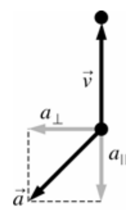
4

KINEMATICS IN TWO DIMENSIONS

Conceptual Questions

4.1. **(a)** As shown in the figure below, the acceleration $\vec{a}$ can be divided into components perpendicular $(\perp)$ and parallel $(||)$ to the velocity. $a_{||}$ will slow the particle down since it is in the opposite direction to $\vec{v}$.

(b) The perpendicular component of $\vec{a}$, $a_{\perp}$, is pointing to the left, and changes the particle direction to the left.



4.2. **(a)** A component of the acceleration either parallel or antiparallel to the velocity would speed up the particle or slow it down, respectively. Since there is no parallel component then the speed isn't changing.
(b) The perpendicular component of $\vec{a}$, $a_{\perp}$, is pointing down, and changes the particle direction in that direction.

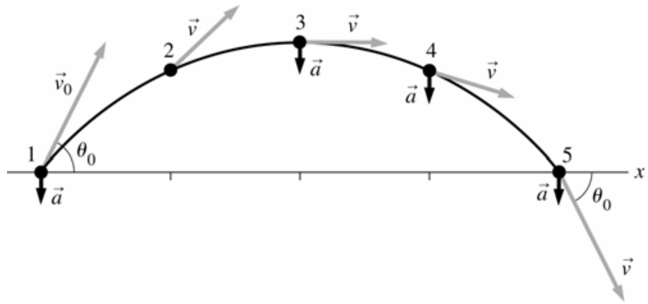
4.3. Approximate Tarzan as a particle in nonuniform circular motion.
(a) As Tarzan just steps off the vine, his velocity is zero, but increasing along his trajectory, so $\vec{a}$ is along the trajectory. The component of $\vec{a}$ that is the centripetal acceleration $a_r = \dfrac{v^2}{r} = 0$ because $v = 0$.

(b) At the bottom of the swing, $a_r = \dfrac{v_t^2}{r} \neq 0$, but the velocity is at a maximum, so $a_t = \dfrac{dv_t}{dt} = 0$, so $\vec{a}$ is not zero and points up.

4.4. A typical trajectory of a projectile is shown in the figure below. The acceleration due to gravity always points down. The velocity changes direction from the launch angle $\theta = \theta_0$ above the $+x$-axis to zero at the top of the trajectory, to $\theta = \theta_0$ below the $+x$-axis when it hits the ground.

(a) At no time are $\vec{v}$ and $\vec{a}$ parallel if $\dfrac{|v_{1_y}|}{v_{1_x}} = \tan 30°$

(b) At the top of the trajectory $\vec{v}$ and $\vec{a}$ are perpendicular.



4.5. For a projectile, only v_x, a_x, and a_y are constant during the flight. Since the acceleration $a_y = -g$ is down, $a_x = 0$ and v_x is constant. The nonzero a_y is constantly changing v_y, so the total speed $v = \sqrt{v_x^2 + v_y^2}$ changes as well. The positions *x* and *y* change, so $r = \sqrt{x^2 + y^2}$ changes, too. Only a_x is zero throughout the flight.

4.6. **(a)** The ball fired upward is a projectile with a horizontal component of initial velocity equal to the cart's speed. Without air friction, there is no horizontal component of the acceleration, so the ball stays over the cart during the whole flight, and lands directly back in the tube.
(b) The cart accelerates after launching the ball, the horizontal component of the ball's velocity is less than the velocity of the cart, so the ball will land behind the cart.

4.7. After the rock is released it is in free fall, so its acceleration is equal to *g*.

4.8. Anita is approaching ball 2 and moving away from where ball 1 was thrown, so ball 1 was thrown with the greater speed. This can be determined numerically as well, treating Anita as a moving reference frame with respect to the ground, so $v_{\text{Anita}} = v_{\text{ball}} - 5$ m/s. For ball 1, Anita measures $10 \text{ m/s} = v_1 - 5 \text{ m/s} \Rightarrow v_1 = 15 \text{ m/s}$. For ball 2, $-10 \text{ m/s} = v_2 - 5 \text{ m/s} \Rightarrow v_2 = -5 \text{ m/s}$. So ball 1 was thrown with greater speed.

4.9. The ball has the same horizontal velocity as the plane whether parked or flying, as does an observer in the plane. When the plane is moving forward at a steady speed, the ball after release appears to fall straight down, landing on the X as it did when the plane was parked.

4.10. Zach should throw his book outward and toward the back of the car. The book has the same initial velocity as do Zach and the car, so throw 1 or 2 will cause the book to land beyond the driveway in the same direction as the car is traveling.

4.11. Since Zach and Yvette are traveling at the same speed they share the same reference frame, so Zach should throw the book straight to her (throw 2.)

4.12. In uniform circular motion the speed is constant, the tangential component of the velocity is constant, the radial component of acceleration is constant, and the tangential component of acceleration is constant. The instantaneous velocity is not constant as its direction changes (even though its magnitude, the speed, is constant). The tangential component of acceleration is zero. The vector quantities have constant magnitudes but changing directions. The components of the velocity and acceleration are not vectors.

4.13. **(a)** $\omega_1 = \omega_2 = \omega_3$. All points on an object turn at the same *angular* rate.
(b) $v_3 > v_1 = v_2$. Since $v = \omega r$ and ω is the same for all of the rotating wheel, the speeds are ranked by how far from the center (r) they are.

4.14. **(a)** ω:+ α:+ Rotation is counterclockwise and increasing in the counterclockwise direction.

(b) ω:− α:+ Rotation is clockwise and decreasing, so the angular acceleration is counterclockwise.

(c) ω:+ α:− Rotation is counterclockwise and decreasing, so the angular acceleration is clockwise.

(d) ω:− α:− Rotation is clockwise and increasing in the clockwise direction.

4.15. **(a)** The instantaneous speed v is zero, and so $\omega = \frac{v}{r} = 0$.

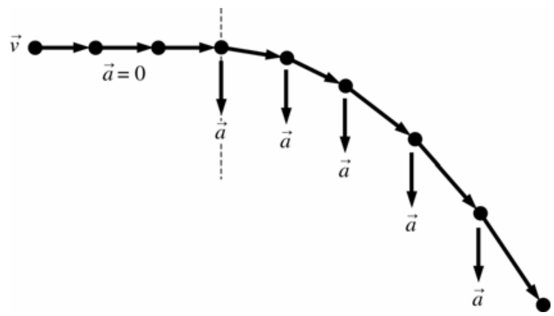
(b) Rotation is beginning in the clockwise direction, so $\alpha < 0$.

Exercises and Problems

Exercises

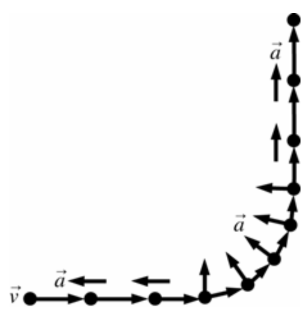
Section 4.1 Motion in Two Dimensions

4.1. **Solve:** **(a)**



(b) A ball rolls along a level table at 3 m/s. It rolls over the edge and falls 1 m to the floor. How far away from the edge of the table does it land?

4.2. **Solve:** **(a)**



(b) A race car slows from an initial speed of 100 mph to 50 mph in order to negotiate a tight turn. After making the $90°$ turn the car accelerates back up to 100 mph in the same time it took to slow down.

4.3. **Solve:** To keep a steady speed there can't be a component of the acceleration parallel to the velocity. To make the particle curve to the right the acceleration must have a component to the right. So the answer is C.

4.4. **Solve:** To make the particle speed up the acceleration needs to have a component that is in the direction of the velocity. To make the particle curve upward the acceleration must have a component upward. So the answer is B.

4.5. Solve: To make the particle speed up the acceleration needs to have a component that is in the direction of the velocity. To make the particle curve downward the acceleration must have a component downward. So the answer is E.

4.6. Model: The puck is a particle and follows the constant-acceleration kinematic equations of motion.
Solve: (a) At $t = 2$ s, the graphs give $v_x = 16$ cm/s and $v_y = 30$ cm/s. The angle made by the vector $\vec{v}$ with the *x*-axis can thus be found as

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{30 \text{ cm/s}}{16 \text{ cm/s}}\right) = 62° \text{ above the } x\text{-axis}$$

(b) After $t = 5$ s, the puck has traveled a distance given by:

$$x_1 = x_0 + \int_0^{5s} v_x\, dt = 0 \text{ m} + \text{area under } v_x\text{-}t \text{ curve} = \tfrac{1}{2}(40 \text{ cm/s})(5 \text{ s}) = 100 \text{ cm}$$
$$y_1 = y_0 + \int_0^{5s} v_y\, dt = 0 \text{ m} + \text{area under } v_y\text{-}t \text{ curve} = (30 \text{ cm/s})(5 \text{ s}) = 150 \text{ cm}$$
$$\Rightarrow r_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{(100 \text{ cm})^2 + (150 \text{ cm})^2} = 180 \text{ cm}$$

4.9. Solve: To find the acceleration we take the derivative of velocity.

$$\vec{v} = (2t\,\hat{i} + (3 - t^2)\,\hat{j}) \text{ m/s} \Rightarrow \vec{a} = (2\hat{i} + (-2t)\,\hat{j}) \text{ m/s}^2$$

Now evaluate that expression at $t = 4$ s:

$$\vec{a}(4 \text{ s}) = (2\hat{i} - 2(4)\,\hat{j}) \text{ m/s}^2 = (2\hat{i} - 8\,\hat{j}) \text{ m/s}^2$$

4.7. Model: Use the particle model for the puck.
Solve: Since the v_x vs *t* and v_y vs *t* graphs are straight lines, the puck is undergoing constant acceleration along the *x*- and y- axes. The components of the puck's acceleration are

$$a_x = \frac{dv_x}{dt} = \frac{\Delta v_x}{\Delta t} = \frac{(10 \text{ m/s} - (-10 \text{ m/s}))}{10 \text{ s} - 0 \text{ s}} = 2.0 \text{ m/s}^2$$
$$a_y = \frac{(10 \text{ m/s} - 0 \text{ m/s})}{(10 \text{ s} - 0 \text{ s})} = 1.0 \text{ m/s}^2$$

The magnitude of the acceleration is $a = \sqrt{a_x^2 + a_y^2} = 2.2 \text{ m/s}^2$.

Assess: The acceleration is constant, so the computations above apply to all times shown, not just at 5 s. The puck turns around at $t = 5$ s in the *x* direction, and constantly accelerates in the *y* direction. Traveling 50 m from the starting point in 10 s is reasonable.

4.8. Solve: (a) At $t = 0$ s, $x = 0$ m and $y = 0$ m, or $\vec{r} = (0\hat{i} + 0\hat{j})$ m. At $t = 4$ s, $x = 0$ m and $y = 0$ m, or $\vec{r} = (0\hat{i} + 0\hat{j})$ m. In other words, the particle is at the origin at both $t = 0$ s and at $t = 4$ s. From the expressions for *x* and *y*,

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \left[\left(\frac{3}{2}t^2 - 4t\right)\hat{i} + (t - 2)\,\hat{j}\right] \text{ m/s}$$

At $t = 0$ s, $\vec{v} = -2\hat{j}$ m/s, $v = 2$ m/s. At $t = 4$ s, $\vec{v} = (8\hat{i} + 2\hat{j})$ m/s, $v = 8.3$ m/s.

(b) At $t = 0$ s, $\vec{v}$ is along $-\hat{j}$, or 90° south of $+x$. At $t = 4$ s,

$$\theta = \tan^{-1}\left(\frac{2 \text{ m/s}}{8 \text{ m/s}}\right) = 14° \text{ north of } +x$$

4.9. Solve: To find the acceleration we take the derivative of velocity.

$$\vec{v} = (2t\,\hat{i} + (3-t^2)\,\hat{j})\text{ m/s} \Rightarrow \vec{a} = (2\hat{i} + (-2t)\,\hat{j})\text{ m/s}^2$$

Now evaluate that expression at $t = 4$ s:

$$\vec{a}(4\text{ s}) = (2\hat{i} - 2(4)\,\hat{j})\text{ m/s}^2 = (2\hat{i} - 8\,\hat{j})\text{ m/s}^2$$

4.10. Solve: (a) To find the position we take the integral of the velocity.

$$\vec{r}(t) = \int \vec{v}\ dt + \vec{r}_0 = \left(-\tfrac{3}{2}t^2\,\hat{i} + \tfrac{2}{3}t^3\,\hat{j}\right)\text{m} \quad + (3.0\hat{i} + 2.0\,\hat{j})\text{ m}$$

Now evaluate that expression at $t = 2$ s:

$$\vec{r}(2\text{ s}) = \left(-\tfrac{3}{2}(2)^2\,\hat{i} + \tfrac{2}{3}(2)^3\,\hat{j}\right)\text{m} \quad + (3.0\hat{i} + 2.0\,\hat{j})\text{ m} = \left(-3.0\hat{i} + \tfrac{22}{3}\,\hat{j}\right)\text{m} = (-3.0\hat{i} + 7.3\hat{j})\text{ m}$$

(b) To find the acceleration we take the derivative of velocity.

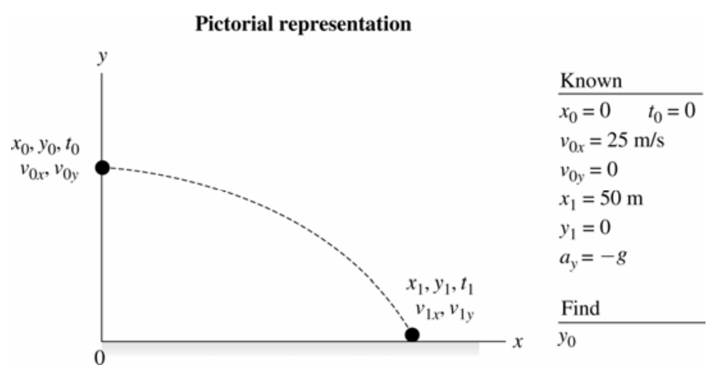
$$\vec{v} = (-3t\,\hat{i} + 2t^2\,\hat{j})\text{ m/s}, \Rightarrow \vec{a} = (-3\hat{i} + 4t\,\hat{j})\text{ m/s}^2$$

Now evaluate that expression at $t = 2$ s:

$$\vec{a}(2\text{ s}) = (-3\hat{i} + 4t\,\hat{j})\text{ m/s}^2 = (-3\hat{i} + 8\,\hat{j})\text{ m/s}^2$$

Section 4.2 Projectile Motion

4.11. Model: The ball is treated as a particle and the effect of air resistance is ignored.
Visualize:



Solve: Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

$$50\text{ m} = 0\text{ m} + (25\text{ m/s})(t_1 - 0\text{ s}) + 0\text{ m} \Rightarrow t_1 = 2.0\text{ s}$$

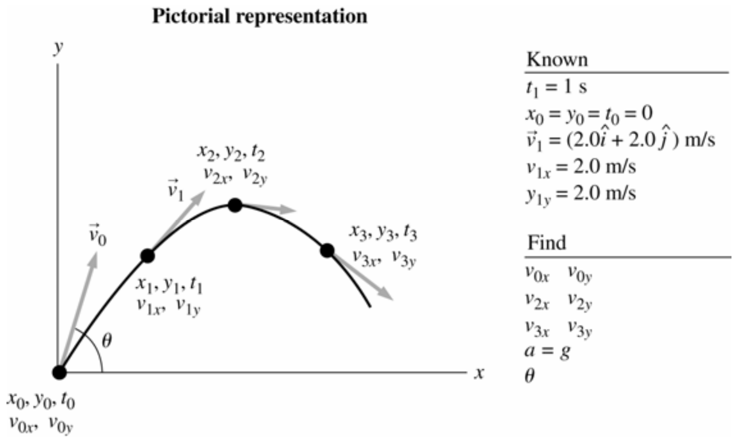
Now, using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$y_1 = 0\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(2.0\text{ s} - 0\text{ s})^2 = -19.6\text{ m}$$

So the ball was thrown from 19.6 m high.
Assess: The minus sign with y_1 indicates that the ball's displacement is in the negative y direction or downward. A magnitude of 19.6 m for the height is reasonable.

4.12. Model: Assume the particle model for the ball, and apply the constant-acceleration kinematic equations of motion in a plane.

Visualize:



Solve: (a) We know the velocity $\vec{v}_1 = (2.0\hat{i} + 2.0\hat{j})$ m/s at $t = 1$ s. The ball is at its highest point at $t = 2$ s, so $v_y = 0$ m/s. The horizontal velocity is constant in projectile motion, so $v_x = 2.0$ m/s at all times. Thus $\vec{v}_2 = 2.0\hat{i}$ m/s at $t = 2$ s. We can see that the *y*-component of velocity *changed* by $\Delta v_y = -2.0$ m/s between $t = 1$ s and $t = 2$ s. Because a_y is constant, v_y changes by -2.0 m/s in *any* 1-s interval. At $t = 3$ s, v_y is 2.0 m/s less than its value of 0 at $t = 2$ s. At $t = 0$ s, v_y must have been 2.0 m/s more than its value of 2.0 m/s at $t = 1$ s. Consequently, at $t = 0$ s,

$$\vec{v}_0 = (2.0\hat{i} + 4.0\hat{j}) \text{ m/s}$$

At $t = 1$ s,

$$\vec{v}(1) = (2.0\hat{i} + 2.0\hat{j}) \text{ m/s}$$

At $t = 2$ s,

$$\vec{v}(2) = (2.0\hat{i} + 0.0\hat{j}) \text{ m/s}$$

At $t = 3$ s,

$$\vec{v}(3) = (2.0\hat{i} - 2.0\hat{j}) \text{ m/s}$$

(b) Because v_y is changing at the rate -2.0 m/s per s, the *y*-component of acceleration is $a_y = -2.0 \text{ m/s}^2$. But $a_y = -g$ for projectile motion, so the value of *g* on Exidor is $g = 2.0 \text{ m/s}^2$.

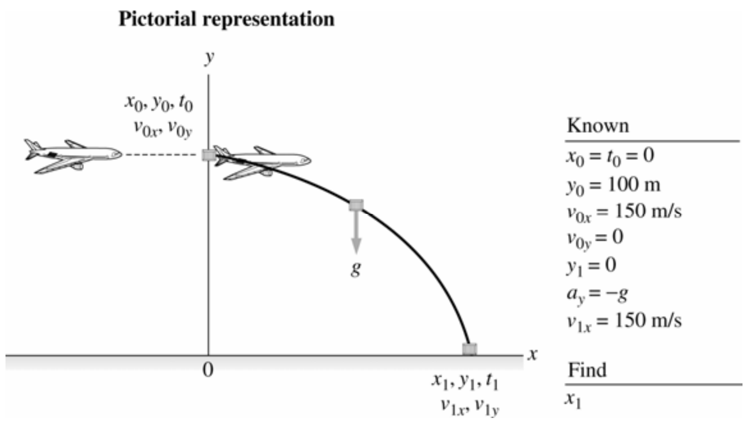
(c) From part (a) the components of $\vec{v}_0$ are $v_{0x} = 2.0$ m/s and $v_{0y} = 4.0$ m/s. This means

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{4.0 \text{ m/s}}{2.0 \text{ m/s}}\right) = 63^\circ \text{ above } +x$$

Assess: The *y*-component of the velocity vector decreases from 2.0 m/s at $t = 1$ s to 0 m/s at $t = 2$ s. This gives an acceleration of -2 m/s^2. All the other values obtained above are also reasonable.

4.13. Model: We will use the particle model for the food package and the constant-acceleration kinematic equations of motion.

Visualize:



Solve: For the horizontal motion,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 = 0 \text{ m} + (150 \text{ m/s})(t_1 - 0 \text{ s}) + 0 \text{ m} = (150 \text{ m/s})t_1$$

We will determine t_1 from the vertical y-motion as follows:

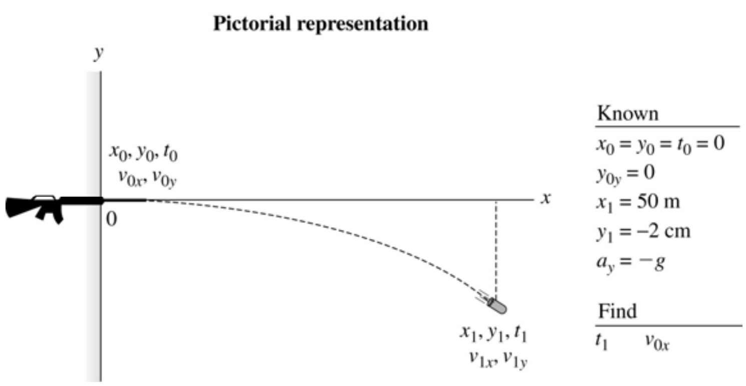
$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$

$$\Rightarrow 0 \text{ m} = 100 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)t_1^2 \Rightarrow t_1 = \sqrt{\frac{200 \text{ m}}{9.8 \text{ m/s}^2}} = 4.518 \text{ s} \approx 4.5 \text{ s}$$

From the above x-equation, the displacement is $x_1 = (150 \text{ m/s})(4.518 \text{ s}) = 678 \text{ m} \approx 680 \text{ m}$.

Assess: The horizontal distance of 678 m covered by a freely falling object from a height of 100 m and with an initial horizontal velocity of 150 m/s (≈ 335 mph) is reasonable.

4.14. Model: The bullet is treated as a particle and the effect of air resistance on the motion of the bullet is neglected.
Visualize:



Solve: **(a)** Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$, we obtain

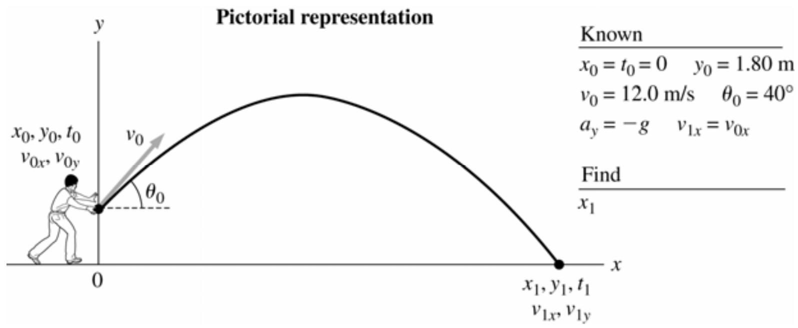
$$(-2.0\times10^{-2} \text{ m}) = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s})^2 \Rightarrow t_1 = 0.0639 \text{ s} \approx 0.064 \text{ s}$$

(b) Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

$$(50 \text{ m}) = 0 \text{ m} + v_{0x}(0.0639 \text{ s} - 0 \text{ s}) + 0 \text{ m} \Rightarrow v_{0x} = 782 \text{ m/s} \approx 780 \text{ m/s}$$

Assess: The bullet falls 2 cm during a horizontal displacement of 50 m. This implies a large initial velocity, and a value of 782 m/s is understandable.

4.15. Model: Assume the particle model and motion under constant-acceleration kinematic equations in a plane.
Visualize:



Solve: (a) Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$0\text{ m} = 1.80\text{ m} + v_0 \sin 40°(t_1 - 0\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 0\text{ s})^2$$
$$= 1.80\text{ m} + (7.713\text{ m/s})t_1 - (4.9\text{ m/s}^2)t_1^2 \Rightarrow t_1 = -0.206\text{ s and }1.780\text{ s}$$

The negative value of t_1 is unphysical for the current situation. Using $t_1 = 1.780$ s and $x_1 = x_0 + v_{0x}(t_1 - t_0)$, we get

$$x_1 = 0 + (v_0 \cos 40°\text{ m/s})(1.780\text{ s} - 0\text{ s}) = (12.0\text{ m/s})\cos 40°(1.78\text{ s}) = 16.36\text{ m} \approx 16.4\text{ m}$$

(b) We can repeat the calculation for each angle. A general result for the flight time at angle θ is

$$t_1 = \left(12\sin\theta + \sqrt{144\sin^2\theta + 35.28}\right)\Big/ 9.8\text{ s}$$

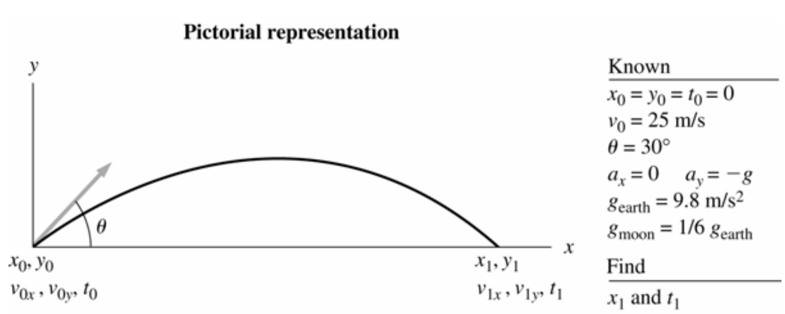
and the distance traveled is $x_1 = (12.0)\cos\theta \times t_1$. We can put the results in a table.

θ	t_1	x_1
40.0°	1.780 s	16.36 m
42.5°	1.853 s	16.39 m
45.0°	1.923 s	16.31 m
47.5°	1.990 s	16.13 m

Maximum distance is achieved at $\theta \approx 42.5°$.

Assess: The well-known "fact" that maximum distance is achieved at 45° is true only when the projectile is launched and lands at the *same* height. That isn't true here. The extra $0.03\text{ m} = 3\text{ cm}$ obtained by increasing the angle from 40.0° to 42.5° could easily mean the difference between first and second place in a world-class meet.

4.16. Model: The golf ball is a particle following projectile motion.
Visualize:



(a) The distance traveled is $x_1 = v_{0x}t_1 = v_0\ \cos\theta \times t_1$. The flight time is found from the *y*-equation, using the fact that the ball starts and ends at $y = 0$:

$$y_1 - y_0 = 0 = v_0 \sin\theta\, t_1 - \tfrac{1}{2}gt_1^2 = (v_0 \sin\theta - \tfrac{1}{2}gt_1)\, t_1 \Rightarrow t_1 = \frac{2v_0 \sin\theta}{g}$$

Thus the distance traveled is

$$x_1 = v_0 \cos\theta \times \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

For $\theta = 30°$, the distances are

$$(x_1)_{\text{earth}} = \frac{2v_0^2 \sin\theta \cos\theta}{g_{\text{earth}}} = \frac{2(25\text{ m/s})^2 \sin 30° \cos 30°}{9.80\text{ m/s}^2} = 55.2\text{ m}$$

$$(x_1)_{\text{moon}} = \frac{2v_0^2 \sin\theta \cos\theta}{g_{\text{moon}}} = \frac{2v_0^2 \sin\theta \cos\theta}{\frac{1}{6}g_{\text{earth}}} = 6 \times \frac{2v_0^2 \sin\theta \cos\theta}{g_{\text{earth}}} = 6(x_1)_{\text{earth}} = 331.2\text{ m}$$

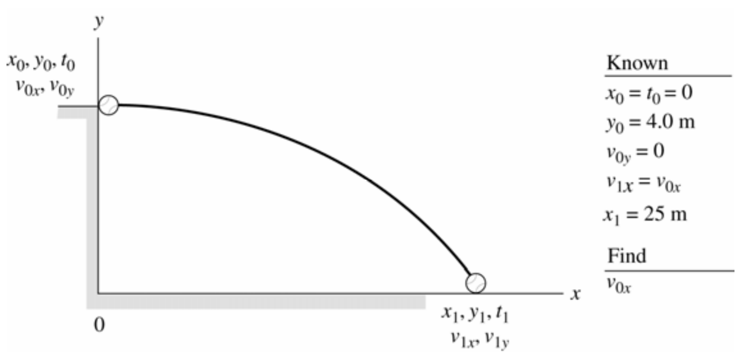
The golf ball travels $331.2\text{ m} - 55.2\text{ m} = 276\text{ m}$ farther on the moon than on earth.
(b) The flight times are

$$(t_1)_{\text{earth}} = \frac{2v_0 \sin\theta}{g_{\text{earth}}} = 2.55\text{ s}$$

$$(t_1)_{\text{moon}} = \frac{2v_0 \sin\theta}{g_{\text{moon}}} = \frac{2v_0 \sin\theta}{\frac{1}{6}g_{\text{earth}}} = 6(t_1)_{\text{earth}} = 15.30\text{ s}$$

The ball spends $15.30\text{ s} - 2.55\text{ s} = 12.75\text{ s}$ longer in flight on the moon.

4.17. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.
Visualize:



Solve: **(a)** The time for the ball to fall is calculated as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$

$$\Rightarrow 0\text{ m} = 4\text{ m} + 0\text{ m} + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 0\text{ s})^2 \Rightarrow t_1 = 0.9035\text{ s}$$

Using this result for the horizontal velocity:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 25\text{ m} = 0\text{ m} + \mathrm{v}_{0x}(0.9035\text{ s} - 0\text{ s}) \Rightarrow v_{0x} = 27.7\text{ m/s}$$

The friend's pitching speed is 28 m/s.
(b) We have $v_{0y} = \pm v_0 \sin\theta$, where we will use the plus sign for up 5° and the minus sign for down 5°. We can write

$$y_1 = y_0 \pm v_0 \sin\theta(t_1 - t_0) - \frac{g}{2}(t_1 - t_0)^2 \Rightarrow 0\text{ m} = 4\text{ m} \pm v_0 \sin\theta\, t_1 - \frac{g}{2}t_1^2$$

Let us first find t_1 from $x_1 = x_0 + v_{0x}(t_1 - t_0)$:

$$25\text{ m} = 0\text{ m} + v_0 \cos\theta\, t_1 \Rightarrow t_1 = \frac{25\text{ m}}{v_0 \cos\theta}$$

Now substituting t_1 into the *y*-equation above yields

$$0\text{ m} = 4\text{ m} \pm v_0 \sin\theta\left(\frac{25\text{ m}}{v_0\cos\theta}\right) - \frac{g}{2}\left(\frac{25\text{ m}}{v_0\cos\theta}\right)^2$$

$$\Rightarrow v_0^2 = \frac{g(25\text{ m})^2}{2\cos^2\theta}\left\{\frac{1}{4\text{ m} \pm (25\text{ m})\tan\theta}\right\} = 22.3\text{ m/s and }44.2\text{ m/s}$$

The range of speeds is 22 m/s to 44 m/s, which is the same as 50 mph to 92 mph.
Assess: These are reasonable speeds for baseball pitchers.

Section 4.3 Relative Motion

4.18. Model: Assume motion along the *x*-direction (downstream to the right). Call the speed of the boat with respect to the water $(v_x)_{\text{BW}}$, the speed of the water with respect to the earth $(v_x)_{\text{WE}}$, and the speed of the boat with respect to the earth $(v_x)_{\text{BE}}$.
Solve: We seek $(v_x)_{\text{WE}}$.

Downstream: $$(v_x)_{\text{BE}} = (v_x)_{\text{BW}} + (v_x)_{\text{WE}} = \frac{30\text{ km}}{3.0\text{ h}} = 10\text{ km/h}$$

Upstream: $$(v_x)_{\text{BE}} = -(v_x)_{\text{BW}} + (v_x)_{\text{WE}} = \frac{30\text{ km}}{5.0\text{ h}} = -6.0\text{ km/h}$$

Add the two equations to get $2(v_x)_{\text{WE}} = 4.0\text{ km/h}$, so the river flows at 2.0 m/s.
Assess: This means that the boat goes at 8.0 m/s relative to the water. Both these numbers sound reasonable.

4.19. Model: Assume motion along the *x*-direction. Let $\Delta x = x_1 - x_0$ be the distance between the gate and the baggage claim. Call your walking speed $(v_x)_{\text{YS}}$, the speed of the moving sidewalk with respect to the floor $(v_x)_{\text{SF}}$, and the speed of you with respect to the floor $(v_x)_{\text{YF}}$ while walking and riding.
Solve: We seek Δt, the time it takes to go Δx while walking on the moving sidewalk.

Walking alone: $$(v_x)_{\text{YS}} = \frac{\Delta x}{50\text{ s}}$$

Standing while riding: $$(v_x)_{\text{SF}} = \frac{\Delta x}{75\text{ s}}$$

Walking while riding: $$(v_x)_{\text{YF}} = \frac{\Delta x}{\Delta t} = (v_x)_{\text{YS}} + (v_x)_{\text{SF}} = \frac{\Delta x}{50\text{ s}} + \frac{\Delta x}{75\text{ s}}$$

Cancel Δx and solve for Δt:

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x}{50\text{ s}} + \frac{\Delta x}{75\text{ s}} \Rightarrow \Delta t = 30\text{ s}$$

Assess: A Δt smaller than 50 s was expected.

4.20. Model: Let the *x*-direction be east and the *y*-direction be north. Use subscripts M, W, and E for Mary, the water, and the earth, respectively. Let the origin be Mary's starting point on the south bank.
Visualize: In the reference frame of the water Mary has no east-west motion; in that frame she travels 100 m across the river at 2.0 m/s so $\Delta t = 50\text{ s}$.

Solve:

(a)
$$\begin{aligned}\vec{r}_{\text{ME}} &= \vec{r}_{\text{MW}} + \vec{r}_{\text{WE}}\\ &= \vec{v}_{\text{MW}}\Delta t + \vec{v}_{\text{WE}}\Delta t\\ &= (2.0\text{ m/s})\hat{j}(50\text{ s}) + (1.0\text{ m/s})\hat{\imath}(50\text{ s})\\ &= (50\text{ m})\hat{i} + (100\text{ m})\hat{j}\end{aligned}$$

So she lands 50 m east (downstream) from where she intended.

(b) $v_{\text{ME}} = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(1.0\text{ m/s})^2 + (2.0\text{ m/s})^2} = 2.236\text{ m/s} \approx 2.2\text{ m/s}$

Assess: Most of Mary's speed with respect to the shore is due to her rowing rather than the current.

4.21. Model: We define the *x*-axis along the direction of east and the *y*-axis along the direction of north.
Solve: (a) The kayaker's speed of 3.0 m/s is relative to the water. Since he's being swept toward the east, he needs to point at angle θ west of north. His velocity with respect to the water is

$$\vec{v}_{\text{KW}} = (3.0\text{ m/s, }\theta\text{ west of north}) = (-3.0\sin\theta\text{ m/s})\hat{i} + (3.0\cos\theta\text{ m/s})\hat{j}$$

We can find his velocity with respect to the earth $\vec{v}_{\text{KE}} = \vec{v}_{\text{KW}} + \vec{v}_{\text{WE}}$, with $\vec{v}_{\text{WE}} = (2.0\text{ m/s})\hat{i}$. Thus

$$\vec{v}_{\text{KE}} = ((-3.0\sin\theta + 2.0)\text{ m/s})\hat{i} + (3.0\cos\theta\text{ m/s})\hat{j}$$

In order to go straight north in the earth frame, the kayaker needs $(v_x)_{\text{KE}} = 0$. This will be true if

$$\sin\theta = \frac{2.0}{3.0} \quad\Rightarrow\quad \theta = \sin^{-1}\left(\frac{2.0}{3.0}\right) = 41.8^\circ$$

Thus he must paddle in a direction 42° west of north.
(b) His northward speed is $v_y = 3.0\cos(41.8^\circ)\text{ m/s} = 2.236\text{ m/s}$. The time to cross is

$$t = \frac{100\text{ m}}{2.236\text{ m/s}} = 44.7\text{ s}$$

The kayaker takes 45 s to cross.

4.22. Model: Let the *x*-direction be east and the *y*-direction be north. Use subscripts S, T, and G for Susan, Trent, and Ground respectively.
Visualize: $\vec{v}_{\text{TS}} = \vec{v}_{\text{TG}} + \vec{v}_{\text{GS}}$ where $\vec{v}_{\text{GS}} = -\vec{v}_{\text{SG}} = (-60\text{ mph})\hat{j}$.
Solve:

$$v_{\text{TS}} = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(45\text{ mph})^2 + (-60\text{ mph})^2} = 75\text{ mph}$$

Assess: We expected the relative speed between Trent and Susan to be greater than either of their speeds relative to the ground.

Section 4.4 Uniform Circular Motion

4.23. Solve: Since $\omega = (d\theta/dt)$ we have

$$\theta_{\text{f}} = \theta_{\text{i}} + \text{ area under the } \omega\text{-versus-}t\text{ graph between } t_{\text{i}} \text{ and } t_{\text{f}}$$

From $t = 0$ s to $t = 2$ s, the area is $(20\text{ rad/s})(2\text{ s}) = 40\text{ rad}$. From $t = 2$ s to $t = 3$ s, the area is 20 rad. From $t = 3$ s to $t = 4$ s, the area is 10 rad.
Thus, the area under the ω-versus-*t* graph during the total time interval of 4 s is 66 rad or $(65\text{ rad})\times(1\text{ rev}/2\pi\text{ rad}) = 10\text{ rev}$.

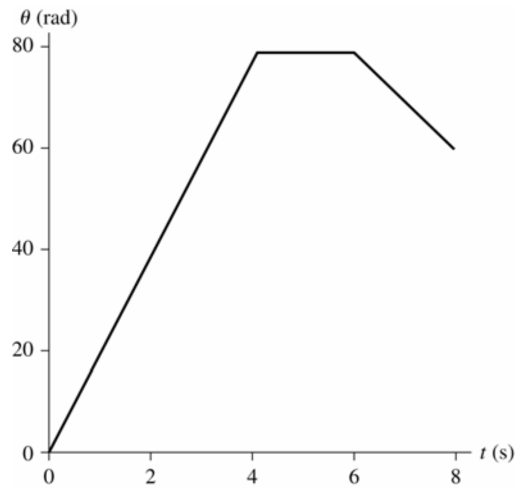
4.24. Visualize: The angular velocity is the slope of the angular position graph.
Solve:

(a) The slope of the graph at $t = 1\text{ s}$ is $\dfrac{4\pi\text{ rad}}{2\text{ s}} = 2\pi\text{ rad/s}$.

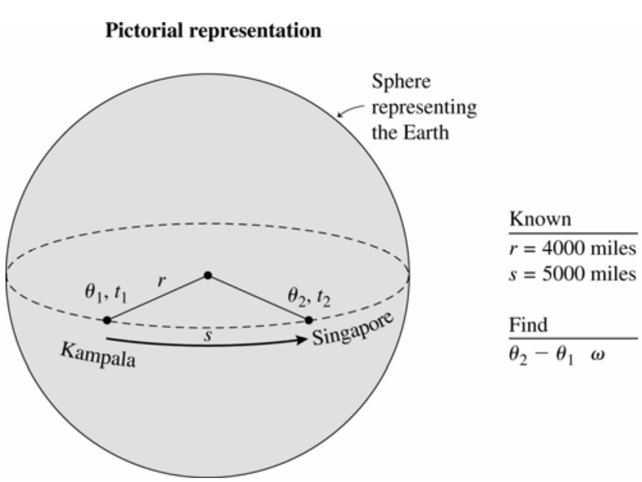
(b) The slope of the graph at $t = 4\text{s}$ is 0 rad/s.

(c) The slope of the graph at $t = 7\text{s}$ is $\dfrac{-6\pi\text{ rad}}{3\text{ s}} = -2\pi\text{ rad/s}$.

4.25. Solve: The angular position graph is the area under the angular velocity graph. At $t = 4$ s the area is 80 rad. Between 4 s and 6 s the angular velocity is zero so the angular position doesn't change. Between 6 s and 8 s the area is 20 rad, but it is below the axis, so we subtract it. The area under the ω versus t graph during the total time interval of 8 s is 80 rad –20 rad = 60 rad. This is where we end up on the θ axis at 8 s.



4.26. Model: The airplane is to be treated as a particle.
Visualize:



Solve: The angle you turn through is

$$\theta_2 - \theta_1 = \frac{s}{r} = \frac{5000\text{ miles}}{4000\text{ miles}} = 1.2500\text{ rad } = 1.2500\text{ rad} \times \frac{180^\circ}{\pi\text{ rad}} = 71.62^\circ$$

The plane's angular velocity is

$$\omega = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{71.62°}{9 \text{ h}} = 8.0°/\text{h}$$

Assess: An angular displacement of approximately one-fifth of a complete rotation is reasonable because the separation between Kampala and Singapore is approximately one-fifth of the earth's circumference.

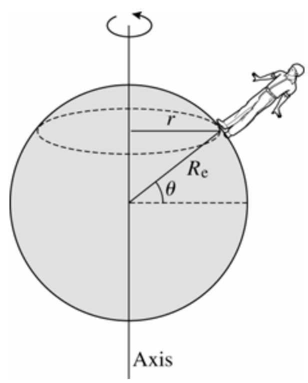
4.27. Model: Treat the record on a turntable as a particle rotating at 45 rpm.
Solve: (a) The angular velocity is

$$\omega = 45 \text{ rpm} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 1.5\pi \text{ rad/s} \approx 4.7 \text{ rad/s}$$

(b) The period is

$$T = \frac{2\pi \text{ rad}}{|\omega|} = \frac{2\pi \text{ rad}}{1.5\pi \text{ rad/s}} = 1.33 \text{ s} \approx 1.3 \text{ s}$$

4.28. Model: The earth is a rigid, rotating, and spherical body.
Visualize:



Solve: At a latitude of θ degrees, the radius is $r = R_e \cos\theta$ with $R_e = 6400 \text{ km} = 6.400 \times 10^6$ m.

(a) In Miami $\theta = 26°$, and we have $r = (6.400 \times 10^6 \text{ m})(\cos 26°) = 5.752 \times 10^6$ m. The angular velocity of the earth is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600 \text{ s}} = 7.272 \times 10^{-5} \text{ rad/s}$$

Thus, $v_{\text{student}} = r\omega = (5.752 \times 10^6 \text{ m})(7.272 \times 10^{-5} \text{ rad/s}) = 418 \text{ m/s} \approx 420 \text{ m/s}$.

(b) In Fairbanks $\theta = 65°$, so $r = (6.400 \times 10^6 \text{ m})\cos 65° = 2.705 \times 10^6$ m and $v_{\text{student}} = r\omega = (2.705 \times 10^6 \text{ m})$ $(7.272 \times 10^{-5} \text{ rad/s}) = 197 \text{ m/s} \approx 200 \text{ m/s}$.

4.29. Solve: The plane must fly as fast as the earth's surface moves, but in the opposite direction. That is, the plane must fly from east to west. The speed is

$$v = \omega r = \left(\frac{2\pi \text{ rad}}{24 \text{ h}}\right)(6.4 \times 10^3 \text{ km}) = 1680 \frac{\text{km}}{\text{h}} = 1680 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ mile}}{1.609 \text{ km}} = 1040 \text{ mph}$$

4.30. Model: Assume the beach is at sea level so that $r_S = 6400$ km for the surfer and $r_C = 6403$ km for the climber. The angular velocity for each of them is $2\pi \text{ rad}/24 \text{ h} = 7.27 \times 10^{-5}$ rad/s.
Visualize: $v = r\omega$.

Solve:

$$\Delta v = v_C - v_S = r_C\omega - r_S\omega = (r_C - r_S)\omega = (3000\text{ m})(7.27\times 10^{-5}\text{rad/s}) = 0.22\text{ m/s} = 22\text{ cm/s}$$

Assess: The difference in speed is small because ω is small.

Section 4.5 Centripetal Acceleration

4.31. Visualize: The magnitude of centripetal acceleration is given by $a = \dfrac{v^2}{r}$.

Solve: The centripetal acceleration is given as 1.5 times the acceleration of gravity, so

$$a = (1.5)(9.80\text{ m/s}^2) = 14.7\text{ m/s}^2$$

The radius of the turn is given by

$$r = \frac{v^2}{a} = \frac{(25\text{ m/s})^2}{14.7\text{ m/s}^2} = 43\text{ m}$$

Assess: This seems reasonable.

4.32. Model: The rider is assumed to be a particle.

Solve: Since $a_r = v^2/r$, we have

$$v^2 = a_r r = (98\text{ m/s}^2)(12\text{ m}) \Rightarrow v = 34\text{ m/s}$$

Assess: $34\text{ m/s} \approx 70\text{ mph}$ is a large yet understandable speed.

4.33. Model: The earth is a particle orbiting around the sun.

Solve: (a) The magnitude of the earth's velocity is displacement divided by time:

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.5\times 10^{11}\text{ m})}{365\text{ days}\times\dfrac{24\text{ h}}{1\text{ day}}\times\dfrac{3600\text{ s}}{1\text{ h}}} = 3.0\times 10^4\text{ m/s}$$

(b) Since $v = r\omega$, the angular velocity is

$$\omega = \frac{v}{r} = \frac{3.0\times 10^4\text{ m/s}}{1.5\times 10^{11}\text{ m}} = 2.0\times 10^{-7}\text{ rad/s}$$

(c) The centripetal acceleration is

$$a_r = \frac{v^2}{r} = \frac{(3.0\times 10^4\text{ m/s})^2}{1.5\times 10^{11}\text{ m}} = 6.0\times 10^{-3}\text{ m/s}^2$$

Assess: A tangential velocity of 3.0×10^4 m/s or 30 km/s is large, but needed for the earth to go through a displacement of $2\pi(1.5\times 10^{11}\text{ m}) \approx 9.4\times 10^8\text{ km}$ in 1 year.

4.34. Model: Model the DVD as a rotating rigid body.

Visualize: The formula for centripetal acceleration is $a = \omega^2 r$. Use ratios so that all the quantities that don't change cancel out.

Solve:

(a) The angular velocity ω doesn't change in this part.

$$\frac{a_2}{a_1} = \frac{\omega^2 r_2}{\omega^2 r_1} = \frac{\omega^2(2r_1)}{\omega^2 r_1} = 2 \Rightarrow a_2 = 2a_1 = 2(20\text{ m/s}^2) = 40\text{ m/s}^2$$

(b) Call a_3 the acceleration of the first speck when ω is doubled. $\omega_3 = 2\omega_1$. The distance r from the center doesn't change in this part.

$$\frac{a_3}{a_1} = \frac{\omega_3^2 r}{\omega_1^2 r} = \frac{(2\omega_1)^2 r}{\omega_1^2 r} = 2^2 = 4 \Rightarrow a_3 = 4a_1 = 4(20 \text{ m/s}^2) = 80 \text{ m/s}^2$$

Assess: This ratio technique is very powerful; it's harder to make mistakes and the ratios reveal relationships between quantities.

4.35. Solve: The pebble's angular velocity $\omega = (3.0 \text{ rev/s})(2\pi \text{ rad/rev}) = 18.9 \text{ rad/s}$. The speed of the pebble as it moves around a circle of radius $r = 30 \text{ cm} = 0.30 \text{ m}$ is

$$v = \omega r = (18.9 \text{ rad/s})(0.30 \text{ m}) = 5.7 \text{ m/s}$$

The radial acceleration is

$$a_r = \frac{v^2}{r} = \frac{(5.7 \text{ m/s})^2}{0.30 \text{ m}} = 108 \text{ m/s}^2$$

Section 4.6 Nonuniform Circular Motion

4.36. Model: The crankshaft is a rotating rigid body.
Visualize: The angular acceleration is the slope of the angular velocity graph.
Solve:

(a) The slope of the graph at $t = 1 \text{ s}$ is $\dfrac{200 \text{ rad/s}}{2 \text{ s}} = 100 \text{ rad/s}^2$.

(b) The slope of the graph at $t = 3 \text{ s}$ is 0 rad/s^2.

(c) The slope of the graph at $t = 5 \text{ s}$ is $-\dfrac{150 \text{ rad/s}}{3 \text{ s}} = -50 \text{ rad/s}^2$.

4.37. Model: The turntable is a rotating rigid body.
Visualize: The angular velocity is the area under the α vs. t graph. Use the formula for the area of a trapezoid.
Solve: Because the turntable starts from rest $\omega_0 = 0$.

(a) The area under the graph from $t = 0 \text{ s}$ to $t = 1 \text{ s}$ is $(5 \text{ rad/s}^2 + 2.5 \text{ rad/s}^2)(1 \text{ s})/2 = 3.75 \text{ rad/s}$

(b) The area under the graph from $t = 0 \text{ s}$ to $t = 2 \text{ s}$ is $(5 \text{ rad/s}^2 + 0 \text{ rad/s}^2)(2 \text{ s})/2 = 5.0 \text{ rad/s}$

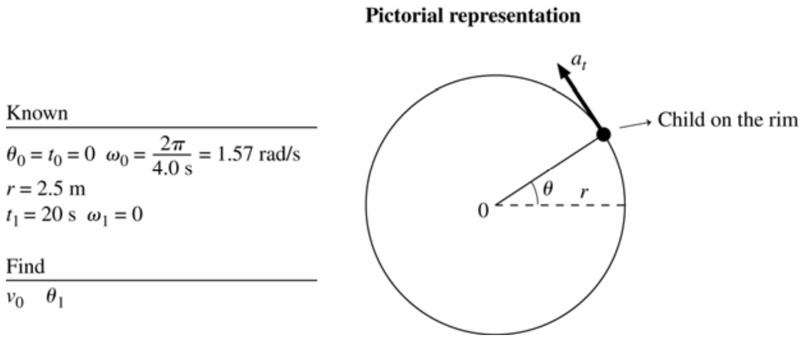
(c) The area under the graph from is not increasing after $t = 2 \text{ s}$ so ω stays the same 5.0 rad/s.

4.38. Visualize: The angular position is the slope of the area under the ω vs. t graph.
Solve: The area under the graph is $20 \text{ rad} + 40 \text{ rad} = 60 \text{ rad}$. Convert to revolutions. $60 \text{ rad}(1 \text{ rev}/2\pi \text{ rad}) = 9.5 \text{ rev}$.

4.39. Solve: Since $\omega_\text{f} = \omega_\text{i} + (\text{area under } \alpha \text{ vs } t \text{ curve})$, at $t = 3 \text{ s}$, the angular velocity is

$$\begin{aligned}\omega_\text{f} &= 60 \text{ rpm} + \tfrac{1}{2}(4.0 \text{ rad/s}^2)(3 \text{ s} - 1 \text{ s}) \\ &= 60 \text{ rpm} + (4 \text{ rad/s})\left(\frac{20 \text{ rev}}{2\pi \text{ rad}} \times \frac{60 \text{ s}}{1 \text{ min}}\right) \\ &= 60 \text{ rpm} + 38 \text{ rpm} = 98 \text{ rpm}\end{aligned}$$

4.40. Model: Model the child on the merry-go-round as a particle in nonuniform circular motion.
Visualize:



Solve: **(a)** The speed of the child is $v_0 = r\omega = (2.5\text{ m})(1.57\text{ rad/s}) = 3.9\text{ m/s}$.

(b) The merry-go-round slows from 1.57 rad/s to 0 in 20 s. Thus

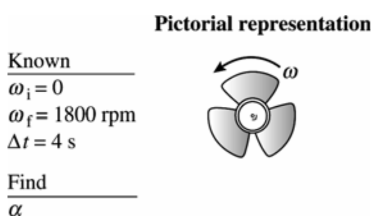
$$\omega_1 = 0 = \omega_0 + \frac{a_t}{r}t_1 \Rightarrow a_t = -\frac{r\omega_0}{t_1} = -\frac{(2.5\text{ m})(1.57\text{ rad/s})}{20\text{ s}} = -0.197\text{ m/s}^2$$

During these 20 s, the wheel turns through angle

$$\theta_1 = \theta_0 + \omega_0 t_1 + \frac{a_t}{2r}t_1^2 = 0 + (1.57\text{ rad/s})(20\text{ s}) - \frac{0.197\text{ m/s}^2}{2(2.5\text{ m})}(20\text{ s})^2 = 15.6\text{ rad}$$

In terms of revolutions, $\theta_1 = (15.6\text{ rad})(1\text{ rev}/2\pi\text{ rad}) = 2.5\text{ rev}$.

4.41. Model: The fan is in nonuniform circular motion.
Visualize:

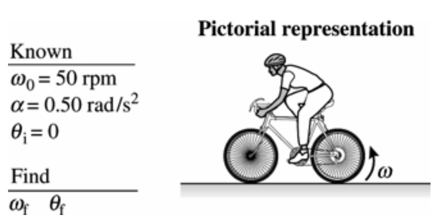


Solve: Note $1800\text{ rev/min}\left(\frac{\text{min}}{60\text{ s}}\right) = 30\text{ rev/s}$. Thus $\omega_f = \omega_i + \alpha\Delta t \Rightarrow 30\text{ rev/s} = 0\text{ rev/s} + \alpha(4.0\text{ s}) \Rightarrow \alpha = 7.5\text{ rev/s}^2$.

This can be expressed as $(7.5\text{ rev/s})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 47\text{ rad/s}^2$.

Assess: An increase in the angular velocity of a fan blade by 7.5 rev/s each second seems reasonable.

4.42. Model: The wheel is in nonuniform circular motion.
Visualize:



Solve: **(a)** Express ω_i in rad/s:

$$\omega_i = (50 \text{ rev/min})\left(\frac{\text{min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) \Rightarrow 5.2 \text{ rad/s}$$

After 10 s, $\omega_f = \omega_i + \alpha\Delta t \Rightarrow \omega_f = 5.2 \text{ rad/s} + (0.50 \text{ rad/s}^2)(10 \text{ s})^2 = 55 \text{ rad/s}$. Converting to rpm,

$$(55 \text{ rad/s})\left(\frac{60 \text{ s}}{\text{min}}\right)\left(\frac{\text{rev}}{2\pi \text{ rad}}\right) = 53 \text{ rpm}$$

(b) In 10 s, the wheel has turned a number of radians

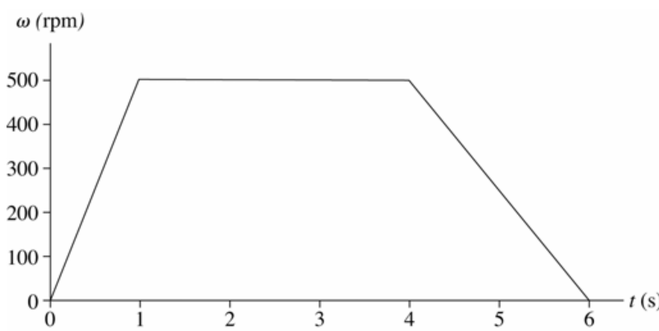
$$\theta_f = \theta_i + \omega_0\Delta t + \tfrac{1}{2}\alpha\Delta t^2 \Rightarrow \theta_f = 0 \text{ rad/sec} + (5.2 \text{ rad/s})(10 \text{ s}) + \tfrac{1}{2}(0.50 \text{ rad/s}^2)(10 \text{ s})^2 = 77 \text{ radians}.$$

Converting, $77 \text{ rad} = 12.3 \text{ revolutions}$.

Assess: Making a bicycle wheel turn just over 12 revolutions in 10 s when it is initially turning almost one revolution per second to begin with seems attainable by a cyclist.

4.43. Model: The DVD is a rotating rigid body.

Visualize: The angular displacement (angle turned) is the area under the angular velocity graph.



Solve:

$$\Delta\theta = \tfrac{1}{2}(1 \text{ s})(500 \text{ rpm}) + (3 \text{ s})(500 \text{ rpm}) + \tfrac{1}{2}(2.0 \text{ s})(500 \text{ rpm}) = (4.5 \text{ s})\left(500 \ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\,\text{min}}{60 \text{ s}}\right) = 38 \text{ rev}$$

Assess: 38 revolutions seem like a reasonable amount in 6 seconds.

Problems

4.44. Model: Assume the spaceship is a particle. The acceleration is constant, so we can use the kinematic equations.

Visualize: We apply the kinematic equation $s_f = s_i + v_0\Delta t + \dfrac{1}{2}a(\Delta t)^2$ in each direction. $\Delta t = 35 \text{ min} = 2100 \text{ s}$.

Solve:

$$x_f = 6.0\times10^5 \text{ km} + (9.5 \text{ km/s})(2100 \text{ s}) + \frac{1}{2}(0.040 \text{ km/s}^2)(2100 \text{ s})^2 s = 708000 \text{ km}$$

$$y_f = -4.0\times10^5 \text{ km} + (0 \text{ km/s})(2100 \text{ s}) + \frac{1}{2}(0 \text{ km/s}^2)(2100 \text{ s})^2 s = -400000 \text{ km}$$

$$z_f = 2.0\times10^5 \text{ km} + (0 \text{ km/s})(2100 \text{ s}) + \frac{1}{2}(-.020 \text{ km/s}^2)(2100 \text{ s})^2 s = -156000 \text{ km}$$

Rounding to two sig figs gives $\vec{r}_f = (710\hat{i} - 400\hat{j} - 160\hat{k})\times10^3$ km.

Assess: The *y*-component didn't change because there was no velocity or acceleration in the *y*-direction.

4.45. Visualize: Focus on the *y*-component first (in the $\hat{j}$ direction).
Solve:

$$a_y = -cv_y \Rightarrow \frac{dv_y}{dt} = -cv_y \Rightarrow v_y = e^{-ct}$$

Plug this into the full equation to integrate.

$$\vec{v}(t) = \int \vec{a}dt + \vec{v}_0 = \int (bt\hat{i} - ce^{-ct}\hat{j})dt + v_{0x}\hat{i} + v_{0y}\hat{j} = \tfrac{1}{2}bt^2\hat{i} + e^{-ct}\hat{j} + v_{0x}\hat{i} + v_{0y}\hat{j}$$

Rearranging terms gives

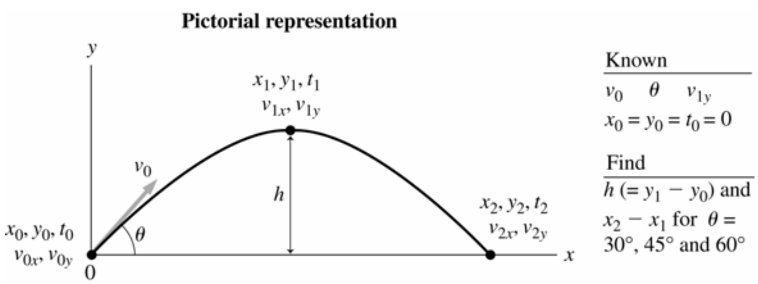
$$\vec{v}(t) = \left(\tfrac{1}{2}bt^2 + v_{0x}\right)\hat{i} + (e^{-ct} + v_{0y})\hat{j}$$

4.46. Solve: From the expression for R, $R_{\text{max}} = v_0^2/g$. Therefore,

$$R = \frac{R_{\text{max}}}{2} = \frac{v_0^2 \sin 2\theta}{g} \Rightarrow \sin 2\theta = \frac{1}{2} \Rightarrow \theta = 15° \text{ and } 75°$$

Assess: The discussion in the chapter explains why launch angles θ and $(90° - \theta)$ give the same range.

4.47. Model: Assume particle motion in a plane and constant-acceleration kinematics for the projectile.
Visualize:



Solve: (a) We know that $v_{0y} = v_0 \sin\theta$, $a_y = -g$, and $v_{1y} = 0$ m/s. Using $v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0)$,

$$0 \text{ m}^2/\text{s}^2 = v_0^2 \sin^2\theta + 2(-g)h \Rightarrow h = \frac{v_0^2 \sin^2\theta}{2g}$$

(b) Using the equation for range and the above expression for $\theta = 30.0°$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 30.0°}{2(9.8 \text{ m/s}^2)} = 14.4 \text{ m}$$

$$(x_2 - x_0) = \frac{v_0^2 \sin 2\theta}{g} = \frac{(33.6 \text{ m/s})^2 \sin(2 \times 30.0°)}{(9.8 \text{ m/s}^2)} = 99.8 \text{ m}$$

For $\theta = 45.0°$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 45.0°}{2(9.8 \text{ m/s}^2)} = 28.8 \text{ m}$$

$$(x_2 - x_0) = \frac{(33.6 \text{ m/s})^2 \sin(2 \times 45.0°)}{(9.8 \text{ m/s}^2)} = 115.2 \text{ m}$$

For $\theta = 60.0°$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 60.0°}{2(9.8 \text{ m/s}^2)} = 43.2 \text{ m}$$

$$(x_2 - x_0) = \frac{(33.6 \text{ m/s})^2 \sin(2\times 60.0°)}{2(9.8 \text{ m/s}^2)} = 99.8 \text{ m}$$

Assess: The projectile's range, being proportional to $\sin(2\theta)$, is maximum at a launch angle of 45°, but the maximum height reached is proportional to $\sin^2(\theta)$. These dependencies are seen in this problem.

4.48. Visualize: The projectile starts at the origin at $t = 0$. $v_{0x} = v_0\cos\theta,\quad v_{0y} = v_0\sin\theta$

Solve: (a)

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$
$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

Apply the conditions above to get

$$x = v_0\cos\theta\ t + \tfrac{1}{2}(-a)t^2$$
$$y = v_0\sin\theta\ t + \tfrac{1}{2}(-g)t^2$$

The range is equal to x when $y = 0$, so find t from the y equation.

$$y = v_0\sin\theta\ t + \tfrac{1}{2}(-g)t^2 = 0 \Rightarrow t = 0,\ \frac{2v_0\sin\theta}{g}$$

The second value gives the range, so plug this t into the x equation.

$$x = v_0\cos\theta\left(\frac{2v_0\sin\theta}{g}\right) + \tfrac{1}{2}(-a)\left(\frac{2v_0\sin\theta}{g}\right)^2 = \frac{2v_0^2}{g}\sin\theta\left(\cos\theta - \tfrac{a}{g}\sin\theta\right)$$

This is the equation for the range in terms of a, g, and θ. To find the θ that gives maximum range in terms of a and g we take the derivative of the x equation with respect to θ and set it equal to zero. Use the product rule.

$$\frac{dx}{d\theta} = \frac{2v_0^2}{g}\left[\sin\theta\left(-\sin\theta - \tfrac{a}{g}\cos\theta\right) + \cos\theta\left(\cos\theta - \tfrac{a}{g}\sin\theta\right)\right]$$
$$= \frac{2v_0^2}{g}\left[-\sin^2\theta + \cos^2\theta - 2\tfrac{a}{g}\sin\theta\cos\theta\right]$$

Now apply two double-angle trig identities.

$$\frac{dx}{d\theta} = \frac{2v_0^2}{g}\left(\cos 2\theta - \tfrac{a}{g}\sin 2\theta\right)$$

Set this equal to zero to get

$$\tfrac{g}{a} = \tan 2\theta \Rightarrow \theta = \tfrac{1}{2}\tan^{-1}\left(\tfrac{g}{a}\right)$$

(b) Insert $a = 0.10g$.

$$\theta = \tfrac{1}{2}\tan^{-1}\left(\tfrac{g}{0.10g}\right) = \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{0.10}\right) = 42°$$

4.49. Model: Treat the kangaroo as a particle.

Visualize: We are asked to find the take-off speed and horizontal speed of the kangaroo given its initial angle, 20°, and its range. Since the horizontal speed is given by $v_x = v_0\cos\theta$ and the time of flight is given by $\Delta t = 2v_0\sin\theta/g$, the range of the kangaroo is given by the product of these: $\Delta x = 2v_0\sin\theta\cos\theta/g$.

Solve: (a) We can solve the above formula for v_0 and then plug in the range and angle to find the take-off speed:

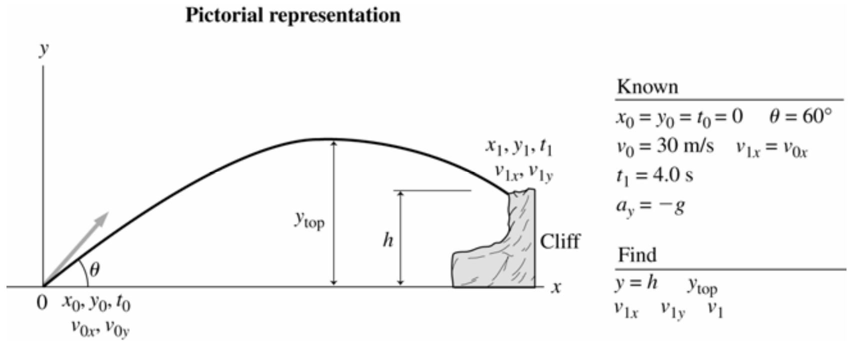
$$v_0 = g\Delta x/(2\sin\theta\cos\theta) = (9.8 \text{ m/s}^2)(10 \text{ m})/(2\ \sin 20°\cos\ 20°) = 12.3 \text{ m/s}$$

Its take-off speed is 12 m/s, to two significant figures.

(b) $h = \dfrac{v_0^2 \sin^2\theta}{2g} = \dfrac{(12.3 \text{ m/s})^2 \sin^2 20°}{2(9.8 \text{ m/s}^2)} = 0.90 \text{ m}$

Assess: These numbers seem reasonable for a kangaroo.

4.50. Model: The particle model for the ball and the constant-acceleration equations of motion are assumed.
Visualize:



Solve: **(a)** Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$h = 0 \text{ m} + (30 \text{ m/s})\sin 60°(4 \text{ s} - 0 \text{ s}) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(4 \text{ s} - 0 \text{ s})^2 = 25.5 \text{ m}$$

The height of the cliff is 26 m.
(b) Using $(v_y^2)_{top} = v_y^2 + 2a_y(y_{top} - y_0)$,

$$0 \text{ m}^2/\text{s}^2 = (v_0 \sin\theta)^2 + 2(-g)(y_{top}) \Rightarrow y_{top} = \frac{(v_0 \sin\theta)^2}{2\,g} = \frac{[(30 \text{ m/s})\sin 60°]^2}{2(9.8 \text{ m/s}^2)} = 34.4 \text{ m}$$

The maximum height of the ball is 34 m.
(c) The x and y components are

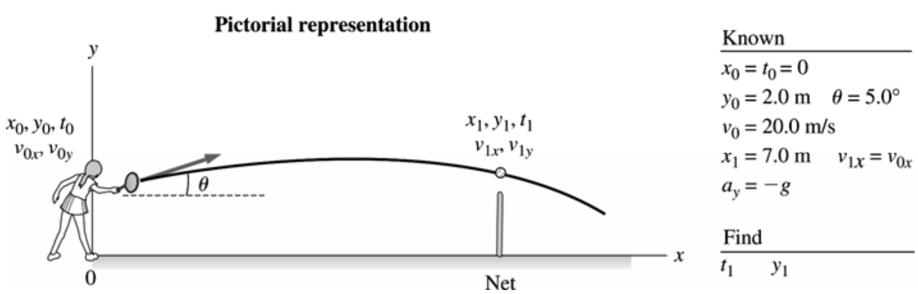
$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = v_0 \sin\theta - gt_1 = (30 \text{ m/s})\sin 60° - (9.8 \text{ m/s}^2) \times (4.0 \text{ s}) = -13.22 \text{ m/s}$$

$$v_{1x} = v_{0y} = v_0 \cos 60° = (30 \text{ m/s})\cos 60° = 15.0 \text{ m/s}$$

$$\Rightarrow v_1 = \sqrt{v_{1x}^2 + v_{1y}^2} = 20.0 \text{ m/s}$$

The impact speed is 20 m/s.
Assess: Compared to a maximum height of 34.4 m, a height of 25.5 for the cliff is reasonable.

4.51. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.
Visualize:



Solve: The initial velocity is

$$v_{0x} = v_0 \cos 5.0° = (20 \text{ m/s})\cos 5.0° = 19.92 \text{ m/s}$$
$$v_{0y} = v_0 \sin 5.0° = (20 \text{ m/s})\sin 5.0° = 1.743 \text{ m/s}$$

The time it takes for the ball to reach the net is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 7.0 \text{ m} = 0 \text{ m} + (19.92 \text{ m/s})(t_1 - 0 \text{ s}) \Rightarrow t = 0.351 \text{ s}$$

The vertical position at $\vec{v} = \vec{v}' + \vec{V}$ is

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$
$$= (2.0 \text{ m}) + (1.743 \text{ m/s})(0.351 \text{ s} - 0 \text{ s}) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(0.351 \text{ s} - 0 \text{ s})^2 = 2.01 \text{ m}$$

Thus the ball clears the net by $1.01 \text{ m} \approx 1.0 \text{ m}$.

Assess: The vertical free fall of the ball, with zero initial velocity, in 0.351 s is 0.6 m. The ball will clear by approximately 0.4 m it is thrown horizontally. The initial launch angle of 5° provides some initial vertical velocity and the ball clears by a larger distance. The above result is reasonable.

4.52. Model: The small ball is a particle in projectile motion with no air resistance.
Visualize: $v_0 = 15 \text{ m/s}$, $\theta_1 = 30°$
Solve: The range is equal to *x* when *y* = 0, so find *t* from the *y* equation.

$$y = v_0 \sin\theta \; t + \tfrac{1}{2}(-g)t^2 = 0 \Rightarrow t = 0, \; \frac{2v_0 \sin\theta}{g}$$

Plug back into the *x* equation.

$$x = v_0 \cos\theta_1 \left(\frac{2v_0 \sin\theta_1}{g}\right) = \frac{2v_0^2}{g}\sin\theta_1 \cos\theta_1$$

We set half this distance equal to the full range for a different angle θ_2.

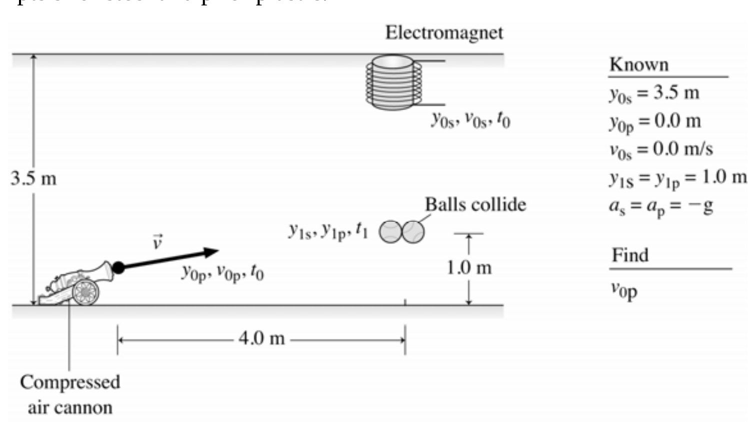
$$\frac{v_0^2}{g}\sin\theta_1\cos\theta_1 = \frac{2v_0^2}{g}\sin\theta_2\cos\theta_2 \Rightarrow \sin\theta_1\cos\theta_1 = 2\sin\theta_2\cos\theta_2$$

Plug in $\theta_1 = 30°$ on the left and use a double-angle trig identity on the right.

$$\frac{\sqrt{3}}{4} = \sin 2\theta_2 \Rightarrow \theta_2 = 13°, \; 167°$$

We report the smaller angle as the one physically applicable in this situation.
Assess: The new angle is just smaller than half the original angle; this seems reasonable. Checking work shows the range for $13°$ is half the range for $30°$.

4.53. Model: Both balls are particles in projectile motion with no air resistance. The steel ball is in one-dimensional free fall and the plastic ball is in two-dimensional projectile motion.
Visualize: Use subscripts s for steel and p for plastic.



Solve:

$$y_s = y_{0s} + v_{0s}\ t + \tfrac{1}{2}a_s t^2$$

$$y_p = y_{0p} + (v_{y0})_p t + \tfrac{1}{2}a_p t^2$$

Insert the known quantities and we have a system of two equations with two unknowns.

$$y_s = (3.5\text{ m}) - \tfrac{1}{2}gt^2$$

$$y_p = (v_{y0})_p t - \tfrac{1}{2}gt^2$$

The balls meet at $y_{1s} = y_{1p} = 1.0$ m. Find the time t_1 from the equation for the steel ball.

$$1.0\text{ m} = (3.5\text{ m}) - \tfrac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{5.0\text{ m}}{9.8\text{ m/s}^2}} = 0.7143\text{ s}$$

Use this time in the vertical motion equation for the plastic ball.

$$1.0\text{ m} = v_{0p}(0.7143\text{ s}) - \tfrac{1}{2}g(0.7143\text{ s})^2 \Rightarrow (v_{y0})_p = 4.9\text{ m/s}$$

The plastic ball also moves horizontally at a constant rate:

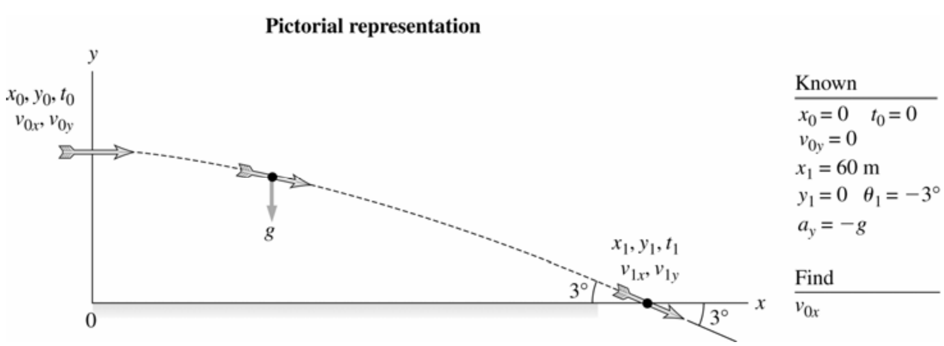
$$x_p = x_{0p} + (v_{x0})_p t_1 \Rightarrow (v_{x0})_p = x_p/t_1 = (4.0\text{ m})/(0.7143\text{ s}) = 5.6\text{ m/s}$$

We use the Pythagorean theorem to find the initial speed of the ball.

$$v_{0p} = \sqrt{(v_{x0})_p^2 + (v_{y0})_p^2} = \sqrt{(5.6\text{ m/s})^2 + (4.9\text{ m/s})^2} = 7.4\text{ m/s}$$

Assess: 7.4 m/s seems like a reasonable speed for an air cannon to launch a plastic ball. The masses of the balls did not matter.

4.54. Model: Use the particle model for the arrow and the constant-acceleration kinematic equations.
Visualize:



Solve: Using $v_{1y} = v_{0y} + a_y(t_1 - t_0)$, we get

$$v_{1y} = 0\text{ m/s} - gt_1 \Rightarrow v_{1y} = -gt_1$$

Also using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$,

$$60\text{ m} = 0\text{ m} + v_{0x}t_1 + 0\text{ m} \Rightarrow v_{0x} = \frac{60\text{ m}}{t_1} = v_{1x}$$

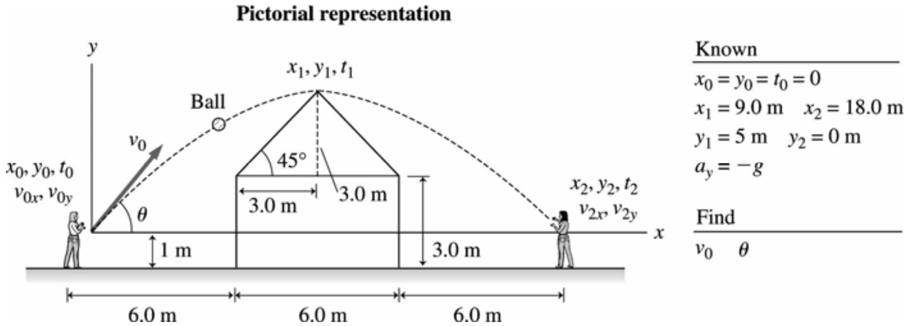
Since $v_{1y}/v_{1x} = -\tan 3.0° = -0.0524$, using the components of v_0 gives

$$\frac{-gt_1}{(60\text{ m}/t_1)} = -0.0524 \Rightarrow t_1 = \sqrt{\frac{(0.0524)(60\text{ m})}{(9.8\text{ m/s}^2)}} = 0.566\text{ s}$$

Having found t_1, we can go back to the *x*-equation to obtain $v_{0x} = 60\text{ m}/0.566\text{ s} = 106\text{ m/s} \approx 110\text{ m/s}$.

Assess: In view of the fact that the arrow took only 0.566 s to cover a horizontal distance of 60 m, a speed of 106 m/s or 237 mph for the arrow is understandable.

4.55. Model: Use the particle model for the ball and the constant-acceleration kinematic equations.
Visualize:



Solve: (a) The distance from the ground to the peak of the house is 6.0 m. From the throw position this distance is 5.0 m. Using the kinematic equation $v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0)$,

$$0 \text{ m}^2/\text{s}^2 = v_{0y}^2 + 2(-9.8 \text{ m/s}^2)(5.0 \text{ m} - 0 \text{ m}) \Rightarrow v_{0y} = 9.899 \text{ m/s}$$

The time for up- and down-motion is calculated as follows:

$$y_2 = y_0 + v_{0y}(t_2 - t_0) + \tfrac{1}{2}a_y(t_2 - t_0)^2 \Rightarrow 0 \text{ m} = 0 \text{ m} + (9.899 \text{ m/s})t_2 - \tfrac{1}{2}(9.8 \text{ m/s}^2)t_2^2 \Rightarrow t_2 = 0 \text{ s and } 2.02 \text{ s}$$

The zero solution is not of interest. Having found the time $t_2 = 2.02$ s, we can now find the horizontal velocity needed to cover a displacement of 18.0 m:

$$x_2 = x_0 + v_{0x}(t_2 - t_0) \Rightarrow 18.0 \text{ m} = 0 \text{ m} + v_{0x}(2.02 \text{ s} - 0 \text{ s}) \Rightarrow v_{0x} = 8.911 \text{ m/s}$$

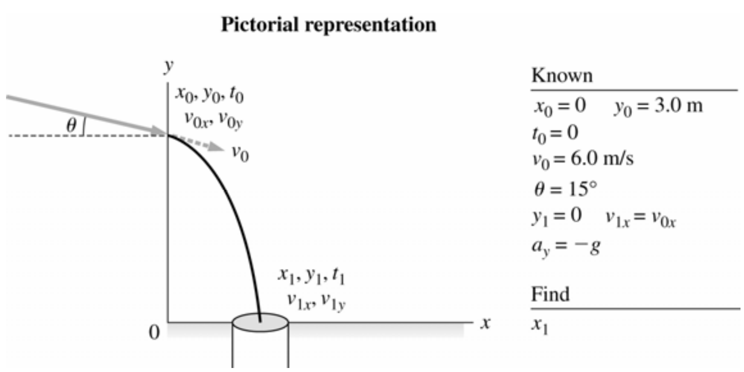
$$\Rightarrow v_0 = \sqrt{(8.911 \text{ m/s})^2 + (9.899 \text{ m/s})^2} = 13.3 \text{ m/s} \approx 13 \text{ m/s}$$

(b) The direction of $\vec{v}_0$ is given by

$$\theta = \tan^{-1}\frac{v_{0y}}{v_{0x}} = \tan^{-1}\frac{9.899}{8.911} = 48°$$

Assess: Since the maximum range corresponds to an angle of 45°, the value of 48° corresponding to a range of 18 m and at a modest speed of 13.3 m/s is reasonable.

4.56. Model: We will assume a particle model for the sand, and use the constant-acceleration kinematic equations.
Visualize:



Solve: Using the equation $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

$$x_1 = 0 \text{ m} + (v_0 \cos 15°)(t_1 - 0 \text{ s}) + 0 \text{ m} = (60 \text{ m/s})(\cos 15°)t_1$$

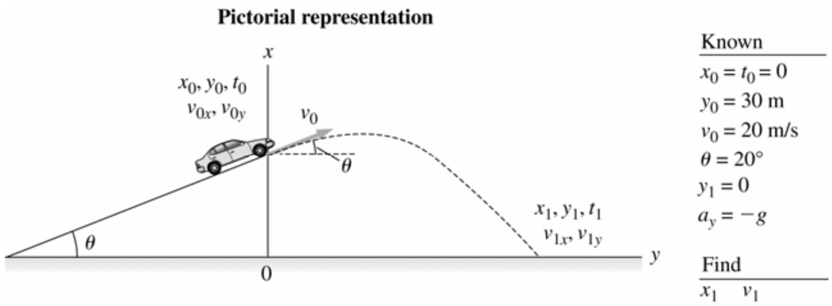
We can find t_1 from the y-equation, but note that $v_{0y} = -v_0 \sin 15°$ because the sand is launched at an angle below horizontal.

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow 0 \text{ m} = 3.0 \text{ m} - (v_0 \sin 15°)t_1 - \tfrac{1}{2}gt_1^2$$
$$= 3.0 \text{ m} - (6.0 \text{ m/s})(\sin 15°)t_1 - \tfrac{1}{2}(9.8 \text{ m/s}^2)t_1^2$$
$$\Rightarrow 4.9t_1^2 + 1.55t_1 - 3.0 = 0 \Rightarrow t_1 = 0.6399 \text{ s and} -0.956 \text{ s (unphysical)}$$

Substituting this value of t_1 in the x-equation gives the distance

$$d = x_1 = (6.0 \text{ m/s})\cos 15°(0.6399 \text{ s}) = 3.71 \text{ m} \approx 3.7 \text{ m}$$

4.57. Model: We will use the particle model and the constant-acceleration kinematic equations for the car.
Visualize:



Solve: (a) The initial velocity is

$$v_{0x} = v_0 \cos\theta = (20 \text{ m/s})\cos 20° = 18.79 \text{ m/s}$$
$$v_{0y} = v_0 \sin\theta = (20 \text{ m/s})\sin 20° = 6.840 \text{ m/s}$$

Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$,

$$0 \text{ m} = 30 \text{ m} + (6.840 \text{ m/s})(t_1 - 0 \text{ s}) + \tfrac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s})^2 \Rightarrow 4.9t_1^2 - 6.840t_1 - 30 = 0$$

The positive root to this equation is $t_1 = 3.269$ s. The negative root is physically unreasonable in the present case. Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2$, we get

$$x_1 = 0 \text{ m} + (18.79 \text{ m/s})(3.269 \text{ s} - 0 \text{ s}) + 0 = 61.4 \text{ m}$$

The car lands 61 m from the base of the cliff.
(b) The components of the final velocity are $v_{1x} = v_{0x} = 18.79$ m/s and

$$v_{1y} = v_{0y} + a_y(t_1 - t_0) = 6.840 \text{ m/s} - (9.8 \text{ m/s}^2)(3.269 \text{ s} - 0 \text{ s}) = -25.2 \text{ m/s}$$
$$\Rightarrow v = \sqrt{(18.79 \text{ m/s}) + (-25.2 \text{ m/s})^2} = 31.4 \text{ m/s}$$

The car's impact speed is 31 m/s.
Assess: A car traveling at 45 mph and being driven off a 30-m high cliff will land at a distance of approximately 200 feet (61.4 m). This distance is reasonable.

4.58. Model: Assuming constant acceleration allows us to use the kinematic equations.
Visualize: We apply the kinematic equations during the free-fall flight to find the velocity as the javelin left the hand. Then use $v_f^2 = v_i^2 + 2a_s\Delta s$ where $\Delta s = 0.70$m.

Solve: The range is $\Delta x = 62\text{m}$.

$$\Delta x = (v_0)_x \Delta t = v_0 \cos\theta \Delta t \Rightarrow \Delta t = \frac{\Delta x}{v_0 \cos\theta}$$

$$y_\text{f} = y_\text{i} + (v_0 \sin\theta)\Delta t + \frac{1}{2} a_y (\Delta t)^2$$

Insert our new expression for Δt.

$$\Delta y = (v_0 \sin\theta) \frac{\Delta x}{v_0 \cos\theta} + \frac{1}{2}(-g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2$$

Solve for v_0.

$$\Delta y = (\tan\theta)\Delta x + \frac{1}{2}(-g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2$$

$$\frac{1}{2}(g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2 = (\tan\theta)\Delta x - \Delta y$$

$$v_0^2 = \frac{g}{2}\left(\frac{\Delta x}{\cos\theta}\right)^2 \left(\frac{1}{(\tan\theta)\Delta x - \Delta y}\right)$$

$$v_0 = \sqrt{\frac{g}{2}\left(\frac{\Delta x}{\cos\theta}\right)^2 \left(\frac{1}{(\tan\theta)\Delta x - \Delta y}\right)}$$

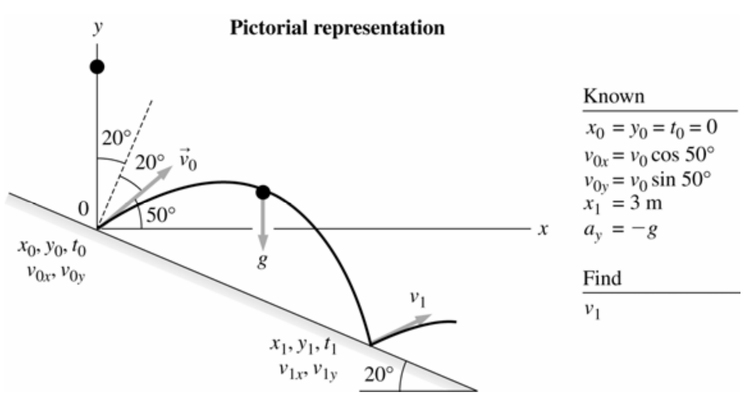
$$= \sqrt{\frac{9.8 \text{ m/s}^2}{2}\left(\frac{62 \text{ m}}{\cos 30°}\right)^2 \left(\frac{1}{(\tan 30°)(62 \text{ m}) - (-2 \text{ m})}\right)} = 25.78 \text{ m/s}$$

This is the speed as the javelin leaves the hand. It now becomes v_f as we consider the time during the throw (as the hand accelerates it from rest).

$$a = \frac{v_\text{f}^2 - v_\text{i}^2}{2\Delta s} = \frac{(25.78 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(0.70 \text{ m})} = 470 \text{ m/s}^2$$

Assess: This is a healthy acceleration, but what is required for a good throw.

4.59. **Model:** Treat the ball as a particle and apply the constant-acceleration equations of kinematics.
Visualize:



Solve: After the first bounce, the ball leaves the surface at 40° relative to the vertical or 50° relative to the horizontal. We first calculate the time t_1 between the second bounce and the first bounce as follows:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \Rightarrow 3.0 \text{ m} = 0 \text{ m} + (v_0 \cos 50°)t_1 + 0 \text{ m} \Rightarrow t_1 = \frac{3.0 \text{ m}}{v_0 \cos 50°}$$

In this time, the ball undergoes a vertical displacement of $y_1 - y_0 = -(3.0 \text{ m})\tan 20° = -1.092 \text{ m}$. Substituting these values in the equation for the vertical displacement yields:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \tfrac{1}{2}a_y(t_1 - t_0)^2$$

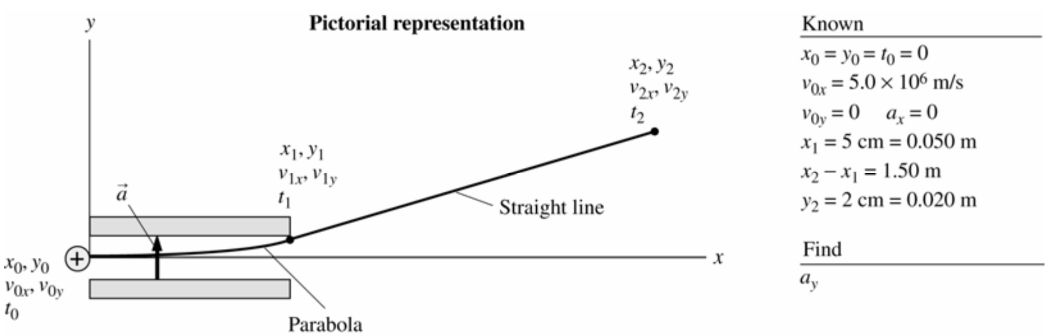
$$-1.092 \text{ m} = 0 \text{ m} + (v_0 \sin 50°)t_1 - \tfrac{1}{2}gt_1^2 = (v_0 \sin 50°)\left(\frac{3.0 \text{ m}}{v_0 \cos 50°}\right) - \tfrac{1}{2}(9.8 \text{ m/s}^2)\left(\frac{3.0 \text{ m}}{v_0 \cos 50°}\right)^2$$

$$\Rightarrow -1.092 \text{ m} - 3.575 \text{ m} = \frac{-106.73 \text{ m}^3/\text{s}^2}{v_0^2} \Rightarrow v_0 = 4.78 \text{ m/s}, \text{ or } v_0 = 4.8 \text{ m/s}$$

Assess: A speed of 4.8 m/s or 10.7 mph on the first bounce is reasonable.

4.60. Model: The ions are particles that move in a plane. They have vertical acceleration while between the acceleration plates, and they move with constant velocity from the plates to the tumor. The flight time will be so small, because of the large speeds, that we'll ignore any deflection due to gravity.

Visualize:



Solve: There's never a horizontal acceleration, so the horizontal motion is constant velocity motion at $v_x = 5.0 \times 10^6 \text{ m/s}$. The times to pass between the 5.0-cm-long acceleration plates and from the plates to the tumor are

$$t_1 - t_0 = t_1 = \frac{0.050 \text{ m}}{5.0 \times 10^6 \text{ m/s}} = 1.00 \times 10^{-8} \text{ s}$$

$$t_2 - t_1 = \frac{1.50 \text{ m}}{5.0 \times 10^6 \text{ m/s}} = 3.00 \times 10^{-7} \text{ s}$$

Upon leaving the acceleration plates, the ion has been deflected sideways to position y_1 and has velocity v_{1y}. These are

$$y_1 = y_0 + v_{0y}t_1 + \tfrac{1}{2}a_y t_1^2 = \tfrac{1}{2}a_y t_1^2$$

$$v_{1y} = v_{0y} + a_y t_1 = a_y t_1$$

In traveling from the plates to the tumor, with no vertical acceleration, the ion reaches position

$$y_2 = y_1 + v_{1y}(t_2 - t_1) = \tfrac{1}{2}a_y t_1^2 + (a_y t_1)(t_2 - t_1) = (\tfrac{1}{2}t_1^2 + t_1(t_2 - t_1))\, a_y$$

We know $y_2 = 2.0 \text{ cm} = 0.020 \text{ m},$ so we can solve for the acceleration a_y that the ion had while between the plates:

$$a_y = \frac{y_2}{\frac{1}{2}t_1^2 + t_1(t_2 - t_1)} = \frac{0.020 \text{ m}}{\frac{1}{2}(1.00\times10^{-8} \text{ s})^2 + (1.00\times10^{-8} \text{ s})(3.00\times10^{-7} \text{ s})} = 6.6\times10^{12} \text{ m/s}^2$$

Assess: This acceleration is roughly 10^{12} times larger than the acceleration due to gravity. This justifies our assumption that the acceleration due to gravity can be neglected.

4.61. Model: Both ships have a common origin at $t = 0$ s. Use subscripts A, B, and E for the ships and the earth. $\vec{r}_{AB} = \vec{r}_{AE} + \vec{r}_{EB}$

Solve: (a) The velocity vectors of the two ships are:

$$\vec{v}_{AE} = (20 \text{ mph})[\cos 30°\hat{i} - \sin 30°\hat{j}] = (17.32 \text{ mph})\hat{i} - (10.0 \text{ mph})\hat{j}$$

$$\vec{v}_{BE} = (25 \text{ mph})[\cos 20°\hat{i} + \sin 20°\hat{j}] = (23.49 \text{ mph})\hat{i} + (8.55 \text{ mph})\hat{j}$$

Since $\vec{r} = \vec{v}\Delta t,$

$$\vec{r}_{AE} = \vec{v}_{AE}(2 \text{ h}) = (34.64 \text{ miles})\hat{i} - (20.0 \text{ miles})\hat{j}$$

$$\vec{r}_{BE} = \vec{v}_{BE}(2 \text{ h}) = (46.98 \text{ miles})\hat{i} + (17.10 \text{ miles})\hat{j}$$

$$\vec{r}_{AB} = \vec{r}_{AE} + \vec{r}_{EB} = \vec{r}_{AE} - \vec{r}_{BE} = (-12.34 \text{ miles})\hat{i} - (37.10 \text{ miles})\hat{j} \Rightarrow R = 39.1 \text{ miles}$$

The distance between the ships two hours after they depart is 39 miles.

(b) Because $\vec{v}_{AB} = \vec{v}_{AE} + \vec{v}_{EB}$

$$\vec{v}_{AB} = \vec{v}_{AE} + \vec{v}_{EB} = \vec{v}_{AE} - \vec{v}_{BE} = -(6.17 \text{ mph})\hat{i} - (18.55 \text{ mph})\hat{j} \Rightarrow v_{AB} = 19.5 \text{ mph} \approx 20 \text{ mph}$$

The speed of ship A as seen by ship B is 19.5 mph.

Assess: The value of the speed is reasonable.

4.62. Model: Use subscripts C, R, and G for car, rain, and ground respectively.

Solve: The Galilean transformation of velocity is $\vec{v}_{RG} = \vec{v}_{RC} + \vec{v}_{CG}$. While driving north, $\vec{v}_{CG} = (25 \text{ m/s})\hat{i}$ and $v_{RG} = -v_R \cos\theta\, \hat{j} - v_R \sin\theta\, \hat{i}$. Thus,

$$\vec{v}_{RC} = \vec{v}_{RG} - \vec{v}_{CG} = (-v_R \sin\theta - 25 \text{ m/s})\hat{i} - v_R \cos\theta\,\hat{j}$$

Since the observer in the car finds the raindrops making an angle of 38° with the vertical, we have

$$\frac{v_R \sin\theta + 25 \text{ m/s}}{v_R \cos\theta} = \tan 38°$$

While driving south, $\vec{v}_{CG} = -(25 \text{ m/s})\hat{i}$, and $\vec{v} = -v_R \cos\theta\, \hat{j} - v_R \sin\theta\, \hat{i}$. Thus,

$$\vec{v}_{RG} = (-v_R \sin\theta + 25 \text{ m/s})\hat{i} - v_R \cos\theta\, \hat{j}$$

Since the observer in the car finds the raindrops falling vertically straight, we have

$$\frac{-v_R \sin\theta + 25 \text{ m/s}}{v_R \cos\theta} = \tan 0° = 0 \Rightarrow v_R \sin\theta = 25 \text{ m/s}$$

Substituting this value of $v_R \sin\theta$ into the expression obtained for driving north yields:

$$\frac{25 \text{ m/s} + 25 \text{ m/s}}{v_R \cos\theta} = \tan 38° \Rightarrow v_R \cos\theta = \frac{50 \text{ m/s}}{\tan 38°} = 64.0 \text{ m/s}$$

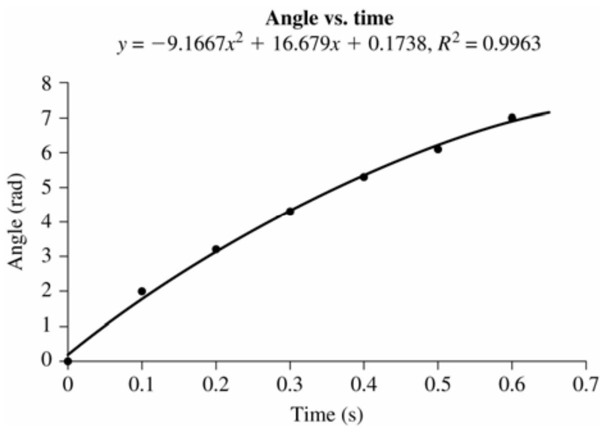
Therefore, we have for the velocity of the raindrops:

$$(v_R \sin\theta)^2 + (v_R \cos\theta)^2 = (25 \text{ m/s})^2 + (64.0 \text{ m/s})^2 \Rightarrow v_R^2 = 4721 (\text{m/s})^2 \Rightarrow v_R = 68.7 \text{ m/s}$$

$$\tan\theta = \frac{v_R \sin\theta}{v_R \cos\theta} = \frac{25 \text{ m/s}}{64 \text{ m/s}} \Rightarrow \theta = 21.3°$$

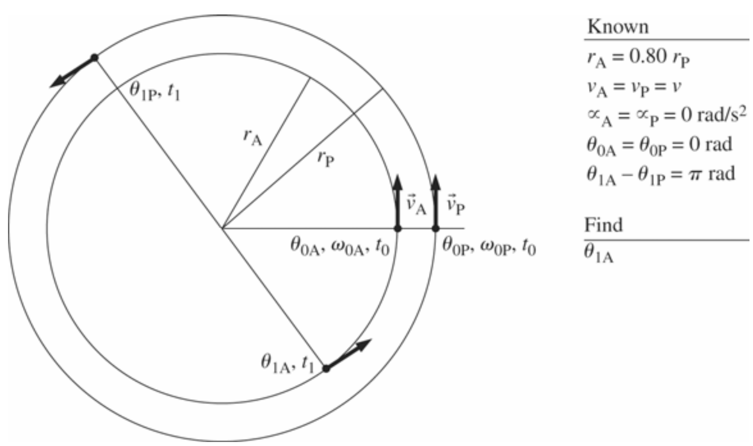
The raindrops fall at 69 m/s while making an angle of 21° with the vertical.

4.63. Model: Model the shaft as a rotating rigid body in the counterclockwise direction. Use the kinematic equations for constant angular acceleration.
Visualize: First graph the data in a spreadsheet and see if it looks linear.



Solve: The data looks moderately linear, but when the spreadsheet program puts on a second order polynomial trendline the fit (R^2) is much better and the intercept is closer to zero. We conclude that the graph isn't linear, that the circular motion is not uniform. The angular velocity is the slope of this graph, and the slope is decreasing, so ω is decreasing. This means the angular acceleration α is negative.

4.64. Model: Paul and Annie are particles and run at the same speed v. Neither has an angular acceleration.
Visualize: Use subscripts s for steel and p for plastic.



Solve:

$$\theta_A = \theta_{0A} + \omega_{0A}\ t + \tfrac{1}{2}\alpha_A t^2$$
$$\theta_P = \theta_{0P} + \omega_{0P}\ t + \tfrac{1}{2}\alpha_P t^2$$

Insert the known quantities.

$$\theta_{\mathrm{A}} = \omega_{0\mathrm{A}} t = \frac{v_{\mathrm{A}}}{r_{\mathrm{A}}} t = \frac{v}{0.80 r_{\mathrm{P}}} t$$

$$\theta_{\mathrm{P}} = \omega_{0\mathrm{P}} t = \frac{v_{\mathrm{P}}}{r_{\mathrm{P}}} t = \frac{v}{r_{\mathrm{P}}} t$$

Solve for t_1 in terms of r_{P} and v using $\theta_{\mathrm{A}} - \theta_{\mathrm{P}} = \pi$ rad when the velocity vectors point in opposite directions.

$$\pi \text{ rad} = \theta_{\mathrm{A}} - \theta_{\mathrm{P}} = \frac{v t_1}{r_{\mathrm{P}}}\left(\frac{1}{0.80} - 1\right) \Rightarrow t_1 = \frac{r_{\mathrm{P}}}{v}(4\pi \text{ rad})$$

Now plug the value of t_1 into the angle equation for Annie.

$$\theta_{1\mathrm{A}} = \frac{v}{0.80 r_{\mathrm{P}}} t_1 = \frac{v}{0.80 r_{\mathrm{P}}}\left(\frac{r_{\mathrm{P}}}{v}(4\pi \text{ rad})\right) = 5\pi \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \frac{5}{2} \text{ rev}$$

Assess: 5/2 rev seems like a reasonable number of times around the track before she is a half lap ahead of Paul.

4.65. **Model:** We will use the particle model for the test tube that is in nonuniform circular motion.
Solve: **(a)** The radial acceleration is

$$a_r = r\omega^2 = (0.1 \text{ m})\left(4000 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}}\right)^2 = 1.75 \times 10^4 \text{m/s}^2$$

(b) An object falling 1 meter has a speed calculated as follows:

$$v_1^2 = v_0^2 + 2a_y(y_1 - y_0) = 0 \text{ m} + 2(-9.8 \text{ m/s}^2)(-1.0 \text{ m}) \Rightarrow v_1 = 4.43 \text{ m/s}$$

When this object is stopped in 1×10^{-3} s upon hitting the floor,

$$v_2 = v_1 + a_y(t_2 - t_1) \Rightarrow 0 \text{ m/s} = -4.43 \text{ m/s} + a_y(1 \times 10^{-3} \text{ s}) \Rightarrow a_y = 4.4 \times 10^3 \text{ m/s}^2$$

This result is one-fourth of the above radial acceleration.
Assess: The radial acceleration of the centrifuge is large, but it is also true that falling objects are subjected to large accelerations when they are stopped by hard surfaces.

4.66. **Model:** We will use the particle model for the astronaut undergoing nonuniform circular motion.
Solve: **(a)** The initial conditions are $\omega_0 = 0$ rad/s, $\theta_0 = 0$ rad, $t_0 = 0$ s, and $r = 6.0$ m. After 30 s,

$$\omega_1 = \frac{1 \text{ rev}}{1.3 \text{ s}} = \frac{1}{1.3} \times \frac{\text{rev}}{\text{s}} \times \frac{2\pi \text{ rad}}{\text{rev}} = 4.83 \text{ rad/s}$$

Using these values at $t_1 = 30$ s,

$$\omega_1 = \omega_0 + (a_t/r)(t_1 - t_0) = 0 + (a_t/r)t_1$$

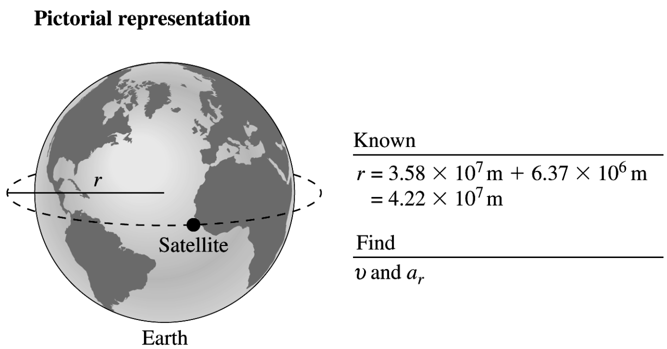
$$\Rightarrow a_t = (6.0 \text{ m})(4.83 \text{ rad/s})\left(\frac{1}{30 \text{ s}}\right) = 0.97 \text{ m/s}^2$$

(b) The radial acceleration is

$$a_r = r\omega_1^2 = (6.0 \text{ m})(4.83 \text{ rad/s})^2 \frac{g}{(9.8 \text{ m/s}^2)} = 14.3g \approx 14g$$

Assess: The above acceleration is typical of what astronauts experience during liftoff.

4.67. Model: The satellite is a particle in uniform circular motion.
Visualize:



Solve: **(a)** The satellite makes one complete revolution in 24 h about the center of the earth. The radius of the motion of the satellite is

$$r = 6.37\times10^6 \text{ m} + 3.58\times10^7 \text{ m} = 4.22\times10^7 \text{ m}$$

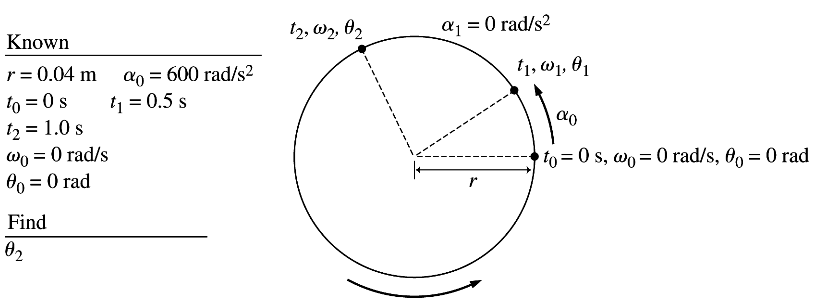
The speed of the satellite is $v = \dfrac{\text{(distance traveled)}}{\text{(time taken)}} = \dfrac{2\pi r}{24 \text{ h}} = 3.07\times10^3 \text{ m/s}.$

(b) The acceleration of the satellite is centripetal, with magnitude

$$a_r = \frac{v^2}{r} = \frac{(3.07\times10^3 \text{ m/s})^2}{4.22\times10^7 \text{ m}} = 0.223 \text{ m/s}^2$$

Assess: The small centripetal acceleration makes sense when realized it is for an object traveling in a circle with radius $\approx 26{,}400$ miles.

4.68. Model: The magnetic computer disk is a rigid rotating body.
Visualize:



Solve: **(a)** Using the rotational kinematic equation $\omega_f = \omega_i + \alpha\Delta t,$ we get

$$\omega_1 = 0 \text{ rad} + (600 \text{ rad/s}^2)(0.5 \text{ s} - 0 \text{ s}) = 300 \text{ rad/s}$$

$$\omega_2 = (300 \text{ rad/s}) + (0 \text{ rad/s}^2)(1.0 \text{ s} - 0.5 \text{ s}) = 300 \text{ rad/s}$$

The speed of the painted dot $v_2 = r\omega_2 = (0.04 \text{ m})(300 \text{ rad/s}) = 12 \text{ m/s}.$

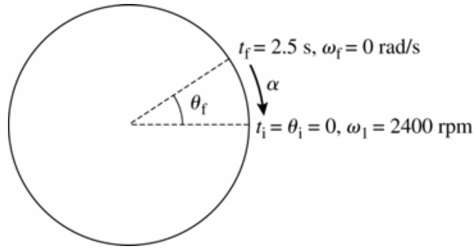
(b) The number of revolutions during the time interval t_0 to t_2 is

$$\theta_1 = \theta_0 + \omega_0(t_1 - t_0) + \frac{1}{2}\alpha_0(t_1 - t_0)^2 = 0 \text{ rad} + 0 \text{ rad} + \frac{1}{2}(600 \text{ rad/s}^2)(0.5 \text{ s} - 0 \text{ s})^2 = 75 \text{ rad}$$

$$\theta_2 = \theta_1 + \omega_1(t_2 - t_1) + \frac{1}{2}\alpha_1(t_2 - t_1)^2$$

$$= 75 \text{ rad} + (300 \text{ rad/s})(1.0 \text{ s} - 0.5 \text{ s}) + 0 \text{ rad} = 225 \text{ rad} = (225 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 36 \text{ rev}$$

4.69. Model: The drill is a rigid rotating body.
Visualize:



The figure shows the drill's motion from the top.
Solve: (a) The kinematic equation $\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes, after using

$\omega_i = 2400 \text{ rpm} = (2400)(2\pi)/60 = 251.3 \text{ rad/s}$, $t_f - t_i = 2.5 \text{ s} - 0 \text{ s} = 2.5 \text{ s}$, and $\omega_f = 0$ rad/s,

$$0 \text{ rad} = 251.3 \text{ rad/s} + \alpha(2.5 \text{ s}) \Rightarrow \alpha = -100 \text{ rad/s}^2$$

(b) Applying the kinematic equation for angular position yields:

$$\begin{aligned}\theta_f &= \theta_i + \omega_i(t_f - t_i) + \frac{1}{2}\alpha(t_f - t_i)^2 \\ &= 0 \text{ rad} + (251.3 \text{ rad/s})(2.5 \text{ s} - 0 \text{ s}) + \frac{1}{2}(-100 \text{ rad/s}^2)(2.5 \text{ s} - 0 \text{ s})^2 \\ &= 3.2 \times 10^2 \text{ rad} = 50 \text{ rev}\end{aligned}$$

4.70. Model: Model the turbine as a rotating rigid body. Assume the angular acceleration is constant. $\Delta t = T$.

Visualize: First find α from $\dfrac{\Delta\omega}{T}$ and then use $\Delta\theta = \omega_0 T + \dfrac{1}{2}\alpha T^2$.

Solve:

$$\alpha = \frac{\Delta\omega}{T} = \frac{0 - \omega_0}{T} = \frac{-\omega_0}{T}$$

$$\Delta\theta = \omega_0 T + \frac{1}{2}\alpha T^2 = \omega_0 T + \frac{1}{2}\left(\frac{-\omega_0}{T}\right)T^2 = \frac{1}{2}\omega_0 T$$

While we were thinking of SI units above, any set of consistent units will do. We want the answer in revolutions, so we'll use the data in the units given.

$$\Delta\theta = \frac{1}{2}\omega_0 T = \frac{1}{2}\left(3800\frac{\text{rev}}{\text{min}}\right)(10 \text{ min}) = 19000 \text{ rev}$$

Assess: That is a lot of revolutions, but the turbine was spinning fast and it took a long time to slow down.

4.71. Model: Model the tire as a rotating rigid body. Assume the angular acceleration is constant. The radius of the tire is 32 cm.
Visualize: $\omega_i = 3.5 \text{ rev/s} = 22 \text{ rad/s}$; $\omega_f = 6.0 \text{ rev/s} = 37.7 \text{ rad/s}$.

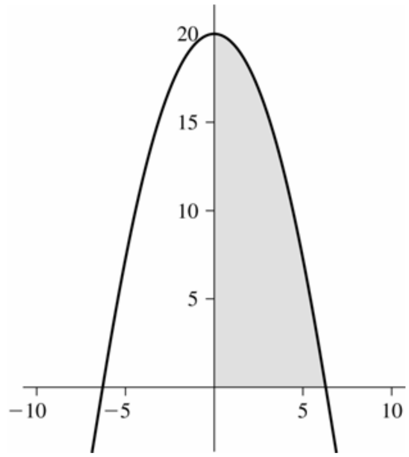
Solve: $\Delta\theta = \dfrac{\Delta x}{r}$.

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{\omega_f^2 - \omega_i^2}{2\dfrac{\Delta x}{r}} = \frac{(37.7 \text{ rad/s})^2 - (22 \text{ rad/s})^2}{2\dfrac{200 \text{ m}}{0.32 \text{ m}}} = 0.75 \text{ rad/s}^2$$

Assess: The units all check out.

4.72. Model: Model the motor as a rotating rigid body. The angular acceleration is not constant, but we still know that the angular acceleration is the derivative of the angular velocity and that the change in angle is the integral of the angular velocity.

Visualize: The area under the ω vs. t graph is shown in the accompanying figure.



Solve:

(a) The motor reverses direction at a turning point, when $\omega = 0$.

$$20 \text{ rad/s} = \frac{1}{2}t^2 \text{ rad/s} \Rightarrow t = \sqrt{40} \text{ s} = 6.325 \text{ s} \approx 6.3 \text{ s}$$

(b) The angular position is the area under the angular velocity graph, but we need calculus to do this for a non-linear graph.

$$\Delta\theta = \int_0^{6.325} 20 - \frac{1}{2}t^2 \; dt = \left[20t - \frac{1}{6}t^3 \right]_0^{6.325} = 84 \text{ rad}$$

Assess: $\Delta\theta$ seems reasonable given the angular speed and the time.

4.73. Model: Model the rider as a particle and the wheel as a rotating rigid body. Let $t = 0$ as the wheel starts from rest. Assume α is constant.

Visualize: We integrate the angular acceleration to get the angular velocity and integrate that to get the angular position.

Solve:

(a) We want the answers expressed in terms of α and $\Delta\theta$.

$$\omega = \int \alpha dt = \alpha t + \omega_0$$

But $\omega_0 = 0$. Now integrate again.

$$\theta = \int \alpha t \, dt = \frac{1}{2}\alpha t^2 + \theta_0$$

$$\Delta\theta = \frac{1}{2}\alpha t^2 \Rightarrow t^2 = \frac{2\Delta\theta}{\alpha}$$

$$\omega = \alpha t = \alpha\sqrt{\frac{2\Delta\theta}{\alpha}} = \sqrt{2\alpha\Delta\theta}$$

$$v = \omega R = \sqrt{2\alpha\Delta\theta}\, R$$

(b) Centripetal acceleration is $a = \omega^2 R$.

$$a = \omega^2 R = 2\alpha\Delta\theta R$$

Assess: The units check out.

4.74. Model: Model the gear as a rotating rigid body. The angular acceleration is not constant, but we still know that the angular acceleration is the derivative of the angular velocity.
Visualize: The area under the ω vs. t graph is shown in the accompanying figure.
Solve:
(a) Take the derivative of $\omega(t)$.

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(2.0 + \frac{1}{2}t^2\right) = t$$

$$\alpha(t = 4\text{ s}) = 4\text{ rad/s}^2$$

(b)

$$a_t = \alpha r = (4\text{ rad/s}^2)(0.060\text{ m}) = 0.24\text{ m/s}^2$$

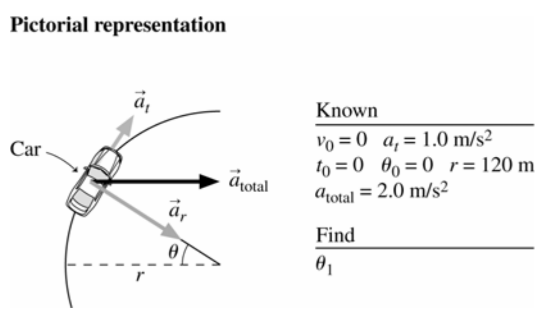
Assess: We don't need to know ω_0 or θ_0 because we don't need those constants of integration when we are taking derivatives.

4.75. Model: Model the gear as a rotating rigid body. Assume the angular acceleration is constant. $\Delta t = t$.
Visualize: We are given $\alpha = (20 - t)\text{ rad/s}^2, \omega_0 = 300\text{ rpm}$.
Solve:

$$\begin{aligned}
\omega &= \omega_0 + \int_{t_0}^{t} \alpha\, dt \\
&= 300\text{ rpm} + \int_0^{20\text{ s}} \left[(20 - t)\text{rad/s}^2\right]dt \\
&= 300\text{ rpm} + \left[20t - \frac{t^2}{1}\right]_0^{20\text{ s}} \\
&= 300\text{ rpm} + \left[400 - 200\right]\text{rad/s} \\
&= 300\text{ rpm} + 200\text{rad/s}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 1940\text{ rpm}
\end{aligned}$$

Assess: We see that the angular acceleration is positive before $t = 20$ s, but negative after that.

4.76. Model: Model the car as a particle in nonuniform circular motion.
Visualize:



Note that the tangential acceleration stays the same at 1.0 m/s^2. As the tangential velocity increases, the radial acceleration increases as well. After a time t_1, as the car goes through an angle $\theta_1 - \theta_0$, the total acceleration will increase to 2.0 m/s^2. Our objective is to find this angle.

Solve: Using $v_1 = v_0 + a_\text{t}(t_1 - t_0)$, we get

$$v_1 = 0 \text{ m/s} + (1.0 \text{ m/s}^2)(t_1 - 0 \text{ s}) = (1.0 \text{ m/s}^2)\, t_1$$

$$\Rightarrow a_r = \frac{rv_1^2}{r^2} = \frac{(1.0 \text{ m/s}^2)^2 t_1^2}{120 \text{ m}} = \frac{t_1^2}{120}(\text{m/s}^4)$$

$$\Rightarrow a_\text{total} = 2.0 \text{ m/s}^2 = \sqrt{a_t^2 + a_r^2} = \sqrt{(1.0 \text{ m/s}^2)^2 + \left[\frac{t_1^2}{120}(\text{m/s}^4)\right]^2} \Rightarrow t_1 = 14.4 \text{ s}$$

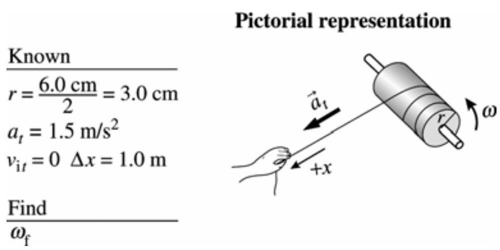
We can now determine the angle θ_1 using

$$\theta_1 = \theta_0 + \omega_0(t_1 - t_0) + \tfrac{1}{2}\left(\frac{a_t}{r}\right)(t_1 - t_0)^2$$

$$= 0 \text{ rad} + 0 \text{ rad} + \frac{1}{2}\frac{(1.0 \text{ m/s}^2)}{(120 \text{ m})}(14.4 \text{ s})^2 = 0.864 \text{ rad} = 49.5°$$

The car will have traveled through an angle of 50°.

4.77. Model: The string is wrapped around the spool in such a way that it does not pile up on itself, and unwinds without slipping.
Visualize:



Solve: Since the string unwinds without slipping, the angular distance the spool turns as the string is pulled 1.0 m is $\Delta\theta = \dfrac{\Delta x}{r} = \dfrac{1.0 \text{ m}}{3.0 \times 10^{-2} \text{ m}} = 33 \text{ radians.}$

The angular acceleration of the spool due to the pull on the string is

$$\alpha = \frac{a_t}{r} = \frac{1.5 \text{ m/s}^2}{3.0 \times 10^{-2} \text{ m}} = 50 \text{ rad/s}^2$$

The angular velocity of the spool after pulling the string is found with kinematics.

$$\omega_\text{f}^2 = \omega_\text{i}^2 + 2\alpha\Delta\theta \Rightarrow \omega_\text{f}^2 = 0 \text{ rad}^2/\text{s}^2 + 2(50 \text{ rad/s}^2)(33 \text{ rad})$$

$$\Rightarrow \omega_\text{f}^2 = 57 \text{ rad/s}$$

Converting to revolutions per minute,

$$(57 \text{ rad/s})\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right) = 5.5 \times 10^2 \text{ rpm}$$

Assess: The angular speed of $57 \text{ rad/s} \approx 9 \text{ rev/s}$ is reasonable for a medium-sized spool.

4.78. Solve: (a) A golfer hits an iron shot with a new club as she approaches the green. She is pretty sure, based on past experience, that she hit the ball with a speed of 50 m/s, but she is not sure at what angle the golf ball took flight. She observed that the ball traveled 100 m before hitting the ground. What angle did she hit the ball?
(b) From the second equation,

$$(4.9 \text{ m/s}^2)t_1^2 - (50\sin\theta \text{ m/s})\, t_1 = 0 \Rightarrow t_1 = 0 \text{ s and } t_1 = \frac{(50 \text{ m/s})\sin\theta}{4.9 \text{ m/s}^2}$$

Using the above value for t_1 in the first equation yields:

$$100\text{ m}=\frac{(50\cos\theta)(50\sin\theta)\text{ m}^2/\text{s}^2}{4.9\text{ m/s}^2}$$

$$\Rightarrow 2\cos\theta\sin\theta=\sin 2\theta=\frac{9.8}{25}=0.392\Rightarrow 2\theta=23.1\Rightarrow\theta=11.5°$$

Assess: Although the original speed is reasonably high $(50\text{ m/s}=112\text{ mph})$, the ball travels a distance of only 100 m, implying either a small launch angle around 10° or an angle closer to 80°. The calculated angle of 11.5° is thus pretty reasonable.

4.79. Solve: (a) A submarine moving east at 3.0 m/s sees an enemy ship 100 m north of its path. The submarine's torpedo tube happens to be stuck in a position pointing 45° west of north. The tube fires a torpedo with a speed of 6.0 m/s relative to the submarine. How far east or west of the ship should the sub be when it fires?
(b) Relative to the water, the torpedo will have velocity components

$$v_x=-6.0\cos 45°\text{ m/s}+3.0\text{ m/s}=-4.24\text{ m/s}+3\text{ m/s}=-1.24\text{ m/s}$$

$$v_y=+6.0\cos 45°\text{ m/s}=+4.2\text{ m/s}$$

The time to travel north to the ship is

$$100\text{ m}=(4.2\text{ m/s})\,t_1\Rightarrow t_1=24\text{ s}$$

Thus, $x=(1.24\text{ m/s})(24\text{ s})=-30\text{ m}$. That is, the ship should be 30 m west of the submarine.

4.80. Solve: (a) A 1000 kg race car enters a 50 m radius curve and accelerates around the curve for 10.0 s. The forward force provided by the car's wheels is 1500 N. After 10.0 s the car has moved 125 m around the track. Find the initial and final angular velocities.
(b) From Newton's second law,

$$F_t=ma_t\Rightarrow 1500\text{ N}=(1000\text{ kg})a_t\Rightarrow a_t=1.5\text{ m/s}^2\qquad \Delta\theta=\frac{\Delta s}{r}=\frac{125\text{ m}}{50\text{ m}}=2.5\text{ rad}$$

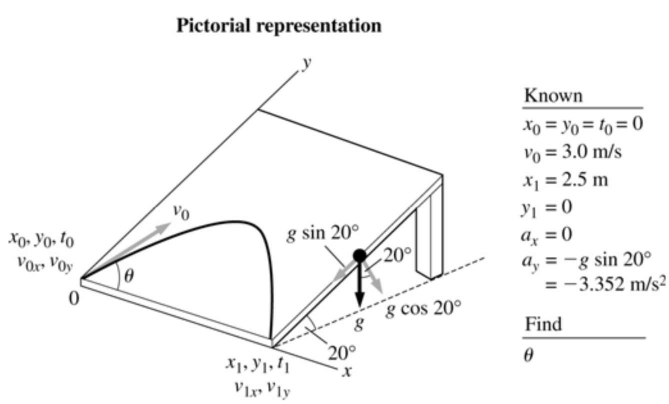
$$\theta_\text{f}=\theta_\text{i}+\omega_\text{i}t+\frac{a_t}{2r}t^2\Rightarrow 2.5\text{ rad}=0\text{ rad}+\omega_i(10\text{ s})+\frac{1.5\text{ m/s}^2}{2(50\text{ m})}(10\text{ s})^2\Rightarrow\omega_\text{i}=0.10\text{ rad/s}$$

$$\omega_\text{f}=\omega_\text{i}+\frac{a_t}{r}t=0.1\text{ rad/s}+\frac{1.5\text{ m/s}^2}{50\text{ m}}(10\text{ s})=0.40\text{ rad/s}$$

Challenge Problems

4.81. Model: We will use the particle model for the ball's motion under constant-acceleration kinematic equations. Note that the ball's motion on the smooth, flat board is $a_y=-g\sin 20°=-3.352\text{ m/s}^2$.

Visualize:



Solve: The ball's initial velocity is

$$v_{0x} = v_0 \cos\theta = (3.0 \text{ m/s})\cos\theta \qquad v_{0y} = v_0 \sin\theta = (3.0 \text{ m/s})\sin\theta$$

Using $x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$,

$$2.5 \text{ m} = 0 \text{ m} + (3.0 \text{ m/s})\cos\theta(t_1 - 0 \text{ s}) + 0 \text{ m} \Rightarrow t_1 = \frac{(2.5 \text{ m})}{(3.0 \text{ m/s})\cos\theta} = \frac{0.833 \text{ s}}{\cos\theta}$$

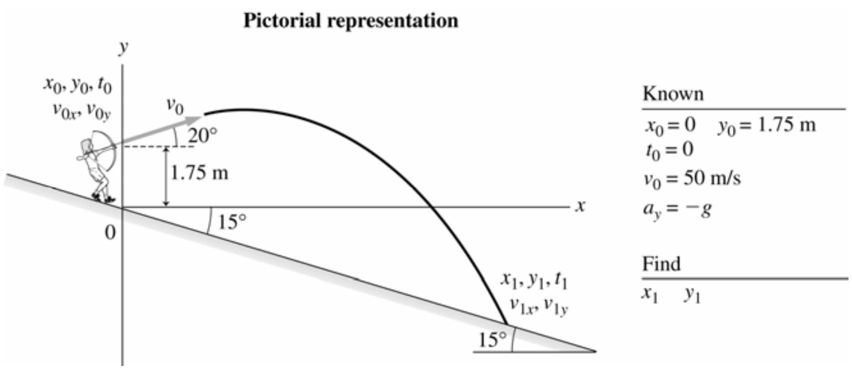
Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$ and the above equation for t_1,

$$0 \text{ m} = 0 \text{ m} + (3.0 \text{ m/s})\sin\theta\left(\frac{0.833 \text{ s}}{\cos\theta}\right) - \frac{1}{2}(3.352 \text{ m/s}^2)\frac{(0.833 \text{ s})^2}{\cos^2\theta}$$

$$\Rightarrow (2.5 \text{ m})\frac{\sin\theta}{\cos\theta} = \frac{1.164}{\cos^2\theta} \Rightarrow 2.5\sin\theta\cos\theta = 1.164 \Rightarrow 2\theta = 68.6° \Rightarrow \theta = 34.3°$$

4.82. Model: Use the particle model for the arrow and the constant-acceleration kinematic equations. We will assume that the archer shoots from 1.75 m above the slope (about 5′ 9″).

Visualize:



Solve: For the *y*-motion:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2 \Rightarrow y_1 = 1.75 \text{ m} + (v_0 \sin 20°)t_1 - \frac{1}{2}gt_1^2$$

$$\Rightarrow y_1 = 1.75 \text{ m} + (50 \text{ m/s})\sin 20° t_1 - \frac{1}{2}gt_1^2$$

For the *x*-motion:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 = 0 \text{ m} + (v_0 \cos 20°)t_1 + 0 \text{ m} = (50 \text{ m/s})(\cos 20°)t_1$$

Because $y_1/x_1 = -\tan 15° = -0.268$,

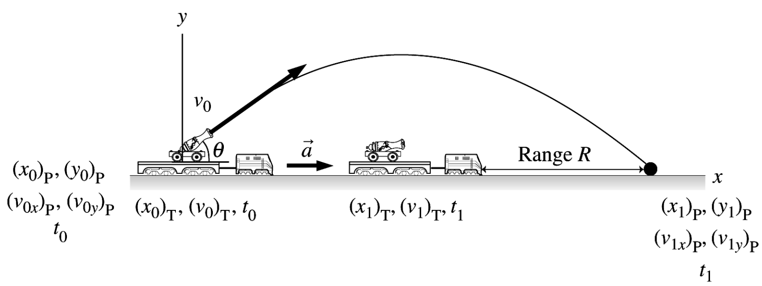
$$\frac{1.75 \text{ m} + (50 \text{ m/s})(\sin 20°)t_1 - \frac{1}{2}gt_1^2}{(50 \text{ m/s})(\cos 20°)t_1} = -0.268 \Rightarrow t_1 = 6.12 \text{ s and } -0.058 \text{ s (unphysical)}$$

Using $t_1 = 6.12$ s in the *x*- and *y*-equations above, we get $y_1 = -77.0$ m and $x_1 = 287$ m. This means the distance down the slope is $\sqrt{x_1^2 + y_1^2} = \sqrt{(287 \text{ m})^2 + (-77.0 \text{ m})^2} = 297$ m.

Assess: With an initial speed of 112 mph (50 m/s) for the arrow, which is shot from a 15° slope at an angle of 20° above the horizontal, a horizontal distance of 287 m and a vertical distance of 77.0 m are reasonable numbers.

4.84. Model: The train and projectile are treated in the particle model. The height of the cannon above the tracks is ignored.

Visualize:



Known	Find
$(x_0)_P = (y_0)_P = (x_0)_T = 0$ m $\quad t_0 = 0$ s	θ that maximizes
$(v_{0x})_P = v_0\cos\theta$ relative to train	$R = (x_1)_P - (x_1)_T$
$(v_{0y})_P = v_0\sin\theta$ relative to train	
$(v_0)_T = v_{train}$	
$(y_1)_P = 0$	
a	

Solve: In the ground reference frame, the projectile is launched with velocity components

$$(v_{0x})_P = v_0\cos\theta + v_{train}$$

$$(v_{0y})_P = v_0\sin\theta$$

While the projectile is in free fall, $v_{fy} = v_{iy} - g\Delta t$. The time for the projectile to rise to the highest point is (with $v_{fy} = 0$) $\Delta t = \dfrac{v_{iy}}{g}$. So the time to vertically rise and fall is

$$t_1 = 2\Delta t = 2\frac{(v_{0y})_P}{g} = \frac{2v_0}{g}\sin\theta$$

During this time the projectile travels a horizontal distance

$$(x_1)_P = (v_{0x})_P t_1 = \frac{2v_0^2}{g}\sin\theta\cos\theta + \frac{2v_0 v_{train}}{g}\sin\theta$$

During the same time, the train travels a horizontal distance

$$(x_1)_T = (v_{0x})_T t_1 + \frac{1}{2}at_1^2 = \frac{2v_0 v_{train}}{g}\sin\theta + \frac{2v_0^2 a}{g^2}\sin^2\theta$$

The range R is the difference between the two horizontal distances:

$$R = (x_1)_P - (x_1)_T = \frac{2v_0^2}{g}\left(\sin\theta\cos\theta - \frac{a}{g}\sin^2\theta\right)$$

Note that the range is independent of v_{train} the train's steady motion. This makes sense, since the train and projectile share that motion when the projectile is launched.

Maximizing the range R requires $\dfrac{dR}{d\theta} = 0$. Thus (ignoring the constant $\dfrac{2v_0^2}{g}$)

$$\frac{dR}{d\theta} = \cos^2\theta - \sin^2\theta - \frac{a}{g}(2\sin\theta\cos\theta) = \cos 2\theta - \frac{a}{g}\sin 2\theta = 0$$

Solving for θ,

$$\frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta = \frac{g}{a} \Rightarrow \theta = \frac{1}{2}\tan^{-1}\left(\frac{g}{a}\right)$$

Note that

$$a > 0 \text{ (train speeding up) gives } \theta < 45°$$

and

$$a < 0 \text{ (train slowing down) gives } \theta > 45°$$

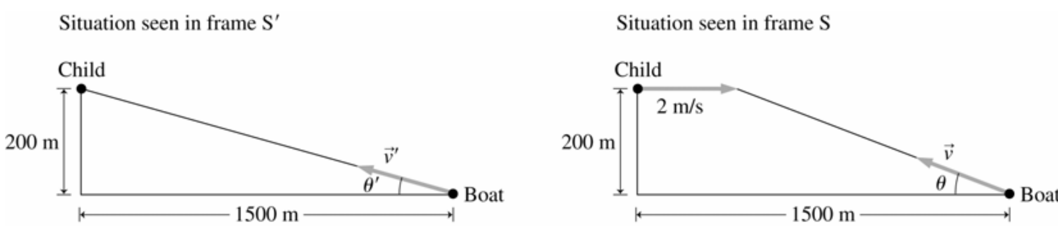
since $\tan^{-1}\left(\frac{g}{a}\right)$ will be in the 2^{nd} quadrant.

Assess: As a check, see what the angle θ is for the limiting case in which the train does not accelerate:

$$a = 0 \Rightarrow \theta = \frac{1}{2}\tan^{-1}(\infty) = \frac{1}{2}\times 90° = 45°$$

This is the expected answer.

4.85. Model: Assume the river flows toward the east. Use subscripts B, W, E for the boat, water, and earth respectively. The child is at rest with respect to the water; this means they are in the same reference frame.
Visualize:



Solve: The boat can go directly to the child at angle $\theta = \tan^{-1}(200/1500) = 7.595°$. The boat's speed is 8.0 m/s, so the components of the boat's velocity in with respect to the water are

$$(v_x)_{\text{BW}} = -(8.0 \text{ m/s})\cos 7.595° = -7.93 \text{ m/s}$$

$$(v_y)_{\text{BW}} = (8.0 \text{ m/s})\sin 7.595° = 1.06 \text{ m/s}$$

The river flows with velocity $(v_x)_{\text{WE}} = 2.0\hat{i}$ m/s relative to the earth. In the earth's frame, which is also the frame of the riverbank and the boat dock, the boat's velocity is

$$(v_x)_{\text{BE}} = -5.93 \text{ m/s} \text{ and } (v_y)_{\text{BW}} = 1.06 \text{ m/s}$$

Thus the boat's angle with respect to the riverbank is $\theta = \tan^{-1}(5.93/1.06) = 10.1° \approx 10°$
Assess: The boat, like the child, is being swept downstream. This moves the boat's angle away from the shore.