UNIVERSIDAD DISTRITAL
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Proyecto Curricular de Ingeniería Electrónica

SISTEMAS DINÁMICOS
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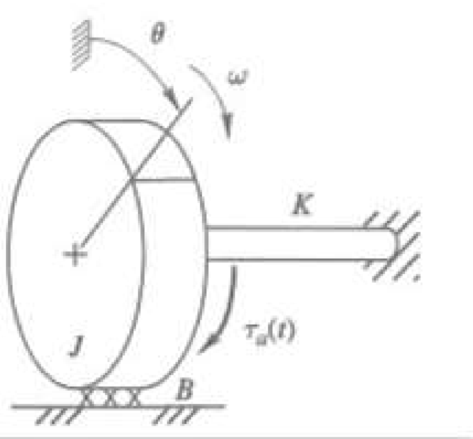
20191005107

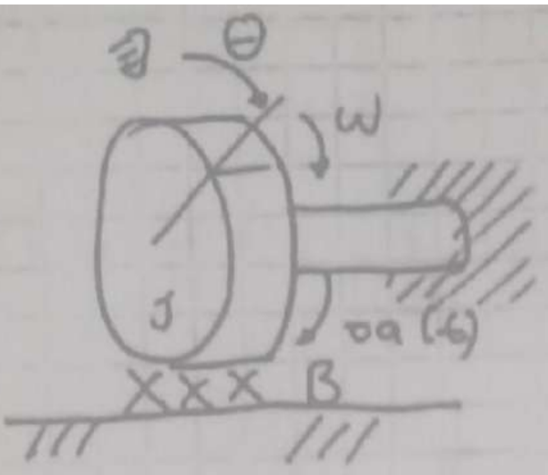
Parcial 2

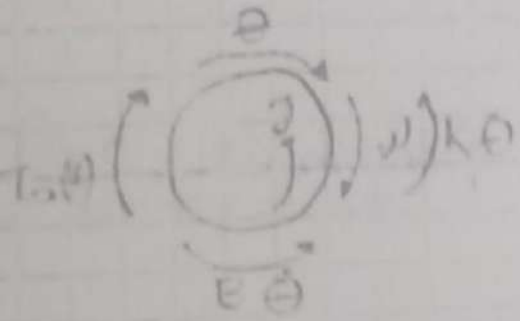
1. Para el sistema rotacional en la figura, determine:

   a. La respresentación en el espacio de estados (b) junto a su diagrama de bloques, (c) así como el diagrama de flijo de señal.

   b. La función de transferencia







$$J\ddot{\theta} + B\dot{\theta} + K\theta = \tau_a$$

$$\ddot{\theta} = \frac{\tau_a}{J} - \frac{B\dot{\theta}}{J} - \frac{K\theta}{J}$$

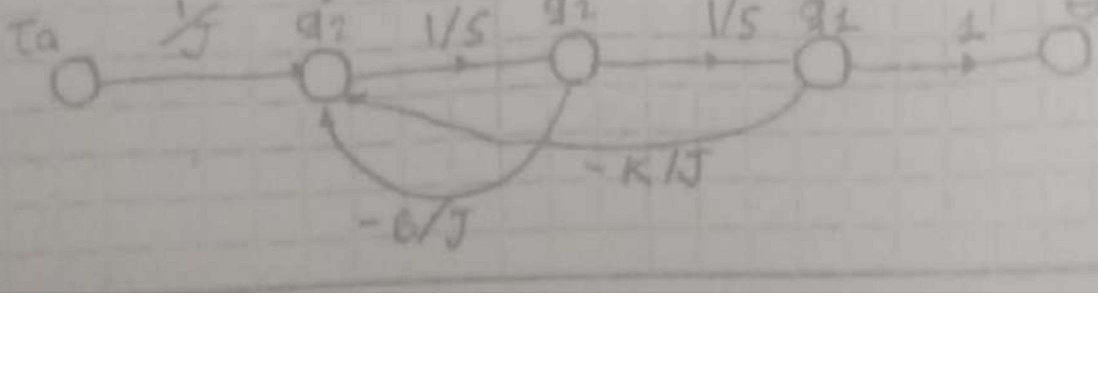
$q_1 = \theta$

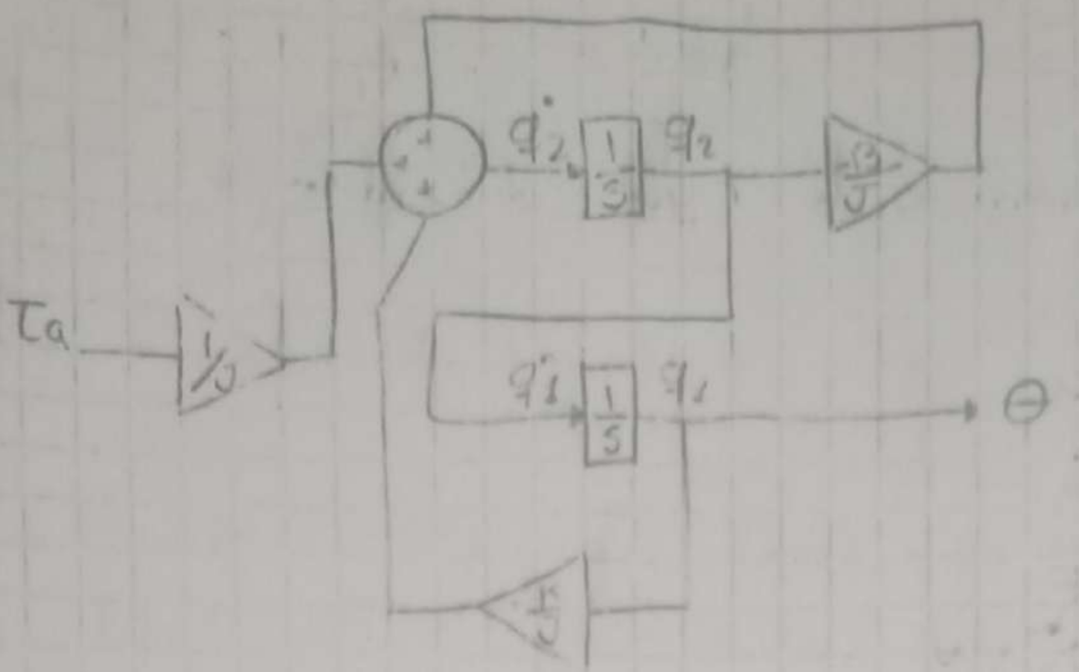
$q_2 = \dot{q}_1 = \dot{\theta}$

$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$





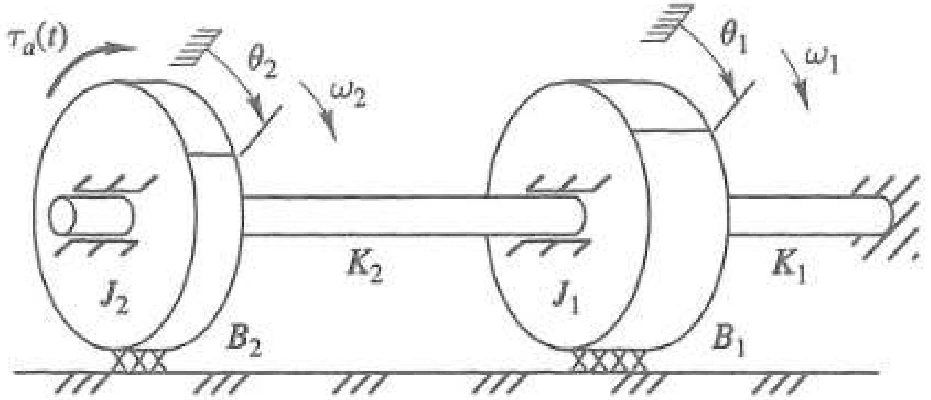
$$\theta(s)(Js^2 + Bs + K) = \tau a(s)$$

$$\frac{\theta(s)}{\tau a(s)} = \frac{1}{Js^2 + Bs + K}$$

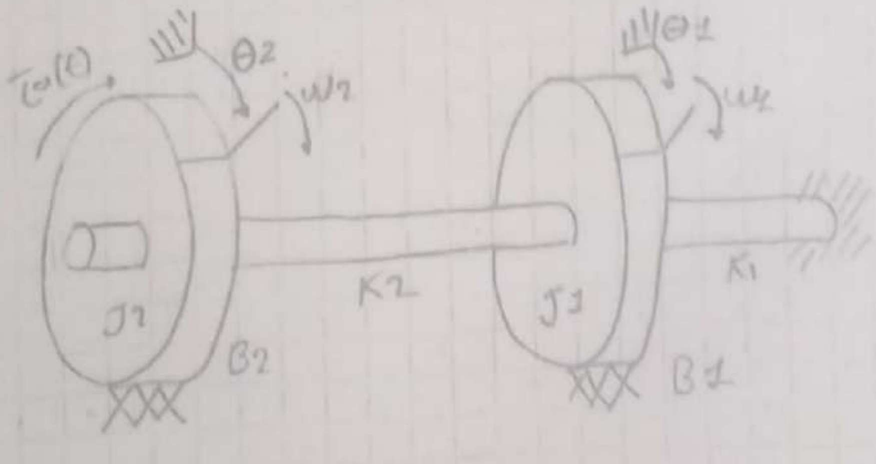
2. Para el sistema rotacional en la figura, asuma $\theta_2 > \theta_L$ y determine:

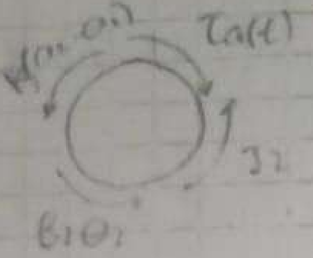
a. La función de transferencia relacionando θ_2 y τ_a.

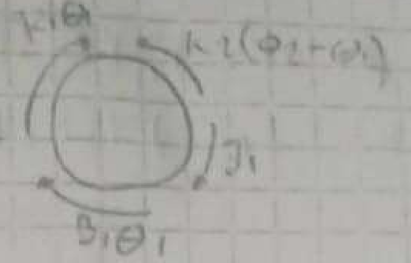
b. La respresentación en el espacio de estados (b) junto a su diagrama de bloques, (c) así como el diagrama de flijo de señal. Todo en términos de θ_2



2.







$$B_2\dot{\theta}_2 + k_1(\theta_2-\theta_1) + J_2\ddot{\theta}_2 = T_a(t) \quad ①$$

$$k_2(\theta_2-\theta_1) = J_1\ddot{\theta}_1 + K_1\theta_1 + B\dot{\theta}_1 \quad ②$$

$$K_2\theta_2(s) - K_2\theta_1(s) = s^2 J_1\theta_1(s) + k_1\theta_1(s) + sB_1\theta_1(s)$$

$$\theta_1(s) = \frac{k_2\theta_2(s)}{s^2J_1 + sB_1 + K_1 + K_2}$$

$$\theta_2(s)\,sB_2 + \theta_2(s)K_2 - K_2\theta_1(s) + J_2 s^2\theta_2(s) = T_a(s)$$

$$\theta_2(s)(s^2J_2 + sB_2 + K_2) - \frac{K_2^2\theta_2(s)}{s^2J_1 + sB_1 + K_1 + K_2} = T_a(s)$$

$$\theta_2(s)\left(\frac{(s^2J_2 + sB_2 + K_2)(s^2J_1 + sB_1 + K_1 + K_2) - K_2^2}{s^2J_1 + sB_1 + K_1 + K_2}\right) = T_a(s)$$

$$\frac{\theta_2(s)}{T_a(s)} = \frac{s^2J_1 + sB_1 + K_1 + K_2}{(s^2J_2 + sB_2 + K_2)(s^2J_1 + sB_1 + K_1 + K_2) - K_2^2}$$

$$\theta_2(s)\big(J_1J_2s^4 + s^3(B_1J_2 + B_2J_1) + s^2(B_1B_2 + K_1J_2 + J_2K_2 + K_2J_1) + s(B_2K_1 + B_2K_2 + B_1K_2) + (K_2K_1)\big)$$

$$=$$

$$T_a(s)(s^2J_1 + sB_1 + K_1 + K_2)$$

$$\theta^{IV} J_1 J_2 + \theta^{III}(B_1 J_2 + B_2 J_1) + \theta^{II}(B_1 B_2 + k_1 J_2 + J_2 k_2 + k_2 J_1) +$$
$$\theta^{I}(B_2 k_1 + B_1 k_2 + B_1 k_2) + (k_2 k_1)\theta$$
$$=$$
$$\tau a^{II} J_1 + \tau a^{I} B_1 + \tau a(k_1 + k_2)$$

$$x_1 = y - \beta_0 u$$
$$x_2 = \dot{y} - \beta_0 \dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u$$
$$x_3 = \ddot{y} - \beta_0 \ddot{u} - \beta_1 \dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$$
$$x_4 = \dddot{y} - \beta_0 \dddot{u} - \beta_1 \ddot{u} - \beta_2 \dot{u} - \beta_3 u = \dot{x}_3 - \beta_3 u$$

$$\beta_0 = b_0$$
$$\beta_1 = b_1 - a_1 \beta_0$$
$$\beta_2 = b_2 - a_1 \beta_1 - a_2 \beta_0$$
$$\beta_3 = b_3 - a_1 \beta_2 - a_2 \beta_1 - a_3 \beta_0$$
$$\beta_4 = b_4 - a_1 \beta_3 - a_2 \beta_2 - a_3 \beta_1 - a_4 \beta_0$$

$$y^{IV} + a_1 y^{III} + a_2 y^{II} + a_3 y^{I} + a_4 y = b_0 u^{IV} + b_1 u^{III} + b_2 u^{II} + b_3 u^{I} + b_4 u$$

$$b_0 = 0 \qquad b_1 = 0 \qquad b_2 = \frac{1}{J_2} \qquad b_3 = \frac{B_1}{J_1 J_2} \qquad b_4 = \frac{k_1 + k_2}{J_1 J_2}$$

$$a_1 = \frac{B_1 J_2 + B_2 J_1}{J_1 J_2} \qquad a_2 = \frac{B_1 B_2 + k_1 J_2 + J_2 k_2 + k_2 J_1}{J_1 J_2}$$

$$a_3 = \frac{B_1 k_1 + B_2 k_2 + B_1 k_2}{J_1 J_2} \qquad a_4 = \frac{k_2 k_1}{J_1 J_2}$$

$\beta_0 = 0$ $\quad \beta_3 = \frac{B_1}{J_1 J_2} - \left(\frac{B_1 J_2 + B_2 J_1}{J_1 J_2^2}\right)$

$\beta_1 = 0$

$\beta_2 = \frac{1}{J_2}$ $\quad \beta_3 = \frac{-B_2 J_1}{J_1 J_2^2} = -\frac{B_2}{J_2^2}$

$\beta_4 = \frac{k_1 + k_2}{J_1 J_2} - \left(\frac{B_1}{J} \frac{J_2 + B_2 J_1}{J_2}\right)\left(-\frac{B_2}{J_2^2}\right) - \left(\frac{B_1 B_2 + k_1 J_2 + J_2 k_2 + k_2 J_1}{J_1 J_2^2}\right)$

$= \frac{k_1 + k_2}{J_1 J_2} + \frac{B_1 B_2 J_2 + B_2^2 J}{J_1 J_2^3} - \frac{B_1 B_2 J_2 - k_1 J_2^2 - J_2^2 k_2 - k_2 J_1 J_2}{J_1 J_2^3}$

$= \frac{k_1 J_2^2 + k_1 J_2^2 + B_2 B_1 J_2 + B_2^2 J_1 - B_1 B_2 J_2 - k_1 J_2^2 - J_2^2 k_2 - k_2 J_1 J_2}{J_1 J_2^3}$

$= \frac{B_2^2 J_1 - k_2 J_1 J_2}{J_1 J_2^3} = \frac{B_2^2 - k_2 J_2}{J_2^3}$

$\dot{x}_4 = \ddddot{y} - \beta_0 \dddot{u} - \beta_1 \ddot{u} - \beta_2 \ddot{u} - \beta_3 \dot{u}$

$\dot{x}_4 = y^{IV} - \frac{u^{II}}{J_2} + \frac{B_2}{J_2^2} u^{I}$

$\dot{x}_4 = -a_4 x_1 - a_3 x_2 - a_2 x_1 - a_1 x_4 + \beta_4 u$

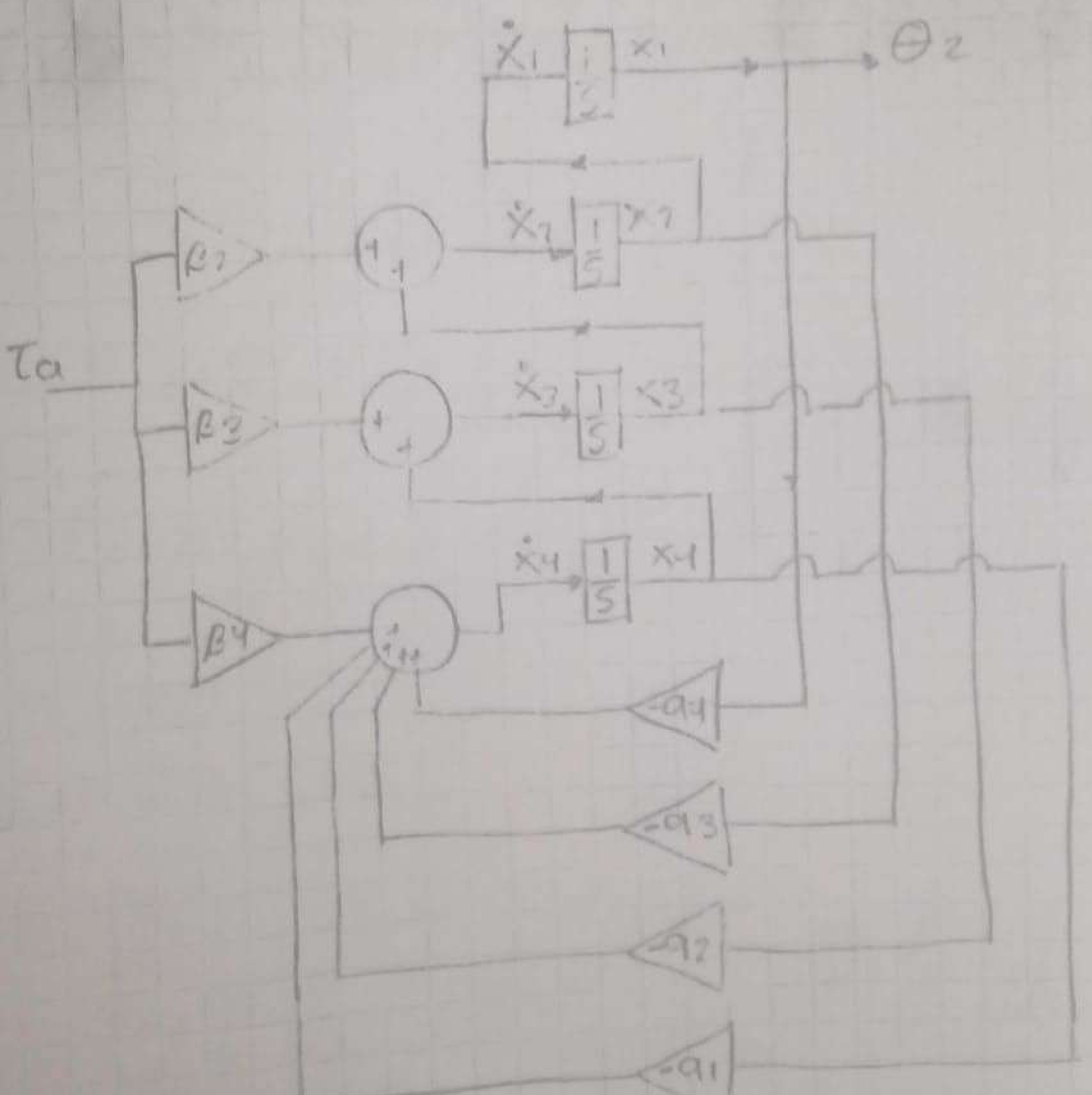
$\dot{x}_1 = x_2$

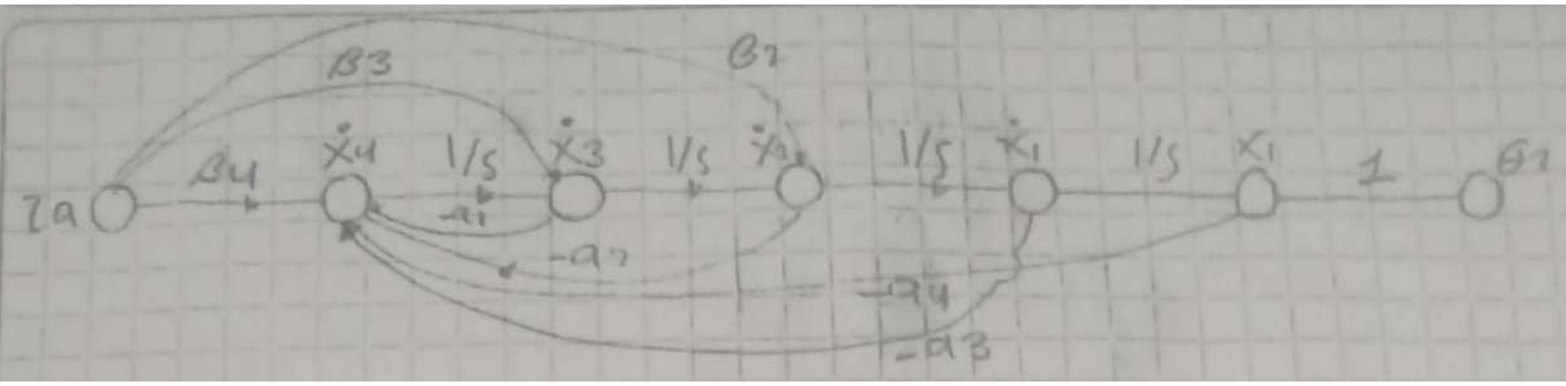
$\dot{x}_2 = x_3 + \frac{T_a}{J_2}$

$\dot{x}_3 = x_4 - \frac{B_2}{J_2^2} T_a$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$





Ia
β4
ẋ4
1/s
ẋ3
1/s
ẋ2
1/s
ẋ1
1/s
x1
1
θr
β3
β2
-a1
-a2
-a4
-a3

3) $K_1 = 0$

$$a_1 = \frac{B_1 J_2 + B_2 J_1}{J_1 J_2}$$

$$a_2 = \frac{B_1 B_2 + J_2 K_2 + J_1 K_1}{J_1 J_2}$$

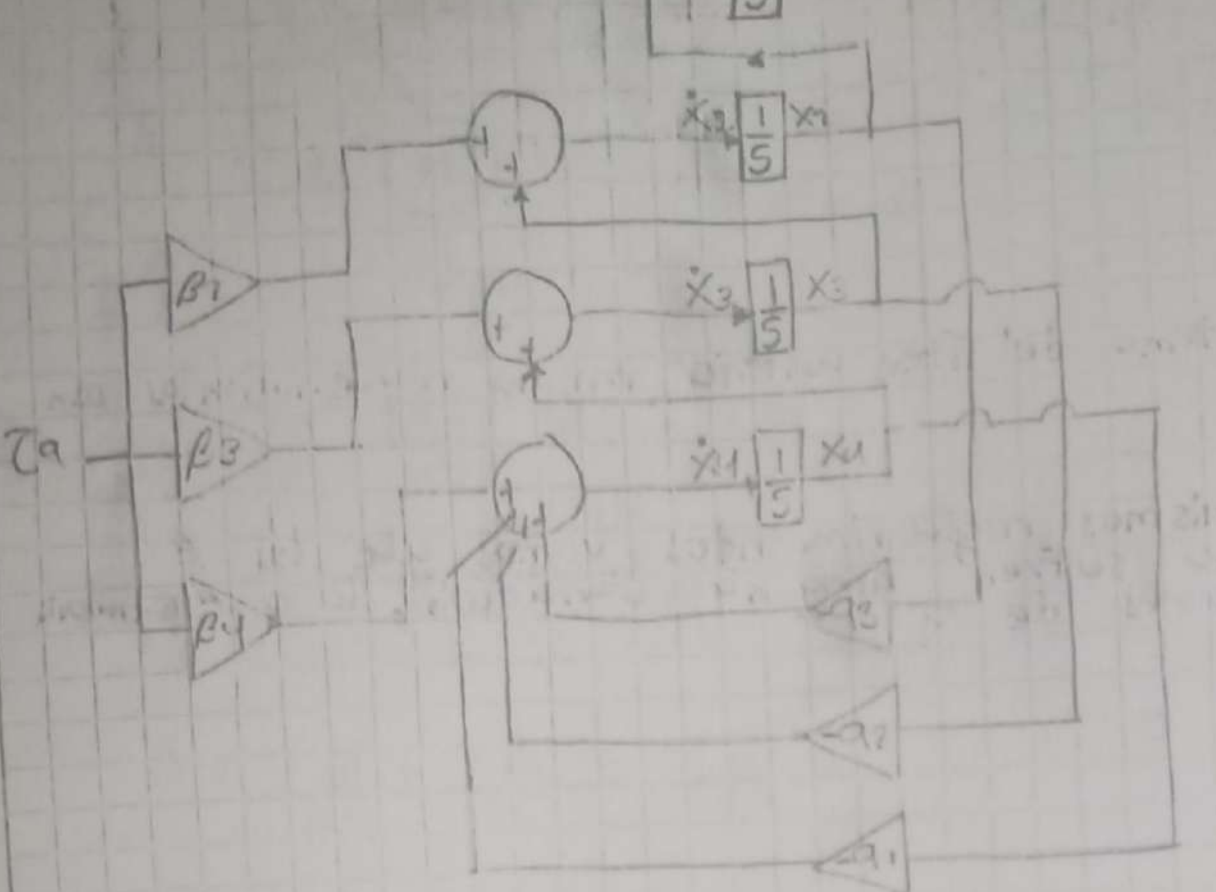
$$a_3 = \frac{B_2 K_2 + B_1 K_2}{J_1 J_2}$$

$$a_4 = 0$$

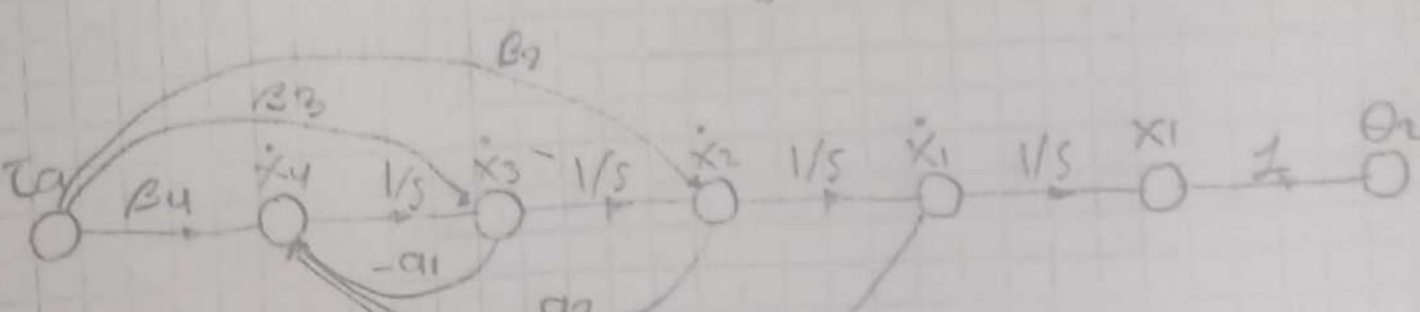
- Espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$



$\dot{x}_1$
1/S
x_1
θ_2
$\dot{x}_2$
x_2
$\dot{x}_3$
x_3
$\dot{x}_4$
x_4
τa
β_2
β_3
β_4
$-a_3$
$-a_2$
$-a_1$

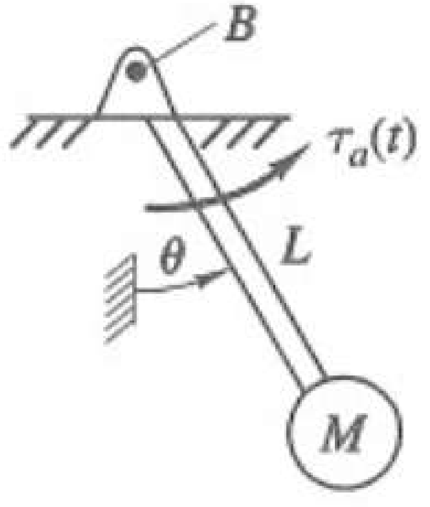


β_2
β_3
τa
β_4
$\dot{x}_4$
1/S
$\dot{x}_3$
1/S
$\dot{x}_2$
1/S
$\dot{x}_1$
1/S
x_1
1
θ_2
$-a_1$
$-a_2$
$-a_3$

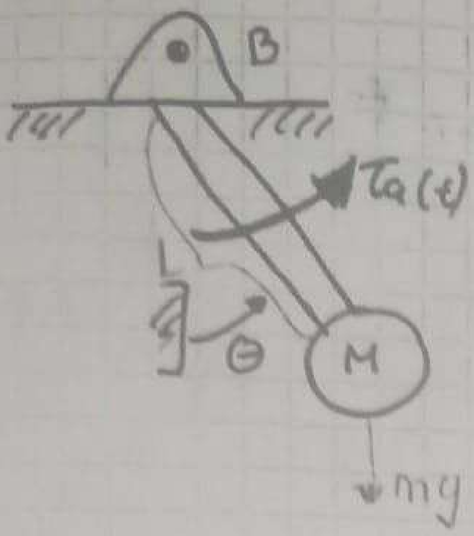
4. Para el sistema rotacional en la figura, determine:

a. La función de transferencia relacionando θ y τ_a.

b. La respresentación en el espacio de estados (c) junto a su diagrama de bloques, (d) así como el diagrama de flujo de señal.



4.



$$\tau_a - MgL\,\text{Sen}\,\theta - B\dot{\theta} = ML^2\ddot{\theta}$$

Se linealiza con $\theta << 1$

$$\text{Sen}\,\theta \approx \theta$$

$$\ddot{\theta} = \frac{\tau_a}{ML^2} - \frac{MgL}{ML^2}\theta - \frac{B\dot{\theta}}{ML^2}$$

$$\ddot{\theta} = \frac{\tau_a}{ML^2} - \frac{g\theta}{L} - \frac{B\dot{\theta}}{ML^2}$$

$$\ddot{\theta} + \frac{g}{L}\theta + \frac{B\dot{\theta}}{ML^2} = \frac{\tau_a}{ML^2}$$

$$\theta(s)\left(s^2 + \frac{g}{L} + \frac{sB}{ML^2}\right) = \frac{\tau_a(s)}{ML^2}$$

$$\frac{\theta(s)}{\tau_a(s)} = \frac{\frac{1}{ML^2}}{\frac{ML^2s^2 + gML + sB}{ML^2}}$$

$$\frac{\theta(s)}{\tau_a(s)} = \frac{1}{ML^2s^2 + sB + gML}$$

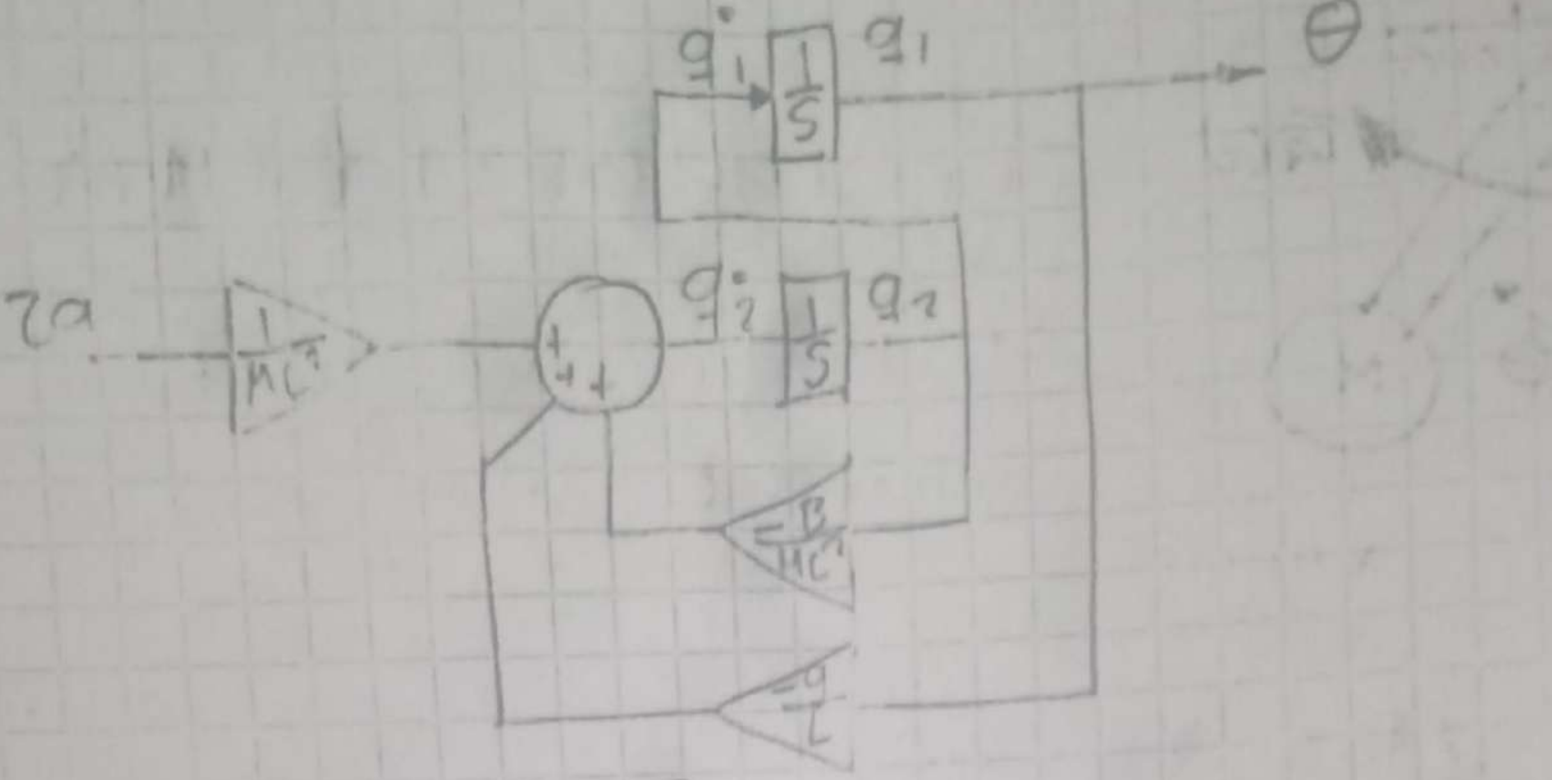
$\theta = q_1$

$\dot{\theta} = \dot{q}_1 = q_2$

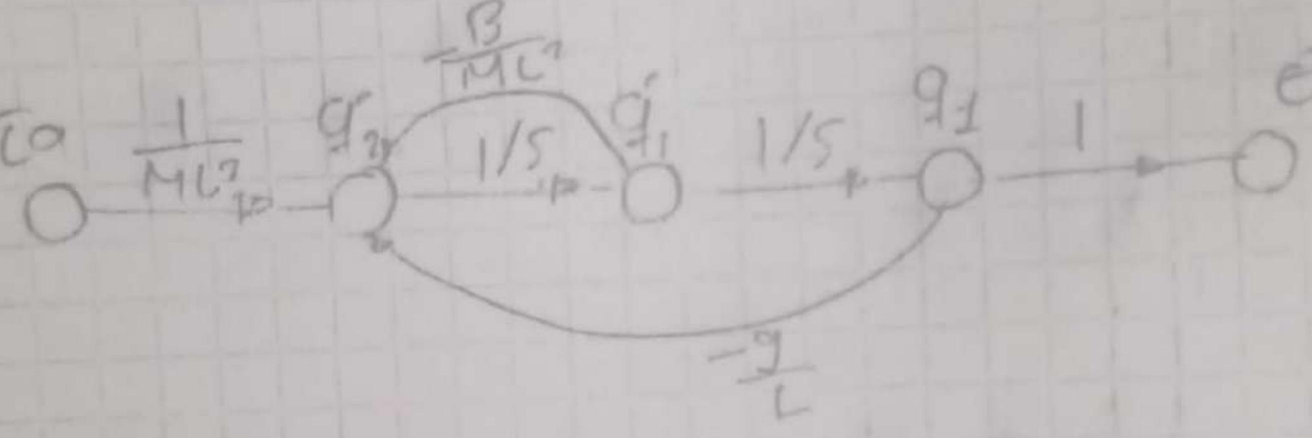
$\dot{q}_2 = \ddot{\theta}$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/L & -\frac{B}{ML^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ML^2} \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$



q̇1
1/S
q1
θ
τa
1/ML²
q̇2
1/S
q2
-B/ML²
-g/L



τa
1/ML²
q̇2
-B/ML²
1/S
q̇1
1/S
q1
1
θ
-g/L