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CSC 28 - Section 01

1.1) From the given ordered pairs (3, 5); (8, 4); (6, 6); (9, 11); (6, 3); (3, 0); (1, 2) find the following relations. Also, find the domain and range of each relation.

(a) Is greater than (2 points)

R is >

R = {(8,4), (6,3), (3,0)}

Domain: {3,6,8}

Range: {0,3,4}

(b) Is less than (2 points)

R is <

R = {(3,5), (9,11), (1,2)}

Domain: {1,3,9}

Range: {2,5,11}

(c) Is less than, by at least two (2 points)

R is < (*By at least two this time)

R = {(3,5), (9,11)}

Domain: {3,9}

Range: {5,11}

(d) Is equal to (2 points)

R is =

R = {(6,6)}

Domain: {6}

Range: {6}

1.2)

(a) Determine the domain and range of the relation R defined by

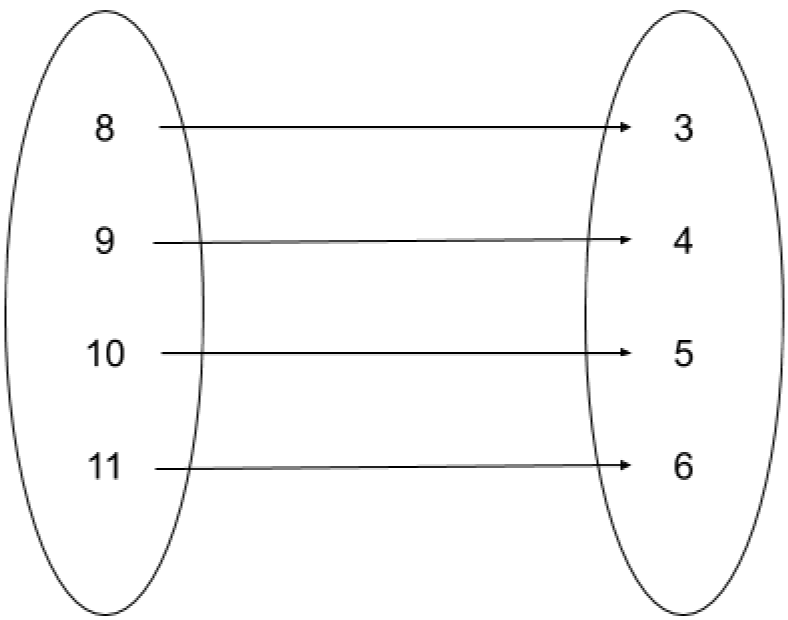
R = {x + 2, x - 3}; x ∈ {6, 7, 8, 9} (domain and range: 4 points) and

Domain: {8,9,10,11}

Range: {3,4,5,6}

(b) draw the arrow diagram for relation R. (4 points)

R = {(8,3),(9,4),(10,5),(11,6)}



(c) What is the range of the relation R if the co-domain includes additional values 7, 12? (1 points)

Before:

{3,4,5,6}

After:

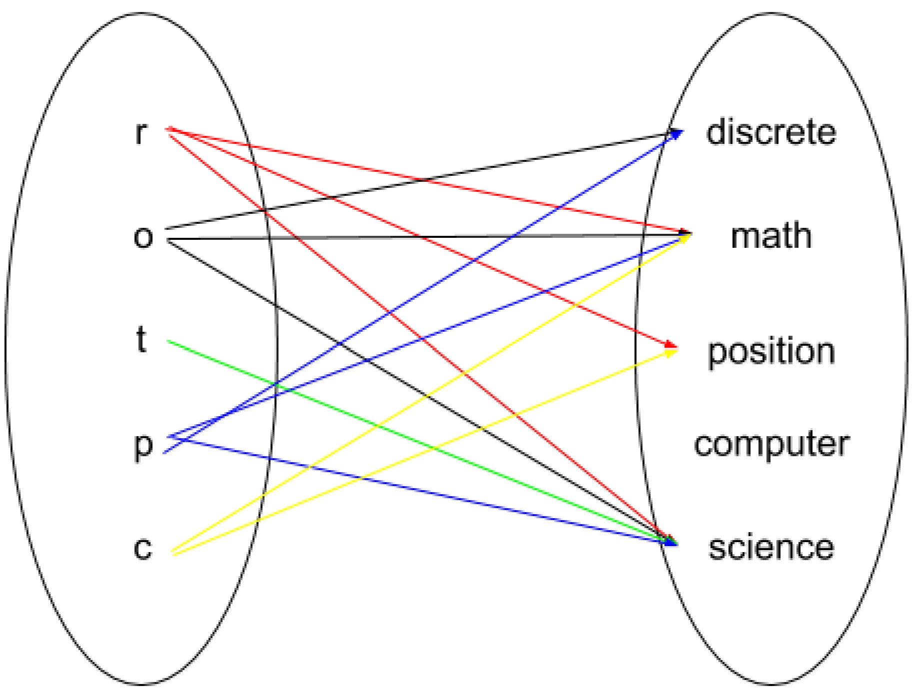
{3,4,5,6,7,12}

So, range is {3,4,5,6,7,12}

2) Draw the arrow diagram and the matrix representation for the following two relations:

(a) Define the set A = {r, o, t, p, c} and B = {discrete, math, position, computer, science}. Define the relation R ⊆ A × B such that (letter, word) is in the relation if that letter doesn't occur somewhere in the word. Be careful - recheck your answer again to see if you by mistake reversed the definition for any pair (if you mistakenly used 'does occur' for 'doesn't occur'). Even if you know the answer, psychological distraction can play a role. It can happen to me too!

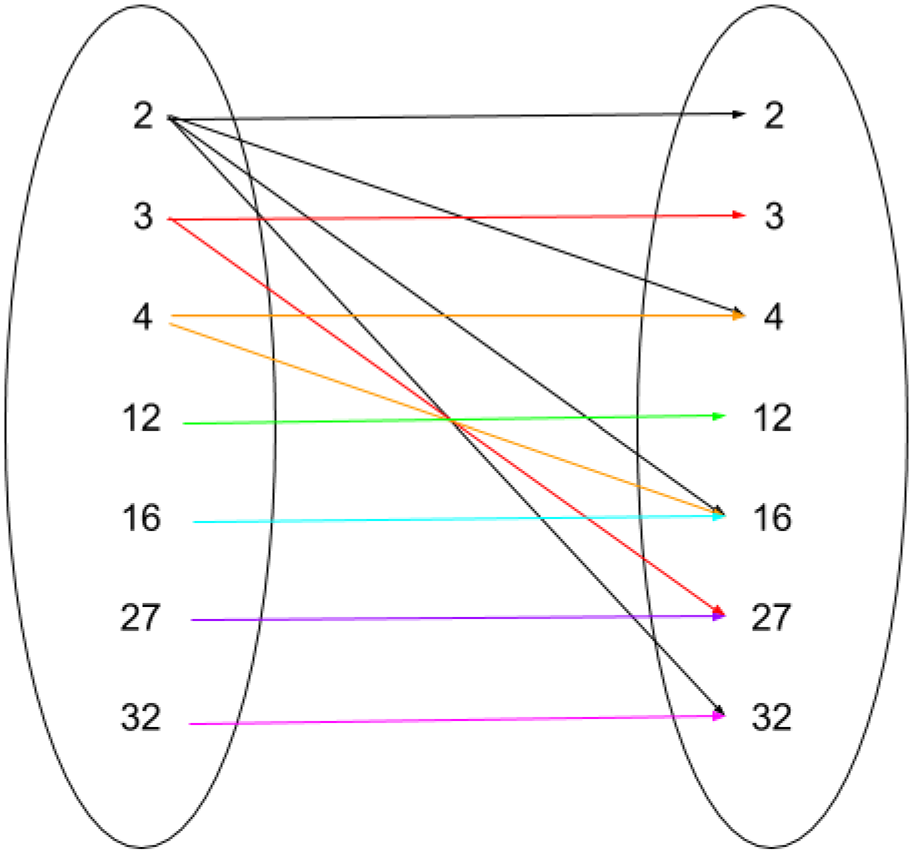
(Arrow Diagram 5 points + Matrix representation 5 points)



	discrete	math	position	computer	science
r	0	1	1	0	1
o	1	1	0	0	1
t	0	0	0	0	1
p	1	1	0	0	1
c	0	1	1	0	0

(b) The domain for the relation M is the set {2, 3, 4, 12, 16, 27, 32, 48}. For x, y in the domain, xMy if there is a positive integer n such that x power n = y. Be careful - include only (x, y) pairs (not (x, n) or n, y) pairs). Even if you know the answer, psychological distraction can play a role. It can happen to me too!

(Arrow Diagram 5 points + Matrix representation 5 points)



	2	3	4	12	16	27	32	48
2	1	0	1	0	1	0	1	0
3	0	1	0	0	0	1	0	0
4	0	0	1	0	1	0	0	0
12	0	0	0	1	0	0	0	0
16	0	0	0	0	1	0	0	0
27	0	0	0	0	0	1	0	0
32	0	0	0	0	0	0	1	0
48	0	0	0	0	0	0	0	1

3) Give the arrow diagram for each of the two matrices, then express the relation as a set of ordered pairs (The rows and columns are numbered 1 through 3 or 4. In other words, the domain and codomain contain the row and column numbers.)

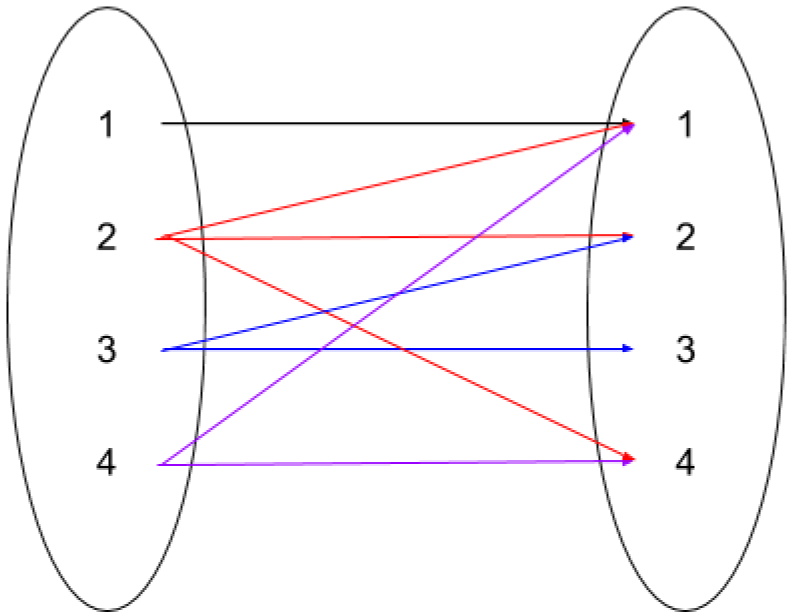
(a)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

OR

	1	2	3	4
1	1	0	0	0
2	1	1	0	1
3	0	1	1	0
4	1	0	0	1

Arrow Diagram (8 points) + Relation as ordered pairs (8 points) + correct domain and co-domain (1 point)



R = {(1,1), (2,1), (2,2), (2,4), (3,2), (3,3), (4,1), (4,4)}

Domain: {1,2,3,4}

Range (Co-domain): {1,2,3,4}

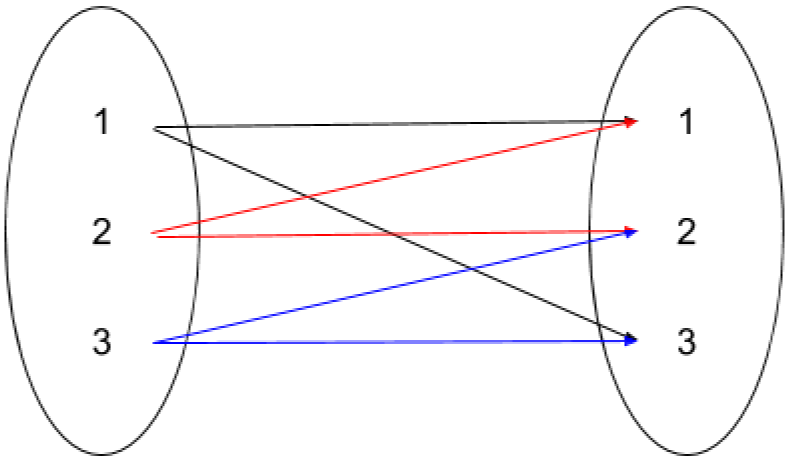
(b)

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

OR

	1	2	3
1	1	0	1
2	1	1	0
3	0	1	1

Arrow Diagram (6 points) + Relation as ordered pairs (6 points) + correct domain and co-domain (1 point)



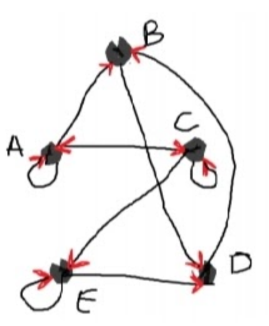
R = {(1,1), (1,3), (2,1), (2,2), (3,2), (3,3)}

Domain: {1,2,3}

Range (Co-domain): {1,2,3}

4) Give the matrix representation for the relation depicted in each arrow diagram. Then express the relation as a set of ordered pairs. Assume the domain and co-domain for each graph contain the letters in the corresponding graph.

(a)

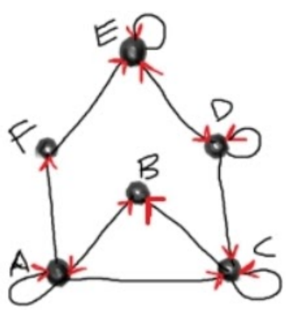


(matrix representation 10 points) + (relation as ordered pairs 10 points)

	A	B	C	D	E
A	1	1	1	0	0
B	0	0	0	1	0
C	1	0	1	0	1
D	0	1	0	0	0
E	0	0	0	1	1

R = {(A,A), (A,B), (A,C), (B,D), (C,A), (C, C), (C, E), (D, B), (E,D), (E,E)}

(b)



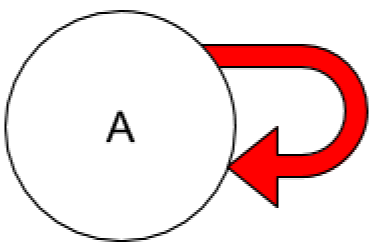
(matrix representation 10 points) + (relation as ordered pairs 10 points)

	A	B	C	D	E	F
A	1	1	1	0	0	1
B	1	0	0	0	0	0
C	0	1	1	0	0	0
D	0	0	1	1	1	0
E	0	0	0	1	1	0
F	0	0	0	0	1	0

R = {(A,A), (A,B), (A,C), (A,F), (B,A), (C,B), (C,C), (D,C), (D,D), (D,E), (E,D), (E,E), (F,E)}

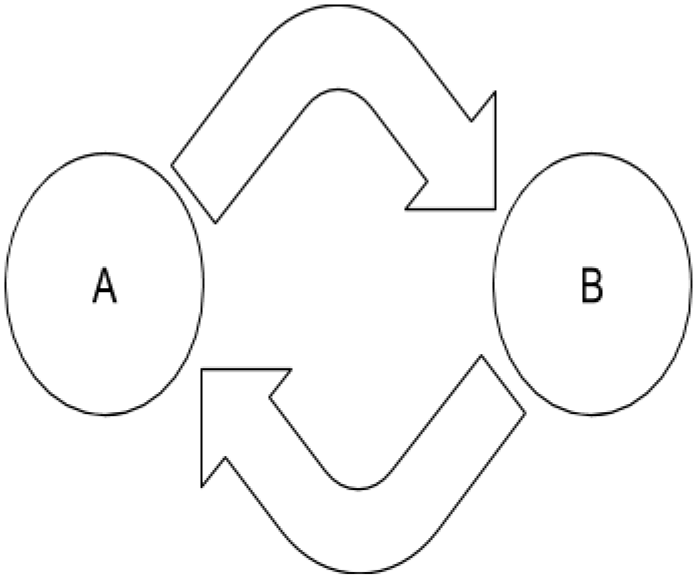
5) What are the types of Relations, Explain each with one example on your own. (3 * 2 points)

Reflexive - A relation R on set B is reflexive if aRa for every a that is an element of B. That is, (a,a) is an element of R for every a that is an element of set B. Thus, a relation R is not reflexive if there exists an a that is an element of B such that (a, a) is not an element of R. For a certain relation to be reflexive, you can think of an element as being equal to itself or referring to itself, as shown in the image below.



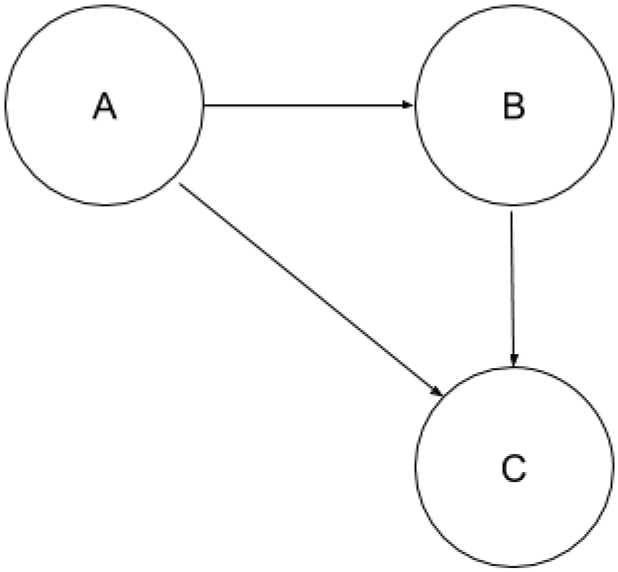
For example, assuming we had the set A ={x,y,z}, if we had a relation R such that (x,x) is an element of R, (y,y) is an element of R, and (z,z) is an element of R, then you can say that the relation is reflexive, regardless of what other elements there are (like (x,y) and (y,z)). If one of those three elements mentioned above is missing, then the relation is no longer reflexive.

Symmetric - A relation R on a set A is symmetric if whenever aRb then bRa, that is, if whenever (a, b) is an element of R then (b, a) must also be an element of R. For a certain relation to be symmetric, you can think of two values referring to each other simultaneously, as shown in the image below.



For example, for the set A = {1,2,3,4}, if there is a relation such that R = {(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)}, then that relation would be symmetric. If either (1,2) or (2,1) were missing, then that relation would no longer be symmetric. All elements that are reflexive (like (1,1) or (3,3)) are considered to be symmetric.

Transitive - A relation R on a set A is transitive if whenever aRb and bRc then aRc, that is, if whenever (a, b), (b, c) are elements of R then (a, c) is an element of R. Thus R is not transitive if there exists a, b, c that are elements of R such that (a, b) and (b, c) are elements of R but (a, c) is not an element of R. For a certain relation to be transitive, you can think of a case where if item a is related to item b and item b is related to item c, then it must be the case that item a is related to item c, as shown in the image below.



For example, for the set C = {1,2,3,4}, if there is a relation such that R = {(1,2), (2,3), (1,3)}, then that relation would be transitive as there is a path from (1,2) to (2,3) that implies (1,3). If element (1,3) were missing or if there was no path that implies (1,3), then that relation would no longer be transitive.

6) True/ False (To get full marks explain the answer in one or two sentences) (5 * 1 points)

(a) If R is the set of all females in a family, then the relation "is sister of" is reflexive over R.

False. Sisterhood is a relationship between two females, not a relationship with one's self. Monique may be a sister of Hannah, but Monique cannot be a sister of herself.

(b) If R is the set of all females in a family, then the relation "is sister of" is not symmetric over R.

False. As stated previously, sisterhood is a relationship between two females. If Monique is a sister of Hannah, then it follows that Hannah is a sister of Monique as well. This is a symmetric relation by default.

(c) If R is the set of mothers and S is the set of children in a family then a relation F on RxS is a symmetric relation.

False. If you cross the sets of mothers with the sets of children in a family and think about it, to say that Mary is a mother of Dolan, it would need to follow that Dolan is a mother of Mary as well.

(d) “Is the same height as” is a reflexive relation.

False. If you state that one person is the same height as someone else, the relationship could not be reflexive as it is between two people.

(e) “Is descendent of” is a transitive relation.

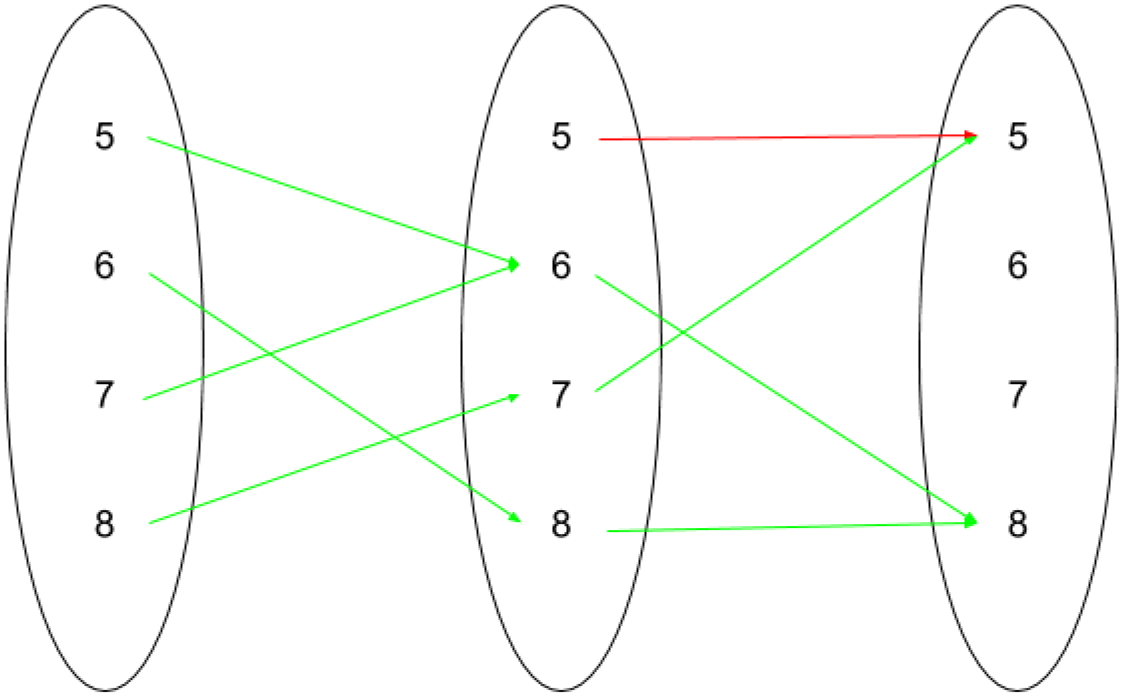
True. If John is a descendent of Herald and Herald is a descendent of Macbeth, then it would follow that John is a descendent of Macbeth.

7) Let A= {5, 6, 7, 8}, Let R: A-> A and S: A-> A. Let R= {(5, 6) (6, 8) (7, 6) (8, 7)} and S= {(5, 5) (6, 8) (7, 5) (8, 8)}. Find the answers for the following and draw the arrow diagram for each result.

(a) R 0 S (5 points)

RoS = {(5,8), (6,8), (7,8), (8,5)}

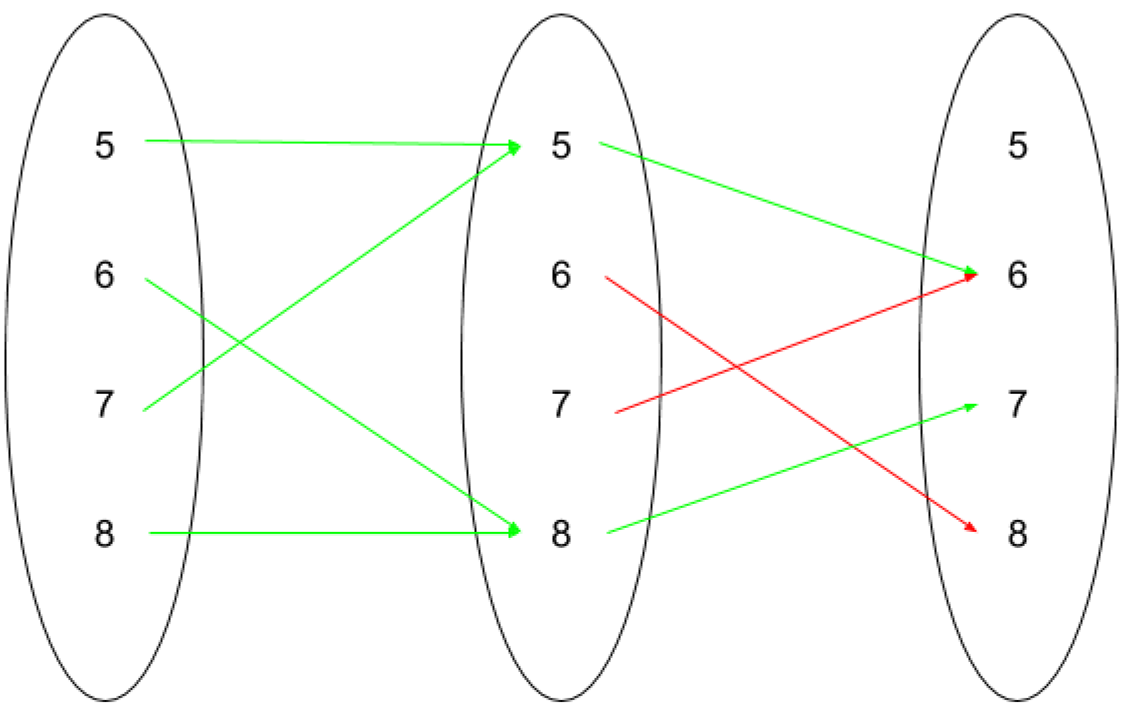
*Red arrows do not contribute to RoS, but green arrows do.



(b) S 0 R (5 points)

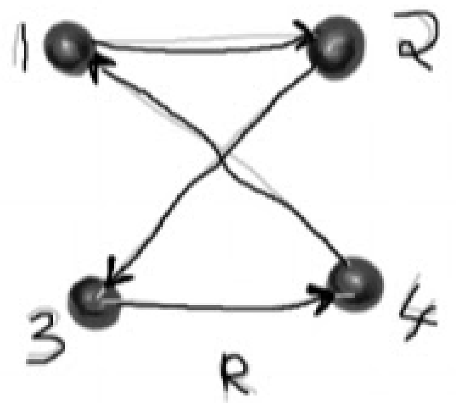
SoR = {(5,6), (6,7), (7,6), (8,7)}

*Red arrows do not contribute to SoR, but green arrows do.



8)Below is the arrow diagram (graph) for relation R with the domain {1, 2, 3, 4}. Define a new relation A to be R o R

(a) Express relation A as a set of related pairs. (5 points)

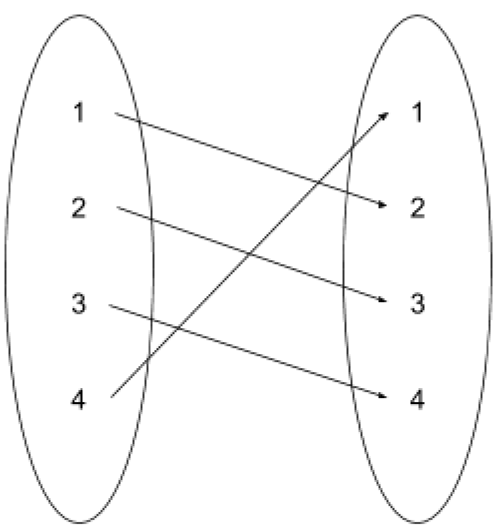


Seeing how A is a composition of RoR, we get:

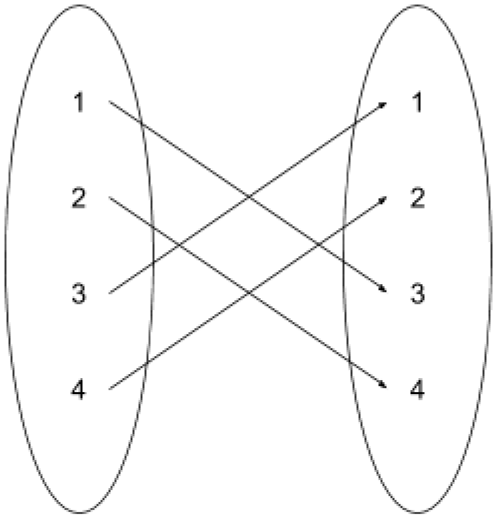
A = {(1,3), (2,4), (3,1), (4,2)}

(b)Draw the arrow diagram for R o A. (5 points)

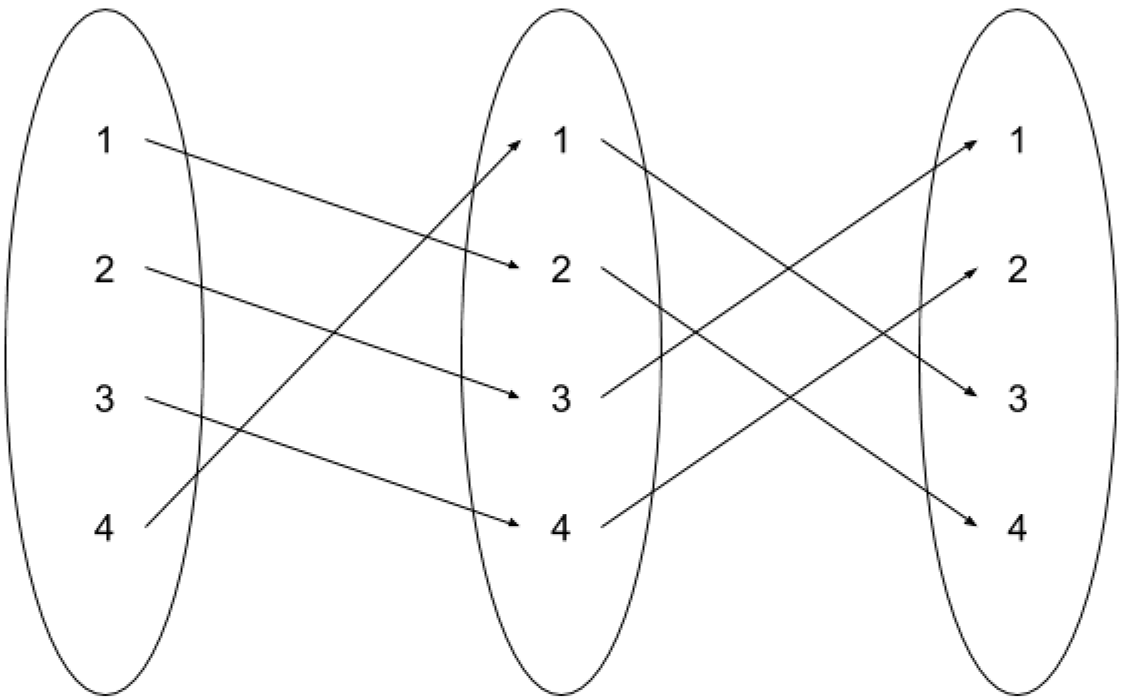
Assuming that we get the following arrow diagram for R:



And that we get the following for A:



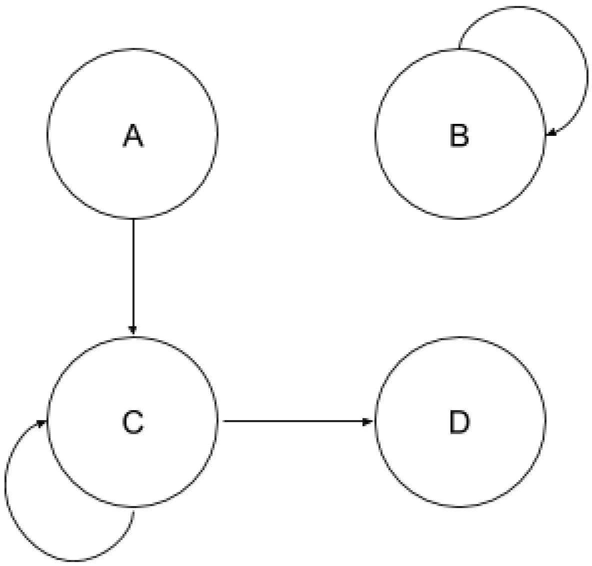
Then we should get the following arrow diagram for RoA:



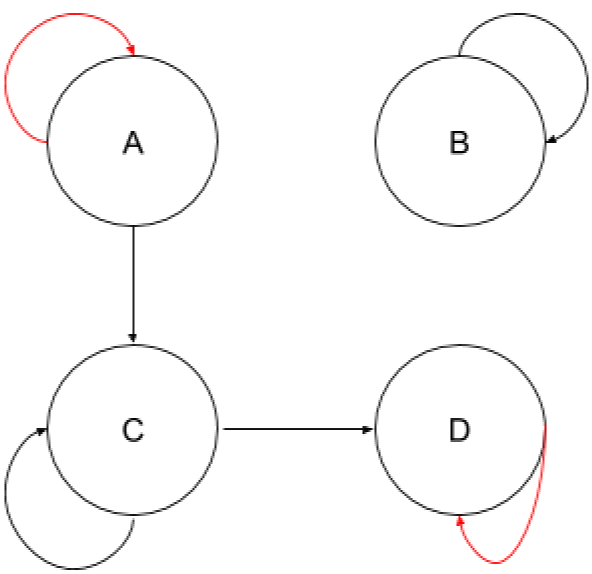
9) What are the three types of closures in relation and explain with an example. (3 * 2 points)

Reflexive closure: A reflexive closure is a type of closure done on a relation of a set to make sure that it is reflexive. For example, let's say we had the set {a,b,c,d} and a relation R = {(a,c), (b,b), (c,c), (c,d)}. The relation is not reflexive because it is missing related pairs where an element is reflexive upon itself. To fix this, a reflexive closure would be needed. For a reflexive closure, we would simply add the pairs (a,a) and (d,d) to make the whole relation reflexive. To help visualize this, you can look at the digraphs below.

Below is a digraph before reflexive closure:

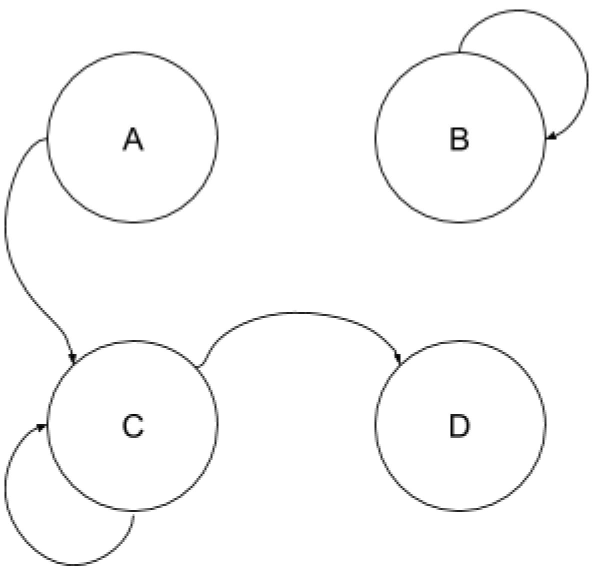


Below is a digraph after reflexive closure:

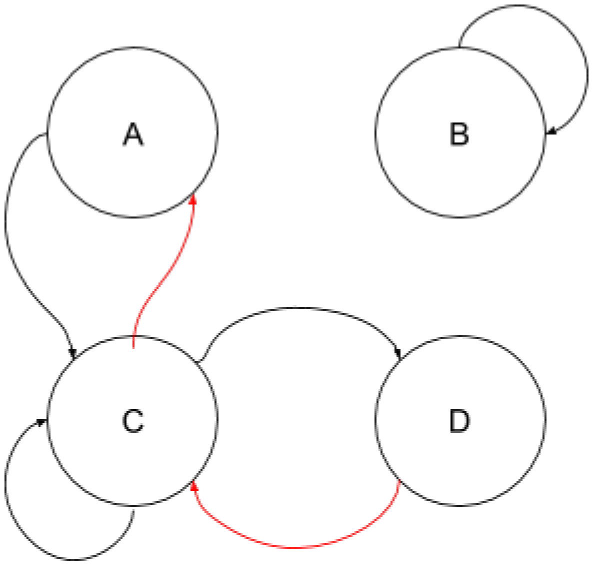


Symmetric closure: A symmetric closure is a type of closure done on a relation of a set to make sure that it is symmetric. For example, let's say we had the set {a,b,c,d} and a relation R = {(a,c), (b,b), (c,c), (c,d)}. The relation is not symmetric because it is missing related pairs where if an element is related to another in a certain order, then the inverse order connection would also be true. To fix this, a symmetric closure would be needed. For a symmetric closure, we would simply add the pairs (c,a) and (d,c) to make the whole relation symmetric. To help visualize this, you can look at the digraphs below.

Below is a digraph before symmetric closure:

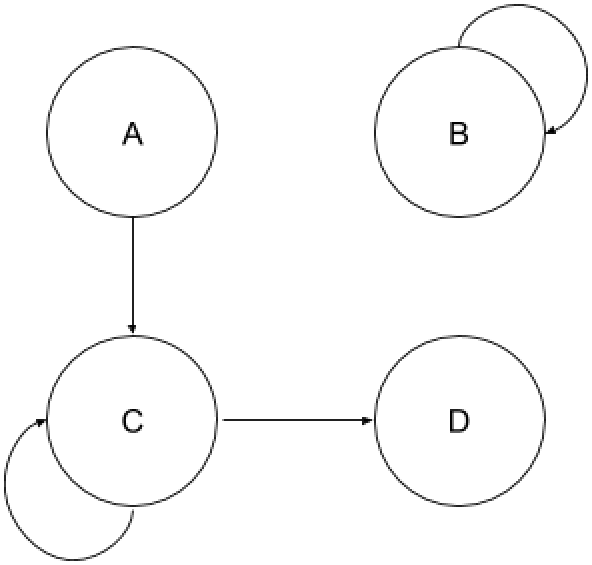


Below is a digraph after symmetric closure:

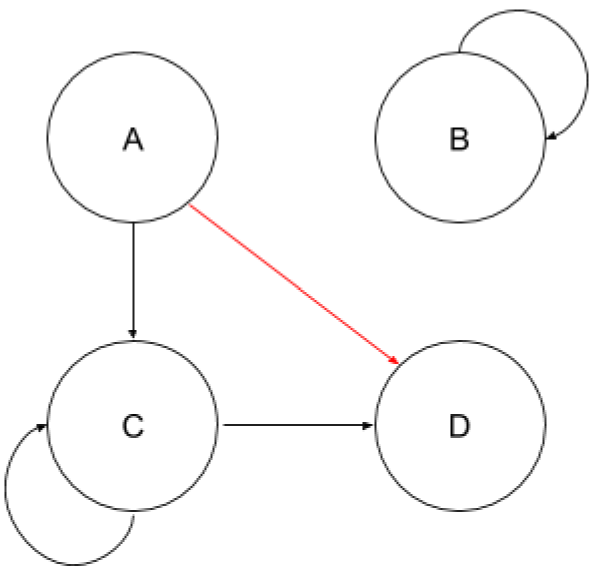


Transitive closure: A transitive closure is a type of closure done on a relation of a set to make sure that it is transitive. For example, let's say we had the set {a,b,c,d} and a relation R = {(a,c), (b,b), (c,c), (c,d)}. The relation is not transitive because it is missing related pairs where if a primary element is related to a secondary element and if a secondary element is related to a tertiary element, then it would follow that the primary element is related to the tertiary element. To fix this, a transitive closure would be needed. For a transitive closure, we would simply add the pair (a,d) to make the whole relation transitive. To help visualize this, you can look at the digraphs below.

Below is a digraph before transitive closure:



Below is a digraph after transitive closure:



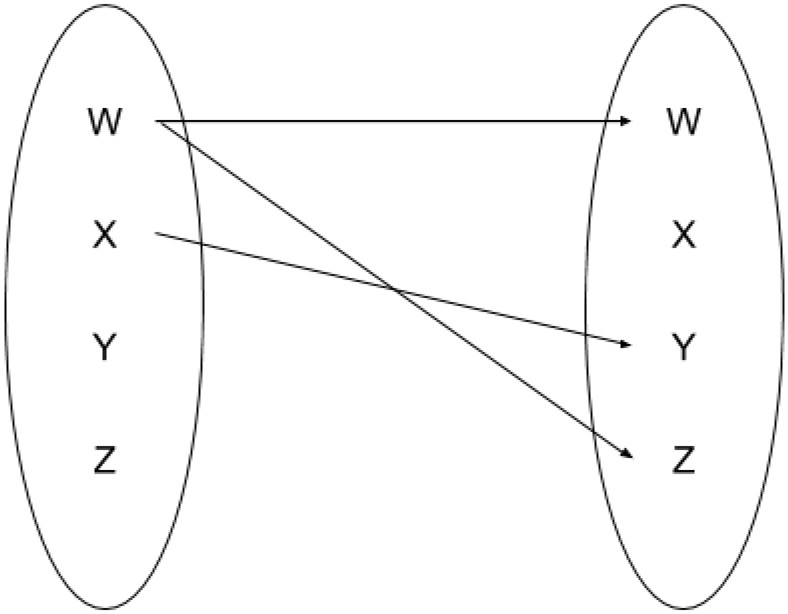
10) Given R= {w, x, y, z} and relation A= {(w, w) (x, y) (w, z)}.

(a) Is relation A reflexive closure? If not, what are the sets that should be added to make it a proper reflexive closure? (2 points)

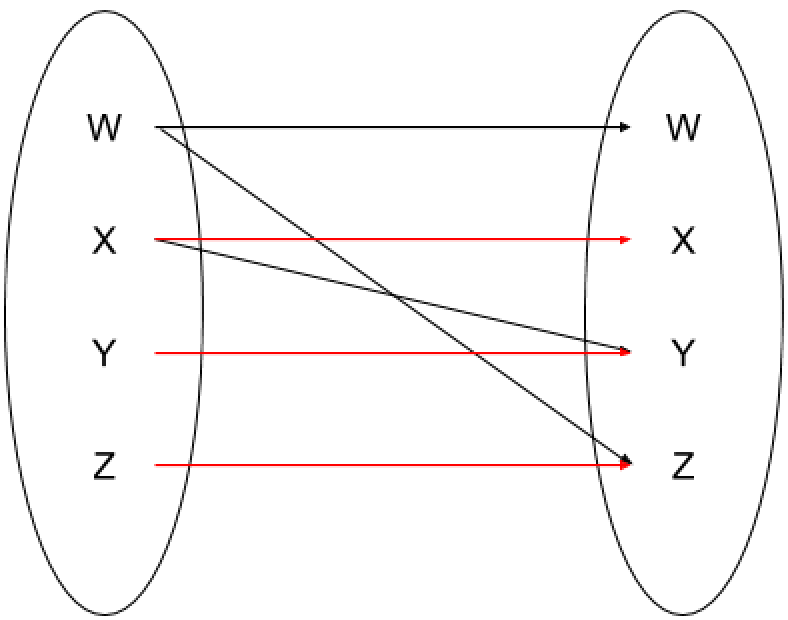
Explain in detail and then draw an arrow diagram for the result.

Relation A is not a reflexive closure because it is missing sets where elements are reflexive upon themselves. The sets that should be added are: (x,x), (y,y), and (z,z).

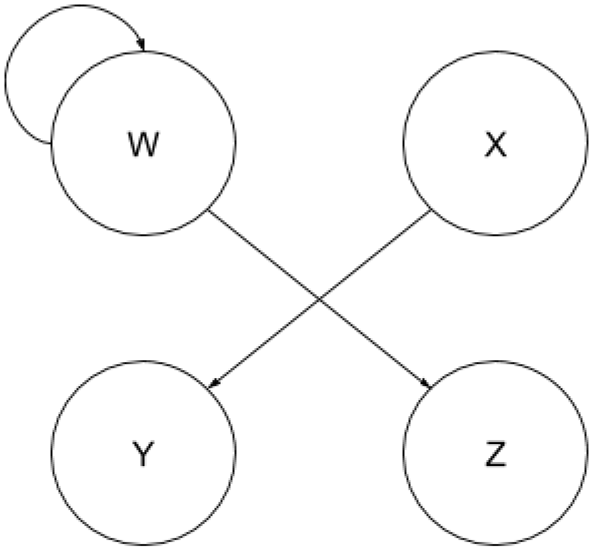
Arrow diagram before reflexive closure:



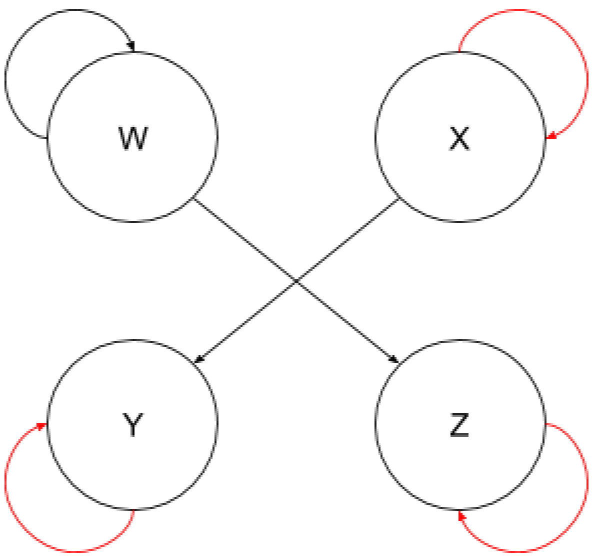
Arrow diagram after reflexive closure:



Digraph before reflexive closure:



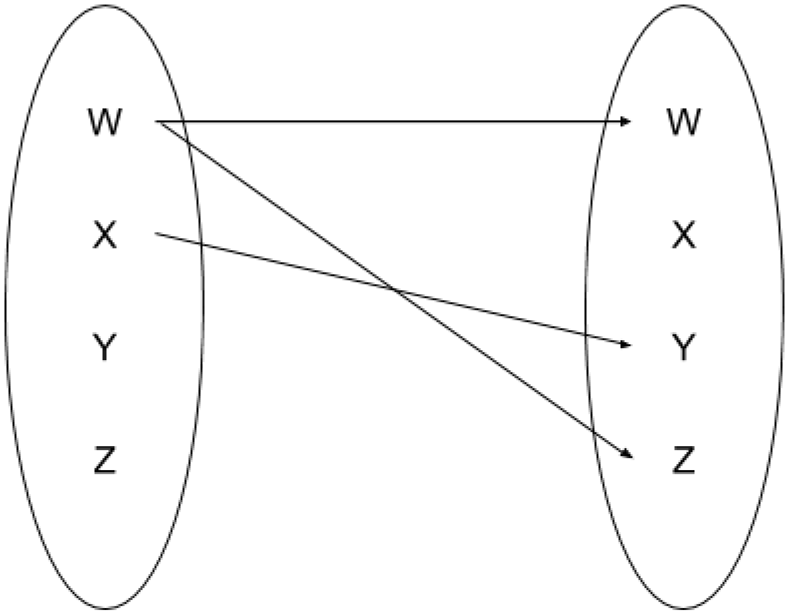
Digraph after reflexive closure:



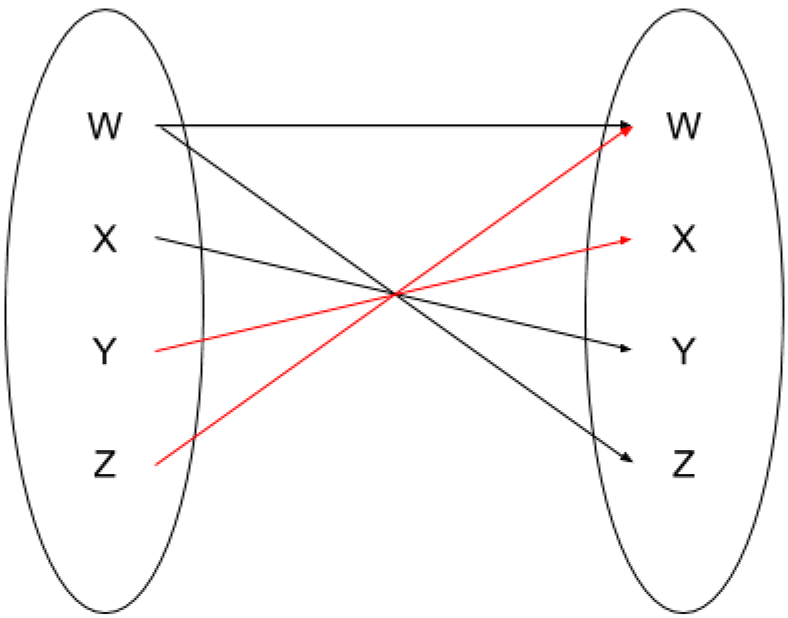
(b) Is relation A symmetric closure? if not what are the sets should be added to make it a proper symmetric closure? Explain in detail and then draw an arrow diagram for the result. (2 points)

Relation A is not a symmetric closure because it is missing sets where elements are symmetric to each other. The sets that should be added are: (y,x) and (z,w).

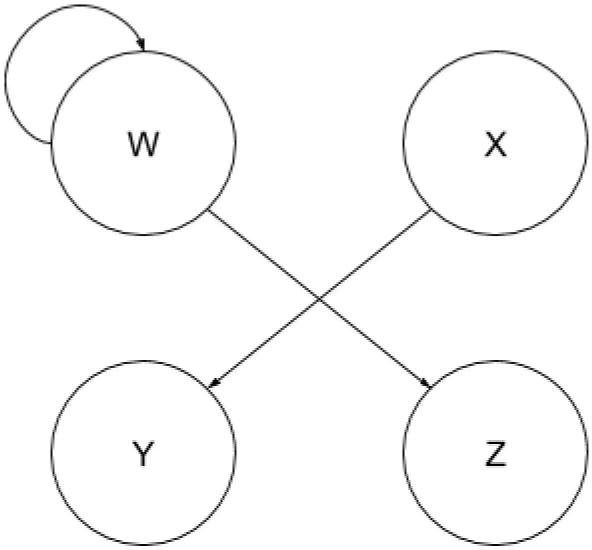
Arrow diagram before symmetric closure:



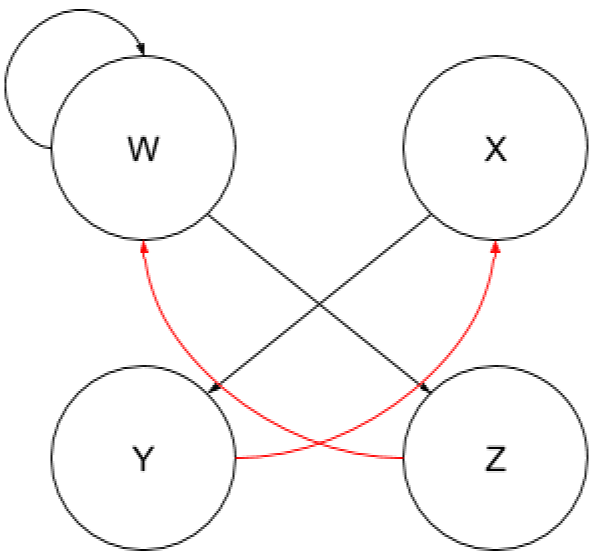
Arrow diagram after symmetric closure:



Digraph before symmetric closure:



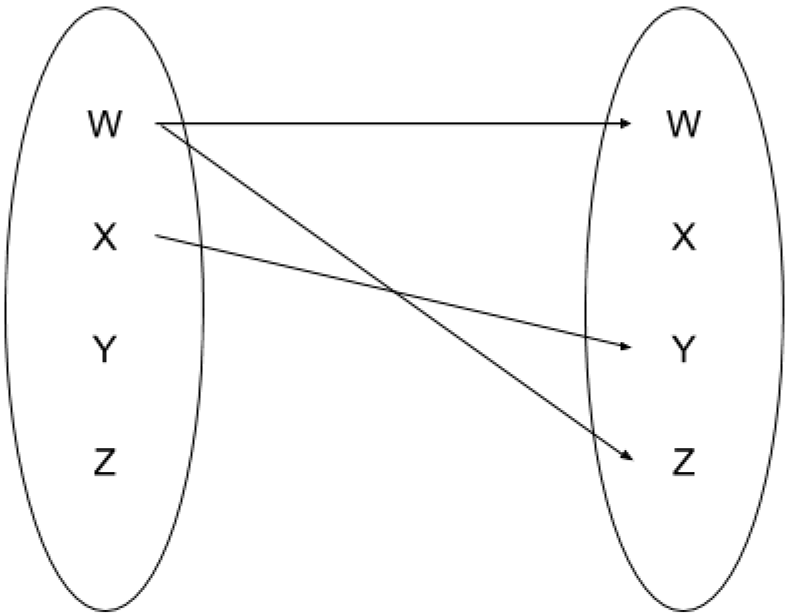
Digraph after symmetric closure:



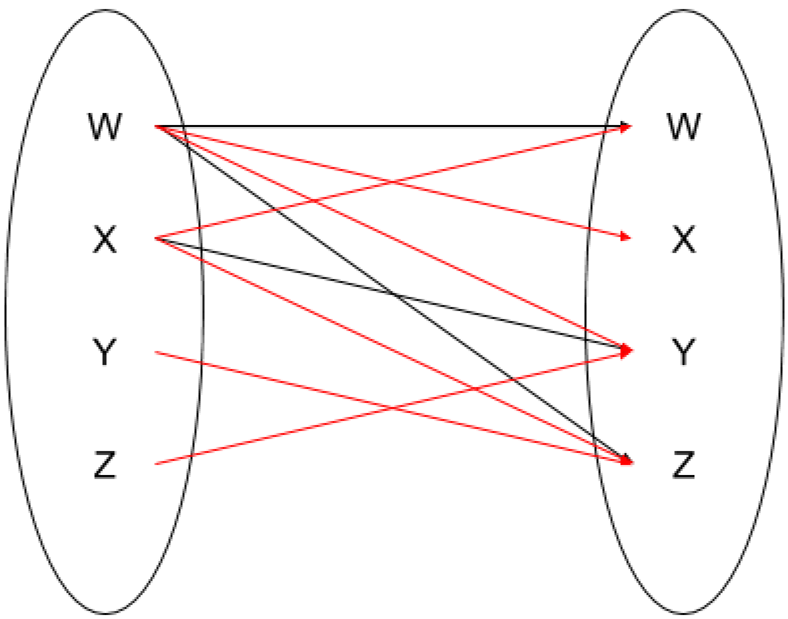
(c) Is relation A transitive closure? If not, what are the sets that should be added to make it a proper transitive closure? Explain in detail and then draw an arrow diagram for the result. (2 points)

Relation A is not a transitive closure because it is missing sets where elements that are transitive to each other have other longer paths that imply their existence. The sets that should be added are: (w,y), (y,z), (x,z), (z,y), (w,x), (x,w).

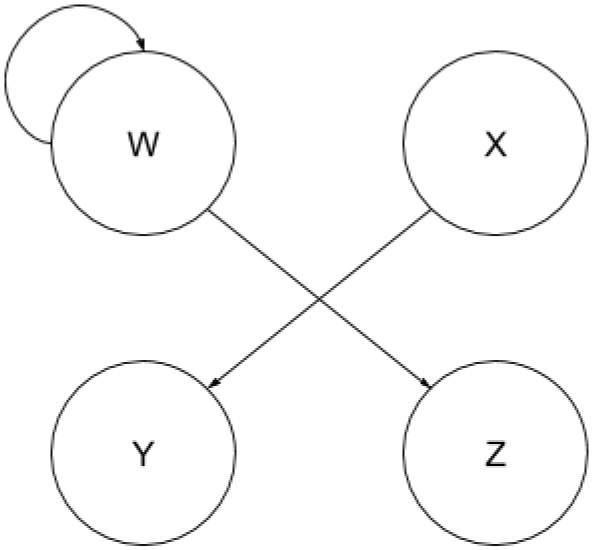
Arrow diagram before transitive closure:



Arrow diagram after transitive closure:



Digraph before transitive closure:



Digraph after transitive closure: