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CSC 28 - Section 05

1) (i) Let p denote "He is rich" and let q denote "He is happy." Write each statement in symbolic form using p and q. Note that "He is poor" and "He is unhappy" are equivalent to ¬p and ¬q, respectively.

(a) If he is rich, then he is unhappy.

$p \rightarrow \neg q$

(b) He is neither rich nor happy.

$\neg p \wedge \neg q$

(c) It is necessary to be poor in order to be happy.

$\neg p \leftrightarrow q$

(d) To be poor is to be unhappy.

$\neg p \rightarrow \neg q$

(ii) Let p be "It is cold" and let q be "It is raining". Give a simple verbal sentence which describes each of the following statements:

(a) ¬p

It is not cold.

(b) p ∧ q

It is cold and it is raining.

(c) p ∨ q

It is cold or it is raining.

(d) q ∨ ¬p. In each case, translate ∧, ∨, and ~ to read "and," "or," and "It is false that" or "not," respectively, and then simplify the English sentence.

It is raining or it is not cold.

(4 + 4) points + 2 points for neatness.

2) Construct the truth table of:

$$\neg\Big(\big((\neg p)\vee(\neg(q\leftrightarrow(\neg q)))\big)\wedge\big((p\leftrightarrow p)\rightarrow(q\leftrightarrow(\neg q))\big)\Big)$$

(2 points) + 1 point for neatness.

p	q	¬p	¬q	p↔p	q↔(¬q)	¬(q↔(¬q))	(¬p)∨(¬(q↔(¬q)))	(p↔p)→(q↔(¬q))	((¬p)∨(¬(q↔(¬q))))∧((p↔p)→(q↔(¬q)))	¬(((¬p)∨(¬(q↔(¬q))))∧((p↔p)→(q↔(¬q))))
T	T	F	F	T	F	T	T	F	F	T
T	F	F	T	T	F	T	T	F	F	T
F	T	T	F	T	F	T	T	F	F	T
F	F	T	T	T	F	T	T	F	F	T

3) (a) Verify that the proposition (p ⋁ ¬(p ⋀ q) ⋁ q) and proposition ((¬p ↔ ¬q) ↔ (q ↔ r))

is a tautology or not. (using truth tables)

The proposition (p ⋁ ¬(p ⋀ q) ⋁ q) is a tautology according to the truth table below:

p	q	p∧q	¬(p∧q)	p∨¬(p∧q)	¬(p∧q)∨q	p∨¬(p∧q)∨q
T	T	T	F	T	T	T
T	F	F	T	T	T	T
F	T	F	T	T	T	T
F	F	F	T	T	T	T

The proposition ((¬p ↔ ¬q) ↔ (q ↔ r)) is not a tautology according to the truth table below:

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \oplus r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \oplus r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

(b) Consider the conditional proposition p → q. The simple propositions q → p, ¬p → ¬q and ¬q → ¬p are called, respectively, the converse, inverse, and contrapositive of the conditional p → q. Find if any of these propositions are logically equivalent to p → q? (using truth tables)

(2 + 2 points) + 2 points for neatness.

Truth table for p → q (Conditional Statement):

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Truth table for q → p (Converse) *For this truth table, you can think of moving the p values to the right of the q values:

p	q	p	q → p
T	T	T	T
T	F	T	T
F	T	F	F
F	F	F	T

Truth table for ¬p → ¬q (Inverse):

p	q	¬p	¬q	¬p → ¬q
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Truth table for ¬q → ¬p (Contrapositive):

p	q	¬q	¬p	¬q → ¬p
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

The only proposition that is logically equivalent to $p \rightarrow q$ would be $\neg q \rightarrow \neg p$ based on the results of the truth tables.

4) Give the inverse, contrapositive, and converse for each of the following statements:

(a) If it rained last night, then the sidewalk is wet.

Inverse: If it had not rained last night, then the sidewalk is not wet.

Contrapositive: If the sidewalk is not wet, then it had not rained last night.

Converse: If the sidewalk is wet, then it rained last night.

(b) If I ride a roller coaster then I will get sick.

Inverse: If I don’t ride a roller coaster, then I will not get sick.

Contrapositive: If I don’t get sick, then I will not ride a roller coaster.

Converse: If I get sick, then I will ride a roller coaster.

(c) If 2 is a prime number, then 7 is an even number.

Inverse: If 2 is not a prime number, then 7 is not an even number.

Contrapositive: If 7 is not an even number, then 2 is not a prime number.

Converse: If 7 is an even number, then 2 is a prime number.

(d) If two angles are congruent, then they have the same measure.

Inverse: If two angles are not congruent, then they do not have the same measure.

Contrapositive: If they do not have the same measure, then the two angles are not congruent.

Converse: If they have the same measure, then the two angles are congruent.

(e) If $8 < 6$, then $6 < 4$.

Inverse: If $8 \nless$ (is not less than) 6, then $6 \nless$ (is not less than) 4.

Contrapositive: If $6 \nless$ (is not less than) 4, then $8 \nless$ (is not less than) 6.

Converse: If $6 < 4$, then $8 < 6$.

(5 points + 1 point for neatness)

5) Translate each English sentence into a logical expression using the propositional variables defined below. Then negate the entire logical expression using parentheses and the negation operation. Apply De Morgan's law to the resulting expression and translate the final logical expression back into English.

R: the candidate has written permission from his guardian.

S: the candidate is at least 20 years old

T: the candidate is at least 14 years old

(a) The candidate has written permission from his guardian and is at least 14 years old.

$R \wedge T$

$\neg(R \wedge T) = \neg R \vee \neg T$

The candidate does not have written permission from his guardian or is not at least 14 years old.

(b) The candidate has written permission from his guardian or is at least 20 years old.

$R \vee S$

$\neg(R \vee S) = \neg R \wedge \neg S$

The candidate does not have written permission from his guardian and is not at least 20 years old.

(2 points) + 1 point for neatness

6) Prove without truth tables to show the following (Hint: use a series of known logical equivalences to go from one proposition to the other):

(a) $[\neg(p \wedge q) \vee (p \wedge q)] \equiv T$

(b) $\neg(\neg p \wedge q) \equiv p \vee \neg q$

(c) $\neg p \wedge (p \vee q) \equiv \neg p \wedge q$

(d) $\neg(\neg p \vee (p \vee q)) \to q$ is a tautology

$$\urcorner(\urcorner P \cup (P \cup Q)) \to Q$$

A. [¬(p∧q)∨(p∧q)]≡T

a) $[\neg(p \wedge q) \vee (p \wedge q)] \equiv T$

$[\neg(p \wedge q) \vee (p \wedge q)] \equiv (p \wedge q) \supset (p \wedge q)$ $\quad \neg p \vee q \equiv p \supset q$

$\equiv T$ $\quad p \supset p \equiv T$

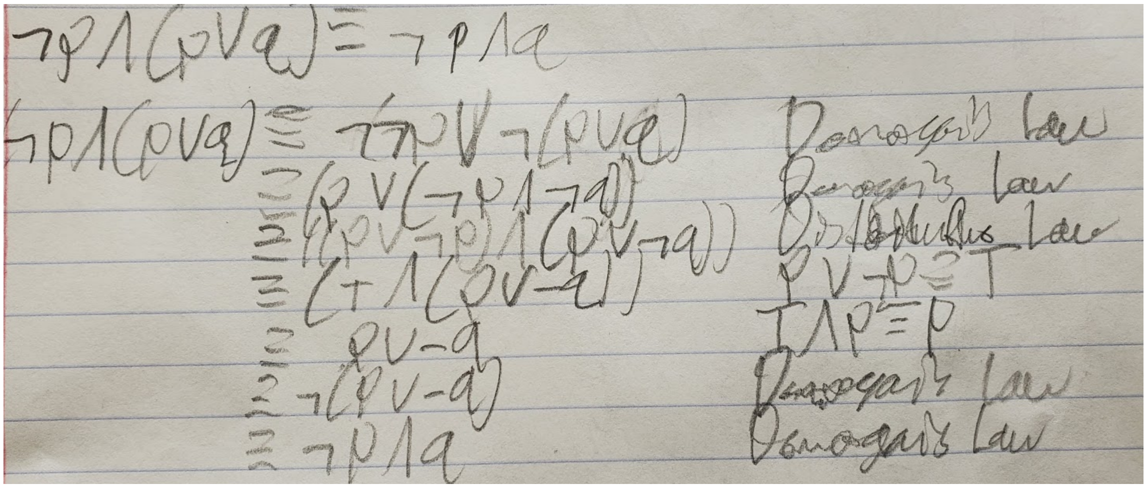
B. ¬(¬p∧q)≡p∨¬q

b) $\neg(\neg p \wedge q) \equiv p \vee \neg q$

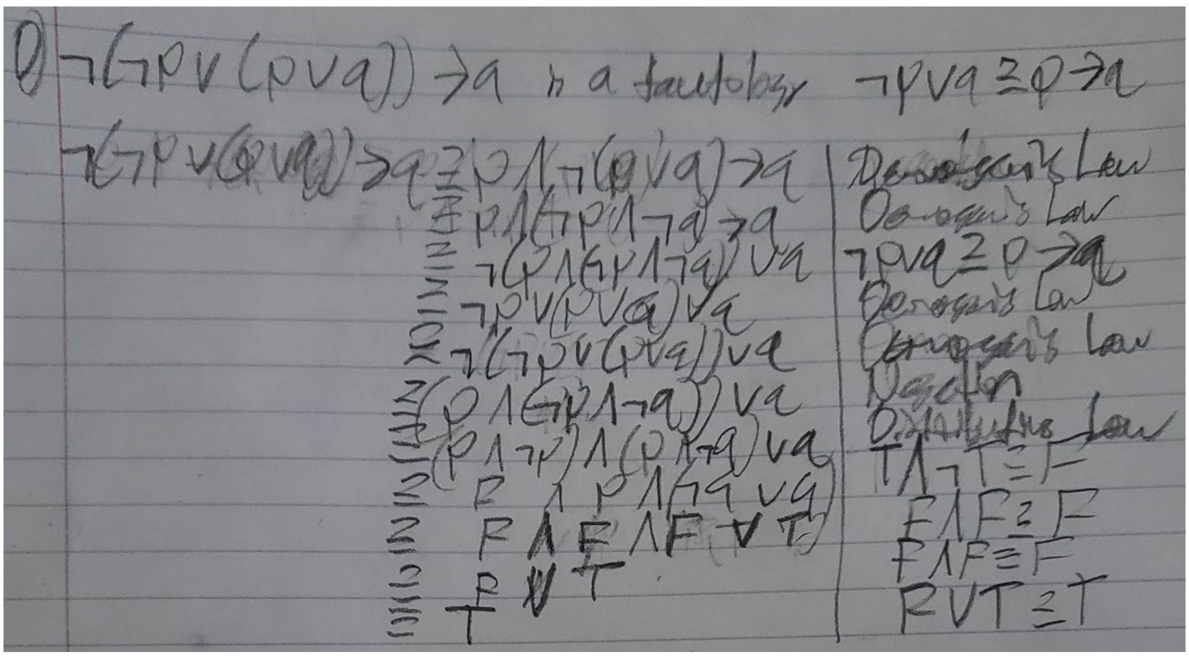
$\neg(\neg p \wedge q) \equiv \neg(\neg p) \vee \neg q$ $\quad$ Demorgan's Law

$\equiv p \vee \neg q$ $\quad$ Demorgan's Law

C. ¬p∧(p∨q)≡¬p∧q



D. $\neg(\neg p \vee (p \vee q)) \rightarrow q$ is a tautology



(4 points) + 1 point for neatness.

7) Translate the following predicate logic to English. Let H(x) represent "x is happy," let C(y) represent "y is a computer," and let O(x, y) represent "x owns y." (The domain of discourse for x consists of people, and the domain for y consists of inanimate objects.)

(a) Jack owns a computer

O(Jack, Computer)

(b) If Jack owns a computer, then he’s happy

$O(Jack, Computer) \rightarrow H(Jack)$

(c) Everything Jack owns is a computer

D = Domain of objects

$(\forall y \in D, C(y) \rightarrow O(Jack, y))$

(d) Everyone is happy

D = Domain of people

$(\forall x \in D, H(x))$

(e) Everyone is unhappy

D = Domain of people

$(\forall x \in D, \neg H(x))$

(f) Someone is unhappy

D = Domain of people

$(\exists x \in D, \neg H(x))$

(6 points) + 1 point for neatness

8) In the following question, the domain is a set of students who show up for a test. Define the following predicates:

R(x): x showed up with a Red-pen

B(x): x showed up with a Blue-pen

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

(a) At least one of the students showed up with a Red-pen.

S = Domain of students

$(\exists x \in S, R(x))$

$\neg(\exists x \in S, R(x))$

$\forall x \in S, \neg R(x)$

All students did not show up with a red pen.

(b) Every student who showed up with a Blue-pen also had a Red-pen.

$p \rightarrow q \equiv \neg p \vee q$

S = Domain of students

$(\forall x \in S, B(x) \rightarrow R(x))$

$\neg(\forall x \in S, B(x) \rightarrow R(x))$

$\exists x \in S, \neg B(x) \rightarrow \neg R(x)$

At least one student who did not show up with a blue pen also did not have a red pen.

(2 points) + 1 point for neatness.

9) Which of the following arguments are invalid and which are valid? Prove your answer by replacing each proposition with a variable to obtain the form of the argument. Then prove that the form is valid or invalid. The symbol with dots has the meaning: “therefore or implies”.

(a) Rohit has football or basketball or both
Rohit has basketball or volleyball or both

∴ Rohit has football or volleyball

(b) Rohit studied for the exam or Rohit failed the exam or both.
Rohit passed the exam.

∴ Rohit studied for the exam.

(c) 2 is an odd integer or 2 is a negative integer.
2 is not a negetive integer.

∴ 2 is an odd integer

(3 points) + 1 points for neatness.

a)

Let P = Rohit has football.

Let Q = Rohit has basketball.

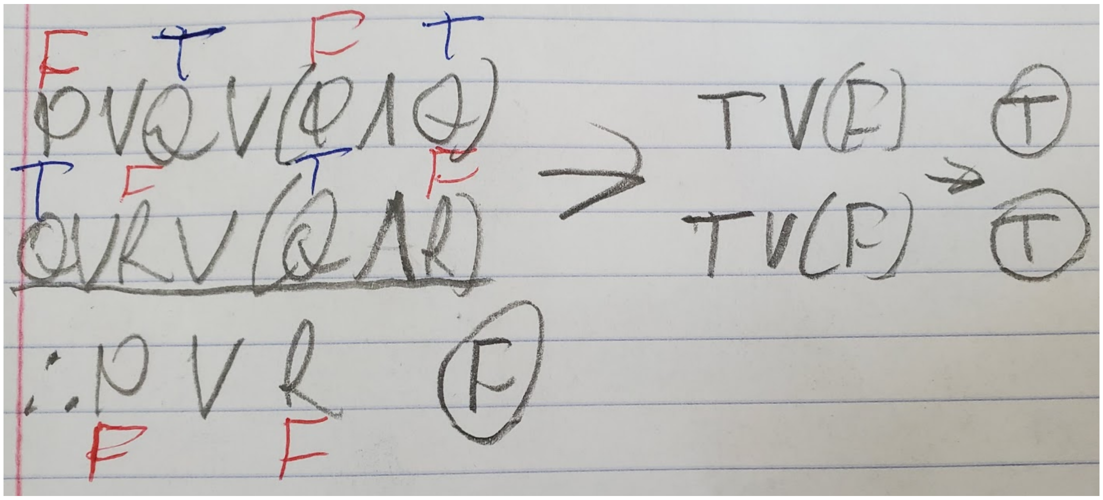
Let R = Rohit has Volleyball.

$P \vee Q \vee (P \wedge Q)$

$Q \vee R \vee (Q \wedge R)$

$\therefore P \vee R$

This argument is invalid as it would be possible to make all the premises true and the conclusion false.



b)

Let P = Rohit studied for the exam.

Let Q = Rohit failed the exam.

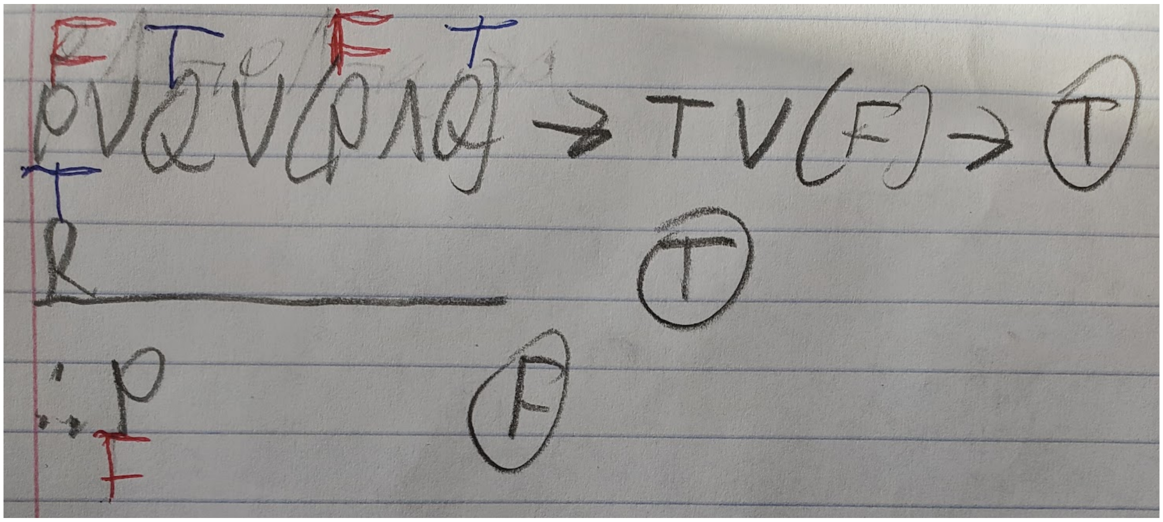
Let R = Rohit passed the exam.

$P \lor Q \lor (P \land Q)$

R

∴ P

This argument is invalid as it would be possible to make all the premises true and the conclusion false.



c)

Let P = 2 is an odd integer.

Let Q = 2 is a negative integer.

P∨Q

¬Q

∴P

This argument is valid as it would not be possible to make all the premises true and the conclusion false.

$F \quad F$

$P \vee Q \rightarrow F$

F

$\neg Q \quad T$

$\therefore P \quad F$

F

10) Give the form of each argument. Then prove whether the argument is valid or invalid.

For valid arguments, <u>use the rules of inference to prove validity.</u>

(a) If Rohit gets money then rohit will buy car and house.
Rohit won't buy house.

∴ Rohit will not get money

(b) Rohit will buy car and house only if Rohit gets money.
Rohit not going to get money.
Rohit will buy house.

∴ Rohit will not buy car

(2 points) + 1 point for neatness

a)

Let P = Rohit gets money.

Let Q = Rohit buys a car.

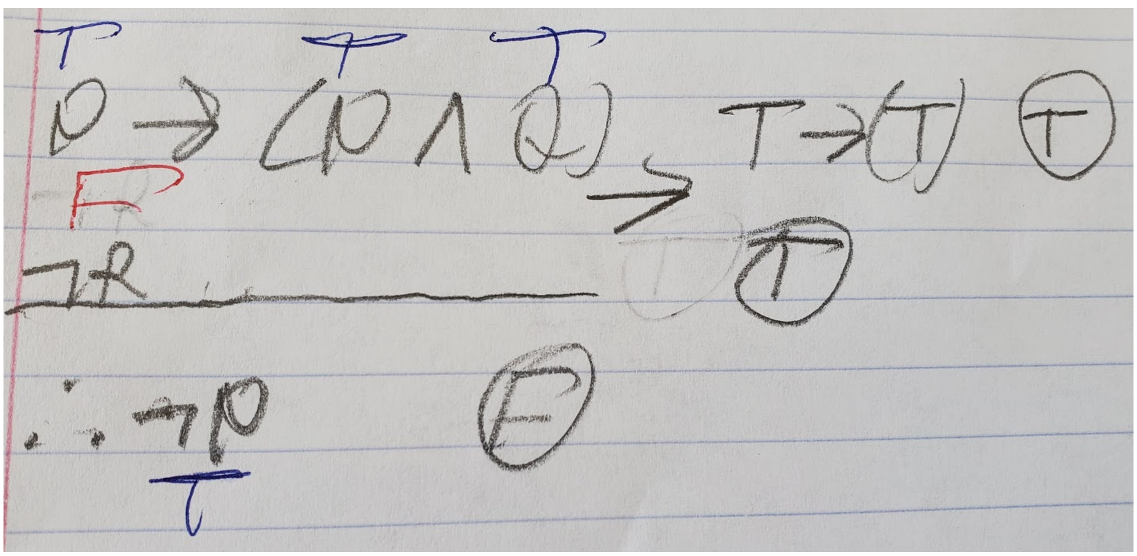
Let R = Rohit buys a house.

$P \rightarrow (P \wedge Q)$

$\neg R$

$\therefore \neg P$

This argument is invalid as it would be possible to make all the premises true and the conclusion false.



b)

Let P = Rohit gets money.

Let Q = Rohit buys a car.

Let R = Rohit buys a house.

$(Q \wedge R) \rightarrow P$

¬P

R

∴¬Q

This argument is valid as it would not be possible to make all the premises true and the conclusion false.