Efficiency and Asymptotic Analysis (20 points)

1. [10 points] Rank the following functions of n by order of growth (starting with the slowest growing) using positive integers (1, 2, . . .). Functions with the same order of growth should be ranked equal.

$\log n^3$	n	$n^2 \log n$	$2^{\lg n^2}$	$\log \sqrt{n}$	$n + \log n^4$	$2^{\log 16}$	n^{-1}	16	$n^{\log 4}$

2. [1 point] Express the function $n^3/1000 - 100n^2 - 100n + 3$ in terms of Θ-notation.

3. Are the following statements true or false. Briefly explain your answer.
   (a) [1 point] $n - 2\log n = \Omega(n)$

   (b) [1 point] $n^2 \log n = \Theta(n^2)$

   (c) [1 point] If $f(n) = O(g(n))$ and $g(n) = O(h(n))$, then $f(n) = O(h(n))$.

   (d) [1 point] $2n^2 + 4n - 17 = O(n^3)$.

4. Assume that n is a positive integer. For each of the following algorithm segments, compute the actual number of additions, subtractions, multiplications, and divisions that must be performed when the algorithm segment is executed (ignore the operations directly performed by the **for** statement). Then compute the asymptotic order of the sum.

   (a) [2 points]

   ```
   for k = 2 to n
       for j = 1 to 3n
           x = a[k] - b[j]
   ```

   (b) [3 points]

   ```
   r = 0
   for i = 1 to n - 1
       p = 1
       q = 1
       for j = i + 1 to n
           p = p · c[j]
           q = q · (c[j])^2
       r = p + q
   ```

Recursion (6 points)

5. [3 points] Using iteration to solve the following recursion:

$$T(n) = 2T(n/2) + n.$$

6. [3 points] Using iteration to solve the following recursion:

$$T(n) = 3T(n/2) + 1.$$