Expected value of Discrete Random Variables

- Suppose the PMF of X is given by
$$p(a) = P(X = a).$$

- Expected value (expectation or mean): defined by
$$\mu = E(X) = \sum ap(a).$$

- Expected value of a function of random variable:
$$E[h(X)] = \sum g(a)p(a)$$
for any function g.

- Rules of expected value: for two constants k and b, there is
$$E(kX + b) = kE(X) + b = k\mu + b.$$

- Variance.
$$V(X) = E[(X - \mu)^2].$$

- Rules of variance: for any two constants k and b, there is
$$V(kX + b) = k^2 V(X).$$

Q1) The PMF of X is given by

a	0	1	2	3	4	5
p(a)	0.002	0.001	0.002	0.005	0.02	0.04
a		6	7	8	9	10
p(a)		0.18	0.37	0.25	0.12	0.01

a. The expected value is

b. $E(X^2)$

c. The variance is

d. The standard deviation is

Q2) The PMF of X is

$$p(x) = \begin{cases} 1-p, & \text{x=0} \\ p & \text{x=1} \\ 0 & \text{otherwise} \end{cases}$$

a. $E(X) =$

b. $E(X^2) =$

c. $V(X) =$

Q3) The PMF of X is

$$p(x) = \begin{cases} kx & \text{x} = 1, 2, 3, \cdots, n \\ 0, & \text{otherwise} \end{cases}$$

Find k.

Q4) The PMF of X is

x	4	6	8
$p(x)$	0.5	0.3	0.2

a. $E(X) =$

- Suppose $Y = h(X) = 20 + 3X + 0.5X^2$.

b. $E(Y) =$

Q5) The PMF of X is

x	1	2	3	4
$p(x)$	0.1	0.2	0.3	0.4

- Let $Y = h(X) = 800X - 900$. Then,

a. $E(Y) =$

b. $V(Y) =$

Q6) Flip a dice twice and let X be the sum of the two outcomes. Then, X is a *discrete random variable*.

- Compute the probability mass function of X.
- Use PMF to compute $P(2 \leq X \leq 4)$, $P(2 \leq X < 4)$, $P(X \geq 4)$.
- Compute the cumulative distribution function (CDF) of X.
- Compute $E(X)$.
- Compute $V(X)$.
- Compute the standard deviation of X.

Q7) Suppose the CDF of a random variable X is given by

$$F(x) = \begin{cases} 0, & \text{when } x < 0 \\ 0.2, & \text{when } 0 \leq x < 1 \\ 0.35, & \text{when } 1 \leq x < 2, \\ 0.65, & \text{when } 2 \leq x < 3 \\ 0.85 & \text{when } 3 \leq x < 4 \\ 1, & \text{when } x \geq 4. \end{cases}$$

- Use CDF to compute $P(1 \leq X \leq 3)$, $P(1 < X < 3)$, $P(1 < X \leq 3)$, $P(1 \leq X < 3)$.
- Compute the PMF of X.
- Compute $E(X)$.
- Compute $V(X)$.
- Compute the standard deviation of X.