hSTAT 50 HW #6
Calculus Review

1. a) Write out $\sum_{i=1}^{n} (2x_i^2 + 3i)$
   b) Write $5 + 10 + 15 + 20 + \cdots$ in Sigma notation.

A.

$$\boxed{(2x_1^2 + 3\cdot 1) + (2x_2^2 + 3\cdot 2) + (2x_3^2 + 3\cdot 3) + \ldots + (2x_n^2 + 3\cdot n)}$$

B.

$$\boxed{\sum_{i=1}^{n} (5 + 5(i-1))}$$

2. Given $f(x) = \sqrt{3x^3 + e^{2x}}$, find $f'(x)$

$$f(x) = (3x^3 + e^{2x})^{1/2}$$

$$\boxed{f'(x) = \frac{1}{2}(3x^3 + e^{2x})^{-1/2}\,[9x^2 + e^{2x}\cdot 2]}$$

3. Find the following anti derivatives:
   a) $\int e^{4x}\,dx$
   b) $\int (x^2 + 3x + 2)\,dx$
   c) $\int (x^2 e^{-x^3} + 2x)\,dx$
   d) $\int \frac{x}{\sqrt{x^2+1}}\,dx$

A.

a) $\int e^{4x}\,dx$

$u = 4x$

$du = 4\,dx$

$dx = \frac{du}{4}$

$\int e^{u}\,\frac{du}{4}$

$\frac{1}{4}\int e^{u}\,du \Rightarrow \frac{1}{4}e^{u} \Rightarrow \boxed{\frac{1}{4}e^{4x}+C}$

B.

b) $\int (x^2+3x+2)\,dx$

$\int x^2 = \frac{x^3}{3}$ $\quad \int 3x = \frac{3x^2}{2}$ $\quad \int 2 = 2x$

$= \boxed{\frac{x^3}{3}+\frac{3x^2}{2}+2x+C}$

C.

c) $\int (x^2 e^{-x^3}+2x)\,dx$ $\quad \int u\,dv = uv - \int v\,du$

$\int x^2 e^{-x^3} \Rightarrow$ For this part one should do u-substitution first!

$u = -x^3$

$du = -3x^2\,dx$

$dx = \frac{du}{-3x^2}$

$\int 2x \Rightarrow \frac{2x^2}{2} = x^2$

$x^2\int e^{u}\frac{du}{-3x^2} \Rightarrow \frac{x^2}{-3x^2}\int e^{u}\,du \Rightarrow \frac{1}{-3}\int e^{u}\,du \Rightarrow \frac{1}{-3}e^{u}$

$\frac{1}{-3}e^{-x^3}$

$\Rightarrow \boxed{\frac{e^{-x^3}}{-3}+x^2+C}$

D.

d) $\int \frac{x}{\sqrt{x^2+1}}\,dx$ $\quad u = x^2+1$ $\quad \int \frac{1}{x} = \ln|x| + C$

$du = 2x\,dx$

$dx = \frac{du}{2x}$ $\quad \int \frac{1}{\sqrt{x}} = \int \frac{1}{x^{1/2}} \Rightarrow 2\sqrt{x} + C$

$x \int \frac{1}{\sqrt{u}} \frac{du}{2x} \Rightarrow \frac{\cancel{x}}{2\cancel{x}} \int \frac{1}{\sqrt{u}}\,du \Rightarrow \frac{1}{2} \int \frac{1}{\sqrt{u}}\,du$

$\frac{1}{a^b} \Rightarrow a^{-b}$

$\frac{1}{2}\, 2\sqrt{u} \Rightarrow \frac{1}{\cancel{2}} \cancel{2} \sqrt{x^2+1}$ $\quad \frac{1}{x^{1/2}} \Rightarrow x^{-1/2}$

$= \boxed{\sqrt{x^2+1} + C}$

$\frac{x^{-1/2+1}}{-1/2+1}$

$\frac{x^{1/2}}{1/2} \Rightarrow 2x^{1/2}$

4. a) Find $\int 2xe^{-2x}\,dx$ using Integration By Parts, then find $\int_0^\infty 2xe^{-2x}\,dx$

b) Find $\int 2x^2 e^{-2x}\,dx$

c) $\int \ln(x^2)\,dx$

A.

4. a) Find $\int 2xe^{-2x}\,dx$ using integration by parts. Then find $\int_0^\infty 2xe^{-2x}\,dx$

$\int 2xe^{-2x}\,dx$ $\qquad$ $\int u\,dv = uv - \int v\,du$

↓

$2\int xe^{-2x}\,dx$ $\qquad$ $u = x$, $u' = 1\,dx$ $\qquad$ $v = \frac{e^{-2x}}{-2}$, $v' = e^{-2x}\,dx$

Log
Inv Trig
Alg
Trig
Exp

$\int e^{-2x}$ $\qquad$ $u = -2x$, $du = -2\,dx$

↓

$\int e^{u}\,du$ $\qquad$ $dx = \frac{du}{-2}$

↓

$\frac{e^{-2x}}{-2}$

$\frac{xe^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}\cdot 1$ $\qquad$ $\int \frac{e^{-2x}}{-2} \to \frac{e^{-2x}}{4}$ $\qquad$ Part 1

$\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}$ $\qquad$ $\boxed{\int 2xe^{-2x}\,dx = -xe^{-2x} - \frac{e^{-2x}}{2} + C}$

$2\left(\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right) \to \boxed{-xe^{-2x} - \frac{e^{-2x}}{2} + C}$

Part 2

$\int_0^\infty 2xe^{-2x}\,dx$ ~~Using~~ work from above

$\lim_{R\to\infty}$ $-xe^{-2x} - \frac{e^{-2x}}{2}\Big|_0^R$ $\qquad$ $\boxed{\int_0^\infty 2xe^{-2x}\,dx = \frac{1}{2}}$

$\left(-Re^{-2R} - \frac{e^{-2R}}{2}\right) - \left(0 - \frac{1}{2}\right)$

$-\infty e^{-2\infty} - \frac{e^{-2\infty}}{2} + \frac{1}{2} \to 0 - \left(-\frac{1}{2}\right) \to \frac{1}{2}$

B.

b) Find $\int 2x^2 e^{-2x}\,dx$ $\quad \int u\,dv = uv - \int v\,du$

$$\int 2x^2 e^{-2x}\,dx$$

$$\downarrow$$

$$2\int x^2 e^{-2x} \qquad u = x^2 \qquad v = \frac{e^{-2x}}{-2}$$

$$u' = 2x\,dx \qquad v' = e^{-2x}\,dx$$

$$x^2\frac{e^{-2x}}{-2} - \int \frac{e^{-2x}}{-2}\, 2x\,dx$$

$$\downarrow$$

$$\int -e^{-2x} x\,dx \rightarrow \int -x e^{-2x}\,dx$$

$$u = -x \qquad v = \frac{e^{-2x}}{-2}$$

$$u' = -1 \qquad v' = e^{-2x}\,dx$$

$$+x\frac{e^{-2x}}{+2} - \int \frac{e^{-2x}}{+2} \times 1$$

$$\int \frac{e^{-2x}}{2} \rightarrow \frac{e^{-2x}}{-4}$$

$$2\left(x^2\frac{e^{-2x}}{-2} - \left(x\frac{e^{-2x}}{2} - \left(\frac{e^{-2x}}{-4}\right)\right)\right)$$

$$-x^2 e^{-2x} - \left(x e^{-2x} + \left(\frac{e^{-2x}}{+2}\right)\right)$$

$$-x^2 e^{-2x} - \left(x e^{-2x} + \frac{e^{-2x}}{2}\right)$$

$$= \boxed{-x^2 e^{-2x} - x e^{-2x} - \frac{e^{-2x}}{2} + C}$$

C.

c) $\int \ln(x^2)\,dx$ $\quad \int u\,dv = uv - \int v\,du$

Log
Inv Trig
Alg
Trig
Exp

$u = \ln(x^2)$ $\quad v = x$

$u' = \frac{2}{x}$ $\quad v' = 1$

$\log_a(x^b) = b \cdot \log_a(x)$ assuming $x \geq 0$

$\ln(x^2)x - \int \cancel{x}\frac{2}{\cancel{x}}$

$\int 2 \to 2x$

$x\ln(x^2) - 2x \to = \boxed{2x\ln(x) - 2x + C}$

5. a) Find $\sum_{i=0}^{\infty} 15\left(\frac{1}{3}\right)^i$

b) Find $\sum_{i=1}^{\infty} 3x^2(.2)^i$

c) Find $\sum_{i=6}^{\infty} 3x^2(.2)^i$

A.

a) Find $\sum_{i=0}^{\infty} 15\left(\frac{1}{3}\right)^i$ $\quad$ Geometric test

$|r| < 1$ $\to$ converges

$|r| \geq 1 \Rightarrow$ diverges

$15 \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^i$ $\quad r$

converges

$\frac{1}{1 - 0.33} = \frac{1}{0.66} = \frac{1}{\frac{2}{3}} \to \frac{3}{2}$

$15 \cdot \frac{3}{2}$

$\boxed{\frac{45}{2}}$

B.

b) Find $\sum_{i=1}^{\infty} 3x^2(0.2)^i$ Geometric test

$$3x^2 \sum_{i=1}^{\infty} (0.2)^i \quad i=1$$

$$\frac{1}{1-0.2} \Rightarrow \frac{1}{0.8} \Rightarrow \frac{1}{\frac{8}{10}} \cdot 10 \Rightarrow \frac{10}{8}$$

$$\boxed{\frac{15x^2}{4}} \leftarrow 3x^2\left(\frac{5}{4}\right) \qquad \frac{5}{4}$$

C.

① Find
$$\sum_{i=6} 3x^2(0.2)^i$$

$3x^2(0.2)^0 = 3x^2$

$3x^2(0.2)^1 = 3x^2(0.2)$

$3x^2(0.2)^2 = 3x^2(0.04)$

$3x^2(0.2)^3 = 3x^2(0.008)$

$3x^2(0.2)^4 = 3x^2(0.0016)$

$3x^2(0.2)^5 = 3x^2(0.00032)$

add them

$\frac{15x^2}{4} -$ $3x^2 + 0.6x^2 + 0.12x^2 + 0.024x^2$
$+ 0.0048x^2 + 0.00096x^2$

$\frac{15x^2}{4} - 3.74976x^2$

$(3.75 - 3.74976)x^2$

$= \boxed{0.00024x^2}$

0.00024

6. Evaluate the following double integrals:

a) $\int_0^1 \int_0^1 (x+y)\,dx\,dy$

b) $\int_1^3 \int_2^4 (x^2y + y + 2x)\,dy\,dx$

c) $f(x,y) = 2x^2 - 3y^2$
over $1 \le x \le 3,\ 0 \le y \le 1$

d) $f(x,y) = e^{x+2y}$
over $0 \le x \le 2,\ 1 \le y \le 2$

A.

a) $\int_0^1 \left(\int_0^1 (w+v)\, dw \right) dv$

$\int_0^1 (w+v)\, dw \rightarrow \left(\frac{w^2}{2} + wv \right) \Big|_0^1 \quad \left(\frac{1}{2} + 1 \cdot v \right) - 0$

$\int_0^1 \left(\frac{1}{2} + v \right) dv \rightarrow \frac{v}{2} + \frac{v^2}{2} \Big|_0^1$

$\left(\frac{1}{2} + \frac{1^2}{2} \right) - 0 = 1$

$\boxed{\int_0^1 \left(\int_0^1 (w+v)\, dw \right) dv = 1}$

B.

b) $$\int_1^3 \left(\int_2^4 (x^2y + y + 2x)\, dy \right) dx$$

$$\left(\frac{x^2y^2}{2} + \frac{y^2}{2} + 2xy \right) \Big|_2^4$$

$$\left(\frac{x^2 16}{2} + \frac{16}{2} + 2x4 \right) - \left(\frac{x^2 4}{2} + \frac{4}{2} + 2x2 \right)$$

$$(8x^2 + 8 + 8x) - (2x^2 + 2 + 4x)$$

$$8x^2 + 8 + 8x - 2x^2 - 2 - 4x$$

$$\int_1^3 (6x^2 + 6 + 4x)\, dx$$

$$\left(\frac{6x^3}{3} + 6x + \frac{4x^2}{2} \right) \Big|_1^3$$

$$(2x^3 + 6x + 2x^2) \Big|_1^3$$

$$(2(3)^3 + 6(3) + 2(3)^2) - (2(1)^3 + 6(1) + 2(1)^2)$$

$$(2(27) + 18 + 18) - (2 + 6 + 2)$$

$$(54 + 18 + 18) - (10)$$

$$(90) - (10) = 80$$

$$\boxed{\int_1^3 \left(\int_2^4 (x^2y + y + 2x)\, dy \right) dx = 80}$$

C.

c) $f(x,y) = 2x^2 - 3y^2$

over $1 \le x \le 3$, $0 \le y \le 1$

$f(x,y) = 2x^2 - 3y^2$

$$\int_0^1 \left(\int_1^3 (2x^2 - 3y^2)\,dx \right) dy$$

$$\left(\frac{2x^3}{3} - 3y^2 x \right) \Big|_1^3$$

$$\left(\frac{2(3)^3}{3} - 3y^2(3) \right) - \left(\frac{2(1)^3}{3} - 3(1)^2(1) \right)$$

$$\left(\frac{2(27)}{3} - 9y^2 \right) - \left(\frac{2}{3} - 3 \right) \qquad \frac{2}{3} - \frac{9}{3} = -\frac{7}{3}$$

$$(18 - 9y^2) - \left(\frac{2}{3} - 3 \right)$$

$$18 - 9y^2 + \frac{7}{3}$$

$$\int_0^1 (20.\bar{3} - 9y^2)\,dy$$

$$\left(20.\bar{3}y - \frac{9y^3}{3} \right) \Big|_0^1$$

$$\left(20.\bar{3}(1) - \frac{9(1)^3}{3} \right) - 0 = 20.\bar{3} - 3 = \boxed{17.\bar{3}}$$

D.

A) $f(x,y) = e^{x+2y}$ over $0 \le x \le 2,\ 1 \le y \le 2$

$f(x,y) = e^{x+2y}$

$$\int_1^2 \left(\int_0^2 (e^{x+2y})\, dx \right) dy$$

e^{2+2y} $u = 2+2y$
$du = 2dy$
$dy = \frac{du}{2}$

$e^{2y} \int e^{x}\, dx$

$\int e^u \frac{du}{2}$

$e^{2y} e^{x} \to e^{2y+x}$ or e^{x+2y}

$\frac{e^{2+2y}}{2} \Big|_1^2$

$e^{x+2y} \Big|_0^2$

$\frac{e^{2+4}}{2} - \frac{e^{2+2}}{2}$ $\frac{e^6}{2} - \frac{e^4}{2}$

$(e^{2+2y}) - (e^{0+2y})$

$$\int_1^2 (e^{2+2y} - e^{2y})\, dy \to \int_1^2 (e^{2+2y})\, dy - \int_1^2 e^{2y}\, dy$$

$\frac{e^{2y}}{2} \Big|_1^2$

or

e^{2+2y}

e^{2y}

$\frac{e^4 - e^2}{2}$

$$\int_1^2 (e^{2y}(e^2 - 1))\, dy$$

$u = 2y$
$du = 2\, dy$
$dy = \frac{du}{2}$

$\frac{e^6 - e^4}{2} - \frac{e^4 - e^2}{2}$

$$= \boxed{\frac{e^6 + 2e^4 + e^2}{2}}$$