Tenemos que: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda(\vec{r}-\vec{r}')\,dl'}{|\vec{r}-\vec{r}'|^3}$ para el problema de un anillo cargado.

Además, vimos que $dl' = a\,d\phi$ y $\vec{r}' = a\cos\phi\,\hat{\imath} + a\sin\phi\,\hat{\jmath}$. Por tanto, podemos reescribir la integral como:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\int_0^{2\pi}\frac{\lambda(\vec{r}-a\cos\phi\,\hat{\imath}-a\,\text{sen}\,\phi\,\hat{\jmath})\,a\,d\phi}{|\vec{r}-a\cos\phi\,\hat{\imath}-a\,\text{sen}\,\phi\,\hat{\jmath}|^3}$$

Sea $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, para $x, y, z \in \mathbb{R}$ arbitrarios. Luego:

$$= \frac{\lambda\cdot a}{4\pi\varepsilon_0}\cdot\int_0^{2\pi}\frac{(x\hat{\imath}+y\hat{\jmath}+z\hat{k}-a\cos\phi\,\hat{\imath}-a\,\text{sen}\,\phi\,\hat{\jmath})\,d\phi}{|x\hat{\imath}+y\hat{\jmath}+z\hat{k}-a\cos\phi\,\hat{\imath}-a\,\text{sen}\,\phi\,\hat{\jmath}|^3}$$

Note que $Q = \lambda\cdot L = \lambda\cdot(2\pi a) \Rightarrow \frac{Q}{2\pi} = \lambda\cdot a$. Entonces:

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2\pi}\cdot\int_0^{2\pi}\frac{(x-a\cos\phi)\hat{\imath}+(y-a\,\text{sen}\,\phi)\hat{\jmath}+z\hat{k}\,d\phi}{\left(\sqrt{(x-a\cos\phi)^2+(y-a\,\text{sen}\,\phi)^2+z^2}\right)^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2\pi}\cdot\int_0^{2\pi}\frac{(x-a\cos\phi)\hat{\imath}+(y-a\,\text{sen}\,\phi)\hat{\jmath}+z\hat{k}\,d\phi}{(x^2+y^2+z^2+a^2(\cos^2\phi+\text{sen}^2\phi)-2ax\cos\phi-2ay\,\text{sen}\,\phi)^{3/2}}$$

Si separamos la integral en sus componentes $\hat{\imath}, \hat{\jmath}, \hat{k}$, obtenemos (teniendo en cuenta que $\cos^2\phi + \text{sen}^2\phi = 1$):

$$\vec{E}_x(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2\pi}\cdot\int_0^{2\pi}\frac{(x-a\cos\phi)\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\,\text{sen}\,\phi)^{3/2}}$$

$$\vec{E}_y(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2\pi}\cdot\int_0^{2\pi}\frac{(y-a\,\text{sen}\,\phi)\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\,\text{sen}\,\phi)^{3/2}}$$

$$\vec{E}_z(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2\pi}\cdot\int_0^{2\pi}\frac{z\,d\phi}{(x^2+y^2+z^2+a^2-2ax\cos\phi-2ay\,\text{sen}\,\phi)^{3/2}}$$