STAT 50 HW #10

Section 4.3 #'s 1, 3, 7, 13, 17

1.

Let X ~ Poisson(4). Find

a. **P(X = 1)**

$X \sim \text{Poisson}(4)$

a) $P(X=1)$

$$P(x) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x \text{ a non-neg integer} \\ 0, & \text{otherwise} \end{cases}$$

$$P(X=1) = \frac{e^{-4}4^{1}}{1!} = \boxed{0.0733}$$

$4 \cdot 3 \cdot 2$

b. **P(X = 0)**

b) $P(X=0)$

$$P(X=0) = \frac{e^{-4}4^{0}}{0!} = \boxed{0.0183}$$

c. **P(X < 2)**

c) $P(X<2) = P(X=0) + P(X=1)$

$$= 0.0183 + 0.0733 = \boxed{0.0916}$$

d. **P(X > 1)**

d) $P(X>1) = 1 - P(X \le 1)$

$$= 1 - (P(X=0) + P(X=1))$$

$$= 1 - 0.0916 = \boxed{0.9084}$$

e. μ_x

e) $\mu_x = \sum_{x=0}^{\infty} x\,p(x) = \sum_{x=0}^{\infty} \frac{x e^{-\lambda} \lambda^x}{x!}$

$\mu_x = \sigma_x^2 = \lambda$, so $\boxed{\mu_x = 4}$

f. σ_x

f) σ_x ← std dev σ_x^2 ← var

$E(X^2) = \lambda^2 + \lambda$ $\sigma_x^2 = E(X^2) - (E(X))^2$

$= \lambda^2 + \lambda - \lambda^2 = \lambda$

$\lambda = 4$ $\sigma_x^2 = 4$ $\sigma_x = \sqrt{\sigma_x^2} = \sqrt{4} = \boxed{2}$

3.

The number of large packages delivered by a courier service follows a Poisson distribution with a rate of 5 per day. Let X be the number of large packages delivered on a given day. Find

a. P(X = 6)

$X \sim \text{Poisson}(5)$

a) $P(X=6)$

$P(X=6) = \frac{e^{-5} 5^6}{6!} = \boxed{0.1462}$

b. P(X ≤ 2)

b) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$P(X=0) = \frac{e^{-5} 5^0}{0!} = 0.0067$

$P(X=1) = \frac{e^{-5} 5^1}{1!} = 0.0337$

$P(X=2) = \frac{e^{-5} 5^2}{2!} = 0.0842$

$0.0067 + 0.0337 + 0.0842 = \boxed{0.1247}$

c. P(5 < X< 8)

c) $P(5<x<8) = P(x=6) + P(x=7)$

$P(x=6) = \frac{e^{-5}5^{6}}{6!} = 0.1462$

$P(x=7) = \frac{e^{-5}5^{7}}{7!} = 0.1044$

$0.1462 + 0.1044 = \boxed{0.2507}$

d. μ_x

$x \sim \text{Poisson}(5)$

d) $\mu_x \rightarrow$ If $x \sim \text{Poisson}(\lambda)$

then, $\mu_x = \lambda$

$\sigma_x^2 = \lambda$

So, $\boxed{\mu_x = 5}$

e. σ_x

e) σ_x $\quad \sigma_x = \sqrt{\sigma_x^2}$

$\sqrt{5}$

$\boxed{\sigma_x = 2.2361}$

7.

The number of hits on a certain website follows a Poisson distribution with a mean rate of 4 per minute.

a. **What is the probability that 5 hits are received in a given minute?**

$X \sim \text{Poisson}(4)$

a) $P(X=5)$

$P(X=5) = \dfrac{e^{-4}4^5}{5!} = \boxed{0.1563}$

b. **What is the probability that 9 hits are received in 1.5 minutes?**

b) $P(X=9)$ in 1.5 min? $\mu_X = 4 \times 1.5 = 6$

$P(X=9) = \dfrac{e^{-6}6^9}{9!} = \boxed{0.0688}$

c. **What is the probability that fewer than 3 hits are received in a period of 30 seconds?**

c) $P(X<3)$ in 0.5 min? $\mu_X = 4 \times 0.5 = 2$

$P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$P(X=0)$

$\dfrac{e^{-2}2^0}{0!} = 0.1353 \quad \dfrac{e^{-2}2^1}{1!} = 0.2707 \quad \dfrac{e^{-2}2^2}{2!} = 0.2707$

$0.1353 + 0.2707 + 0.2707 = \boxed{0.6767}$

13.

The number of defective components produced by a certain process in one day has a Poisson distribution with mean 20. Each defective component has probability 0.60 of being repairable.

a. **Find the probability that exactly 15 defective components are produced.**

$X \sim \text{Poisson}(20)$ defective components have 0.60 prob of being repairable.

P(defective)

a) $P(X=15)$

$P(X=15) = \dfrac{e^{-20}\,20^{15}}{15!} = \boxed{0.0516}$

b. **Given that exactly 15 defective components are produced, find the probability that exactly 10 of them are repairable.**

b) Given that exactly 15 defective components are produced, find the prob that 10 of them are repairable.

$P(Y=10) \rightarrow X \sim \text{Bin}(15, 0.6)$ for $Y=10$

$P(Y=10) = \binom{15}{10}(0.6)^{10}(0.4)^{5}$

${}_{15}C_{10} = \dfrac{15!}{(15-10)!\,10!} = \dfrac{15\cdot14\cdot13\cdot12\cdot11\cdot\cancel{10!}}{\cancel{10!}\cdot 5\cdot4\cdot3\cdot2\cdot1} = 3003$

$3003(0.006)(0.0102) = \boxed{0.1859}$

c. **Let N be the number of defective components produced, and let X be the number of them that are repairable. Given the value of N, what is the distribution of X?**

c) N = # of def comp. produced

X = # that are repairable

Given N, what is the distribution of X?

$\downarrow$

$\boxed{X \sim \text{Bin}(N, 0.6)}$

d. **Find the probability that exactly 15 defective components are produced, with exactly 10 of them being repairable.**

d) Find prob that exactly 15 def. are observed with 10 lasting reasonable.

$P(X=15) * P(Y=10)$

$0.0516 * 0.1859 = \boxed{0.0096}$

17.

Mom and Grandma are each baking chocolate chip cookies. Each gives you two cookies. One of Mom's cookies has 14 chips in it and the other has 11. Grandma's cookies have 6 and 8 chips.

a. Estimate the mean number of chips in one of Mom's cookies.

17. Mom → 14, 11

Grandma → 6, 6

a) μ_{mom}? $\mu_{mom} = \frac{14+11}{2} = \frac{25}{2} = \boxed{12.5 \text{ chips}}$

b. Estimate the mean number of chips in one of Grandma's cookies.

b) μ_{GMA}? $\mu_{GMA} = \frac{6+6}{2} = \frac{14}{2} = \boxed{7 \text{ chips}}$

c. Find the uncertainty in the estimate for Mom's cookies.

c) Uncertainty in estimate for mom's cookies?

$\sigma_{\bar{x}} = \frac{\sigma_x}{6} = \frac{\sigma_x}{6} = \frac{\sqrt{\lambda}}{6} = \sqrt{\frac{\lambda}{6}}$ ← 2 for 2 cookies $X \sim \text{Poisson}(12.5)$

$\sqrt{\frac{12.5}{2}} = \boxed{2.5}$

d. Find the uncertainty in the estimate for Grandma's cookies.

d) Uncertainty in estimate for grandma's cookies?

$$\sigma_{\hat{\lambda}} = \sqrt{\frac{7}{12}} = \boxed{1.9}$$

e. **Estimate how many more chips there are on the average in one of Mom's cookies than in one of Grandma's. Find the uncertainty in this estimate.**

e) Estimate how many more chips on average in one of Mom's cookies than in one of Grandma's then find the uncertainty.

$$12.5 - 7 = 5.5$$

$$12.5 + 7 = 19.5 \rightarrow \sqrt{\frac{19.5}{2}} = 3.1$$

So, we get $\boxed{5.5 \pm 3.1}$