

$$\pi_{a_1} \circ \pi_{a_2} \circ \dots \circ \pi_n \equiv \pi_{a_1, a_2, \dots, a_n} \quad (3)$$

$$\sigma_{pa_1} \circ \sigma_{pa_2} \circ \dots \circ \sigma_{pa_n} \equiv \sigma_{pa_1 \wedge pa_2 \wedge \dots \wedge pa_n} \quad (4)$$

$$\pi_{a_1, \dots, a_n} \circ \sigma_{pa_i} \equiv \sigma_{pa_i} \circ \pi_{a_1, \dots, a_n} \quad (5)$$

$$\pi_{\delta \uplus \delta'} \circ \bowtie \equiv \bowtie_{\alpha} (\pi_{\delta}, \pi_{\delta'}) \quad \text{avec } \alpha \in \delta \text{ et } \delta', \delta \subseteq \Delta, \delta' \subseteq \Delta' \quad (6)$$

$$\sigma_{p\alpha \wedge q\beta} \circ \bowtie \equiv \bowtie (\sigma_{p\alpha}, \sigma_{q\beta}) \quad \text{avec } \alpha \in \Delta, \beta \in \Delta' \quad (7)$$

$$id \equiv decrypt_{\alpha, c} \circ crypt_{\alpha, c} \quad \text{si } \alpha \in \Delta \quad (8)$$

$$id \equiv defrag \circ frag_{\delta} \quad \text{si } \delta \subseteq \Delta \quad (9)$$

$$\pi_{\delta} \circ decrypt_{\alpha, c} \equiv decrypt_{\alpha, c} \circ \pi_{\delta} \quad \text{si } \alpha \in \delta \quad (10)$$

$$\pi_{\delta} \circ decrypt_{\alpha, c} \equiv \pi_{\delta} \quad \text{si } \alpha \notin \delta \quad (11)$$

$$\pi_{\delta} \circ defrag_{\delta'} \equiv defrag_{\delta'} (\pi_{\delta \cap \delta'}, \pi_{\delta \setminus \delta'}) \quad (12)$$

$$\sigma_{p_{\delta}} \circ decrypt_{\alpha, c} \equiv decrypt_{\alpha, c} \circ \sigma_{p_{\delta}} \quad \text{si } \alpha \notin \delta \quad (13)$$

$$\sigma_{p_{\delta}} \circ decrypt_{\alpha, c} \equiv decrypt_{\alpha, c} \circ \sigma_{c \Rightarrow p_{\delta}} \quad \text{si } \alpha \in \delta \quad (14)$$

$$\sigma_{g_{\delta \cap \delta'} \wedge d_{\delta \setminus \delta'} \wedge p_{\delta}} \circ defrag_{\delta'} \equiv \sigma_{p_{\delta}} \circ defrag_{\delta'} (\sigma_{g_{\delta \cap \delta'}}, \sigma_{d_{\delta \setminus \delta'}}) \quad (15)$$

$$\sigma_{g_{\delta} \wedge vrai \wedge vrai} \circ defrag_{\delta'} \equiv defrag_{\delta'} (\sigma_{g_{\delta}}, id) \quad \text{avec } \delta \subseteq \delta' \quad (16)$$

$$\sigma_{vrai \wedge d_{\delta} \wedge vrai} \circ defrag_{\delta'} \equiv defrag_{\delta'} (id, \sigma_{d_{\delta}}) \quad \text{avec } \delta \cap \delta' = \emptyset \quad (17)$$

$$count_{\delta} \circ decrypt_{\alpha, c} \equiv decrypt_{\alpha, c} \circ count_{c \Rightarrow \delta} \quad \text{si } \alpha \in \delta \quad (18)$$

$$count_{\delta} \circ decrypt_{\alpha, c} \equiv count_{\delta} \quad \text{si } \alpha \notin \delta \quad (19)$$

$$count_{\delta} \circ defrag_{\delta'} (id, \pi_{\emptyset}) \equiv defrag_{\delta'} (count_{\delta}, \pi_{\emptyset}) \quad \text{si } \delta \subseteq \delta' \quad (20)$$

$$count_{\delta} \circ defrag_{\delta'} (\pi_{\emptyset}, id) \equiv defrag_{\delta'} (\pi_{\emptyset}, count_{\delta}) \quad \text{si } \delta \cap \delta' = \emptyset \quad (21)$$

$$f \circ id \equiv id \circ f \equiv f \quad (22)$$

$$defrag_{\delta, \delta'}(id, id, id) \circ frag_{\delta, \delta'} \equiv$$

$$defrag_{\delta}(id, defrag_{\delta'}(id, id) \circ frag_{\delta'}) \circ frag_{\delta} \quad \text{si } \delta' \subseteq (\Delta \setminus \delta) \quad (23)$$

$$frag_{\delta} \circ decrypt_{\alpha, c} \equiv (decrypt_{\alpha, c}, id) \circ frag_{\delta} \quad \text{si } \alpha \in \delta \quad (24)$$

$$frag_{\delta} \circ decrypt_{\alpha, c} \equiv (id, decrypt_{\alpha, c}) \circ frag_{\delta} \quad \text{si } \alpha \notin \delta \quad (25)$$

$$decrypt_{\alpha, c} \circ defrag_{\delta} \equiv defrag_{\delta}(decrypt_{\alpha, c}, id) \quad \text{si } \alpha \in \delta \quad (26)$$

$$decrypt_{\alpha, c} \circ defrag_{\delta} \equiv defrag_{\delta}(id, decrypt_{\alpha, c}) \quad \text{si } \alpha \notin \delta \quad (27)$$

$$frag_{\delta} \circ crypt_{\alpha, c} \equiv (crypt_{\alpha, c}, id) \circ frag_{\delta} \quad \text{si } \alpha \in \delta \quad (28)$$

$$frag_{\delta} \circ crypt_{\alpha, c} \equiv (id, crypt_{\alpha, c}) \circ frag_{\delta} \quad \text{si } \alpha \notin \delta \quad (29)$$