1 Sequences

1. Determine whether or not the sequence below converges. If it does, find the limit.

$$\{a_n\} = \ln(n+1) - \ln n$$

$\{a_n\} = \ln(n+1) - \ln n = \ln\left(\frac{n+1}{n}\right)$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \ln\left(\frac{n+1}{n}\right) = \ln(1) = 0$

$\lim\limits_{n\to\infty} a_n = 0$

2. Determine whether or not the sequence below converges. If it does, find the limit.

$$\{b_n\} = \sqrt[n]{n}$$

$\{b_n\} = \sqrt[n]{n} = (n)^{1/n} = n^{1/n} = \{1, \sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \ldots\}$

Since $\{b_n\}$ is decreasing and bounded (below), $\{b_n\}$ must converge.

$\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} n^{1/n} = \lim\limits_{n\to\infty} e^{\ln(n^{1/n})} = \lim\limits_{n\to\infty} e^{\frac{1}{n}\cdot\ln n} = 1$

$\lim\limits_{n\to\infty} b_n = 1$

2 Series

1. Find the sum of the series shown below using a calculator and using the formula.

$$\sum_{n=1}^{8} \frac{5^n}{(-7)^{n+1}}$$

$$\{a_n\} = \frac{5^n}{(-7)^{n+1}} = \frac{5^n}{(-7)\cdot(-7)^n} = -\frac{1}{7}\left(-\frac{5}{7}\right)^n$$

$$\sum_{n=1}^{8} -\frac{1}{7}\left(-\frac{5}{7}\right)^n = -\frac{1}{7}\sum_{n=1}^{8}\left(-\frac{5}{7}\right)^n = -\frac{1}{7}\left[\frac{-\frac{5}{7}\left(1-\left(-\frac{5}{7}\right)^8\right)}{1-\left(-\frac{5}{7}\right)}\right]$$

$$\sum_{n=1}^{8} \frac{5^n}{(-7)^{n+1}} = 0.0555$$

2. Determine the convergence of the series shown below. If it converges, find the limit.

$$\sum \sin n$$

$$\{b_n\} = \sin n$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \sin n \neq 0$$

$\sum \sin n$ is divergent

3. Determine the convergence of the series shown below. If it converges, find the limit.

$$\sum_{n=1}^{\infty} \frac{2^n + 5^n}{e^{2n}}$$

$$\{a_n\} = \frac{2^n+5^n}{e^{2n}} = \frac{2^n+5^n}{(e^2)^n} = \left(\frac{2}{e^2}\right)^n + \left(\frac{5}{e^2}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{2^n+5^n}{e^{2n}} = \sum_{n=1}^{\infty}\left(\frac{2}{e^2}\right)^n + \sum_{n=1}^{\infty}\left(\frac{5}{e^2}\right)^n = \frac{2/e^2}{1-2/e^2} + \frac{5/e^2}{1-5/e^2}$$

$$\sum_{n=1}^{\infty} \frac{2^n+5^n}{e^{2n}} = \frac{2}{e^2-2} + \frac{5}{e^2-5} \approx 2.464$$

4. Find all values of x that makes the series shown below convergent.

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n}$$

$$\{b_n\} = \frac{(x-2)^n}{3^n} = \left[\frac{(x-2)}{3}\right]^n \text{ converges for } -1 < r < 1$$

$$-1 < \frac{x-2}{3} < 1 \rightarrow -3 < x-2 < 3 \rightarrow -1 < x < 5$$

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n} \text{ converges for } -1 < x < 5$$

5. Determine whether or not the series converges. If it does, find the limit. (Use the partial sums and observe that this is a telescoping series)

$$\sum_{n=2}^{\infty} \frac{1}{n^3 - n}$$

$$\{a_n\} = \frac{1}{n^3-n} = \frac{1}{n(n+1)(n-1)} = -\frac{1}{n} + \frac{1}{2(n+1)} + \frac{1}{2(n-1)}$$

$$\{s_n\} = \frac{1}{4} + \frac{1}{2n} - \frac{1}{n} + \frac{1}{2(n+1)} = \frac{1}{4} - \frac{1}{2n(n+1)}$$

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}\left(\frac{1}{4} - \frac{1}{2n(n+1)}\right) = \frac{1}{4}$$

$$\sum_{n=2}^{\infty} \frac{1}{n^3-n} = \frac{1}{4}$$