Nombre Santiago Andrés Macías Paz — Fecha 26 día 05 mes 2024 año

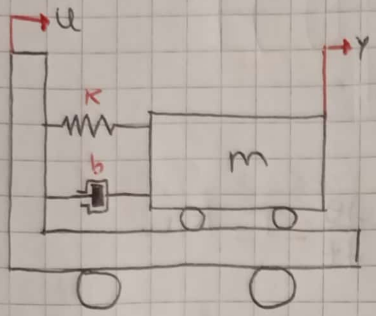
Profesor Henry Borrero Guerrero — Materia Sistemas Dinamicos

Institución Universidad Distrital Francisco José de Caldas — Curso — Nota

Primer Punto parcial
segundo corte

① Ogata

3-3



$$ma = \Sigma F$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = b\frac{du}{dt} + ku$$

$$[ms^2 + bs + k]Y(s) = (bs + k)U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k}$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u$$

$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u \qquad a_1 = \frac{b}{m} \quad a_2 = \frac{k}{m} \quad b_0 = 0 \quad b_1 = \frac{b}{m} \quad b_2 = \frac{k}{m}$$

$$\beta_0 = b_0 = 0$$

$$\beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$$

$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$$

$$x_1 = y - \beta_0 u = y$$

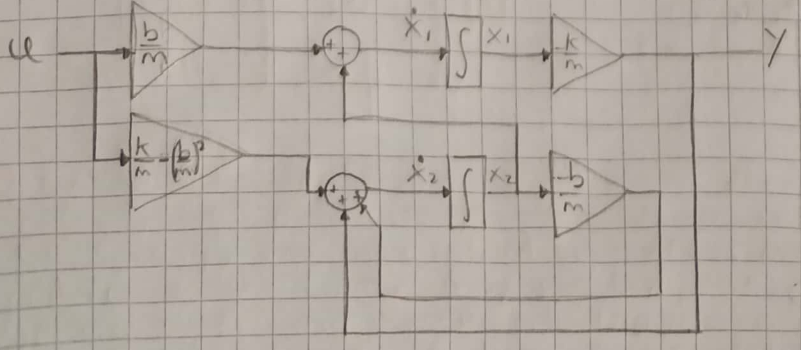
$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m}u$$

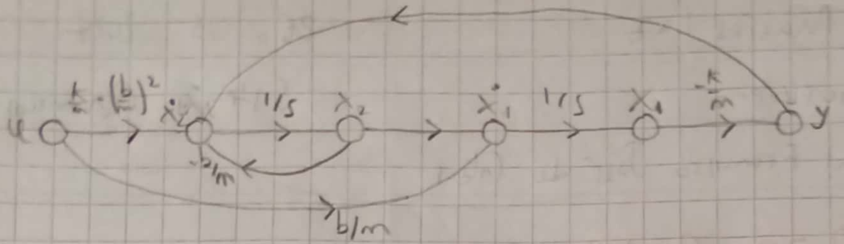
$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m}u$$

$$\dot{x}_2 = -a_2x_1 - a_1x_2 + \beta_2 u = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right]u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}b/m\\ \frac{k}{m} - \left(\frac{b}{m}\right)^2\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$





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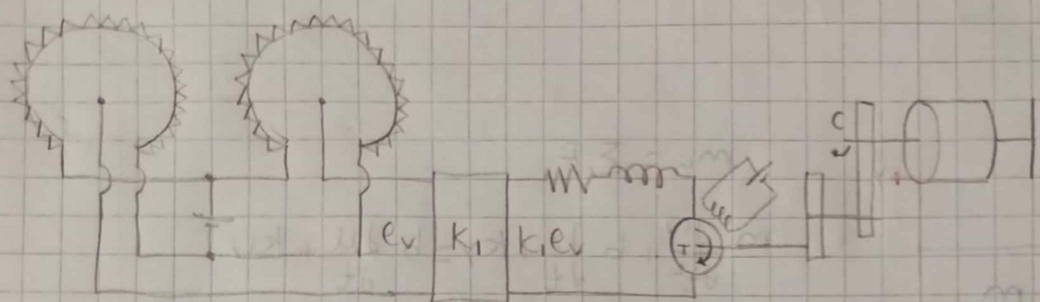


$$L_a \frac{di_a}{dt} + R_a i_a + k_3 \frac{d\theta}{dt} = k_1 e_v \qquad J_0 \frac{d^2\theta}{dt^2} \quad b_0 \frac{d\theta}{dt} = T = k_2 i_a$$

$$\frac{\Theta(s)}{E_v(s)} = \frac{k_1 k_2}{s\,(L_a s + R_a)\,(J_0 s + b_0) + k_2 k_3 s}$$

$$G(s) = \frac{k_0 k_1 k_2 n}{s\,(L_a s + R_a)(J_0 s + b_0) + k_2 k_3} = \frac{k_0 k_1 k_2 n / R_a}{J_0 s^2 + \left(b_0 + \frac{k_2 k_3}{R_a}\right) s}$$

$J = J_0 / n^2$

$B = [b_0 + (k_2 k_3 / R_a)] / n^2$

$K = k_0 k_1 k_2 / n R_a$

$$G(s) = \frac{K}{J s^2 + B s} = \frac{K_m}{s\,(T_m s + 1)}$$

$$K_m = \frac{K}{B} \qquad T_m = \frac{J}{B} = \frac{R_a J_0}{R_a b_0 + k_2 k_3}$$

$eK = J\ddot{C} + B\dot{C}$

$\ddot{C} = \frac{eK}{J} - \frac{B\dot{C}}{J}$

Variables de estados

$q_1 = C$

$q_2 = \dot{q}_1 = \dot{C}$

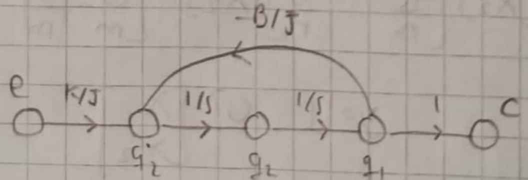
$\dot{q}_2 = \ddot{q}_1 = \ddot{C}$

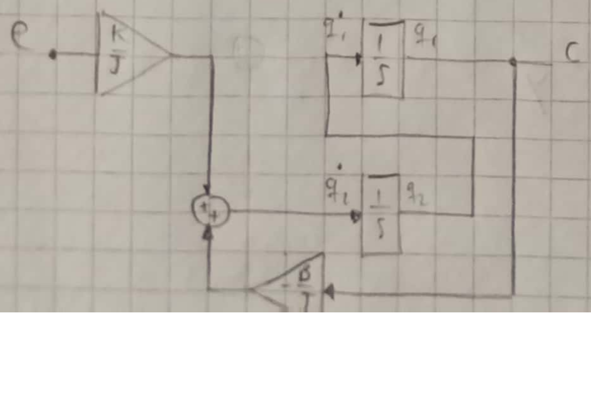
Espacio de estados

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -B/J & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K/J \end{bmatrix} e$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + [0]\, e$$

Diagrama de flujo de señal





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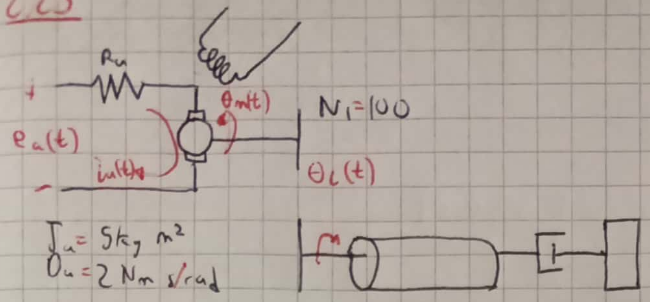
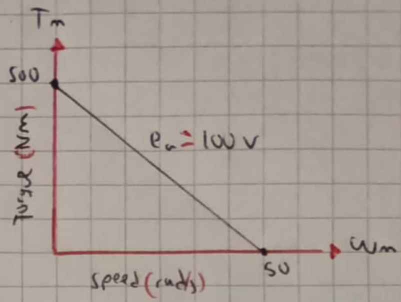
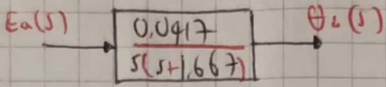


$J_a = 5\ kg\ m^2$
$D_a = 2\ Nm\ s/rad$

$J_L = 700\ kg\ m^2$ $D_L = 800\ Nm\ s/rad$





$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12 \qquad D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10$$

$T_{stall} = 500$
$\omega_{no\ load} = 50$
$e_a = 100$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{500}{100} = 5 \qquad K_b = \frac{e_a}{\omega_{no\text{-}load}} = \frac{100}{50} = 2$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{5/12}{s\left\{s + \frac{1}{12}[10 + (5)(2)]\right\}} = \frac{0{,}417}{s(s+1{,}667)}$$

$$\frac{\theta_L(s)}{E_a(s)} = \frac{0{,}0417}{s(s+1{,}667)}$$

$$\theta_L(s)\,[s^2 + 1{,}667 s] = 0{,}0417\ E_a(s)$$

$$\ddot{\theta} + \dot{\theta}\,1{,}667 = 0{,}0417\ E_a \qquad q_1 = \theta$$
$$q_2 = \dot{q} = \dot{\theta}$$

$$\ddot{\theta} = 0{,}0417\ E_a - \dot{\theta}\,1{,}667$$

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -1{,}667\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix} + \begin{bmatrix}0\\ 0{,}0417\end{bmatrix}E_a$$

$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix}$$

Diagrama de Bloques

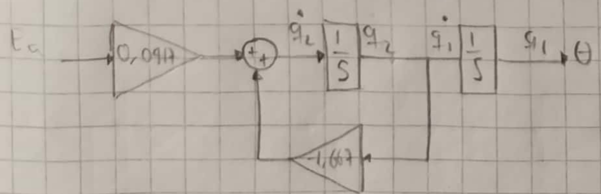
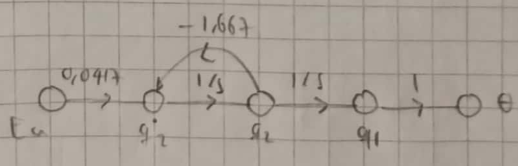


Diagrama de flujo



4) En el libro de Ogata se hace un planteamiento muy diferente al de Nise pues plantea las ecuaciones usando muchas constantes, sin embargo una vez introduce nuevos parametros la expresión se reduce a una forma muy similar, pues tanto su expresión en el espacio de estados como sus Diagramas son muy similares, lo cual nos muestra como se llega a las realizaciones de los mismos sistemas.

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Parcial

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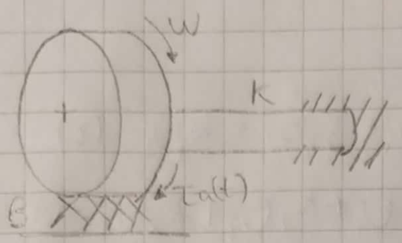
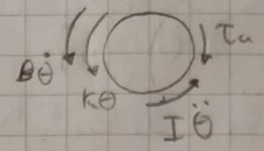


$$\tau_a = J\ddot{\theta} + B\dot{\theta} + k\theta$$

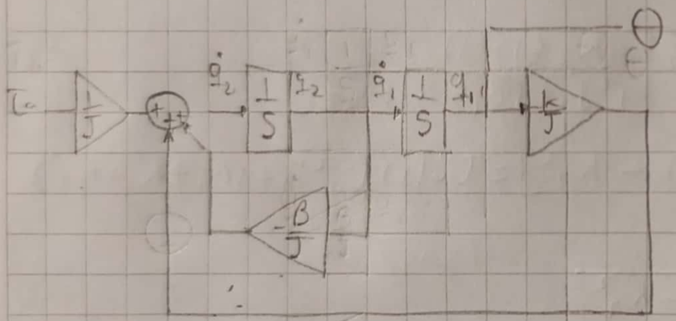


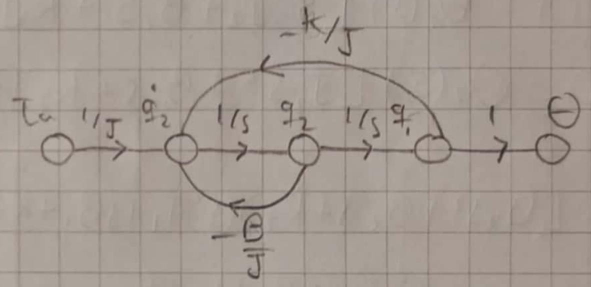
$$q_1 = \theta$$
$$q_2 = \dot{\theta}$$

$$\ddot{\theta} = \frac{\tau_a - B\dot{\theta} - k\theta}{J}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/J & -B/J \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau_a$$

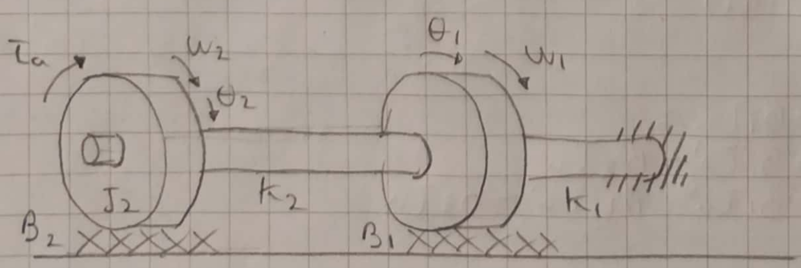
$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

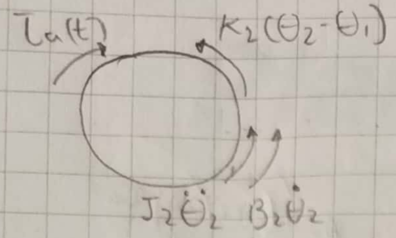


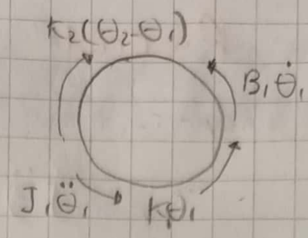


$$T_a(s) = \Theta(s)\left[s^2 J + sB + k\right] \qquad \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + k}$$

2 Asuma $\theta_2 > \theta_1$







$$\tau_a = J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + k_2(\theta_2-\theta_1) \quad (1)$$

$$k_2(\theta_2-\theta_1) = B_1\dot{\theta}_1 + J_1\ddot{\theta}_1 + k_1\theta_1 \quad (2)$$

$$B_1\dot{\theta}_1 + J_1\ddot{\theta}_1 + k_1\theta_1 - k_2\theta_2 + k_2\theta_1 = 0$$

$$\theta_1(s)[s^2J_1 + sB_1 + k_1+k_2] = k_2\theta_2(s)$$

$$\theta_1(s) = \frac{k_2\theta_2(s)}{s^2J_1 + sB_1 + k_1 + k_2} \quad (3)$$

Reemplazo 3 en 1

$$\tau_a(s) = \theta_2(s)[s^2J_2 + B_2s + k_2] - k_2\theta_1(s)$$

$$\tau_a(s) = \theta_2(s)[J_2s^2 + B_2s + k_2] - k_2\left[\frac{k_2\theta_2(s)}{s^2J_1 + sB_1 + k_1 + k_2}\right]$$

$$\theta_2(s)[(J_2s^2 + B_2s + k_2)(s^2J_1 + sB_1 + k_1 + k_2) - k_2^2] = \tau_a(s)(s^2J_1 + sB_1 + k_1 + k_2)$$

$$\frac{\theta_2(s)}{\tau_a(s)} = \frac{s^2J_1 + sB_1 + k_1 + k_2}{(J_2s^2 + B_2s + k_2)(s^2J_1 + sB_1 + k_1 + k_2) - k_1^2}$$

$$\theta_2(s)[J_2J_1s^4 + s^3(B_1J_2 + B_2J_1) + s^2(J_2k_1 + J_2k_2 + B_1B_2 + k_2J_1) + s(B_2k_1 + B_2k_2 + B_1k_2) + k_2k_1]$$
$$= \tau_a(s)[s^2J_1 + sB_1 + k_1 + k_2]$$

$$\Downarrow \mathcal{L}^{-1}$$

$$\theta_2^{IV}(J_2J_1) + \theta_2'''(B_1J_2 + B_2J_1) + \theta_2''(J_2k_1 + J_2k_2 + B_1B_2 + k_2J_1) + \theta_2'(B_2k_1 + B_2k_2 + B_1k_2) + \theta_2k_2k_1$$
$$= \tau_a''J_1 + \tau_a'B_1 + \tau_a(k_1 + k_2)$$

$$x_1 = \theta_2 - \beta_0\tau_a$$
$$x_2 = \theta_2' - \beta_0\tau_a' - \beta_1\tau_a$$
$$x_3 = \theta_2'' - \beta_0\tau_a'' - \beta_1\tau_a' - \beta_2\tau_a$$
$$x_4 = \theta_2''' - \beta_0\tau_a''' - \beta_1\tau_a'' - \beta_2\tau_a' - \beta_3\tau_a$$

$$\beta_0 = b_0$$
$$\beta_1 = b_1 - a_1\beta_0$$
$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$$
$$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0$$
$$\beta_4 = b_4 - a_1\beta_3 - a_2\beta_2 - a_3\beta_1 - a_4\beta_0$$

Función de transferencia

$$y^{IV} + a_1 y''' + a_2 y'' + a_3 y' + a_4 y = b_0 u^{IV} + b_1 u''' + b_2 u'' + b_3 u' + b_4 u$$

$a_1 = \frac{B_1 J_2 + B_2 J_1}{J_1 J_2}$ $\quad b_0 = 0 \quad b_1 = 0$

$a_2 = \frac{J_2 k_1 + J_2 k_2 + B_1 B_2 + k_2 J_1}{J_1 J_2}$ $\quad b_2 = \frac{1}{J_2} \quad b_3 = \frac{B_1}{J_2 J_1} \quad b_4 = \frac{k_1 + k_2}{J_2 J_1}$

$a_3 = \frac{B_2 k_1 + B_2 k_2 + B_1 k_2}{J_1 J_2}$

$$\dot{x}_1 = x_2 + \beta_1 u$$
$$\dot{x}_2 = x_3 + \beta_2 u$$
$$\dot{x}_3 = x_4 + \beta_3 u$$

$a_4 = \frac{k_2 k_1}{J_2 J_1}$

$\beta_0 = 0 \quad \beta_1 = 0 \quad \beta_2 = \frac{1}{J_2}$

$\beta_3 = \frac{B_1}{J_1 J_2} - \frac{B_1 J_2 + B_2 J_1}{J_1 J_2^2} \quad \beta_3 = \frac{-B_2}{J_2^2}$

$\beta_4 = \frac{k_1 + k_2}{J_1 J_2} + \frac{(B_1 J_2 + B_2 J_1) B_2}{J_1 J_2^3} - \frac{J_2 k_1 + J_2 k_2 + B_1 B_2 + k_2 J_1}{J_1 J_2^2}$

$\beta_4 = \frac{B_1 B_2}{J_1 J_2^2} + \frac{B_2^2}{J_2^3} - \frac{B_1 B_2}{J_1 J_2^2} - \frac{k_2}{J_2^2} = \frac{B_2^2}{J_2^3} - \frac{k_2}{J_2^2}$

$$\dot{x}_4 = \theta_2^{IV} - \beta_0 u^{IV} - \beta_1 u''' - \beta_2 u'' - \beta_3 u'$$

$$\dot{x}_4 = \theta_2^{IV} - \frac{u''}{J^2} + \frac{u' B_2}{J^2}$$

$$\dot{x}_4 = -a_4 x_1 - a_3 x_2 - a_2 x_3 - a_1 x_4 + \beta_4 u$$

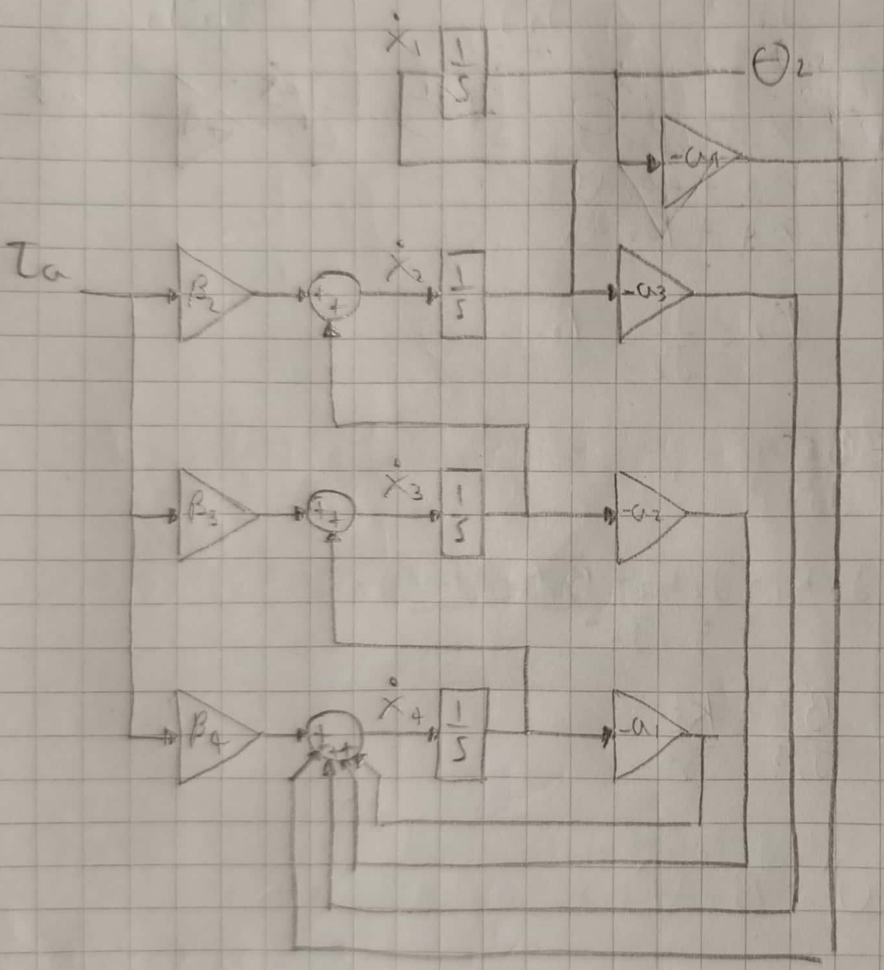
$\dot{x}_1 = x_2$

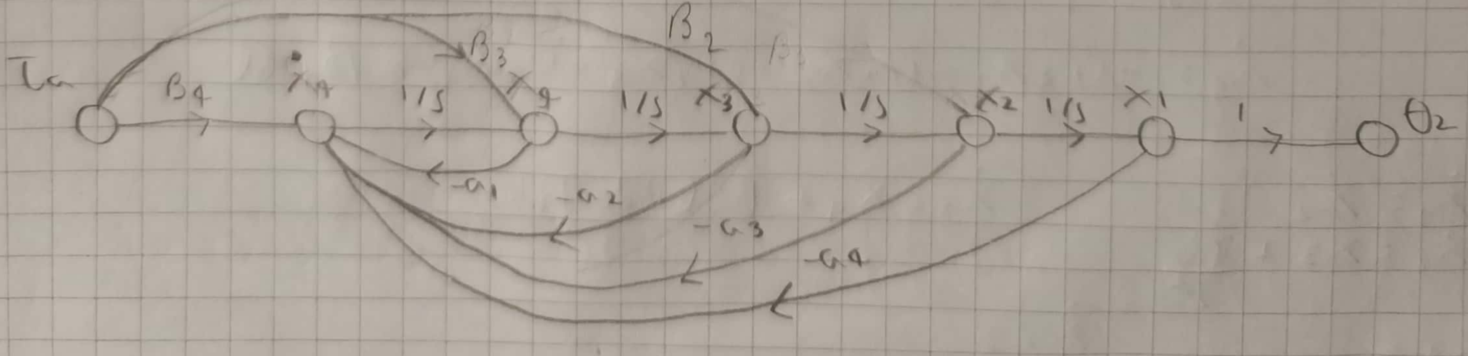
$\dot{x}_2 = x_3 + \frac{T_a}{J^2}$

$\dot{x}_3 = x_4 - T_a \frac{(B_2^2 - J_2 k_2)}{J_2^3}$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\-a_4&-a_3&-a_2&-a_1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0\\ \beta_2\\ \beta_3\\ \beta_4\end{bmatrix}\tau_a$$

$$\theta_2 = [1\ 0\ 0\ 0]\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$$





③ Como los β calculados en el anterior punto no dependen de k_1, no cambian, los coeficientes de a siguiendo de la siguiente forma:

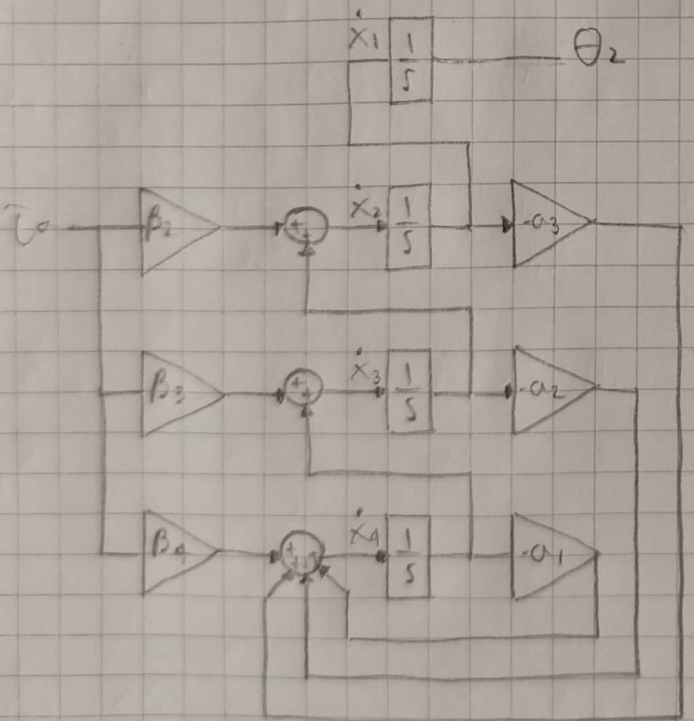
$$a_1 = \frac{B_1 J_2 + B_2 J_1}{J_1 J_2} \qquad a_2 = \frac{J_2 k_2 + B_1 B_2 + k_2 J_1}{J_1 J_2}$$

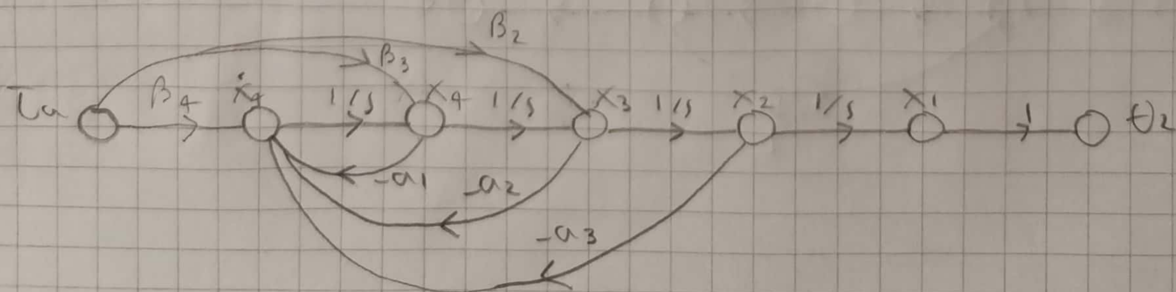
$$a_3 = \frac{B_2 k_2 + B_1 k_2}{J_1 J_2} \qquad a_4 = 0$$

Con esto en mente obtenemos el siguiente espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$





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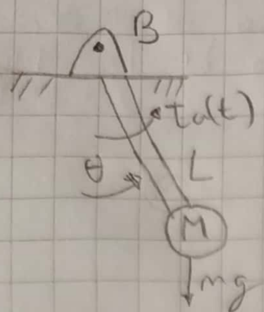


$$\tau_a - mgL\,\text{sen}\,\theta - B\dot{\theta} = mL^2\ddot{\theta}$$

Linealizamos el sistema con $\theta << 1$ tal que $\text{sen}(\theta) \approx \theta$

$$\ddot{\theta} = \frac{\tau_a}{mL^2} - \frac{g}{L}\theta - \frac{B}{mL^2}\dot{\theta}$$

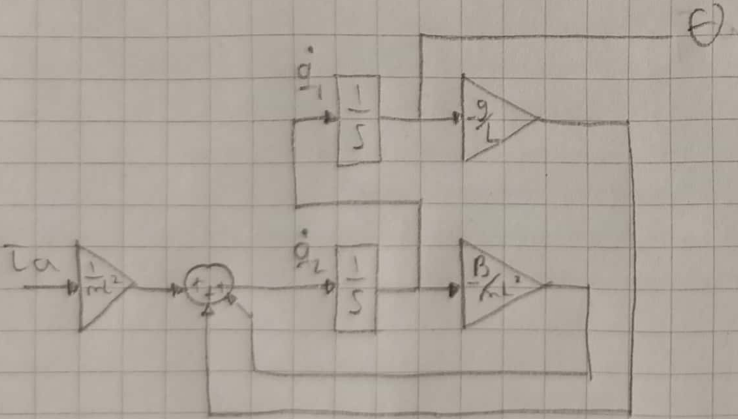
$q_1 = \theta$
$q_2 = \dot{q}_1$

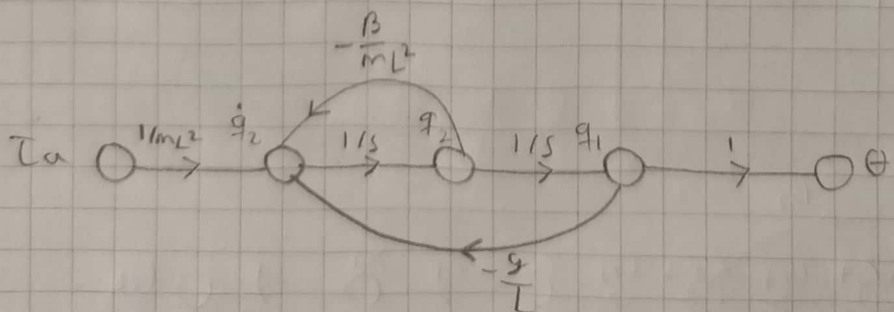
$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/L & -\frac{B}{mL^2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/mL^2 \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$\tau_a = \Theta(s)\,[\,s^2 mL^2 + sB + mgL\,]$$

$$\frac{\Theta(s)}{\tau_a(s)} = \frac{1}{s^2 mL^2 + sB + mgL}$$





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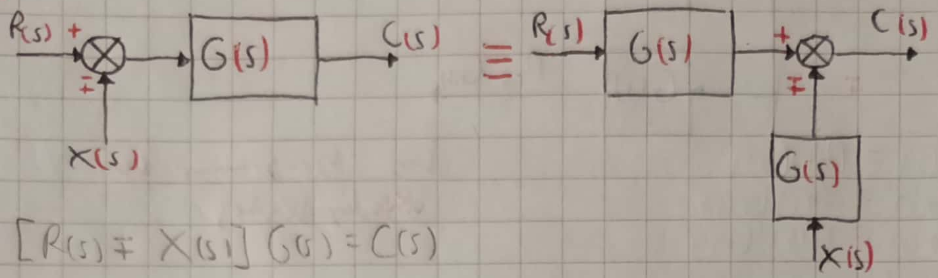
Bonificación Algebra de bloques Parcial

1)

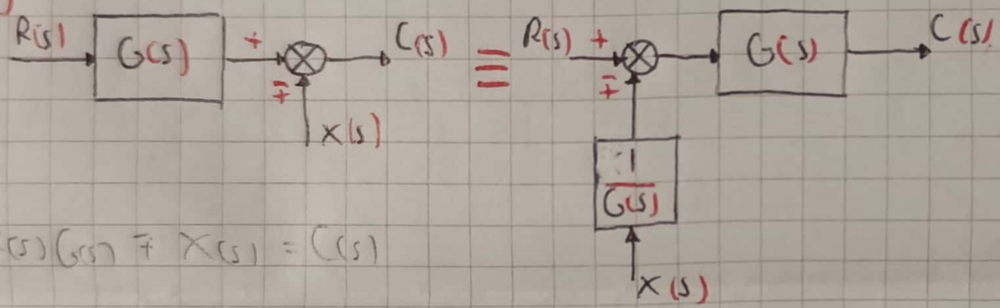


$[R(s) \mp X(s)]\, G(s) = C(s)$

$R(s)G(s) \mp X(s)G(s) = C(s) \equiv R(s)G(s) \mp X(s)G(s) = C(s)$

Los diagramas son equivalentes

2)



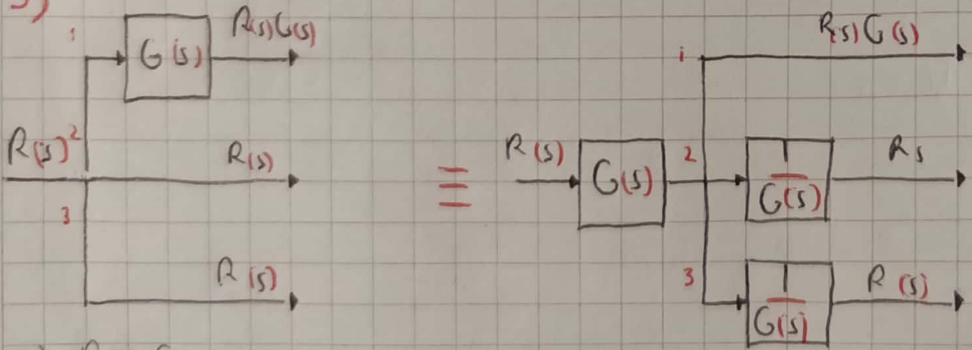
$R(s)G(s) \mp X(s) = C(s)$

Los diagramas son equivalentes

$\left[R(s) \mp \frac{X(s)}{G(s)}\right] G(s) = C(s)$

$R(s)G(s) \mp X(s) = C(s) \equiv G(s)R(s) \mp X(s) = C(s)$

3)



1) $R(s) \cdot G(s) = R_s G(s)$

2) $[R_s G(s)] \frac{1}{G(s)} = R(s)$

3) $[R_s G(s)] \frac{1}{G(s)} = R_s$

1) $R(s)G(s)$
2) $R(s)$
3) $R(s)$

1) $G(s)R(s) = R(s)G(s)$
2) $R(s) = R(s)$
3) $R(s) = R(s)$

Los diagramas son equivalentes

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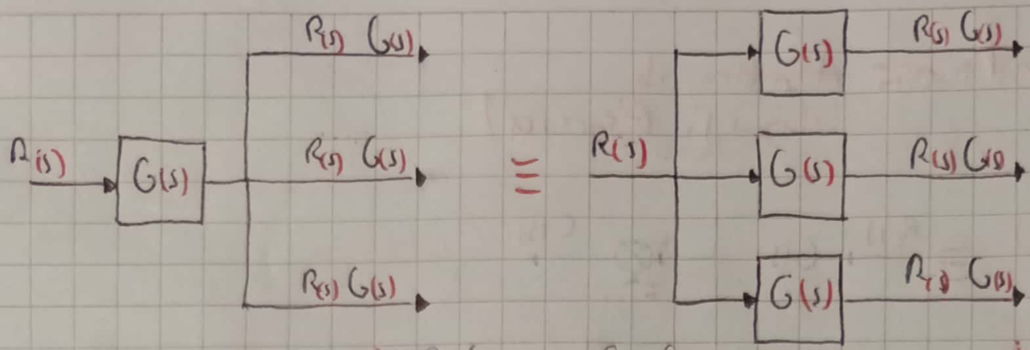
4)



1) $R(s)G(s) = R(s)G(s)$
2) $R(s)G(s) = R(s)G(s)$
3) $R(s)G(s) = R(s)G(s)$

Los diagramas son equivalentes