

Video 2: Partial Fractions

$$X(s) = \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{A}{s+2+j2} + \frac{A^*}{s+2-j2}$$

$$k_1 = s X(s) \Big|_{s=0} = \frac{s(2s^3 + 8s^2 + 4s + 8)}{s(s+1)(s^2 + 4s + 8)} \Big|_{s=0} = \frac{8}{8} \Rightarrow \boxed{k_1 = 1}$$

$$k_2 = (s+1) X(s) \Big|_{s=-1} = \frac{(s+1)(2s^3 + 8s^2 + 4s + 8)}{(s+1)(s^2 + 4s + 8)} \Big|_{s=-1}$$

$$k_2 = \frac{2(-1)^3 + 8(-1)^2 + 4(-1) + 8}{(-1)(-1^2 + 4(-1) + 8)} = \frac{-2 + 8 - 4 + 8}{(-1)(1 - 4 + 8)} = \frac{10}{-5} \Rightarrow \boxed{k_2 = -2}$$

$$A(s+2+j2) X(s) \Big|_{s=-2-j2} = (s+2+j2) \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)} \Big|_{s=-2-j2}$$

$$A = \frac{(s+2+j2) \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s^2 + 4s + 8)}}{(s+2+j2)(s+2-j2)} \Big|_{s=-2-j2} = \frac{2s^3 + 8s^2 + 4s + 8}{s(s+1)(s+2-j2)} \Big|_{s=-2-j2}$$

Per partes

$$\begin{aligned} 2s^3 &= 2[(-2)^3 + 3(-2)^2(-j2) + 3(-2)(-j2)^2 + (-j2)^3] \\ &= 2[-8 - j24 + 24 + j8] = 2[16 - j16] \\ &= 32 - j32 \end{aligned}$$

$$\begin{aligned} 8s^2 &= 8[-2-j2]^2 \\ &= 8[(-2)^2 + 2(-2)(-j2) + (j2)^2] = 8[4 + 8j - 4] \\ &= 64j \end{aligned}$$

$$A = \frac{32 - j32 + 4(-2-j2) + 8 + 64j}{(2-j2)(-1-j2)(-2j)} = \frac{32 + j24}{24 + j8}$$

$$= \frac{4 + j3}{3 + j} = \frac{4 + j3}{3 + j} \cdot \frac{3 - j}{3 - j} \Rightarrow \boxed{A = 1.5 + j0.5}$$

$$X(s) = \frac{1}{s} + \frac{-2}{s+1} + \frac{1, s + j0,5}{s+2+j2} + \frac{1, s - j0,5}{s+2-j2}$$