$$\pi_{a_1} \circ \pi_{a_2} \circ \cdots \circ \pi_n \equiv \pi_{a_1,a_2,\ldots,a_n} \tag{1}$$

$$\sigma_{pa_1} \circ \sigma_{pa_2} \circ \cdots \circ \sigma_{pa_n} \equiv \sigma_{pa_1 \wedge pa_2 \wedge \cdots \wedge pa_n} \tag{2}$$

$$\pi_{a_1,\ldots,a_n} \circ \sigma_{pa_i} \equiv \sigma_{pa_i} \circ \pi_{a_1,\ldots,a_n} \tag{3}$$

$$\pi_{\delta + \delta'} \circ \bowtie \; \equiv \; \bowtie_\alpha (\pi_\delta, \pi_{\delta'}) \qquad \text{avec } \alpha \in \delta \text{ et } \delta', \delta \subseteq \Delta, \delta' \subseteq \Delta' \tag{4}$$

$$\sigma_{p\alpha \wedge q\beta} \circ \bowtie \; \equiv \; \bowtie (\sigma_{p\alpha}, \sigma_{q\beta}) \qquad \text{avec } \alpha \in \Delta, \beta \in \Delta' \tag{5}$$

$$\pi_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv decrypt_{\alpha,\mathsf{c}} \circ \pi_\delta \qquad \text{si } \alpha \in \delta \tag{6}$$

$$\pi_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv \pi_\delta \qquad \text{si } \alpha \notin \delta \tag{7}$$

$$\pi_\delta \circ defrag_{\delta'} \equiv defrag_{\delta'} (\pi_{\delta \cap \delta'}, \pi_{\delta \setminus \delta'}) \tag{8}$$

$$\sigma_{p\delta} \circ decrypt_{\alpha,\mathsf{c}} \equiv decrypt_{\alpha,\mathsf{c}} \circ \sigma_{p\delta} \qquad \text{si } \alpha \notin \delta \tag{9}$$

$$\sigma_{p\delta} \circ decrypt_{\alpha,\mathsf{c}} \equiv decrypt_{\alpha,\mathsf{c}} \circ \sigma_{\mathsf{c} \Rightarrow p\delta} \qquad \text{si } \alpha \in \delta \tag{10}$$

$$\sigma_{g_{\delta \cap \delta'} \wedge d_{\delta \setminus \delta'} \wedge p\delta} \circ defrag_{\delta'} \equiv \sigma_{p\delta} \circ defrag_{\delta'} (\sigma_{g_{\delta \cap \delta'}}, \sigma_{d_{\delta \setminus \delta'}}) \tag{11}$$

$$\sigma_{g_\delta \wedge vrai \wedge vrai} \circ defrag_{\delta'} \equiv defrag_{\delta'} (\sigma_{g_\delta}, id) \qquad \text{avec } \delta \subseteq \delta' \tag{12}$$

$$\sigma_{vrai \wedge d_\delta \wedge vrai} \circ defrag_{\delta'} \equiv defrag_{\delta'} (id, \sigma_{d_\delta}) \qquad \text{avec } \delta \cap \delta' = \emptyset \tag{13}$$

$$count_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv decrypt_{\alpha,\mathsf{c}} \circ count_{\mathsf{c} \Rightarrow \delta} \qquad \text{si } \alpha \in \delta \tag{14}$$

$$count_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv count_\delta \qquad \text{si } \alpha \notin \delta \tag{15}$$

$$count_\delta \circ defrag_{\delta'} (id, \pi_\emptyset) \equiv$$
$$defrag_{\delta'} (count_\delta, \pi_\emptyset) \qquad \text{si } \delta \subseteq \delta' \tag{16}$$

$$count_\delta \circ defrag_{\delta'} (\pi_\emptyset, id) \equiv$$
$$defrag_{\delta'} (\pi_\emptyset, count_\delta) \qquad \text{si } \delta \cap \delta' = \emptyset \tag{17}$$

$$f \circ id \equiv id \circ f \equiv f \tag{18}$$

$$defrag_{\delta,\delta'}(id, id, id) \circ frag_{\delta,\delta'} \equiv$$
$$defrag_\delta (id, defrag_{\delta'} (id, id) \circ frag_{\delta'}) \circ frag_\delta \qquad \text{si } \delta' \subseteq (\Delta \setminus \delta) \tag{19}$$

$$frag_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv (decrypt_{\alpha,\mathsf{c}}, id) \circ frag_\delta \qquad \text{si } \alpha \in \delta \tag{20}$$

$$frag_\delta \circ decrypt_{\alpha,\mathsf{c}} \equiv (id, decrypt_{\alpha,\mathsf{c}}) \circ frag_\delta \qquad \text{si } \alpha \notin \delta \tag{21}$$

$$decrypt_{\alpha,\mathsf{c}} \circ defrag_\delta \equiv defrag_\delta (decrypt_{\alpha,\mathsf{c}}, id) \qquad \text{si } \alpha \in \delta \tag{22}$$

$$decrypt_{\alpha,\mathsf{c}} \circ defrag_\delta \equiv defrag_\delta (id, decrypt_{\alpha,\mathsf{c}}) \qquad \text{si } \alpha \notin \delta \tag{23}$$

$$frag_\delta \circ crypt_{\alpha,\mathsf{c}} \equiv (crypt_{\alpha,\mathsf{c}}, id) \circ frag_\delta \qquad \text{si } \alpha \in \delta \tag{24}$$

$$frag_\delta \circ crypt_{\alpha,\mathsf{c}} \equiv (id, crypt_{\alpha,\mathsf{c}}) \circ frag_\delta \qquad \text{si } \alpha \notin \delta \tag{25}$$