1) A Student wants some books from the library. He has to choose from 4 different comic books, 5 different fiction books, and 4 different medical books: (5*2 = 10 points)
a) In how many ways can a student choose one of each kind of book? Explain your answer.

By the product rule, you would multiply the number of different comic books by the number of different fiction books and the number of different medical books, assuming you could only choose one of each. This gives us: 4*5*4 possible combinations, which is equal to 80. The student can choose one of each kind of book 80 different ways.

b) In how many ways can a student just choose one of the books? Explain your answer.

Because the student is only choosing one of the books, you can refer to the sum rule. You would get 4 + 5 + 4 possible combinations, which is equal to 13. The student can choose just one of the books 13 different ways.

2) There are 7 red and 9 black marbles. There should be 10 of the marbles placed in a box. How many different combinations can there be in one box if: (5*4 = 20 points)
a) There must be an equal number of both colors of marbles.

Assuming that there must be 5 red marbles and 5 black marbles and order does not matter, we use the formula for combinations where order doesn't matter, which is:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

${}_7C_5 * {}_9C_5$

$\frac{7!}{(7-5)!5!} * \frac{9!}{(9-5)!5!} = \frac{5040}{240} * \frac{362880}{2880} = 21*126 = 2646$

*(2646 possible combinations)

b) There must be at least 6 red marbles in a box.

Assuming that there must be 6 red marbles and 4 black marbles and order does not matter, we use the formula for combinations where order doesn't matter, which is:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

${}_7C_6 * {}_9C_4$

$\frac{7!}{(7-6)!6!} * \frac{9!}{(9-4)!4!} = \frac{5040}{720} * \frac{362880}{2880} = 7*126 = 882$

*(882 possible combinations)

c) All the red marbles should be used.

Assuming that there must be 7 red marbles and 3 black marbles and order does not matter, we use the formula for combinations where order doesn't matter, which is:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

${}_7C_7$ * ${}_9C_3$

$\frac{7!}{(7-7)!7!} * \frac{9!}{(9-3)!3!} = \frac{5040}{5040} * \frac{362880}{4320} = 1*84 = 84$

*(84 possible combinations)

d) All the black marbles should be used.

Assuming that there must be 1 red marble and 9 black marbles and order does not matter, we use the formula for combinations where order doesn't matter, which is:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

${}_7C_1$ * ${}_9C_9$

$\frac{7!}{(7-1)!1!} * \frac{9!}{(9-9)!9!} = \frac{5040}{720} * \frac{362880}{362880} = 7*1 = 7$

*(7 possible combinations)

3) There are 5 buses between Sac State and 65th Street, and 4 train lines between 65th Street and Folsom. Find the number of ways that a man can travel by bus:
(a) from Sac State to Folsom by way of 65th Street;

There are 5 bus routes from Sac State to 65th Street and 4 train lines from 65th street to Folsom, so one would have to take one bus route from Sac State to 65th Street and one train route from 65th Street to Folsom. Here, we can just use the product rule. By the product rule, you would multiply the number of different bus routes from Sac State to 65th Street by the number of different train lines from 65th street to Folsom. Assuming you could only choose one of each at a time. This gives us: 5*4 possible combinations, which is equal to 20. The man could travel from Sac State to Folsom 20 different ways. You could also use the combinations formula below, where order doesn't matter:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

${}_5C_1 * {}_4C_1$

$$\frac{5!}{(5-1)!1!} * \frac{4!}{(4-1)!1!} = \frac{120}{24} * \frac{24}{6} = 5*4 = 20$$

*(20 different routes possible)

(b) roundtrip from Sac State to Folsom by the way of 65th Street;

A roundtrip means a journey to one or more places and back again. So we are trying to determine the number of ways a man can go from Sac State to Folsom and back again. We will also be assuming that repetition is allowed in that he is not prohibited from using the same bus routes and train lines twice. Again we can try using the product rule:
5 (bus routes from Sac State to 65th) * 4 (Train lines from 65th to Folsom) * 4 (Train lines from Folsom to 65th) * 5 (bus routes from 65th to Sac State) = 400

You could also use the combinations formula below, where order doesn't matter:

$$nCr = \frac{n!}{(n-r)!r!}$$

Which gives us:

${}_5C_1 * {}_4C_1 * {}_4C_1 * {}_5C_1$

$$\frac{5!}{(5-1)!1!} * \frac{4!}{(4-1)!1!} * \frac{4!}{(4-1)!1!} * \frac{5!}{(5-1)!1!} = \frac{120}{24} * \frac{24}{6} * \frac{24}{6} * \frac{120}{24} = 5*4*4*5 = 400$$

*(400 different routes possible)

(c) roundtrip from Sac State to Folsom by way of 65th Street but without using a transportation mode more than once. (5*3 = 15 points)

A roundtrip means a journey to one or more places and back again. So we are trying to determine the number of ways a man can go from Sac State to Folsom and back again. This time, repetition is not allowed. Since repetition is not allowed, we reduce the number of possible bus routes and train lines when the man returns. Again we can try using the product rule:
5 (bus routes from Sac State to 65th) * 4 (Train lines from 65th to Folsom) * 3 (Train lines from Folsom to 65th) * 4 (bus routes from 65th to Sac State) = 240

*(240 different routes possible)

4) a. How many distinguishable ways can the letters of the word MISSISSIPPI be arranged in order?

This is a permutation, so we would use a formula like the one below:

$$\frac{n!}{n_1!n_2!n_3!} = \left(\frac{n}{n_1}\right)\left(\frac{n-n_1}{n_2}\right)\left(\frac{n-n_1-n_2}{n_3}\right) = \frac{n!}{n_1!(n-n_1)!} * \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} * \frac{(n-n_1-n_2)!}{(n-n_1-n_2-n_3)!n_3!}$$

We should get:

M = 1, I = 4, S = 4, P = 2

$$\frac{11!}{1!4!4!2!} = \left(\frac{11}{1}\right)\left(\frac{10}{4}\right)\left(\frac{6}{4}\right)\left(\frac{2}{2}\right) = \frac{11}{1!(10)!} * \frac{10!}{4!(6)!} * \frac{6!}{4!(2)!} * \frac{2!}{2!(0)!}$$

After cancelling the common terms on the right of the expression above, you should be left with:

$\frac{11!}{1!4!4!2!} = \frac{3991680}{1152} = 34650$ (There are 34650 distinguishable ways to arrange the letters of the word MISSISSIPPI).

b. How many distinguishable orderings of the letters of MISSISSIPPI begin with M and end with I? (5*2 = 10 points)

There are 11 letters of which 1 is M, 4 are I's, 4 are S's, and 2 are P's. Since we know that the first letter will be an M and the last an I, we are left with 9 letters or which 3 are I's, 4 are S's, and 2 are P's. We end up with:

I = 3, S = 4, P = 2

$$\frac{9!}{3!4!2!} = \left(\frac{9}{3}\right)\left(\frac{6}{4}\right)\left(\frac{2}{2}\right) = \frac{9}{3!(6)!} * \frac{6!}{4!(2)!} * \frac{2!}{2!(0)!}$$

After cancelling the common terms on the right of the expression above, you should be left with:

$\frac{9!}{3!4!2!} = \frac{362880}{288} = 1260$ (There are 1260 distinguishable ways to arrange the letters of the word MISSISSIPPI, assuming you start with M and end with I).

5) A team is selected with 12 players including the captain (5*3 = 15 points)

a) How many different combinations of 3 can be chosen?

Assuming that there must be 3 players, where 11 are not the captain and one is, and order does not matter, we use the formula for combinations where order doesn't matter (we will be looking at combinations of 3 players from 12 solely as status does not matter), which is:

nCr = $\frac{n!}{(n-r)!r!}$

Which gives us:

$_{12}C_3$

$\frac{12!}{(12-3)!3!} = \frac{1320}{6} = 220$

*(220 different combinations possible)

b) How many of these combinations include captain?

Assuming that there must be 3 players, where 11 are not the captain and one is, and order does not matter, we use the formula for combinations where order doesn't matter, which is:

$$nCr = \frac{n!}{(n-r)!r!}$$

Which gives us:

${}_{11}C_2 * {}_1C_1$

$$\frac{11!}{(11-2)!2!} * \frac{1!}{(1-1)!1!} = \frac{110}{2} * \frac{1}{1} = 55*1 = 55$$

*(55 different combinations with the captain possible)

c) How many do not include the captain?

Assuming that there must be 3 players, where 11 are not the captain and one is, and order does not matter, we use the formula for combinations where order doesn't matter, which is:

$$nCr = \frac{n!}{(n-r)!r!}$$

Which gives us:

${}_{11}C_3$

$$\frac{11!}{(11-3)!3!} = \frac{990}{6} = 165$$

*(165 different combinations without the captain possible)

6) A computer programming team of 5 should be formed from 9 employees. Two of the employees are managers. However, to avoid dispute problems, the 2 managers cannot both be chosen. Find the number of teams that can be formed? (6 points)

We have 2 managers and 7 regular employees. We can try to solve this problem by figuring out every possible combination regardless of status, and then remove the combinations with both managers in them.

We use:

$$nCr = \frac{n!}{(n-r)!r!}$$

Which gives us:

${}_9C_5$

$$\frac{9!}{(9-5)!5!} = \frac{378}{3} = 126$$ (*Every possible combination)

${}_2C_2 * {}_7C_3$

$$\frac{2!}{(2-2)!2!} * \frac{7!}{(7-3)!3!} = \frac{2}{1} * \frac{35}{1} = 1*35 = 35$$ (*The combinations with both managers)

126 - 35 = 91
So, we have 91 possible teams where 2 managers aren’t in at the same time.

We could also just add the possibilities of teams having one manager or no managers at all.
nCr = $\frac{n!}{(n-r)!r!}$
Which gives us:
$_2C_1 * {}_7C_4$
$\frac{2!}{(2-1)!1!} * \frac{7!}{(7-4)!4!} = \frac{2}{1} * \frac{35}{1} = 2*35 = 70$
$_7C_5$
$\frac{7!}{(7-5)!5!} = \frac{42}{2} = 21$
70 + 21 = 91
Again, we have 91 possible teams where 2 managers aren’t in at the same time.

7) A photo has to be captured with 8 different celebrities. There are also some chairs available. So, they have the option to either sit on the chair or stand while taking pictures. How many different photos are possible? (6 points)
Assuming that the photo has to be captured with 8 celebrities at one time and that there are two possible ways to take a picture with each, we can use the product rule.
2*2*2*2*2*2*2*2 = 256 different types of photos possible.

8) I have to create one computer password. A password is of length 5 characters, first two of which are distinct numbers, next character can be any upper-case letter, and the remaining 2 characters can be any digit or letter (upper- or lower-case)? How many combinations are allowed? Note: Repetition of characters are not allowed. (6 points)

Here, we can refer to the product rule principle. The first two characters we use are numbers, but we can’t repeat characters, so we get 10 possibilities for the first character and 9 for the second character. The next value is an uppercase letter, so we get 26 possibilities. Then, the final two characters can be any digit or letter that is lower or uppercase. Again, repetitions are not allowed so we get 8 + 51 possibilities for the 4th character and 7 + 50 possibilities for the last character. The formula we would use is:
10 * 9 * 26 * 59 * 57 = 7869420

*(7869420 different combinations allowed)

9) You have 3 red pens and 7 blue pens. If you line up all the 10 pens one pen per day for 10 days, where the pens are indistinguishable by color, how many weeks (plus days) will it take to complete all combinations? (6 points)

We have 3 red pens and 7 blue pens and we know that we will be lining them all up in combinations of 10, but at a rate of one pen a day. We can think of this as trying to solve for permutations of different kinds of objects.

We use:
$$\frac{n!}{n_1!n_2!}$$

Which gives us:
$$\frac{10!}{3!*7!} = \frac{3628800}{30240} = 120$$
$$\left(\frac{n}{n_1}\right)\left(\frac{n-n_1}{n_2}\right) = \frac{n!}{n_1!(n-n_1)!} * \frac{(n-n_1)!}{n_2!(n-n_1-n_2)!}$$
We should get:
Red = 3, Blue = 7
$$\frac{10!}{3!7!} = \left(\frac{10}{3}\right)\left(\frac{7}{7}\right) = \frac{10}{3!(7)!} * \frac{7!}{7!(0)!}$$
After cancelling the common terms on the right of the expression above, you should be left with:
$$\frac{10!}{3!7!} = \frac{3628800}{30240} = 120$$

So, it would take us 120 days (or about 17 weeks) to complete all combinations.

10) There are 4 entry and exit points (A single point can be used either for entry or exit). In how many ways can a person enter and leave a space if he or she has to use different points? What if the person can use the same points? (6 points)

a) For the first question, we are trying to figure out the number of ways a person can enter and exit a building assuming they have to use different doors. A person can enter 4 ways, but can’t leave through the same door, so they only have 3 ways to leave after. Here, we can try the product rule.
   4 possible ways to enter * 3 possible ways to leave = 12 ways a person can enter and exit.

b) For the second question, we are trying to figure out the same thing, except the person is allowed to leave the way they entered this time. So they have 4 ways to enter and 4 ways to leave.
   4 possible ways to enter * 4 possible ways to leave = 16 ways a person can enter and exit.