STAT 50 HW #4 Section 2.3
1.
Let A and B be events with P(A) = 0.8 and P(A ∩ B) = 0.2. For what value of P(B) will A and B be independent?

If A is not equal to 0, then:
$P(A \cap B) = P(A)\, P(B|A)$

$0.2 = 0.8\, P(B|A)$
$$\frac{0.2}{0.8} = \frac{(0.8)P(B|A)}{0.8}$$
$$P(B|A) = \frac{0.2}{0.8} = 0.25$$
Because the events are independent, we can interpret P(B|A) to be equal to P(B) and thus interpret P(B) to be equal to 0.25.

3.
A box contains 15 resistors. Ten of them are labeled 50 Ω and the other five are labeled 100 Ω.

a. **What is the probability that the first resistor is 100 Ω?**

P(1st resistor is 100Ω) = Number of 100 Ω resistors/Total number of resistors
P(1st resistor is 100Ω) = 5/15 = 1/3 = 0.3333

b. **What is the probability that the second resistor is 100 Ω, given that the first resistor is 50 Ω?**

A = Event that 1st resistor drawn is 50 Ω
B = Event that 2nd resistor drawn is 100 Ω
P(B|A) = ?
*Seeing how the 1st resistor was already drawn, the total number of resistors has been reduced, but we still have 5 100 Ω resistors as the first resistor drawn was 50 Ω.

5/14 = 0.357!

c. **What is the probability that the second resistor is 100 Ω, given that the first resistor is 100 Ω?**

A = Event that 1st resistor drawn is 100 Ω
B = Event that 2nd resistor drawn is 100 Ω
P(B|A) = ?

*Seeing how the 1st resistor was already drawn, the total number of resistors has been reduced, but now we have 4 100 Ω resistors as the first resistor drawn was 100 Ω.

4/14 = 2/7 = 0.286!

7.
Suppose that 90% of bolts and 85% of nails meet specifications. One bolt and one nail are chosen independently.

a. **What is the probability that both meet specifications?**

P(A) = Probability that a bolt meets the specifications.
P(B) = Probability that a nail meets the specifications.
P(A∩B) = Probability that a bolt and a nail meet the specifications.
P(A∩B) = P(A)P(B) = (0.90)(0.85) = 0.765

b. **What is the probability that neither meets specifications?**

P(A) = Probability that a bolt does not meet the specifications.
P(A) = 1 - 0.90 = 0.10
P(B) = Probability that a nail does not meet the specifications.
P(B) = 1 - 0.85 = 0.15
P(A∩B) = Probability that neither a bolt nor a nail meet the specifications.
P(A∩B) = P(A)P(B) = (0.10)(0.15) = 0.015

c. **What is the probability that at least one of them meets specifications?**

P(A) = Probability that a bolt meets the specifications.
P(B) = Probability that a nail meets the specifications.
P(A∪B) = Probability that at least one of them would meet the specifications.
P(A∪B) = P(A) + P(B) - P(A∩B) *We use this because A and B are chosen independently!
P(A∪B) = 0.90 + 0.85 - 0.765 = 0.985

9.
At a certain car dealership, 20% of customers who bought a new vehicle bought an SUV, and 3% of them bought a black SUV. Given that a customer bought an SUV, what is the probability that it was black?

P(A) = Probability that a customer bought an SUV.
P(B) = Probability that a customer bought a black SUV.
P(B|A) = Probability that a customer bought a black SUV given that they bought an SUV.

P(B|A) = P(B∩A)/P(A) = 0.03/0.20 = 0.15

13.

A particular automatic sprinkler system has two different types of activation devices for each sprinkler head. One type has a reliability of 0.9; that is, the probability that it will activate the sprinkler when it should is 0.9. The other type, which operates independently of the first type, has a reliability of 0.8. If either device is triggered, the sprinkler will activate. Suppose a fire starts near a sprinkler head.

a. **What is the probability that the sprinkler head will be activated?**

P(A) = Probability that type 1 activates = 0.90
P(B) = Probability that type 2 activates = 0.80
P(Sprinkler activates) = P(A ∪ B) = P(A) + P(B) - P(A∩B)
P(Sprinkler activates) = 0.90 + 0.80 - 0.72 = 0.98

b. **What is the probability that the sprinkler head will not be activated?**

P(Sprinkler does not activate) = 1 - P(Sprinkler activates)
1 - 0.98 = 0.02

c. **What is the probability that both activation devices will work properly?**

P(Type 1 and type 2 work) = P(A∩B) = P(Type 1 activates)*P(Type 2 activates)
P(Type 1 and type 2 work) = 0.90*0.80 = 0.72

d. **What is the probability that only the device with reliability 0.9 will work properly?**

P(Only type 1 works) = P(A)*P(B^C) = 0.9 * 0.2 = 0.18

15.

A population of 600 semiconductor wafers contains wafers from three lots. The wafers are categorized by lot and by whether they conform to a thickness specification. The following table presents the number of wafers in each category. A wafer is chosen at random from the population.

Lot	Conforming	Nonconforming
A	88	12
B	165	35
C	260	40

a. **If the wafer is from Lot A, what is the probability that it is conforming?**

*I just created another table for ease and convenience.

Lot	Conforming	Nonconforming	Total:
A	88	12	100
B	165	35	200
C	260	40	300
Total:	513	87	600

P(Wafer conforms|Wafer is from lot A) = $\frac{P(Wafer\ conforms \cap Wafer\ is\ from\ lot\ A)}{P(Wafer\ is\ from\ lot\ A)} = \frac{88/600}{100/600} =$ 88/100 = 0.88

P(Wafer conforms|Wafer is from lot A) = 0.88

b. If the wafer is conforming, what is the probability that it is from Lot A?

P(Wafer is from lot A|Wafer conforms) = $\frac{P(Wafer\ is\ from\ lot\ A \cap Wafer\ conforms)}{P(Wafer\ conforms)} = \frac{88/600}{513/600} =$ 88/513 = 0.1715

P(Wafer is from lot A|Wafer conforms) = 0.1715

c. If the wafer is conforming, what is the probability that it is not from Lot C?

P(Wafer is not from lot C|Wafer conforms) = $\frac{P(Wafer\ is\ not\ from\ lot\ C \cap Wafer\ conforms)}{P(Wafer\ conforms)} =$ $\frac{(165+88)/600}{513/600} = \frac{253/600}{513/600}$ =253/513 = 0.4932

P(Wafer is not from lot C|Wafer conforms) = 0.4932

d. If the wafer is not from Lot C, what is the probability that it is conforming?

P(Wafer conforms|Wafer is not from lot C) = $\frac{P(Wafer\ conforms \cap Wafer\ is\ not\ from\ lot\ C)}{P(Wafer\ is\ not\ from\ lot\ C)} =$ $\frac{(88+165)/600}{(100+200)/600} = \frac{253/600}{300/600}$ = 253/300 = 0.8433

P(Wafer conforms|Wafer is not from lot C) = 0.8433

17.

A geneticist is studying two genes. Each gene can be either dominant or recessive. A sample of 100 individuals is categorized as follows.

Gene 1	Gene 2	
	Dominant	Recessive
Dominant	56	24
Recessive	14	6

a. **What is the probability that a randomly sampled individual, gene 1 is dominant?**

P(Gene 1 is dominant) = (56 + 24)/100 = 80/100 = 0.80

b. **What is the probability that a randomly sampled individual, gene 2 is dominant?**

P(Gene 2 is dominant) = (56 + 14)/100 = 70/100 = 0.70

c. **Given that gene 1 is dominant, what is the probability that gene 2 is dominant?**

P(Gene 2 is dominant|Gene 1 is dominant) = $\frac{P(Gene\ 2\ is\ dominant \cap Gene\ 1\ is\ dominant)}{P(Gene\ 1\ is\ dominant)} = \frac{56/100}{80/100} = 56/80$
56/80 = 0.70
P(Gene 2 is dominant|Gene 1 is dominant) = 0.70

d. **These genes are said to be in linkage equilibrium if the event that gene 1 is dominant is independent of the event that gene 2 is dominant. Are these genes in linkage equilibrium?**

Yes, because P(Gene 2 is dominant) = P(Gene 2 is dominant|Gene 1 is dominant).

18.
A car dealer sold 750 automobiles last year. The following table categorizes the cars sold by size and color and presents the number of cars in each category. A car is to be chosen at random from the 750 for which the owner will win a lifetime of free oil changes.

Size	Color			
	White	Black	Red	Grey
Small	102	71	33	134
Midsize	86	63	36	105
Large	26	32	22	40

a. **If the car is small, what is the probability that it is black?**

P(Car is black|Car is small) = $\frac{P(Car\ is\ black \cap Car\ is\ small)}{P(Car\ is\ small)} = \frac{71/750}{340/750} = 71/340 = 0.2088$

b. **If the car is white, what is the probability that it is midsize?**

P(Car is midsize|Car is white) = $\frac{P(\text{Car is midsize} \cap \text{Car is white})}{P(\text{Car is white})} = \frac{86/750}{214/750} = 86/214 =$ 0.4019

c. **If the car is large, what is the probability that it is red?**

P(Car is red|Car is large) = $\frac{P(\text{Car is red} \cap \text{Car is large})}{P(\text{Car is large})} = \frac{22/750}{120/750} = 22/120 = 0.1833$

d. **If the car is red, what is the probability that it is large?**

P(Car is large|Car is red) = $\frac{P(\text{Car is large} \cap \text{Car is red})}{P(\text{Car is red})} = \frac{22/750}{91/750} = 22/91 = 0.2418$

e. **If the car is not small, what is the probability that it is not grey?**

P(Car is not grey|Car is not small) = $\frac{P(\text{Car is not grey} \cap \text{Car is not small})}{P(\text{Car is not small})} = \frac{265/750}{410/750} = 265/410$ = 0.6463

20.
An automobile insurance company divides customers into three categories, good risks, medium risks, and poor risks. Assume that 70% of the customers are good risks, 20% are medium risks, and 10% are poor risks. Assume that during the course of a year, a good risk customer has probability 0.005 of filing an accident claim, a medium risk customer has probability 0.01, and a poor risk customer has probability 0.025. A customer is chosen at random.

	Good risk	Medium risk	Poor risk
Amount(%)	0.70	0.20	0.10
Prob. of filing a claim.	0.005	0.01	0.025

a. **What is the probability that the customer is a good risk and has filed a claim?**

P(Good risk∩Filed claim) = 0.005

b. **What is the probability that the customer has filed a claim?**

P(Filed claim) = (0.70*0.005)+(0.20*0.01)+(0.10*0.025) = 0.008

c. Given that the customer has filed a claim, what is the probability that the customer is a good risk?

P(Customer is a good risk|Filed claim) = $\frac{P(Customer\ is\ a\ good\ risk \cap Filed\ claim)}{P(Filed\ claim)} =$

$$\frac{0.70*0.005}{(0.70*0.005)+(0.20*0.01)+(0.10*0.025)} = \frac{0.0035}{0.008} = 0.4375$$

23.

A lot of 10 components contains 3 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. Find P(A).

P(A) = 3/10

b. Find P(B|A).

P(B|A) = 2/9

c. Find P(A ∩ B).

P(A∩B) = (3/10)*(2/9) = 0.06

d. Find P(A c ∩ B).

P(A c) = 1 - 3/10 = 7/10
P(B) = 3/9
P(A c ∩ B) = 7/10*3/9 = 0.2333

e. Find P(B). Are A and B independent? Explain.

P(B) = 3/10
They are not independent. B would depend on A because if the first component is defective and not replaced, then there would be two defective components left after the first selection, which changes the probability of B.

24.

A lot of 1000 components contains 300 that are defective. Two components are drawn at random and tested. Let A be the event that the first component drawn is defective, and let B be the event that the second component drawn is defective.

a. **Find P(A).**

$P(A) = 300/1000$

b. **Find P(B|A).**

$P(B|A) = 299/999$

c. **Find P(A ∩ B).**

$P(A \cap B) = (300/1000)*(299/999) = 0.0898$

d. **Find P(A c ∩ B).**

$P(A\ c) = 1 - 300/1000 = 700/1000$
$P(B) = 299/999$
$P(A\ c \cap B) = 700/1000*300/999 = 0.2102$

e. **Find P(B).**

$P(B) = 300/1000 * 299/999 + 700/1000 * 300/999 = 0.3$

f. **Find P(A|B).**

$P(A \cap B) / [P(A \cap B) + P(A' \cap B)] = 0.0898/[0.0898 + 0.2102] = 0.2993$

g. **Are A and B independent? Is it reasonable to treat A and B as though they were independent? Explain.**

No. P(B) is not equal to P(B|A) in this regard. As such, it would be unreasonable to treat A and B as though they were independent.

36.
A system contains two components, A and B, connected in series, as shown in the diagram.

Assume A and B function independently. For the system to function, both components must function.

a. **If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions.**

P(System functions) = P(A works∩B works) = (1 - 0.05)*(1 - 0.03) = (0.95)*(0.97) = 0.9215

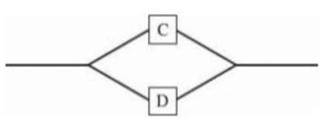
b. **If both A and B have probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?**

P(System functions) = P(A works∩B works) = (1 - p)*(1 - p) = 0.90
(1 - p)^2 = 0.90
(1 - p) = 0.9487
-p = 0.9487 - 1
p = 0.0513

c. **If three components are connected in series, and each has probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?**

P(System functions) = P(A works∩B works∩C works) = (1 - p)(1 - p)(1 - p) = 0.90
(1 - p)^3 = 0.90
(1 - p) = 0.9655
-p = 0.9655 - 1
p = 0.0345

37.
A system contains two components, C and D, connected in parallel as shown in the diagram.



Assume C and D function independently. For the system to function, either C or D must function.

a. **If the probability that C fails is 0.08 and the probability that D fails is 0.12, find the probability that the system functions.**

P(System functions) = (C works∪D works) = P(C works) + P(D works) - P(C works∩D works)
= (1 - 0.08) + (1 - 0.12) - ((1 - 0.08)*(1 - 0.12)) = 0.92 + 0.88 - (0.92*0.88) = 0.9904

b. **If both C and D have probability p of failing, what must the value of p be so that the probability that the system functions is 0.99?**

P(System functions) = 1 - P(System fails) = P(C fails) * P(D fails) = 1 - 0.99 = 0.01
P(System fails) = p^2 = 0.01
p = 0.1

c. **If three components are connected in parallel, function independently, and each has probability p of failing, what must the value of p be so that the probability that the system functions is 0.99?**

P(System functions) = 1 - P(System fails) = P(C fails) * P(D fails) * P(E fails)= 1 - 0.99 = 0.01
P(System fails) = p^3 = 0.01
p = 0.2154

d. **If components function independently, and each component has probability 0.5 of failing, what is the minimum number of components that must be connected in parallel so that the probability that the system functions is at least 0.99?**

P(System functions) = 0.99 = 1 - [P(One fails)]^n
P(One fails) = 0.5
1 - [0.5]^n = 0.99
-[0.5]^n = 0.99-1
0.5^n = 0.01
n*ln(0.5) = ln(0.01)
n = ln(0.01)/ln(0.5) = 6.6439, so ≈ 7 components needed

19?
The following table presents the 100 senators of the 115th U.S. Congress on January 3, 2017, classified by political party affiliation and gender.

	Male	Female
Democrat	30	16
Republican	47	5
Independent	2	0

A senator is selected at random from this group. Compute the following probabilities.

a. **The senator is a male Republican.**

P(Male Republican) = 47/100 = 0.47

b. **The senator is a Democrat or a female.**

P(Democrat or female) = P(Democrat) + P(female) - P(Democrat∩female) = 46/100 + 21/100 - 16/100 = 51/100 = 0.51

P(Democrat or female) = 0.51

c. **The senator is a Republican.**

P(Republican) = 52/100 = 0.52

d. **The senator is not a Republican.**

P(Not republican) = 1 - P(Republican) = 1 - 0.52 = 0.48

e. **The senator is a Democrat.**

P(Democrat) = 46/100 = 0.46

f. **The senator is an Independent.**

P(Independent) = 2/100 = 0.02

g. **The senator is a Democrat or an Independent.**

P(Democrat or Independent) = P(Democrat) + P(Independent) - P(Democrat∩Independent) = 46/100 + 2/100 - 0 = 48/100 = 0.48
P(Democrat or Independent) = 0.48

32
A quality-control program at a plastic bottle production line involves inspecting finished bottles for flaws such as microscopic holes. The proportion of bottles that actually have such a flaw is only 0.0002. If a bottle has a flaw, the probability is 0.995 that it will fail the inspection. If a bottle does not have a flaw, the probability is 0.99 that it will pass the inspection.

a. **If a bottle fails inspection, what is the probability that it has a flaw?**

P(Bottle has flaw|Bottle fails inspection) = $\frac{P(Bottle\ has\ flaw \cap Bottle\ fails\ inspection)}{P(Bottle\ fails\ inspection)} = \frac{0.0002}{0.995} =$ 0.0002
P(Bottle has flaw|Bottle fails inspection) = 0.0002

b. **Which of the following is the more correct interpretation of the answer to part (a)?**
i) Most bottles that fail inspection do not have a flaw.
ii) Most bottles that pass inspection do have a flaw.

The more correct interpretation is that most bottles that fail inspection don't have a flaw.

c. **If a bottle passes inspection, what is the probability that it does not have a flaw?**

P(No flaw|Passes inspection) = $\frac{P(No\,flaw \cap Passes\ inspection)}{P(Passes\ inspection)} = \frac{0.99}{0.995} = 0.9949$

d. **Which of the following is the more correct interpretation of the answer to part (c)?**
i) Most bottles that fail inspection do have a flaw.
ii) Most bottles that pass inspection do not have a flaw.

The more correct interpretation is that most bottles that pass inspection don't have a flaw.

e. **Explain why a small probability in part (a) is not a problem, so long as the probability in part (c) is large.**

A small probability in part (a) is not a problem as it only makes a microscopic contribution to the results, changing almost nothing when compared to the probability in part (c), which is significantly large.