Quiz 5

① a) Let $\vec{u} = \begin{bmatrix} a \\ b \end{bmatrix}$ $a \geq 0$, $b \geq 0$ and let $\vec{v} = \begin{bmatrix} c \\ d \end{bmatrix}$ $c \geq 0$, $d \geq 0$

then $\vec{u}+\vec{v} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$ and note that $a+c \geq 0$ and $b+d \geq 0$ $\rightarrow$ So $\vec{u}+\vec{v}$ is in V.

b) Let $\vec{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and let $c=-1$, then $c\vec{u} = -1\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $-1<0$, $-2<0$ thus

$(1 \geq 0$ and $2 \geq 0)$

$\hookrightarrow$ thus $c\vec{u}$ is not in V

② i) let $a=0$ then $0 \cdot t^2 = 0$ $(0+0t+0t^2)$ (the zero vector in $\mathbb{P}_2$) is in H.

ii) Consider $\vec{p}(t) = at^2$ and $\vec{q}(t) = bt^2$ where a, b is in $\mathbb{R}$.

Then $\vec{p}(t)+\vec{q}(t) = at^2+bt^2 = (a+b)t^2$ where $a+b$ is in $\mathbb{R}$ $\rightarrow$ $\vec{p}(t)+\vec{q}(t)$

$= c \cdot t^2$ where c is in $\mathbb{R}$, and $a+b=c$

Thus, $\vec{p}(t)+\vec{q}(t)$ is in H.

iii) Let c be a scalar (in $\mathbb{R}$) then $c \cdot \vec{p}(t) = c \cdot at^2 = (c \cdot a)t^2$

and $a \cdot c$ is in $\mathbb{R}$, thus for any scalar, c $c \cdot \vec{p}(t)$ is in H.

Therefore, H is a subspace of $\mathbb{P}_2$.

③ i) non-zero vector in Col A = $\begin{bmatrix} 6 \\ -3 \\ -9 \\ 9 \end{bmatrix}$

ii) $R_1 \leftrightarrow R_2$, $2R_1+R_2 \rightarrow R_2$, $-3R_1+R_3 \rightarrow R_3$, $3R_1+R_4 \rightarrow R_4$

$\begin{bmatrix} -3 & 2 \\ 6 & -4 \\ -9 & 6 \\ 9 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} -3 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Consider $A\vec{x}=\vec{0}$ $\rightarrow \left[\begin{array}{cc|c} -3 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -2/3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\rightarrow x_1 - 2/3 x_2 = 0$

$\rightarrow x_1 = \frac{2}{3}x_2$

$\hookrightarrow \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2/3\, x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$

Let $x_2 = 3$, then

$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$