

# Lab 03: Projectile Motion

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**Objective Statement:** Describe the purpose of the lab in a few sentences, and state any hypothesis formed ahead of doing the experiment.

The purpose of this lab is to determine the gravitational acceleration by investigating a projectile in motion. We hypothesized that if gravity is roughly constant or about the same everywhere on Earth, then we should be able to get a result close to  $980.00 \frac{cm}{s^2}$  within the uncertainty of our measurements.

1. Measure the x and y position of the ball for 10 different points. Hint: in order to do so, you will have to establish the origin. I recommend using the  $x = 0$  for the initial point, and  $y = 0$  for the final point. Leave your measurements in units of grids, and do not make any conversions. Also, calculate  $v_y$  and fill the table on the right. Hint: each point is 1 flash.

t (flashes)	x (grids)	y (grids)
0	2.5	26.5
1	8.5	29
2	14.5	30.1
3	20.5	30.1
4	26.7	28.9
5	33	26
6	39	22
7	45	16.5
8	51	10
9	57.3	1.8

t (flashes)	$v_y$ (grids/flash)
.5	2.5
1.5	1.1
2.5	0
3.5	-1.2
4.5	-2.9
5.5	-4
6.5	-5.5
7.5	-6.5
8.5	-8.2

Origin was established at the bottom right of the grid, to ensure all x and y values remained positive.

2. What are your uncertainty estimates for  $\delta x$  and  $\delta y$  (include explanations):

Our uncertainty estimates are  $\pm 0.5$  grids for both  $\delta x$  and  $\delta y$ . The shadows of the ball itself appears to cover approximately 1 grid, and due to the fact that eyeball estimation of position between grid points is imprecise at best. Those two factors combined make it easy to argue that a ball could “really be” anywhere in a  $\pm$  range of 0.5 grids.

$$\delta x = 0.5 \text{ grids}$$

$$\delta y = 0.5 \text{ grids}$$

3. Use the error propagation technique to derive the expression for  $\delta v_y$ , and calculate its value (show work):

$$\delta v_y = \sqrt{\left(\frac{\partial v_y}{\partial y_1} \cdot \delta y_1\right)^2 + \left(\frac{\partial v_y}{\partial y_2} \cdot \delta y_2\right)^2}$$

$$\frac{\partial v_y}{\partial y_1} = (a - y_1) \frac{d}{dy_1} = -1 \text{ grids}$$

$$\frac{\partial v_y}{\partial y_2} = (y_2 - a) \frac{d}{dy_2} = 1 \text{ grids}$$

$$\delta y_1 = \delta y_2 = 0.5 \text{ grids}$$

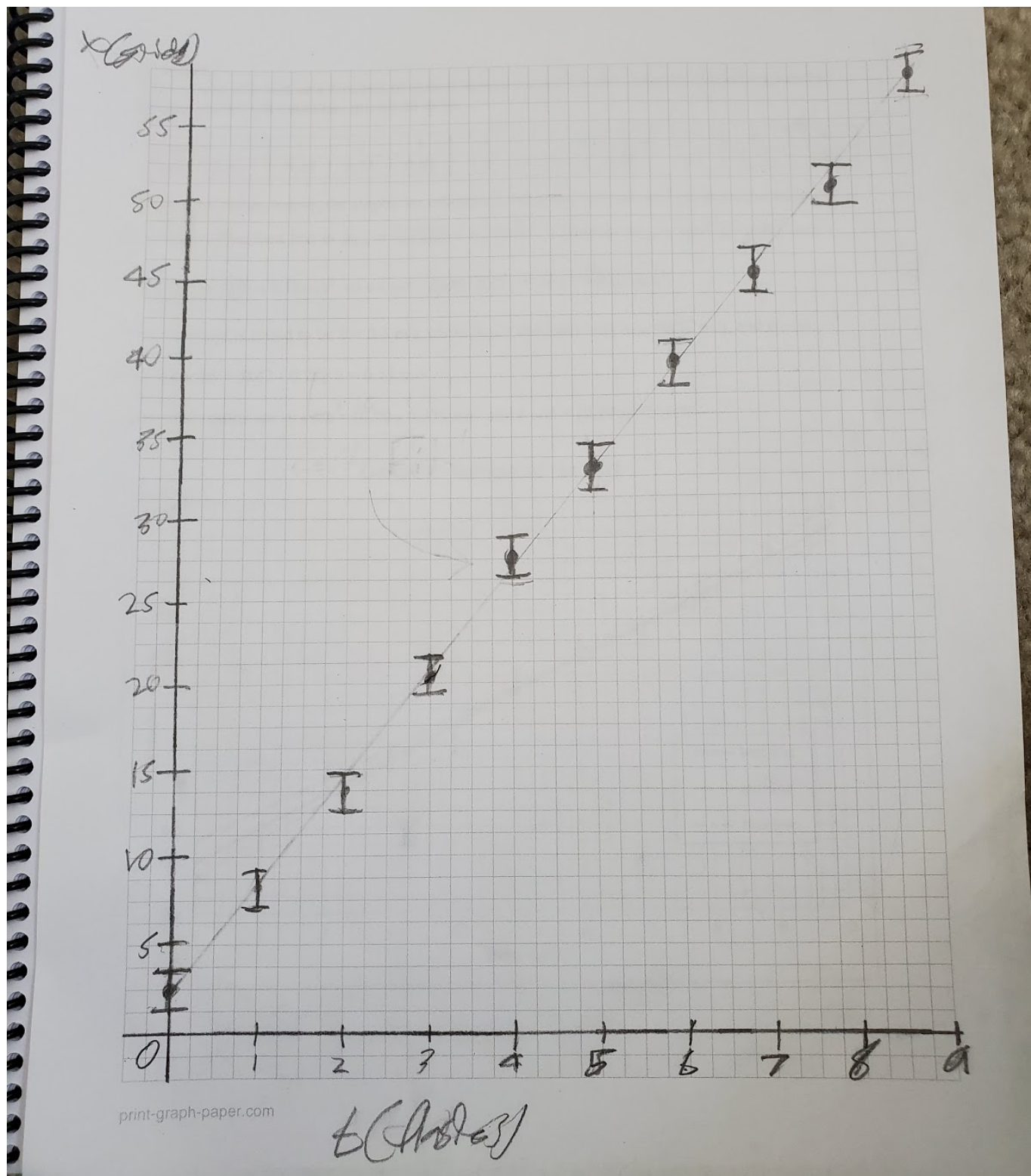
$$\delta v_y = \sqrt{(-1 \cdot 0.5)^2 + (1 \cdot 0.5)^2}$$

$$\delta v_y = \sqrt{0.25 + 0.25}$$

$$\delta v_y = 0.7071 \text{ grids} = (\sqrt{2})(0.5)$$

$$\boxed{\delta v_y = 1 \text{ grids}}$$

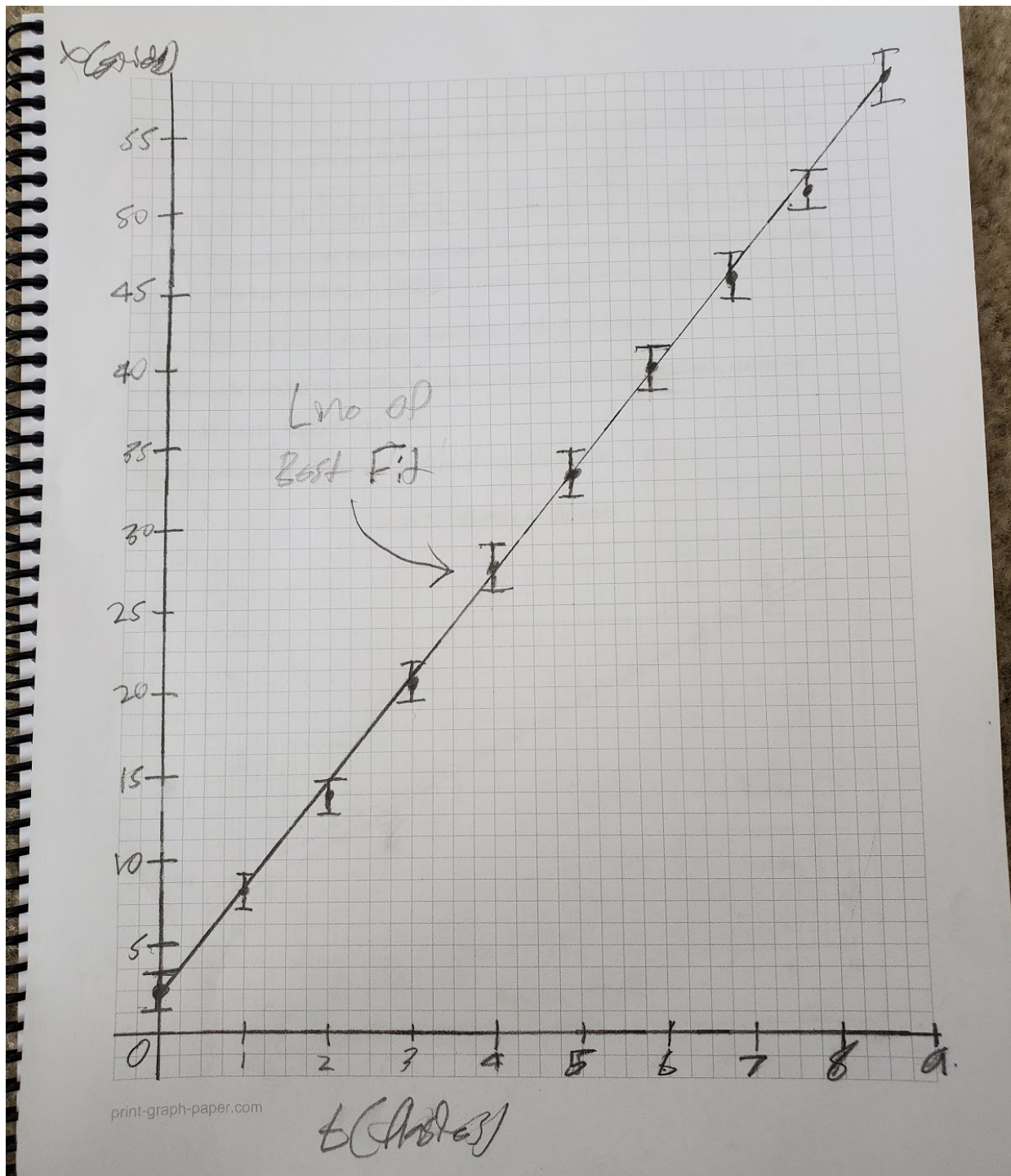
4. On a full-size of graph paper, make a graph of  $x$  vs.  $t$  ( $x$  on the vertical axis,  $t$  on the horizontal axis). Please do this step by hand! Include error bars for  $x$  ( $\pm \delta x$ ), but assume that  $t$  is measured exactly.



$$x = 6.08t + 2.5$$

5. Do the points seem to describe a straight line? If so, draw the line of best-fit (see pg. 27-28 of manual). Calculate the slope of this line. Show your work! What does this slope represent?

The points do seem to describe a straight line. The slope of the line represents a linear approximation of the horizontal velocity of the ball fired from the spring cannon.





$$M = \frac{57.3 - 1.5}{9 - 0} = \frac{54.8}{9} = 6.08$$

$$(0, 2.5) \quad (9, 57.3)$$

$x$        $y$

$$V = mt + b$$

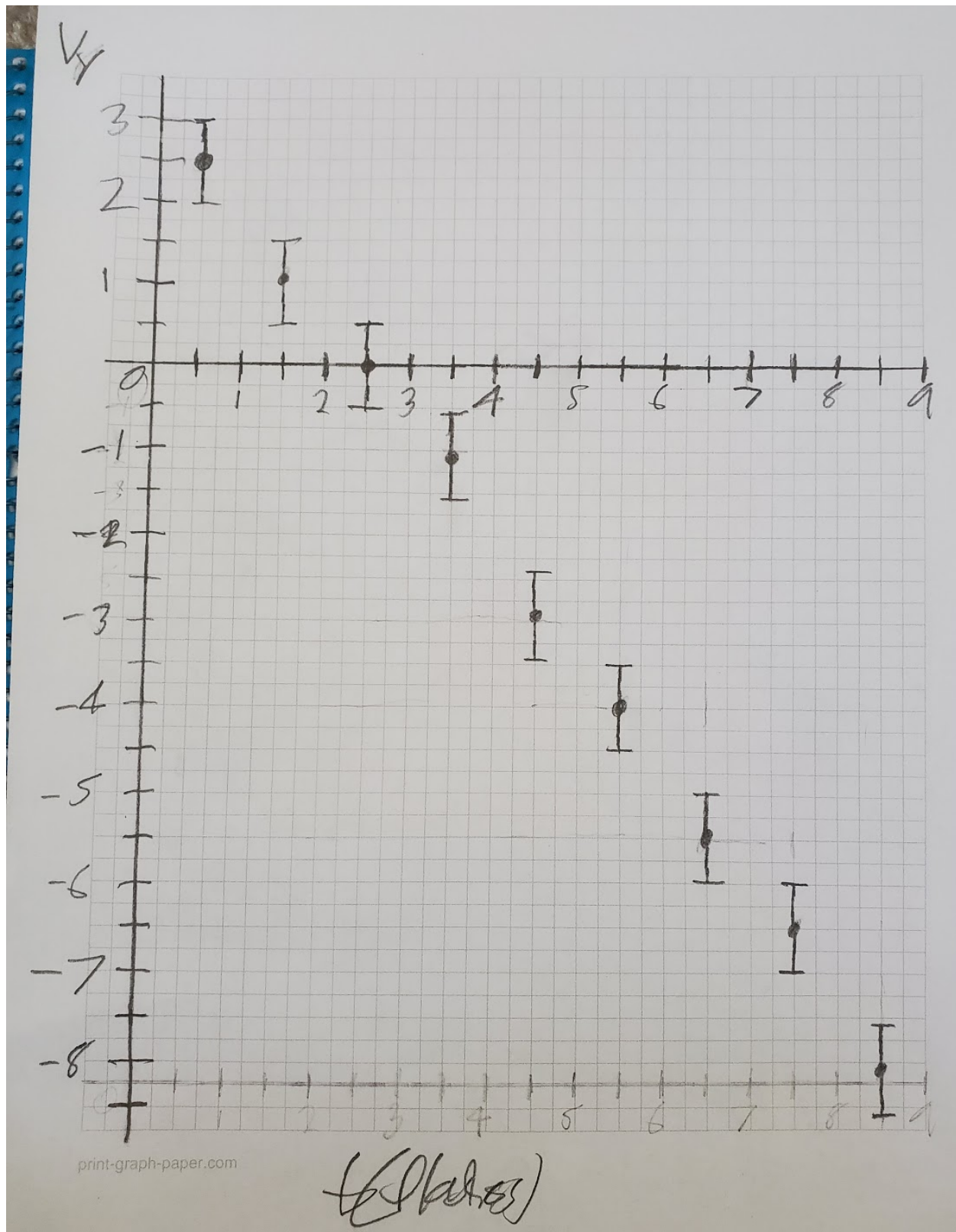
$$25 = \frac{54.8}{9} (9) + b$$

$$x = 6.08t + 2.5$$

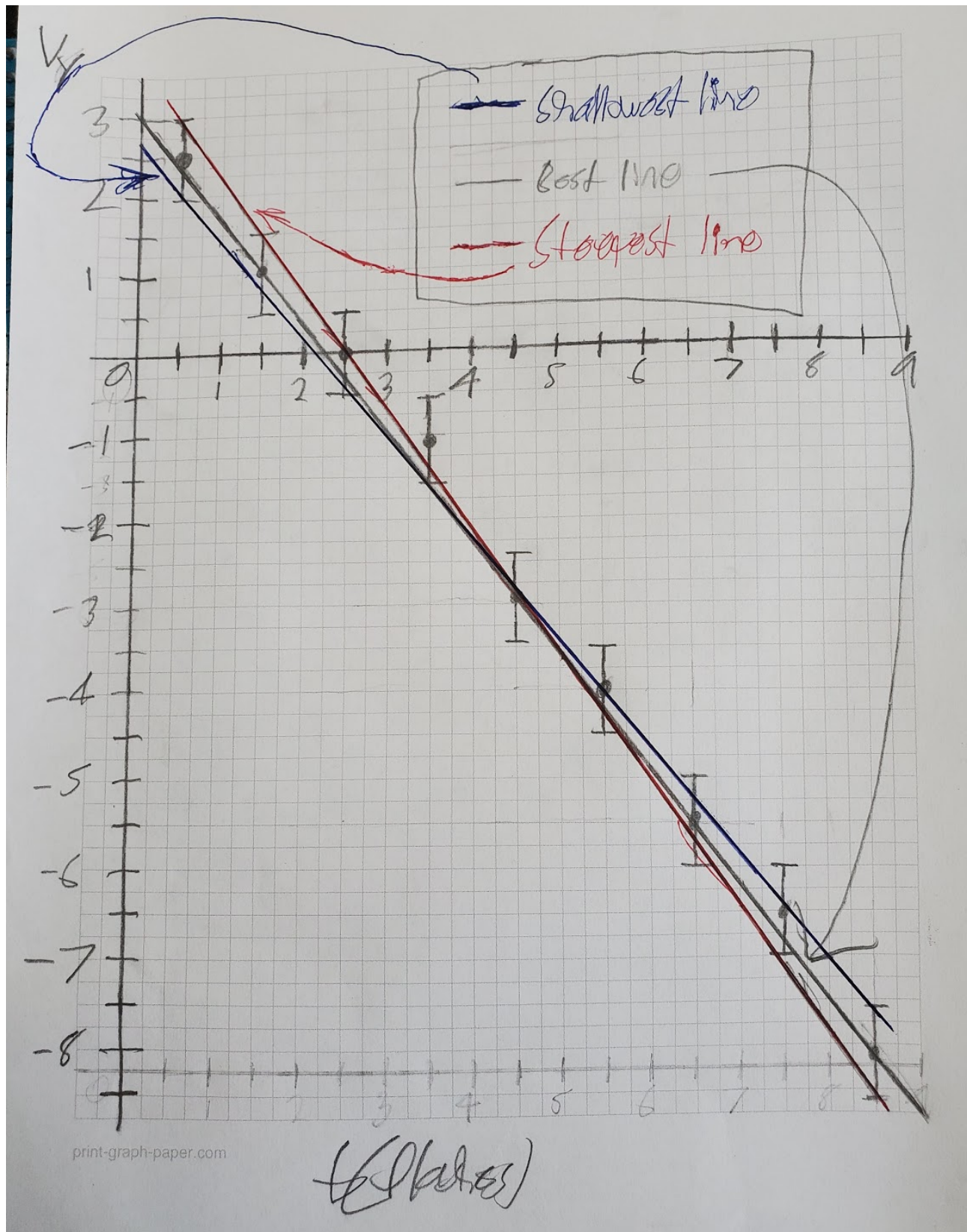
$$b = 2.5$$

New Best fit line:  $x = 6.08t + 2.5$

6. On a separate full-size of graph paper, make a graph (by hand) of  $v_y$  vs.  $t$  ( $v_y$  on the vertical axis,  $t$  on the horizontal axis). Please do this step by hand! Include error bars for  $v_y$  ( $\pm \delta v_y$ ), but assume that  $t$  is measured exactly.



7. On your graph, graph the best-fit line, as well as steepest/shallowest fit lines (see pg. 27-28 of manual).





8. List the corresponding equations for each fit line.

Best fit line:  $x = -1.31t + 3$

Steepest fit line:  $x = -1.44t + 3.62$

Shallowest fit line:  $x = -1.21t + 2.6$

9. Describe how to calculate  $v_{0y} \pm \delta v_{0y}$  and  $a_y \pm \delta a_y$ , and then actually calculate these values. Show all calculations for uncertainties.

$v_{0y}$  is the initial vertical velocity and  $\delta v_{0y}$  is the uncertainty of the initial vertical velocity. It can be calculated plugging in other known values to the formula  $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ .  $\delta v_{0y}$  can be determined through the application of an error propagation (see problem 3 or the image posted below).

$$29 - 26.5 = v_{0y}(1) - \frac{1}{2}(9.8)(1)^2 \rightarrow v_{0y} = 7.25 \pm 1 \frac{\text{grids}}{\text{flash}} = 287 \frac{\text{cm}}{\text{s}}$$

Handwritten calculations for error propagation of  $v_{0y}$ :

$$\delta v_y = \sqrt{\left(\frac{\partial v_y}{\partial y_1} \cdot \delta y_1\right)^2 + \left(\frac{\partial v_y}{\partial y_2} \cdot \delta y_2\right)^2}$$

$$\frac{\partial v_y}{\partial y_1} = (a - y_1) \frac{d}{dy_1} = -1 \text{ grids}$$

$$\frac{\partial v_y}{\partial y_2} = (y_2 - a) \frac{d}{dy_2} = 1 \text{ grids}$$

$$\delta y_1 = \delta y_2 = 0.5 \text{ grids}$$

$$\delta v_y = \sqrt{(-1 \cdot 0.5)^2 + (1 \cdot 0.5)^2}$$

$$\delta v_y = \sqrt{0.25 + 0.25}$$

$$\delta v_y = 0.7071 \text{ grids} = (\sqrt{2})(0.5)$$

$\delta v_y = 1 \text{ grids}$



$a_y$  is the vertical acceleration that comes from the slope of the line of best fit, which is -1.31 in this case. You find vertical acceleration by subtracting initial vertical velocity from final vertical velocity and then divide that by final time minus initial time.  $\delta a_y$  is the uncertainty of vertical acceleration that comes from taking the absolute value of the steepest slope minus the shallowest slope and then dividing by two.

$$a_y = \frac{V_{y2} - V_{y1}}{t_2 - t_1} = \frac{(-8.75 - 3)}{9 - 0} = -1.31$$

$$\delta a_y = \frac{|max - min|}{2} = \frac{|-1.4 - (-1.205)|}{2} = 0.0975$$

**10. Convert  $a_y$  and  $\delta a_y$  to  $cm/sec^2$  and apply the parallax correction. Show all work! See Parallax file on Canvas for values of  $d$  and  $D$ .**

$$a_y = -1.31 \frac{grids}{flash^2} \text{ so, } -1.31 \frac{grids}{flash^2} \left( 1.998 \frac{cm}{grid} \right) \left( 20 \frac{flash}{s} \right)^2 = -1050 \frac{cm}{s^2}$$

$$\delta a_y = 0.0975 \frac{grids}{flash^2} \text{ so, } 0.0975 \frac{grids}{flash^2} \left( 1.998 \frac{cm}{grid} \right) \left( 20 \frac{flash}{s} \right)^2 = 80 \frac{cm}{s^2}$$

$$d = 17.5cm$$

$$D = 234.4cm$$

$$\text{Parallax conversion: } \frac{\Delta x_T}{\Delta x_A} = \frac{(D - d)}{D} = 1 - \frac{d}{D}$$

$$a_y = (-1050 \frac{cm}{s^2}) \left( 1 - \frac{17.5cm}{234.4cm} \right) = 970 \frac{cm}{s^2}$$

$$\delta a_y = (80 \frac{cm}{s^2}) \left( 1 - \frac{17.5cm}{234.4cm} \right) = 70 \frac{cm}{s^2}$$

**11. Calculate % error between your value of  $a_y$  and the expected value.**

$$\% \text{ error} = \frac{|Approximate Value - Exact Value|}{|Exact Value|} * 100$$

$$\% \text{ error} = \frac{|970 - 980|}{|980|} * 100 = 1.02\%$$

**12. List all possible reasons for this error, and classify them as random or systematic. Remember, there is no such thing as "human error"! See pg.15-16 of manual.**

One reason for this error would be Parallax, which in this case could be systematic. Parallax occurs when there is some distance between the measuring scale and the indicator used to obtain a measurement. Another could be failure to account for a factor, which is systematic. In this case, that factor could be the initial position of the ball at “flash 0”, which had to be made up and added into our data. We also encountered random error through using a limited data set, had we used more cannon shots, we could have reduced the random error associated with varying ball launches. Finally, the method of measuring the location of the ball against the grid introduced error, had the grid used a tighter weave of strings, we could have obtained more precise measurements regarding the balls location during its flight path.

**Conclusion: Summarize your findings (*including relevant numbers and their uncertainties*) and compare them to your objective statement/hypothesis.**

Our initial calculation for acceleration due to gravity was  $1050 \pm 80 \frac{\text{cm}}{\text{s}^2}$ . After applying a Parallax conversion to reduce some of the systemic error, our final result was an  $a_y$  value of  $970 \pm 70 \frac{\text{cm}}{\text{s}^2}$ . The result, with error, is within 2% of the expected result of  $a_y = 980.00 \frac{\text{cm}}{\text{s}^2}$ . Therefore, our initial hypothesis that gravity would act as a constant regardless of location was supported. Unfortunately, our experiment has significant amounts of error, both systemic and random which prevented obtaining better results. We had few data points, 10 total, which means we cannot be certain of random error introduced by the shot of the cannon, or discrepancies in the flight of the projectile. Furthermore, our measurements for the location of the ball were only accurate to  $\pm .5$  grids, or approximately 1 centimeter in any direction, which had a cascading effect on further calculations. We also had to accept several measurements at face value, such as the dimensions of the grid, the timing of the strobe flash, and the position of the camera. Meaning if there were any mistakes in the collection of that information, it has been applied to our calculations.