## Math 30 Final Exam Monday, May 11, 2020

## Professor Michael VanValkenburgh

Name:	

Instructions: (Updated for Online Teaching.) Show all your work, and clearly indicate your answers. You may use the class textbook, your lecture notes, and your homework, but you may not get help from any other source. This is for your long-term benefit.

Also, by submitting your work you are agreeing to NOT discuss the contents of this exam with anyone until your grade is posted.

Problem	Your score	Possible Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total		90

- 1. Consider the function  $f(x) = 3x^2 2x$ .
  - (a) Use the definition of the derivative (as a limit) to find f'(1).
  - (b) Find the equation of the tangent line to the graph of f at  $x_0 = 1$ .
  - (c) Find where the line in (b) intersects the x-axis.
  - (d) Make a sketch illustrating your answer to part (c).

- 2. Consider the function  $h(x) = \sqrt{1 + \sqrt{x}}$ .
  - (a) Write h as a composition of simpler functions g(u) and f(x).
  - (b) Find g'(u) and f'(x).
  - (c) Find h'(x).

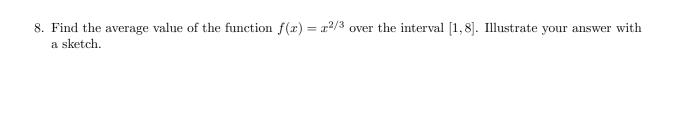
- 3. Consider the function  $f(x) = 3x^3 + x^2 + 1$ .
  - (a) Find the intervals on which f is increasing and decreasing.
  - (b) Find those x where f(x) has a local maximum or local minimum.
  - (c) Find the inflection points.
  - (d) Find where the graph of f is concave up and concave down.
  - (e) Sketch the graph of f.

4. The curve given by  $x^4 + y^4 = 17$  is a "squarish circle." Find the equation of the tangent line to the curve at (x, y) = (1, -2).

5.	Suppose a cylinder has constant volume 100 cubic inches. Suppose its radius is decreasing at a constant rate of $1/3$ inch per second. What is the rate of increase of the length when the radius is $1/2$ inch? (I am imagining a baker rolling out a long tube of dough.)

6. Write the integral  $\int_0^1 4x^3 dx$  as a limit of sums of areas of rectangles with equal width and using "right endpoints." Make a sketch illustrating your answer.

7. Evaluate the definite integral  $\int_1^2 \frac{e^x}{1 + e^{2x}} dx$  using *u*-substitution.



- 9. Consider the function  $f(x) = x \tan^{-1} x$ .  $(\tan^{-1} x)$  is another name for  $\arctan x$ .)
  - (a) Find f'(x).
  - (b) Use part (a) to evaluate the indefinite integral  $\int \tan^{-1} x \, dx$ .