a) Sea

$$C^N_r = \binom{N}{r} = \frac{(N)!}{r!\,(N-r)!}$$

Como son particulas por exclusion hacemos combinación sin repetición.

tomamos $N = N$ $\quad N_0 = r$

$$C^N_{N_0} = \frac{N!}{N_0!\,(N_0 + N_1 - N_0)!} = \frac{N!}{N_0!\,N_1!}$$

b)

$$S(N_1 N_0) = k_B \ln(\Omega)$$

$$= k_B \ln\left(\frac{N!}{N_0!\,N_1!}\right)$$

$$= k_B\left(\ln N! - (\ln N_0! + \ln N_1!)\right)$$

$$= k_B\left(N\ln(N) - N - (N_0\ln(N_0) - N_0 + N_1\ln(N_1) - N_1)\right)$$

Sabemos que $-(-N_0 - N_1) = N$

$$= k_B\left(N\ln(N) - \sum_{i=0}^{1} N_i \ln(N_i)\right)$$

c)

$x = \frac{N_1}{N}$ $\quad x(N_1 + N_2) = N_1$

$N_1(x-1) + xN_2 = 0$

$N_1 = \frac{-xN_2}{x-1}$

$N - N_1 = N_0$

$N - xN = N_0$

Dado que $S = k_B \ln \Omega$ & $\Omega = \frac{N!}{N_0!\,N_1!}$

$$S = k_B \ln\left(\frac{N!}{N_0!\,N_1!}\right)$$

$$= k_B\left(\ln N! - (\ln N_0! + \ln N_1!)\right)$$

$$\approx k_B\left[(N\ln(N) - N) - (N_0\ln(N_0) - N_0 + N_1\ln(N_1) - N_1)\right], \quad -N_1 - N_0 = -N$$

$$= k_B\left[N\ln(N) - \left[(N - xN)\ln\underbrace{(N - xN)}_{N(1-x)} + \underbrace{xN\ln(xN)}_{xN\ln x + xN\ln N}\right]\right]$$

$$= -k_B N\left[-N\cancel{\ln N} + \cancel{(1-x)\ln N} + (1-x)\ln(1-x) + x\ln x + \cancel{x\ln N}\right]$$

$$S = -k_B N\left[(1-x)\ln(1-x) + x\ln x\right]$$

e) Tenemos

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N = \left(\frac{\partial S}{\partial x}\right)_N \left(\frac{\partial x}{\partial E}\right)$$

&

$$x = \frac{1}{N(\varepsilon_0 - \varepsilon_1)}(\varepsilon - N\varepsilon_0) \quad ; \quad \Delta\varepsilon = \varepsilon_1 - \varepsilon_0$$

$$\frac{\partial S}{\partial x} = -k_B N\left[\ln(x) - \ln(1-x)\right] = -k_B N \ln\left(\frac{x}{1-x}\right)$$

$$\frac{\partial x}{\partial E} = \frac{1}{N\Delta E}$$

$$\frac{1}{T} = \frac{-k_B N \ln\left(\frac{x}{1-x}\right)}{N\Delta E}$$

$$\frac{\Delta E}{-k_B T} = \ln\left(\frac{x}{1-x}\right)$$

$$e^{\Delta E/-k_B T} = \frac{x}{1-x}$$

$$e^{\Delta E/-k_B T} - x e^{\Delta E/-k_B T} = x$$

$$x\left(1 + e^{\Delta E/-k_B T}\right) = e^{\Delta E/-k_B T}$$

$$x = \frac{e^{\Delta E/-k_B T}}{1 + e^{\Delta E/-k_B T}}$$

$$x = \frac{1}{\left(1 + e^{\Delta E/-k_B T}\right) e^{\Delta E/k_B T}}$$

$$x(T) = \frac{1}{1 + e^{\Delta E/k_B T}}$$

f) Tenemos que

$$\lim_{t \to \infty} x(T) = \frac{1}{2}$$

$$S\left(\tfrac{1}{2}\right) = -k_B N\left[\left(1-\tfrac{1}{2}\right)\ln\left(1-\tfrac{1}{2}\right) + x\ln\tfrac{1}{2}\right]$$

$$= -k_B N\left(\tfrac{1}{2}\ln\left(\tfrac{1}{2}\right) + \tfrac{1}{2}\ln\left(\tfrac{1}{2}\right)\right)$$

$$= k_B N \ln\left(\tfrac{1}{2}\right)^{-1}$$

$$S\left(\tfrac{1}{2}\right) = k_B N \ln 2$$

g)

$$\Delta S = N k_B \ln\left(\frac{V_f}{V_i}\right) = N k_B \ln\left(\frac{2V}{V}\right)$$

$$\Delta S = N k_B \ln(2)$$

Comprobamos que fue el mismo resultado.