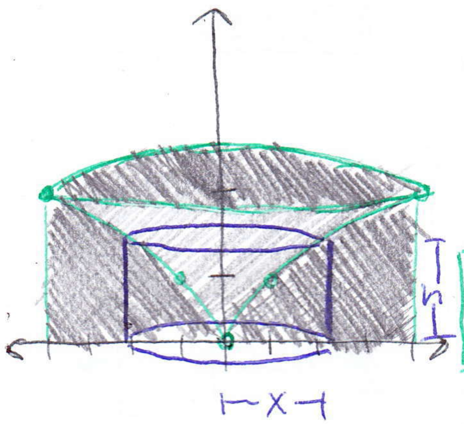
1.

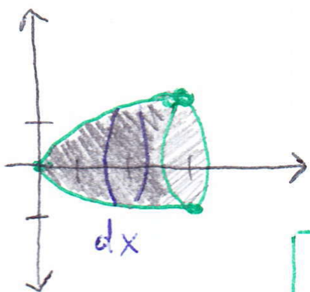


$$V=\int_a^b 2\pi r h w=\int_0^4 2\pi x\sqrt{x}\,dx$$

$$V=2\pi\int_0^4 x^{3/2}\,dx=2\pi\left(\frac{2}{5}x^{5/2}\Big|_0^4\right)$$

$$\boxed{V=\frac{128\pi}{5}\ un^3}$$

2.

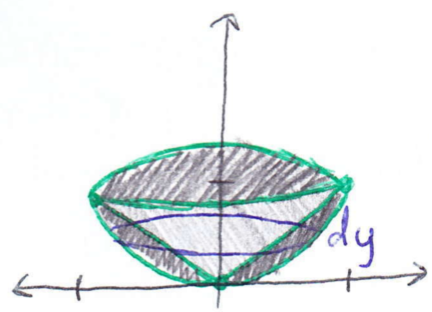


$$A(x)=\pi r_x^2=\pi\left(\frac{1}{\sqrt{3}}\sqrt{x}\right)^2=\frac{\pi}{3}x$$

$$V=\int_a^b A(x)\,dx=\int_0^3\frac{\pi}{3}x\,dx=\frac{\pi}{3}\int_0^3 x\,dx$$

$$\boxed{V=\frac{3\pi}{2}\ un^3}$$

3.



$$A(y)=\pi(r_o^2-r_i^2)=\pi\left[(\sqrt{y})^2-(y)^2\right]=\pi(y-y^2)$$

$$V=\int_a^b A(y)\,dy=\int_0^1\pi(y-y^2)\,dy=\pi\int_0^1(y-y^2)\,dy$$

$$V=\pi\left[\frac{1}{2}y^2\Big|_0^1-\frac{1}{3}y^3\Big|_0^1\right]=\pi\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$\boxed{V=\frac{\pi}{6}\ un^3}$$

4. $$\int x^2\sin x\,dx=-x^2\cos x+2\int x\cos x\,dx$$

* $u=x^2$, $du=2x\,dx$
$v=-\cos x$, $dv=\sin x\,dx$

* $u=x$, $du=dx$
$v=\sin x$, $dv=\cos x\,dx$

$$=-x^2\cos x+2\left(x\sin x-\int\sin x\,dx\right)$$

$$=\boxed{-x^2\cos x+2(x\sin x+\cos x)+C}$$

5. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta\,d\theta = \int_0^{\pi/4} \frac{\sin^2\theta\cos\theta}{\cos\theta}\,d\theta$

$*$ Let $x = \sin\theta$
$dx = \cos\theta\,d\theta$

$= \int_0^{\pi/4} \sin^2\theta\,d\theta = \int_0^{\pi/4} \frac{1-\cos 2\theta}{2}\,d\theta$

$= \frac{1}{2}\int_0^{\pi/4} (1-\cos 2\theta)\,d\theta = \frac{1}{2}\left[\theta\Big|_0^{\pi/4} - \frac{1}{2}\sin 2\theta\Big|_0^{\pi/4}\right]$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{8} - \frac{1}{4} = \boxed{\frac{\pi-2}{8}}$

6. $\int \frac{\tan^3\theta}{\cos^3\theta}\,d\theta = \int \frac{\sin^3\theta}{\cos^3\theta} \cdot \frac{1}{\cos^3\theta}\,d\theta = \int \frac{\sin^3\theta}{\cos^6\theta}\,d\theta$

$= \int \frac{\sin^2\theta}{\cos^6\theta} \cdot \sin\theta\,d\theta = \int \frac{(1-\cos^2\theta)}{\cos^6\theta} \cdot \sin\theta\,d\theta$

$*u = \cos\theta,\ du = -\sin\theta\,d\theta$

$= \int \frac{u^2-1}{u^6}\,du = \int (u^{-4} - u^{-6})\,du = -\frac{1}{3}u^{-3} + \frac{1}{5}u^{-5}$

$= \boxed{\frac{1}{5}\cos^{-5}\theta - \frac{1}{3}\cos^{-3}\theta + C}$

7. 

$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^{1/2} \sqrt{1+(2x)^2}\,dx = \int_0^{1/2} \sqrt{1+4x^2}\,dx$

$L = \int_0^{\pi/4} \sqrt{1+\tan^2\theta} \cdot \frac{1}{2}\sec^2\theta\,d\theta$

$*$ Let $x = \frac{1}{2}\tan\theta$
$dx = \frac{1}{2}\sec^2\theta\,d\theta$

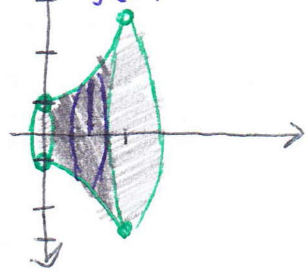
$L = \int_0^{\pi/4} \frac{1}{2}\sec^3\theta\,d\theta = \frac{1}{2}\int_0^{\pi/4} \sec^3\theta\,d\theta$

$*$ Use the fact that
$\int \sec^3 x\,dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|)$

$L = \frac{1}{4}\left[\sec\theta\tan\theta\Big|_0^{\pi/4} + \ln|\sec\theta + \tan\theta|\Big|_0^{\pi/4}\right]$

$L = \frac{1}{4}\left[\sqrt{2}(1) + \ln(\sqrt{2}+1)\right] = \boxed{\frac{\sqrt{2}}{4} + \frac{\ln(\sqrt{2}+1)}{4}\ \text{un}}$

8. $f(x)=e^x$



$$S=\int_a^b 2\pi r\,ds=\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S=\int_0^1 2\pi e^x\sqrt{1+(e^x)^2}\,dx=\int_0^1 2\pi e^x\sqrt{1+e^{2x}}\,dx$$

$*$ Let $u=e^x$, $du=e^x dx$

$$S=2\pi\int_1^e\sqrt{1+u^2}\,du$$

$*$ Let $u=\tan\theta$, $du=\sec^2\theta\,d\theta$

$$=2\pi\int_{\pi/4}^{\tan^{-1}e}\sqrt{1+\tan^2\theta}\,\sec^2\theta\,d\theta$$

$$S=2\pi\int_{\pi/4}^{\tan^{-1}e}\sec^3\theta\,d\theta=2\pi\left[\tfrac{1}{2}\sec\theta\tan\theta\Big|_{\pi/4}^{\tan^{-1}e}+\tfrac{1}{2}\ln\left|\sec\theta+\tan\theta\right|\Big|_{\pi/4}^{\tan^{-1}e}\right]$$

$$\boxed{S=\pi\left[\sec(\tan^{-1}e)\cdot e-\sqrt{2}+\ln[\sec(\tan^{-1}e+e)]-\ln(\sqrt{2}+1)\right]\text{ un}^2}$$

9. For $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{2n}=\sum_{n=1}^{\infty}\left[\left(\frac{n}{n+1}\right)^2\right]^n$, we have

$$\lim_{n\to\infty}\sqrt[n]{\left[\left(\frac{n}{n+1}\right)^2\right]^n}=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^2=\left[\lim_{n\to\infty}\left(\frac{n}{n+1}\right)\right]^2=1,$$

So the Root Test is inconclusive here.

However, recall that $e=\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n$, so $\frac{1}{e}=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n$

and thus, $\frac{1}{e^2}=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^{2n}$. So $a_n\not\to 0$ for $\sum a_n$

and $\boxed{\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{2n}\text{ is divergent}}$

10. For $\sum_{n=1}^{\infty}(-1)^n \frac{n^2-1}{n^3+1}$, we can apply the Alternating Series Test because ① $\lim_{n\to\infty}\frac{n^2-1}{n^3+1}=0$ and ② $b_n=\frac{n^2-1}{n^3+1}$ is a decreasing sequence. So the sum

$$\boxed{\sum_{n=1}^{\infty}(-1)^n \frac{n^2-1}{n^3+1} \text{ converges by Alt. Series Test}}$$

11. For $\sum_{k=1}^{\infty}\frac{(k!)^k}{k^{4k}}=\sum_{k=1}^{\infty}\left(\frac{k!}{k^4}\right)^k$, we have

$\lim_{k\to\infty}\sqrt[k]{\left(\frac{k!}{k^4}\right)^k}=\lim_{k\to\infty}\frac{k!}{k^4}=\infty$, which is greater than 1.

$$\boxed{\text{By the Root Test, } \sum_{k=1}^{\infty}\frac{(k!)^k}{k^{4k}} \text{ is divergent}}$$

12. The general Taylor series for e^t is $\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(t-a)^n$.
Notice for $f(x)=e^{-2x}$, we have

$f(x)=e^{-2x}$ so the Taylor series for $f(x)=e^{-2x}$ is

$$\boxed{\sum_{n=0}^{\infty}\frac{(-2)^n e^{-2}}{n!}(x-1)^n}$$

$f'(x)=-2e^{-2x}$

$f''(x)=4e^{-2x}$

$f'''(x)=-8e^{-2x}$

$\vdots$

$f^{(n)}(x)=(-2)^n e^{-2x}$