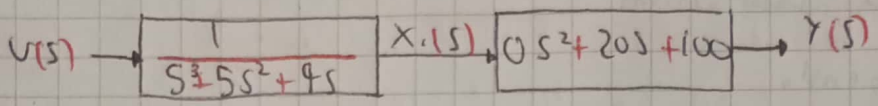
Apuntes Video "State feedback control system design"

$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$ $\begin{cases} \%OS = 95\% \\ t_s = 0{,}74 \text{ seg} \end{cases}$



$\frac{X_1(S)}{U(S)} = \frac{1}{S^3+5S^2+4S}$

$[S^3+5S^2+4S]\,X_1(S) = U(S)$

$\dddot{X}_1 + 5\ddot{X}_1 + 4\dot{X}_1 = u$

$x_1 = x_1$
$x_2 = \dot{x}_1$
$x_3 = \dot{x}_2$
$\dot{x}_3 = \ddot{x}_1$

$\boxed{\dot{x}_3 = -5x_3 - 4x_2 + u}$ ①

$Y(S) = [20S^2 + 20S + 100]\,X_1(S)$

$Y(S) = 20\dot{x}_1 + 100x_1 \Rightarrow \boxed{y = 20x_2 + 100x_1}$ ②

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \cdot 100$

$\ln(0{,}095) = \ln\left[e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right]$

$-2{,}3534 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}}$

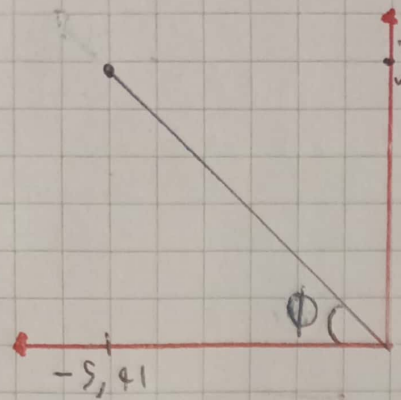
$5{,}5407(1-\zeta^2) = \zeta^2\pi^2$

$5{,}5407 = \zeta^2(\pi^2 + 5{,}5407)$

$\zeta^2 = \frac{5{,}5407}{\pi^2 + 5{,}5407} \Rightarrow \zeta = 0{,}5496$

$s = \sigma + j\omega_d$

$s = \zeta\omega_n + j\omega_d$



$\zeta = \cos\phi \qquad t_s = 0{,}74 \qquad 0{,}74 = \frac{4}{\sigma}$

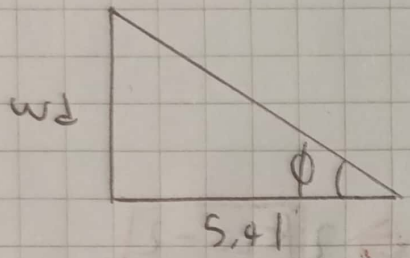
$\phi = 53{,}16°$

$\sigma = 5{,}405 \qquad \sigma = \zeta\omega_n$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$5{,}405 = 0{,}5976\,\omega_n$

$\omega_n = 9{,}02\ rad/s$



$\tan\phi = \frac{\omega_d}{5{,}41}$

$\omega_d = 7{,}214$