\mu_x)^2$ = E(x^2) - (E(x))^2

$\sigma_x^2 = 1^2(1/15) + 2^2(2/15) + 3^2(3/15) + 4^2(4/15) + 5^2(5/15) - (11/3)^2$

σ_x^2 = 1/15 + 8/15 + 27/15 + 64/15 + 125/15 - 121/9

σ_x^2 = 225/15 - 121/9

σ_x^2 = 15 - 121/9

σ_x^2 = 135/9 - 121/9

σ_x^2 = 14/9

e. Find the standard deviation of the number of times the packet is sent.

$\sigma_x = \sqrt{\sigma_x^2} = \sqrt{14/9} = 1.247$

13.

Resistors labeled 100 Ω have true resistances that are between 80 Ω and 120 Ω. Let X be the resistance of a randomly chosen resistor. The probability density function of X is given by

$$f(x) = \begin{cases} \frac{x-80}{800} & 80 < x < 120 \\ 0 & \text{otherwise} \end{cases}$$

a. What proportion of resistors have resistances less than 90 Ω?

$P(x < 90) = P(x \leq 90) = \int_{80}^{90} \frac{x-80}{800} = \int \frac{x-80}{800} = \frac{1}{800} \int x - 80 = \frac{1}{800}(\frac{x^2}{2} - 80x) = \frac{1}{800}(\frac{x^2}{2} - 80x)\Big|_{80}^{90}$

$= \frac{1}{800}(\frac{90^2}{2} - 80(90)) - \frac{1}{800}(\frac{80^2}{2} - 80(80))$

$= \frac{1}{800}(\frac{90^2}{2} - 80(90)) - \frac{1}{800}(\frac{80^2}{2} - 80(80))$

$= \frac{1}{800}(\frac{8100}{2} - 7200) - \frac{1}{800}(\frac{6400}{2} - 6400)$

$= \frac{1}{800}(4050 - 7200) - \frac{1}{800}(3200 - 6400)$

$= \frac{1}{800}(-3150) - \frac{1}{800}(-3200)$

= -3.9375 -(-4) = 0.0625 or $\frac{1}{16}$

b. Find the mean resistance.

$\mu_x = E(x) = \int_{-\infty}^{\infty} xf(x)dx = \int_{80}^{120} x(\frac{x-80}{800}) = \int x(\frac{x-80}{800}) = \frac{1}{800}\int x(x-80)$

$= \frac{1}{800}(\frac{x^3}{3} - \frac{80x^2}{2}) = \frac{1}{800}(\frac{x^3}{3} - \frac{80x^2}{2})\Big|_{80}^{120}$

$= \frac{1}{800}(\frac{120^3}{3} - \frac{80(120)^2}{2}) - \frac{1}{800}(\frac{80^3}{3} - \frac{80(80)^2}{2})$
$= \frac{1}{800}(\frac{1728000}{3} - \frac{80(14400)}{2}) - \frac{1}{800}(\frac{512000}{3} - \frac{80(6400)}{2})$
$= \frac{1}{800}(\frac{1728000}{3} - \frac{1152000)}{2}) - \frac{1}{800}(\frac{512000}{3} - \frac{512000)}{2})$
$= \frac{1}{800}(576000 - 57600) - \frac{1}{800}(170666.66 - 256000)$
$= \frac{1}{800}(0) - \frac{1}{800}(-85333.33)$
$= 106.66\Omega$

c. Find the standard deviation of the resistances.

$\sigma_x = \sqrt{\sigma_x^2}$

$$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - (\mu_x)^2 = E(x^2) - (E(x))^2$$

$$= \int_{80}^{120} x^2(\frac{x-80}{800})dx - (106.66)^2$$

$$= \int_{80}^{120} (\frac{x^3-80x^2}{800})dx - 11377.77$$

$$= \frac{1}{800}(\frac{x^4}{4} - \frac{80x^3}{3})\Big|_{80}^{120} - 11377.77$$

$$= \frac{1}{800}(\frac{(120)^4}{4} - \frac{80(120)^3}{3}) - \frac{1}{800}(\frac{(80)^4}{4} - \frac{80(80)^3}{3}) - 11377.77$$

$=7200-(-4266.66) - 11377.77$

$=11466.66 - 11377.77 = 88.88$

$\sigma_x = \sqrt{88.88} = 9.4281\ \Omega$

d. Find the cumulative distribution function of the resistances.

$$f(x)=\begin{cases}\dfrac{x-80}{800} & 80<x<120\\ 0 & \text{otherwise}\end{cases}$$

For $x<80$ $F(x)=0$ 13, A

For $80<x<120$

$$F(x)=\int_{80}^{x}\frac{x-80}{800}\,dx$$

$$\int_{80}^{x} x\,dx \to \frac{x^2}{2}-3200 \qquad \frac{1}{800}\left(\frac{x^2}{2}-3200-(80x-6400)\right)$$

$$\int_{80}^{x} 80\,dx = 80x-6400$$

$$\frac{x^2}{1600}-\frac{x}{10}+4$$

For $x\geq 120$, $F(x)=1$

$$\boxed{F(x)=\begin{cases}0 & x<80\\ \dfrac{x^2}{1600}-\dfrac{x}{10}+4 & 80\leq x<120\\ 1 & x\geq 120\end{cases}}$$

15.

The lifetime of a transistor in a certain application is random with probability density function

$$f(t)=\begin{cases}0.1e^{-0.1t} & t>0\\ 0 & t\leq 0\end{cases}$$

a. **Find the mean lifetime.**

*This one was a little complex for me, which is why I did by hand.

$\int x \cdot 0.1e^{-0.1x}\,dx \qquad x = \frac{u}{-0.1}$

$0.1 \int x e^{-0.1x}\,dx \qquad u = -0.1x \qquad du = -0.1\,dx$

$0.1 \int \left(\frac{u}{-0.1}\right) e^{u}\,\frac{du}{-0.1} \qquad dx = \frac{du}{-0.1}$

$-10 \int e^{u}\,du$

$-10 e^{u})$

$-10 e^{-0.1x}$

$\int_0^\infty x\, 0.1 e^{-0.1x}\,dx$

$(-10e^{-0.1(\infty)}) - (-10e^{-0.1(0)})$

$(-10(0)) - (-10(1))$

$0 - (-10) = 10$

$\boxed{\mu_x = E(X) = 10 \text{ months}}$

b. Find the standard deviation of the lifetimes.

$\sigma_x = \sqrt{\frac{1}{0.1^2}} = \frac{1}{0.1} = 10$ months

c. Find the cumulative distribution function of the lifetime.

CDF = 1 - $e^{-0.1x}$

$$f(t) = \begin{cases} 0.1e^{-0.1t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

The PDF suggests that a random variable to follow an exponential function with parameter $\lambda = 0.1$

$$\boxed{F(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-0.1t} & t \geq 0 \end{cases}}$$

d. Find the probability that the lifetime will be less than 12 months.

$$P(x < 12) = \int_0^{12} 0.1e^{-0.1x}\,dx$$

$$0.1\int_0^{12} e^{-0.1x}\,dx \qquad x = \frac{u}{0.1}, \quad u = -0.1x, \quad du = -0.1dx, \quad dx = \frac{du}{-0.1}$$

$$0.1\int \frac{1}{-0.1} e^u\,du$$

$$0.1\left(-10\int e^u\,du\right)$$

$$0.1(-10)e^u$$

$$-e^{-0.1x}$$

$$\left(-e^{-0.1(12)}\right) - \left(-e^{-0.1(0)}\right)$$

$$\left(-e^{-1.2}\right) - \left(-(1)\right)$$

$$(-0.301) + 1 = \boxed{0.6988}$$

19.

The level of impurity (in percent) in the product of a certain chemical process is a random variable with probability density function

$$\begin{cases} \frac{3}{64}x^2(4-x) & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

a. **What is the probability that the impurity level is greater than 3%?**

$$\int_0^{x} \frac{3}{64} x^2(4-x)\,dx$$

$$\downarrow$$

$$\int_3^4 \frac{3}{64} x^2(4-x)$$

$$\downarrow$$

$$\frac{4(3)x^2}{64} - \frac{3x^3}{64}$$

$$\downarrow$$

$$\frac{12x^3}{64(3)} - \frac{3x^4}{64(4)} \Big|_3^4$$

$$\left(\frac{12(4)^3}{192} - \frac{3(4)^4}{64(4)}\right) - \left(\frac{12(3)^3}{192} - \frac{3(3)^4}{64(4)}\right)$$

$$\boxed{= 0.2617}$$

b. **What is the probability that the impurity level is between 2% and 3%?**

$$\int_0^{x} \frac{3}{64} x^4 (4-x)\,dx$$

$$\int_2^3 \frac{3}{24} x^4 (4-x)\,dx$$

$$\frac{4(3)x^2}{64} - \frac{3x^3}{64}$$

$$\frac{12x^3}{64(3)} - \frac{3x^4}{64(4)}\Bigg|_2^3$$

$$\left(\frac{12(3)^3}{192} - \frac{3(3)^4}{64(4)}\right) - \left(\frac{12(2)^3}{192} - \frac{3(2)^4}{64(4)}\right)$$

$$\boxed{= 0.4257}$$

c. Find the mean impurity level.

$\mu_x = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$\frac{3}{64}x^2(4-x)$

$\int_0^4 x\left(\frac{3}{64}x^2(4-x)\right)dx$

$\frac{3}{64}\int_0^4 x^3(4-x)\,dx$

$\frac{3}{64}\int_0^4 (4x^3 - x^4)\,dx$

$\frac{3}{64}\left(\frac{4x^4}{4} - \frac{x^5}{5}\right)\Big|_0^4$

$\frac{12x^4}{256} - \frac{3x^5}{320}\Big|_0^4$

$\frac{12(4)^4}{256} - \frac{3(4)^5}{320}$

$12 - 9.6 = 2.4$

$\boxed{\mu_x = 2.4\%}$

d. Find the variance of the impurity levels.

$$\int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu_x^2$$

$$\int_0^4 x^2 \frac{3}{64}x^2(4-x)\,dx - (2.4)^2$$

$$\int_0^4 \frac{3}{64}x^2(4x^2-x^3)\,dx - (2.4)^2$$

$$\int_0^4 \left(\frac{12x^4}{64} - \frac{3x^5}{64}\right)dx - (2.4)^2$$

$$\left(\frac{12x^5}{64(5)} - \frac{3x^6}{64(6)}\right)\Bigg|_0^4 - 5.76$$

$$\frac{12(4)^5}{320} - \frac{3(4)^6}{384} - 5.76$$

$$38.4 - 32 - 5.76 = \boxed{0.64}$$

e. Find the cumulative distribution function of the impurity level.

$$\begin{cases} \frac{3}{64} x^2(4-x) & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

For $x<0$, $F(x)=0$

For $0<x<4$

$$F(x) = \int_0^x \frac{3}{64} x^2(4-x)\,dx$$

$$\frac{3}{64}\int_0^x (4x^2 - x^3)\,dx$$

$$\int_0^x 4x^2\,dx \rightarrow \frac{4x^3}{3} \qquad \int_0^x x^3 \rightarrow \frac{x^4}{4}$$

$$\frac{3}{64}\left(\frac{4x^3}{3} - \frac{x^4}{4}\right) \rightarrow \frac{x^3}{16} - \frac{3x^4}{256}$$

For $x \geq 4$, $F(x)=1$

$$\boxed{F(x) = \begin{cases} 0 & x<0 \\ \frac{x^3}{16} - \frac{3x^4}{256} & 0 \leq x < 4 \\ 1 & x \geq 4 \end{cases}}$$

20.

The main bearing clearance (in mm) in a certain type of engine is a random variable with probability density function

$$f(x) = \begin{cases} 625x & 0 < x \leq 0.04 \\ 50 - 625x & 0.04 < x \leq 0.08 \\ 0 & \text{otherwise} \end{cases}$$

a. **What is the probability that the clearance is less than 0.02 mm?**

$$\int_{\infty}^{0} 625x\,dx$$

$$\int_{0}^{0.02} 625x\,dx$$

$$\frac{625x^2}{2}\Big|_{0}^{0.02}$$

$$\frac{625(0.02)^2}{2} - 0 = \boxed{0.125}$$

b. Find the mean clearance.

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\int_{0}^{0.08} x f(x)\,dx$$

$$\int_{0}^{0.04} x(625x)\,dx + \int_{0.04}^{0.08} x(50-625x)\,dx$$

$$625x^2 \qquad\qquad 50x - 625x^2$$

$$\frac{625x^3}{3}\Big|_{0}^{0.04} \qquad\qquad \frac{50x^2}{2} - \frac{625x^3}{3}\Big|_{0.04}^{0.08}$$

$$0.01333 \qquad\qquad 0.02666$$

$$0.01333 + 0.02666 = \boxed{0.0399}$$

c. Find the standard deviation of the clearances.

Variance $= \int_{0}^{0.04} 625x^3\,dx + \int_{0.04}^{0.06} (50x^2 - 625x^3)\,dx - \mu^2$

$= \frac{625x^4}{4} \Big|_{0}^{0.04} + \frac{50x^3}{3} - \frac{625x^4}{4} \Big|_{0.04}^{0.08} - 0.04^2$

$= 0.0002667$

$\sqrt{0.0002667} = \boxed{0.01633}$

d. Find the cumulative distribution function of the clearance.

$$f(x) = \begin{cases} 625x & 0 < x \leq 0.04 \\ 50 - 625x & 0.04 < x \leq 0.08 \\ 0 & \text{otherwise} \end{cases}$$

For $x \leq 0$, $F(x) = 0$

For $0 < x \leq 0.04$

$$F(x) = \int_0^x 625x\,dx \rightarrow 625x^2/2$$

For $0.04 < x \leq 0.08$, $F(x) = \int_0^{0.04} 625x\,dx + \int_{0.04}^{x} (50 - 625x)\,dx$

$$= 50x - \frac{625x^2}{2} - 1$$

For $x > 0.08$, $F(x) = 1$

$$\boxed{F(x) = \begin{cases} 0 & x \leq 0 \\ \dfrac{625x^2}{2} & 0 < x \leq 0.04 \\ 50x - \dfrac{625x^2}{2} - 1 & 0.04 < x \leq 0.08 \\ 1 & x > 0.08 \end{cases}}$$

e. Find the median clearance.

HW #3

$$\int_{-\infty}^{x_m} f(x)\,dx = 0.5$$

Since $F(x)$ is described by different expressions for $x \le 0.04$ and $x > 0.04$ we compute $F(0.4)$

$$F(0.4) = \int 6.25x = 0.5$$

$$6.25\frac{x^2}{2} = 0.5$$

$$6.25\frac{(0.04)^2}{2} = 0.5$$

So

$$\boxed{x_m = 0.04}$$

f. **The specification for the clearance is 0.015 to 0.063 mm. What is the probability that the specification is met?**

$P(0.015 < X < 0.063)$

$$\int_{0.015}^{0.04} 625x\,dx + \int_{0.04}^{0.063} (50-625x)\,dx$$

$$\frac{625x^2}{2}\Big|_{0.015}^{0.04} + 50x - \frac{625x^2}{2}\Big|_{0.04}^{0.063}$$

$$= \boxed{0.9097}$$

22.

The concentration of a reactant is a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{2e^{-2x}}{1-e^{-2}} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. **What is the probability that the concentration is greater than 0.5?**

$$\int_0^1 \frac{2e^{-2x}}{1-e^{-2}}\,dx$$

$$\int_{0.5}^{1} \frac{2e^{-2x}}{1-e^{-2}}$$

$\downarrow$

$$\frac{e^{-2x}}{e^{-2}-1}\Big|_{0.5}^{1} = \boxed{0.2689}$$

b. **Find the mean concentration.**

$$\mu_x = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$\int_0^1 x\left(\frac{2e^{-2x}}{1-e^{-2}}\right)dx$$

$$\Downarrow$$

$$-\frac{-e^2+3}{2(e^2-1)}\Big|_0^1 = \boxed{0.34348}$$

c. **Find the probability that the concentration is within ±0.1 of the mean.**

$$\mu_x = 0.34346$$

$$P(\mu-0.1 < x < \mu+0.1) = \int_{0.24348}^{0.44348} \frac{2e^{-2x}}{1-e^{-2}}\,dx$$

$$\Downarrow$$

$$\frac{-e^{-2x}}{1-e^{-2}}\Big|_{0.24346}^{0.44346}$$

$$\Downarrow$$

$$\boxed{0.23429}$$

d. **Find the standard deviation σ of the concentrations.**

Variance $\Rightarrow \int_0^1 \frac{2x^2 e^{-2x}}{1-e^{-2}} dx - \mu^2$

$= \frac{e^2}{e^2-1} \int_0^1 2x^2 e^{-2x} dx - \mu^2$

$= \frac{e^2}{4e^2-4} \int_0^2 u^2 e^{-u} du - \mu^2$

$= \frac{e^2}{4e^2-4} \; -u^2 e^{-u} \Big|_0^2 + \int_0^2 2u e^{-u} du \; - \frac{e^2-3}{2e^2-2}$

$= \frac{e^2}{4e^2-4} \; -4e^{-2} + 2 \; 1-3e^{-2} - \frac{e^2-3}{2e^2-2}$

$= \frac{e^2-5}{2e^2-2} - \frac{e^2-3}{2e^2-2}$

$= 0.0684845$

$\sqrt{0.068...} = \boxed{0.26265}$

e. Find the probability that the concentration is within ±σ of the mean.

$$P(\mu-\sigma < X < \mu+\sigma) = \int_{0.0808}^{0.606} \frac{2e^{-2x}}{1-e^{-2}}\,dx$$

$$= \frac{-e^{-2x}}{1-e^{-2}}\Bigg|_{0.0808}^{0.0606}$$

$$= \frac{e^{-0.16167} - e^{-1.21}}{1-e^{-2}}$$

$$\boxed{= 0.63979}$$

f. **Find the cumulative distribution function of the concentration.**

$$f(x) = \begin{cases} \frac{2e^{-2x}}{1-e^{-2}} & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

For $x \le 0$, $F(x) = 0$

For $0<x<1$ $F(x) = \int_0^x \frac{2e^{-2x}}{1-e^{-2}}\,dx = \frac{1-e^{-2x}}{1-e^{-2}}$

For $x \ge 1$, $F(x) = 1$

$$\boxed{F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1-e^{-2x}}{1-e^{-2}} & 0<x<1 \\ 1 & x \ge 1 \end{cases}}$$

25.

The repair time (in hours) for a certain machine is a random variable with probability density function

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

a. **What is the probability that the repair time is less than 2 hours?**

$P(X \le 2)\ \int_0^2 x e^{-x}\,dx \qquad \int u\,dv = uv - \int v\,du$

$\downarrow$

integrate by parts $\qquad u = x \quad du = 1 \qquad v = -e^{-x} \quad dv = e^{-x}$

$-xe^{-x} - \int -e^{-x} \Big|_0^2$

$\downarrow$

$-xe^{-x} - e^{-x} \Big|_0^2$

$\downarrow$

$\boxed{0.59399}$

b. What is the probability that the repair time is between 1.5 and 3 hours?

$$\int_{1.5}^{3} x e^{-x}\,dx$$

same work as before just with different bounds

$$-xe^{-x} - e^{-x} \Big|_{1.5}^{3}$$

$$\boxed{0.35867}$$

c. **Find the mean repair time.**

$\mu_x = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$

$\int_0^\infty x(xe^{-x})\,dx$

$\int u\,dv = uv - \int v\,du$

$\int_0^\infty x^2 e^{-x}$

$u = x^2 \quad v = -e^{-x}$

$du = 2x \quad dv = e^{-x}$

$-x^2 e^{-x} - \int -e^{-x}\,2x$

$-x^2 e^{-x} + 2\int$

$u = x \quad v = -e^{-x}$

$du = 1 \quad dv = e^{-x}$

$-2(-e^{-x}x - e^{-x})$

$\int -e^{-x}$

$xe^{-x} - \int(-e^{-x})$

$-e^{-x}$

$-x^2 e^{-x} - 2(-e^{-x} - xe^{-x})\Big|_0^\infty$

$0 - (-2)$

$= \boxed{2 \text{ hours}}$

d. Find the cumulative distribution function of the repair times.

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

For $x < 0$, $F(x) = 0$

For $x > 0$, $F(x) = \int_0^x xe^{-x}\,dx$

$$\Downarrow$$

$$1-(x+1)e^{-x}$$

$$\boxed{F(x) = \begin{cases} 0 & x \leq 0 \\ 1-(x+1)e^{-x} & x > 0 \end{cases}}$$

Section 2.5

2.

The bottom of a cylindrical container has an area of 10 cm2 . The container is filled to a height whose mean is 5cm, and whose standard deviation is 0.1 cm. Let V denote the volume of fluid in the container.

a. **Find** μ_V **.**

$\mu_v = A * H = 10 * 5 = 50cm^3$

b. **Find** σ_V **.**

$\sigma_v = 0.1 * 10 = 1cm^3$

3.

The lifetime of a certain transistor in a certain application has mean 900 hours and standard deviation 30 hours. Find the mean and standard deviation of the length of time that four bulbs will last.

$$\mu_x = E(x) = 900 + 900 + 900 + 900 = 4(900) = 3600 \textit{ hours}$$
$$\sigma_x = \sqrt{(30)^2 + (30)^2 + (30)^2 + (30)^2} = \sqrt{3600} = 60 \textit{ hours}$$

7.

The molarity of a solute in solution is defined to be the number of moles of solute per liter of solution (1 mole = 6.02 × 10 23 molecules). If X is the molarity of a solution of magnesium chloride (MgCl2), and Y is the molarity of a solution of ferric chloride (FeCl3), the molarity of chloride ion (Cl−) in a solution made of equal parts of the solutions of MgCl2 and FeCl3 is given by M = X + 1.5Y. Assume that X has mean 0.125 and standard deviation 0.05, and that Y has mean 0.350 and standard deviation 0.10.

a. **Find µM.**

$$\mu_M = \mu_x + 1.5(\mu_y) = 0.125 + 1.5(0.350) = 0.65$$

b. **Assuming X and Y to be independent, find σM.**

Variance of (x+y) = Var(x)+Var(y)
Var (ax) = a^2Var(x)

$$\sigma_M^2 = (x + 1.5y)$$
$$\sigma_M^2 = (x)^2 + (1.5)^2(y)^2$$
$$\sigma_M^2 = (0.05)^2 + (2.25)(0.10)^2 = 0.025$$
$$\sigma_M = \sqrt{0.025} = 0.158$$

8.

A machine that fills bottles with a beverage has a fill volume whose mean is 20.01 ounces, with a standard deviation of 0.02 ounces. A case consists of 24 bottles randomly sampled from the output of the machine.

a. **Find the mean of the total volume of the beverage in the case.**

$$\mu_x = \mathrm{E}(24\mathrm{x}) = 24{*}\mathrm{E}(\mathrm{x}) = 24(20.01) = 480.24$$

b. **Find the standard deviation of the total volume of the beverage in the case.**

$\sigma_x = \sqrt{Var(24x)} = \sqrt{24^2 * Var(x)} = 24 * \sqrt{V(x)} = 24 * \sigma_x$

$\sigma_x = 24*0.02 = 0.48$

c. Find the mean of the average volume per bottle of the beverage in the case.

$E(\overline{x}) = \frac{E(24x)}{24} = \frac{24}{24} * E(x) = 1 * 20.01 = 20.01$

d. Find the standard deviation of the volume per bottle of the beverage in the case.

$\sigma_x = \sqrt{\frac{\sigma_x^2}{n-1}} = \frac{0.02}{\sqrt{23}} = 0.00417$

e. How many bottles must be included in a case for the standard deviation of the average volume per bottle to be 0.0025 ounces?

$\sigma_x = \frac{\sigma_x}{\sqrt{n-1}} = 0.0025$

$\sqrt{n-1} * 0.0025 = 0.02$

$\sqrt{n-1} = \frac{0.02}{0.0025} = 8$

$n = 8^2 + 1 = 64 + 1 = 65$

11.

A certain commercial jet plane uses a mean of 0.15 gallons of fuel per passenger-mile, with a standard deviation of 0.01 gallons.

a. Find the mean number of gallons the plane uses to fly 8000 miles if it carries 210 passengers.

$\mu_x = (8000)(210)\mu = 8000\,(210 * 0.15) = 252000\, gallons$

b. Assume the amounts of fuel used are independent for each passenger-mile traveled. Find the standard deviation of the number of gallons of fuel the plane uses to fly 8000 miles while carrying 210 passengers.

$\sigma_x = \sqrt{(8000)(210 * \sigma_x^2)} = \sqrt{(8000)(210 * (0.01)^2)} = 12.961$

c. The plane used X gallons of fuel to carry 210 passengers 8000 miles. The fuel efficiency is estimated as X/(210 × 8000). Find the mean of this estimate.

$E(k) = \frac{E(x)}{210*8000} = \frac{252000}{210*8000} = 0.15 gallons$

d. **Assuming the amounts of fuel used are independent for each passenger-mile, find the standard deviation of the estimate in part (c).**

$$\sigma_k = \sqrt{\frac{Var(x)}{210*8000}} = \sqrt{\frac{(\sigma_x)^2}{210*8000}} = \sqrt{\frac{12.961}{210*8000}} = 7.71 * 10^6$$

15.

Measurements are made on the length and width (in cm) of a rectangular component. Because of measurement error, the measurements are random variables. Let X denote the length measurement and let Y denote the width measurement. Assume that the probability density function of X is

$$f(x) = \begin{cases} 10 & 9.95 < x < 10.05 \\ 0 & \text{otherwise} \end{cases}$$

and that the probability density function of Y is

$$g(y) = \begin{cases} 5 & 4.9 < y < 5.1 \\ 0 & \text{otherwise} \end{cases}$$

Assume that the measurements X and Y are independent.

a. **Find P(X < 9.98).**

P(x < 9.98) = 10(9.98-9.95) = 10(0.03) = 0.3

b. **Find P(Y > 5.01).**

P(y > 5.01) = 5(5.1-5.01) = 0.45

c. **Find P(X < 9.98 and Y > 5.01).**

P(X < 9.98 and Y > 5.01) = P(X < 9.98) * P(y > 5.01)
= 0.3 * 0.45 = 0.135
P(X < 9.98 and Y > 5.01) = 0.135

d. **Find μ_x.**

E(x) = $\int_{-\infty}^{\infty} xf(x)dx$

$= \int_{9.95}^{10.05} x(10)dx$

$=10 \frac{x^2}{2} \Big|_{9.95}^{10.05}$

=5(10.05)^2 - 5(9.95)^2

=10

e. Find μ_y.

$$E(y) = \int_{-\infty}^{\infty} yf(y)dy$$

$$= \int_{4.9}^{5.1} y(5)dy$$

$$=5\ \frac{y^2}{2}\ \Big|_{4.9}^{5.1}$$

=2.5(5.1)^2 - 2.5(4.9)^2

=5

16.

The thickness X of a wooden shim (in mm) has probability density function

$$f(x) = \begin{cases} \frac{3}{4} - \frac{3(x-5)^2}{4} & 4 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

a. Find μ_x.

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx$$

$$= \int_{4}^{6} x\left(\tfrac{3}{4} - \tfrac{3(x-5)^2}{4}\right)dx$$

$$= \frac{3}{4}\int_{4}^{6} x\left(1 - \frac{\frac{3(x-5)^2}{4}}{\frac{3}{4}}\right)\mathrm{dx}$$

$$= \frac{3}{4}\int_{4}^{6} x\,(-x^2 + 10x - 24\,\mathrm{dx})$$

$$= \frac{3}{4}\int_{4}^{6} (-x^3 + 10x^2 - 24x\,\mathrm{dx})$$

$$= \frac{3}{4}\left(-\frac{x^4}{4} + 10\frac{x^3}{3} - 12x^2\right)\Big|_4^6$$

$$= \left(-\frac{3x^4}{16} + \frac{5x^3}{2} - 9x^2\right)\Big|_4^6$$

=5

b. Find σ_x^2.

$$\sigma_x^2 = \int_{-\infty}^{\infty}(x-\mu_x)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - (\mu_x)^2 = E(x^2) - (E(x))^2$$

$$=\int_4^6 x^2\left(\tfrac{3}{4} - \tfrac{3(x-5)^2}{4}\right)dx - (5)^2$$

$$=\int_4^6 \left(\tfrac{3}{4}x^2 - \tfrac{3(x-5)^2}{4}(x^2)\right)dx - 25$$

$$=\left(-\tfrac{3x^5}{20} + \tfrac{15x^4}{8} - 6x^3\right)\Big|_4^6 - 25$$

$$=\left(-\tfrac{3(6)^5}{20} + \tfrac{15(6)^4}{8} - 6(6)^3\right) - \left(-\tfrac{3(4)^5}{20} + \tfrac{15(4)^4}{8} - 6(4)^3\right) - 25$$

$$=(126/5) - 25 = 0.2$$

c. **Let Y denote the thickness of a shim in inches (1 mm = 0.0394 inches). Find μ_y and σ_y^2.**

$\mu_y = (0.0394)\mu_x = (0.0394) * 5 = 0.197$
$\sigma_y^2 = (0.0394)^2\sigma_x^2 = (0.0394)^2 0.2 = 0.0003$

d. **If three shims are selected independently and stacked one atop another, find the mean and variance of the total thickness.**

$\mu_x = 3 * 5 = 15$
$\sigma_x^2 = 3(0.2) = 0.6$