STAT 50 HW #14 Sections 5.1 and 5.3

Section 5.1 #'s 1, 5, 7, 10, 11, 17, 21, 23

1.

Find the value of zα/2 to use in expression (5.1) to construct a confidence interval with level

a. **95%**

1. a) 95% $\quad \bar{x} \pm z_{\alpha/2}\,\sigma_{\bar{x}}$ where $\sigma_{\bar{x}} = \sigma/\sqrt{n}$

$\bar{x} \pm 1.96 \frac{s}{\sqrt{n}} \to \boxed{1.96}$

$1-0.95 = 0.05$

$\alpha/2 = \frac{0.05}{2} = 0.025$

from table

$1-0.025 = 0.975 \quad z_{0.025}$

b. **98%**

b) 98% $\quad 1-0.98 = 0.02$

$\alpha/2 = \frac{0.02}{2} = 0.01 \to z_{0.01} \to 0.99$

$\boxed{2.33}$

c. **99%**

c) 99% $\quad 1-0.99 = 0.01$

$\frac{0.01}{2} = 0.005 \to z_{0.005} \to 1-0.005$

0.995

$\boxed{2.57}$

d. **80%**

d) 80% $\quad 1-0.80 = 0.2$

$\frac{0.2}{2} = 0.1 \to z_{0.1} \to 1-0.1 = 0.9$

$\boxed{1.28}$

5.

A sample of 114 patients were given a drug to lower cholesterol. A 95% confidence interval for the mean reduction in cholesterol (in mmol/L) was (0.88, 1.02).

a. What was the sample mean reduction?

5. Sample of 114
95% confidence interval for mean reduction was
(0.88, 1.02)
a) Sample mean reduction?

$$\frac{0.88 + 1.02}{2} = \boxed{0.95}$$

b. What was the sample standard deviation of the reduction amounts?

b) Sample std dev of reduction amounts?

$0.95 - 0.88 = 0.07$ $\boxed{s = 0.3813}$

$0.95 \pm 1.96 \frac{s}{\sqrt{114}}$ $1.96 \frac{s}{10.6} = \frac{0.07}{1.96}$ $\frac{s}{10.6} = 0.03$

7.

In a sample of 100 steel canisters, the mean wall thickness was 8.1 mm with a standard deviation of 0.5 mm.

a. Find a 95% confidence interval for the mean wall thickness of this type of canister.

7. $n = 100$
mean = 8.1 mm, $\bar{x}$ $\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$
std dev = 0.5 mm, s
a) Find 95% C.I.

$8.1 \pm 1.96 \frac{0.5}{\sqrt{100}}$

$8.1 \pm 0.098 \rightarrow \boxed{(8.002, 8.198)}$

b. Find a 99% confidence interval for the mean wall thickness of this type of canister.

b) Find 99% C.I.

$8.1 \pm 2.58 \frac{0.5}{\sqrt{100}}$

$8.1 \pm 0.129 \rightarrow \boxed{(7.971, 8.229)}$

c. **An engineer claims that the mean thickness is between 8.02 and 8.18 mm. With what level of confidence can this statement be made?**

c) Engineer claims mean thickness between 8.02 and 8.18 mm. With what level of confidence can this statement be made?

$\frac{8.02 + 8.18}{2} = 8.1$ $\quad 8.1 - 8.02 = 0.08$

$z\frac{0.5}{10} = 0.08$ $\quad z = 1.6 \to 0.9452$

$z = \frac{0.08}{\frac{0.5}{10}}$

$1 - 0.9452 = 0.0548$

$z_{0.0548} \to 0.0548 \cdot 2$

0.1096

$0.8904 = 1 - 0.1096$

$\boxed{89.04\%}$

d. **How many canisters must be sampled so that a 95% confidence interval will specify the mean wall thickness to within ±0.08 mm?**

d) # to sample so 95% C.I. specifies mean within ±0.08 mm?

$1.96\frac{0.5}{\sqrt{n}} = \frac{0.08}{1.96}$

$\sqrt{n}\frac{0.5}{\sqrt{n}} = 0.04\sqrt{n}$

$\sqrt{n} = \frac{0.5}{0.04} \to \sqrt{n} = 12.25$

$n = 150.06 \uparrow$

$\boxed{n = 151}$

Got it

e. **How many canisters must be sampled so that a 99% confidence interval will specify the mean wall thickness to within ±0.08 mm?**

e) # to sample so 99% C.I. specifies mean within ± 0.08 mm?

$2.58\frac{0.5}{\sqrt{n}} = \frac{0.08}{2.58}$

$\sqrt{n}\frac{0.5}{\sqrt{n}} = 0.0310\sqrt{n}$

$\frac{0.5}{0.0310} = \sqrt{n}$

$\sqrt{n} = 16.125$

$n = 260.01 \uparrow$

$\boxed{n = 261}$

10.

Oven thermostats were tested by setting them to 350°F and measuring the actual temperature of the oven. In a sample of 67 thermostats, the average temperature was 348.2°F and the standard deviation was 5.1°F.

a. **Find a 90% confidence interval for the mean oven temperature.**

10. $n = 67$
$\bar{x} = 348.2$
$s = 5.1$
a) Find 90% C.I.
$348.2 \pm 1.645 \frac{5.1}{\sqrt{67}}$
$348.2 \pm 1.024 \rightarrow \boxed{(347.175, 349.225)}$

b. **Find a 95% confidence interval for the mean oven temperature.**

b) Find 95% C.I.
$348.2 \pm 1.96 \frac{5.1}{\sqrt{67}}$
$348.2 \pm 1.22 \rightarrow \boxed{(346.97, 349.42)}$

c. **What is the confidence level of the interval (347.5, 348.9)?**

c) C.I. of (347.5, 348.9)
$\frac{347.5 + 348.9}{2} = 348.2$ $\quad 348.2 - 347.5 = 0.7$
$z \frac{5.1}{\sqrt{67}} = 0.7 \rightarrow z = \frac{0.7}{\frac{5.1}{\sqrt{67}}} \rightarrow z = 1.12 \rightarrow 0.8686$
$1 - 0.8686 = 0.1314$
$0.1314 \cdot 2 = 0.2628$
$1 - 0.2628 = 0.7372$
$\boxed{73.72\%}$

d. **How many thermostats must be sampled so that a 90% confidence interval specifies the mean to within ±0.8°F?**

d) # so that 90% C.I. specifies mean within ± 0.8

$$\cancel{1.645}\frac{5.1}{\sqrt{n}} = \frac{0.8}{1.645} \qquad \frac{5.1}{\sqrt{n}} = 0.486$$

$$5.1 = 0.486\sqrt{n}$$

$$\sqrt{n} = 10.48$$

$$n = 109.97\uparrow \rightarrow \boxed{n = 110}$$

e. **How many thermostats must be sampled so that a 95% confidence interval specifies the mean to within ±0.8°F?**

e) # so that 95% C.I. specifies mean within ± 0.8

$$\cancel{1.96}\frac{5.1}{\sqrt{n}} = \frac{0.8}{1.96} \qquad \frac{5.1}{\sqrt{n}} = 0.408$$

$$5.1 = 0.408\sqrt{n}$$

$$\sqrt{n} = 12.445$$

$$n = 156.125\uparrow \rightarrow \boxed{n = 157}$$

11.

In a sample of 80 light bulbs, the mean lifetime was 1217 hours with a standard deviation of 52 hours.

a. **Find a 95% confidence interval for the mean lifetime of this type of light bulb.**

11. $n = 80$

$\bar{x} = 1217$

$s = 52$

a) Find 95% C.I.

$$1217 \pm 1.96\frac{52}{\sqrt{80}}$$

$$1217 \pm 11.395 \rightarrow \boxed{(1205.605, 1228.395)}$$

b. **Find a 98% confidence interval for the mean lifetime of this type of light bulb.**

b) Find a 98% C.I.

$1217 \pm 2.33 \frac{52}{\sqrt{80}}$

$1217 \pm 13.54 \rightarrow \boxed{(1203.454,\ 1230.546)}$

c. **An engineer claims that the mean lifetime is between 1208 and 1226 hours. With what level of confidence can this statement be made?**

c) mean between 1208 and 1226 hrs. Level of confidence?

$\frac{1208+1226}{2} = 1217 \qquad 1217-1208 = 9$

$z \frac{52}{\sqrt{80}} = 9 \qquad z = 1.55 \rightarrow 0.4394$

$\frac{9}{\frac{52}{\sqrt{80}}}$

$1-0.4394 = 0.0606$

$0.0606 \cdot 2 = 0.1212$

$\boxed{87.88\%} \leftarrow 1-0.1212 = 0.8788$

d. **How many light bulbs must be sampled so that a 95% confidence interval will specify the mean lifetime to within ±8 hours?**

d) # so 95% C.I. specifies mean within ±8?

$1.96 \frac{52}{\sqrt{n}} = 8 \qquad \frac{52}{\sqrt{n}} = 4.08 \cdot \sqrt{n} \qquad \frac{52}{4.08} = \cancel{\frac{4.08\sqrt{n}}{4.08}}$

$\sqrt{n} = 12.74 \rightarrow n = 162.3076$

$\boxed{n = 163}$

e. **How many light bulbs must be sampled so that a 98% confidence interval will specify the mean lifetime to within ±8 hours?**

b) # so 98% C.I. specifies mean within ±8?

$2.33 \frac{52}{\sqrt{n}} = 8$ $\quad \frac{52}{\sqrt{n}} = 3.43$ $\quad \sqrt{n}\,\frac{52}{\sqrt{n}} = 3.43\sqrt{n}$ $\quad \frac{52}{3.43} = \frac{3.43\sqrt{n}}{3.43}$

$\sqrt{n} = 15.146$ $\quad n = 229.370$

$\boxed{n = 230}$

17.

Refer to Exercise 10.

a. **Find a 99% upper confidence bound for the mean temperature.**

17. $n = 67$

$\bar{x} = 348.2$

$s = 5.1$

a) Find 99% UCB for mean

$\bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}$ $\quad 100(1-\alpha)\%$

$348.2 + 2.33 \frac{5.1}{\sqrt{67}} \rightarrow 348.2 + 1.45 = \boxed{349.6517}$

b. **The claim is made that the mean temperature is less than 349.5°F. With what level of confidence can this statement be made?**

b) mean less than 349.5. Level of confidence for this statement?

$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{349.5 - 348.2}{5.1/\sqrt{67}} \approx 2.09$

$P(Z < 2.09) = 0.9817 = \boxed{98.17\%}$

21.

An investigator computes a 95% confidence interval for a population mean on the basis of a sample of size 70. If she wishes to compute a 95% confidence interval that is half as wide, how large a sample does she need?

21. 95% C.I. for pop. mean on sample of 70.
If she wants a 95% C.I. that is half as wide
How large a sample does she need?

m.o.e.:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \qquad \text{width} = 2E = 2 z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

If we want width to halve, $\sqrt{n}$ must double and thus n needs to quadruple

$$n^* = 4n = 4(70) = 280$$

$$\boxed{n = 280}$$

23.

Based on a large sample of capacitors of a certain type, a 95% confidence interval for the mean capacitance, in µF, was computed to be (0.213, 0.241). Find a 90% confidence interval for the mean capacitance of this type of capacitor.

23. Logs Lands
95% C.I. for mean in MF → (0.213, 0.241)
Find 90% C.I. for mean

$\frac{0.213+0.241}{2} = 0.227$ $\quad 1-0.95 = 0.05/2 = 0.025$

$z_{\alpha/2}$ $\quad 1-0.025 = 0.975$ ↓ 1.96

M.O.E.:

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.96\frac{\sigma}{\sqrt{n}}$

$\frac{0.241-0.213}{2} = 0.014$ or $0.227-0.213 = 0.014$

$1.96\frac{\sigma}{\sqrt{n}} = 0.014 \rightarrow \frac{\sigma}{\sqrt{n}} = \frac{0.014}{1.96} \approx 0.0071$

$z_{\alpha/2}$:

$1-0.90 = 0.1/2 = 0.05$ $\quad 1.645$

$1-0.05 = 0.95$

M.O.E.:

$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} = 1.645 \cdot 0.0071 \approx 0.0117$

$0.227 - 0.0117 = 0.2153$
$0.227 + 0.0117 = 0.2388$

$\boxed{(0.2153, 0.2388)}$

Section 5.3 #'s 1, 7, 8 a & b only, 9, 11

1.

Find the value of tn−1,α/2 needed to construct a two-sided confidence interval of the given level with the given sample size:

a. **Level 90%, sample size 12.**

Section 5.3

1. Find $t_{n-1,\alpha/2}$ needed for a two-sided confidence interval of the given level with the given size.

a) Level 90%, sample size 12

$1-0.90=0.10 \rightarrow 0.10/2=0.05$

$n-1 \rightarrow 12-1=11$ degrees of freedom

$=\boxed{1.796}$

b. Level 95%, sample size 7.

b) Level 95%, sample size 7

$1-0.95=0.05 \rightarrow 0.05/2=0.025$

$n-1 \rightarrow 7-1=6$ degrees of freedom

$=\boxed{2.447}$

c. Level 99%, sample size 2.

c) Level 99%, sample size 2

$1-0.99=0.01 \rightarrow 0.01/2=0.005$

$n-1 \rightarrow 2-1=1$ degrees of freedom

$=\boxed{63,657}$

d. Level 95%, sample size 29.

d) Level 95%, sample size 29

$1-0.95=0.05 \rightarrow 0.05/2=0.025$

$n-1 \rightarrow 29-1=28$ degrees of freedom

$=\boxed{2.048}$

7.

The article "An Automatic Visual System for Marble Tile Classification" (L. Carrino, W. Polini, and S. Turchetta, Journal of Engineering Manufacture, 2002:1095–1108) describes a measure for the shade of marble tile in which the amount of light reflected by the tile is measured on a scale of 0–255. A perfectly black tile would reflect no light and measure 0, and a perfectly white tile would measure 255. A sample of nine Mezza Perla tiles were measured, with the following results:

204.999

206.149

202.102

207.048

203.496

206.343

203.496

206.676

205.831

Is it appropriate to use the Student's t statistic to construct a 95% confidence interval for the mean shade of Mezza Perla tile? If so, construct the confidence interval. If not, explain why not.

7. $n = 9$

204.994, 206.141, 207.102, 207.048, 203.496, 206.343, 203.446, 206.676, 205.831

mean = 205.12

$204.994 - 205.12 = -0.127^2 = 0.016$

$206.141 - 205.12 = 1.022^2 = 1.045$

$202.102 - 205.12 = -3.02^2 = 9.148$

$207.048 - 205.12 = 1.92^2 = 3.69$

$203.496 - 205.12 = -1.63^2 = 2.65$

$206.343 - 205.12 = 1.21^2 = 1.47$

$203.446 - 205.12 = -1.63^2 = 2.65$

$206.676 - 205.12 = 1.54^2 = 2.4$

$205.831 - 205.12 = 0.704^2 = 0.496$

a 95%

$1 - 0.95 = 0.05/2$

0.025

std dev

$\frac{23}{8} = 2.94 = 1.717$

$t_{8, 0.025}$

↓

2.306

$205.12 \pm 2.306\left(\frac{1.717}{\sqrt{9}}\right)$

$205.12 \pm 1.32 \rightarrow (203.807, 206.447)$

$\boxed{\text{Yes, there are no apparent outliers. } (203.81, 206.45)}$

8.

A chemist made six independent measurements of the sublimation point of carbon dioxide (the temperature at which it changes to dry ice). She obtained a sample mean of 196.64 K with a standard deviation of 0.68 K.

a. **Use the Student's t distribution to find a 95% confidence interval for the sublimation point of carbon dioxide.**

8. $n=6$

$\bar{x} = 196.64$

$s = 0.68$

a) Use t dist. to find 95% C.I.

$1-0.95 = 0.05/2 \rightarrow 0.025$

$t_{5, 0.025} \rightarrow 2.571$

$196.64 \pm 2.571\left(\frac{0.68}{\sqrt{6}}\right)$

$196.64 \pm 0.7137 \rightarrow \boxed{(195.93, 197.35)}$

b. **Use the Student's t distribution to find a 98% confidence interval for the sublimation point of carbon dioxide.**

b) Use t dist. to find 98% C.I.

$1-0.98 = 0.02/2 = 0.01$

$t_{5, 0.01} \rightarrow 3.365$

$196.64 \pm 3.365\left(\frac{0.68}{\sqrt{6}}\right)$

$196.64 \pm 0.934 \rightarrow \boxed{(195.71, 197.57)}$

c. ~~**If the six measurements had been 196.35, 196.32, 196.4 198.02, 196.36, 196.39, would the confidence intervals above be valid? Explain.**~~

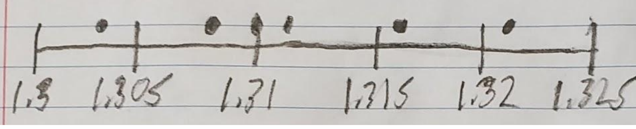
9.

Six measurements are taken of the thickness of a piece of 18-gauge sheet metal. The measurements (in mm) are: 1.316, 1.308, 1.321, 1.303, 1.311, and 1.310.

a. **Make a dotplot of the six values.**

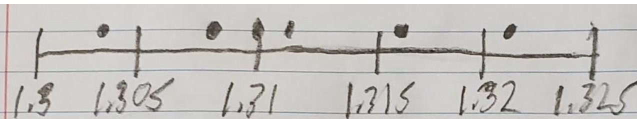
a. 1.316, 1.308, 1.321, 1.303, 1.311, and 1.310

a) Make a dotplot



1−0.a

b. Should the t curve be used to find a 99% confidence interval for the thickness? If so, find the confidence interval. If not, explain why not.



$1-0.99 = 0.01/2$

$\downarrow$

0.005

b) Should t curve be used?

mean = 1.3115

$1.316 - 1.3115 = 0.0045^2 = 2.025\times10^{-5}$
$1.308 - 1.3115 = -0.0035^2 = 1.225\times10^{-5}$
$1.321 - 1.3115 = 0.0095^2 = 9.025\times10^{-5}$
$1.303 - 1.3115 = -0.0085^2 = 7.225\times10^{-5}$
$1.311 - 1.3115 = -0.0005^2 = 2.5\times10^{-7}$
$1.310 - 1.3115 = -0.0015^2 = 2.25\times10^{-6}$

Std dev

$\sqrt{\frac{1.975\times10^{-4}}{5}} = 0.006$

$t_{5, 0.005}$

$\downarrow$

4.032

$1.3115 \pm 4.032\left(\frac{0.006}{\sqrt{6}}\right)$

Cont'd

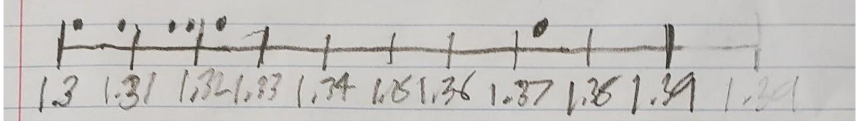
$1.3115 \pm 0.01 \rightarrow (1.301, 1.321)$

Yes, because there are no outliers.

$(1.301, 1.321)$

c. **Six independent measurements are taken of the thickness of another piece of sheet metal. The measurements this time are: 1.317, 1.318, 1.301, 1.307, 1.374, 1.323. Make a dotplot of these values.**

c) 1.317, 1.318, 1.301, 1.307, 1.374, 1.323



d. **Should the t curve be used to find a 95% confidence interval for the thickness of this metal? If so, find the confidence interval. If not, explain why not.**

d) No, there appears to be an outlier at point 1.374!

11.

The article "Effect of Granular Subbase Thickness on Airfield Pavement Structural Response" (K. Gopalakrishnan and M. Thompson, Journal of Materials in Civil Engineering, 2008:331–342) presents a study of the effect of the subbase thickness on the amount of surface deflection caused by aircraft landing on an airport runway. In six applications of a 160 kN load on a runway with a subbase thickness of 864 mm, the average surface deflection was 2.03 mm with a standard deviation of 0.090 mm. Find a 90% confidence interval for the mean deflection caused by a 160 kN load.

11. $n=6$ Find 90% C.I.

$\bar{x} = 2.03$

$s = 0.090$ $1-0.9 = 0.1/2 = 0.05$

$t_{5,0.05} \rightarrow 2.015$

$2.03 \pm 2.015\left(\frac{0.09}{\sqrt{6}}\right)$

$2.03 \pm 0.074 \rightarrow \boxed{(1.956, 2.104)}$