INSTITUTO TECNOLOGICO DE BUENOS AIRES
· UNIVERSIDAD PRIVADA ·

MATERIA: Relaciones Útiles PAG: 1 DE:
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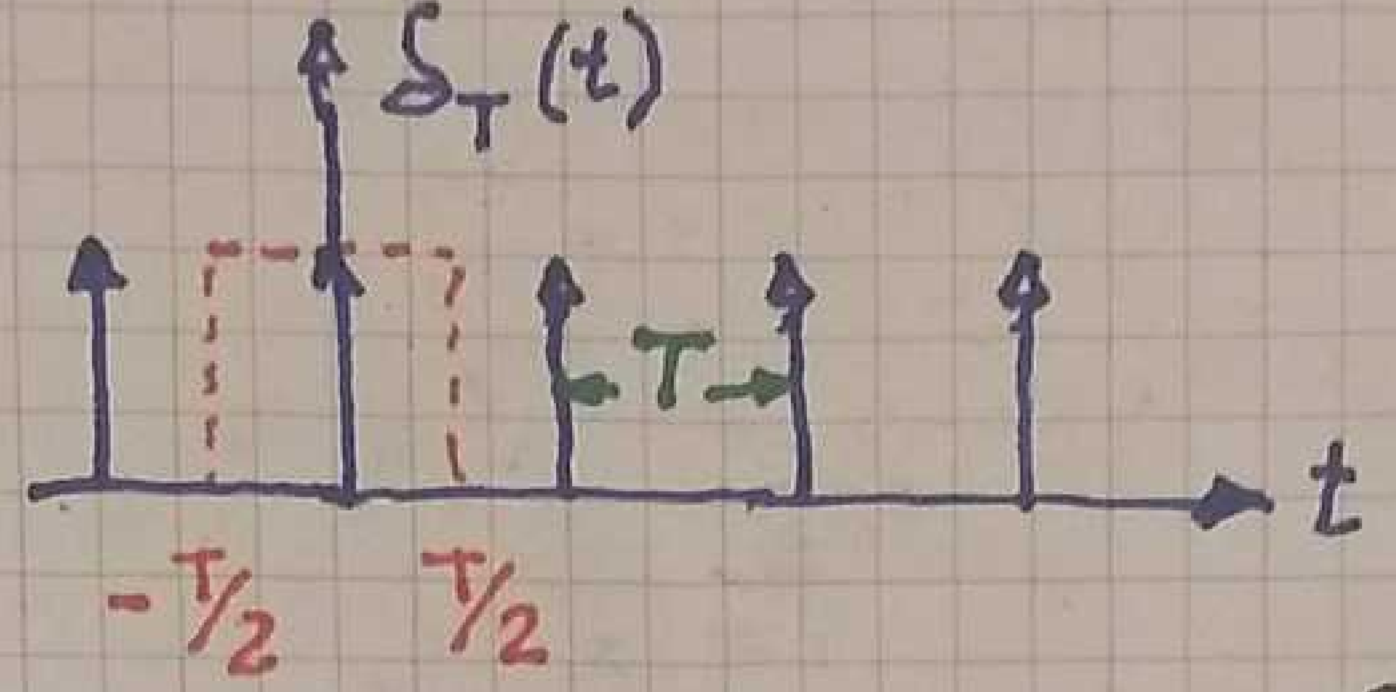
LEGAJO Nº: COMISION: CARRERA:
APELLIDO Y NOMBRE:

Transformada de un tren periodico de deltas



$$\delta_T(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT) = \sum_{m=-\infty}^{\infty} C_m \cdot e^{jm\omega_s t}$$

PERIODICA $\Rightarrow$ Serie de Fourier

$$C_m = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jm\omega_s t}\, dt \qquad f(t) = \delta_T(t)$$

$$C_m = \int_{-T/2}^{T/2} \delta_T(t)\, e^{-jm\omega_s t}\, dt = \frac{1}{T}\, e^{jm\omega_s t}\Big|_{t=0} = \frac{1}{T} e^{0} = \boxed{\frac{1}{T}}$$ (1)

$C_m = \frac{1}{T}$ (no depende de m)

$$\boxed{\delta_T(t) = \frac{1}{T}\sum_{m=-\infty}^{\infty} e^{jm\omega_s t}}$$

Expresion ALTERNATIVA de $\delta_T(t)$

(1) Recordar que:

$$\int f(t)\cdot \delta(t - t_0)\, dt = f(t_0)$$

RESUMIENDO:

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - mT) = \frac{1}{T}\sum_{m=-\infty}^{\infty} e^{jm\omega_s t}$$

Ahora podemos encontrar la T.F. de $\delta_T(t)$

$$\mathcal{F}\left[\delta_T(t)\right] = \mathcal{F}\left[\frac{1}{T}\sum_{m=-\infty}^{\infty} e^{jm\omega_s t}\right]$$

$$= \frac{1}{T}\sum_{m=-\infty}^{\infty} \mathcal{F}\left[e^{jm\omega_s t}\right]$$

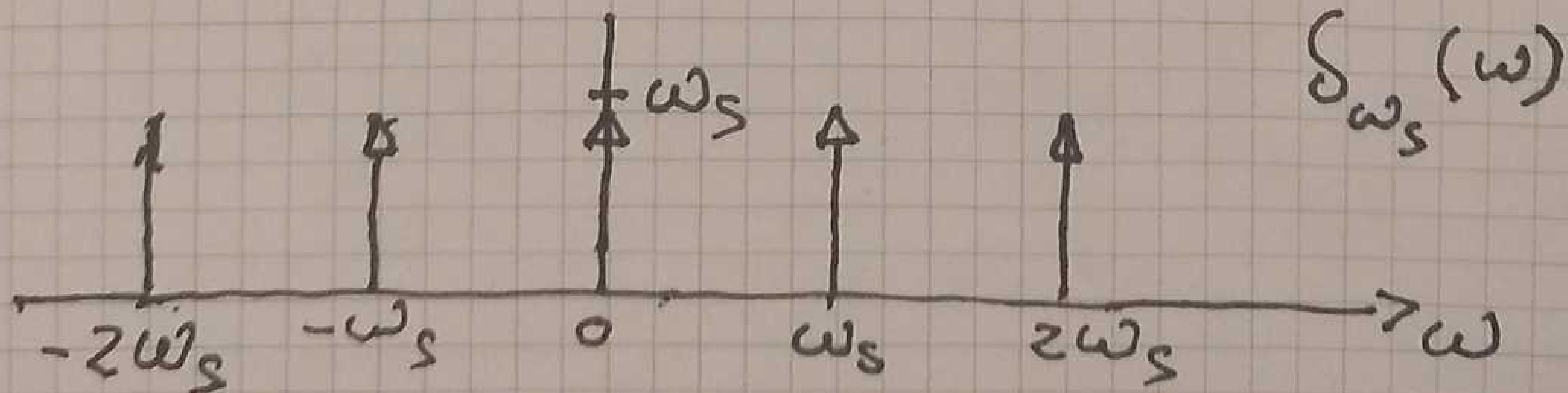
Como $\mathcal{F}\left[e^{jm\omega_s t}\right] = 2\pi\,\delta(\omega - m\omega_s)$ VER Apendice 1

$$\mathcal{F}\left[\delta_T(t)\right] = \frac{1}{T}\sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega - m\omega_s)$$

$$= \frac{2\pi}{T}\sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)$$

$$\boxed{\mathcal{F}\left[\delta_T(t)\right] = \omega_s \cdot \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)}$$

Espectro de un tren de deltas.



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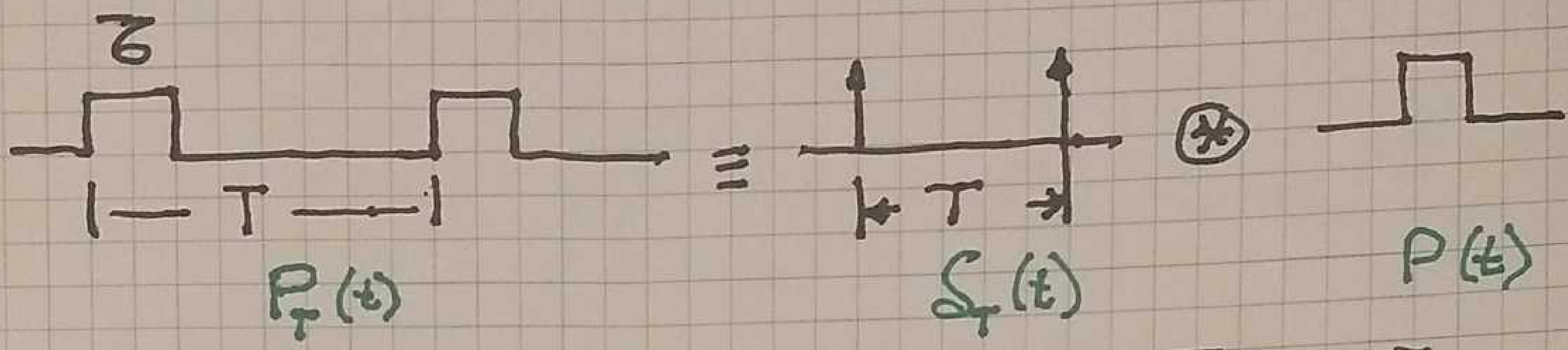
TRANSFORMADA DE UN TREN DE PULSOS



$$\underbrace{\delta_T(t) \circledast p(t)}_{P_T(t)} \longleftrightarrow \underbrace{\mathcal{F}[\delta_T(t)] \cdot \mathcal{F}[p(t)]}_{\mathcal{F}(P_T(t))}$$

$$\mathcal{F}[\delta_T(t)] = \omega_s \cdot \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)$$

$$\mathcal{F}[p(t)] = \tau \cdot \mathrm{Sinc}\left(\frac{\tau}{2}\omega\right)$$ Apendice 2

$$\mathcal{F}(P_T(t)) = \omega_s \sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s) \cdot \tau\, \mathrm{Sinc}\left(\frac{\tau}{2}\omega\right)$$

$$\left(\tau \cdot \omega_s = \tau \cdot \frac{2\pi}{T} = 2\pi \cdot \frac{\tau}{T}\right)$$

$$\mathcal{F}(P_T(t)) = 2\pi \frac{\tau}{T}\left[\sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)\right] \mathrm{Sinc}\left(\frac{\tau}{2}\omega\right)$$

Ceros $\mathrm{Sinc}\left(\frac{\tau}{2}\omega\right) \Rightarrow \mathrm{Sen}\left(\frac{\tau}{2}\omega_z\right) = 0$

$$\frac{\tau}{2}\omega_z = K\pi \qquad \boxed{\omega_z = \frac{\pi}{\left(\tau/2\right)} K}$$

Ceros de la Sinc

$$\boxed{\omega_z = \frac{2\pi}{\tau} \cdot K \qquad K = \pm 1, \pm 2, \ldots}$$

$$\mathcal{F}\left[P_T(t)\right] = P_T(\omega)$$

$$P_T(\omega) = 2\pi \cdot \frac{\tau}{T}\left[\sum_{m=-\infty}^{\infty} \delta(\omega - m\omega_s)\right] \cdot \mathrm{Sinc}\left(\frac{\tau \cdot \omega}{2}\right)$$

La función $P_T(\omega)$ es $\neq \emptyset$ solo cuando $\omega = m\omega_s$. Por lo tanto $P_T(\omega)$ sera un tren de deltas ponderadas por $\mathrm{Sinc}\left(\frac{\tau\cdot\omega}{2}\right)$ en $\omega = m\omega_s$. Entonces:

$$P_T(\omega) = 2\pi \frac{\tau}{T} \sum \mathrm{Sinc}\left(\frac{\tau \overbrace{m\omega_s}^{\omega}}{2}\right) \cdot \delta(\omega - m\omega_s)$$

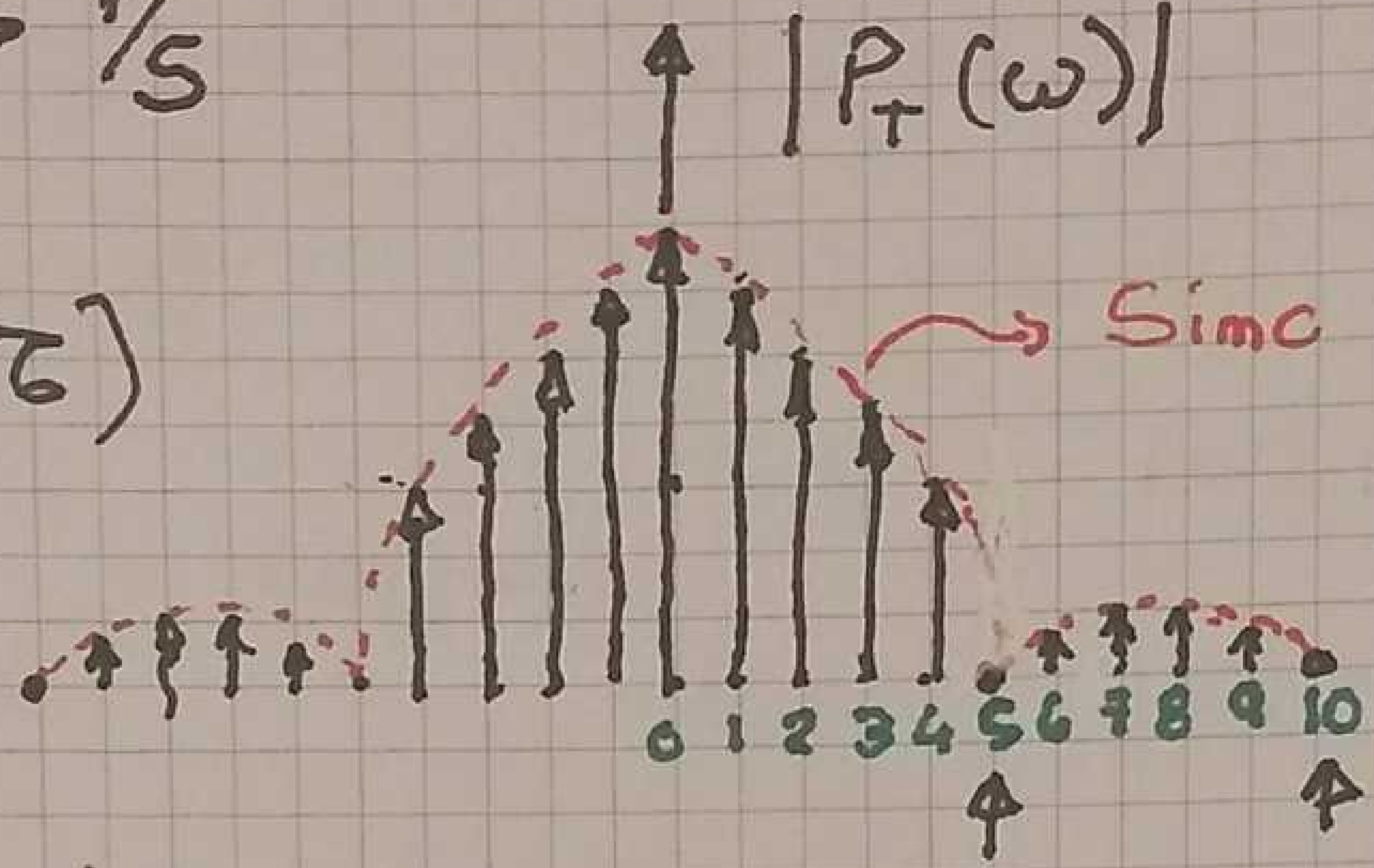
Notese que los ceros $\omega_z = \frac{2\pi}{\tau} k$ coinciden con $m\omega_s$ solo cuando: $\omega_z = m\omega_s = \frac{2\pi}{\tau} k$

es decir que: $m\omega_s = m\frac{2\pi}{T} = \frac{2\pi}{\tau} k$

$$\Rightarrow \quad m = \frac{T}{\tau} k \quad \therefore \quad \frac{T}{\tau} \text{ debe ser entero}$$

Ej $\tau/T = 1/5$

$(T = 5\tau)$



Ceros en $m = \frac{T}{\tau} K = \frac{5\cancel{\tau}}{\cancel{\tau}} K$; $K = \pm 1, \pm 2 \ldots$

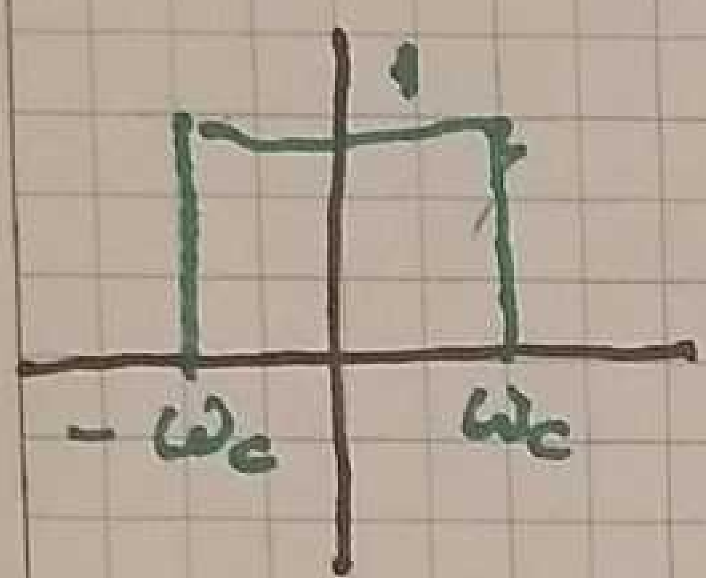
$m = 5 \cdot K$ $\quad$ m ES ENTERO

1° cero $m_1 = \pm 5$

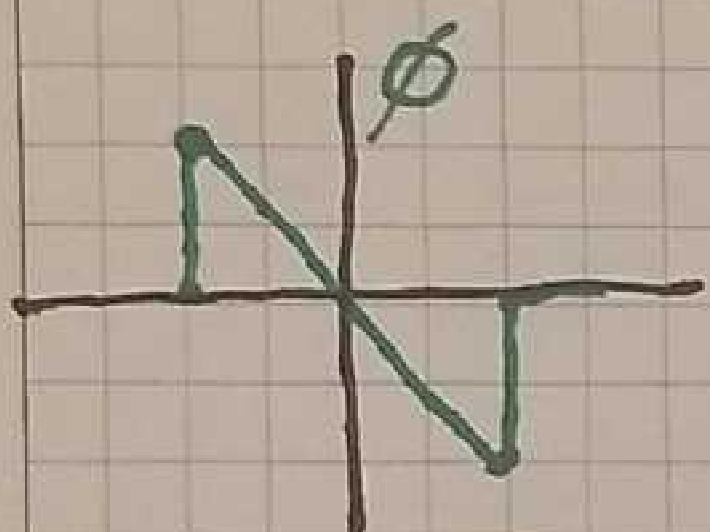
2° cero $m_2 = \pm 10$

FILTRO Recuperador IDEAL

$$H_I(j\omega) \begin{cases} e^{-j\omega t_0} & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$



$$|H_I(j\omega)| \begin{cases} 1 & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$



$$\phi_I(j\omega) \begin{cases} -\omega t_0 & |\omega| \leq \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

$$t_0 > 0$$

Respuesta impulsiva del filtro IDEAL

$$h_I(t) = \mathcal{F}^{-1}\left[H_I(j\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_I(j\omega) \cdot e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega t_0} \cdot e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega(t-t_0)}\, d\omega$$

$$h_I(t) = \frac{1}{2\pi} \cdot \frac{1}{j(t-t_0)} \cdot e^{j\omega(t-t_0)} \Big|_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi} \cdot \frac{1}{j(t-t_0)} \left[e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)} \right]$$

$$= \frac{1}{2\pi} \frac{1}{j(t-t_0)} \left[e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)} \right] \times \frac{\omega_c}{\omega_c}$$

$$= \frac{\omega_c}{\pi} \; \frac{e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)}}{2j} \; \frac{1}{\omega_c(t-t_0)}$$

$$= \frac{\omega_c}{\pi} \; \frac{\text{Sen}\left[\omega_c(t-t_0)\right]}{\omega_c(t-t_0)} .$$

$$\boxed{h_I(t) = \frac{\omega_c}{\pi} \; \text{Sinc}\left(\omega_c(t-t_0)\right)}$$

Respuesta impulsiva del FILTRO Ideal.

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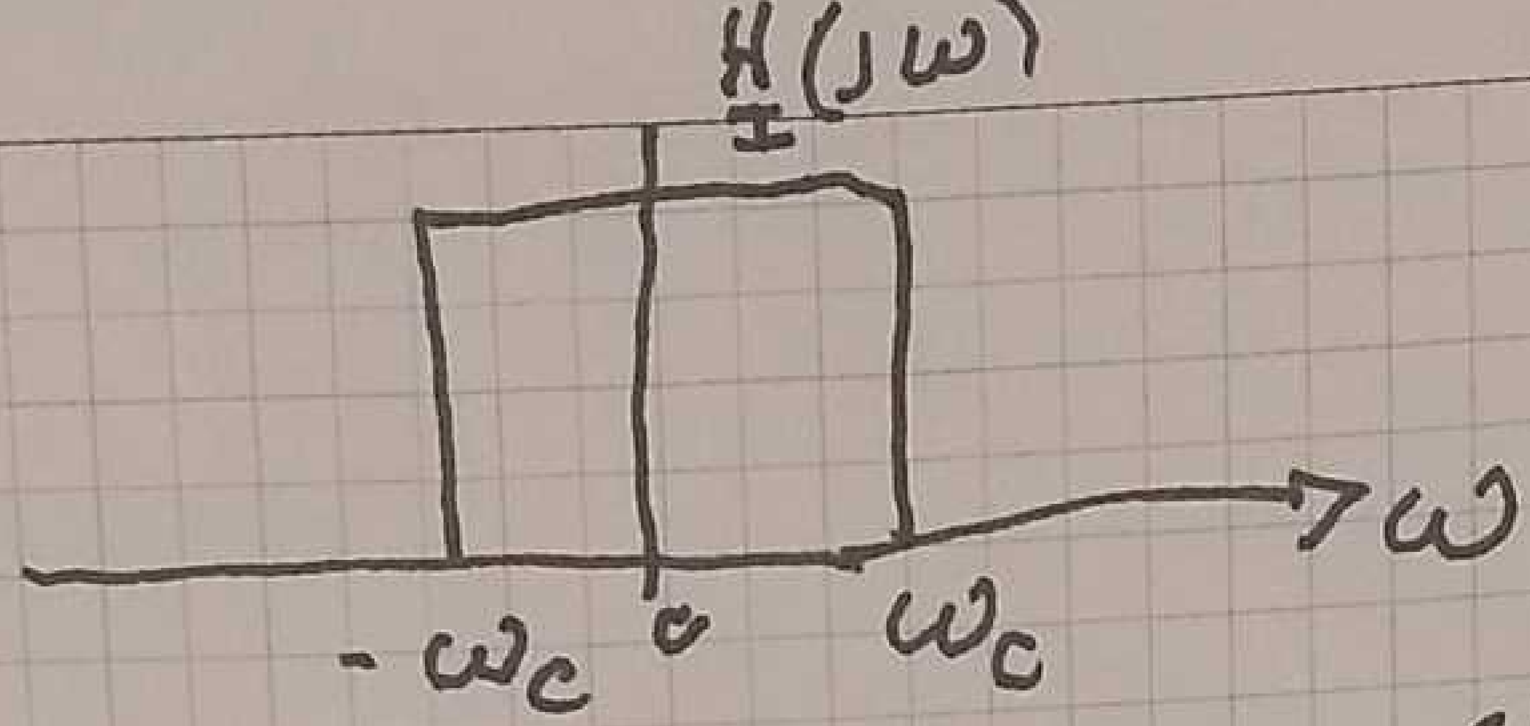
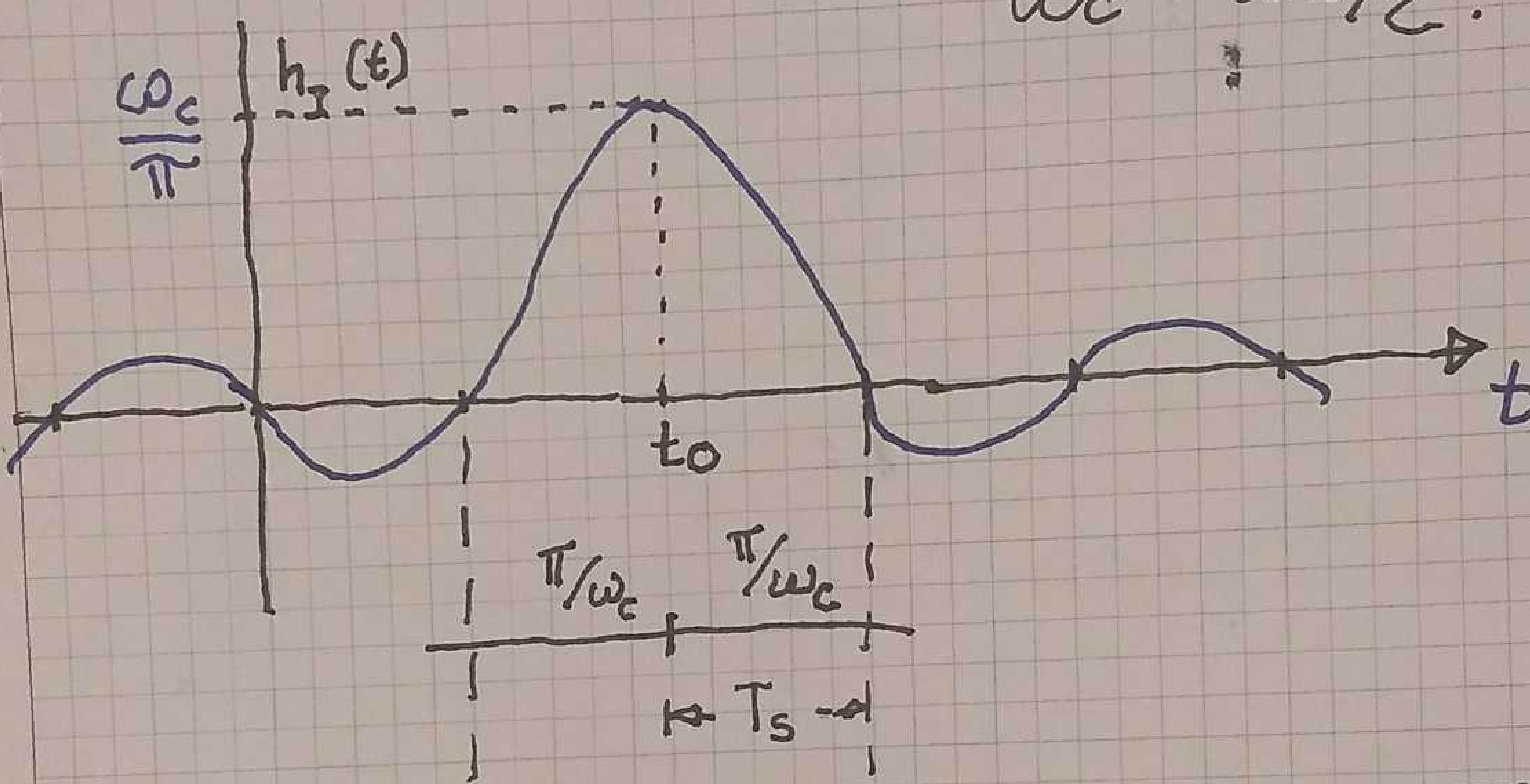


$\omega_c = \omega_s/2.$



$$T_s = \frac{2\pi}{\omega_s} = \frac{\pi}{\omega_s/2} = \frac{\pi}{\omega_c} \quad \therefore \quad \boxed{T_s = \frac{\pi}{\omega_c}}$$

$$h_I(t) = \frac{\omega_c}{\pi}\,\mathrm{Sinc}\left(\omega_c(t-t_0)\right)$$

ceros: $\omega_c\cdot(t-t_0) = K\pi \Rightarrow t-t_0 = K$

$$t = K\underbrace{\frac{\pi}{\omega_c}}_{T_s} + t_0$$

$t = KT_s \pm t_0$ si $t_0 = 0$ $t = KT_s$

$(\phi(\omega) = 0)$

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Apendice 1

$$\mathcal{F}[\delta(t-t_0)] = \int_{-\infty}^{\infty} \delta(t-t_0) e^{-j\omega t}\, dt$$

$$= e^{-j\omega t}\Big|_{t=t_0} = e^{-j\omega t_0}$$

$$\boxed{\mathcal{F}[\delta(t-t_0)] = e^{-j\omega t_0}}$$

$$\mathcal{F}^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0) e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} e^{j\omega t}\Big|_{\omega=\omega_0} = \frac{1}{2\pi}\cdot e^{j\omega_0 t}$$

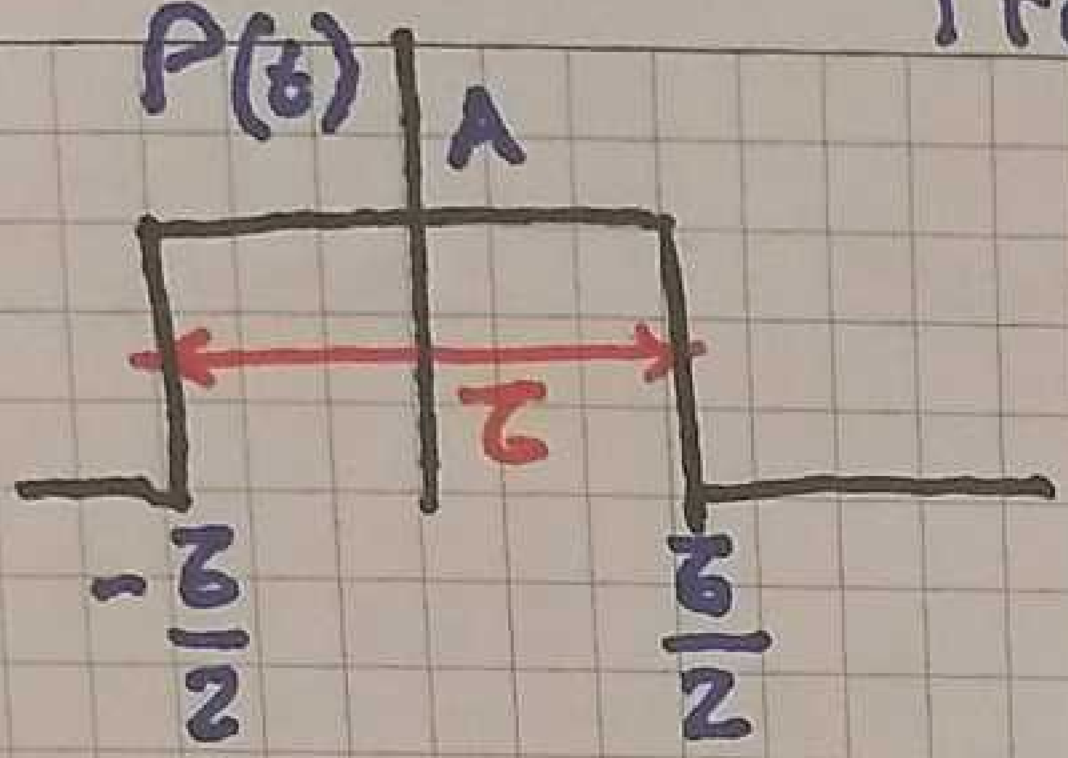
$$\mathcal{F}^{-1}[\delta(\omega-\omega_0)] = \frac{1}{2\pi} e^{j\omega_0 t}$$

$$\delta(\omega-\omega_0) = \frac{1}{2\pi}\mathcal{F}[e^{j\omega_0 t}]$$

$$\boxed{\mathcal{F}[e^{j\omega_0 t}] = 2\pi\,\delta(\omega-\omega_0)}$$

APENDICE 2

Transformada de un Pulso



$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$P(\omega) = \int_{-\infty}^{\infty} P(t)\, e^{-j\omega t}\, dt = \int_{-\tau/2}^{\tau/2} A e^{-j\omega t}\, dt$$

$$= \frac{A}{-j\omega} e^{-j\omega t} \Big|_{-\tau/2}^{+\tau/2} = \frac{A}{-j\omega}\left(e^{-j\omega \tau/2} - e^{-j\omega(-\frac{\tau}{2})}\right)$$

$$= \frac{A}{-j\omega} \cdot \left(e^{-j\omega \tau/2} - e^{+j\omega \tau/2}\right)$$

$$= \frac{A}{j\omega}\left(e^{+j\omega \tau/2} - e^{-j\omega \tau/2}\right) \times \frac{2}{2}$$

$$= \frac{2A}{\omega} \cdot \frac{\left(e^{+j\omega \tau/2} - e^{-j\omega \tau/2}\right)}{2j}$$

$$= \frac{2A}{\omega}\, \operatorname{Sen}\left(\frac{\omega \tau}{2}\right) = \frac{2A}{\omega} \boxed{\frac{\tau}{\tau}} \operatorname{Sen}\left(\frac{\omega \tau}{2}\right)$$

$$= A\tau \, \frac{\operatorname{Sen}\left(\omega \tau/2\right)}{\omega \tau/2}$$

$$P(\omega) = A\tau \, \operatorname{Sinc}\left(\frac{\omega \tau}{2}\right)$$

$$\operatorname{Sinc}(x) = \frac{\operatorname{Sen}(x)}{x}$$

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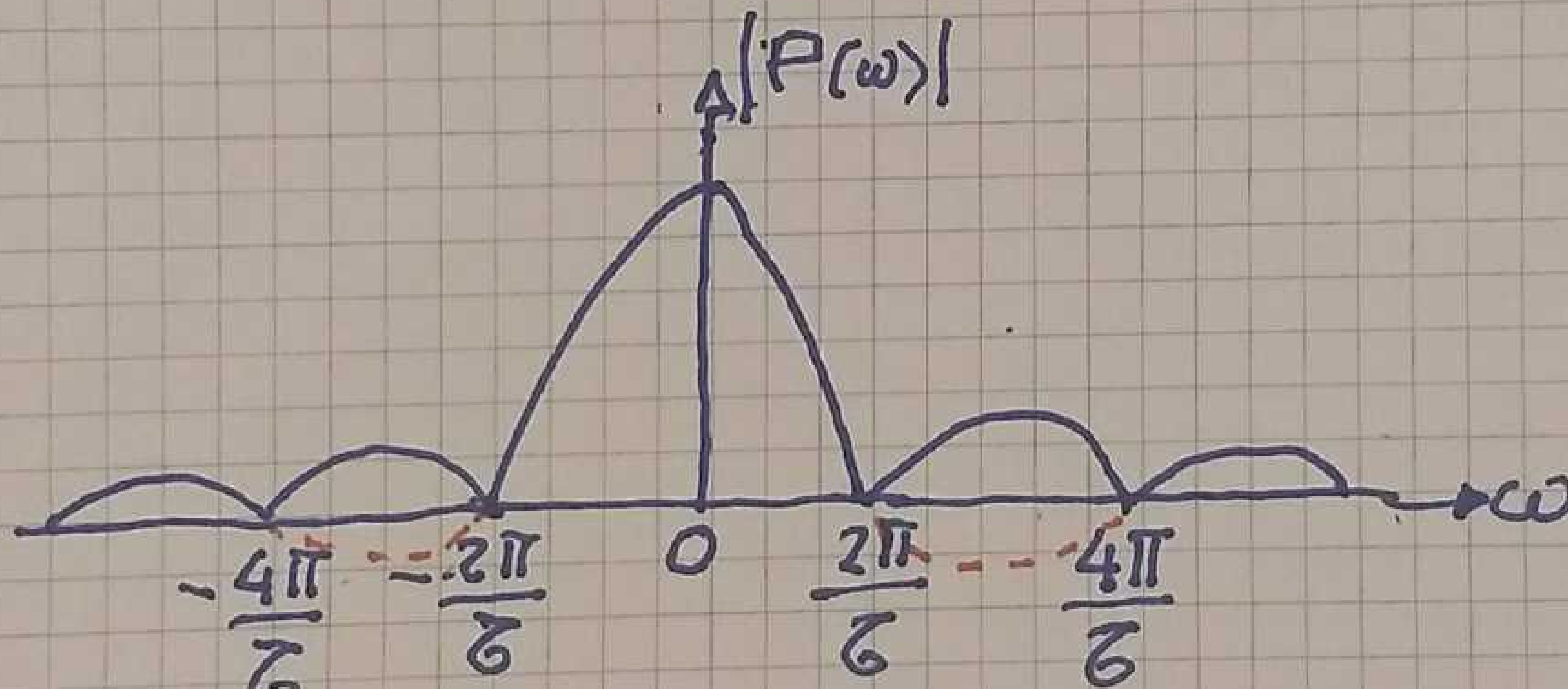
Ceros: $\operatorname{Sen}\left(\frac{\omega\tau}{2}\right) = 0$

$\frac{\omega\tau}{2} = k\pi \qquad \omega = \frac{2\pi}{\tau}k \qquad k = \pm 1, \pm 2 \ldots$

$P(\omega) = A\tau \cdot \frac{\operatorname{Sen}\left(\frac{\omega\tau}{2}\right)}{\omega\frac{\tau}{2}}$



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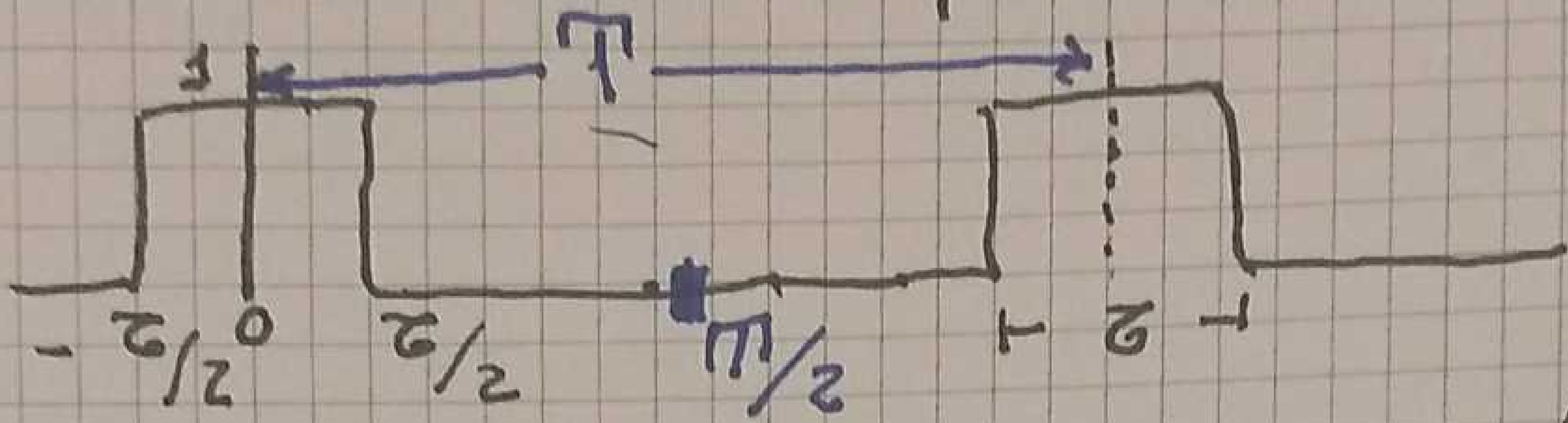
APELLIDO Y NOMBRE: Apendice 3

Transformada de un tren periodico de pulsos usando la serie de fourier.



$$f(t) = \sum_{m=-\infty}^{\infty} C_m \, e^{jm\omega_s t} \qquad C_m = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\cdot e^{-jm\omega_s t}\, dt$$

$$C_m = \frac{1}{T}\int_{-\tau/2}^{\tau/2} e^{-jm\omega_s t}\, dt = \frac{\tau/2}{T}\cdot \mathrm{Sinc}\left(m\,\omega_s \frac{\tau}{2}\right)$$