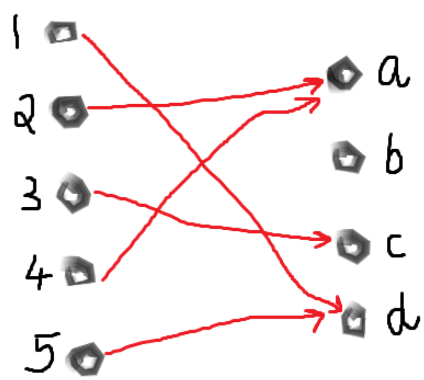
Functions

1) Define what is a function with example? The drawing below shows the arrow diagram for a function f. Give answers for what is domain, codomain, and range of function f.



Answer:

Suppose we assign elements of set A to elements of set B, then this is a function.

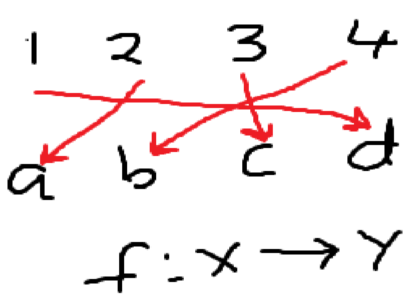
Domain: {1, 2, 3, 4, 5}

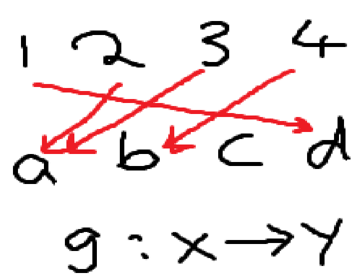
Co-Domain: {a, b, c, d}

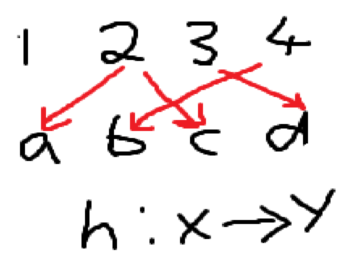
Range: {a, c, d}

2) (a) Which of the following diagrams represent a function? Explain in detail.

Let X= {1, 2, 3, 4} and Y= {a, b, c, d}.







(b) Are the three expressions given below well-defined functions from R to R? Explain your answer in detail.

$$f(x) = \frac{1}{x-2}$$

$$g(x) = \sqrt{x^2+2}$$

$$h(x) = \pm\sqrt{x^2+5}$$

(a) Answer:

f is a function. All elements are mapped from domain and no element is mapped to more than one in co-domain. g is also a function. h is not a function. 1 has not been mapped. 2 has been mapped more than once.

(b) Answer:

f is undefined for x = 2.

g is well defined for every real number. g results in positive numbers based on the implicit assumption of positive roots.

h is not well defined since here both positive and negative roots are allowed.

3) (a) Consider the function f: A→A Given by f (0) = 0 and f (a+1) = f(a)+2a+1. **Find f (6).**

(b) Give recursive definitions for the functions described below.

(i) f: B→B gives the number of butterflies in your terrarium 'b' years after you built it, assuming you started with 3 butterflies and the number of butterflies doubles each year.

(ii) g: B→B gives the number of Punches you do 'b' days after you started your Punching challenge, assuming you could do 7 Punches on day zero and you can do 2 more Punches each day.

(a)Answer:

f(6) = f(5 + 1) = f(5) + 2*5 +1 = f(4+1) +11 = f(4) + 8 + 1 + 11 = f(3 +1) +20 = f(3) + 6 + 1 + 20 = f(2+1) + 27 = f(2) + 4 + 1 + 27 = f(1+1) + 32 = f(1) + 2 + 1 + 32 = f(0 +1) + 35 = f(0) + 0 + 1 + 35

= 0 + 36 = 36.

(b)Answer:

f(0) = 3 and f(b+1) = f(b)*2

(c)Answer:

g(0) = 7 and g(b +1) = g(b) + 2

4) (i) The following functions have **{a, b, c, d, e}** as both their domain and codomain. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

$$(a)\ f = \begin{pmatrix} a & b & c & d & e \\ e & e & e & e & e \end{pmatrix}$$

$$(b)\ f = \begin{pmatrix} a & b & c & d & e \\ b & c & a & e & d \end{pmatrix}$$

(Hint:

x	0	1	2	3	4
$f(x)$	3	3	2	4	1

$\Rightarrow f = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 3 & 2 & 4 & 1 \end{pmatrix}.$)

(ii) The following functions have **{1, 2, 3, 4, 5}** as both their domain and codomain. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

$$(c)\ f(x) = 6 - x$$

$$(d)\ f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ (x+1)/2 & \text{if } x \text{ is odd} \end{cases}$$

(i) Answer:

(a) f is neither injective (all domain values are mapped to e) nor surjective (as the co-domain of f is larger than {e}).

(b) f is bijective since it is injective (all domain values map to unique co-domain values) and surjective (all co-domain values in {a,b,c,d,e} are mapped)

(ii) Answer:

F(x) = 6-x is bijective. Since is injective (every value in domain is mapped to a unique co-domain value) and surjective (all values y in co-domain have a map from 6-y in the domain).

F(x) = x/2 if x is even and (x+1)/2 if x is odd. This function is not injective (since 2 and 3 are mapped to the same value 2). F is not surjective (there is no clear definition going from co-domain value y to domain value x.). Hence it is not bijective.

5) Use the definition of the functions **f** below to **Compute f (2.2), f (2.9), f (2.5), f (2), and f (3).**

$$(a)\ f(x) = \left\lceil \frac{x}{2} + 2x + 1 \right\rceil$$

$$(b)\ f(x)\ \left\lfloor x + 1.5 \right\rfloor$$

Answer:

x =	2.2	2.9	2.5	2	3
(a) f(x) =	7	9	8	6	9
(b) f(x) =	3	4	4	3	4

6) (a) Define five types of functions with examples.

F: N -→ N (natural numbers to natural numbers)

- f(x) = 2*x, (bijective)
- f(x) = 2/(x-1) (not a function)
- f(x) = x+1 (injective but not surjective, co-domain value 1 has no map in domain)
- f(x) = x^2 (injective not surjective, co-domain value 3 has no domain map).
- f(x) = x+2 if x is odd, x-1 if x is even (not injective, not surjective)

f: R→R (real numbers to real numbers)

F(x) = x^2 (not injective, but surjective)

(b) match the following relative sizes of the domain (D) and target (T) of functions:

1) $|D| \geq |T|$

2) $|D| \leq |T|$ and $|D| \geq |T|$
$|D| = |T|$

3) $|D| \leq |T|$

(i) bijection

(ii) one-to-one

(iii) onto

Answer:

1 $\lvert D \rvert \geq \lvert T \rvert$ length of D ≥ length of T	**Onto (iii)**
2 $\lvert D \rvert \leq \lvert T \rvert$ & $\lvert D \rvert \geq \lvert T \rvert \Rightarrow \lvert D \rvert = \lvert T \rvert$	**Bijection (i)**
3 $\lvert D \rvert \leq \lvert T \rvert$	**One-to-one (ii)**

(c) At the end of the semester, professor assigns letter grades to each of his students. Is this a function? If so, what sets make up the domain and codomain, and is the function injective, surjective, bijective, or neither?

Answer:
Domain: Set of Students
Co-Domain: Set of grades
Range: Only the unique grades that were mapped for all students.

Every student need not get unique grade (not injective)

If every grade type does not have a map from a student (not surjective).

If every grade type has a map from a student who got it (surjective)

If there are n (say 5) grades and same number n (say 5) students and each student gets a unique grade, then it is bijective.

7) (a) Given g = {(4, x), (5, y), (6, w)}, a function from X = {4, 5, 6} to Y = {w, x, y, z} and

f = {(w, b), (x, b), (y, d), (z, a)}, a function from Y to Z = {a, b, c, d}.Write as a set of ordered pairs and draw the arrow diagram of (f o g).

(b) Let *f* and *g* be functions from the positive real numbers to the positive real numbers defined by the equations given below (observe the brackets carefully). Find the compositions of (f o g), (g o f), (f o f) and (g o g).

$$f(x) = \lfloor 3x \rfloor, \quad g(x) = x^2$$

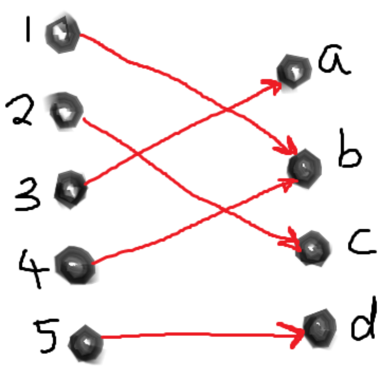
Answer:

(a) (f o g) = { (4, b), (5, d), (6, b)
(b)

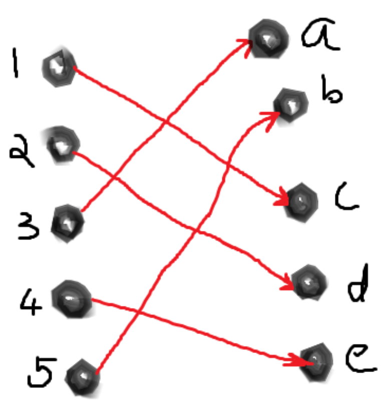
(f o g)	(g o f)	(f o f)	(g o g)
$3\lfloor x^2 \rfloor$	$(\lfloor 3x \rfloor)^2$	$\lfloor 3\lfloor 3x \rfloor \rfloor$	x^4

8) Each of the arrow diagrams below define a function f. For each arrow diagram, indicate whether f^-1 (f inverse) is well-defined. If f^-1 is not well-defined, indicate why. If f^-1 is well-defined, give an arrow diagram showing f^-1.

(a)



(b)



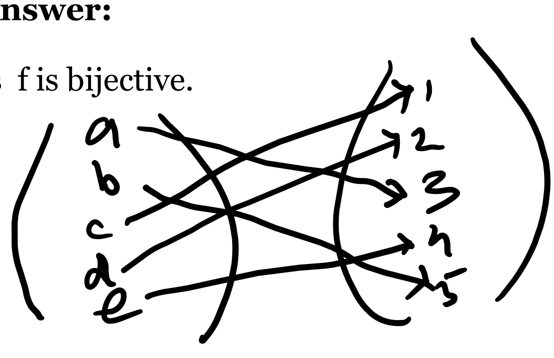
(a) Answer:

f^{-1} is not defined since f is not one-to-one f(1) = b and f(4) = b

(b) Answer:

f^{-1} exists as f is bijective.

It is:



9) Answer the following questions using logarithms**:**

(a) $6^x = 45$, find x (approx)

(b) $\log_5 10 + \log_5 6 = ?$

(c) $\log_2 45 - \log_2 9 = ?$

(d) $\log_9 16 \Rightarrow \boxed{?} \log_9 4$

fill the box

(e) $\log_7 10 + \log_7 8 - \log_7 4 = ?$

Answer:

(a) $6^x = 45 \Rightarrow x \log 6 = \log 45 \Rightarrow x = \log_{10} 45 / \log_{10} 6$

(b) $\log_5 10 + \log_5 6 = \log_5 (6 \times 10) = \log_5 60$

(c) $\log_2 45 - \log_2 9 = \log_2 (45/9) = \log_2 5$

(d) $\log_9 16 \Rightarrow ? \log_9 4 \Rightarrow 2 \log_9 4$

(e) $\log_7 10 + \log_7 8 - \log_7 4 = \log_7 (10 \times 8)/4$

$= \log_7 20$

10) (a) Indicate whether the two functions are equal. If the two functions are not equal, then give an element of the domain on which the two functions have different values.

(i) s: Z → Z, where s(x) = x^3.
h: Z → Z, where h(x) = |x|^3.

(ii) s: Z → Z, where s(x) = x^4.
h: Z → Z, where h(x) = |-x|^4

(b) Express the range of function g.

Let A = {2, 3, 4, 5, 6}.
g: A → Z such that g(x) = 2x + x^2 - 1.

Answer:

(i) Not same for negative value.
(ii) Same.
(iii) Range = {g(2), g(3), g(4), g(5), g(6)} = {7, 14, 23, 34, 47}.