STAT 50 HW #8

Section 4.2 #'s 1, 3, 7, 9, 12, 13, 16, 21

1.
Let X ~ Bin(7, 0.3). Find

a. P(X = 1)

P(X = 1) = $\binom{7}{1}(0.3)^1(1 - 0.3)^6$

$\binom{7}{1}$ or $_7C_1 = \frac{7!}{(7-1)!1!} = 7$

7(0.3)((0.7)^6) = 0.247

<u>P(X = 1) = 0.247</u>

b. P(X = 2)

P(X = 2) = $\binom{7}{2}(0.3)^2(1 - 0.3)^5$

$\binom{7}{2}$ or $_7C_2 = \frac{7!}{(7-2)!2!} = \frac{7!}{5!2!} = \frac{7*6}{2!} = 21$

21((0.3)^2)((0.7)^5) = 0.318

<u>P(X = 2) = 0.318</u>

c. P(X < 1)

P(X < 1) = P(X = 0) = $\binom{7}{0}(0.3)^0(1 - 0.3)^7$

$\binom{7}{0}$ or $_7C_0 = \frac{7!}{(7-0)!0!} = 1$

1((0.3)^0)((0.7)^7) = 0.0824

<u>P(X < 1) = 0.0824</u>

d. P(X > 4)

P(X > 4) = 1 - P(X ≤ 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)

$1 - \frac{7!}{(7-0)!0!}(0.3)^0(1 - 0.3)^7 + \frac{7!}{(7-1)!1!}(0.3)^1(1 - 0.3)^6 +$

$\frac{7!}{(7-2)!2!}(0.3)^2(1 - 0.3)^5 + \frac{7!}{(7-3)!3!}(0.3)^3(1 - 0.3)^4 +$
$\frac{7!}{(7-4)!4!}(0.3)^4(1 - 0.3)^3$
1 - (0.082 + 0.247 + 0.317 + 0.227 + 0.097) = 0.0288

<u>P(X > 4) = 0.0288</u>

e. μ_x

μ_x = n(p) = 7*0.3 = 2.1
μ_x = 2.1

f. σ_x^2

σ_x^2 = np(1-p) = 7*0.3(0.7) = 1.47
σ_x^2 = 1.47

3.
Find the following probabilities:
a. P(X = 2) when X ~ Bin(4, 0.6)

P(X = 2) = $\binom{4}{2}(0.6)^2(1 - 0.6)^2$
$\binom{4}{2}$ or ${}_4C_2 = \frac{4!}{(4-2)!2!} = \frac{4!}{2!2!} = \frac{12}{2} = 6$
6((0.6)^2)((0.4)^2) = 0.3456

<u>P(X = 2) = 0.3456</u>

b. P(X > 2) when X ~ Bin(8, 0.2)

P(X > 2) = 1 - P(X ≤ 2) = 1 - P(X = 0) - P(X = 1) - P(X = 2)
$1 - \frac{8!}{(8-0)!0!}(0.2)^0(1 - 0.2)^8 + \frac{8!}{(8-1)!1!}(0.2)^1(1 - 0.2)^7 +$
$\frac{8!}{(8-2)!2!}(0.2)^2(1 - 0.2)^6$
1 - (0.1677 + 0.3355 + 0.2936) = 0.2031

P(X > 4) = 0.2031

c. P(X ≤ 2) when X ~ Bin(5, 0.4)

P(X ≤ 2) = 1 - P(X > 2) = 1 - P(X = 3) - P(X = 4) - P(X = 5)

$$1 - \frac{5!}{(5-3)!3!}(0.4)^3(1 - 0.4)^2 + \frac{5!}{(5-4)!4!}(0.4)^4(1 - 0.4)^1 + \frac{5!}{(5-5)!5!}(0.4)^5(1 - 0.4)^0$$

1 - (0.2304 + 0.0768 + 0.0102) = 0.6826

P(X ≤ 2) = 0.6826

d. P(3 ≤ X ≤ 5) when X ~ Bin(6, 0.7)

P(3 ≤ X ≤ 5) = 1 - P(X > 5) - P(X < 3) = 1 - P(X = 6) - P(X = 0) - P(X = 1) - P(X = 2)

$$1 - \frac{6!}{(6-6)!6!}(0.7)^6(1 - 0.7)^0 + \frac{6!}{(6-0)!0!}(0.7)^0(1 - 0.7)^6 + \frac{6!}{(6-1)!1!}(0.7)^1(1 - 0.7)^5 + \frac{6!}{(6-2)!2!}(0.7)^2(1 - 0.7)^4$$

1 - (0.1176 + 0.0007 + 0.0102 + 0.0595) = 0.8119

P(3 ≤ X ≤ 5) = 0.8119

7.

Of all the weld failures in a certain assembly, 85% of them occur in the weld metal itself, and the remaining 15% occur in the base metal. A sample of 20 weld failures is examined.

a. What is the probability that exactly five of them are base metal failures?

P(X = 5) when X ~ Bin(20, 0.15)

P(X = 5) = $\binom{20}{5}(0.15)^5(1 - 0.15)^{15}$

$$\binom{20}{5} \ or \ {}_{20}C_5 = \frac{20!}{(20-5)!5!} = \frac{20!}{15!5!} = \frac{19*18*17*16}{6} = 15504$$

15504((0.15)^5)((0.85)^15) = 0.1028

P(X = 5) = 0.1028

b. What is the probability that fewer than four of them are base metal failures?

P(X < 4) when X ~ Bin(20, 0.15)

P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)

$$\frac{20!}{(20-0)!0!}(0.15)^0(1-0.15)^{20}+\frac{20!}{(20-1)!1!}(0.15)^1(1-0.15)^{19}+\frac{20!}{(20-2)!2!}(0.15)^2(1-0.15)^{18}+\frac{20!}{(20-3)!3!}(0.15)^3(1-0.15)^{17}$$

(0.0387 + 0.1367 + 0.2293 + 0.2428) = 0.6477

<u>P(X < 4) = 0.6477</u>

c. **What is the probability that none of them are base metal failures?**

P(X = 0) when X ~ Bin(20, 0.15)

P(X = 0) = $\binom{20}{0}(0.15)^0(1-0.15)^{20}$

$\binom{20}{0}$ or ${}_{20}C_0 = \frac{20!}{(20-0)!0!} = \frac{20!}{20!0!} = 1$

1((0.15)^0)((0.85)^20) = 0.0388

<u>P(X = 0) = 0.0388</u>

d. **Find the mean number of base metal failures.**

μ_x = n(p) = 20*0.15 = 3

μ_x = 3

e. **Find the standard deviation of the number of base metal failures.**

σ_x^2 = np(1-p) = 20*0.15(0.85) = 2.55

σ_x^2 = 2.55

$\sigma_x = \sqrt{\sigma_x^2} = 1.597$

9.

Several million lottery tickets are sold, and 60% of the tickets are held by women. Five winning tickets will be drawn at random.

a. **What is the probability that three or fewer of the winners will be women?**

X ~ Bin(5, 0.6)

$P(X \leq 3) = 1 - P(X > 3) = 1 - P(X = 4) - P(X = 5)$

$$1 - \frac{5!}{(5-4)!4!}(0.6)^4(1 - 0.6)^1 + \frac{5!}{(5-5)!5!}(0.6)^5(1 - 0.6)^0$$

1 - (0.2592 + 0.0777) = 0.6630

<u>$P(X \leq 3) = 0.6630$</u>

b. **What is the probability that three of the winners will be of one gender and two of the winners will be of the other gender?**

$P(X = 3) = \binom{5}{3}(0.6)^3(1 - 0.6)^2$

$$\binom{5}{3} \text{ or } {}_5C_3 = \frac{5!}{(5-3)!3!} = \frac{5!}{2!3!} = \frac{5*4}{2} = 10$$

10((0.6)^3)((0.4)^2) = 0.3456

$P(X = 2) = \binom{5}{2}(0.6)^2(1 - 0.6)^3$

$$\binom{5}{2} \text{ or } {}_5C_2 = \frac{5!}{(5-2)!2!} = \frac{5!}{3!2!} = \frac{5*4}{2} = 10$$

10((0.6)^2)((0.4)^3) = 0.2304

0.2304 + 0.3456 = 0.5760

12.

Of the items manufactured by a certain process, 20% are defective. Of the defective items, 60% can be repaired.

a. **Find the probability that a randomly chosen item is defective and cannot be repaired.**

P(Defective) = 0.2
P(Unrepairable) = 1 - P(Repairable) = 1 - 0.6 = 0.4
P(Unrepairable & Defective) = 0.2*0.4 = 0.08

b. **Find the probability that exactly 2 of 20 randomly chosen items are defective and cannot be repaired.**

X ~ Bin(20, 0.08)

$P(X = 2) = \binom{20}{2}(0.08)^2(1 - 0.08)^{18}$

$\binom{20}{2}$ or ${}_{20}C_{2} = \frac{20!}{(20-2)!2!} = \frac{20!}{18!2!}$ =190

190((0.08)^2)((0.92)^18) = 0.2711

P(X = 2) = 0.2711

13.

Of the bolts manufactured for a certain application, 90% meet the length specification and can be used immediately, 6% are too long and can be used after being cut, and 4% are too short and must be scrapped.

a. **Find the probability that a randomly selected bolt can be used (either immediately or after being cut).**

P(Imm) = 0.90
P(After Cut) = 0.06
P(Imm ∪ After Cut) = 0.90 + 0.06 = 0.96

b. **Find the probability that fewer than 9 out of a sample of 10 bolts can be used (either immediately or after being cut).**

X ~ Bin(10, 0.96)
P(X < 9) = 1 - P(X ≥ 9) = 1 - P(X = 9) + P(X = 10)

$$\frac{10!}{(10-9)!9!}(0.96)^{9}(1 - 0.96)^{1} + \frac{10!}{(10-10)!10!}(0.96)^{10}(1 - 0.96)^{0}$$

1 - (0.2770 + 0.6648) = 0.0582

P(X < 9) = 0.0582

16.

A distributor receives a large shipment of components. The distributor would like to accept the shipment if 10% or fewer of the components are defective and to return it if more than 10% of the components are defective. She decides to sample 10 components, and to return the shipment if more than 1 of the 10 is defective.

a. **If the proportion of defectives in the batch is in fact 10%, what is the probability that she will return the shipment?**

X ~ Bin(10, 0.10)

P(X > 1) = 1 - P(X ≤ 1) = 1 - P(X = 0) - P(X = 1)

$$1 - \frac{10!}{(10-0)!0!}(0.10)^{0}(1 - 0.10)^{10} + \frac{10!}{(10-1)!1!}(0.10)^{1}(1 - 0.10)^{9}$$

1 - (0.3487 + 0.3874) = 0.2639

$\underline{P(X > 1) = 0.2639}$

b. **If the proportion of defectives in the batch is 20%, what is the probability that she will return the shipment?**

X ~ Bin(10, 0.20)

$P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1)$

$$1 - \frac{10!}{(10-0)!0!}(0.20)^{0}(1 - 0.20)^{10} + \frac{10!}{(10-1)!1!}(0.20)^{1}(1 - 0.20)^{9}$$

$1 - (0.1074 + 0.2684) = 0.6242$

$\underline{P(X > 1) = 0.6242}$

c. **If the proportion of defectives in the batch is 2%, what is the probability that she will return the shipment?**

X ~ Bin(10, 0.02)

$P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1)$

$$1 - \frac{10!}{(10-0)!0!}(0.02)^{0}(1 - 0.02)^{10} + \frac{10!}{(10-1)!1!}(0.02)^{1}(1 - 0.02)^{9}$$

$1 - (0.8171 + 0.1667) = 0.0162$

$\underline{P(X > 1) = 0.0162}$

d. **The distributor decides that she will accept the shipment only if none of the sampled items are defective. What is the minimum number of items she should sample if she wants to have a probability no greater than 0.01 of accepting the shipment if 20% of the components in the shipment are defective?**

$$0.01 = \binom{10}{n} 0.2^n (1-0.2)^{10-n}$$

$$\frac{10!}{n!(10-n)!} 0.2^n (0.8)^{10-n}$$

$$\frac{10 \cdot 9 \cdot \cancel{8} \cdot 7 \cdot \cancel{6!}}{\cancel{6!}\, \cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1} \qquad 210(0.2^6)(0.8^4)$$

$$\frac{630}{3} = 210 \qquad 0.0055$$

$$\frac{\cancel{10} \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5!}}{\cancel{8} \cdot 4 \cdot 3 \cdot \cancel{2} \cdot 1\, 5!} = \frac{3024}{12} = 252$$

$$252(0.2)^5(0.8)^5 = 0.026$$

The answer is 6! I got some help from the Sac State Math Lab and it is sort of a trial and error type of problem.

21.
A message consists of a string of bits (0s and 1s). Due to noise in the communications channel, each bit has probability 0.3 of being reversed (i.e., a 1 will be changed to a 0 or a 0 to a 1). To improve the accuracy of the communication, each bit is sent five times, so, for example, 0 is sent as 00000. The receiver assigns the value 0 if three or more of the bits are decoded as 0, and 1 if three or more of the bits are decoded as 1. Assume that errors occur independently.

a. **A 0 is sent (as 00000). What is the probability that the receiver assigns the correct value of 0?**

X ~ Bin(5, 0.7)

$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$$\frac{5!}{(5-3)!3!}(0.7)^3(1 - 0.7)^2 + \frac{5!}{(5-4)!4!}(0.7)^4(1 - 0.7)^1 + \frac{5!}{(5-5)!5!}(0.7)^5(1 - 0.7)^0$$

$(0.3087 + 0.3602 + 0.1681) = 0.8369$

<u>$P(X \geq 3) = 0.8369$</u>

b. **Assume that each bit is sent n times, where n is an odd number, and that the receiver assigns the value decoded in the majority of the bits. What is the minimum value of n necessary so that the probability that the correct value is assigned is at least 0.90?**

$X \sim Bin(n, 0.7)$ n is odd

$0.902 = \binom{n}{0}(0.7)^0(0.3)^{n-0}$

$\frac{n!}{0!(0-n)!}(0.7)^n(0.3)^{5-n}$

$\frac{7!}{7!0!}$ $\frac{7!}{6!1!}$ 7

$\binom{7}{0}(0.7)^0(0.3)^7 = 0.0002$

$7\binom{7}{1}(0.7)^1(0.3)^6 = 0.0035$

$21\binom{7}{2}(0.7)^2(0.3)^5 = 0.0250047$

$\frac{9\cdot 8\cdot 7!}{7!\,2!}$ 36

$35\binom{7}{3}(0.7)^3(0.3)^4 = 0.097$

$1-(0.0002+0.0035+0.025+0.097) = 0.87$

$1\binom{9}{0}(0.7)^0(0.3)^9 = 0.00001$

$9\binom{9}{1}(0.7)^1(0.3)^8 = 0.0004$

$36\binom{9}{2}(0.7)^2(0.3)^7 = 0.0038$

$\frac{9\cdot 8\cdot 7\cdot 6!}{6!\,3!}$ 84

$84\binom{9}{3}(0.7)^3(0.3)^6 = 0.02$ *Add them all

$1-0.025 = 0.97$

0.97

My thoughts may be hard to interpret, but the answer is 9! I was able to figure this out using a similar process to 16d.