STAT 50 HW #3 Sections 2.1 and 2.2
Section 2.1
1.
The probability that a bolt meets a strength specification is 0.87. What is the probability that the bolt does not meet the specification?

Let P(A) be the probability that a bolt meets the strength specification.
Then, P(A^c) would be the probability that a bolt does not meet the specification.
P(A^c) = 1 - P(A) = 1 - 0.87 = 0.13

2.
A die (six faces) has the number 1 painted on three of its faces, the number 2 painted on two of its faces, and the number 3 painted on one face. Assume that each face is equally likely to come up.

a. **Find a sample space for this experiment.**

Let S represent the sample space.
S = {1,2,3}

b. **Find P(odd number).**

P(odd number) = P(1 gets rolled) + P(3 gets rolled)
P(odd number) = 3/6 + 1/6
P(odd number) = 4/6 = 2/3

c. **If the die were loaded so that the face with the 3 on it were twice as likely to come up as each of the other five faces, would this change the sample space? Explain.**

This would not change the sample space. The set of all possible outcomes remains unchanged.
Our sample space still has the same possible outcomes of 1,2,3.
S = {1,2,3}

d. **If the die were loaded so that the face with the 3 on it were twice as likely to come up as each of the other five faces, would this change the value of P(odd number)? Explain.**

Yes.
P(S) = 1
P(Not 3) = 5a
P(3) = 2a

$5a + 2a = 1$ *Solving for a gives us 1/7.
P(odd number) = P(1 gets rolled) + P(3 gets rolled)
= 3/7 + 2/7 = 5/7
P(odd number) = 5/7 now!

3.
A section of an exam contains four True-False questions. A completed exam paper is selected at random, and the four answers are recorded.

a. **List all 16 outcomes in the sample space.**

S = {TTTT, TTTF, TTFT, TFTT, FTTT, TTFF, TFFT, FFTT, TFFF, FFFT, FTFF, FFTF, FTTF, TFTF, FTFT, FFFF}

b. **Assuming the outcomes to be equally likely, find the probability that all the answers are the same.**

P(All answers the same) = 2/16 = 1/8

c. **Assuming the outcomes to be equally likely, find the probability that exactly one of the four answers is "True."**

P(Exactly one answer is true) = 4/16 = 1/4

d. **Assuming the outcomes to be equally likely, find the probability that at most one of the four answers is "True."**

P(At most one is true) = 5/16

5.
Four candidates are to be interviewed for a job. Two of them, numbered 1 and 2, are qualified, and the other two, numbered 3 and 4, are not. The candidates are interviewed at random, and the first qualified candidate interviewed will be hired. The outcomes are the sequences of candidates that are interviewed. So one outcome is 2, and another is 431.

a. **List all the possible outcomes.**

{1, 2, 31, 41, 32, 42, 341, 342, 431, 432}

b. **Let A be the event that only one candidate is interviewed. List the outcomes in A.**

A = {1,2}

c. **Let B be the event that three candidates are interviewed. List the outcomes in B.**

B = {341, 342, 431, 432}

d. **Let C be the event that candidate 3 is interviewed. List the outcomes in C.**

C = {31, 32, 341, 342, 431, 432}

e. **Let D be the event that candidate 2 is not interviewed. List the outcomes in D.**

D = {1, 31, 41, 341, 431}

f. **Let E be the event that candidate 4 is interviewed. Are A and E mutually exclusive? How about B and E, C and E, D and E?**

E = {41, 42, 341, 342, 431, 432}
A and E are mutually exclusive.
B and E are not mutually exclusive.
C and E are not mutually exclusive.
D and E are not mutually exclusive.

9.
Among the cast aluminum parts manufactured on a certain day, 80% were flawless, 15% had only minor flaws, and 5% had major flaws. Find the probability that a randomly chosen part

a. **has a flaw (major or minor).**

P(Flawed) = P(Minor flaw) + P(Major flaw) = 0.15 + 0.05 = 0.20

b. **has no major flaw.**

P(No major flaw) = P(No flaw) + P(Minor flaw) = 0.80 + 0.15 = 0.95

10.
The article "High Cumulative Risk of Lung Cancer Death among Smokers and Nonsmokers" (P. Brennan, et al. American Journal of Epidemiology, 2006:1233–1241) states that the probability is 0.24 that a man who is a heavy smoker will contract lung cancer. True or false:

a. **In a sample of 100 men who are heavy smokers, exactly 24 of them will contract lung cancer.**

False.

b. **In a sample of 100 men who are heavy smokers, the number who will contract lung cancer is likely to be close to 24, but not exactly equal to 24.**

True.

c. **As more and more heavy-smoking men are sampled, the proportion who contract lung cancer will approach 0.24.**

True.

14.
Six hundred paving stones were examined for cracks, and 15 were found to be cracked. The same 600 stones were then examined for discoloration, and 27 were found to be discolored. A total of 562 stones were neither cracked nor discolored. One of the 600 stones is selected at random.

a. **Find the probability that it is cracked, discolored, or both.**

*It helps to make a table, which is why I have one below.

	Discolored	Not Discolored	Total
Cracked	4	11	15
Not Cracked	23	562	585
Total	27	573	600

Let $P(A)$ = Probability that it is cracked.
Let $P(B)$ = Probability that it is discolored.
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 15/600 + 27/600 - 4/600 = 38/600 = 19/300$

b. **Find the probability that it is both cracked and discolored.**

$P(A \cap B) = 4/600 = 1/150$

c. **Find the probability that it is cracked but not discolored.**

$P(A \cap B^c) = 11/600$

Section 2.2

1.

DNA molecules consist of chemically linked sequences of the bases adenine, guanine, cytosine, and thymine, denoted A, G, C, and T. A sequence of three bases is called a codon. A base may appear more than once in a codon.

a. **How many different codons are there?**

There are 4 possible bases and sequences of 3 with repeats allowed.
So we have 4*4*4 = 64 possible codons.

b. **The bases A and G are purines, while C and T are pyrimidines. How many codons are there whose first and third bases are purines and whose second base is a pyrimidine?**

We have 2 choices for the 1st and 3rd bases and 2 choices for the 2nd base.
2*2*2 = 8 possible codons.

c. **How many codons consist of three different bases?**

We have 4 options for the 1st base, 3 for the 2nd, and 2 for the 3rd, so
4*3*2 = 24 possible codons.

2.

A metallurgist is designing an experiment to determine the effect of flux, base metal, and energy input on the hardness of a weld. She wants to study four different fluxes, two different base metals, and three different amounts of energy input. If each run of the experiment involves a choice of one flux, one base metal, and one amount of energy input, how many different runs are possible?

4*2*3 = 24 possible runs.

5.

In horse racing, one can make a trifecta bet by specifying which horse will come in first, which will come in second, and which will come in third, in the correct order. One can

make a box trifecta bet by specifying which three horses will come in first, second, and third, without specifying the order.

a. **In an eight-horse field, how many different ways can one make a trifecta bet?**

Because order matters, this would be a permutation.

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 * 7 * 6 = 336 \text{ possible trifecta bets!}$$

b. **In an eight-horse field, how many different ways can one make a box trifecta bet?**

Because order doesn't matter. This would be a combination.

$${}_8C_3 = \binom{8}{3} = \frac{8!}{(8-3)!3!} = \frac{8*7*6*5!}{5!3!} = 8 * 7 = 56 \text{ possible box trifecta bets!}$$

6.

A college math department consisting of 10 faculty members must choose a department head, an assistant department head, and a faculty senate representative. In how many ways can this be done?

Assuming no one can take 2 roles, we have:
10*9*8 = 720 possible ways

7.

A test consists of 15 questions. Ten are true-false questions, and five are multiple-choice questions that have four choices each. A student must select an answer for each question. In how many ways can this be done?

2*2*2*2*2*2*2*2*2*2*4*4*4*4*4 = $2^{10} * 4^5$ = 1,048,576 possible ways

8.

License plates in a certain state consist of three letters followed by three digits.

a. **How many different license plates can be made?**

There are 26 letters and 10 digits from 0-9 (We aren't told we can't repeat values)
26*26*26*10*10*10 = 17,576,000 different license plates

b. **How many license plates are there that contain neither the letter "Q" nor the digit "9"?**

We are left with 25 letters and 9 digits (Assuming Q and 9 are both not allowed)
25*25*25*9*9*9 = 11,390,625 license plates

c. **A license plate is drawn at random. What is the probability that it contains neither the letter "Q" nor the digit "9"?**

P(Neither a Q nor a 9) = 11,390,625/17,576,000 = 0.64

9.

A computer password consists of eight characters.

a. **How many different passwords are possible if each character may be any lowercase letter or digit?**

We have 8 characters with 26 lowercase letters and 10 digits to choose from
(26+10)*(36)*(36)*(36)*(36)*(36)*(36)*(36) = $36^8 = 2.8211 * 10^{12}$

b. **How many different passwords are possible if each character may be any lowercase letter or digit, and at least one character must be a digit?**

$36^8 - 26^8 = 2.6123 * 10^{12}$

c. **A computer system requires that passwords contain at least one digit. If eight characters are generated at random, and each is equally likely to be any of the 26 letters or 10 digits, what is the probability that a valid password will be generated?**

P(At least 1 digit) = 1 - P(No digit) = $36^8 - 26^8$
$= \left(\frac{36^8 - 26^8}{36^8} \right) = 0.9260$

11.

One drawer in a dresser contains 8 blue socks and 6 white socks. A second drawer contains 4 blue socks and 2 white socks. One sock is chosen from each drawer. What is the probability that they match?

Assuming we have a 3/7 chance of drawing a white sock from the first drawer (6/14) and a 1/3 chance of drawing a white sock from the second drawer (2/6), we have:
(3/7)(1/3) = 3/21 = 1/7
Assuming we have a 4/7 chance of drawing a blue sock from the first drawer (8/14) and a 2/3 chance of drawing a blue sock from the second drawer, we have:
(4/7)(2/3) = 8/21
When we add up both probabilities, we have:
(1/7) + (8/21) = 11/21

12.
A drawer contains 6 red socks, 4 green socks, and 2 black socks. Two socks are chosen at random. What is the probability that they match?

P(2 socks match) = P(2 red) + P(2 green) + P(2 black)
P(2 red) = 6/12 * 5/11 = 30/132
P(2 green) = 4/12 * 3/11 = 12/132
P(2 black) = 2/12 * 1/11 = 2/132
P(2 socks match) = (30/132) + (12/132) + (2/132) = 44/132 = 1/3