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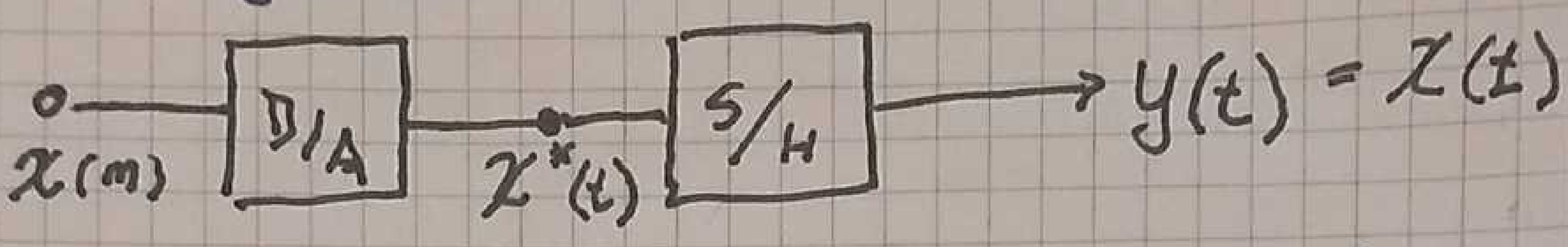
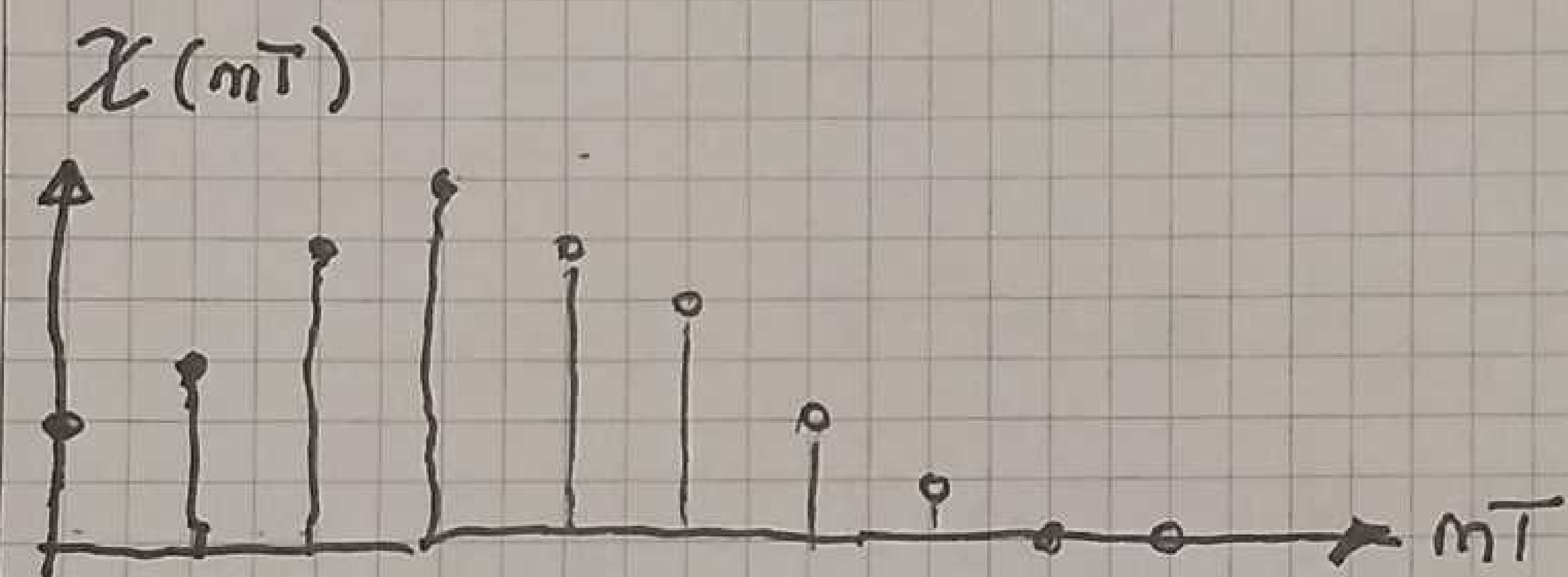
APELLIDO Y NOMBRE:

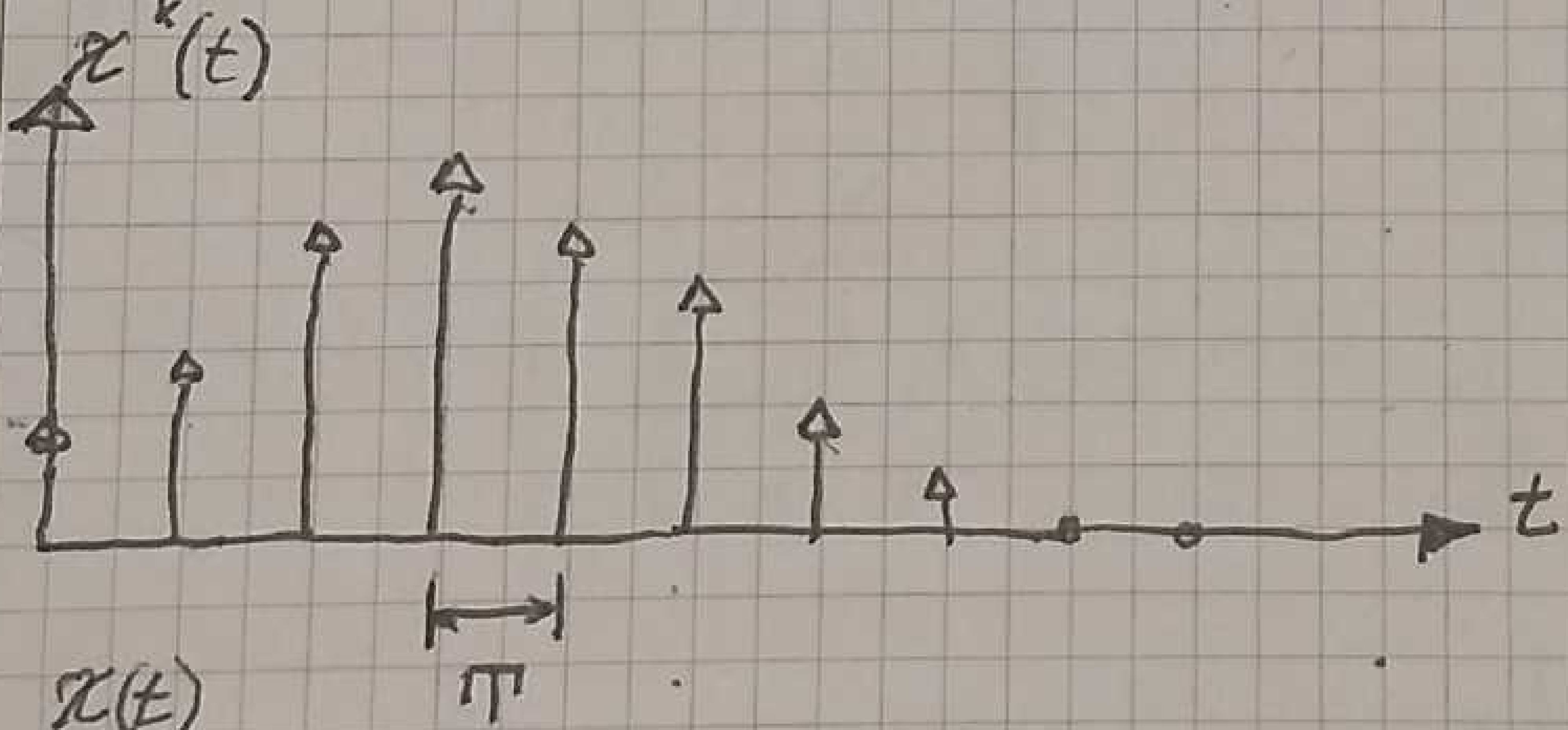
MODELADO DEL ZOH (S/H)

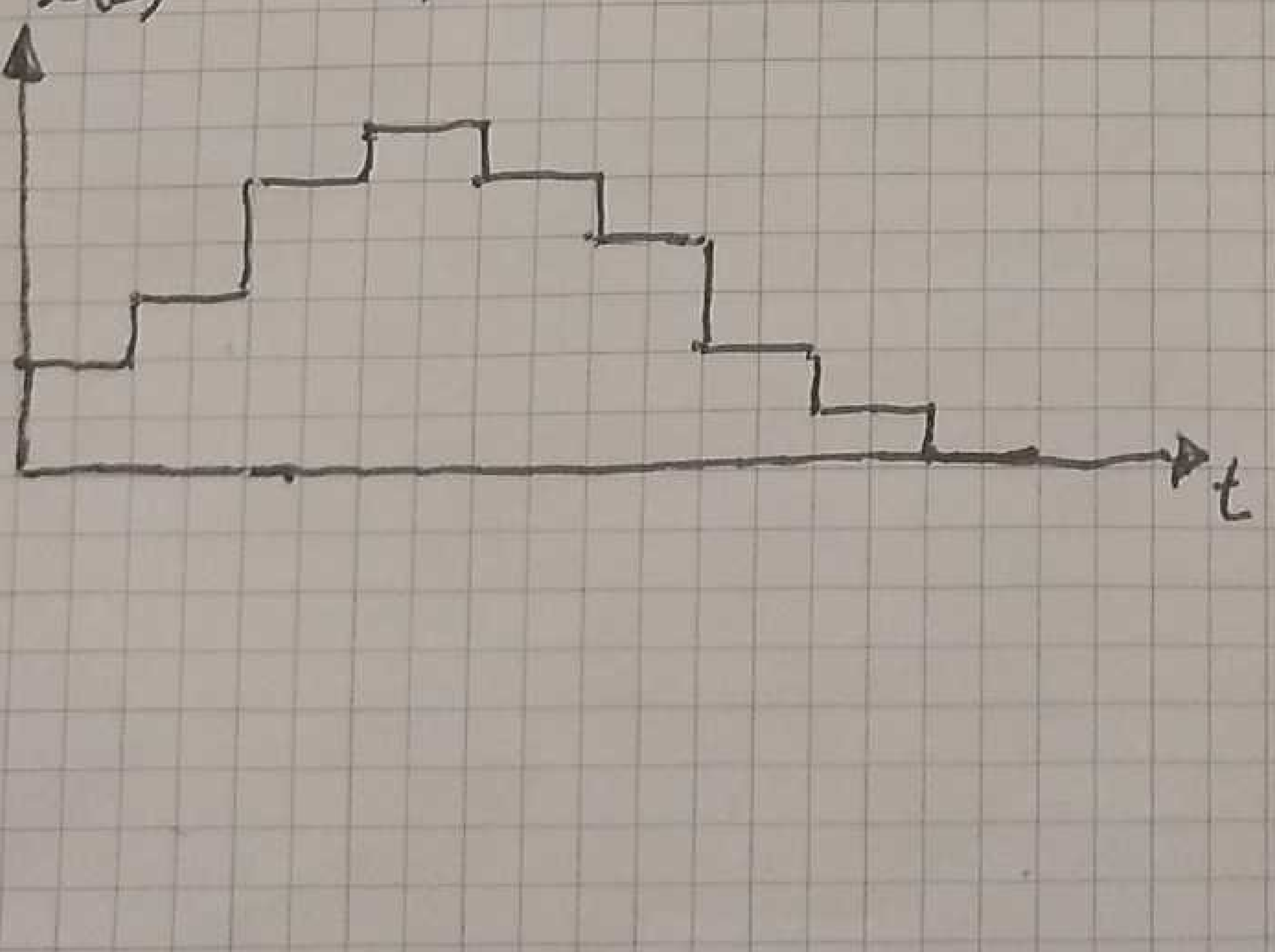
(MUESTREO INSTANTANEO)



$$x^*(t) = \begin{cases} x^*(t)\big|_{t=mT} = x(mT), \quad x^*(t) = 0 \quad t \neq mT \\ x^*(t) = \sum x(t)\,\delta(t-mT) \end{cases}$$







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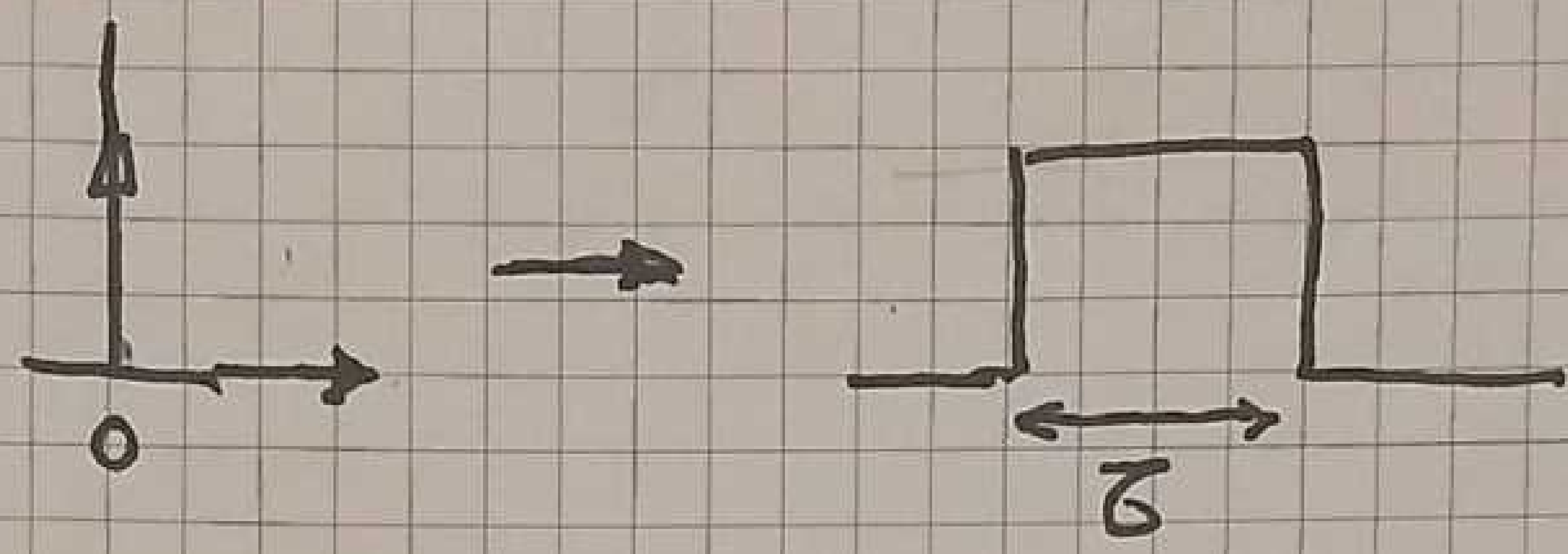
MODELADO del ZOH (S/H)

$$\delta(t) \longrightarrow \boxed{ZOH} \longrightarrow h_{ZOH}(t)$$



Respuesta impulsiva

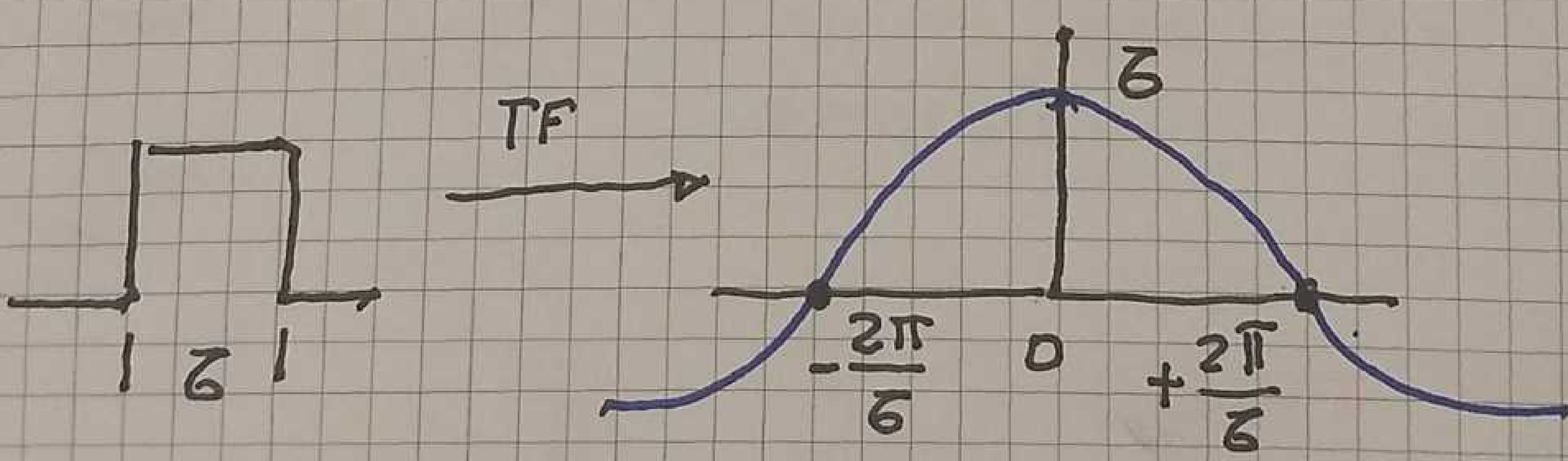
$$x^*(t) \longrightarrow \boxed{ZOH} \longrightarrow y(t)$$

Respuesta a un tren de impulsos

$$y(t) = x^*(t) \circledast h_{ZOH}(t)$$

$$Y(\omega) = X^*(\omega) \cdot H_{ZOH}(\omega) \rightarrow \text{ESPECTRO}$$

$$Y(\omega) = \left[\frac{1}{T}\sum_{m=-\infty}^{\infty} X(\omega - m\omega_s)\right]\left[\tau \, \text{Sinc}\left(\frac{\omega \cdot \tau}{2}\right)\right]$$



$h_{ZOH}(t)$ $\qquad$ $H_{ZOH}(\omega)$

ceros $\longrightarrow$ $\text{Sen}\left(\frac{\omega \tau}{2}\right) = 0$ $\qquad \frac{\omega \tau}{2} = k\pi \qquad k = \pm 1, \pm 2 \ldots$

$$\boxed{\omega = k\,\frac{2\pi}{\tau}} \qquad k = \pm 1, \pm 2, \ldots$$

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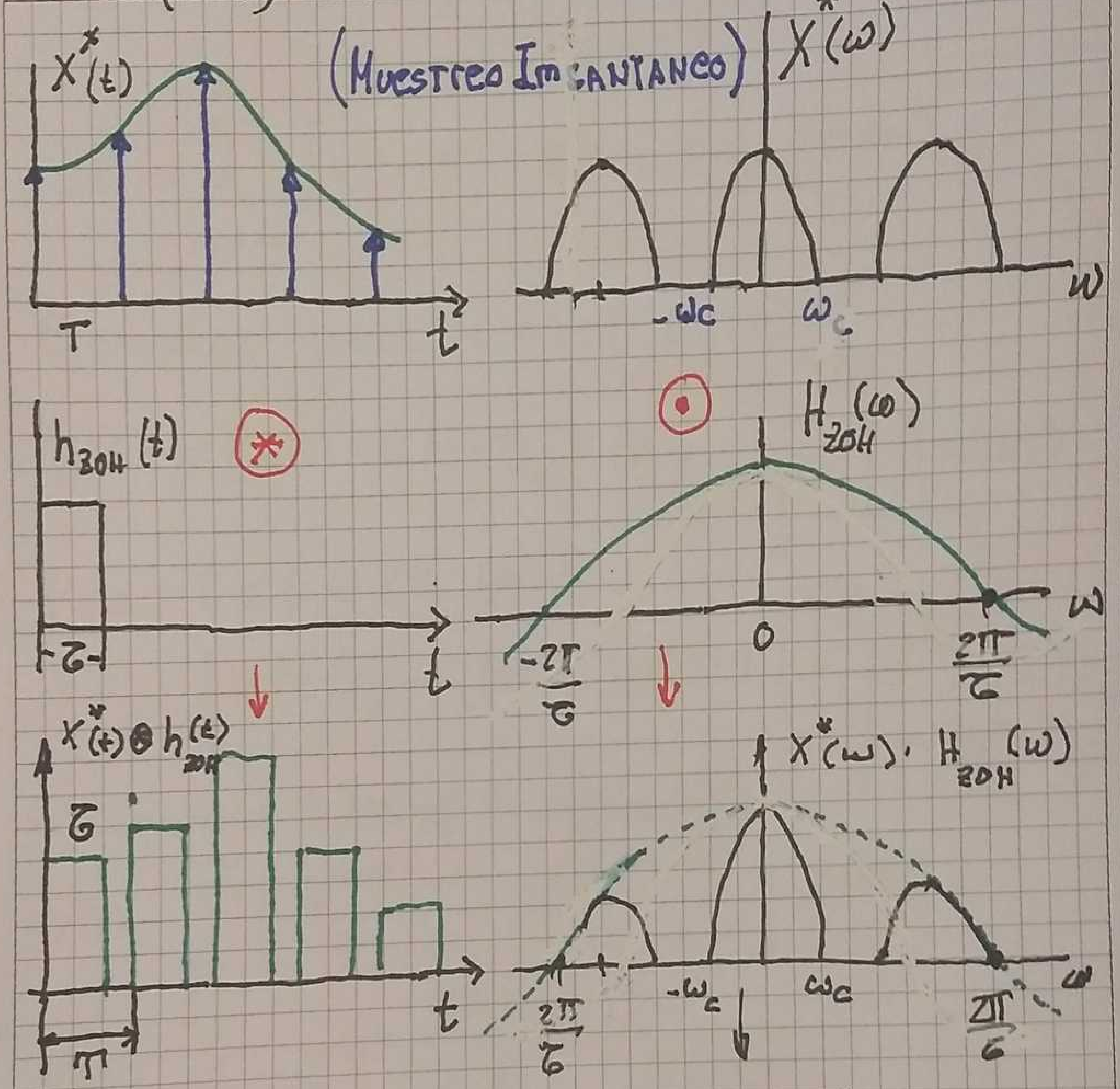
(ZOH) Representacion GRAFICA



El espectro en banda base sufre distorsion.

$\circledast$ conv.

$\odot$ Prod.

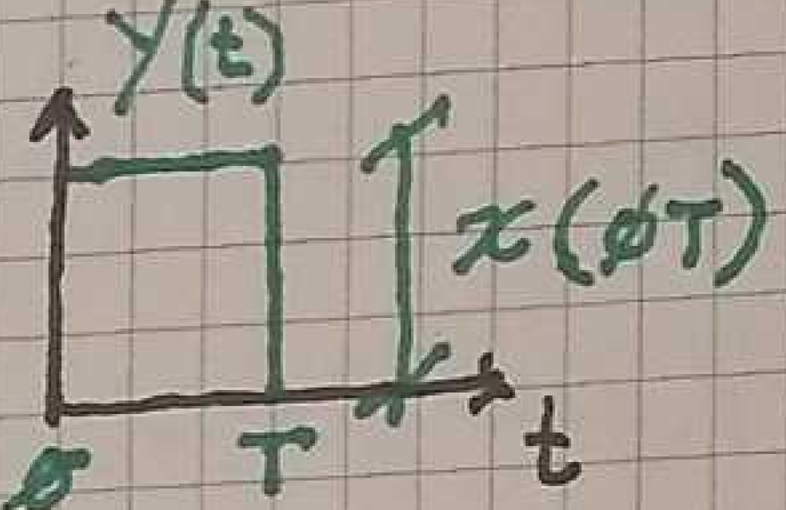
MODELADO ZOH

$(\tau = T)$

$$y(t) = \sum_{m=-\infty}^{\infty} \left[u(t-mT) - u(t-(m+1)T) \right] x(mT) \quad [1]$$

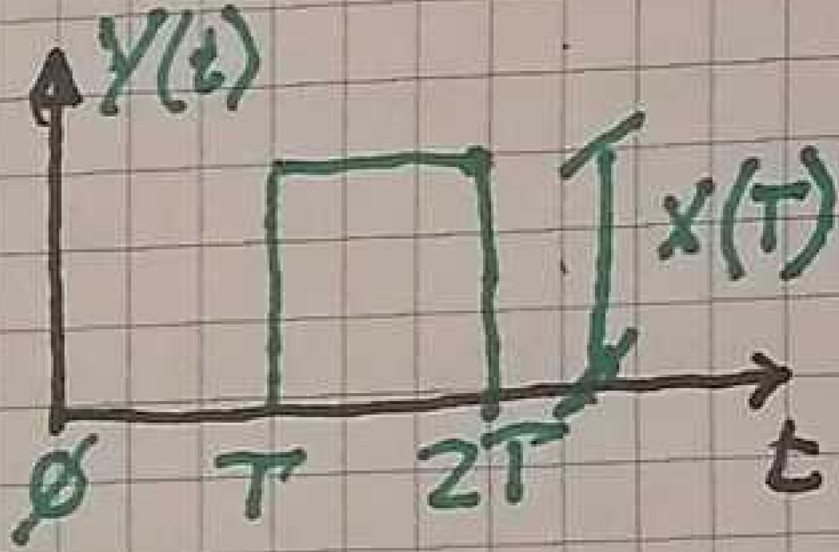
Ejemplo: para $0 \leq t < T$ $(m=0)$

$$y(t) = \left[u(t) - u(t-T) \right] x(\emptyset T)$$



Para $T \leq t \leq 2T$ $(m=1)$

$$y(t) = \left[u(t-T) - u(t-2T) \right] x(T)$$



Si tomamos la transf. de LAPLACE de [1]

$$Y(s) = \sum_{m=-\infty}^{\infty} \left[\frac{e^{-smT}}{s} - \frac{e^{-s(m+1)T}}{s} \right] \cdot x(mT)$$

$$Y(s) = \frac{1-e^{-sT}}{s} \sum_{m=-\infty}^{\infty} e^{-smT} x(mT)$$

$$Y(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} \cdot \underbrace{\sum_{m=-\infty}^{\infty} e^{-j\omega mT} x(mT)}_{\mathcal{F}\left[x^*(t)\right] = X^*(j\omega)} \quad [2]$$

Recordar que:

$$X^*(j\omega) = \frac{1}{T} \sum X(j\omega - jm\,\omega_s) \qquad \omega_s = 2\pi/T$$

$$X^*(j\omega) = \frac{1}{T} X(j\omega) \circledast \sum_{m=-\infty}^{\infty} (j\omega - jm\omega_s)$$

Entonces como:

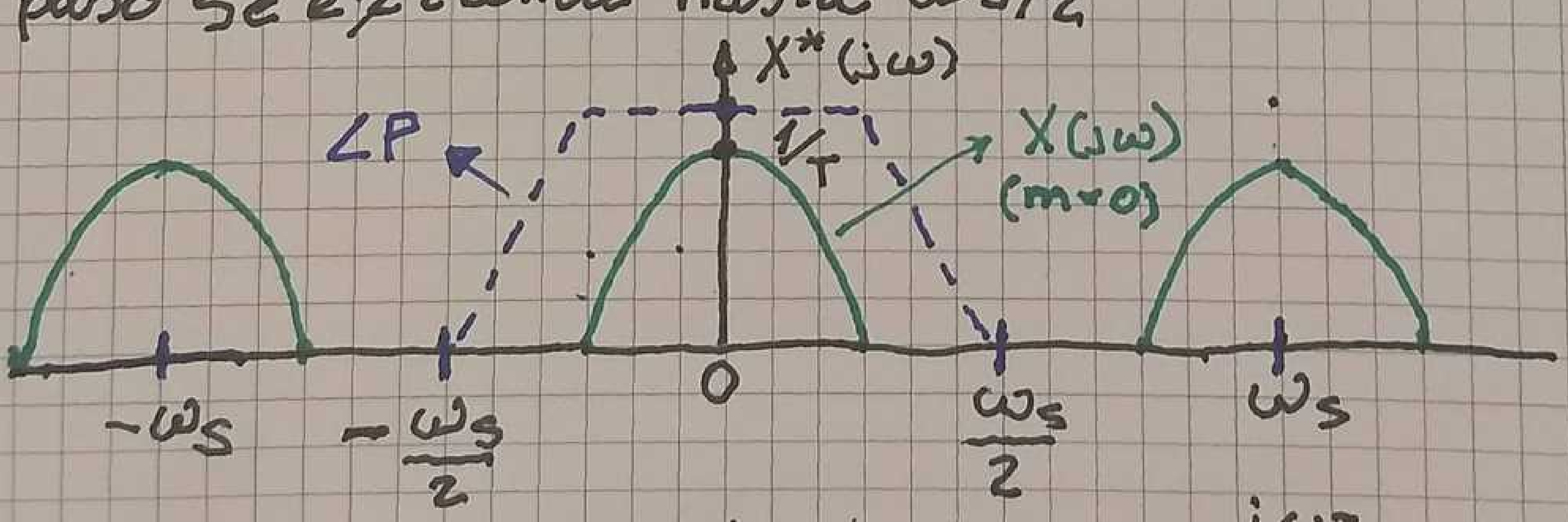
$$Y(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} \cdot X^*(j\omega) \qquad (\text{ver } [2])$$

y $$X^*(j\omega) = \frac{1}{T} X(j\omega) \circledast \sum_{m=-\infty}^{\infty} (j\omega - jm\omega_s)$$

Entonces:

$$Y(j\omega) = \underbrace{\frac{1-e^{-j\omega T}}{j\omega}}_{S/H} \cdot \underbrace{\left[\frac{1}{T} X(j\omega) \circledast \sum_{m=-\infty}^{\infty} (j\omega - jm\omega_s)\right]}_{X^*(j\omega)}$$

El factor entre corchetes representa la repetición periodica del espectro $X(j\omega)$. Para recuperar el espectro en banda base $(m=0)$ debemos poner un filtro pasabajos centrado en $\omega=0$ cuya banda de paso se extiende hasta $\omega_s/2$

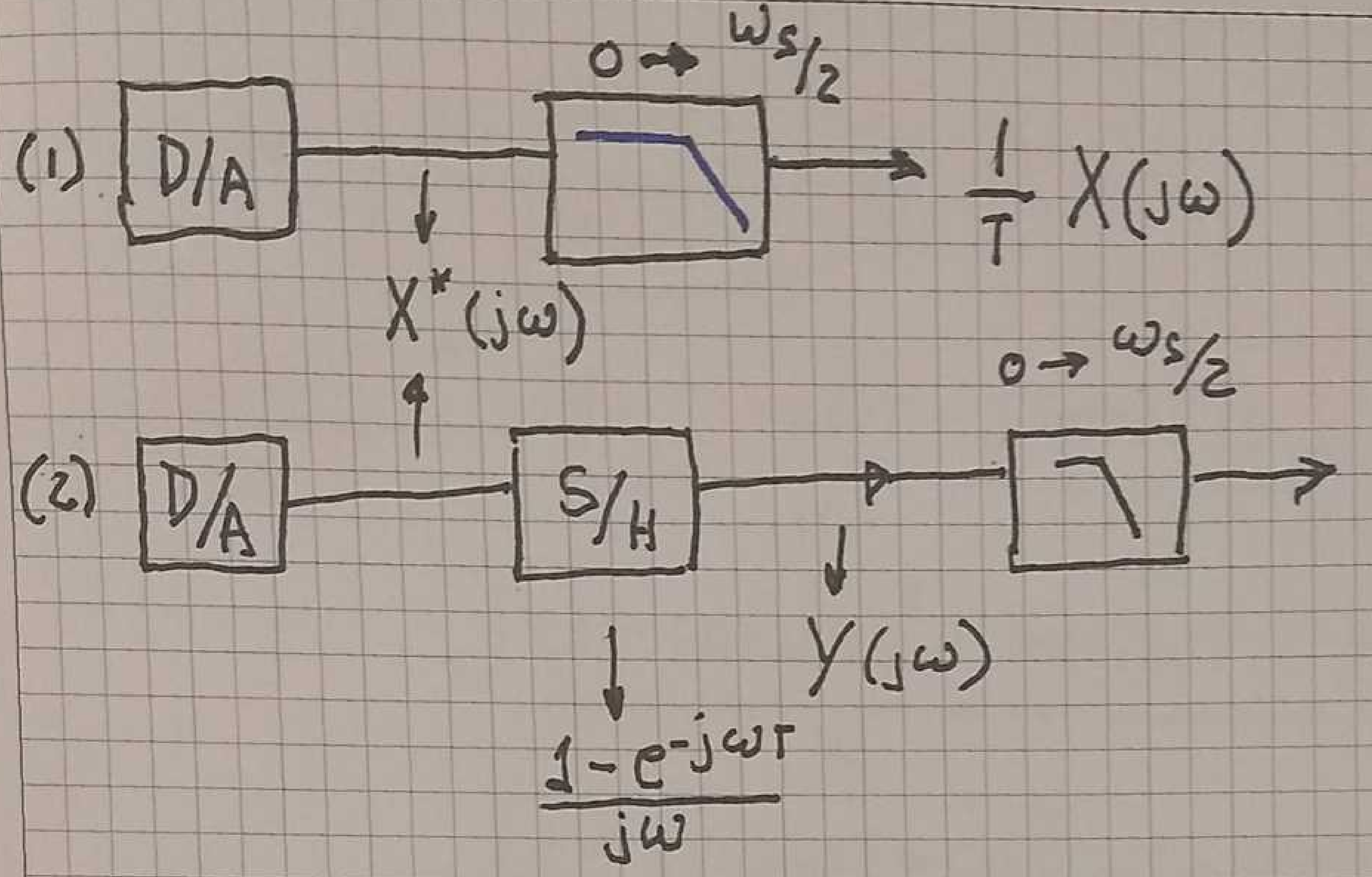


Si tomamos $m=0$ $$\underbrace{Y(j\omega)}_{\text{Espectro c/S|H}} = \underbrace{\frac{1-e^{j\omega T}}{j\omega}}_{\text{Respuesta en frecuencia c/S|H}} \underbrace{\frac{1}{T} X(j\omega)}_{\text{Espectro de la señal muestrada sin S/H}}$$

Espectro c/S|H

Respuesta en frecuencia c/S|H

Espectro de la señal muestrada sin S/H



(1) Sin S/H (2) Con S/H

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Respuesta en frecuencia del S/H

$$|Y(j\omega)| = \underbrace{\left|\frac{1-e^{j\omega T}}{j\omega}\right|} \cdot \frac{1}{T}\,|X(j\omega)|$$

$$\left|\frac{1-e^{j\omega T}}{j\omega}\right| = \left|\frac{1-(\cos\omega T - j\,\mathrm{Sen}\,\omega T)}{j\omega}\right| \quad \text{MODULO}$$

$$= \frac{\left[(1-\cos\omega T)^2 + \mathrm{Sen}^2\omega T\right]^{1/2}}{\omega}$$

$$= \frac{\left[1 - 2\cos\omega T + \cos^2\omega T + \mathrm{Sen}^2\omega T\right]^{1/2}}{\omega}$$

$$= \frac{(2-2\cos\omega T)^{1/2}}{\omega} = \frac{\sqrt{\frac{4(1-\cos\omega T)}{2}}}{\omega}$$

$$\left|\frac{1-e^{j\omega T}}{j\omega}\right| = \frac{\sqrt{\frac{1-\cos\omega T}{2}}}{\omega/2} = \frac{\mathrm{Sen}\,\frac{\omega T}{2}}{\omega/2}$$

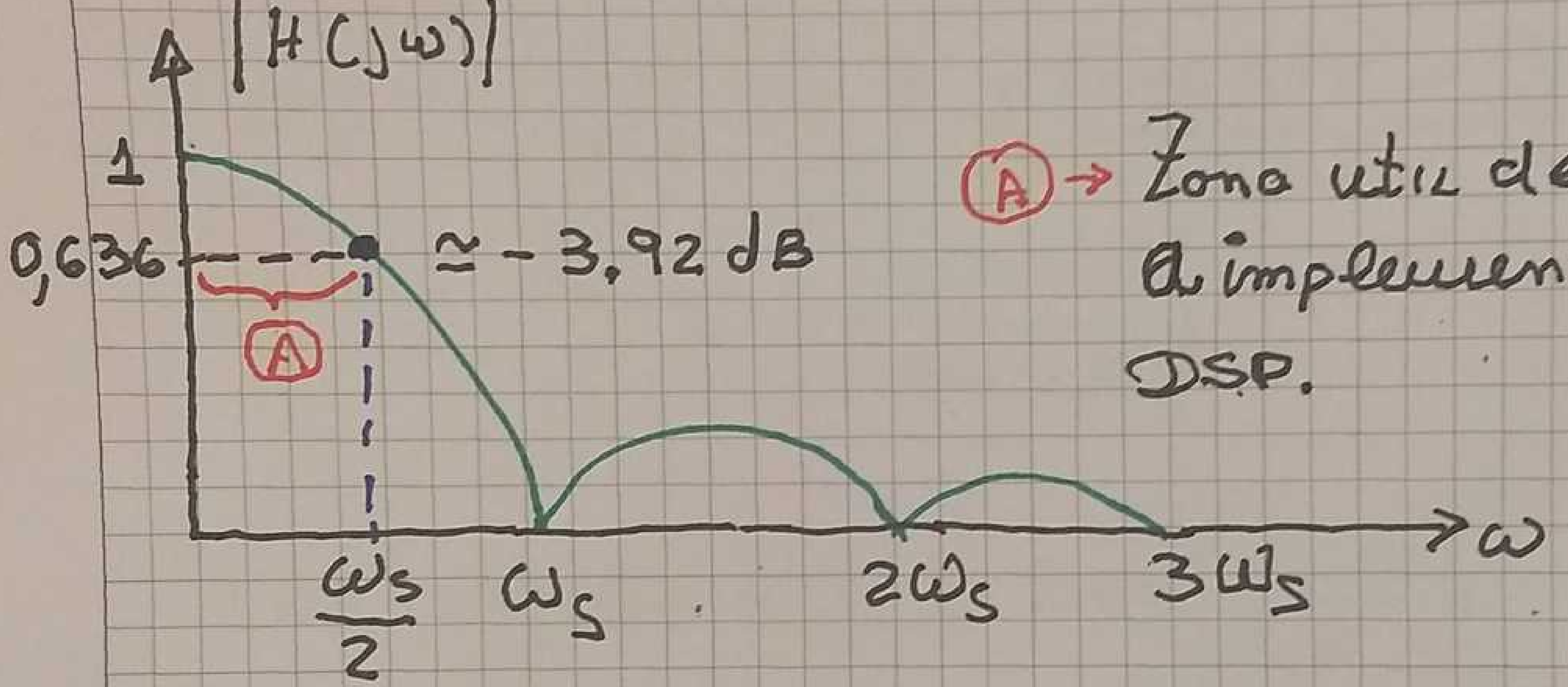
$$\left|\frac{Y(j\omega)}{X(j\omega)}\right| = \frac{1}{T}\,\frac{\mathrm{Sen}\,\frac{\omega T}{2}}{\omega/2} = \boxed{+\frac{\mathrm{Sen}\left(\frac{\omega T}{2}\right)}{\omega T/2}}$$

$$|H(j\omega)| = \frac{\mathrm{Sen}\,z}{z} \qquad z = \frac{\omega T}{2}$$

Ceros de $\frac{\operatorname{sen} x}{x}$ $\begin{cases} \operatorname{Sen} x = 0 & x = k\pi \quad k = \pm 1, \pm 2 \\ \forall x \neq 0 & k \neq 0 \end{cases}$

Osea $\frac{\omega T}{2} = k\pi \rightarrow \omega_k = k\left(\frac{2\pi}{T}\right) = k\,\omega_s$

$\{\omega_k\} = \pm\omega_s, \pm 2\omega_s, \pm 3\omega_s, \dots$



A → Zona util del filtro a implementar en el DSP.

$$\left|H(j\omega)\right|_{\omega = \omega_s/2} = \left|H\left(\frac{\omega_s}{2}\right)\right| = \frac{\operatorname{Sen}\left[\frac{\left(\frac{\omega_s}{2}\right)T}{2}\right]}{\frac{\left(\frac{\omega_s}{2}\right)T}{2}}$$

$$\left|H\left(\frac{\omega_s}{2}\right)\right| = \frac{\operatorname{Sen}\left(\frac{2\pi/\cancel{T}}{2}\right)\frac{\cancel{T}}{\cancel{2}}}{\left(\frac{2\pi/\cancel{T}}{2}\right)\frac{\cancel{T}}{\cancel{2}}} = \frac{\operatorname{Sen}(\pi/2)}{\pi/2} = \frac{1}{\pi/2} = \frac{2}{\pi}$$

$$\left|H(\omega_s/2)\right| = 2/\pi = \boxed{0,6366}$$

$$\left|H(\omega_s/2)\right|_{dB} = \boxed{-3,922\ dB}$$

FASE

$$\phi(\omega) = \operatorname{arctg}\left[\frac{\mathrm{Im}(H(j\omega))}{\mathrm{Re}(H(j\omega))}\right]$$

$$H(j\omega) = \frac{1 - e^{-j\omega T}}{j\omega} = \frac{j\omega - j\omega e^{-\omega T}}{-\omega^2}$$

$$= \frac{j\cancel{\omega} - j\cancel{\omega}(\cos\omega T - j\,\mathrm{Sen}\,\omega T)}{-\omega^{\cancel{2}}}$$

$$H(j\omega) = \frac{+\mathrm{Sen}\,\omega T}{+\omega} + j\,\frac{(\cos\omega T - 1)}{\omega}$$

$$\phi(\omega) = \operatorname{arctg}\left[\frac{\cos\omega T - 1}{\mathrm{Sen}\,\omega T}\right]$$

$$\cos\omega T - 1 = -2\,\mathrm{Sen}^2\left(\frac{\omega T}{2}\right) \quad (x)$$

$$\mathrm{Sen}\,\omega T = 2\,\mathrm{Sen}\left(\frac{\omega T}{2}\right)\cdot\cos\left(\frac{\omega T}{2}\right) \quad (x)$$

$$\phi(\omega) = \operatorname{arctg}\left(\frac{-\cancel{2}\,\mathrm{Sen}^{\cancel{2}}\left(\frac{\omega T}{2}\right)}{\cancel{2}\,\cancel{\mathrm{Sen}}\left(\frac{\omega T}{2}\right)\cos\left(\frac{\omega T}{2}\right)}\right)$$

$$\phi(\omega) = \operatorname{arctg}\left(\frac{-\mathrm{Sen}\left(\frac{\omega T}{2}\right)}{\cos\left(\frac{\omega T}{2}\right)}\right) = \boxed{-\frac{\omega T}{2}}$$

$$\tau = \frac{d\phi(\omega)}{d\omega} = \boxed{-\frac{T}{2}}$$

(x) Ver ATRAS

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = (1-\operatorname{sen}^2\alpha) - \operatorname{sen}^2\alpha$$

$$\boxed{\cos 2\alpha = 1 - 2\operatorname{sen}^2\alpha}$$

$$\therefore \quad \cos \omega T - 1 = \left(\cancel{1} - 2\operatorname{sen}^2\frac{\omega T}{2}\right) - \cancel{1}$$

$$\boxed{\cos \omega T - 1 = -2\operatorname{sen}^2\left(\frac{\omega T}{2}\right)}$$

$$\operatorname{sen} 2\alpha = 2\operatorname{sen}\alpha \cdot \cos\alpha$$

$$\boxed{\operatorname{sen}\omega T = 2\operatorname{sen}\left(\frac{\omega T}{2}\right)\cos\left(\frac{\omega T}{2}\right)}$$