

Proofs (10 points)

1. Bubblesort is a popular, but inefficient, sorting algorithm. It works by repeatedly swapping adjacent elements that are out of order.

BUBBLESORT(A)

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1  for  $i = 1$  to  $A.length - 1$ 
2      for  $j = A.length$  downto  $i + 1$ 
3          if  $A[j] < A[j - 1]$ 
4              exchange  $A[j]$  with  $A[j - 1]$ 
```

- (a) [1 point] Let A' denote the output of BUBBLESORT(A). To prove that BUBBLESORT is correct, we need to prove that it terminates and that

$$A'[1] \leq A'[2] \leq \dots \leq A'[n]$$

where $n = A.length$. In order to show that BUBBLESORT actually sorts, what else do we need to prove?

- (b) [2 points] State precisely a loop invariant for the **for** loop in lines 2–4, and prove that this loop invariant holds. Your proof should use the structure of the loop invariant proof.

- (c) [3 points] Using the termination condition of the loop invariant proved in part (b), state a loop invariant for the **for** loop in lines 1–4 that will allow you to prove inequality above. Your proof should use the structure of the loop invariant proof.

2. Prove by induction.

- (a) [2 points] For all $n \geq 1$, $\sum_{i=1}^n (2i - 1) = n^2$.

(b) [2 points] For all $n \geq 0$, $\sum_{i=0}^n x^i = (1 - x^{n+1}) / (1 - x)$.