Math 100 - HW2

(1)

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{array}\right] \xrightarrow{-R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{array}\right] \xrightarrow{\substack{\frac{1}{3}R_2 \to R_2 \\ \frac{1}{11}R_3 \to R_3}} \left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 1 & \frac{7}{3} & -5/3 \\ 0 & 0 & 1 & -2/11 \end{array}\right]$$

$$\xrightarrow{\substack{-\frac{7}{3}R_3+R_2 \to R_2 \\ 6R_3+R_1 \to R_1}} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 109/11 \\ 0 & 1 & 0 & -41/33 \\ 0 & 0 & 1 & -\frac{2}{11} \end{array}\right] \xrightarrow{2R_2+R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 245/33 \\ 0 & 1 & 0 & -41/33 \\ 0 & 0 & 1 & -2/11 \end{array}\right] \to$$

yes, $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

$$\left[\begin{array}{c} \frac{14}{33} - \frac{55}{33} = \frac{-41}{33} \\ -\frac{12}{11} + \frac{121}{11} = \frac{109}{11} \end{array}\right] \qquad \left[\begin{array}{c} -\frac{82}{33} \; \frac{327}{33} \\ = \frac{245}{33} \end{array}\right]$$

(2)

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \xrightarrow{2R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

inconsistent system thus $\vec{b}$ is <u>not</u> a linear combination of the columns of A.

(3)

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{array}\right] \xrightarrow{R_1+R_3 \to R_3} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ 0 & -3 & h+3 \end{array}\right] \xrightarrow{-3R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & h+3 \end{array}\right] \xrightarrow{\frac{1}{7}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & h+3 \end{array}\right]$$

$$\xrightarrow{5R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2 \\ 0 & -3 & h+3 \end{array}\right] \xrightarrow{3R_2+R_3 \to R_3} \left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2 \\ 0 & 0 & h-3 \end{array}\right] \to$$

$x_1 = -7$

$x_2 = -2$

$0 = h-3 \to h=3$

(4) a) No. $\vec{b} \neq \vec{a}_1 + \vec{a}_2 + \vec{a}_3$ b) 3 vectors

c)
$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{array}\right] \xrightarrow{2R_1+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{array}\right] \xrightarrow{-2R_2+R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right] \xrightarrow{-1R_3\to R_3} \left[\begin{array}{ccc|c} & '' & & \\ & '' & & \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$2R_3+R_2\to R_2$

$-4R_3+R_1\to R_1$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 3 & 0 & -3 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{3}R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right] \rightarrow$$

Yes $\vec{b}$ is in W.

$(-4)\vec{a}_1 + (-1)\vec{a}_2 + (-2)\vec{a}_3 = \vec{b}$

d) The span of these 3 vectors is All linear combinations of these 3 vectors, so there are infinitely many vectors in W.

e) $(1)\vec{a}_1 + (0)\vec{a}_2 + (0)\vec{a}_3 = \vec{a}_1$, A linear combination, thus $\vec{a}_1$ is in W.

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(5) $2\begin{bmatrix} 1 \\ -2 \end{bmatrix} + (-1)\begin{bmatrix} 2 \\ -3 \end{bmatrix} + 1\begin{bmatrix} -3 \\ 1 \end{bmatrix} + (-1)\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(6) a) $\begin{bmatrix} 2 & -1 & -4 & 0 \\ -4 & 5 & 3 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$ b) $2z_1 - z_2 - 4z_3 = 5$

$-4z_1 + 5z_2 + 3z_3 + 2z_4 = 12$

(7)
$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 1 & 5 & 2 & 4 \\ -3 & -7 & 6 & 12 \end{array}\right] \xrightarrow[3R_1+R_3\to R_3]{-R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 0 & 2 & 6 & 6 \\ 0 & 2 & -6 & 6 \end{array}\right] \xrightarrow[\frac{1}{2}R_3\to R_3]{\frac{1}{2}R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 0 & 1 & 3 & 3 \\ 0 & 1 & -3 & 3 \end{array}\right] \rightarrow$$

$-R_2+R_3\to R_3$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & -6 & 0 \end{array}\right] \xrightarrow{-\frac{1}{6}R_3\to R_3} \left[\begin{array}{ccc|c} 1 & 3 & -4 & -2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[4R_3+R_1\to R_1]{-3R_3+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & 3 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{-3R_2+R_1\to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\boxed{\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 0 \end{bmatrix}}$$

(8)

a) $\left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ -2 & 2 & 0 & b_2 \\ 4 & -1 & 3 & b_3 \end{array}\right] \xrightarrow[-4R_1+R_3\to R_3]{2R_1+R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -2 & -2 & b_2+2b_1 \\ 0 & 7 & 7 & b_3-4b_1 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2\to R_2} \left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -1 & -1 & \frac{1}{2}b_2+b_1 \\ 0 & 7 & 7 & b_3-4b_1 \end{array}\right]$.

$7R_2+R_3\to R_3$

$\to \left[\begin{array}{ccc|c} 1 & -2 & -1 & b_1 \\ 0 & -1 & -1 & \frac{1}{2}b_2+b_1 \\ 0 & 0 & 0 & b_3-4b_1+(\frac{7}{2}b_2+7b_1) \end{array}\right] \to 0 = 3b_1+\frac{7}{2}b_2+b_3$

Whatever is chosen must satisfy this equation.

Choose $b_1=1,\ b_2=2,\ b_3=1 \to 0 = 3(1)+\frac{7}{2}(2)+(1)$

$0 = 3+7+1$

no solution for this $\leftarrow$ $0 = 11 \to$ not true

b) The columns of A can <u>NOT</u> span $\mathbb{R}^3$, By (a) the vector $\vec{b} = \begin{bmatrix}1\\2\\1\end{bmatrix}$ is not in the span of the columns of $\vec{A}$, [Even with one vector 'missing', can't span $\mathbb{R}^3$.] There is <u>No</u> linear combination of the columns of A that can form $\vec{b} = \begin{bmatrix}1\\2\\1\end{bmatrix}$.

c) The set of all $\vec{b}$ for which $A\vec{x}=\vec{b}$ has a solution is

Span $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ where $\vec{a}_1 = \begin{bmatrix}1\\-2\\4\end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix}-2\\2\\-1\end{bmatrix}$, $\vec{a}_3 = \begin{bmatrix}-1\\0\\3\end{bmatrix}$ and $A = [\vec{a}_1\ \vec{a}_2\ \vec{a}_3]$.

Also,

The equation $3x+\frac{7}{2}y+z=0$ is a plane in $\mathbb{R}^3$ that contains all of the vectors $\vec{b}$ for which $A\vec{x}=\vec{b}$ has a solution.

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a) True, by Theorem 3

b) True, see Definiton of Span and Theorem 4

c) False, the pivot may be in the augmented column resulting in [0...0|1]

d) True by Theorem 4

e) True by Theorem 4 / Existence of Solutions in Notes

f) True by definition of $A\vec{x}$.