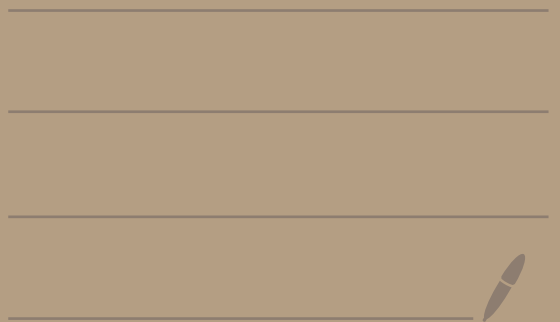


Math 30, Monday, May 4, 2020

1pm class



Your Final Exam is on Monday, May 11.

It will be available at 6am

and is due by 11:59pm

→ should take you \approx 2 hours.

Questions?

Final Exam Review Prof.s are on

Canvas now

(I sent an email yesterday)

Today: The last new topic: "Substitution".

It's just applying the Chain Rule to integrals...

Two types of integrals:

① indefinite integrals:

$$\int x^2 dx = \frac{1}{3} x^3 + C$$

stands for "all antiderivs. of x^2 "

② definite integrals: for these we have
The F.T. of Calc, Part II.

$$\int_a^b f'(x) dx = f(b) - f(a).$$

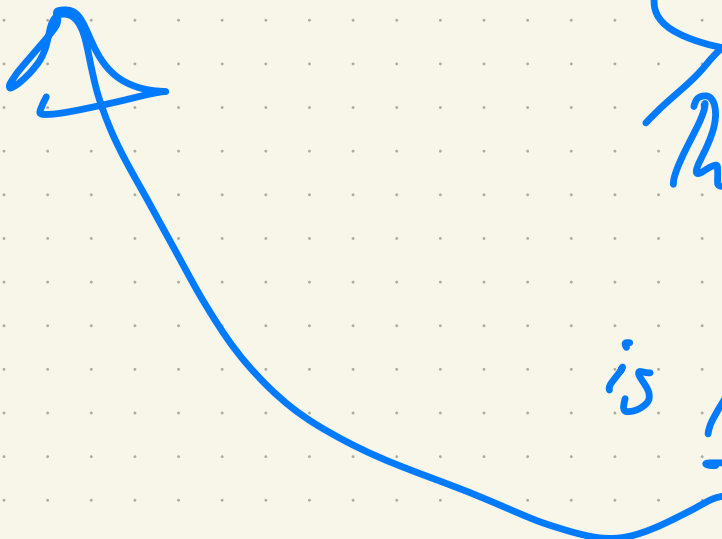
Combine with the Chain Rule:

$$\frac{d}{dx} F(g(x)) = F'(g(x))g'(x)$$

① For indefinite integrals:

$$\int F'(g(x))g'(x)dx = F(g(x)) + C$$

The derivative
of this
is this



② for definite integrals:

$$\int_a^b F'(g(x))g'(x)dx = F(g(b)) - F(g(a))$$

This is called "substitution"

That is,

$$\int_a^b F'(g(x))g'(x) dx$$

Chain Rule ↓

$$= \int_a^b \frac{d}{dx} F(g(x)) dx$$

F.T. of Calc. ↓
Part II

$$= F(g(b)) - F(g(a))$$

Here's why we call it "substitution"
(some say "u-substitution")
for an indefinite integral:

$$\int F'(g(x))g'(x)dx = ?$$

let $u = g(x)$. Think of it
as a new variable.

$$\frac{du}{dx} = g'(x)$$

rewrite it as: $du = g'(x)dx$

"substitute"

$$\begin{aligned} &= \int F'(u) du = F(u) + C \\ &= F(g(x)) + C \end{aligned}$$

Write it like this: (you'll do this a lot)
in Calc. II.

Example. $\int x \cos(x^2) dx = ?$

Try to choose u so that it looks like
a composition / Chain Rule.

Try $u = x^2$) a good guess comes from experience
usually let u be the "inside function"
"move dx over"

$$\frac{du}{dx} = 2x \quad \text{rewrite as} \quad du = 2x dx$$

"divide by 2"
rewrite as: $\frac{1}{2} du = x dx$

Now "substitute": write in terms of u , not x :

$$\int x \cos(x^2) dx = \int \cos u \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \cos u du \quad \text{) easier integral!}$$

Same example but faster:

Ex. $\int x \cos(x^2) dx = ?$

like in Chain Rule

Good guess: let $u = x^2$ (The "inside function")

$$\frac{du}{dx} = 2x, \quad du = 2x dx$$

$$\frac{1}{2} du = x dx$$

So

$$\int x \cos(x^2) dx = \int \cos u \cdot \frac{1}{2} du$$

$$= \frac{1}{2} \int \cos u du \quad \text{easy!}$$

$$= \frac{1}{2} \sin u + C$$

last step:
write in terms
of the original variable

$$\boxed{= \frac{1}{2} \sin(x^2) + C}$$

Final answer:

$$\int x \cos(x^2) dx = \frac{1}{2} \sin(x^2) + C.$$

The deriv. of
this is that

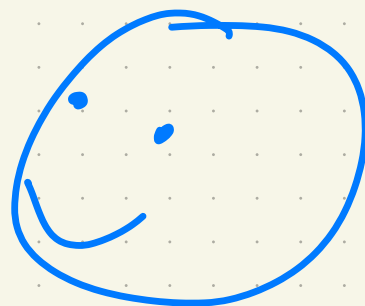
Important: we can check our answer!

$$\frac{d}{dx} \left(\frac{1}{2} \sin(x^2) + C \right)$$

↓ Chain Rule (duh!)

$$= \frac{1}{2} \cos(x^2) \cdot 2x + 0$$

$$= x \cos(x^2)$$



we must be right!

$$\frac{d}{dx} \sin(x^2) = \frac{d}{dx} F(g(x))$$

& use Chain Rule

$$\text{where } F(u) = \sin u \\ g(x) = x^2$$

Ex. $\int \sin^4 x \cos x \, dx = ?$

Try $u = \sin x$ (so $u^4 = \sin^4 x$)

Then $\frac{du}{dx} = \cos x$ so $du = \cos x \, dx$

Now substitute: write the integral
in terms of u only
(no x 's)

$$\int \sin^4 x \cos x \, dx = \int u^4 \, du \rightarrow \text{easy integral!}$$

$$= \frac{1}{5} u^5 + C$$

$$= \frac{1}{5} \sin^5 x + C$$

That is,

$$\int \sin^4 x \cos x \, dx = \frac{1}{5} \sin^5 x + C$$

indefinite integral: "all antiderivatives of $\sin^4 x \cos x$ "

check your answer by differentiating:

$$\frac{d}{dx} \left(\frac{1}{5} \sin^5 x + C \right)$$

$$= \frac{d}{dx} \left(\frac{1}{5} (\sin x)^5 + C \right)$$

Chain Rule (duh!)

$$= \sin^4 x \cdot \cos x + 0$$

Ex. $\int \frac{x+4}{x^2+1} dx = ?$

not so easy to see what u should be...

Trick: split into two integrals.

$$\dots = \int \frac{x}{x^2+1} dx + \int \frac{4}{x^2+1} dx$$

Try: let $u = x^2 + 1$

$$du = 2x dx$$

$$\frac{1}{2} du = x dx$$

$$= \int \frac{1}{u} \cdot \frac{1}{2} du + 4 \int \frac{1}{x^2+1} dx$$

$$\dots = \int \frac{1}{u} \cdot \frac{1}{2} du + 4 \int \frac{1}{x^2+1} dx$$

$$= \frac{1}{2} \int \frac{1}{u} du + 4 \int \frac{1}{x^2+1} dx$$

easier!

recall: $\frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$

$$= \frac{1}{2} \ln|u| + 4 \tan^{-1} x + C$$

last step: write in terms
of the original variable
 $u = x^2 + 1$

$$= \frac{1}{2} \ln(x^2+1) + 4 \tan^{-1} x + C$$

check by differentiating!

Ex. $\int \tan x \, dx = ?$

not easy to find $u \dots$

rewrite it:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

Now try $u = \cos x$

$$du = -\sin x \, dx$$

$$(-1) du = \sin x \, dx$$

$$= \int \frac{1}{u} (-1) du \quad \text{easy integral!}$$

check by differentiating...

$$= -\ln|u| + C = -\ln|\cos x| + C$$

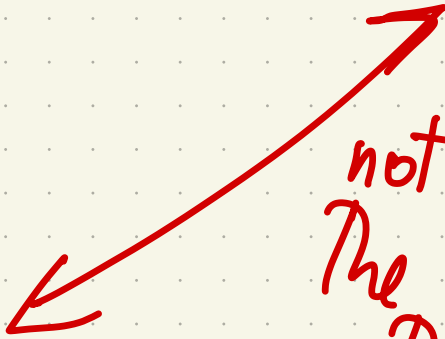
How not to do it:

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx = \int \sin x \cdot \frac{1}{\cos x} \, dx$$

Try $u = \sin x$

$$du = \cos x \, dx$$

not
The same
thing!



You might not choose the right u
on the first try...

Next time: substitution w/
definite integrals

See you on Wednesday!