LASER CALCULATOR: MANIPULATION OF THE WAVENUMBER IN LINEAR MEDIA

Santiago García Rodríguez
Experimental Techniques III. Physics Degree. University of Oviedo

MOTIVATION OF THE EXPERIMENT

Algebraic operations between waves can be physically implemented using linear optics and proper signal processing.
Previously, experiments were conducted by overlapping two wavefronts, which produced the interference patter corresponding to the sum of two wave functions

$$\Psi = \Psi_1 + \Psi_2$$

By introducing phase shifts with retardation plates, subtraction can be found.

By modulating its amplitude with polarizers or coherent superposition, multiplication by a scalar can be achieved. This makes it possible to reproduce the interference pattern of any linear combination of two waves:

$$\Psi = c_1\Psi_1 + c_2\Psi_2 \qquad c_1, c_2 \in \mathbb{R}$$

In this way, addition and subtraction of two waves, as well as scalar multiplication, are implemented. Two basic operations would be missing to be obtained: multiplication and division between two waves.

MATHEMATICAL DEVELOPMENT

The product or quotient of two monochromatic waves results in a sum or difference of their exponents, directly modifying the wavenumber and frequency.

In practice, with linear media, the global phase is not directly observable. In this context, the only degree of freedom accessible to implement these operations is the exponent, whose dependence on the wavenumber is explicit.

The use of nonlinear media would significantly simplify the experiment. However, when restricted to linear media, it is not possible to access the exponent of a wave in real space. By moving to the Fourier domain, the spatial phases are transformed into translations, allowing a direct manipulation of the wavenumber.

Until now, $\Psi(x, t)$ has been expressed; however, the expression $\Psi(k, t)$ is more useful. To achieve this, the Fourier transform of the two-wave product is applied:

$$\mathcal{F}\{A_1 e^{ik_1 x} \cdot A_2 e^{ik_2 x}\}(k) = A_1 A_2 (2\pi)^2 \delta(k - (k_1 + k_2))$$

The two-wave product shifts the value of k to the sum $k_1 + k_2$, produces a translation in the wave number. In addition, the frequency depends on the wavenumber. Therefore, it is only necessary to modify the wavenumber, and the frequency will be determined by $\omega(k)$.

Consequently, implementing wave products using linear media requires operating in a plane equivalent to Fourier space, where a physical translation is interpreted as a modification of the wave number.

OBJECTIVES

- Reproduce translations in Fourier space by appropriately filtering the scattered beams.
- Construct the interference pattern between two waves manipulated differently in the Fourier plane.
- Experimentally determine analogies between algebraic operations and linear optics.

Note: Amplitudes will not be studied in this experiment. However, the resulting amplitude could be achieved with polarizers and coherent superposition.

EXPERIMENTAL DEVICE

In the real world, it is possible to find a plane that behaves approximately the same as Fourier space:

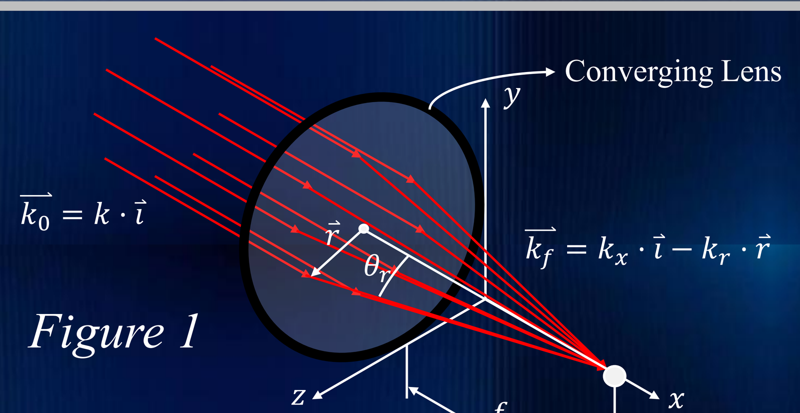


Figure 1

$$f \gg r \to \theta_r \ll 1 \to \begin{cases} \mathrm{tg}(\theta_r) \approx \theta_r = \dfrac{r}{f} \\ \sin(\theta_r) \approx \theta_r = \dfrac{k_r}{k} \end{cases} \to \frac{r}{f} = \frac{k_r}{k}$$

$$\boxed{k_r = \frac{k \cdot r}{f} \quad [1]}$$

Equation [1] transforms a spatial position into a wavenumber. There is a region, corresponding to the plane of the convergent lens in *Figure 1*, where a physical translation is equivalent to a change in the wavenumber, i.e., a Fourier transform.

So far, a wave has been manipulated to illustrate the principle behind the transformation from position to wavenumber. The experiment will be carried out in the plane located at a distant plane f from the converging lens, where all the waves collapse and recover $\vec{k} = k \cdot \bar{\imath}$. The change in $\vec{k}$ will be induced by an element with spatial periodicity: a diffraction grating. The complete device is shown in *Figure 2*.

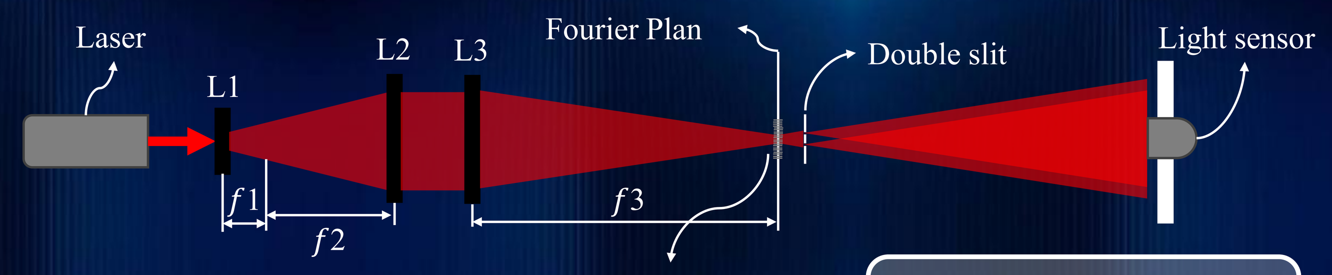


Figure 2

$$\boxed{\Delta k = \frac{2\pi\Delta m}{d} \quad [2]}$$

ORDER FILTERING

If the double slit is removed, the behavior of a diffraction grating in the Fourier plane can be observed (see *Figure 4*): it does not cause interference, it separates each order of diffraction, assigning it to a specific point in space.
The diffraction grating generates beams with different diffraction orders. Using the double slit, two orders must be filtered as shown in *Figure 3*.

Figure 3

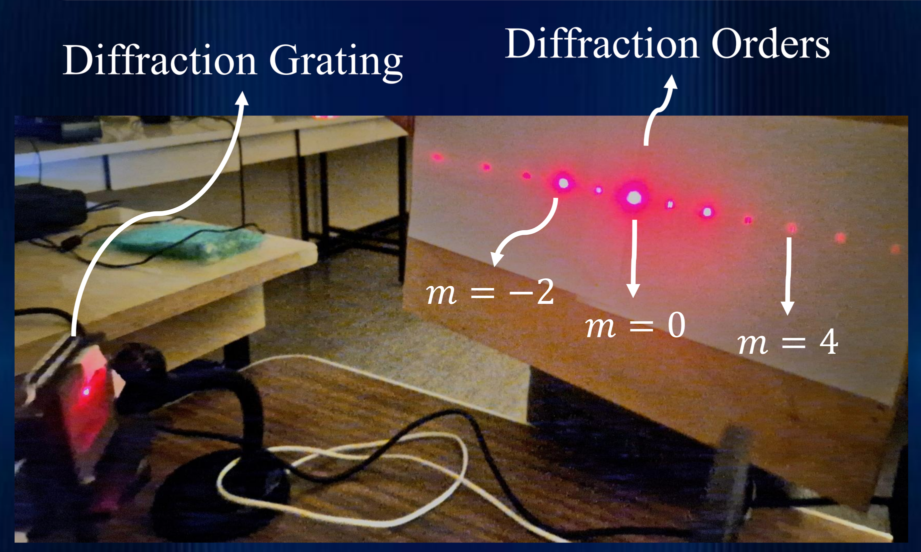


Figure 4

This explains *Equation [2]*. The wavenumber of the beam passing through the slit that filters order $m = -2$ in *Figure 3* has been modified by;

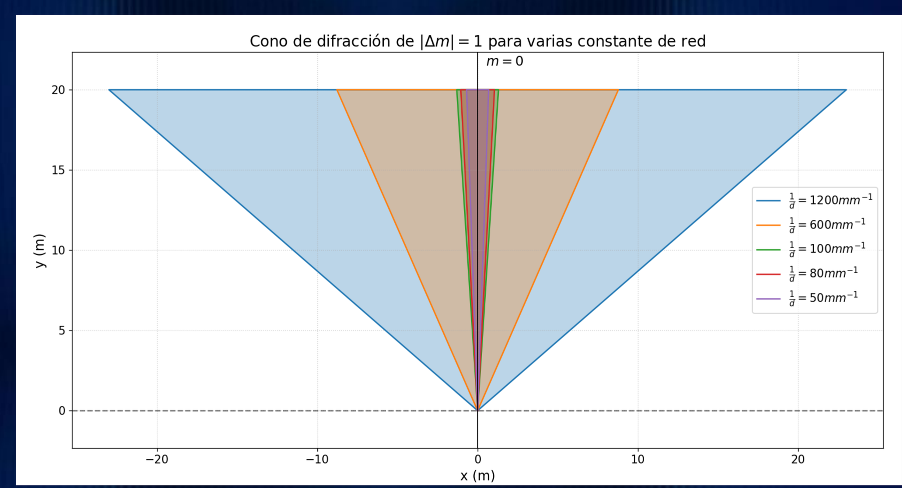
$$\frac{2\pi\Delta m}{d}, \qquad d\text{: grating constant}$$

The diffraction grating represents the second wave function in the product of two waves. After the double slit, the interference pattern is created between one wave with $k_1 = k$ and another with $k_2 = k - \frac{2\pi\Delta m}{d}$.

SUITABLE PARAMETERS

According to *Equation [2]*, if you want to find a significant change in the wavenumber, you should use a diffraction grating with a small grating constant or, conversely, try to filter out orders that are as far apart as possible.

However, looking at *Graph 1*, the second option can be ruled out. For small grating constants, the orders are too dispersed to try to filter two distant orders with double slits that are millimeters apart, which were the available ones.



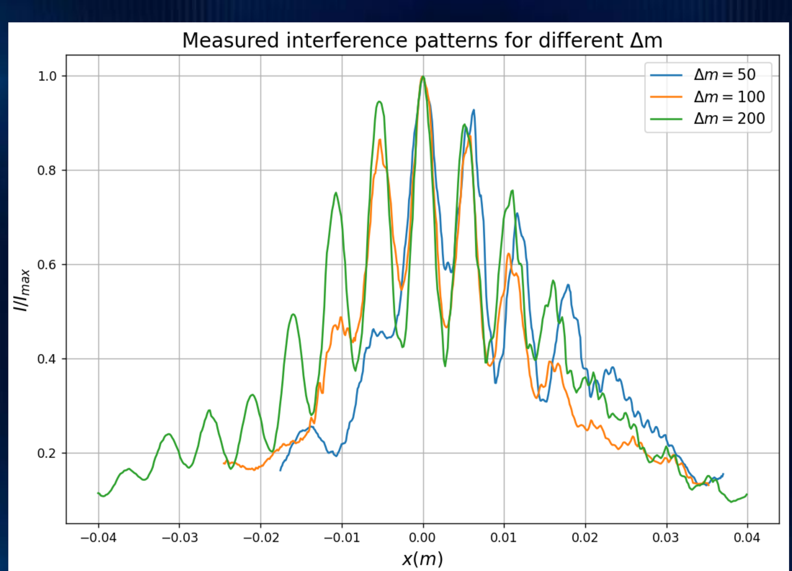
Graph 1

On the other hand, for the change Δk was appreciable: $\frac{\Delta k}{k} > 0{,}05$; two orders would have to be filtered as far apart as possible. To achieve this, the diffraction grating was taken with $d = 0{,}25mm$, lthe greater available. Since, as can be found by Graph 1, this network has a minimum angular dispersion which allows two distant orders to be filtered with double slits separated by less than $1mm$. In this way, an appreciable change in the wavenumber is achieved.

The fact that the orders are very close together is also a disadvantage, as parasitic orders must be considered. When filtering through slits of an appreciable width, several adjacent orders will pass. However, in the interference pattern, the desired orders will still dominate.

RESULTS

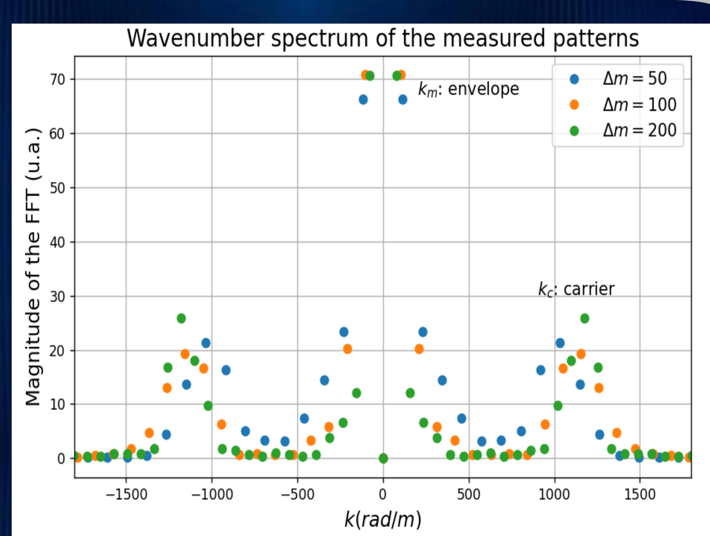
The interference patterns measured (see *Figure 2*) exhibit modulation, local irregularities, and other consequences of the device's limitations. What is clear is that spatial periodicity exists.
The spatial periodicity was found by the spectrum of wavenumbers, applying the FFT. The results were highly revealing (see *Graph 3*) giving two clear maxima. These correspond to the interference oscillation at k_c and the modulation of interference by k_m.



Graph 2

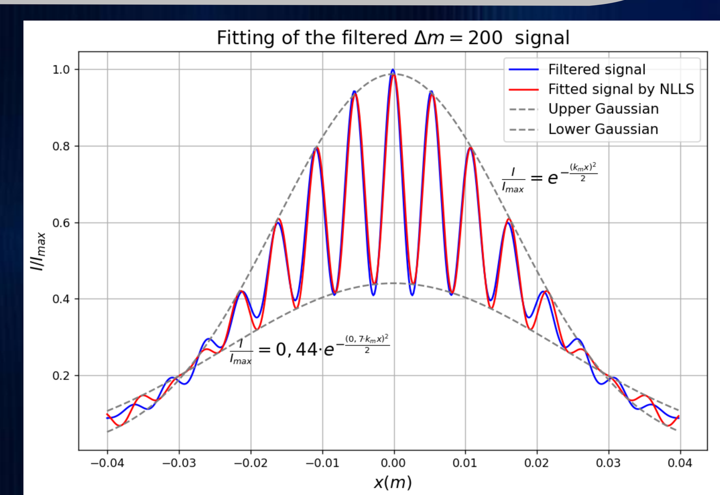
Band-pass filters were applied to the spectrum to extract these two values from the number of waves, eliminating out background contributions and noise. Finally, to accurately measure these wavenumbers, an expression must be found that reproduces the filtered signal as faithfully as possible.

After adjusting several expressions for nonlinear least squares (NLLS), the best behavior was found considering the oscillation given by k_c and modulated by an envelope modeled by Gaussian functions whose deviation depends on k_m. This is shown in Graph 4 and corresponds to the filtered and symmetric interference pattern. Since the experimental setup is symmetric, the pattern is also symmetric; therefore, data from both side were averaged.



Graph 3

Note: The expression used for the fit is not the exact expression, it is the best fit of the expressions that were tested.



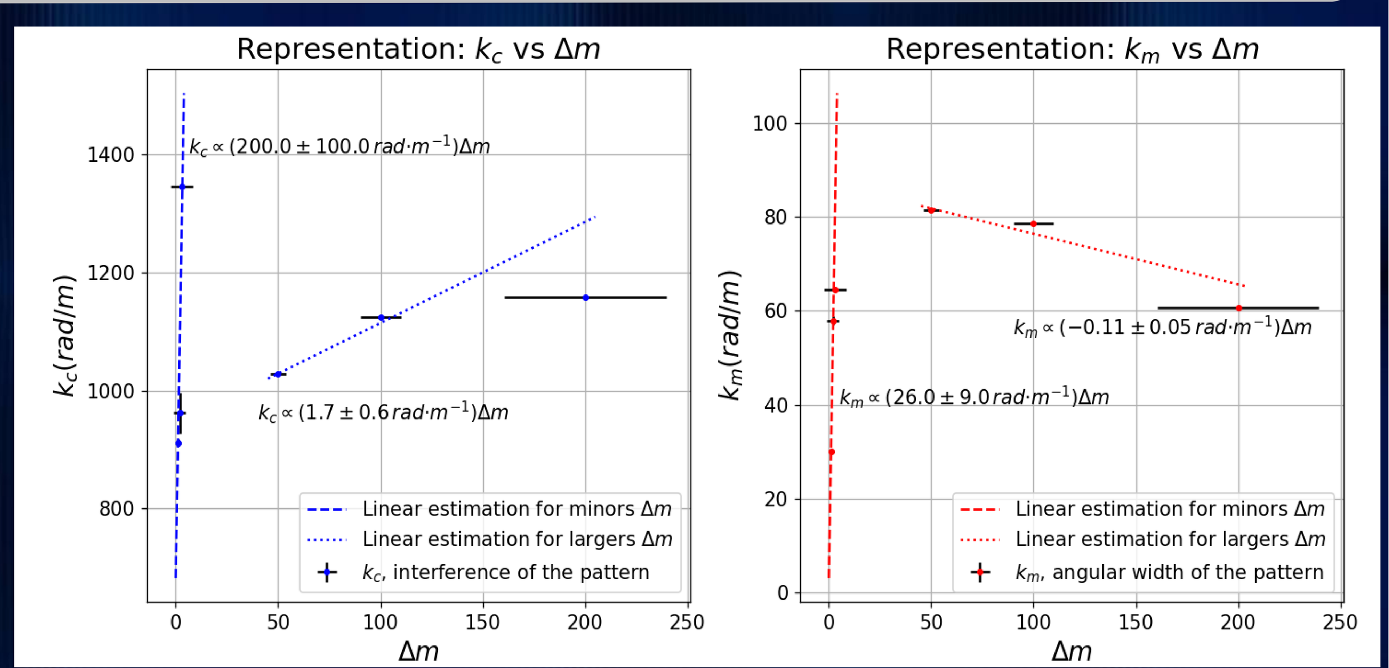
Graph 4

From the fitted models, the values of k_c and k_m were obtained. In addition, the patterns were measured for small Δm values. To make this feasible, another double slit with narrower apertures was used. Therefore, the relevance of this parameter can also be determined. For smaller Δm values, the slits were separated by $0{,}4mm$, and for larger Δm values, by $1{,}25mm$.

The results are represented in *Graph 5*. Several behaviors can be observed depending on the double slit used and the Δm region. Regarding the k_c results, both regions maintain a positive slope, but significantly smaller in the region where the double slits were further apart. Therefore, one possibility is that the slope depends on slit separation. For the results of k_m, the slope goes from positive to negative between the regions measured, suggesting the existence of two distinct regimes based on Δm, connected by a transition point. The sensitivity of k_m to the slit separation is expected to be negligible, as k_m is associated with the beam's angular width. This aspect of the beam is regulated by the diffraction grating and possibly by the width of each slit aperture. However, the latter cannot be determined in the present experiment since this parameter remained constant throughout all measurements.
The error bars and uncertainties in the linear estimates reflect instrumental and alignment limitations of the setup, as well as the propagation of uncertainties considered in the numerical data processing.

CONCLUSIONS

- It has been experimentally demonstrated that the manipulation of the beam in the Fourier plane allows modification of the interference pattern, enabling access to the wavenumber resulting from the two-wave product using only linear media.
- The experiment identifies dependencies between the parameters of the interference pattern, although it is not sufficient to establish a complete analytical parameterization of wave product.
- An approximately linear relationship is observed between k_c and Δm, with the slope depending on slit separation, as well as two roughly linear regimes of k_m depending on Δm.
- The presence of parasitic orders, experimental uncertainties, and the lack of exploration in the influence of additional parameters and broader Δm regions limit the global interpretation of the experiment to a qualitative regime-based analysis.
- This experiment lays the foundations for parameterizing the basic operations between waves and opens the possibility of exploring more complex ones.



Graph 5

BIBLIOGRAPHY

- Goodman, J. W. (2005). *Introduction to Fourier optics* (3rd ed.). Roberts and Company.
- Hecht, E. (2017). *Optics* (5th ed.). Pearson Education.
- Pinnell, J., Klug, A., & Forbes, A. (2020). *Spatial filtering of structured light*. arXiv. https://arxiv.org/abs/2005.06000
- Born, M., & Wolf, E. (1999). *Principles of optics* (7th ed.) Cambridge University Press. https://doi.org/10.1017/CBO9781139644181