Homework answers for Sets and Counting

CSC28 Fall 2020 (Jagan Chidella) [Note Complement notation A^c has been typed A^c]

1. Are these sets equal? (Is A=B=C). Explain your answer.

The 3 sets ARE equal because the members of each set are the same.

A= {1,2,3} B= {3,1,2} C= {3,2,2,1}

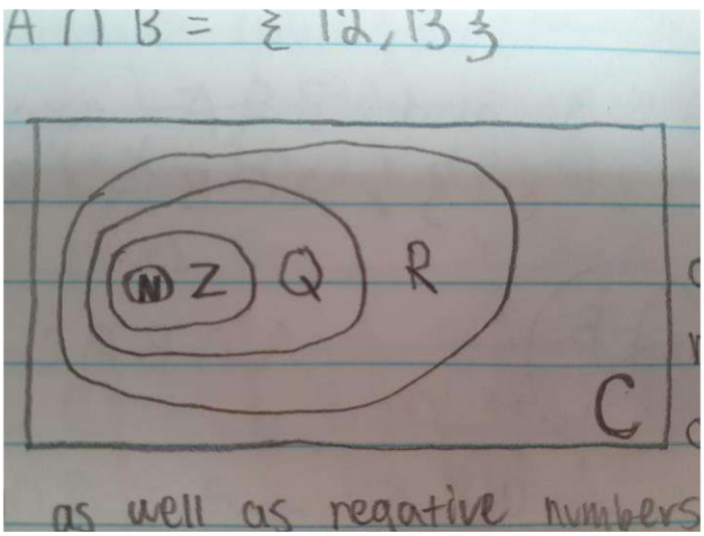
So, A is a subset of B: $A \subseteq B$, and $B \subseteq A$ so A = B. Also, $B \subseteq C$ and $C \subseteq B$ so B=C. If A=B and B=C, then A=B=C.

2. List all the partitions(subsets) of A∩B

A ∩ B = {12,13} so P (A ∩ B) = {{Ø}, {12}, {13}, {12,13}}

3. Show relationship of N, Z, Q, R using example and Venn Diagram.

Example: **N** is the smallest circle of the diagram, it contains natural numbers "1,2,3, ...". **Z** is the set of all integers and it contains N as well as negative numbers "..., -1, 0, 1, ...". **Q** is rational numbers are including Z and N, as well as terminating fractions "..., -1, -½, 0, ½, 1, ...". **R** is real numbers and include Z, N, and Q as well as irrational numbers such as "π". **N ⊆Z ⊆Q ⊆R ⊆C**



4. Examples:
   a. Empty Set: A = Ø or A = {}. Example: The set of students at Sac State with 10 arms is empty.
   b. Disjoint Set: A= {1,2,3} B= {4,5,6}, A is disjoint with B because there are no common elements. A ⊄ B and B ⊄A. Example: the set of students attending Sac State is disjoint with the set of students attending Harvard.
   c. Universal Set: A universal set is the largest fixed set that contains all other sets. Ø ⊆ {Set} ⊆ u.
   d. Subset:

   Example: If A={1,2} and B={2,1}, then A ⊆ B and B ⊆ A.

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The difference between a subset and a proper subset is that a proper subset has at least one element in a set that doesn't belong to the other set.

Example:

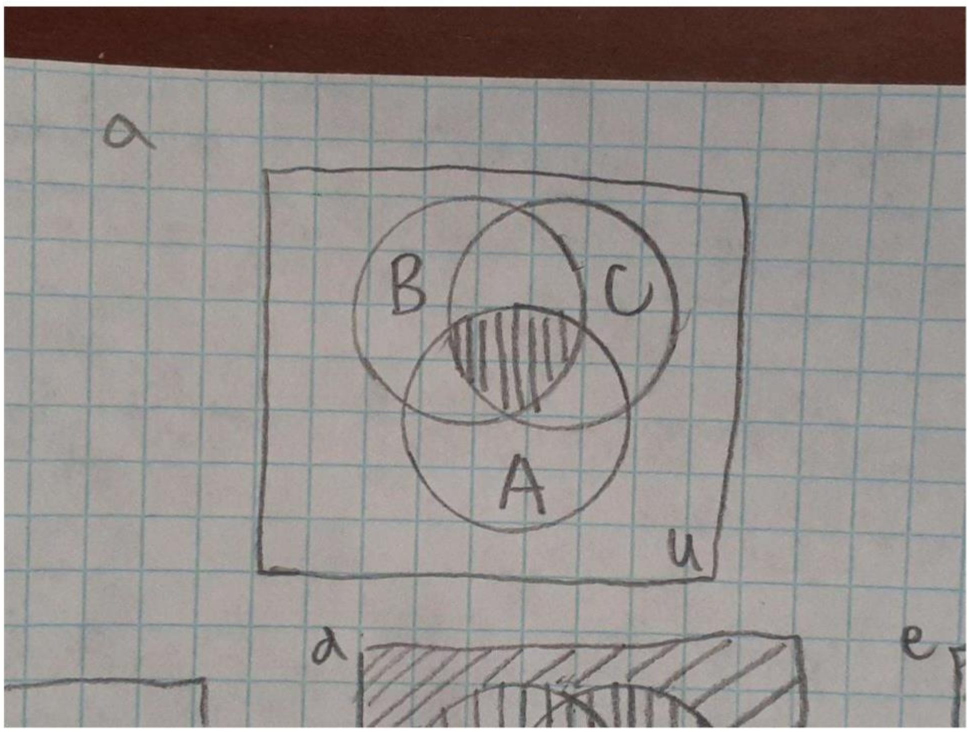
$$A=\{1,2\}\ B=\{2,1\}\ C=\{1,2,3\}$$

$A \subseteq B$ but $A \subset C$, A is a proper subset of C.

e. Equal: You say 2 sets are equal when set A and B are $A \subseteq B$, and $B \subseteq A$, then A=B. Example:

$$A=\{1\}\ B=\{1\},\ A \subseteq B, \text{ and } B \subseteq A, \text{ so } A=B.$$

5. Venn Diagrams:
   a. $A \cap B \cap C = \{3\}$



   b. $(B \cap C) - A = \{4\}$

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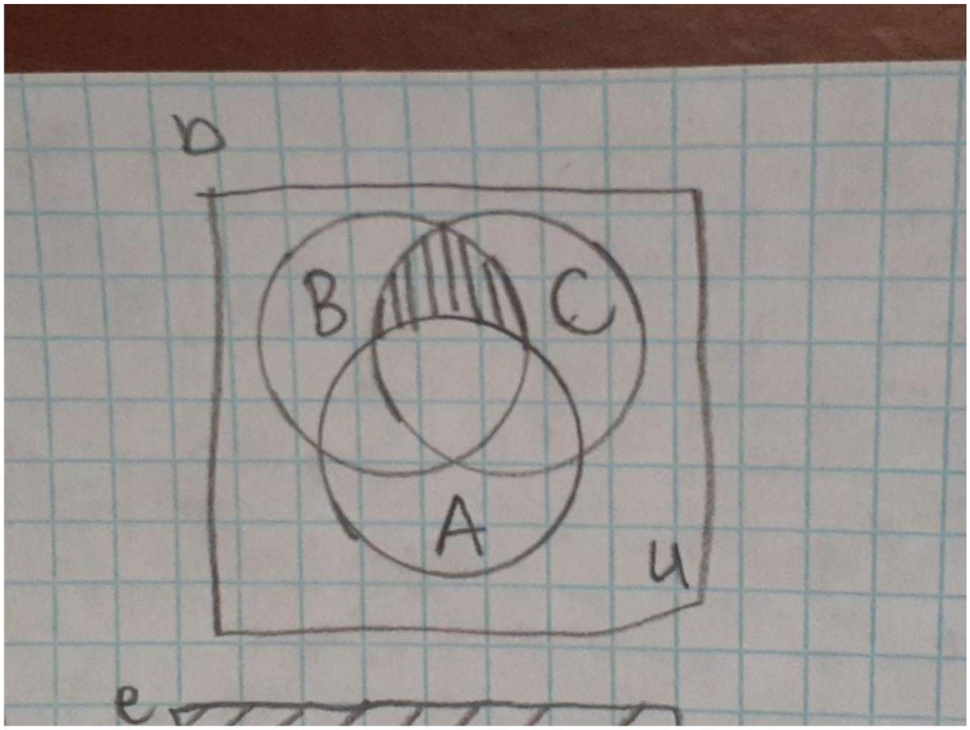
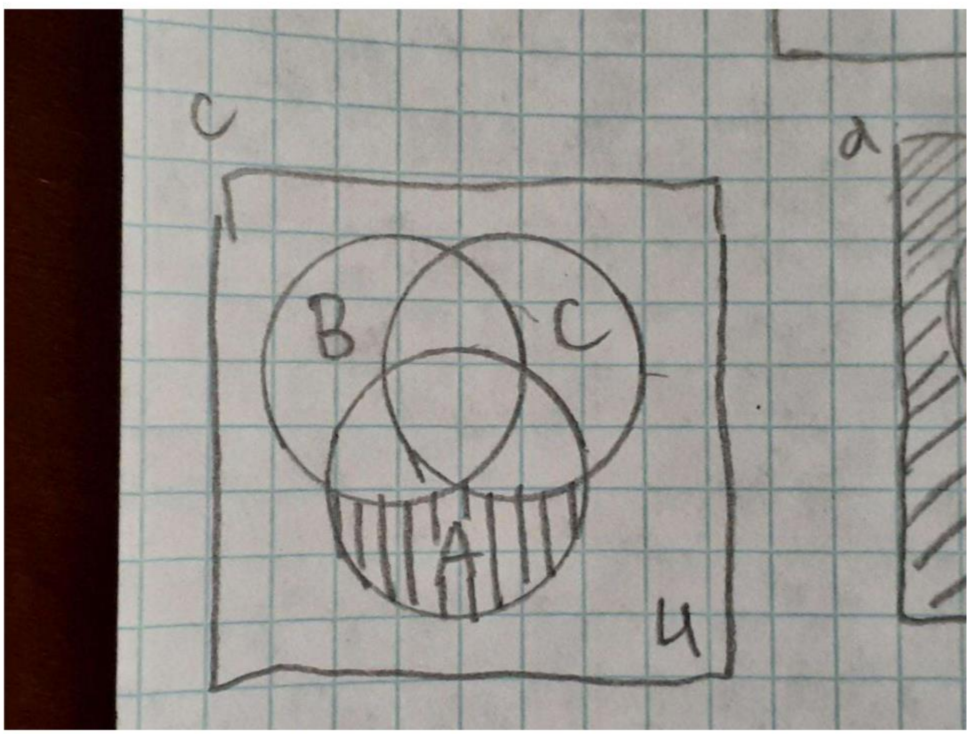


c. A - (B ∪ C) = {5}



d. (A ∩ B)^c ∪ C = {2,3,4,5,6,7,9}

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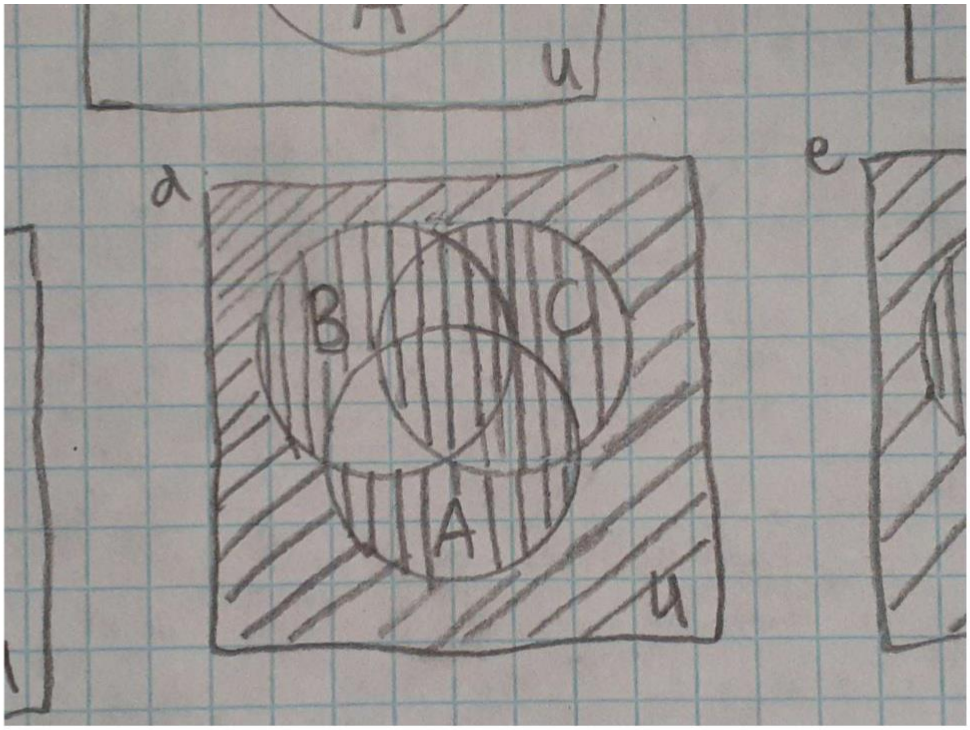
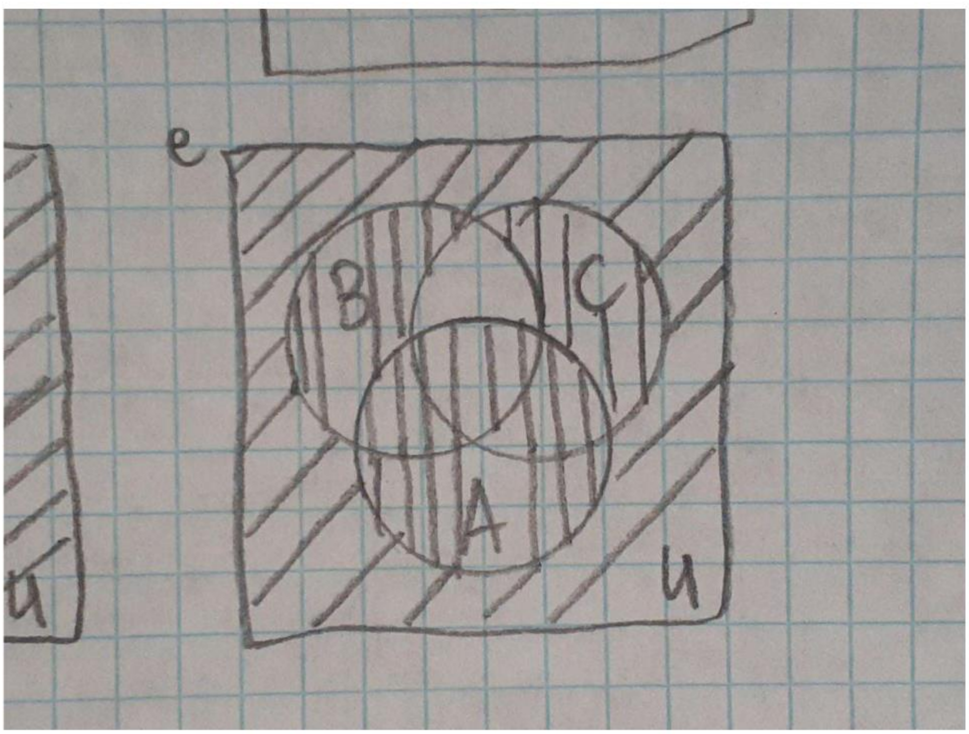


e. $A \cup (B \cap C)^c = \{1,2,3,5,6,7,9\}$



6. If A is "students who wear masks" then A complement (A^c) would be the # of students at Sac State who don't wear masks. So, the answer would be **B.**
7. Prove $X * Y \neq Y * X$

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Example: X= {2,4} Y= {1,3}

X * Y = {(2,1), (2,3), (4,1), (4,3)}

Y * X = {(1,2), (3,2), (1,4), (3,4)}

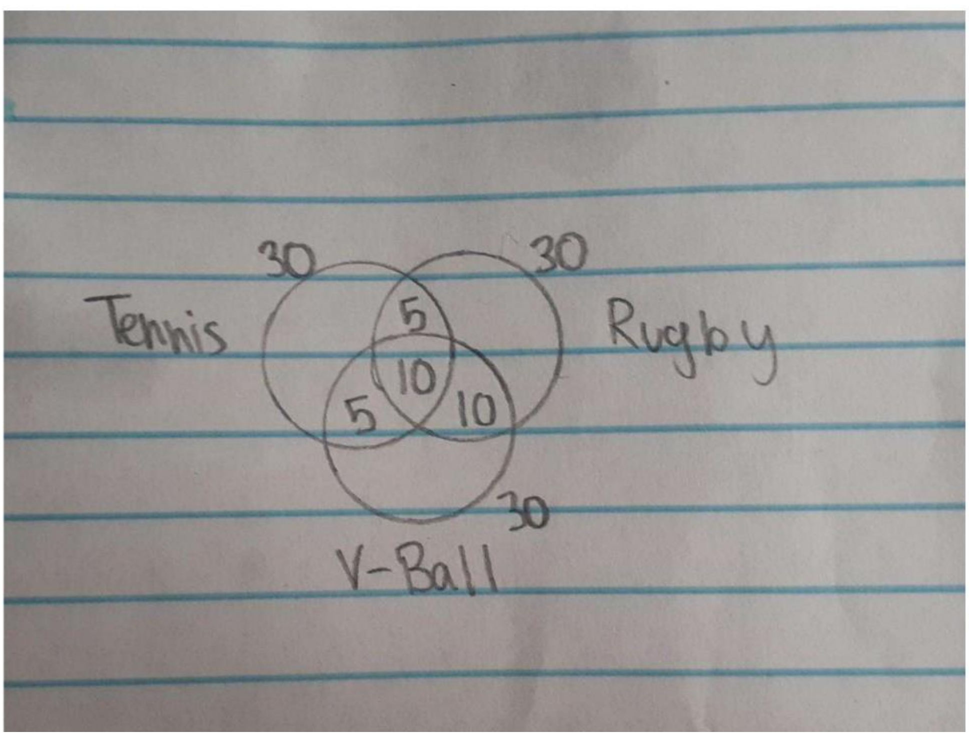
This proves that X * Y ≠ Y * X

8. What is A ⊕ B?
   a. A ⊕ B = {7,10,11} (for detail explanation of 8a see last page)
   b. Also Prove:
      A ∩ B ⊆ A: {**8,9**} ⊆ {7,**8,9**}, true so A ∩ B is in fact a subset of A
      ii. A ⊆ A ∪ B: {**7,8,9**} ⊆ {**7,8,9**,10,11}, true so A is in fact a subset of A ∪ B
      iii. A ∩ B ⊆ B: {**8,9**} ⊆ {**8,9**,10,11}, true so A ∩ B is in fact a subset of B
      iv. B ⊆ A∪B: {**8,9,10,11**} ⊆ {7,**8,9,10,11**}, true so B is in fact a subset of A ∪ B
   c. From this, we have two properties. Property 1: A ∩ B ⊆ A and A ∩ B ⊆ B. Property 2: A ⊆ A ∪ B and B ⊆ A∪B. By combining these two properties together, we can deduce that:

$$A \cap B \subseteq A \subseteq A \cup B$$

$$A \cap B \subseteq B \subseteq A \cup B$$

9.



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a. Total number of students: To find the total number of students, we add the number of students who play each game then we subtract the number of students who play multiple different games. 30+30+30-20-15-15+10 = 50. So, the total number of students is **50**.

Only Tennis: 30 - (10 + 5 + 5) = **10** students only played tennis.

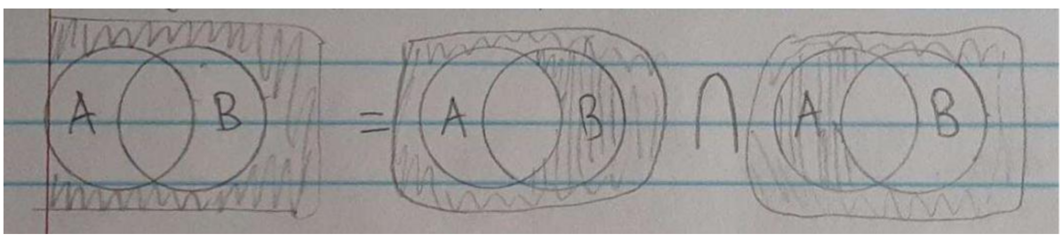
Only Rugby: 30 - (10 + 10 + 5) = **5** students only played rugby.

Only Volleyball: 30 - (10 + 10 + 5) = **5** students only played v-ball.

b. Suppose there were 70 students how many played none of the 3 games? If the original information stayed the same, then the number of students who played the game would stay the same. In the previous part, I found that the total number of students playing at least 1 game is 50 so if there are 70 students. Then 70 – 50 gives us **20** students who didn't play a single game.

10.

a. Prove De Morgan's Law: (A ⋃ B) ^c = A^c ⋂ B^c



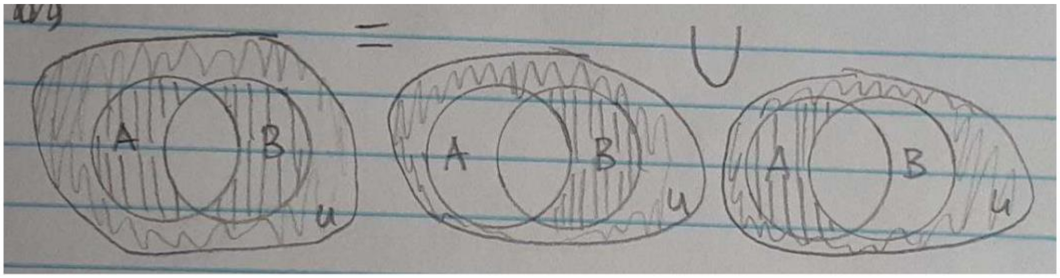
{x | x ∉ (A or B)} = {x| x ∉A and x ∉ B}

Example: say that x ∈ (A ⋃ B) ^c , this also means that x ∉ A ⋃ B, and if it doesn't belong to either A or B, then it also belongs to both A^c and B^c. So (A ⋃ B) ^c = A^c ⋂ B^c becomes true.

Say that x ∈ A^c ⋂ B^c ^, this also means that x ∉ (A ⋃ B), and if it doesn't belong to either A or B, then it also belongs to x ∈ (A ⋃ B) ^c. So (A ⋃ B) ^c = A^c ⋂ B^c becomes true.

(A ⋂ B) ^c = A^c ⋃ B^c

Homework answers for Sets and Counting



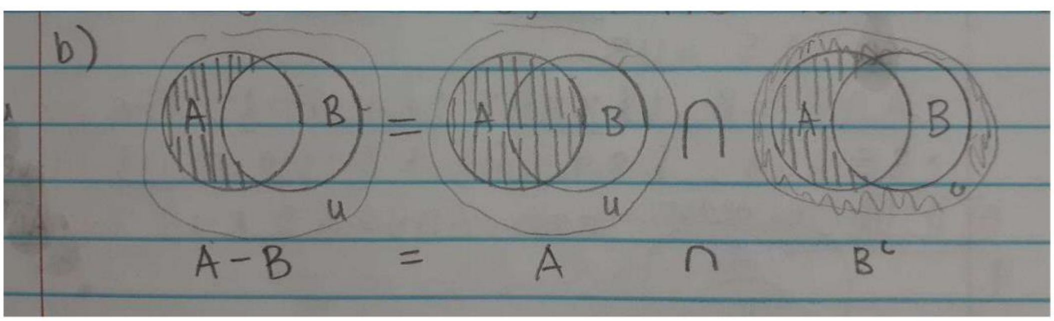
$\{x \mid x \notin (A \text{ and } B)\} = \{x \mid x \notin A \text{ or } x \notin B\}$

Example: say that $x \in (A \cap B)$ ^c , this also means that $x \notin A \cap B$, and if it doesn't belong to both A and B, then it must belong to either A^c or B^c. So $(A \cap B)$ ^c = A^c $\cup$ B^c becomes true.

Say that $x \in$ A^c $\cup$ B^c, this also means that $x \notin (A \cap B)$, and if it doesn't belong to A and B, then it also belongs to $x \in (A \cap B)$ ^c. So $(A \cap B)$ ^c = A^c $\cup$ B^c becomes true.

10 (b).

a. A-B = A $\cap$ B^c



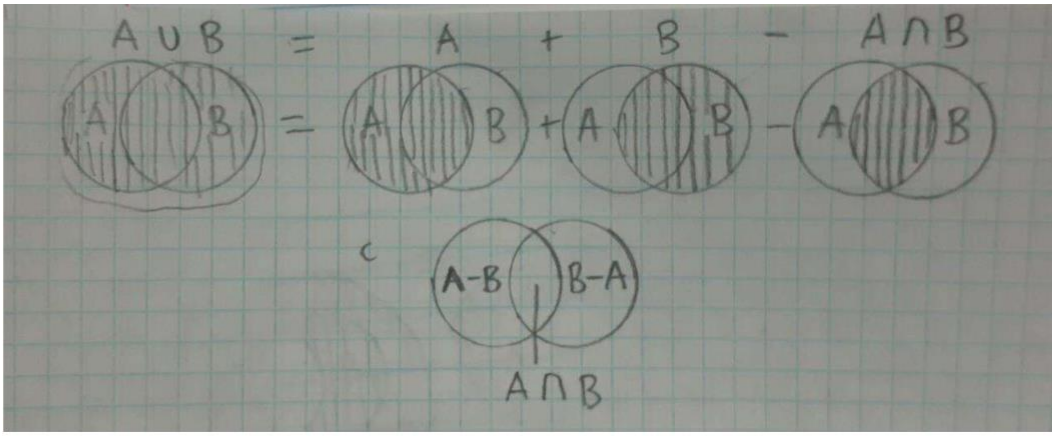
Example: say that $x \in A - B$, this also means that $x \in A$ and $x \notin B$ ($x \in$ B^c), Since $x \in A$ and $x \in$ B^c. Then, this proves that $x \in A \cap$ B^c; therefore, $A \cap$ B^c $\subseteq A - B$ is true.

Example 2: A = {a,b,c,d} B={c,d,e,f}

Then A-B = A $\cap$ B^c = {a,b}

b. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Homework answers for Sets and Counting



Example: $n(A \cup B)$ can also be written as $n(A-B) + n(A \cap B) + n(B-A)$

So we have

- $n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$
    - $n(A-B) = n(A) - n(A \cap B)$
    - $n(B-A) = n(B) - n(A \cap B)$
- $n(A \cup B) = n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$

Which leaves us with: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Example 2: $A = \{a,b,c,d\}$ $B=\{c,d,e,f\}$

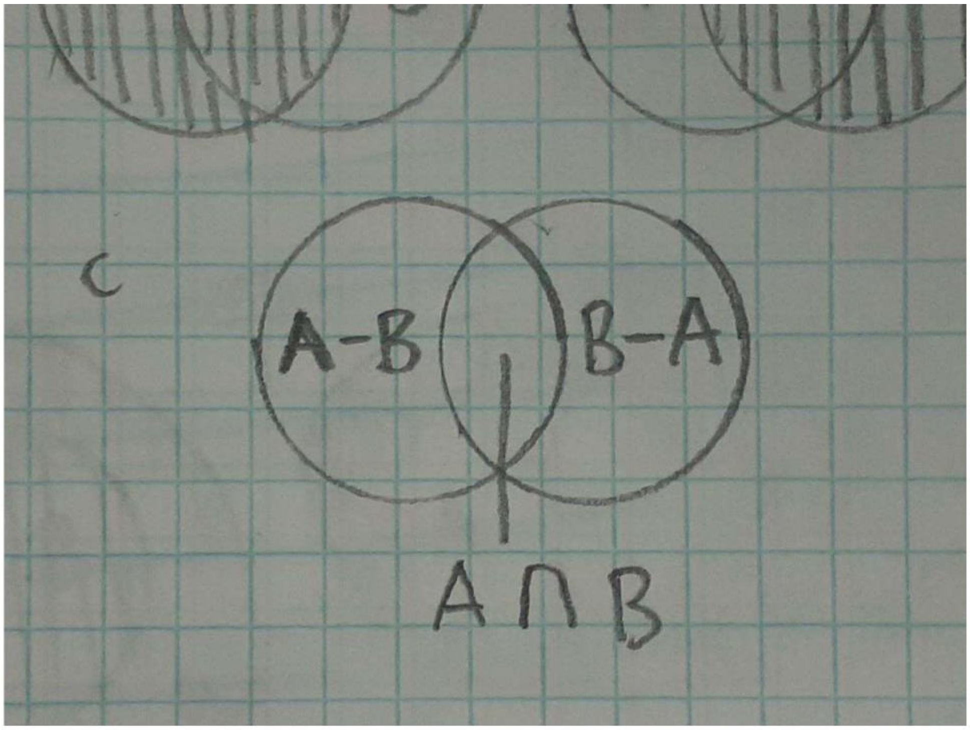
$$n(A)=4 \quad n(B)=4 \quad n(A\cap B)=2$$

$$n(A \cup B) = n(A) + n(B) - n(A\cap B) = 4 + 4 - 2$$

$$= 6$$

c. $n(A \cup B) = n(A-B) + n(A\cap B) + n(B-A)$

Homework answers for Sets and Counting



Example: A ∪ B can be split into 3 parts which are added together, the three parts are: only unique to A, both A and B, and only unique to B. The respective parts can be written as A-B, A∩B, and B-A. Adding each section together gives us the total n(A ∪ B) = n(A-B) + n(A∩B) +n(B-A).

Example 2: A = {a,b,c,d} B={c,d,e,f}

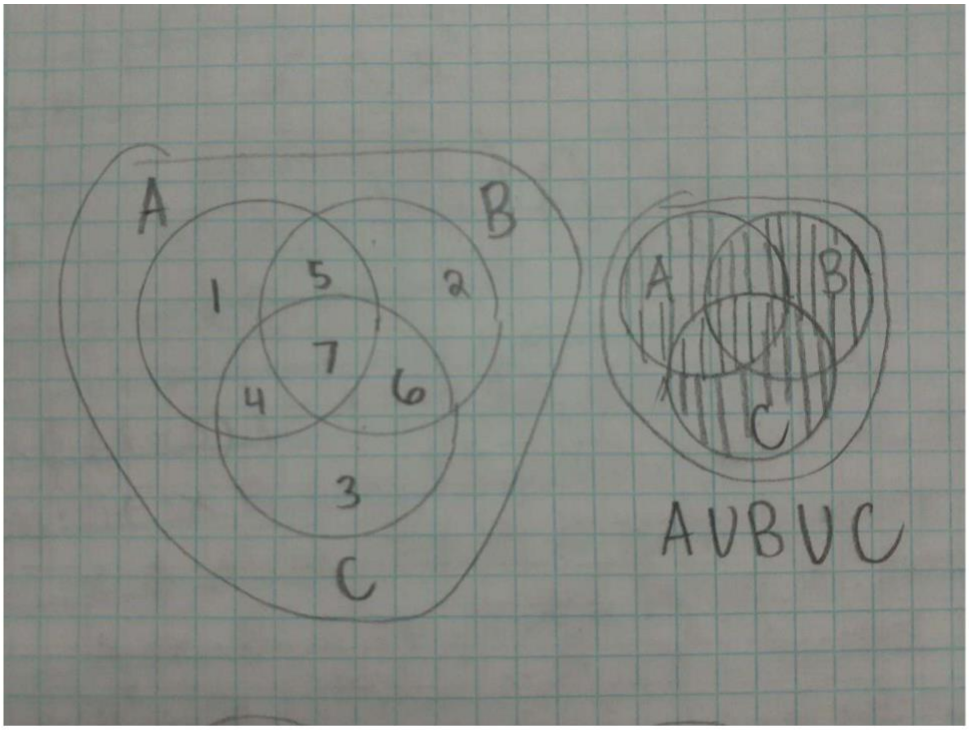
$$n(A-B)=2 \quad n(B-A)=2 \quad n(A\cap B)=2$$

$$n(A \cup B) = n(A-B) + n(A\cap B) + n(B-A) = 2 + 2 + 2$$

$$=6$$

d. $n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$

Homework answers for Sets and Counting



Venn Diagram Key:

1. Only A
2. Only B
3. Only C
4. A and C not B
5. A and B not C
6. B and C not A
7. A and B and C

Example: $n(A \cup B \cup C)$ can be written as $n[(A \cup B) \cup C]$. We can take this a step further and say that C is also n(C) - the intersection of A or B with C. So we get

$$n[(A \cup B) \cup C] = n(A \cup B) + n(C) - n[(A \cup B) \cap C].$$

As we found in the previous problem, $n(A \cup B) = n(A) + n(B) – n(A\cap B)$. We end up with:

$$n(A) + n(B) – n(A\cap B) + n(C) - n[(A \cap C) \cup (B \cap C)]$$

We'll treat $(A \cap C) \cup (B \cap C)$ like it was $A \cup B$ which gives us:

$$n(A) + n(B) + n(C) – n(A\cap B) - [n(A \cap C) + n(B \cap C) - n[(A \cap C) \cap (B \cap C)]]$$

$$n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$$

So $n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$ is true.

Example 2: $A = \{a,b,c,d\}$ $B=\{c,d,e,f\}$ $C=\{e,f,g,h\}$

$$n(A)=4 \quad n(B)=4 \quad n(C)=4$$

$$n(A\cap B)=2 \quad n(A\cap C)=0 \quad n(B\cap C)=2 \quad n(A\cap B\cap C)=0$$

$$n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)=$$
$$4+4+4-2-0-2-0=8$$

e. $(A \cup B)-C = (A-C) \cup (B-C)$

Detail answer to 8a.

8) $A=\{7,8,9\}$ $B=\{8,9,10,11\}$

a) $A \oplus B = (A-B) \cup (B-A)$
$= \{7\} \cup \{10,11\}$
$= \{7,10,11\}$