Quiz 2

$T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}$ Note that $T: \mathbb{R}^2 \to \mathbb{R}^3$

$\mathbb{R}^2 \downarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $\mathbb{R}^3 \downarrow \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

① Find $T(\vec{x})$ where

a) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \to T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1\cdot(1) + (-2)(0) \\ 1\cdot(4) + (3)(0) \\ 1\cdot(5) + (-1)(0) \end{bmatrix} = \begin{bmatrix} 1+0 \\ 4+0 \\ 5+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}$

b) $\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \to T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0\cdot(1) + (-2)(1) \\ 0\cdot(4) + (3)(1) \\ 0\cdot(5) + (-1)(1) \end{bmatrix} = \begin{bmatrix} 0-2 \\ 0+3 \\ 0-1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

c) $\vec{x} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \to T\left(\begin{bmatrix} 5 \\ 6 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 5\cdot(1) + (-2)(6) \\ 5\cdot(4) + (3)(6) \\ 5\cdot(5) + (-1)(6) \end{bmatrix} = \begin{bmatrix} 5-12 \\ 20+18 \\ 25-6 \end{bmatrix} = \begin{bmatrix} -7 \\ 38 \\ 19 \end{bmatrix}$

② Want to find $\vec{y}$ $\left(\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right)$ such that $T(\vec{y}) = A\vec{y} = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

So, $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 4 & 3 & 1 \\ 5 & -1 & 1 \end{array}\right] \xrightarrow[-5R_1+R_3 \to R_3]{-4R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 11 & -3 \\ 0 & 9 & -4 \end{array}\right] \xrightarrow[\frac{1}{9}R_3 \to R_3]{\frac{1}{11}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -3/11 \\ 0 & 1 & -4/9 \end{array}\right] \to$

$-1R_2 + R_3 \to R_3$

$0 \quad -1 \quad +\frac{27}{99}$ $\left(\frac{3}{11}\cdot\frac{9}{9}\right)$

$0 \quad 1 \quad -\frac{44}{99}$ $\left(\frac{-4}{9}\cdot\frac{11}{11}\right)$

$0 \quad 0 \quad -17/99$

$\to \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & -3/11 \\ 0 & 0 & -17/99 \end{array}\right]$

$\hookrightarrow 0 \neq \frac{-17}{99}$

$\hookrightarrow$ inconsistent

$\hookrightarrow$ no solution

$\hookrightarrow$ no $\vec{y}$.

③ Looking for $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ such that $T(\vec{y}) = \begin{bmatrix} 5 \\ 9 \\ 16 \end{bmatrix}$

So, $\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 4 & 3 & 9 \\ 5 & -1 & 16 \end{array}\right] \xrightarrow[\text{(same as \#2)}]{\text{(same as \#2)}} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 11 & -11 \\ 0 & 9 & -9 \end{array}\right] \xrightarrow[\frac{1}{9}R_3 \to R_3]{\frac{1}{11}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow[2R_2+R_1 \to R_1]{-1R_2+R_3 \to R_3} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right] \to$ thus $y_1 = 3$, $y_2 = -1$ $\to \boxed{\vec{y} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}}$

this is consistent

$0=0$

$\hookrightarrow T\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 & -2 \\ 4 & 3 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 9 \\ 16 \end{bmatrix}$