(1)

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ -2 & -7 & 1 & 0 \\ 2 & -4 & 9 & 0 \end{array}\right] \xrightarrow{-2R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ -2 & -7 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \xrightarrow[\frac{1}{3}R_3 \to R_3]{2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -11 & 7 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Could stop here, no free variable = no non-trivial sol.

$-7R_3+R_2 \to R_2$
$-3R_3+R \to R_1$

$$\to \left[\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & -11 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

(2)

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ -4 & -4 & -8 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3 \to R_3]{2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow[-2R_2+R_1 \to R_1]{R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$s$ is a scalar $\quad \vec{u} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

Thus, $x_1 + x_3 = 0 \to x_1 = -x_3$
$x_2 + x_3 = 0 \to x_2 = -x_3$

$$\to \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = s\vec{u}$$

(3)

$$\left[\begin{array}{cccc|c} 1 & -3 & -8 & 5 & 0 \\ 0 & 1 & 2 & -4 & 0 \end{array}\right] \xrightarrow{3R_2+R_1 \to R_1} \left[\begin{array}{cccc|c} 1 & 0 & -2 & -7 & 0 \\ 0 & 1 & 2 & -4 & 0 \end{array}\right] \to \begin{array}{l} x_1 - 2x_3 - 7x_4 = 0 \\ x_2 + 2x_3 - 4x_4 = 0 \end{array}$$

$$\to \begin{array}{l} x_1 = 2x_3 + 7x_4 \\ x_2 = -2x_3 + 4x_4 \end{array} \to \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2x_3 + 7x_4 \\ -2x_3 + 4x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ -2 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 7 \\ 4 \\ 0 \\ 1 \end{bmatrix} = s\vec{u} + t\vec{v}$$

(4)

$$\begin{array}{l} x_1 = 5 + 4x_3 \\ x_2 = -2 - 7x_3 \end{array} \to \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 + 4x_3 \\ -2 - 7x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ -7 \\ 1 \end{bmatrix} = \vec{v} + s\vec{u}$$

(5) Use same steps as #2 (row ops)

① $2R_1+R_2 \to R_2$ ② $\frac{1}{2}R_1 \to R_1$ ③ $-\frac{1}{3}R_3 \to R_3$

$-R_3+R_1 \to R_1$, $R_3 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 8 \\ -4 & -4 & -8 & -16 \\ 0 & -3 & -3 & 12 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \begin{array}{l} x_1+x_3=8 \\ x_2+x_3=-4 \end{array}$$

$$\to \begin{array}{l} x_1 = 8-x_3 \\ x_2 = -4-x_3 \end{array} \to \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8-x_3 \\ -4-x_3 \\ 0+x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = \vec{v} + s\vec{u}$$

✓ This is the answer to #2 with an added constant vector.

✓ The answer to #2 was a line in 3 space going through the origin (0,0,0)

✓ The answer to #5 is parallel to the line from #2 translated (shifted) by $\begin{bmatrix} 8 \\ -4 \\ 0 \end{bmatrix}$

(6) ① $3R_2+R_3 \to R_3$ ② $\frac{1}{2}R_2 \to R_2$ ① $\frac{1}{4}R_3 \to R_3$ ② $-2R_3+R_2 \to R_2$, $-9R_3+R_1 \to R_1$

$$\left[\begin{array}{ccc|c} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 5 & 7 & 9 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 5 & 7 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

→ At this point we can see there will be no free variable, thus the trivial solution is the Only solution, so these vectors are L.I.

(7) These are 4 vector with 3 entries, meaning more columns than rows. So these columns are Linearly Dependent.

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(8) We would like to find an $h$ such that the $3^{rd}$ vector is a linear combination of the first two vectors, ensuring that the 3 vectors are linearly dependent. So,

$$\left[\begin{array}{cc|c} 2 & 4 & -2 \\ -2 & -6 & 2 \\ 4 & 7 & h \end{array}\right] \xrightarrow[-2R_1+R_3 \to R_3]{R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 2 & 4 & -2 \\ 0 & -2 & 0 \\ 0 & -1 & h+4 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2 \to R_2} \left[\begin{array}{cc|c} 2 & 4 & -2 \\ 0 & 1 & 0 \\ 0 & -1 & h+4 \end{array}\right] \xrightarrow{R_2+R_3 \to R_3} \left[\begin{array}{cc|c} 2 & 4 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & h+4 \end{array}\right]$$

$\rightarrow$ $0 = h+4 \Rightarrow h = -4$ thus $3^{rd}$ vector is $\begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix} = -1\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$ ← $1^{st}$ vector

(9) We want to keep the assumption that $\vec{v}_3$ is <u>not</u> a linear combination of $\vec{v}_1$ and $\vec{v}_2$ and show that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ can <u>still</u> be linearly dependent.

So, two options: (a) $\vec{v}_1$ or $\vec{v}_2$ is the zero vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ → L.D.

or

(b) $\vec{v}_1$ and $\vec{v}_2$ are multiples of each other.

(10) a) False, trivial solution is <u>always</u> a solution to a homogeneous system

b) False, could have $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (for example)

c) False, $A\vec{x} = \vec{0}$ <u>always</u> has the trivial solution, it needs to be the <u>only</u> solution.

d) True

e) True. $\vec{w}$ in span$\{\vec{u}, \vec{v}\}$ means $\vec{w}$ is a linear combo of $\vec{u}$ and $\vec{v}$.

f) True

g) False, one of the vectors is a linear combination of other vectors in the set, thus the set is linearly dependent.