

$$① \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 2 & 1 & -3 & 13 \\ -1 & 1 & 0 & -8 \end{array} \right] \xrightarrow{R_2 - 2R_1, R_3 + R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & -1 & 3 & 3 \\ 0 & 2 & -3 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & -1 & 3 & 3 \\ 0 & 2 & -3 & -3 \end{array} \right] \xrightarrow{\text{swap } R_2 \text{ and } R_3} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 2 & -3 & -3 \\ 0 & -1 & 3 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 2 & -3 & -3 \\ 0 & -1 & 3 & 3 \end{array} \right] \xrightarrow{\text{divide } R_2 \text{ by } 2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & -1 & 3 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 6 \end{array} \right] \xrightarrow{\text{divide } R_3 \text{ by } 6} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 5 \\ 0 & 1 & -\frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{aligned} x_1 - x_3 &= 7 \\ x_2 - x_3 &= 1 \\ x_3 &= 0 \end{aligned} \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 7 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \left[\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right] = \left[\begin{array}{c} 1 \\ -1 \\ 0 \end{array} \right] + x_3 \left[\begin{array}{c} 1 \\ -1 \\ 1 \end{array} \right]$$

⑥

CONFUSION

~~coeff~~

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 1 & 0 & 0 \end{array} \right]$$

→ all the steps
would be the
same, but ...

↓
we get this!

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right]$$

$$\begin{aligned} x_1 - 1x_3 &= 0 \\ x_2 - 1x_3 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

$$\begin{aligned} x_1 &= x_3 \\ x_2 &= x_3 \\ x_3 &\in \mathbb{R} \end{aligned}$$

3. W in H ? ✓

1) for $\vec{u}, \vec{v} \in W$, $\vec{u} + \vec{v} \in W$? ✓

2) $c\vec{u} \in W$? ✓ $Ax = 0$ makes it easier

if $A \cdot 0 = 0$ and A can be any matrix, so if c would be anything that $0c = 0$ in X .

3) Let $u \in A(W)$ and $v \in A(W)$

$$A(W) \neq 0 \text{ and } A(W) \neq 0$$

$$\text{so, } A(u+v) = 0$$

4) Let C be in \mathbb{R} and $u \in A(W)$

$$C \in \mathbb{R}^n \text{ and } u$$

$$A(Cu) = C(A(u)) \\ = C0 = 0$$

Thus $Cu \in V$ as well

The set of X 's is a subspace of \mathbb{R}^n

The set of X 's is a subspace. We know this because the zero vector can be found in X as implied by $AX = 0$. For each u and v , we have that $A(U) = 0$ and $A(V) = 0$, so $A(U+V) = 0$. For any scalar C and u in X , Cu would give us $A(Cu) = C(A(u)) = C(0)$.

*My explanation might be weak, but keep in mind I was rushing. I kind of get the concept, but I need time to think.

4. $H_2 \subset \left\{ \begin{bmatrix} -3b-a \\ 3a+2c \\ a+3b+2c \\ 5b-3c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$

Subspace of \mathbb{R}^4 ?

Compose $\begin{bmatrix} -3b-a \\ 3a+2c \\ a+3b+2c \\ 5b-3c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$

IP vs SF
a, b, c to 0
all coeffs at
so 4 coeffs of
a subspace.

basis?

$$\begin{aligned} -a - 3b + 0c &= 0 \\ 3a + 0b + 2c &\rightarrow v_1 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 2 \\ 2 \\ -3 \end{bmatrix} \\ a + 3b + 2c & \\ 5a + 5b - 3c & \end{aligned}$$

Does appear to be a
-dep or also L.S.D.
as all L.T!
Thus $\{v_1, v_2, v_3\}$ is a basis!

$$\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ -3 \end{bmatrix} \right\}$$

$$S \begin{bmatrix} 3 & 3 & 3 & 2 & 0 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & -2 & 2 \\ 2 & 0 & 4 & -2 & -2 \end{bmatrix}$$

a)

$$\begin{bmatrix} 3 & 3 & 3 & 2 & 0 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & -2 & 2 \\ 2 & 0 & 4 & -2 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 3 & 3 & 3 & 2 & 0 \\ 1 & 1 & 1 & -2 & 2 \\ 2 & 0 & 4 & -2 & -2 \end{bmatrix}$$

$\downarrow R_2 - 3R_1, R_3 - R_1, R_4 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & -1 & -4 & -3 \\ 0 & 3 & -1 & -4 & -3 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \xleftarrow{R_3 - 3R_2} \begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & -1 & -4 & -3 \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 2 & 1 \\ 0 & -1 & -4 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

4 rows ok
dim of Col 4 > 4
 $\text{Rank} = 4$

dim Mat A = 5 cols, but 4 are obs
so it has no pivot

dim Mat A₂₁ rank A = 4
dim Mat A₂₁

By bus de 01 A
Looking at a few snapshots
in ab3B34260 we use best

$$\text{best} \rightarrow \{v_1, v_2, v_3, v_4\}$$

Can A?
Not so sure from best!

$$\text{best for row } i : \{1, 2, 2, 1, 1, 3, -1, -4, 3\}$$
$$(0, 9, 4, 16, 12), (0, 0, 0, 2, 0)$$

Nul A? \rightarrow Need RREF?

$$\begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & -1 & -4 & -2 \\ 0 & 0 & 4 & 1 & 1 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & -1 & -4 & -3 \\ 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \leftarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{array}{l} x_1 = 5 \\ x_2 = 0 \\ x_3 = 3 \\ x_4 = 0 \\ \hline \cancel{x_5 = 0} \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Can't find 3k

Const 3)

Basis for col A:

$$\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -2 \end{bmatrix}$$

Basis for row A'

$$\left\{ \begin{bmatrix} 1 & 0 & 2 & 3 & 0 & 0 & 1 & 6 & 12 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 3 & 0 & 0 & 0 \end{bmatrix} \right\}$$

Basis for row A':

$$\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

Row 1 and Row 3 to Row 1

③ Det A is 0 as its 2 Row 1

$$6. A = \begin{bmatrix} 3 & -4 & -1 \\ 0 & -1 & -1 \\ 0 & -4 & 2 \end{bmatrix} \text{ det } \begin{bmatrix} 3-\lambda & -4 & -1 \\ 0 & -1-\lambda & -1 \\ 0 & -4 & 2-\lambda \end{bmatrix}$$

expand across 1st col

$$= 3-\lambda \cdot \det \begin{bmatrix} 1-\lambda & -1 \\ -4 & 2-\lambda \end{bmatrix} - \cancel{0}$$

$$\cancel{(-1-\lambda)(2-\lambda)} - 2 + 1 - 2\lambda + \lambda^2 - \cancel{(\lambda)}$$

$$\lambda^2 - \lambda - 6 \quad (\lambda+2)(\lambda-3)$$

and mul

$$\text{Eigen: } -2, 3$$

(A) multiply by m, 2-2
real m = 3

$$-2 \begin{bmatrix} 3+2-4 & -1 \\ 0 & -1+2 & -1 \\ 0 & -4 & 2+2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -4 & -1 \\ 0 & 1 & -1 \\ 0 & -4 & 4 \end{bmatrix} \xrightarrow{\downarrow} \begin{bmatrix} 3 & -4 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -4 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix} \xrightarrow{\text{row 1} \leftrightarrow \text{row 3}} \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

row 1 / 3

~~cont'd~~

$$\begin{aligned}x_1 - 10 &= 0 \\x_2 + 10 &= 0 \\x_3, x_4 &\text{ free}\end{aligned}$$

$$\left[\begin{array}{cccc|c} x_1 & 1 & 0 & 0 & 1 \\ x_2 & 0 & 1 & 0 & 1 \\ x_3 & 0 & 0 & 1 & 0 \\ x_4 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 3 & -3 & -4 & -1 & 0 \\ 0 & -1 & 3 & -1 & 0 \\ 0 & -4 & 2 & -3 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 8 & -4 & -1 & -1 & 0 \\ 0 & -1 & 1 & -1 & 0 \\ 0 & -4 & 1 & -1 & 0 \end{array} \right]$$

$$\begin{aligned}-4x_2 - 4x_3 &= 0 \\x_2, x_3 &\text{ free} \\x_1 &\text{ free}\end{aligned}$$

$$\left[\begin{array}{cccc|c} 0 & -4 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\begin{aligned}-4x_2 - 4x_3 &= 0 \\x_2 &= -x_3 \\x_2 &= \frac{1}{4}x_3 \\x_3 &\text{ free}\end{aligned}$$

$$\left[\begin{array}{cccc|c} x_1 & 0 & 0 & 0 & 1 \\ x_2 & 0 & 1 & -1 & 0 \\ x_3 & 1 & 0 & 0 & 0 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

last row - 2 : $\left[\begin{array}{c} -1 \\ 1 \\ 0 \end{array} \right]$

first row - 2 : $\left[\begin{array}{c} 0 \\ -1 \\ 0 \end{array} \right]$

2.a) $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ $\det(A - \lambda I) = \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix}$

$$(1-\lambda)(3-\lambda) - 8$$

$$3 - 3\lambda + \lambda^2 - 8$$

$$\cancel{(1-\lambda)}(1-\lambda)^2 - 5 \quad (1+\lambda)(1-\lambda)$$

Ergebnis: -1, 5

~~Was?~~? $A + I = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

$$\begin{array}{l} x_1 + 2x_2 = 0 \\ x_2 = 0 \end{array}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{array}{l} x_1 + 2x_2 = 0 \\ x_2 = 0 \end{array} \quad \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -2 \\ 1 \end{bmatrix} \quad \begin{array}{l} x_1 = 0 \\ -x_1 = -x_2 \end{array}$$

$$A - 5I = \begin{bmatrix} -4 & 4 \\ 2 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{array}{l} x_1 + x_2 = 0 \\ x_2 = 0 \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad D? \rightarrow \text{Faktor!}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \quad D? \rightarrow \text{Faktor!}$$

$$P_2 \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}, 0 = \begin{bmatrix} 1 & 0 \\ 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

7(b) $A = \begin{bmatrix} 3 & 1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ expand along 2nd col

Row 3)

$$\det \begin{bmatrix} 3-\lambda & 1 & 0 \\ -1 & 5-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix}$$

$$0 + (-1-\lambda) \cdot \det \begin{bmatrix} 3-\lambda & 1 \\ -1 & 5-\lambda \end{bmatrix}$$

$$(-1-\lambda)(3-\lambda)$$

$$B = -3\lambda - \lambda^2 + \lambda^2 - 6\lambda$$

$$(A^2 - 8\lambda - 16)(A - 2)$$

$$-\lambda^3 + (\cancel{2\lambda^2} - \cancel{16\lambda}) - 4(\cancel{\lambda^2}) + \cancel{32\lambda} - \cancel{64}$$

$$-\lambda^3 + 4\lambda^2 + 16\lambda - 64$$

8. $W = \{ \text{linear combination of } v_1, v_2, v_3 \}$ ist ein Vektorraum? Warum?

$$v_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$

ausrechnen:

$$v_1, v_2 = (1)(2) + (-2)(1) + (2)(3) = 0$$

$$\|v_1\| = \sqrt{1^2 + (-2)^2 + 2^2} \\ = \sqrt{1 + 4 + 4} \\ = \sqrt{9} = 3$$

$$\|v_2\| = \sqrt{2^2 + 4^2 + 3^2} \\ = \sqrt{4 + 16 + 9} \\ = \sqrt{29}$$

$$\left\{ \frac{1}{3} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{29}} \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix} \right\}$$

$$a. \quad V_1 = \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad V = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$V \cdot V_1 = 3(3) + 1(1) + (-5)(-1) + 1(1) = 12$$

$$V \cdot V_1 = a + b + c + d = 12$$

$$V \cdot V_2 = 3(1) + 1(-1) + 5(1) + 1(-1) = 6$$

$$V_2 \cdot V_2 = 1 + 1 + 1 + 1 = 4$$

$$\frac{2V \cdot V_1}{V_2 \cdot V_2} = \frac{V \cdot V_1}{V_2 \cdot V_2}$$

$$\frac{6}{12} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} + \frac{6}{4} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix} + \frac{3}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 3/2 \\ 1/2 \\ -1/2 \\ 1/2 \end{bmatrix} + \begin{bmatrix} 3/2 \\ 3/2 \\ 3/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} 6/2 \\ 2/2 \\ 2/2 \\ 2/2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$10) A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 2 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}$$

~~4x2~~
no solution

out-solve

$$A^T A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & 0 & 2 \end{bmatrix}_{2 \times 4} \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 1 & 0 \\ 2 & 2 \end{bmatrix}_{4 \times 2} = \begin{bmatrix} 10 & 3 \\ 6 & 6 \end{bmatrix}$$

$$a_{11} = (1)(1)(2) + (0)(0) + (0)(2) = 10$$

$$a_{12} = 1 + (-2) + 0 + 4 = 3$$

$$a_{21} = 1 + (-2) + 0 + 4 = 3$$

$$a_{22} = 1 + 1 + 0 + 4 = 6$$

$$\left[\begin{array}{c|cc} 10 & 3 & 16 \\ \hline 6 & 6 & 0 \end{array} \right]$$

$$\left[\begin{array}{c|cc} 10 & 1 & 16 \\ \hline 6 & 6 & 0 \end{array} \right]$$

$$A^T b = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & -1 & 0 & 2 \end{bmatrix}_{2 \times 4} \begin{bmatrix} 8 \\ 6 \\ 2 \\ 2 \end{bmatrix}_{4 \times 1} = \begin{bmatrix} 18 \\ 0 \end{bmatrix}_{2 \times 1}$$

$$\left[\begin{array}{c|cc} 18 & 0 \\ \hline 0 & 0 & 0 \end{array} \right]$$

$$a_{11} = 0 + 8 + 6 + 9 = 23$$

$$\left[\begin{array}{c|cc} 1 & 6 & 3 & 0 \\ \hline 0 & -1 & 1 & 2 \end{array} \right]$$

$$a_{12} = 0 - 4 + 0 + 4 = 0$$

$$\boxed{\begin{bmatrix} 0 \\ -2 \end{bmatrix}}$$

$$⑥ A^2 = \begin{bmatrix} 1 & 2 \\ 0 & -5 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$$

~~$A^2 = 6 = 401$~~
no with

L5

$$A^T A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & -5 \\ -1 & -5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ -1 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 8 \end{bmatrix}$$

$$a_{11} = 1 + 1 + 0 + 1 = 3$$

$$\begin{bmatrix} 3 & 0 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$a_{12} = 2 + 4 + 0 - 1 = 5$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2/3 \\ -6/8 \end{bmatrix}$$

$$a_{21} = 2 + 4 + 0 - 1 = 5$$

$$a_{22} = 4 + 16 + 25 + 36 = 81$$

$$\boxed{\begin{bmatrix} 1 \\ 2/3 \\ -8/8 \end{bmatrix}}$$

$$A^T b^2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 4 & -5 \\ -1 & -5 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

$$a_{11} = 3 - 1 + 0 + 0 = 2$$

$$a_{21} = 6 - 4 - 10 + 0 = -8$$

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 1 & 2 \\ 0 & 2 & -4 \end{bmatrix}, b = \begin{bmatrix} 7 \\ -3 \\ -8 \end{bmatrix}$$

~~4x3~~ ~~3x3~~