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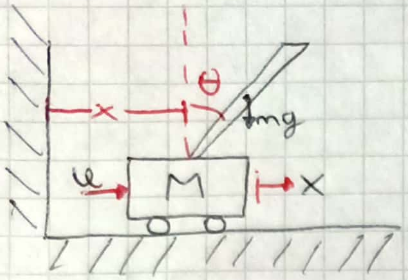
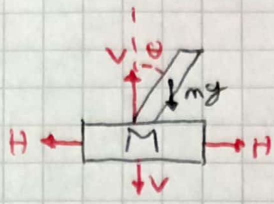
Sistema del pendulo invertido





$$(M+m)\ddot{x} + ml\ddot{\theta} = u$$

$$(I+ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$$

$q_1 = x$ $\quad q_3 = \theta$

$q_2 = \dot{x}$ $\quad q_4 = \dot{\theta}$

$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m}$$

$$(I+ml^2)\ddot{\theta} + \frac{mlu - m^2l^2\ddot{\theta}}{M+m} = mgl\theta$$

$$\ddot{\theta}\left(I + ml^2 - \frac{m^2l^2}{M+m}\right) = mgl\theta - \frac{mlu}{M+m}$$

$$\ddot{\theta}\left(\frac{MI + Im + Mml^2 + m^2l^2 - m^2l^2}{M+m}\right) = \frac{mgl\theta(M+m) - mlu}{M+m}$$

$$\ddot{\theta} = \frac{mgl\theta(M+m) - mlu}{MI + Im + Mml^2} = \frac{mgl(M+m)\theta}{MI + Im + mMl^2} - \frac{mlu}{MI + Im + mMl^2}$$

$$\ddot{x} = \frac{u}{M+m} - \frac{ml}{M+m}\left(\frac{mgl(M+m)}{MI + Im + mMl^2}\theta - \frac{mlu}{MI + Im + mMl^2}\right)$$

$$\ddot{x} = \frac{u}{M+M} + \frac{m^2l^2u}{(M+m)(MI + Im + Mml^2)} - \frac{m^2gl^2(M+m)}{(M+m)(MI + Im + mMl^2)}\theta$$

$$\ddot{x} = u\left(\frac{1}{M+m} + \frac{m^2l^2}{(M+m)(MI + Im + Mml^2)}\right) - \frac{m^2gl^2(M+m)}{(M+m)(MI + Im + mMl^2)}\theta$$

$$\ddot{x} = u\left(\frac{MI + Im + Mml^2 + m^2l^2}{(M+m)(MI + Im + Mml^2)}\right) - \frac{m^2gl^2\cancel{(M+m)}}{\cancel{(M+m)}(MI + Im + Mml^2)}\theta$$

$$\ddot{x} = u\left(\frac{\cancel{(M+m)}[I + ml^2]}{\cancel{(M+m)}(MI + Im + Mml^2)}\right) - \frac{m^2gl^2}{MI + Im + Mml^2}\theta$$

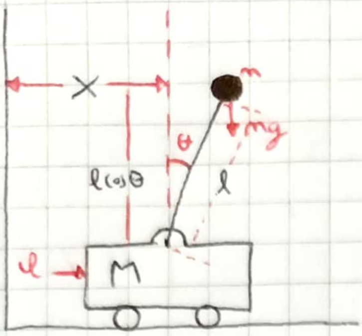
$$\ddot{x} = u\left(\frac{I + ml^2}{MI + Im + Mml^2}\right) - \frac{m^2gl^2}{MI + Im + Mml^2}\theta$$

$$x = q_1 \qquad q_3 = \theta$$
$$\dot{x} = \dot{q}_1 = q_2 \qquad q_4 = \dot{q}_3 = \dot{\theta}$$
$$\dot{q}_2 = \ddot{q}_1 = \ddot{x} \qquad \dot{q}_4 = \ddot{q}_3 = \ddot{\theta}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{mgl(M+m)}{MI + Im + Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-m^2 g l^2}{MI + Im + Mml^2} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{ml}{MI + Im + Mml^2} \\ 0 \\ \frac{I + ml^2}{MI + Im + Mml^2} \end{bmatrix} [u]$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Pendulo invertido



→ Ecuaciones de movimiento

Eje X

$$F = u - b\dot{x}$$

u = Fuerza del motor y las llantas en el eje x

b = constante de fricción

$\dot{x}$ = velocidad del robot

M = masa del carro

x = posición del carro

m = masa del pendulo

$x_p = x + l\sin\theta$ = Posición del pendulo

$$F = ma$$

$$u - b\dot{x} = ma$$

$$u - b\frac{dx}{dt} = M\frac{d^2x}{dt^2} + m\frac{d^2x_p}{dt^2}$$

$$u - b\frac{dx}{dt} = M\frac{d^2x}{dt^2} + m\frac{d^2}{dt^2}(x + l\sin\theta)$$

$$u - b\frac{dx}{dt} = M\frac{d^2x}{dt^2} + m\frac{d^2x}{dt^2} + m\frac{d^2}{dt^2}(l\sin\theta)$$

$$u - b\frac{dx}{dt} = (M+m)\frac{d^2x}{dt^2} + ml\frac{d^2}{dt^2}(\sin(\theta))$$ Resolvemos las derivadas

$$u - b\dot{x} = (M+m)\ddot{x} - ml\sin(\theta)\dot{\theta}^2 + ml\cos(\theta)\ddot{\theta}$$

Eje Y usando segunda ley de Newton

$$F = mg\sin\theta$$

$y_p = l\cos\theta$ = posición en y del pendulo

$$mg\sin\theta = m\cos(\theta)\frac{d^2}{dt^2}x_p - m\sin(\theta)\frac{d^2}{dt^2}y_p$$

$$mg\sin\theta = m\cos(\theta)\frac{d^2}{dt^2}(x\ l\sin(\theta)) - m\sin(\theta)\frac{d^2}{dt^2}(l\cos(\theta))$$ Resolvemos las derivadas

$$mg\sin\theta = m\ddot{x}\cos(\theta) + ml\cos^2(\theta)\ddot{\theta} - ml\sin(\theta)\cos(\theta)\dot{\theta}^2 + ml\sin^2(\theta)\ddot{\theta} + ml\sin(\theta)\cos(\theta)\dot{\theta}^2$$

$$g\sin(\theta) = \ddot{x}\cos(\theta) + l\ddot{\theta}$$

Se linealiza con angulos cercanos a cero tal que:

$$\sin(\theta) \approx \theta \qquad \cos(\theta) \approx 1$$

Eje x

$$u - b\dot{x} = (M+m)\ddot{x} - ml\theta\dot{\theta}^2 + ml\ddot{\theta}$$

$$u - b\dot{x} = (M+m)\ddot{x} + ml\ddot{\theta} \quad (1)$$

Eje y

$$g\theta = \ddot{x} + l\ddot{\theta} \quad (2)$$

→ Representación en el espacio de estados

$$x_1 = \theta \qquad x_3 = x$$
$$x_2 = \dot{\theta} \qquad x_4 = \dot{x}$$

Usamos 1 y 2 para encontrar $\ddot{x}$ y $\ddot{\theta}$ en términos de las variables de estado

$$u - b\dot{x} = M\ddot{x} + m(\ddot{x} + l\ddot{\theta}) \qquad u - b\dot{x} = M\ddot{x} + mg\theta$$

$$\ddot{x} = \frac{1}{M}u - \frac{b}{M}\dot{x} - \frac{mg}{M}\theta \quad (3)$$

$$g\theta = \ddot{x} + l\ddot{\theta} \qquad g\theta = \frac{1}{M}u - \frac{b}{M}\dot{x} - \frac{mg}{M}\theta + l\ddot{\theta}$$

$$\ddot{\theta} = \frac{g}{l}\left(1 + \frac{m}{M}\right)\theta - \frac{1}{Ml}u + \frac{b}{Ml}\dot{x}$$

$$\ddot{\theta} = g\left(\frac{M+m}{lM}\right)\theta - \frac{1}{Ml}u + \frac{b}{Ml}\dot{x} \quad (4)$$

Usando 3 y 4 planteamos el espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ g\left[\frac{M+m}{Ml}\right] & 0 & 0 & \frac{b}{Ml} \\ 0 & 0 & 0 & 1 \\ -\frac{mg}{M} & 0 & 0 & -\frac{b}{M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} [u]$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$