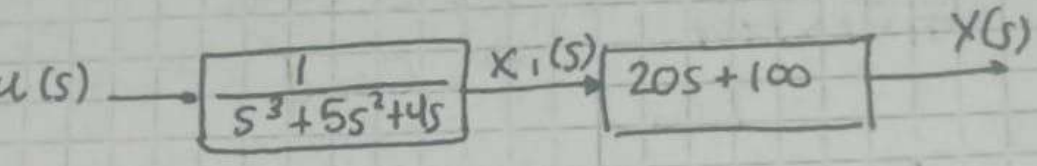
Tarea 5

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)} \quad \begin{cases} 0.5 = 9.5\% \\ t_s = 0.745g \end{cases}$$

$$G(s) = \frac{20s+100}{s^3+5s^2+4s}$$



$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s}$$

$x_1 = x_1$
$x_2 = \dot{x}_1$
$x_3 = \dot{x}_2 = \ddot{x}_1$
$\dot{x}_3 = \dddot{x}_1$

$$u = \dddot{x}_1 + 5\ddot{x}_1 + 4\dot{x}_1 = u$$

$$\dot{x}_3 = -5x_3 - 4x_2 + u \quad (1)$$

$$Y(s) = (20s+100)X_1(s)$$

$$= 20\dot{x}_1 + 100x \Rightarrow \quad Y = 20x_2 + 100x_1 \quad (2)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\%0.5 = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$\ln(0.045) = \ln\left(e^{-(\zeta\pi/\sqrt{1-\zeta^2})}\right)$$

$$-2.3539 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$5.5407(1-\zeta^2) = \zeta^2\pi^2$$

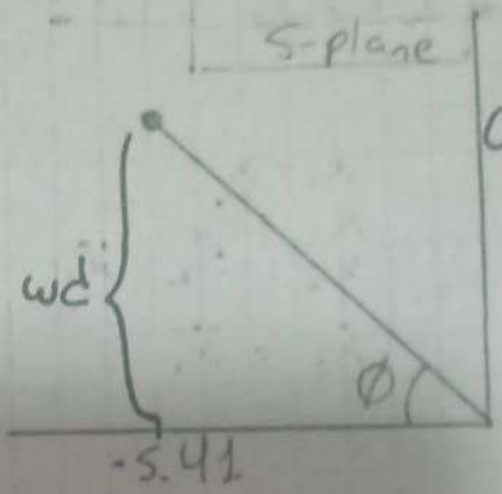
$$5.5407 = \zeta^2(\pi^2 + 5.5407)$$

$$\zeta^2 = \frac{5.5407}{\pi^2 + 5.5407} \Rightarrow \zeta = 0,5496$$

$$\zeta = \cos\phi$$

$s = \sigma + j\omega_d$

$= \zeta\omega_n$



$\cos(\zeta) = \phi$

$\phi = 53.16°$

$t_s = 0.74$

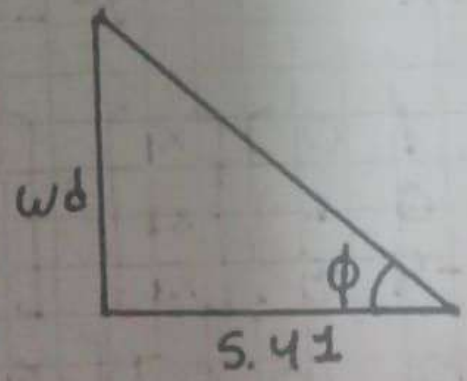
$t_s = \frac{4}{\sigma}$

$$\sigma = \frac{4}{0.74} = 5.405$$

$$5.0405 = \zeta\omega_n$$

$$\frac{5.0405}{0.5496} = \omega_n$$

$$9.02 = \omega_n$$



$\tan\phi = \frac{\omega_d}{5.41}$

$\omega \approx 7.21$