SCALAR PHANTOMS

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We show that, by a minimal modification of the standard SU(3) × SU(2) × U(1) theory, we can account for the dark matter of the universe. With a reasonable choice of an unknown coupling, the galactic mass scale emerges. We comment on the prospects for laboratory detection of the scalar particle involved and the possible production of anti-protons in cosmic rays.

In this paper we speculate on the existence of a scalar particle which may account for the dark matter [‡1] in the universe. Our speculation does not involve any exotic theoretical ideas. Indeed, the discussion will be in context of the SU(2) × U(1) theory of Glashow, Salam and Weinberg. We will also discuss the possibility of laboratory detection of such a particle.

Suppose the standard theory contains a scalar field, which we code name X, transforming as a singlet under SU(3) × SU(2) × U(1). There is no known physical principle against such a field. To the contrary, the presence of such a field is quite likely in the context of grand unification theory [‡2] and of theory with dynamical scalar fields.

The X field can only couple to the Higgs sector of the theory, which is now amended to read

$$-\tfrac{1}{2}\mu^2\varphi^+\varphi + \tfrac{1}{4}\lambda(\varphi^+\varphi)^2 + \tfrac{1}{2}m^2X^2 + \tfrac{1}{4}\eta X^4 + \tfrac{1}{4}\rho X^2\varphi^+\varphi .$$

We have imposed a discrete reflection symmetry $X \leftrightarrow -X$. We also take m^2 to be positive. After spontaneous symmetry breaking, $|\varphi| = v + H(x)$, $v^2 = \mu^2/\lambda$, we have $m_X^2 = m^2 + \frac{1}{2}\rho v^2$ and the coupling term $\frac{1}{2}\rho v H X^2$. Thus, the X particle is absolutely stable.

The physics in our discussion is parameterized in terms of m_X and ρ. A priori, we know nothing about m_X and ρ. As a first guess, we might suppose $\rho \sim \lambda = \frac{1}{8}g^2(m_H/M_W)^2$. In what follows, whenever a numerical estimate is given, we use this "typical" value of ρ. By taking suitable ratios, we find that the combination $(\rho m_W^2/g^2 m_H^2)$ often appears in physical quantities of interest. In the same spirit, we might also suppose m_X to be of the same order of magnitude as m_H. But given the existence of various hierarchy puzzles, we may wish to keep open the possibility of a very light X particle, with $m_X \ll m_H$.

Since the X couples to ordinary matter only via the Higgs field H, it is naturally rather feebly interacting and could easily have escaped detection. At low momentum transfer, the interaction of the X with a quark or lepton is characterized by the amplitude $\sim \rho m_f/m_H^2$, which is to be compared to the interaction amplitude of a neutrino with a fermion, $\sim g^2/m_Z^2$. A dimensionless measure of the relative interaction amplitudes is thus given by $(\rho m_Z^2/g^2 m_H^2)(m_f/m_X)$. This ex-

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[‡1] For an extended review of the dark matter problem, see ref. [1]; for brief reviews, see ref. [2].
[‡2] Barr has discussed fermions which transform as singlets under SU(3) × SU(2) × (1) [3]. The physics involved is quite different from that in our case.

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pression shows clearly that the X interacts feebly with ordinary matter because it interacts via the Higgs which itself couples very weakly to the first generation of fermions.

But then it also follows that in a cosmological setting, where the X could interact with the heavy fermions of the second and third generations, the presence of the X may have a significant impact.

Our strategy consists of first fixing m_X so that the X contributes the critical closure mass density to the universe and then asking for the effect of X dominance on galaxy formation.

In the early universe, the number density of the X particle at time t, n(t), is governed by the standard rate equation

$$dn/dt = -(3\dot{R}/R)n - \langle v\sigma \rangle n^2 + \langle v\sigma \rangle n_0^2. \tag{1}$$

Here R denotes the scale size of the universe and $\langle v\sigma \rangle$ is the thermal averaged cross section for annihilation of an X pair into fermions: $X + X \to f + \bar{f}$. The third term describes the production of X pairs and is fixed by the requirement that in equilibrium $\dot{R} = 0$, $\dot{n} = 0$, and $n = n_0 =$ the number density of X if the X were in equilibrium.

The annihilation cross section is easily calculated to be

$$\begin{aligned} v\sigma &= \left[3(\rho m_f)^2/4\pi\right]\left(4m_X^2 - m_H^2\right)^{-2} \\ &\quad \times \left[\left(m_X^2 - m_f^2\right)^{1/2}/m_X\right]^3 \\ &\approx (3/4\pi)(\rho m_f)^2/\left(4m_X^2 - m_H^2\right)^2, \\ &\quad \text{for } m_X \gtrsim m_f. \end{aligned} \tag{2}$$

We have anticipated that at the temperature of interest (see below) the X particle is non-relativistic. The cross section is dominated by the most massive fermion, which we take to be a quark (hence the factor of 3 for color in eq. (2)) (also, we assume that $m_X < m_H$ so that we do not include the process $X + X \to H + H$).

The physics of eq. (1) is well-understood by now, thanks in part to a work of Lee and Weinberg [4], who tracked the cosmological history of stable massive neutrinos which annihilate in pairs. As the temperature of the universe drops, at some freezing temperature T_f, the annihilation rate cannot keep pace with the expansion rate and n becomes larger that n_0: the X particles go out of chemical equilibrium. (The freezing temperature is to be distinguished from the decoupling temperature, when the collision rate of the X (in processes such as $X + f \to X + f$) drops below the expansion rate of the universe, at which point the X is said to go out of thermal or kinetic equilibrium. See below.) Happily, we can take over the numerical analysis of Lee and Weinberg with suitable modifications. [‡3] The result is that the number density n at freezing is given by [‡4]

$$(n/T^3)_f \simeq 10a^{-0.95}, \tag{3}$$

where

$$a = \left(45/8\pi^3 N_F g\right)^{1/2} m_X \langle v\sigma \rangle. \tag{4}$$

Here N_F is the effective number of relativistic degrees of freedom defined by the total energy density of the universe at freezing

$$\rho = N_F \pi^2 T^4/15. \tag{5}$$

The expansion of the universe is governed as usual by the equation

$$\dot{R}/R = -\dot{T}/T = (8\pi G\rho/3)^{1/2}. \tag{6}$$

The freezing temperature T_f comes out to be about $T_f/m_X \sim \frac{1}{20}$. This number depends logarithmically on quantities such as the annihilation cross section and is quite stable under variations of the un-

‡3 The relevant differences are the following: (1) The X particle is spinless. (2) It decouples at a somewhat higher temperature, so the number of relativistic degrees of freedom is higher. (3) The higher freezing temperature implies that the factor by which the photon density is increased through subsequent annihilation of charged particles is larger.

‡4 Lee and Weinberg [4] were led to study a differential equation of the form $df/dx = a(f^2 - f_0^2)$. The expression given in eq. (3) with the exponent 0.95 results from a numerical fit given by Lee and Weinberg for the range of parameters of interest to them. The value of a in our case differs from that in their case by a factor of about 2. We assume that the relevant exponent will not be changed drastically, an assumption which we have checked by a rough analytic calculation.

known parameters m_X, m_H, and ρ. This justifies our evaluating the annihilation cross section in the non-relativistic limit. For the purpose of a rough estimate, we will simply replace the exponent 0.95 in eq. (3) by 1. The condition that the present mass density of the X should not exceed the critical density, $\Omega_X \leqslant 1$, then works out to be

$$\left(4m_X^2/m_H^2 - 1\right)^2 \leqslant 0.15 N_F^{-1/2} (m_f/\text{GeV})^2 h^2. \quad (7)$$

(Here h is in the uncertainty in the Hubble parameter $H = 100h$(km/sec/Mpc). Observationally, $\frac{1}{2} \lesssim h \lesssim 1$.) For m_X larger than the bottom quark mass, we can take $m_f \sim 5$ GeV. For the regions of interest to us, N_F will turn out to be about 8.25, and so

$$\left(4m_X^2/m_H^2 - 1\right)^2 \leqslant 1.25h^2. \quad (8)$$

A striking prediction of the inflationary universe [5] is that Ω is equal to 1. The X particle could saturate Ω if $m_X \simeq 0.75 m_H$. (Let us first take h to be 1.) Thus for m_H in the range 10–30 GeV, the condition $\Omega_X = 1$ requires that m_X lies in the range 7.5–22 GeV and the freezing temperature T_f is in the 0.3–1 GeV range. The value of N_F cited above is roughly the value relevant for this range. Smaller values of m_H have presumably been ruled out experimentally, but if m_H is significantly below 10 GeV, in eq. (7) m_f would have to be replaced by the charm mass $m_C \sim 1.5$ GeV and m_X would have to be very close to $\frac{1}{2}m_H$ for $\Omega_X = 1$.

Considering all the approximations which went into eq. (8), we can really only take the 1.25 in eq. (8) as a symbol representing number ξ of order 1. If ξ is less than but close to 1, critical density may be achieved for $2m_X \ll m_H$.

In writing down eq. (7) we have already set ρ to its "typical" value. From the dependence of $\langle v\sigma \rangle$ on $(4m_X^2 - m_H^2)^2$ it is clear that the right-hand side of eq. (7) scales like ρ^2. The physics behind this dependence is clear enough: as ρ^2 and $\langle v\sigma \rangle$ increase, the freezing temperature T_f decreases, and so $n_X|_f$ and $\Omega_X|_f$ decrease because of the Boltzmann factor. (Note also that we have used the form of $\langle v\sigma \rangle$ for $m_X \gtrsim m_f$.)

On the other hand, if $h \simeq \frac{1}{2}$, then m_X is required to be about $\frac{1}{2}m_H$ in order for $\Omega_X = 1$.

To study the effect of X on galaxy formation, we want to determine the amount of mass contained within the horizon when the X particle started to dominate the mass density of the universe. We have

$$\rho_X/\rho_\gamma \sim (36/\pi^4)(m_\sigma/T_\gamma)(n\sigma/n_\gamma)$$
$$\sim (m_\sigma/2.7T_\gamma) g^{-1} (n_\sigma/n_\gamma)_f. \quad (9)$$

The factor g accounts for annihilation into photons since the X was frozen. For the range of parameters of interest ($m_X \sim 10$ GeV, $T_f \sim 500$ MeV) we have $g \sim 6$ or so. Thus, the X dominated when the photon temperature was of order

$$T_\gamma^{\text{dom}} \sim (m_X/2.7g)(n_X/n_\gamma)_f. \quad (10)$$

The volume inside the horizon was given roughly by

$$V_H \sim \left(0.04/N_F^{3/2}\right) M_{Pl}^3 T^{-6}. \quad (11)$$

Putting it all together, we find

$$M_{HX} \sim 0.2\left(g^2/N_F^{3/2}\right) m_X^{-2} (n_X/n_\gamma)_f^{-2} M_\odot. \quad (12)$$

(In what follows, masses are given in GeV units.)

For convenience, define $y \equiv m_X/T_f$. Then the freezing temperature is determined approximately by [4]

$$y^{1/2} e^y \simeq 3.5 \times 10^7 h^{-2} m_X. \quad (13)$$

(For $h = 1$, y ranges from 18.5 to 19.3 as m_X ranges from 14 GeV to 28 GeV.) As mentioned before, the ratio T_f/m_X vary only logarithmically. Thus, it is sensible to write the relevant physical quantities in terms of this ratio:

$$T_\gamma^{\text{dom}} \simeq 4.5 \times 10^{-11} h^2 y^2 \simeq 20h^2 \text{ eV}, \quad (14)$$

and

$$M_{HX} \simeq \left(1.3 \times 10^{17} M_\odot\right) h^{-4} y^4$$
$$\simeq 8 \times 10^{11} M_\odot h^{-4}. \quad (15)$$

(The value of T_γ^{dom} can of course be deduced directly from the ratio Ω_X/Ω_B. In eq. (15) we have set $N_F = 1$ in accordance with the value of T_γ^{dom}.)

Galaxies typically have masses around 10^{11} or 10^{12} $M_\odot$. It is amusing that M_{HX} comes out to have this order of magnitude in our calculations.

In our picture, the X particle constitutes what is known as cold dark matter. Galaxies formed before galactic clusters, in accordance [1,2] with observations.

We still have to check that the decoupling temperature T_{dec} at which the X goes out of kinetic equilibrium is lower than freezing temperature T_f. As a rough estimate, we equate the reaction rate $n_f\langle v\sigma\rangle$ for the process $f + X \to f + X$ (where f is some fermion) to the expansion rate of the universe. The cross section $\langle v\sigma\rangle \sim (1/32\pi)\times \rho^2 m_f^2/m_H^4$ is now considerably smaller than the cross section in eq. (2) since we have to use a light quark mass, $m_u \sim 4$ MeV or $m_d \sim 7$ MeV. (We assume the fermion to be relativistic.) But this is more than offset by the fact that n_f is not Boltzmann suppressed. Using our "typical" value for ρ as always, we find

$$T_{dec} \simeq 3\times 10^{-6} m_f^{-2} \simeq 60 \text{ MeV} \quad \text{for } m_f = m_d. \tag{16}$$

Thus, at the freezing temperature the X were indeed in kinetic equilibrium. (The precise value of T_{dec} may be somewhat different since the confinement transition supposedly occurs at $T_{conf} \sim 100$ MeV. Our calculation is too crude for us to say whether or not T_{dec} is above T_{conf}.)

In closing, we comment on the prospect of laboratory detection of the X. The cross section for the scattering of an X particle on an electron in the laboratory frame as a function of T, the kinetic energy of the recoil electron, is given by

$$\mathrm{d}\sigma/\mathrm{d}T = \left\{\rho^2 m_e^2/16\pi\right\} \times\left[p_X^2\left(k^2 - m_H^2\right)^{-2}(T + 2m_e)\right]. \tag{17}$$

Here p_X = incoming momentum of X, $-k^2 = 2m_e T$ = square of four-momentum transferred. For a rough estimate of the corresponding cross section for X scattering on a nucleon, we may replace m_e by m_N in the kinematical part of the cross section (the square bracket in eq. (17)) and m_e by m_u or m_d in the factor which comes from the couplings (the curly bracket in eq. (17)). Thus, X scattering in matter is dominated by scattering on nucleons by a factor $(m_u/m_e)^2$ and $(m_d/m_e)^2$.

For momentum transfer much less than m_H, the cross section depends on T linearly, in contrast to the quadratic dependence for the neutrino (and anti-neutrino) scattering on matter.

Again, for momentum transfer much less than m_H, the total cross section for X scattering on nucleons is approximately

$$\sigma(\mathrm{XN}\to\mathrm{XN}) \sim \left(\rho^2 m_u^2/32\pi m_H^4\right) \times(E_X - m_X + 4m_N)/(E_X + m_X), \tag{18}$$

and approaches a constant for $E_X > m_X$. Near threshold, $E_X \sim m_X$,

$$\sigma(\mathrm{XN}\to\mathrm{XN}) \sim \left(\rho^2 m_u^2/16\pi m_H^4\right) m_N/m_X. \tag{19}$$

For comparison, we note that the cross section for $\nu_\mu e^-$ scattering is of order $(1/120\pi)(g^2/M_W^2)^2\times m_e E_\nu$. Thus, the ratio of the cross sections at some incident beam energy E is roughly $\sigma(\mathrm{XN})/\sigma(\nu e) \sim (1\ \mathrm{GeV}/E)\times 10^{-2}$ for $E > m_X$. For $E \sim m_X$, the ratio is to be multiplied by the factor m_N/m_X. Keep in mind that the ν scattering cross section on nucleon is about $\sim 10^3$ times the ν cross section on electron.

We will show later that for the X to be cosmologically interesting, its mass has to be in the 10 GeV range. If so, then the cross section for X scattering in matter is many orders of magnitude below that for neutrino (or anti-neutrino) scattering in matter. (If for some reason $m_X \ll m_N$, X scattering at very low energy could conceivably compete with ν scattering.)

Of course, the rather feeble interaction of X with ordinary fermions also implies that the production cross section of X in laboratory processes such as pp collisions is very small.

Interestingly, while the interaction of X with ordinary matter is utterly negligible, its interaction with the Higgs particle may be very important. If $m_H > 2m_X$, then the mode $H \to 2X$ may dominate the decay of the Higgs particle. The rate for this decay is given by

$$\Gamma(H\to 2X) = (1/32\pi)\left[(\rho v)^2/m_H\right] \times\left(1 - 4m_X^2/m_H^2\right)^{1/2}, \tag{20}$$

to be compared with the rate for Higgs decay into a muon–anti-muon pair

$$\Gamma(\mathrm{H} \to \mu\bar{\mu}) = (1/8\pi)(m_\mu/v)m_\mathrm{H} \times \left(1 - 4m_\mu^2/m_\mathrm{H}^2\right)^{1/2}, \tag{21}$$

$$\Gamma(\mathrm{H} \to \mathrm{XX})/\Gamma(\mathrm{H} \to \mu\bar{\mu}) \sim \tfrac{1}{16}(m_\mathrm{H}/m_\mu)^2 \text{ (phase space factor)}. \tag{22}$$

Wilczek [6] has pointed out that if there are many heavy quark flavors the dominant mode for Higgs decay may be the two-gluon mode with a relative branching ratio

$$\Gamma(\mathrm{H} \to \mathrm{XX})/\Gamma(\mathrm{H} \to \mu\bar{\mu}) = \tfrac{1}{9}N^2(\bar{g}^2/4\pi^2)^2(m_\mathrm{H}/m_\mu)^2 \times \text{(phase space factor)}. \tag{23}$$

Here N denotes the number of heavy quark flavors. The decay is suppressed by four powers of the strong coupling $\bar{g}$. Some of the ongoing experimental Higgs searches are keyed to the Higgs decaying into a pair of muons. The existence of the X with a mass below $\frac{1}{2}m_\mathrm{H}$ would further reduce the branching ratio into muons. A very massive Higgs, with $m_\mathrm{H} \sim m_\mathrm{W}$ say, may then be very difficult to detect.

An interesting question is whether the pair annihilation of X can account for the observed excess in cosmic rays of anti-protons [7][‡5] in the energy range $\lesssim 1$ GeV. Recently, Silk and Srednicki [9] have addressed the same question for photino annihilation. The physics involved is straightforward and we can read much of what we need from ref. [9]. From the observed rotation velocity of the galaxy, a model for the density distribution of the halo, and the annihilation rate per unit volume $n_\mathrm{X}^2\langle v\sigma\rangle$, we can deduce the observed flux of annihilation product. We will take $m_\mathrm{H} = 20$, $m_\mathrm{X} = 14$ for which $\langle v\sigma\rangle \simeq 1.8 \times 10^{-27}$ cm^3/s. (This is roughly an order of magnitude smaller than the value of $\langle v\sigma\rangle$ for photino annihilation as is consistent with the fact that our m_X is larger than the photino mass considered in ref. [9]. The other difference is that X annihilation much above the $\mathrm{b\bar{b}}$ threshold goes almost entirely into the $\mathrm{b\bar{b}}$ channel.) To obtain the observed $\bar{\mathrm{p}}$ flux, we have to fold in an enhancement factor due to the trapping of cosmic ray in the halo for about 10^8 yr and the average number of $\bar{\mathrm{p}}$ yields from a pair of $\mathrm{b\bar{b}}$ (about 0.3 according to experiment [7][‡5]). We find a flux

$$F_{\bar{\mathrm{p}}} = (2 \times 10^{-6} \times 1.8f)\, v_{200}^4 a_{10}^{-2} K_{29}^{-1} (\mathrm{cm^2\ s\ st})^{-1},$$

compared to the observed value of 3×10^{-6} (cm^2 s st)$^{-1}$. Here v_{200}, a_{10}, K_{29} denote, respectively, the galactic rotation velocity, galactic core radius, and the cosmic diffusion coefficient in units of their standard values. Finally, f is a fudge factor describing the uncertainty in estimating the fraction of $\bar{p}$ in $\mathrm{X} + \mathrm{X} \to \mathrm{b} + \bar{\mathrm{b}} \to \bar{\mathrm{p}} + (\cdots)$ which has energy $\lesssim 1$ GeV. It may be estimated [9] that the mean energy of the $\bar{\mathrm{p}}$ is about $\lesssim 0.1 m_\mathrm{X}$ and so take the relevant fraction to be ~ 1.

In conclusion, we have shown that without introducing any new physical principles, we can modify the standard theory "minimally" to account for the dark matter of the universe. In our discussion, it is crucial that the discrete symmetry $\mathrm{X} \to -\mathrm{X}$ is not broken. If this discrete symmetry is broken, the X will mix with the Higgs and the X particle will decay into fermion pairs and gluon pairs.

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[‡5] For a brief review, see section 3.2 of ref. [8].

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