Homework 3: Aggregte Demand, Aggregate Supply and General Equilibrium

Microeconomics - ECON 401A

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1 Dynamics of a Demand Shock

Consider the market for a single good. Market demand is $X(p) = 1500 - 50p$, where p is the price of the good. There are many potential firms, each with cost function $c(q_k) = 100 + q_k^2/4$.

(a) Calculate the long-run equilibrium, in which firms are free to enter and exit. How much does each firm produce (q^*)? What is the equilibrium price (p^*)? How many firms enter (K^*)?

Each firm k will produce in their efficient point. This means:

$$MCG_k(q_k) = AC_k(q_k)$$

From our cost function $c_k(q_k)$:

$$\begin{aligned} MCG_k(q_k) &= \frac{d}{dq_k}c_k(q_k) = \frac{q_k}{2} \\ AC_k(q_k) &= \frac{c_k(q_k)}{q_k} = \frac{100}{q_k} + \frac{q_k}{4} \end{aligned}$$

Solving for the firm k:

$$\begin{aligned}
MCG_k(q_k) &= AC_k(q_k) \\
\frac{q_k}{2} &= \frac{100}{q_k} + \frac{q_k}{4} \\
\frac{q_k^2}{4} &= 100 \\
q_k &= 20
\end{aligned}$$

So each firm k is producing $q^* = 20$.

We know that the supply function for the firm k is $p = MCG_k(q_k)$. The price p is:

$$\begin{aligned}
p &= MCG_k(q_k) = MCG_k(20) \\
p &= \frac{q_k}{2} = \frac{20}{2} = 10
\end{aligned}$$

The equilibrium price is $p^* = 10$.

At this price, the demand is:

$$\begin{aligned}
X(p) &= 1500 - 50p \\
X(10) &= 1500 - 50(10) \\
X(10) &= 1000
\end{aligned}$$

So the demand is $X(10) = 1000$.

(b) Suppose demand rises. New demand is given by $\hat{X}(p) = 1800 - 50p$. Suppose demand rises. New demand is given by $\hat{X}(p) = 1800 - 50p$.

In the very-short run, each firms' output is fixed. What is equilibrium price(p_v)? How much profit does each firm make (π_v)?

In the very short-run there are only $K = 50$ producing $q_k = 20$ goods each, so the aggregate supply is $Q = 1000$. At that level of aggregated supply, and given the new aggregated demand:

$$\hat{X}(p) = 1800 - 50p = 1000$$

The new equilibrium price is $p_v = 16$

At that price, the profit of each firm is:

$$\pi_v = 16(20) - \left(100 + \frac{20^2}{4}\right)$$

So each firm is making a profit of $\pi_k = 120$. This level of profit is not sustainable because, as time passes, firms have incentives to produce one extra unit and take advantage of that price.

(c) In the short run, each firm can adjust its output to maximize its profits, but there is no entry or exit. What is equilibrium price (p_s)? How much does each firm produce (q_s)? How much profit does each firm make (π_s)?

The number of producers remains the same, so $K = 50$. Now each firm wants to maximize their profit.

$$\pi_k(q_k) = pq_k - c_k(q_k)$$

In order to maximize:

$$\begin{aligned} \frac{d}{dq_k}\pi_k(q_k) &= 0 \implies p - \frac{q_k}{2} = 0 \\ q_k &= 2p \end{aligned}$$

So the optimal aggregated supply is:

$$Q(p) = 50(2p) = 100p$$

In equilibrium $\hat{X}(p) = Q(p)$:

$$1800 - 50p = 100p \implies p_s = 12$$

At this price, the aggregated demand is:

$$\hat{X}(p_s) = 1800 - 50p_s = 1800 - 50(12) = 1200$$

So each firm produces:

$$q_s = \frac{1200}{50} = 24$$

And makes a profit of:

$$\pi_s = (12)(24) - \left(100 - \frac{24^2}{4}\right) = 44$$

As a summary, in the short-run, the new equilibrium is:

$$\begin{aligned} K &= 50 \\ q_s &= 24 \\ p_s &= 12 \\ \pi_s &= 44 \end{aligned}$$

(d) In the long run, there is free entry and exit. How much does each firm produce (q_e)? What is equilibrium price (p_e)? How many firms enter (K_e)?

In the long run, assuming that each firm has the same cost function, each firm will produce $q_c = 20$ at a price $p_c = 10$, which is the efficient point identified prior to the expansion of demand.

To find the quantity of firms producing:

$$K_c = \frac{\hat{X}(p_c)}{q_c} = \frac{1800 - 50(10)}{20} = 65$$

In the long-run there will be $K_c = 65$ firms producing.

2 General Equilibrium

There are two goods and J agents, $j \in \{1, \ldots, J\}$. Each agent has identical utility $u(x_1^j, x_2^j) = \sqrt{x_1^j} + \sqrt{x_2^j}$. Agent j is endowed with (ω_1^j, ω_2^j). We normalize the price of good 2, so the prices of the two goods are $(p_1, p_2) = (p, 1)$. Denote aggregate endowments by $\omega_1 = \sum_j \omega_1^j$ and $\omega_2 = \sum_j \omega_2^j$.

(a) What is agent j's demand, $x_1^j(p, \omega_1^j, \omega_2^j)$ and $x_2^j(p, \omega_1^j, \omega_2^j)$?

Solving the maximization problem:

$$\begin{aligned} \max_{x_1^j, x_2^j} U(x_1^j, x_2^j) &= \sqrt{x_1^J} + \sqrt{x_2^J} \\ s.t. \quad m^j &= p(w_1^j) + w_2^j \\ p(w_1^j) + w_2^j &> p(x_1^j) + x_2^j \end{aligned}$$

Calculating the first order conditions:

$$\begin{aligned} \frac{dU}{dx_1^J} &= \frac{1}{2\sqrt{X_1^J}} - p = 0 \\ \frac{dU}{dx_2^J} &= \frac{1}{2\sqrt{X_2^J}} - \ = 0 \\ \frac{dU}{d} &= p(w_1^j) + w_2^j - p(x_1^j) - x_2^j = 0 \\ x_2^j &= p(w_1^j) + w_2^j - p(x_1^j) \end{aligned}$$

Solving for λ:

$$\begin{aligned}
\frac{1}{2\sqrt{X_2^J}} &= \frac{1}{2p\sqrt{X_1^J}} \\
\sqrt{X_1^J} &= \frac{\sqrt{X_2^J}}{p} \\
\sqrt{X_1^J} &= (\frac{\sqrt{X_2^J}}{p})^2 \\
x_1^j &= \frac{X_2^J}{p^2} \\
x_1^j * p^2 &= x_2^j
\end{aligned}$$

Substituting in the budget constraint:

$$\begin{aligned}
x_2^j &= p(w_1^j) + w_2^j - p(\frac{X_2^J}{p^2}) \\
x_2^j &= p(w_1^j) + w_2^j - (\frac{X_2^J}{p}) \\
x_2^j + (\frac{X_2^J}{p}) &= p(w_1^j) + w_2^j \\
x_2^j &= \frac{p(w_1^j) + w_2^j}{1 + \frac{1}{p}} \\
x_2^j &= \frac{p(w_1^j) + w_2^j}{1 + \frac{1}{p}} \\
x_2^j &= \frac{p(w_1^j) + w_2^j}{\frac{p}{p} + \frac{1}{p}} \\
x_2^j &= \frac{p(w_1^j) + w_2^j}{\frac{p+1}{p}} \\
x_2^j &= \frac{p(p(w_1^j) + w_2^j)}{p+1} = \frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)} \\
x_1^j &= \frac{(p(w_1^j) + w_2^j)}{p(p+1)}
\end{aligned}$$

The j's demand for good 1 is:

$$x_1^j(p, \omega_1^j, \omega_2^j) = \frac{(p(w_1^j) + w_2^j)}{p(p+1)}$$

and for good 2 is:

$$x_2^j(p, \omega_1^j, \omega_2^j) = \frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)}$$

(b) Show that agent j's indirect utility is $v(p, x_1^j, x_2^j) = \left[\frac{1+p}{p}(\omega_1^j + \omega_2^j)\right]^{1/2}$.

$$
\begin{aligned}
v(X_1^j, X_2^j) &= \sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} + \sqrt{\frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)}} \\
v(X_1^j, X_2^j) &= \sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} + p\sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} \\
v(X_1^j, X_2^j) &= (1+p)\sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} \\
v(X_1^j, X_2^j) &= \sqrt{(1+p)^2\frac{(p(w_1^j) + w_2^j)}{p(p+1)}}
\end{aligned}
$$

$$
v(X_1^j, X_2^j) = \sqrt{\frac{1+p}{p}(pw_1^j + w_2^j)}
$$

(c) Equating aggregate demand and the aggregate endowments, solve for equilibrium prices.

$$
\begin{aligned}
X_1 &= \sum_j \frac{p(w_1^j) + w_2^j}{p(p+1)} = \sum_j w_1^j \\
X_1 &= \sum_j \frac{p(w_1^j) + w_2^j}{p(p+1)} = w_1 \\
X_1 &= \frac{p\sum_j (w_1^j) + \sum_j w_2^j}{p(p+1)} = w_1
\end{aligned}
$$

$$
\begin{aligned}
X_1 &= w_1 \\
X_2 &= w_2
\end{aligned}
$$

$$
\begin{aligned}
\frac{p(w_1) + w_2}{p(p+1)} &= w_1 \\
\frac{p(w_1) + w_2}{w_1} &= p(p+1) \\
p + \frac{w_2}{w_1} &= p^2 + p \\
\frac{w_2}{w_1} &= p^2
\end{aligned}
$$

$$\sqrt{\frac{w_2}{w_1}} = p$$

(d) Show that agent j is a net seller of good 1 if

$$\frac{\omega_1^j}{\omega_2^j} > \frac{\omega_1}{\omega_2}.$$

$$\begin{aligned}
\frac{w_2^j}{w_1^j} &< \frac{w_2}{w_1} \\
\frac{w_2^j}{w_1^j} &< p^2 \\
x_1^j &= \frac{(p(w_1^j) + w_2^j)}{p(p+1)} \\
x_1^j p(p+1) &= (p(w_1^j) + w_2^j) \\
x_1^j(p^2+p) &= (p(w_1^j) + w_2^j) \\
\frac{x_1^j(p^2+p)}{w_1^j} - p &= \frac{w_2^j}{w_1^j}
\end{aligned}$$

From that results then:

$$\begin{aligned}
\frac{x_1^j(p^2+p)}{w_1^j} - p &< p^2 \\
\frac{x_1^j(p+1)}{w_1^j} - 1 &< p \\
\frac{x_1^j(p+1)}{w_1^j} &< p+1 \\
(x_1^j)(p+1) &< (p+1)(w_1^j) \\
x_1^j &< w_1^j
\end{aligned}$$

Therefore, if $\frac{\omega_1^j}{\omega_2^j} > \frac{\omega_1}{\omega_2}$, then $x_1^j < w_1^j$, meaning that the optimal consumption of good 1 for agent j is than the initial endowment, so agent j is a net seller of good 1.

(e) Suppose an agent k finds a small amount of good 2, so their endowment ω_2^k rises a little. How does this impact the price p? Show that agent j's utility increases if she is a net seller of good 1.

If w_2^k grows, $w_2 = \sum_j w_2^j$ will grow, and consequently, the price ($\sqrt{\frac{w_2}{w_1}} = p$) will increase.

When agent j is a net seller, it implies an increase in w_1^j compared to the case when she is not a net seller, therefore her utility $(v(X_1^j, X_2^j) = \sqrt{\frac{1+p}{p}(pw_1^j + w_2^j)})$ will increase.

3 Housing Prices, in Theory

4 Housing Prices, in Practice

5 AirBnB Regulation