Math 30, Thursday, May 7, 2020

Review

Questions?

- "lecture 55 review problems"
- " 56 "
- Final Exam Review

& more!

If it was a "normal" class,
you'd work on worksheets and
I'd walk around and help.

Quiz 10:

Compute $\frac{d}{dx}\int_0^x \sin(2t)\,dt$ two ways:

(a) Using F.T. of C. Part I:

$$\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$$

"differentiating an integral"

$$\frac{d}{dx}\int_0^x \sin(2t)\,dt = \boxed{\sin(2x).}$$

That's all!

(b) Using F.T. of C. Part II:

an antiderivative of $\sin(2t)$

First integrate: $\int_0^x \sin(2t)\,dt = \left[-\frac{1}{2}\cos(2t)\right]_0^x$

$$= \left(-\frac{1}{2}\cos(2x)\right) - \left(-\frac{1}{2}\right)$$

$$\int_0^x \sin(2t)\,dt = \left[-\frac{1}{2}\cos(2t)\right]_0^x$$

$$= \left(-\frac{1}{2}\cos(2x)\right) - \left(-\frac{1}{2}\right)$$

$$= -\frac{1}{2}\cos(2x) + \frac{1}{2}.$$

Now differentiate:

$$\frac{d}{dx}\int_0^x \sin(2t)\,dt = \frac{d}{dx}\left(-\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$$

$$= \sin(2x)$$

same thing as before!

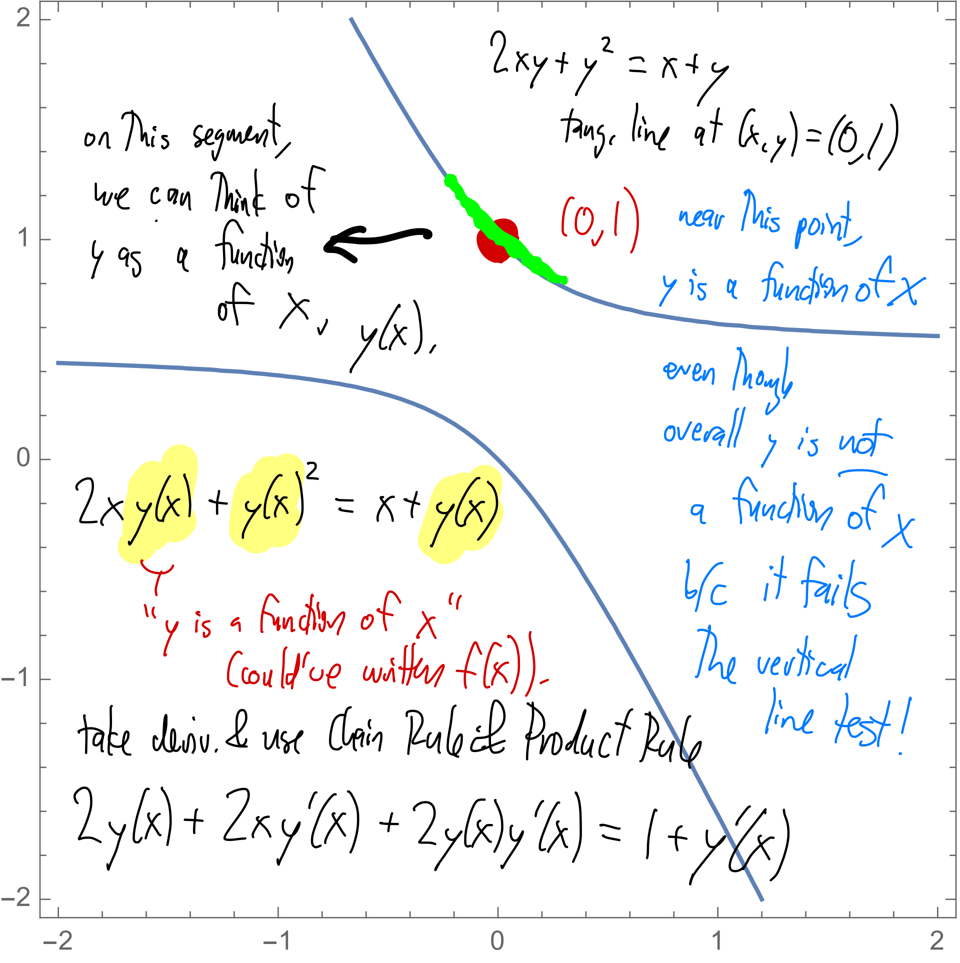
Other questions?

#4 on Practice Final.

curve's equation is

$$2xy + y^2 = x + y.$$

Find the eq$^{=}$ of tangent line at the point $(x,y) = (0,1)$ & plot it.



$$2xy + y^2 = x + y$$

tang. line at $(x,y) = (0,1)$

$(0,1)$

on this segment, we can think of y as a function of x, $y(x)$.

near this point, y is a function of x

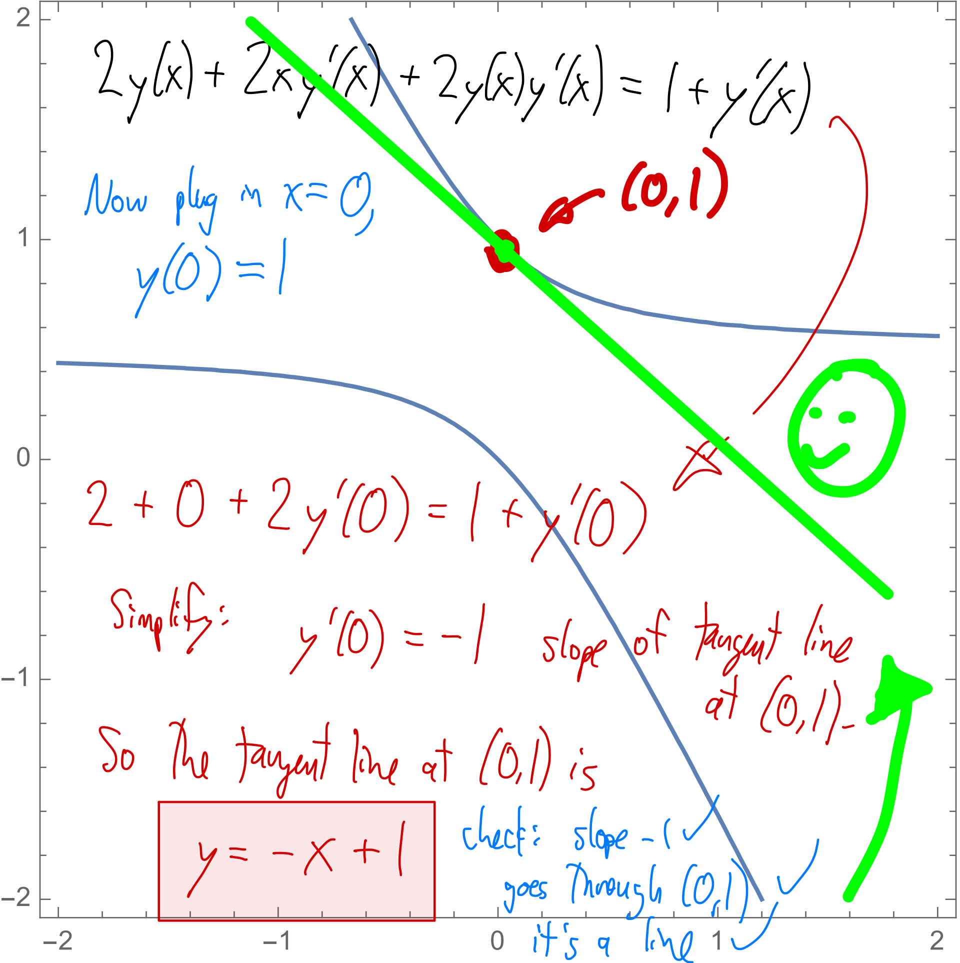
even though overall y is not a function of x b/c it fails the vertical line test!

$$2x\,y(x) + y(x)^2 = x + y(x)$$

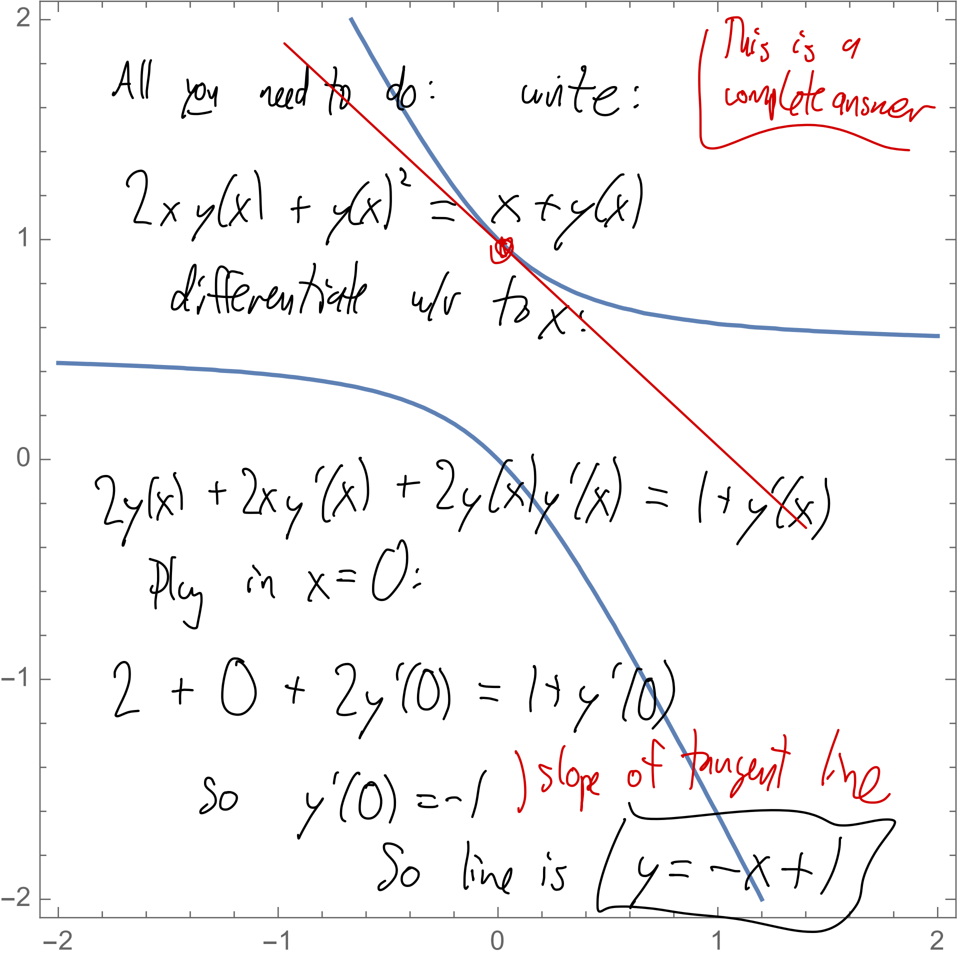
"y is a function of x" (could've written $f(x)$).

take deriv. & use Chain Rule & Product Rule

$$2y(x) + 2x\,y'(x) + 2y(x)\,y'(x) = 1 + y'(x)$$



$2y(x) + 2xy'(x) + 2y(x)y'(x) = 1 + y'(x)$
Now plug in $x = 0$,
$y(0) = 1$
(0,1)
$2 + 0 + 2y'(0) = 1 + y'(0)$
Simplify:
$y'(0) = -1$ slope of tangent line at (0,1).
So the tangent line at (0,1) is
$\boxed{y = -x + 1}$
check: slope -1 ✓
goes through (0,1) ✓
it's a line ✓
2
1
0
−1
−2
−2
−1
0
1
2



All you need to do: write:

This is a complete answer

$$2x\,y(x) + y(x)^2 = x + y(x)$$

differentiate w/r to x:

$$2y(x) + 2x\,y'(x) + 2y(x)\,y'(x) = 1 + y'(x)$$

Plug in $x = 0$:

$$2 + 0 + 2y'(0) = 1 + y'(0)$$

So $y'(0) = -1$) slope of tangent line

So line is $\boxed{y = -x + 1}$

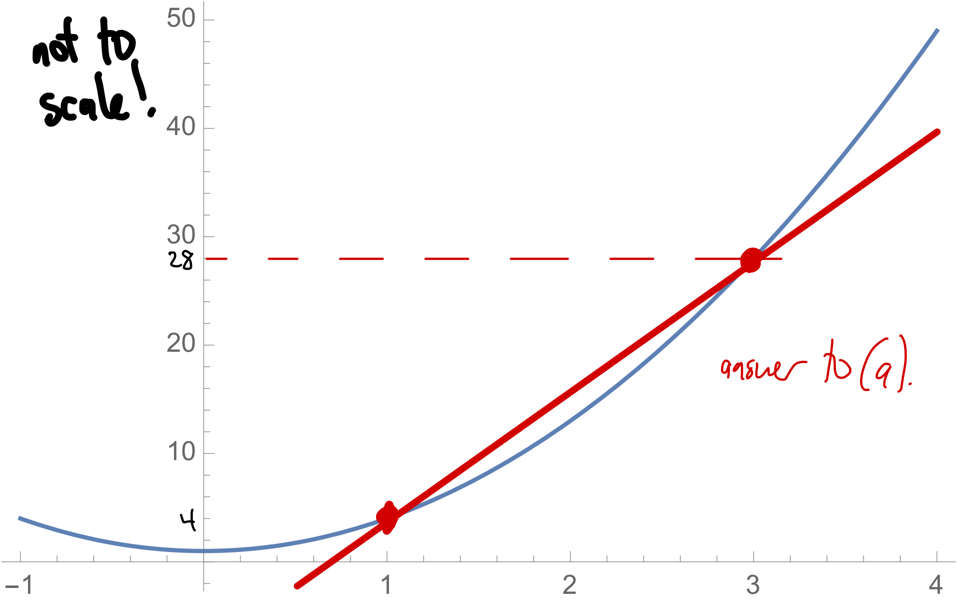
Next Question: #1 on Practice Final.

1. $f(x) = 3x^2 + 1$ (graph on next page)

a. Find slope connecting $(1, f(1))$ to $(3, f(3))$.

$$m = \frac{f(3) - f(1)}{3-1} \quad \begin{matrix} \text{"rise"} \\ \text{"run"} \end{matrix}$$

$$= \frac{28-4}{2} = 12$$


not to scale!
50
40
30
28
20
10
4
−1
1
2
3
4
answer to (a).

(b) Given any $h \neq 1$, find slope between $(1, f(1))$ and $(h, f(h))$.

$$m = \frac{f(h) - f(1)}{h-1} \quad \begin{matrix}\text{"rise"} \\ \text{"run"}\end{matrix}$$

$$= \frac{(3h^2+1) - (3+1)}{h-1}$$

The definition of the derivative!

$$= \frac{3(h^2-1)}{h-1} = \frac{3(h+1)(h-1)}{h-1}$$

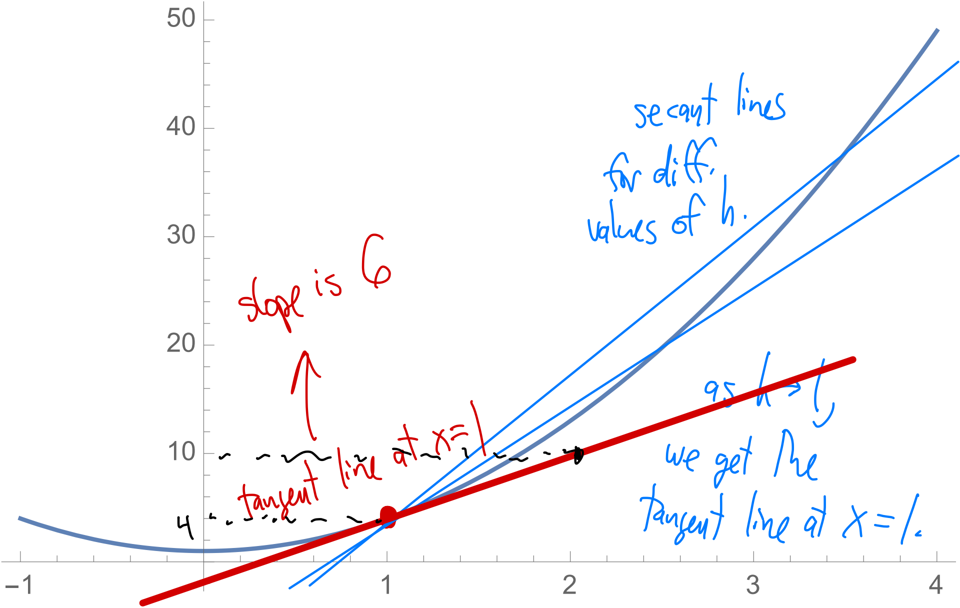
$$= 3(h+1).$$

$f'(1)$

$\parallel$

(c) $$\lim_{h \to 1} \frac{f(h) - f(1)}{h-1} = 6$$

(check using Power Rule)



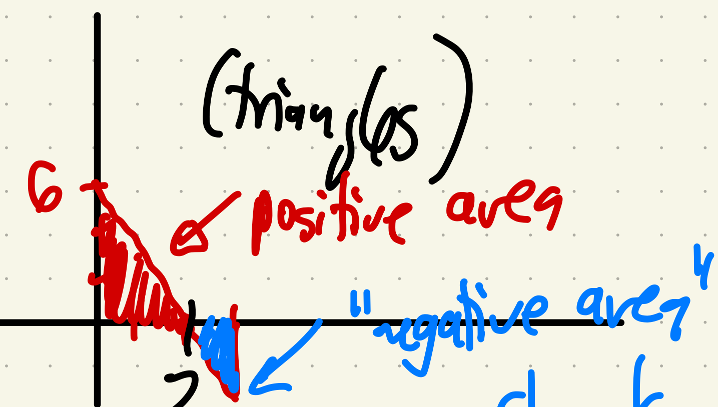
secant lines for diff. values of h.
slope is 6
tangent line at x=1
as h → 1, we get the tangent line at x=1.
4
10

Other Questions?

May 1 Worksheet "Integration Worksheet"

1. $\int_0^3 (6-3x)\,dx$ two diff. ways:

a. Sketch & find areas



$\frac{12}{2} - \frac{3}{2} = \frac{9}{2}$ ☺

Check

b. Use F.T. of Calc. Part II:

arithmetic

an antideriv. of $g(x) = 6 - 3x$

is $f(x) = 6x - \frac{3}{2}x^2$ $\boxed{\frac{9}{2}}$

$\int_0 \;= \left[6x - \frac{3}{2}x^2\right]_0^3 = 18 - \frac{27}{2} - 0$

More like this tomorrow!