EMITTANCE MEASUREMENT WITH QUADRUPOLE SCAN

Attempt using BTM.QNO20 + SEM02 on BCMS beam

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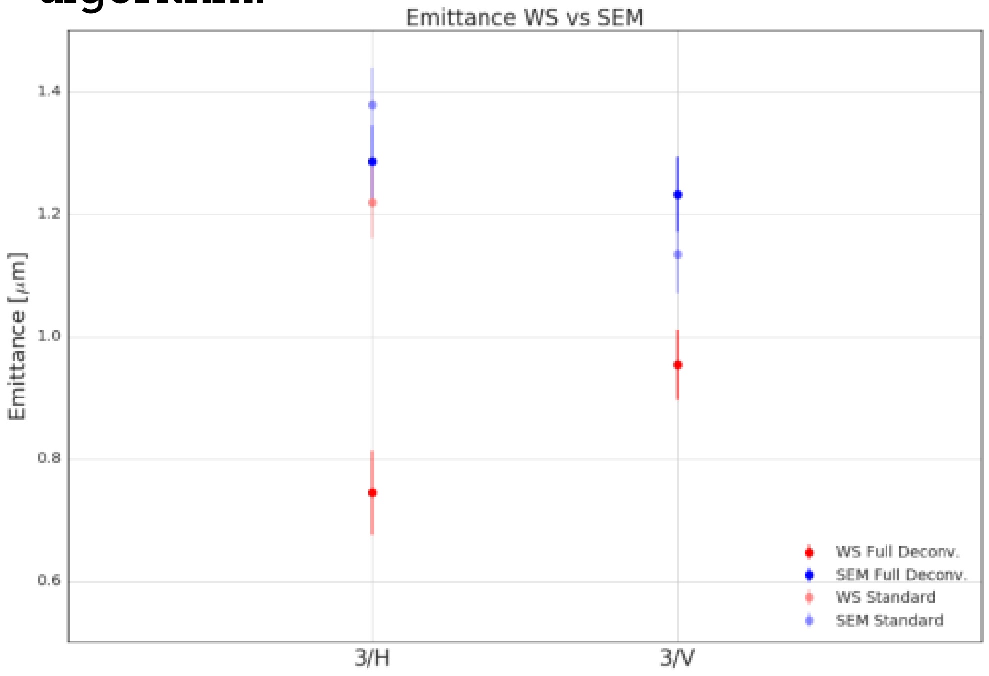
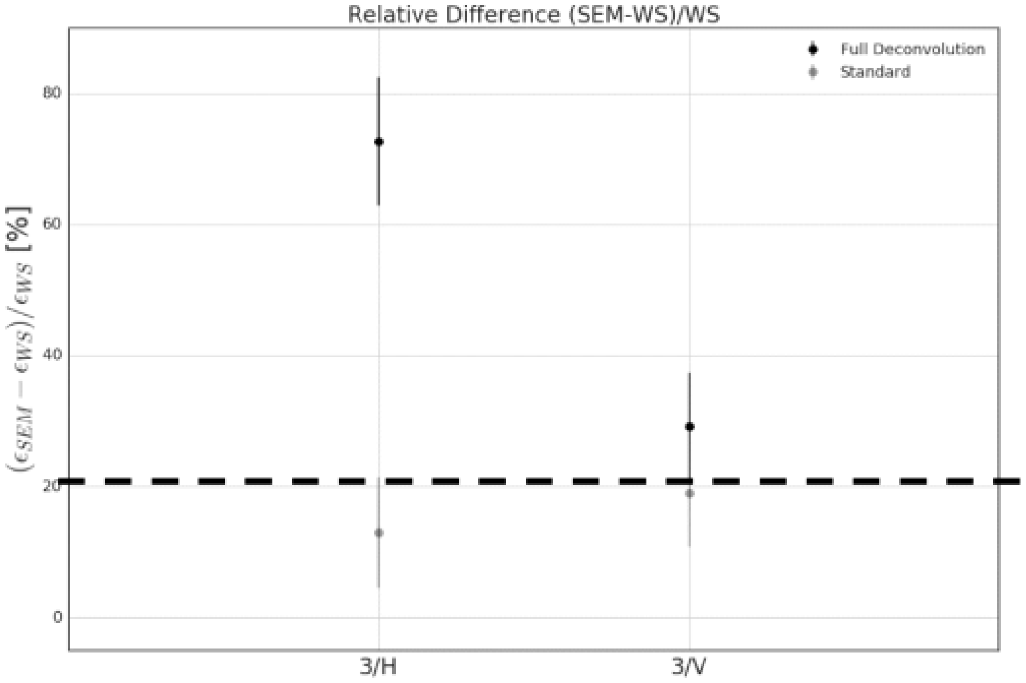
MOTIVATION

LIU PS Beam Dynamics WG (2 Novemeber 2017):
https://indico.cern.ch/event/676527/contributions/2769084/attachments/1551316/2437384/2017.11.02_WSvsSEM.pdf

BCMS 1.5 eVs

- **Emittance measured** at the **SEM grid** (BTM line) is systematically **larger** by **~20%** than the correspondent measurement at the **WS** with the standard algorithm
- Difference increases up to ~70% in the horizontal plane when applying the full deconvolution algorithm.





What happens with quadrupole scan at the SEM grids?

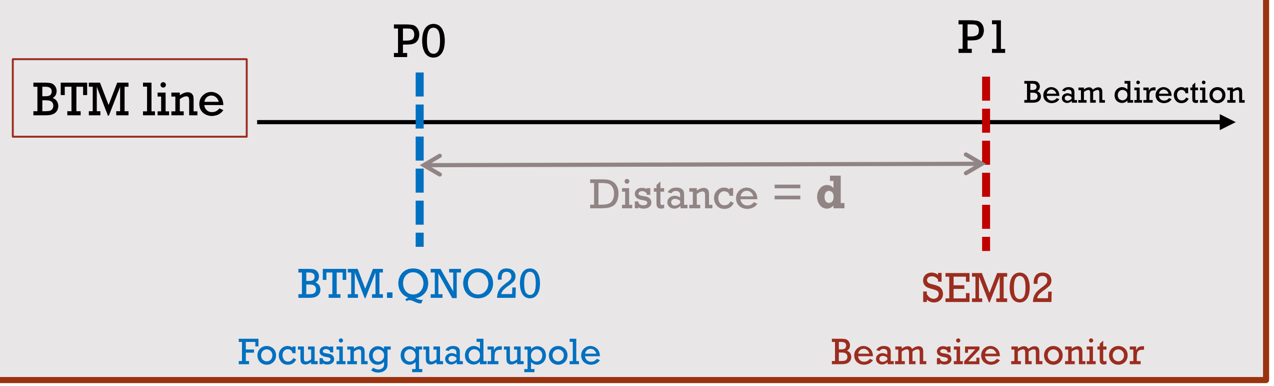
* Ref: *"Particle Accelerator Physics"*, Helmut Wiedemann. Chapter 5.1, Measurement of beam emittance.

METHOD*

ASSUMPTIONS:
- No dispersion
- No space charge

Determine **beam emittance at** the location **P0** by using:

- beam size monitor at P1 (downstream from P0)
- quadrupole between P0 and P1.



At any point, the beam can be described with the so called "**beam matrix**", **Σ**:

Average values of all particles within a fraction of the beam

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \text{ where } \begin{cases} \sigma_{11} = \langle x_i^2 \rangle = \epsilon\beta \\ \sigma_{22} = \langle x'^2_i \rangle = \epsilon\gamma \\ \sigma_{12} = \sigma_{21} = \langle x_i x'_i \rangle = -\epsilon\alpha \end{cases}$$

Square of **rms beam size**

Twiss parameters

Geometrical emittance: $\epsilon = \det(\Sigma) = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} = \sqrt{\langle x_i^2 \rangle \langle x'^2_i \rangle - \langle x_i^2 x'^2_i \rangle}$ Valid for any particle distribution.

Phase space area: $V = \pi\epsilon$

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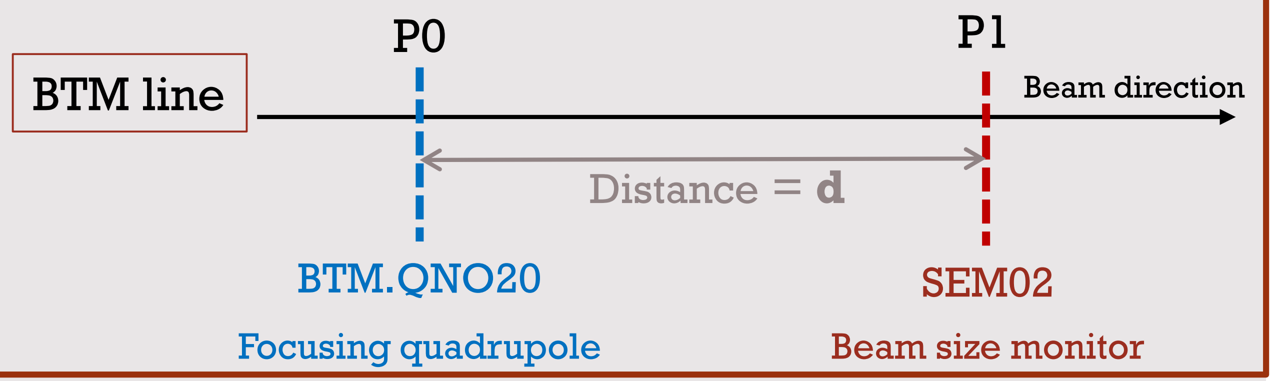

Beam matrices: Σ^{quad} Σ^{sem}

In general, the beam matrix transforms as:

$\Sigma^{sem} = M\,\Sigma^{quad} M^T$ where ***M*** is the **transport matrix**

Valid for *Thin Lens approx.*

k = quad. strength [m^{-2}]
l = effective length [m]
d = drift length [m]

Between P0 and P1 there are a quadrupole and a drift space:

$$M = \underbrace{\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}_{drift} \underbrace{\begin{pmatrix} 1 & 0 \\ kl & 1 \end{pmatrix}}_{quadrupole} = \begin{pmatrix} 1+dkl & d \\ kl & 1 \end{pmatrix}$$

Do the matrix product and consider the equation for the $\boldsymbol{\sigma_{11}^{sem}}$ element → **square** of the **beam rms size** at the SEM grid.

$$\sigma_{11}^{sem} = (1+dkl)^2\sigma_{11}^{quad} + 2(1+dlk)d\sigma_{12}^{quad} + d^2\sigma_{22}^{quad}$$

* Ref: *"Particle Accelerator Physics"*, Helmut Wiedemann. Chapter 5.1, Measurement of beam emittance.

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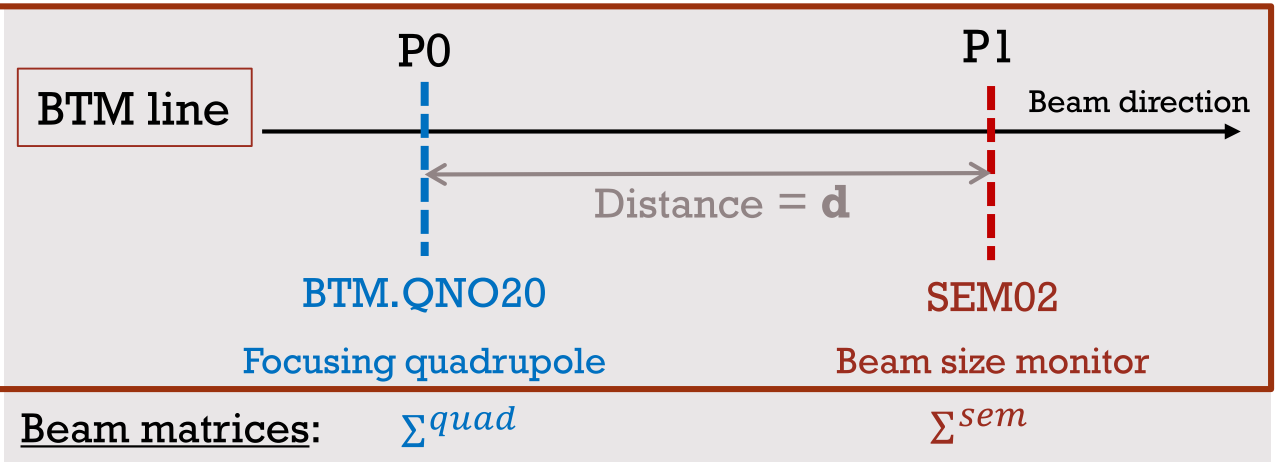

At the quadrupole location, we can vary the *quad. strength* $\boldsymbol{k}$. So let's rewrite the previous equation as function of k:

$$\boxed{\sigma_{11}^{sem}} = \underbrace{\boxed{\left(\sigma_{11}^{quad} d^2 l^2\right)}}_{A} \boldsymbol{k^2} + \underbrace{\boxed{\left(2dl\sigma_{11}^{quad} + 2d^2 l\sigma_{12}^{quad}\right)}}_{B} \boldsymbol{k} + \underbrace{\boxed{\sigma_{11}^{quad} + 2d\sigma_{12}^{quad} + d^2\sigma_{22}^{quad}}}_{C}$$

Beam size squared at the SEM grid

Coefficients of a parabola fit to ***(beam size)² VS k***

* <u>Ref:</u> *"Particle Accelerator Physics"*, Helmut Wiedemann. Chapter 5.1, Measurement of beam emittance.

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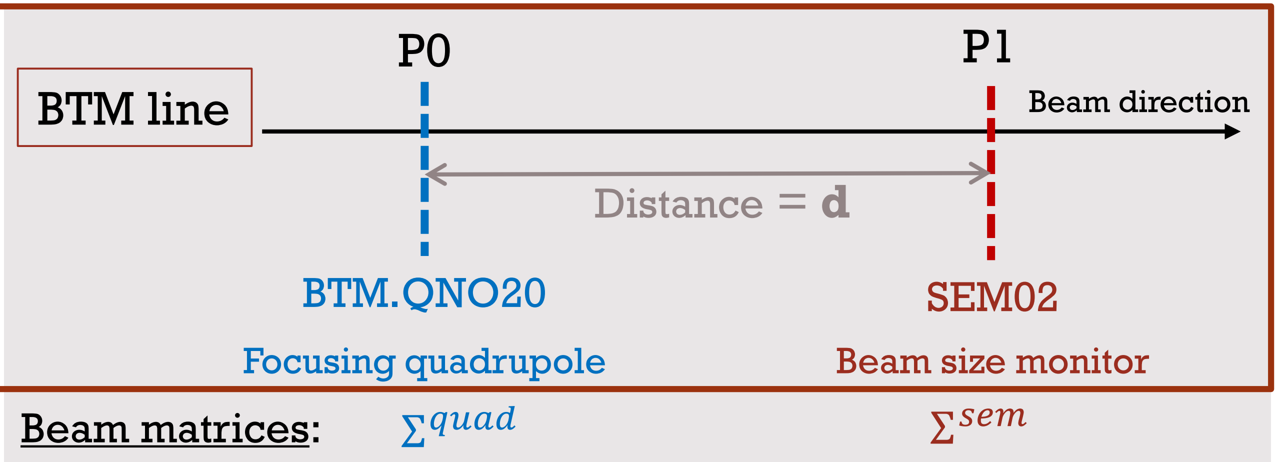

We can now derive the sigma elements of the beam matrix at the quadrupole location:

$$\sigma_{11}^{quad} = \frac{A}{d^2 l^2} \qquad \sigma_{12}^{quad} = \frac{B - 2dl\sigma_{11}^{quad}}{2d^2 l} \qquad \sigma_{22}^{quad} = \frac{C - \sigma_{11}^{quad} - 2d\sigma_{12}^{quad}}{d^2}$$

And finally calculate the emittance:

$$\epsilon = \sqrt{\sigma_{11}^{quad}\sigma_{22}^{quad} - \left(\sigma_{12}^{quad}\right)^2}$$

and the **normalized emittance:** $\epsilon^N = \beta\gamma\epsilon$

$\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

* <u>Ref:</u> *"Particle Accelerator Physics"*, Helmut Wiedemann. Chapter 5.1, Measurement of beam emittance.

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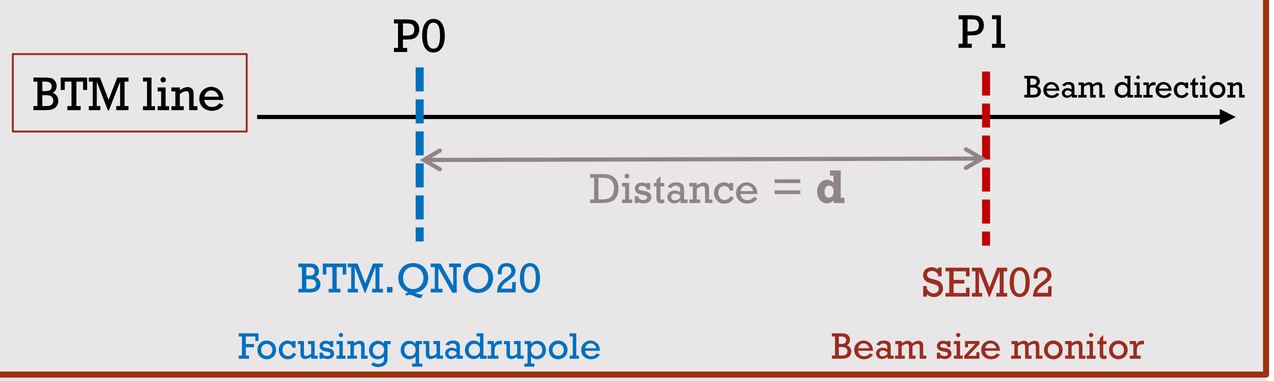


<u>Beam matrices</u>: Σ^{quad} Σ^{sem}

The beam matrix defines also the <u>Twiss parameters</u> at the *beginning of the quadrupole*:

$$\alpha^{quad} = -\frac{\sigma_{12}^{quad}}{\epsilon}$$

$$\beta^{quad} = \frac{\sigma_{11}^{quad}}{\epsilon}$$

$$\gamma^{quad} = \frac{\sigma_{22}^{quad}}{\epsilon}$$

COMMENTS
- Chose setting with **focus closed to** the **SEM grid**
- **Careful at the focus** – beam very small and possible space charge effects
- Guarantee **large beam size variation with quadrupole strength**, to be able to accurately fit the 3 parameters.

MEASUREMENT SETUP

- Beam type: **LHC_BCMS_1.5eVs_2017** ***(MD2748)*** → ***I~80E10 particles***
- Study in the **Ring3-horizontal plane** of the **BTM line**
- Focusing (in H-plane) quadrupole: **BTM.QNO20**
- Beam profile measured at **SEM02**

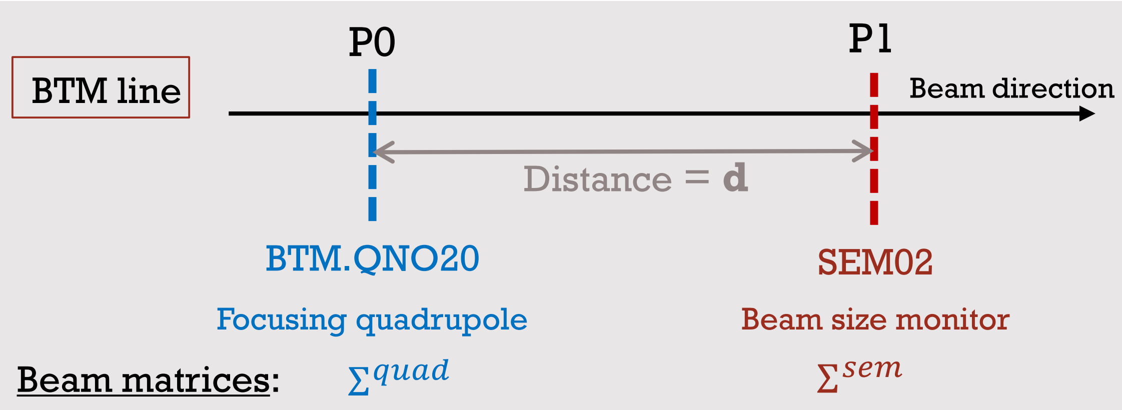
STRATEGY

- Find the maximum applicable current to BTM.QNO20

  200 A *(CCV value)* → 0.97778 m^{-2}

- Measure the **Dispersion** in BTM line:
  - Nominal optics
  - Dispersion-free optics at Sem02
- **Systematically measure the profile** at SEM02 as a function of k

ANALYSIS SETUP



NUMBERS

- Distance BTM.QNO20 – SEM

  d = 6.9043 m (from this table)

  d = 6.8305 m (from Layout DB)

	MADX file (2016)	GEODE correction to MADX	Position
BTM.QNO20	3.69809 m	-0.00097 m	**3.69712 m**
SEM02	10.52859 m	+0.07280 m	**10.60139 m**

- Effective length

  ***l* = 0.560 m**

- Conversion factor (cf): Current→Quad. strength

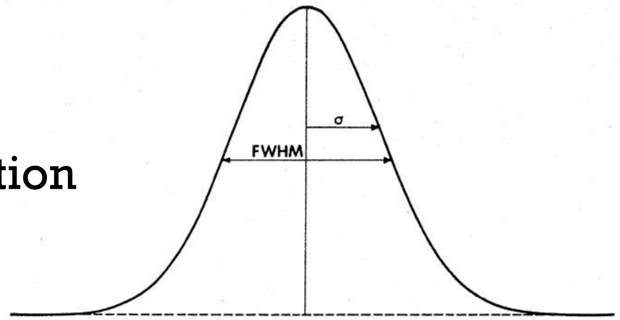
  cf = 0.93745 [m^{-2}]/191.75 [A]

Beam parameters	
Kinetic energy	1.4 GeV
Relativistic beta	0.9160
Gamma (Lorentz's factor)	2.4925

DEFINITION

What do we use as "**rms beam size**" at the SEM grid?

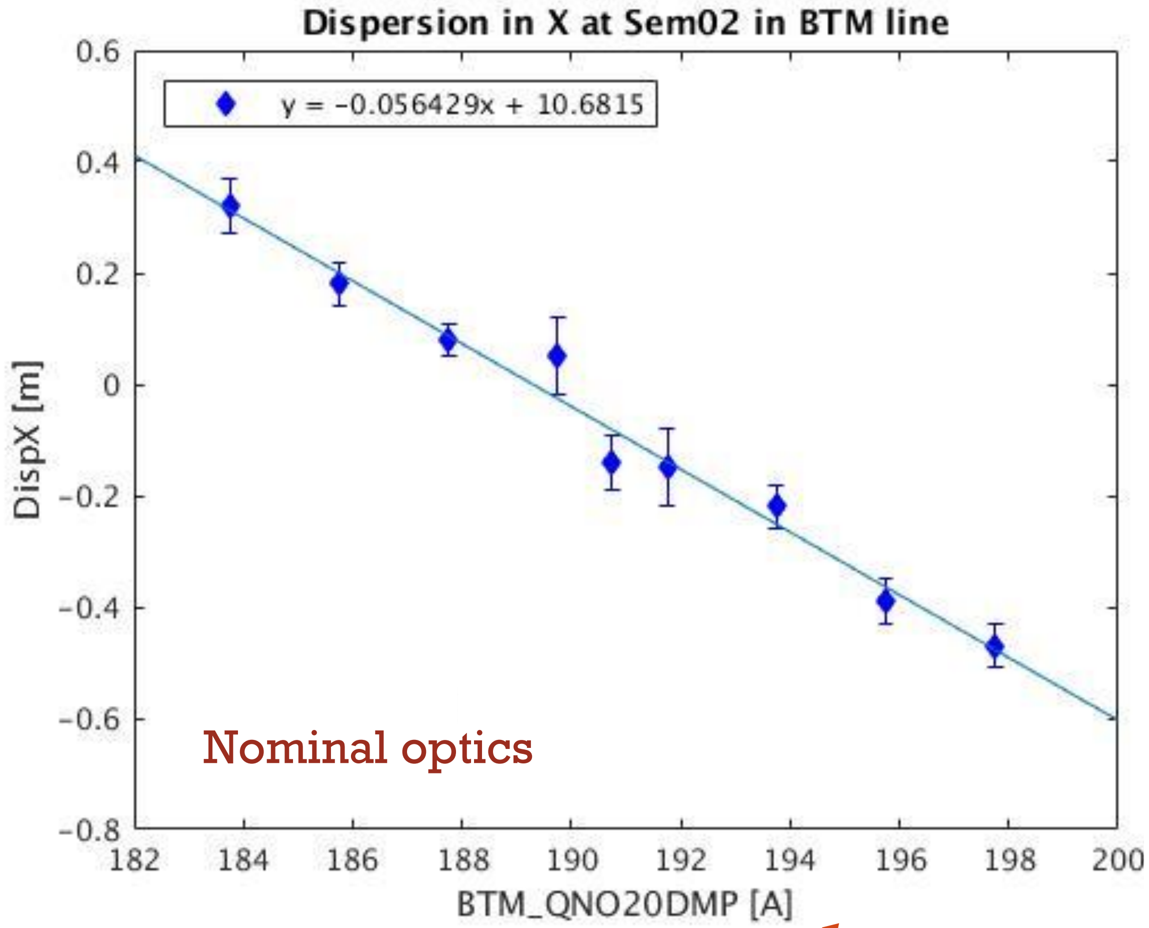
- **Measured** sigma→ sigma of Gaussian fit on data
- **Betatronic** sigma→ measured sigma – dispersive contribution
- **Deconvoluted** sigma→ from G. Sterbini deconv. algorythm

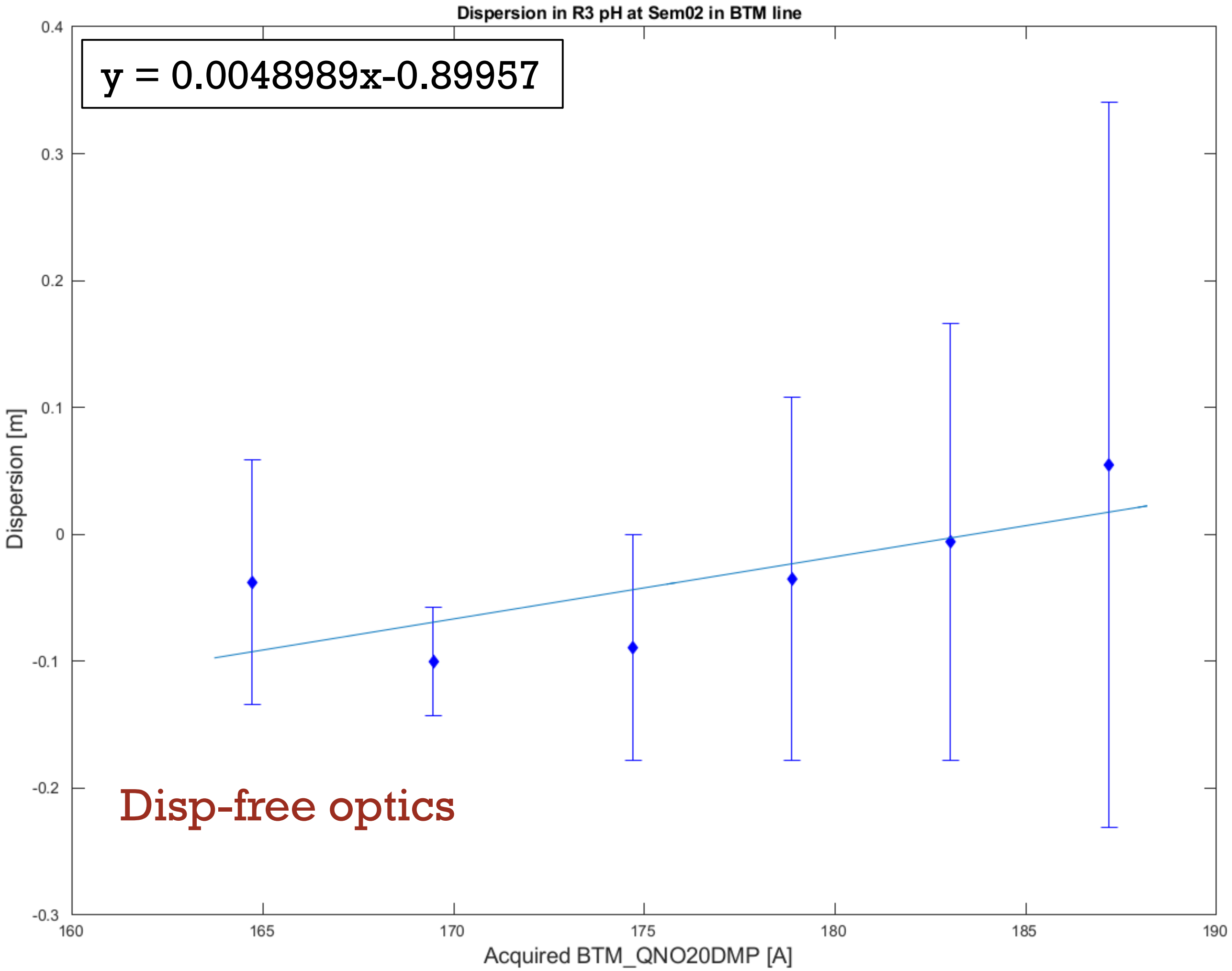


DISPERSION

Dispersion **R3-Hplane** using the same beam type as for the emittance measurement.

Linear fit used in the analysis to evaluate the dispersive contribution for each measured profile.



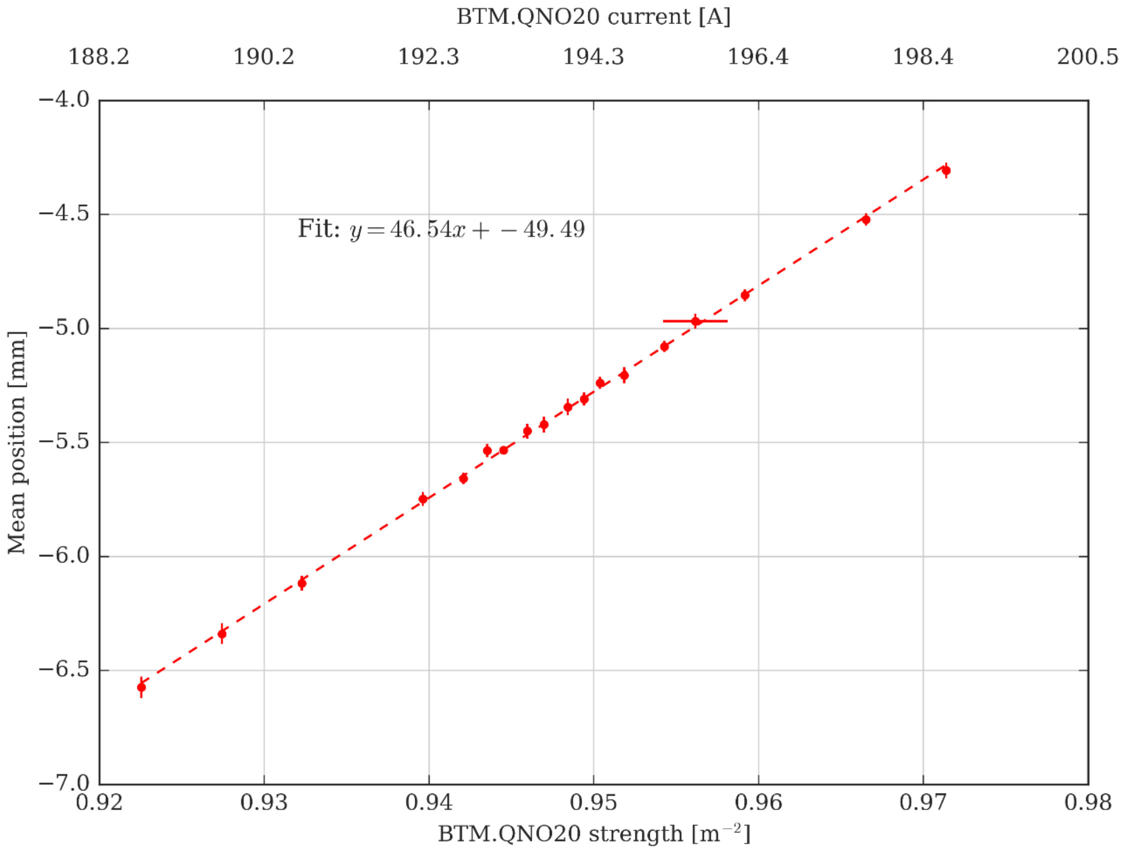


Acquired current!

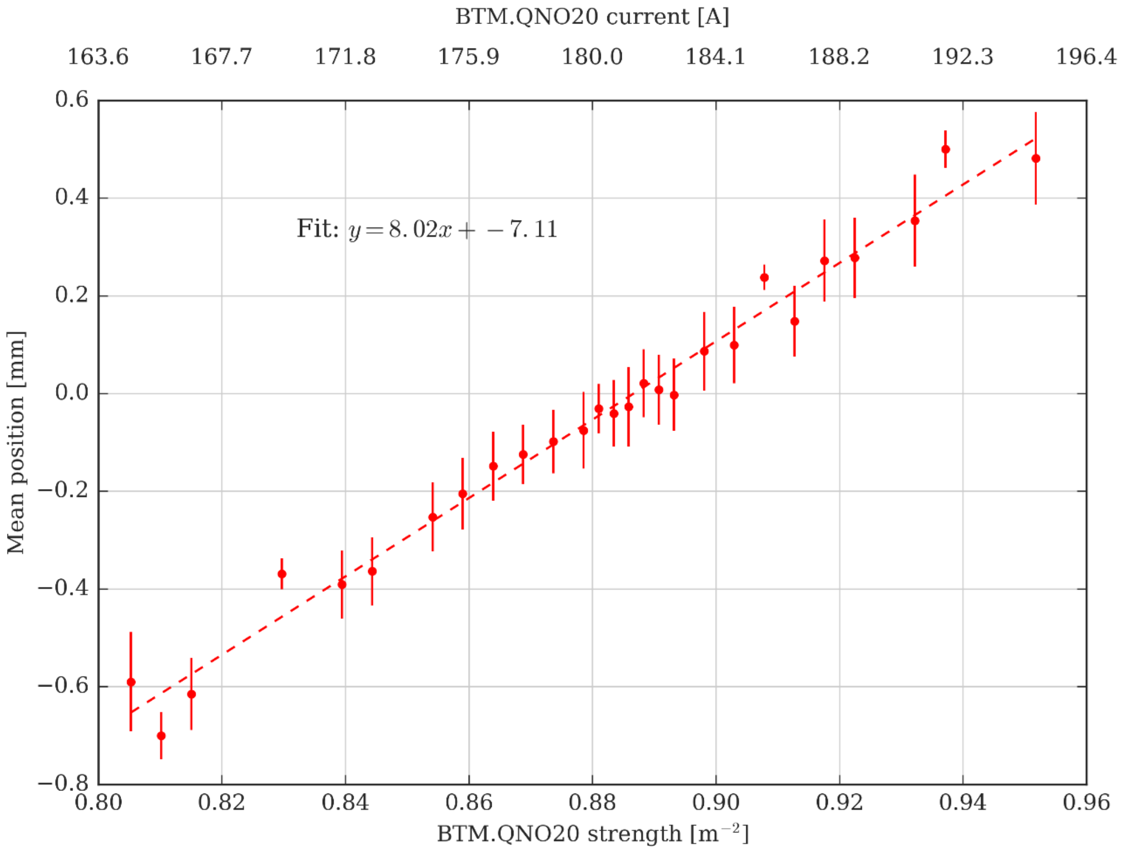
BEAM POSITION VS K

The beam at SEM2 as a function of the quad. strength.
Much better steering in disp-free optics.

Nominal optics



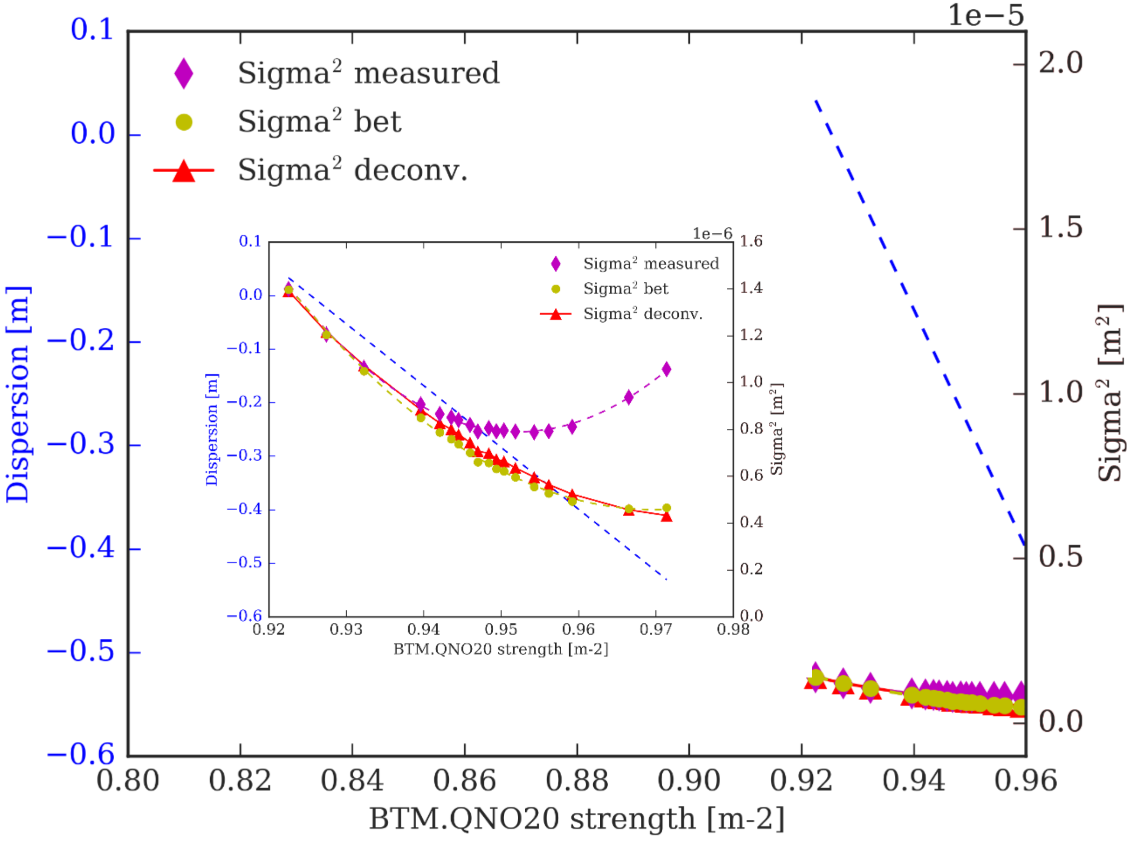
Disp-free optics



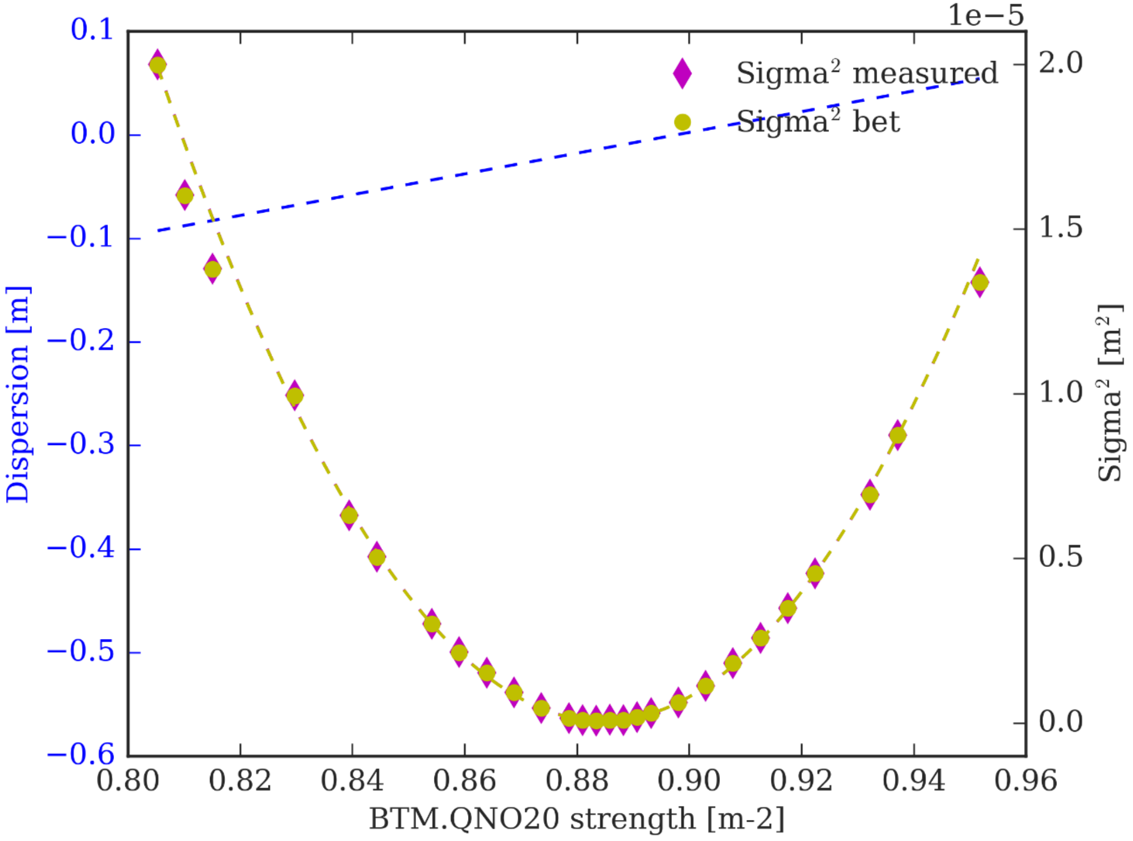
SIGMA² VS K

Parabola fit to extract the beam emittance

Nominal optics



Disp-free optics



RESULTS: normalized emittance

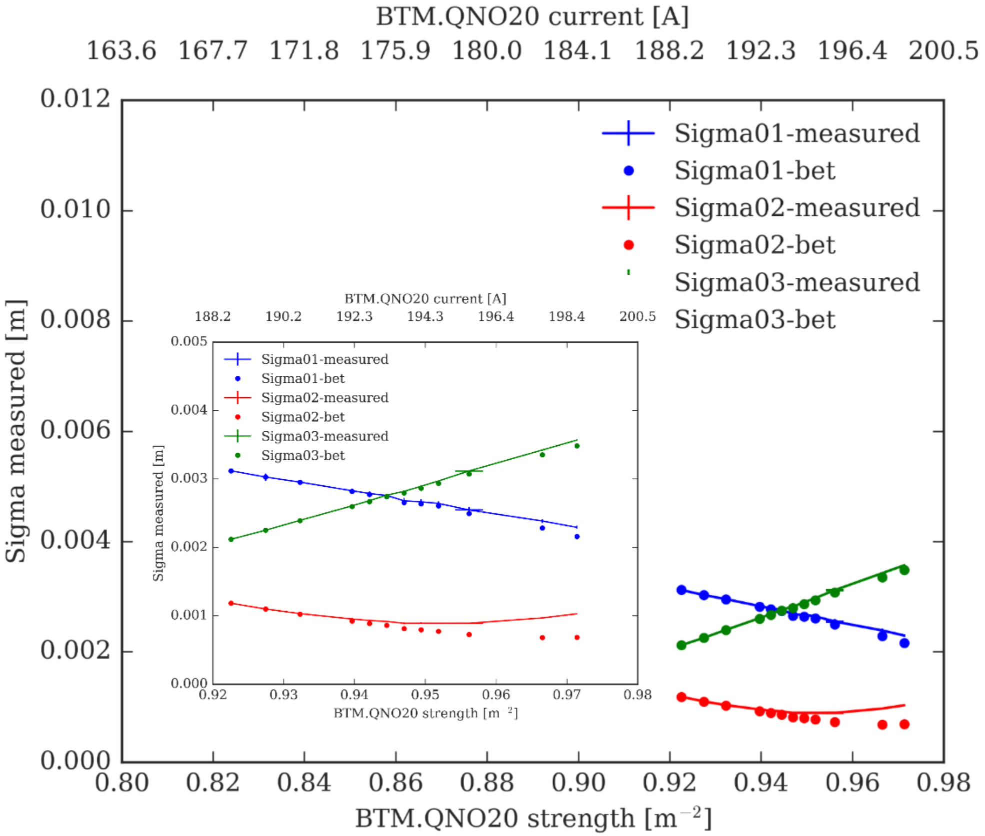
from	NOMINAL OPTICS	DISP-FREE OPTICS
Measured sigma	2.02E-6 (m.rad)	1.19E-6 (m.rad)
Betatronic sigma	**1.20E-6 (m.rad)**	**1.19E-6 (m.rad)**
Deconvoluted sigma	0.97E-6 (m.rad)	*No convergency*

- Good agreement on normalized emittance from "betatronic sigma"
- Issue on emittance error (under investigation)
- No Wire scanner measurement to be compared with

COMPARISON WITH 3 SEMGRID METHOD

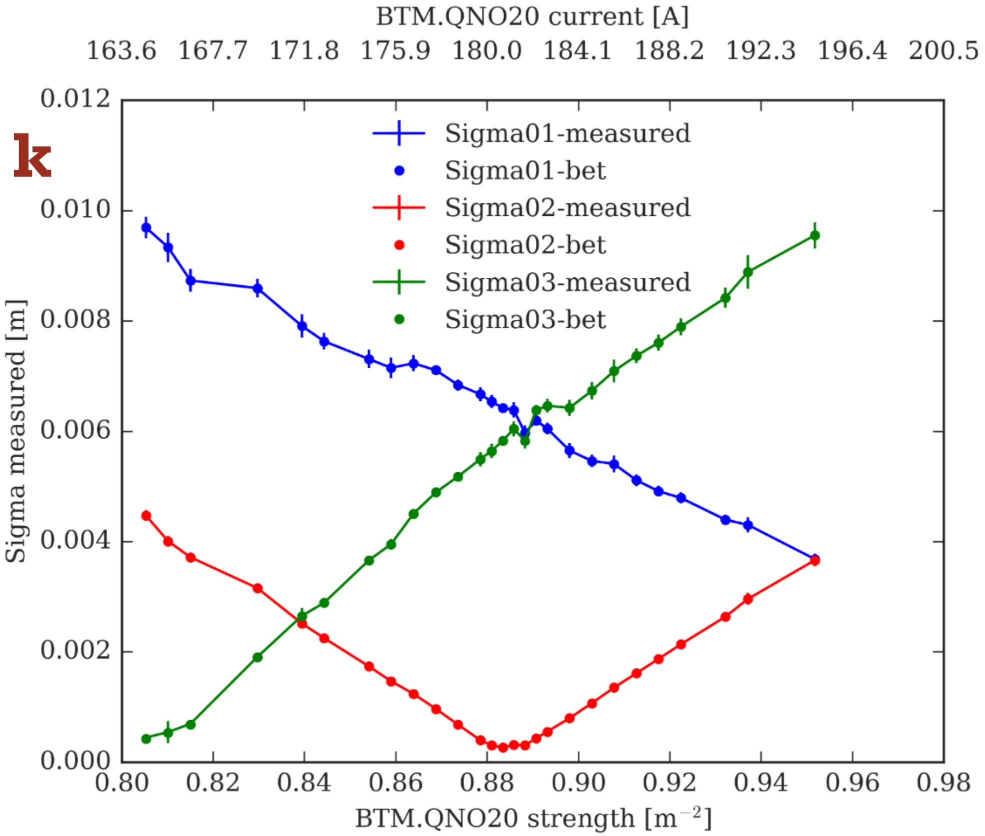
The same set of data used for the quadrupole scan but analysed for the 3 SEM grids

Nominal optics



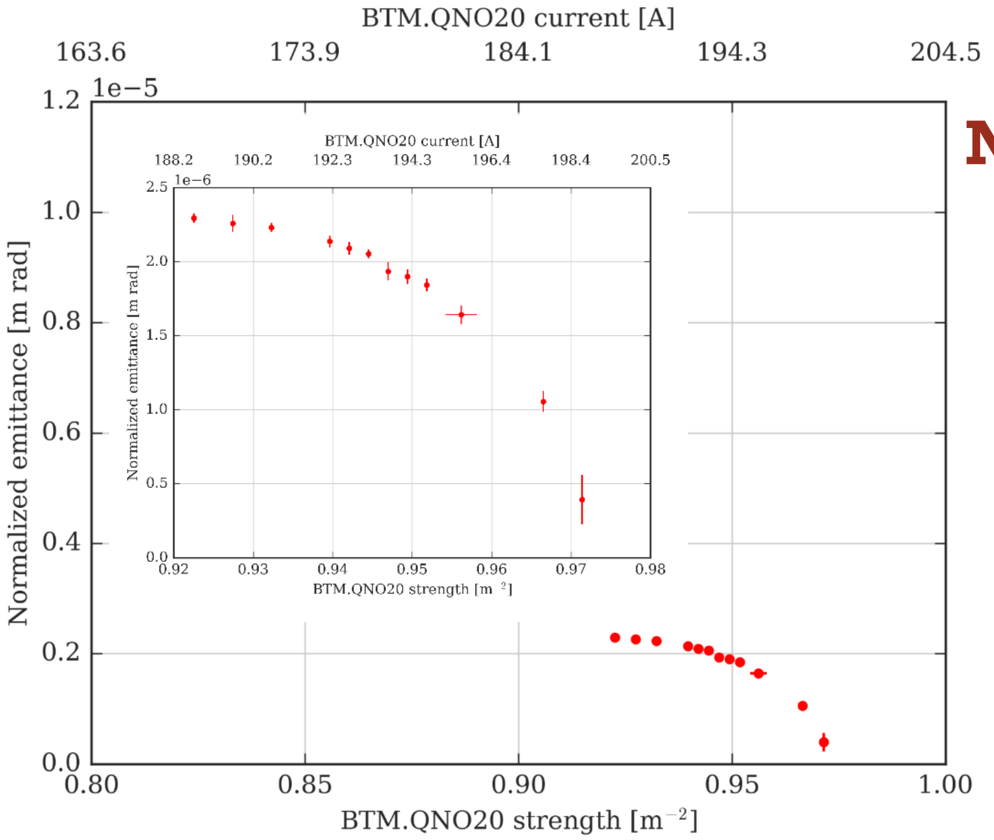
Sigma VS k

Disp-free optics



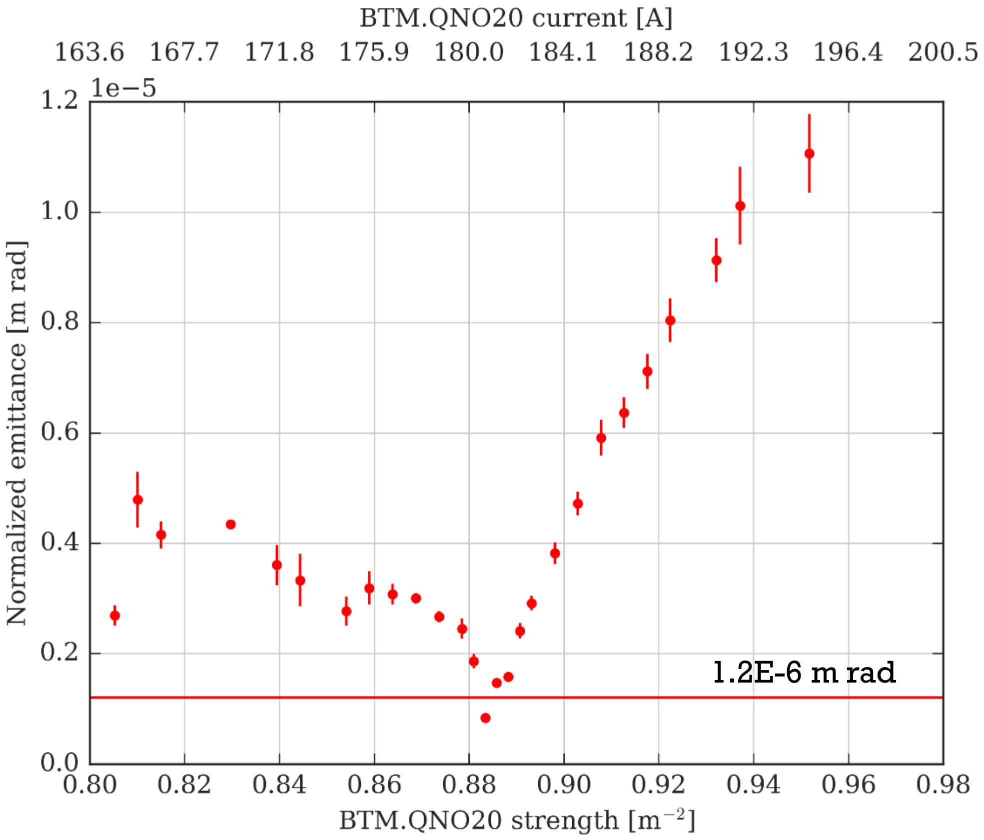
COMPARISON WITH 3 SEMGRID METHOD

Nominal optics



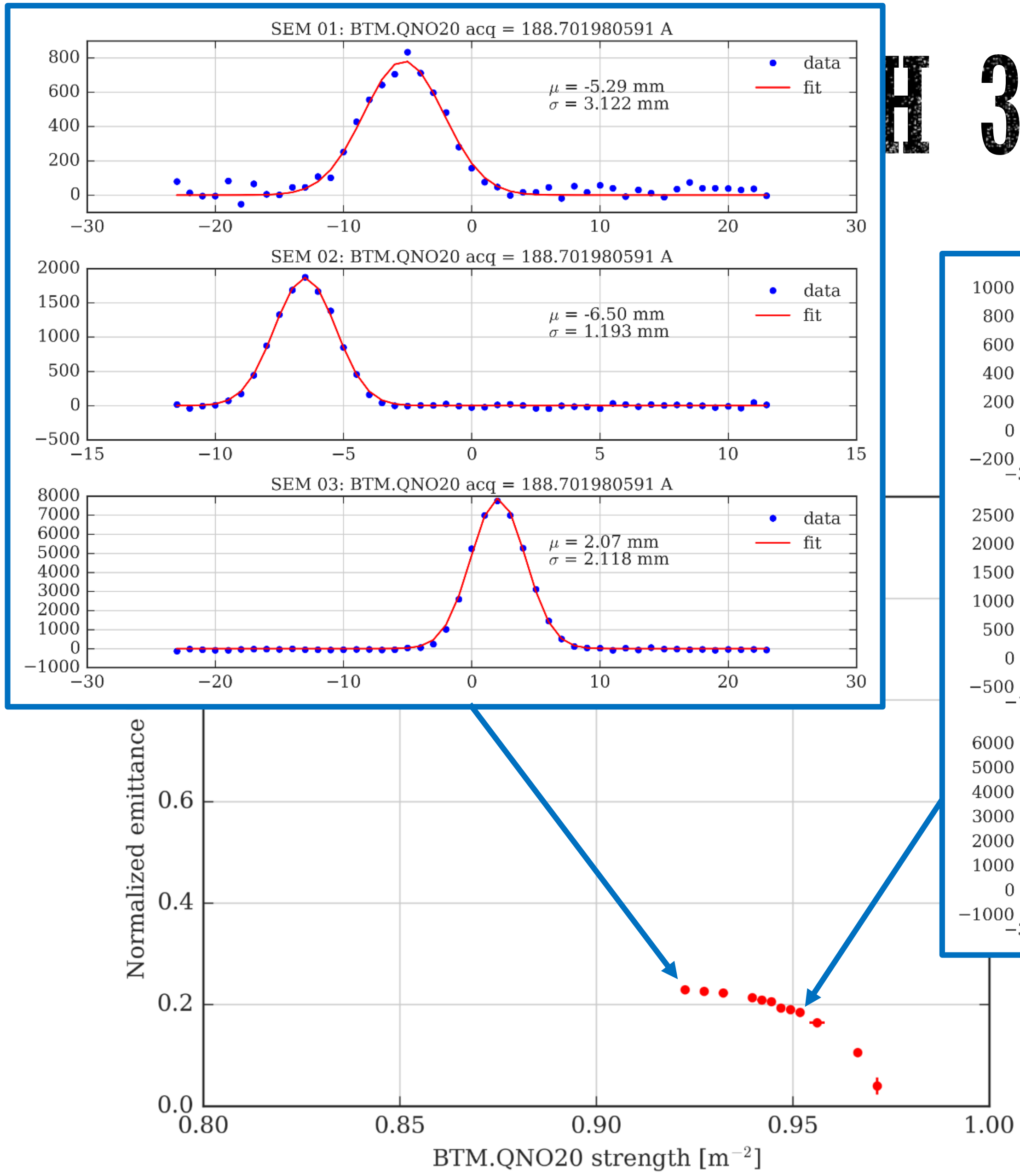
Norm. emit.
VS
k

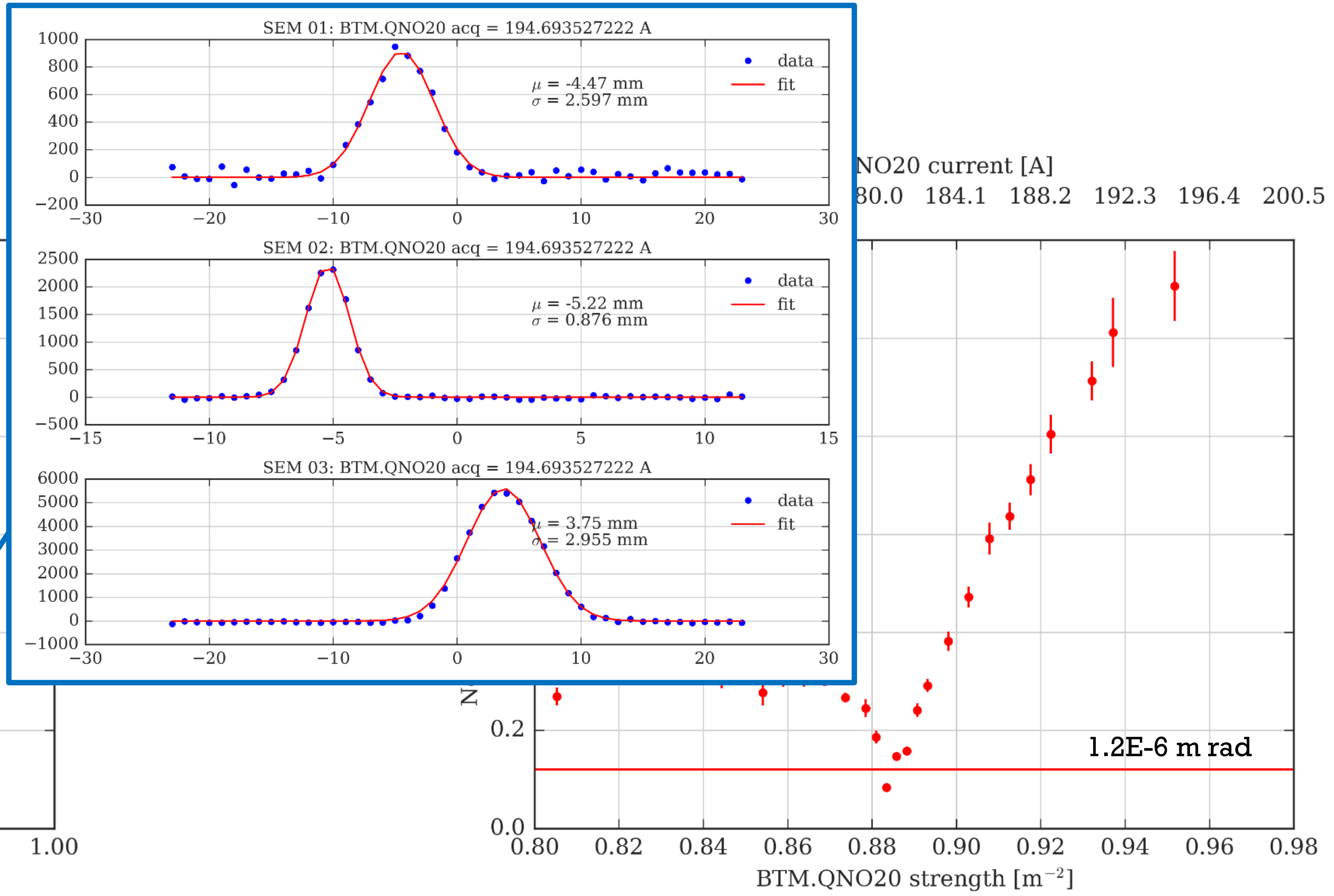
Disp-free optics



<u>From MADX</u>: **no dependence** of normalized emittance on k!

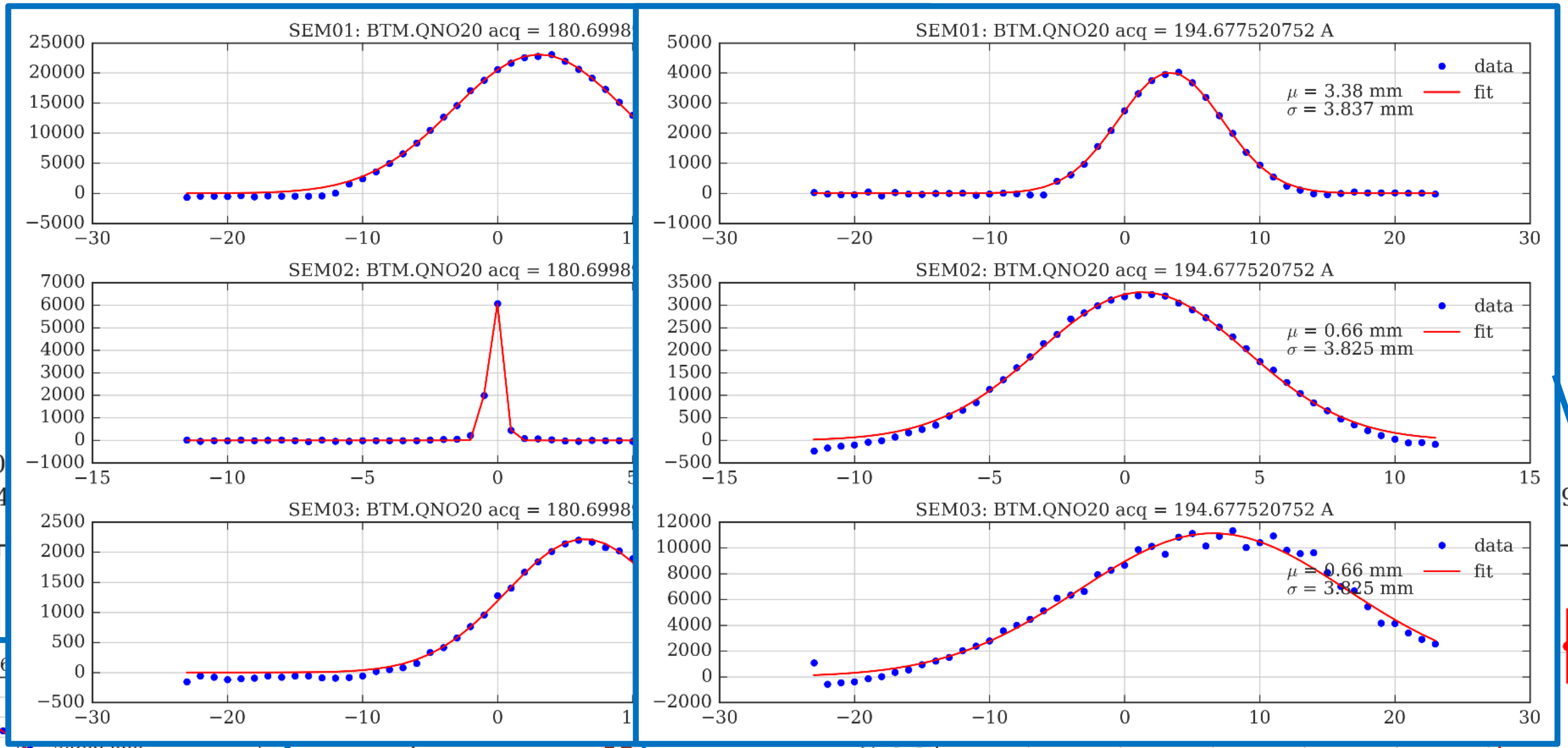
H 3 SEMGRID METHOD

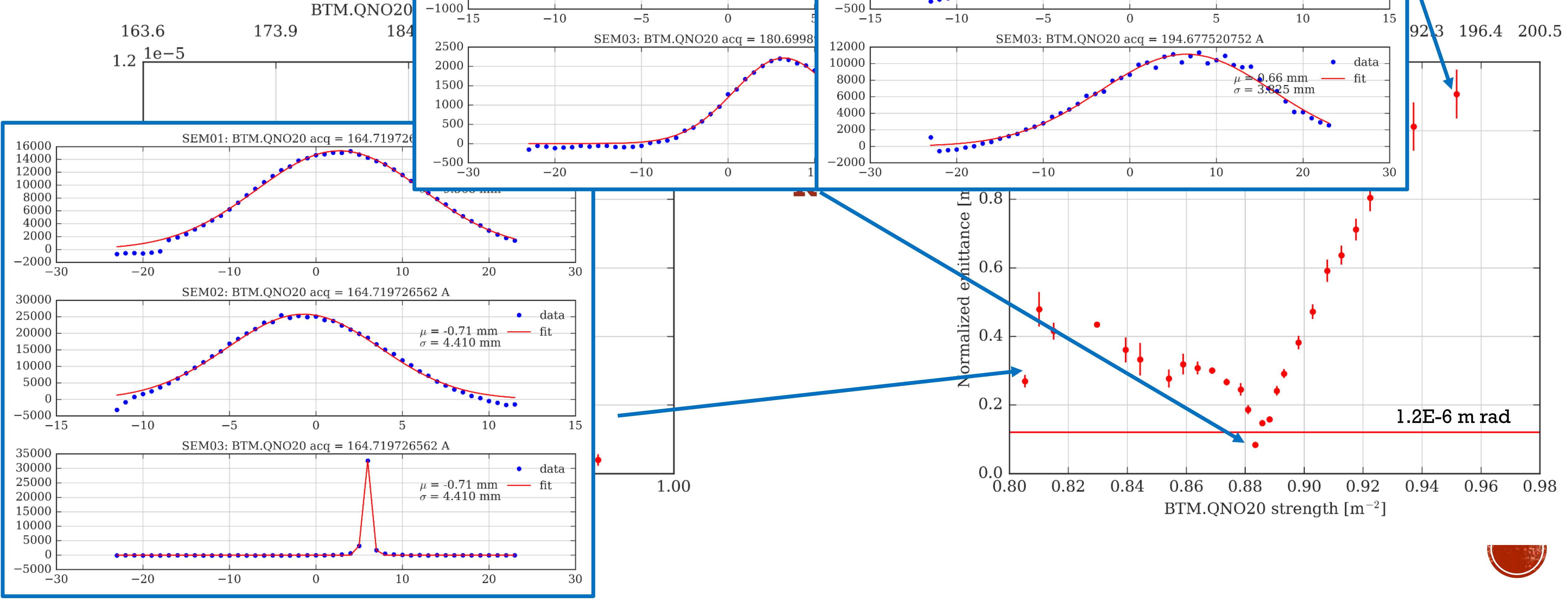




COMPARISO

Nominal optics





CONCLUSIONS AND OUTLOOK

- Test of quadrupole (Q) scan method to understand the discrepancy of the measured emittance with the wires scanner in the ring and SEM grids in BTM line.
- Q-scan method showed good agreement between normalized emittance measured at SEM 02 in BTM line with 2 different optics (nominal and disp-free)
- 3-SEM grids method applied on the same set of data showed a dependency of the normalized emittance on the quadrupole strength (from MADX expected to be flat)

Still a lot to be done:

- Estimate the error on normalized emittance
- Estimate the dependence of the Q-method results on the parameters (d, l and parabola coeff.)
- Complete the comparison adding a wire scanner measurement
- Understand emittance dependence on quadrupole strength in 3-SEM grid methods

DISPERSION FREE OPTICS

BT.QNO10	186.00 A
BT.QNO20	130.00 A
BT.QNO30	60.36 A
BT.QNO40	218.64 A
BT.QNO50	207.47 A
BTM.QNO05	137.53 A
BTM.QNO10	-155.72 A
BTM.QNO20	176.75 A

RESULTS: normalized emittance

Nominal optics: HORIZONTAL plane R3

```
************ measured ************
EMITTANCE = 8.86755084732e-07 m
EMITTANCE error = 1.94963487822e-05 m
csi11= 0.0000474617, err_csi11= 0.0000007757, err_per= 0.02
csi12= -0.0000321813, err_csi12= 0.0000004292, err_per= 0.01
csi22= 0.0000218370, err_csi22= 0.0000002542, err_per= 0.01
alpha = 36.2910298255 beta = 53.5229321939 gamma = 24.6256845761
NORMALIZED EMITTANCE = 2.02457713661e-06 m
NORMALIZED EMITTANCE error = 4.45126987951e-05 m

************ betatronic ************
EMITTANCE = 5.24001657041e-07 m
EMITTANCE error = 2.1074634076e-05 m
csi11= 0.0000285910, err_csi11= 0.0000008102, err_per= 0.03
csi12= -0.0000196672, err_csi12= 0.0000004477, err_per= 0.02
csi22= 0.0000135383, err_csi22= 0.0000002649, err_per= 0.02
alpha = 37.5326340179 beta = 54.5627482139 gamma = 25.8362832237
NORMALIZED EMITTANCE = 1.19636390324e-06 m
NORMALIZED EMITTANCE error = 4.81161292979e-05 m
```

NORMALIZED EMITTANCE = 1.2E-06 m

```
************ deconvoluted ************
EMITTANCE = 4.24970468554e-07 m
EMITTANCE error = 1.09737053265e-05 m
csi11= 0.0000208023, err_csi11= 0.0000004665, err_per= 0.02
csi12= -0.0000144124, err_csi12= 0.0000002577, err_per= 0.02
csi22= 0.0000099940, err_csi22= 0.0000001525, err_per= 0.02
alpha = 33.9139369504 beta = 48.949954124 gamma = 23.5169805585
NORMALIZED EMITTANCE = 9.7026282587e-07 m
NORMALIZED EMITTANCE error = 2.50543958422e-05 m
```

Disp-free optics: HORIZONTAL plane R3

```
************ measured ************
EMITTANCE = 5.22983121506e-07 m
EMITTANCE error = 0.000764063017779 m
csi11= 0.0002105661, err_csi11= 0.0000045800, err_per= 0.02
csi12= -0.0001348233, err_csi12= 0.0000023614, err_per= 0.02
csi22= 0.0000863273, err_csi22= 0.0000013172, err_per= 0.02
alpha = 257.796699867 beta = 402.625046033 gamma = 165.067074483
NORMALIZED EMITTANCE = 1.1940384542e-06 m
NORMALIZED EMITTANCE error = 0.00174445519778 m

************ betatronic ************
EMITTANCE = 5.21301983623e-07 m
EMITTANCE error = 0.000765925739589 m
csi11= 0.0002104212, err_csi11= 0.0000045796, err_per= 0.02
csi12= -0.0001347295, err_csi12= 0.0000023612, err_per= 0.02
csi22= 0.0000862666, err_csi22= 0.0000013170, err_per= 0.02
alpha = 258.448128388 beta = 403.645525828 gamma = 165.482907137
NORMALIZED EMITTANCE = 1.19020019787e-06 m
NORMALIZED EMITTANCE error = 0.00174870803383 m
```

NORMALIZED EMITTANCE = 1.2E-06 m

From MADX model
1) Normalized emittance (betatronic)=1.5E-6 m
→Ask to VINCENZO about the plot
2) I used beta_coeff from Vincenzo and I obtained 1.84E-6 m rad

Disp-free optics: VERTICAL plane R3

```
************ measured ************
EMITTANCE = 4.21615980603e-07 m
EMITTANCE error = 5.61668063954e-07 m
csi11= 0.0000136428, err_csi11= 0.0000001552, err_per= 0.01
csi12= 0.0000027205, err_csi12= 0.0000000580, err_per= 0.02
csi22= 0.0000005555, err_csi22= 0.0000000251, err_per= 0.05
alpha = -6.45258107414 beta = 32.3583900336 gamma = 1.31761198484 NORMALIZED
EMITTANCE = 9.62604093793e-07 m
NORMALIZED EMITTANCE error = 1.28236120685e-06 m
************ betatronic ************
EMITTANCE = 4.21591345617e-07 m
EMITTANCE error = 5.61676445951e-07 m
csi11= 0.0000136414, err_csi11= 0.0000001552, err_per= 0.01
csi12= 0.0000027202, err_csi12= 0.0000000580, err_per= 0.02
csi22= 0.0000005555, err_csi22= 0.0000000251, err_per= 0.05 alpha = -6.45230540646
beta = 32.3568410601 gamma = 1.31756511642
```

NORMALIZED EMITTANCE = 9.62547848918e-07 m
NORMALIZED EMITTANCE error = 1.28238034404e-06 m