MATH 30, 4/13/2020: APPLIED OPTIMIZATION PROBLEMS

- Read and understand the problem.
- Draw a diagram.
- Introduce notation.
- Write the quantity Q to be optimized in terms of your notation.
- Write Q as a function of a single variable, $Q = f(x)$.
- Find the global maximum and/or minimum of f.

In this worksheet, you can use a calculator in the last step if you want.

(1) Draw the parabolas $y = x^2 + 1$ and $y = x - x^2$ on the same axes. What is the minimum vertical distance between these parabolas?

(2) An island is 2 miles due north of its closest point along a straight shoreline. A visitor is staying at a cabin on the shore that is 6 miles west of that point. The visitor is planning to go from the cabin to the island. Suppose the visitor runs at a rate of 8 mph and swims at a rate of 3 mph. How far should the visitor run before swimming to minimize the time it takes to reach the island?

(3) You are constructing a box for your cat to sleep in. The plush material for the square bottom of the box costs \$5 per square foot (ft^2) and the material for the sides costs \$2/ft^2. You need a box with volume 4 ft^3. Find the dimensions of the box that minimize cost. Use x to represent the length of the side of the box.

(4) A cylindrical can without a top is made to contain V cm^3 of liquid. Find the dimensions of the can that will minimize the cost of the metal to make the can.

(5) An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta}$$

where μ is a constant called the coefficient of friction. For what value of θ is F smallest?

(6) Owners of a car rental company have determined that if they charge customers p dollars per day to rent a car, where $50 \leq p \leq 200$, then the number of cars n they rent per day can be modeled by the linear function $n(p) = 1000 - 5p$. If they charge \$50 per day or less, they will rent all their cars. If they charge \$200 per day or more, they will not rent any cars. Assuming the owners plan to charge customers between \$50 per day and \$200 per day to rent a car, how much should they charge to maximize their revenue?