Derivación

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→ Show that the $D^4 f$ operator is given by:

$$D^4 f(x_j) \cong \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4}$$

→ Sumamos las series de Taylor para f_{i+1} y f_{i-1}, ignorando órdenes $> h^4$ $(\mathcal{O}(h^4))$:

$$\Rightarrow f_{i+1} + f_{i-1} = 2f_i + h^2 f''_i + \frac{h^4}{4} f_i^{IV}$$

$$\Rightarrow f_i^{IV} = \frac{4}{h^4}\left[f_{i+1} + f_{i-1} - 2f_i + h^2 f_i''\right]$$

→ Por la fórmula alternativa del 3.7.1:

$$f_i'' = \frac{f_{i+2} - 2f_i + f_{i-2}}{4h^2} \Rightarrow f_i^{IV} = \frac{4}{h^4}\left[f_{i+1} + f_{i-1} - 2f_i + \left(\frac{f_{i+2} - 2f_i + f_{i-2}}{4}\right)\right] \cong \frac{f(x_{j+2}) - 4f(x_{j+1}) + 6f(x_j) - 4f(x_{j-1}) + f(x_{j-2})}{h^4} + \mathcal{O}((h^5))$$

$\boxed{\text{Q.e.d}}$