Homework 3: Aggregte Demand, Aggregate Supply and General Equilibrium

MICROECONOMICS - ECON 401A

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1 Dynamics of a Demand Shock

Consider the market for a single good. Market demand is $X(p) = 1500 - 50p$, where p is the price of the good. There are many potential firms, each with cost function $c(q_k) = 100 + q_k^2/4$.

> (a) Calculate the long-run equilibrium, in which firms are free to enter and exit. How much does each firm produce (q^*)? What is the equilibrium price (p^*)? How many firms enter (K^*)?

Each firm k will produce in their efficient point. This means:

$$MCG_k(q_k) = AC_k(q_k)$$

From our cost function $c_k(q_k)$:

$$MCG_k(q_k) = \frac{d}{dq_k}c_k(q_k) = \frac{q_k}{2}$$
$$AC_k(q_k) = \frac{c_k(q_k)}{q_k} = \frac{100}{q_k} + \frac{q_k}{4}$$

Solving for the firm k:

$$
\begin{aligned}
MCG_k(q_k) &= AC_k(q_k) \\
\frac{q_k}{2} &= \frac{100}{q_k} + \frac{q_k}{4} \\
\frac{q_k^2}{4} &= 100 \\
q_k &= 20
\end{aligned}
$$

So each firm k is producing $q^* = 20$.

We know that the supply function for the firm k is $p = MCG_k(q_k)$. The price p is:

$$
\begin{aligned}
p &= MCG_k(q_k) = MCG_k(20) \\
p &= \frac{q_k}{2} = \frac{20}{2} = 10
\end{aligned}
$$

The equilibrium price is $p^* = 10$.

At this price, the demand is:

$$
\begin{aligned}
X(p) &= 1500 - 50p \\
X(10) &= 1500 - 50(10) \\
X(10) &= 1000
\end{aligned}
$$

So the demand is $X(10) = 1000$.

(b) Suppose demand rises. New demand is given by $\hat{X}(p) = 1800 - 50p$. Suppose demand rises. New demand is given by $\hat{X}(p) = 1800 - 50p$.

In the very-short run, each firms' output is fixed. What is equilibrium price(p_v)? How much profit does each firm make (π_v)?

In the very short-run there are only $K = 50$ producing $q_k = 20$ goods each, so the aggregate supply is $Q = 1000$. At that level of aggregated supply, and given the new aggregated demand:

$$
\hat{X}(p) = 1800 - 50p = 1000
$$

The new equilibrium price is $p_v = 16$

At that price, the profit of each firm is:

$$
\pi_v = 16(20) - \left(100 + \frac{20^2}{4} \right)
$$

So each firm is making a profit of $\pi_k = 120$. This level of profit is not sustainable because, as time passes, firms have incentives to produce one extra unit and take advantage of that price.

(c) In the short run, each firm can adjust its output to maximize its profits, but there is no entry or exit. What is equilibrium price (p_s)? How much does each firm produce (q_s)? How much profit does each firm make (π_s)?

The number of producers remains the same, so $K = 50$. Now each firm wants to maximize their profit.

$$\pi_k(q_k) = pq_k - c_k(q_k)$$

In order to maximize:

$$\frac{d}{dq_k}\pi_k(q_k) = 0 \implies p - \frac{q_k}{2} = 0$$
$$q_k = 2p$$

So the optimal aggregated supply is:

$$Q(p) = 50(2p) = 100p$$

In equilibrium $\hat{X}(p) = Q(p)$:

$$1800 - 50p = 100p \implies p_s = 12$$

At this price, the aggregated demand is:

$$\hat{X}(p_s) = 1800 - 50p_s = 1800 - 50(12) = 1200$$

So each firm produces:

$$q_s = \frac{1200}{50} = 24$$

And makes a profit of:

$$\pi_s = (12)(24) - \left(100 - \frac{24^2}{4}\right) = 44$$

As a summary, in the short-run, the new equilibrium is:

$$
\begin{aligned}
K &= 50 \\
q_s &= 24 \\
p_s &= 12 \\
\pi_s &= 44
\end{aligned}
$$

> (d) In the long run, there is free entry and exit. How much does each firm produce (q_e)? What is equilibrium price (p_e)? How many firms enter (K_e)?

In the long run, assuming that each firm has the same cost function, each firm will produce $q_c = 20$ at a price $p_c = 10$, which is the efficient point identified prior to the expansion of demand.

To find the quantity of firms producing:

$$K_c = \frac{\hat{X}(p_c)}{q_c} = \frac{1800 - 50(10)}{20} = 65$$

> In the long-run there will be $K_c = 65$ firms producing.

2 General Equilibrium

There are two goods and J agents, $j \in \{1, \ldots, J\}$. Each agent has identical utility $u(x_1^j, x_2^j) = \sqrt{x_1^j} + \sqrt{x_2^j}$. Agent j is endowed with (ω_1^j, ω_2^j). We normalize the price of good 2, so the prices of the two goods are $(p_1, p_2) = (p, 1)$. Denote aggregate endowments by $\omega_1 = \sum_j \omega_1^j$ and $\omega_2 = \sum_j \omega_2^j$.

> (a) What is agent j's demand, $x_1^j(p, \omega_1^j, \omega_2^j)$ and $x_2^j(p, \omega_1^j, \omega_2^j)$?

Solving the maximization problem:

$$
\begin{aligned}
\max_{x_1^j, x_2^j} U(x_1^j, x_2^j) &= \sqrt{x_1^J} + \sqrt{x_2^J} \\
s.t. \quad m^j &= p(w_1^j) + w_2^j \\
p(w_1^j) + w_2^j &> p(x_1^j) + x_2^j
\end{aligned}
$$

Calculating the first order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_1^j} &= \frac{1}{2\sqrt{X_1^J}} - p = 0 \\
\frac{\partial \mathcal{L}}{\partial x_2^j} &= \frac{1}{2\sqrt{X_2^J}} - = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= p(w_1^j) + w_2^j - p(x_1^j) - x_2^j = 0 \\
x_2^j &= p(w_1^j) + w_2^j - p(x_1^j)
\end{aligned}
$$

Solving for λ:

$$\frac{1}{2\sqrt{X_2^J}} = \frac{1}{2p\sqrt{X_1^J}}$$

$$\sqrt{X_1^J} = \frac{\sqrt{X_2^J}}{p}$$

$$\sqrt{X_1^J} = (\frac{\sqrt{X_2^J}}{p})^2$$

$$x_1^j = \frac{X_2^J}{p^2}$$

$$x_1^j * p^2 = x_2^j$$

Substituting in the budget constraint:

$$x_2^j = p(w_1^j) + w_2^j - p(\frac{X_2^J}{p^2})$$

$$x_2^j = p(w_1^j) + w_2^j - (\frac{X_2^J}{p})$$

$$x_2^j + (\frac{X_2^J}{p}) = p(w_1^j) + w_2^j$$

$$x_2^j = \frac{p(w_1^j) + w_2^j}{1 + \frac{1}{p}}$$

$$x_2^j = \frac{p(w_1^j) + w_2^j}{1 + \frac{1}{p}}$$

$$x_2^j = \frac{p(w_1^j) + w_2^j}{\frac{p}{p} + \frac{1}{p}}$$

$$x_2^j = \frac{p(w_1^j) + w_2^j}{\frac{p+1}{p}}$$

$$x_2^j = \frac{p(p(w_1^j) + w_2^j)}{p+1} = \frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)}$$

$$x_1^j = \frac{(p(w_1^j) + w_2^j)}{p(p+1)}$$

The j's demand for good 1 is:

$$x_1^j(p, \omega_1^j, \omega_2^j) = \frac{(p(w_1^j) + w_2^j)}{p(p+1)}$$

and for good 2 is:

$$x_2^j(p, \omega_1^j, \omega_2^j) = \frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)}$$

(b) Show that agent j's indirect utility is $v(p, x_1^j, x_2^j) = \left[\frac{1+p}{p}(\omega_1^j + \omega_2^j)\right]^{1/2}$.

5

$$v(X_1^j, X_2^j) = \sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} + \sqrt{\frac{p^2(p(w_1^j) + w_2^j)}{p(p+1)}}$$

$$v(X_1^j, X_2^j) = \sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}} + p\sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}}$$

$$v(X_1^j, X_2^j) = (1+p)\sqrt{\frac{(p(w_1^j) + w_2^j)}{p(p+1)}}$$

$$v(X_1^j, X_2^j) = \sqrt{(1+p)^2\frac{(p(w_1^j) + w_2^j)}{p(p+1)}}$$

$$v(X_1^j, X_2^j) = \sqrt{\frac{1+p}{p}}(pw_1^j + w_2^j)$$

(c) Equating aggregate demand and the aggregate endowments, solve for equilibrium prices.

$$X_1 = \sum_j \frac{p(w_1^j) + w_2^j}{p(p+1)} = \sum_j w_1^j$$

$$X_1 = \sum_j \frac{p(w_1^j) + w_2^j}{p(p+1)} = w_1$$

$$X_1 = \frac{p\sum_j(w_1^j) + \sum_j w_2^j}{p(p+1)} = w_1$$

$$X_1 = w_1$$
$$X_2 = w_2$$

$$\frac{p(w_1) + w_2}{p(p+1)} = w_1$$

$$\frac{p(w_1) + w_2}{w_1} = p(p+1)$$

$$p + \frac{w_2}{w_1} = p^2 + p$$

$$\frac{w_2}{w_1} = p^2$$

$$\sqrt{\frac{w_2}{w_1}} \quad = \quad p$$

(d) Show that agent j is a net seller of good 1 if

$$\frac{\omega_1^j}{\omega_2^j} > \frac{\omega_1}{\omega_2}.$$

$$\frac{w_2^j}{w_1^j} \quad < \quad \frac{w_2}{w_1}$$

$$\frac{w_2^j}{w_1^j} \quad < \quad p^2$$

$$x_1^j \quad = \quad \frac{(p(w_1^j) + w_2^j)}{p(p+1)}$$

$$x_1^j p(p+1) \quad = \quad (p(w_1^j) + w_2^j)$$

$$x_1^j(p^2 + p) \quad = \quad (p(w_1^j) + w_2^j)$$

$$\frac{x_1^j(p^2 + p)}{w_1^j} - p \quad = \quad \frac{w_2^j}{w_1^j}$$

From that results then:

$$\frac{x_1^j(p^2 + p)}{w_1^j} - p \quad < \quad p^2$$

$$\frac{x_1^j(p+1)}{w_1^j} - 1 \quad < \quad p$$

$$\frac{x_1^j(p+1)}{w_1^j} \quad < \quad p+1$$

$$(x_1^j)(p+1) \quad < \quad (p+1)(w_1^j)$$

$$x_1^j \quad < \quad w_1^j$$

Therefore, if $\frac{\omega_1^j}{\omega_2^j} > \frac{\omega_1}{\omega_2}$, then $x_1^j < w_1^j$, meaning that the optimal consumption of good 1 for agent j is lower than the initial endowment, so agent j is a net seller of good 1.

(e) Suppose an agent k finds a small amount of good 2, so their endowment w_2^k rises a little. How does this impact the price p? Show that agent j's utility increases if she is a net seller of good 1.

If w_2^k grows, $w_2 = \sum_j w_2^j$ will grow, and consequently, the price $(\sqrt{\frac{w_2}{w_1}} = p)$ will increase.

When agent j is a net seller, it implies an increase in w_1^j compared to the case when she is not a net seller, therefore her utility $(v(X_1^j, X_2^j) = \sqrt{\frac{1+p}{p}(pw_1^j + w_2^j)})$ will increase.

3   Housing Prices, in Theory

A unit mass of people indexed by $v \sim U[0,1]$ must choose to live in either Los Angeles or Kansas City. Each city has housing stock $3/4$, so there is enough housing between the two cities but not in any one city. House prices are determined by a competitive market of landlords; they have no costs if they rent out their house.

Agents have utilities $u_{LA} = b + v - p_{LA}$ and $u_{KC} = b - p_{KC}$ from living in the two cities, where p_{LA} and p_{KC} are the prices of renting in the two cities, $v \sim U[0,1]$ indicates how much the agent likes California weather, and b is the benefit of having a house (we assume this is positive, so no-one chooses to be homeless).

> (a) Suppose $p_{LA} = p_{KC} = 0$. Is this an equilibrium? Is there excess demand/supply in either market?

When $p_{LA} = p_{KC} = 0$, everyone would desire to live in LA, since $b > 0$ and v is uniformly distributed on $[0,1]$. This suggest each individual values the additional utility of living in LA. at least a little over living in KC. Therefore $u_{LA} > u_{KC}$. Here we are assuming $v \neq 0$. In the case $v = 0$, the individual will be indifferent.

When $p_{LA} = p_{KC} = 0$, all individuals would choose to live in LA. But, housing is enough only for $3/4$ of the population in each city.

In LA the demand of housing would be 1 or the whole population.

> The excess of demand in LA is $1/4$, and the excees of supply in KC is $3/4$, because only $1/4$ would not be able to live in LA.
>
> $p_{LA} = p_{KC} = 0$ is not an equilibrium, as market does not clear in both cities.

> (b) What are the equilibrium prices p_{LA}, p_{KC}?

In order to establish indifference condition:

$$
\begin{aligned}
u_{LA} &= u_{KC} \\
b + v - p_{LA} &= b - p_{KC}
\end{aligned}
$$

$$v = p_{LA} - p_{KC} \tag{1}$$

The condition 1 tell us the value of v at which individual is indifferentt between living in LA and KC. Individuals with v grater than this value will prefer LA and, on the contrary, individuals with v lower than this value will prefer KC.

Since each city can only accommodate $3/4$ of the population, the v^* be the cutoff point that determines the proportion of people preferring LA should be $3/4$ of the population. In Context of a uniform distribution thi is the 75th percentile, which is 0.75:

$$
\begin{aligned}
v^* &= 0.75 \\
p_{LA} - p_{KC} &= 0.75
\end{aligned}
$$

$$p_{LA} = p_{KC} + 0.75 \qquad (2)$$

The condition 2 give us a range of possible prices combinations instead of a single unique solution for each cities rental price.

(c) What happens to house prices in LA and KC if we build a few more houses in LA or KC?

Regarding the equilibrium condition 1, if housing supply increase in LA ($\Delta H_{LA} > 0$), the price of housing in LA will decrease ($p_{LA_2} < p_{LA_1}$) and the price of housing in KC ($p_{KC_2} = p_{KC_1}$) will remain the same. therefore, the new equilibrium condition will be:

$$v_2^* \;\; = \;\; p_{LA_2} - p_{KC_2} < v_1^*$$

This mens agents with lower valoration of LA weather might prefer LA over KC, because La becomes more affordable conig those with v value just above the new lower v_2^* will prefer LA.

If housing supply in KC increase ($\Delta H_{KC} > 0$),

$$v_2^* \;\; = \;\; p_{LA_2} - p_{KC_2} > v_1^*$$

With a higher v_2^*, fewer people comy tose with v value above the nwe v_2^* will find LA more a ttractive than KC. Hence more people choose KC.

Δs in supply in either city, the change in rental prices which in turn shifts the indifference point v^*. This shift in the supply the proportion of population preferring one city over the other, based on their valoration of LA.

(d) Suppose we can build new housing at constant marginal cost $c = 1/10$ in both cities. What is the long-run equilibrium price? How many people live in each city?

(e) How much does social welfare increase from the extra building in (d)?

4 Housing Prices, in Practice

It is commonly believed by the general public that an increase in housing supply can increase prices. Figure 1 shows a Tweet about one case in Boston. Figure 2 presents a survey that shows people generally think increasing supply lowers prices, except when it comes to housing.

Does the argument in Figure 1 have merit? What does this argument overlook? How does this mesh with the traditional supply-demand analysis? [1 page maximum]

In traditional Supply - Demand analysis, an increase in supply, ceteris paribus, tends to lower prices. The housing price paradox, as perceived by the public, fundamentally questions the intuition of this perspective by arguing that an increase in the number of houses in a particular area would subsequently lead to increased

demand for these houses.

By acknowledging that the housing market is not a peripheral market with unique characteristics which could justify deviations from conventional economic analysis, we will approach this narrative and attempt to reason through it by leveraging the idea of perception bias about how markets work.

While the housing market shares fundamental economic principles with other markets, it possesses unique characteristics that contribute to the housing price paradox. Unlike commodities or the stock market, the housing market experiences slower adjustments in prices, introducing a time lag from the moment of a change in value to its reflection in prices. Consequently, it becomes challenging for market participants to have timely and accurate knowledge of the fair value of an asset at any given time. Another distinctive feature of the housing market is the heterogeneity of assets. Unlike an ounce of gold, which remains identical regardless of the exchange (NYSE or LSE), two identical houses may have significantly different prices due to a series of externalities.

As Figure 2 suggests, agents are generally good at predicting how changes in supply impact prices, but this ability appears to diminish as the complexity of the market increases. The price adjustment time lag combined with the heterogeneity of assets creates a perception bias regarding the factors that drive home prices. Agents tend to rationalize and simplify price changes by unconsciously implying that correlation implies causation. In other words, the belief that a new, more expensive property would drive up area value represents linear thinking that overlooks a fundamental economic insight. Suppliers produce as long as they believe they can sell the goods for a price that exceeds their marginal cost of production. This implies that, in reality, new, more expensive houses don't serve as price leaders, but rather as catalysts that accelerate the adjustment of prices to the market level, and since it wouldn't make economic sense for a company to develop construction at a time when prices are falling, the adjustment effect will inherently trend upward.

5   AirBnB Regulation

One can think of AirBnB as a market. What are the key aspects of the goods being sold? What are the types of buyers and sellers? What are their alternatives if they aren't on AirBnB?

You are the mayor of Santa Monica. You are worried that AirBnB is pushing up prices for long-term rentals for residents, changing the character of the city and causing a nuisance for residents. On the other hand, tourists are important for the city and you AirBnB is useful in more efficiently using real estate. What are some of the possible dimensions for regulations? What would you recommend?

The Airbnb market operates through the interaction between supply, represented by homeowners offering short-term lodging services, and demand, comprised of individuals seeking temporary accommodations in a specific city. The supply side involves people aiming to generate income by renting their property for short durations, either because they are not utilizing the space or have additional room to spare. Conversely, the demand typically consists of tourists, business travelers, or short-term visitors seeking affordable and flexible accommodation in the city.

Credibility in the Airbnb market is established through a robust system of reviews, evaluating factors such as accessibility, hospitality, and pricing. Additionally, the platform incorporates security features to safeguard the interests of both hosts and renters, addressing aspects like payments, damage policies, and protection against scams.

Therefore, Airbnb functions as a trusted marketplace where hosts and renters can easily engage and obtain the desired services. It exists as an alternative to other accommodation options such as hotels (often with higher prices), rental agencies, hostels, or vacation rentals by owners, each varying in terms of pricing

and facilities offered. Airbnb serves as a valuable market for lodge owners seeking revenue from underutilized properties, particularly in expensive cities.

However, the proliferation of Airbnb listings in major cities has raised concerns, particularly regarding increased housing prices and reduced availability of affordable housing. Consequently, cities like Los Angeles, Amsterdam, or Paris have implemented various restrictions to balance the benefits of the market with the need to protect residents within the housing market, which is a significant and relatively inelastic market.

One notable regulation involves limiting the number of available days for renting, aiming to curb the shift from long-term leasing to short-term leasing. This restriction addresses the tendency of property owners to favor short-term rentals for higher daily rates, potentially driving up long-term rental prices. By imposing limits, property owners have fewer incentives to transition from long-term to short-term rentals.

Another regulatory approach, as observed in Amsterdam, involves zoning restrictions. For instance, in Santa Monica, regulations could be implemented to restrict short-term rentals in residential areas, preserving residential prices without hindering the influx of tourists. This approach mirrors successful strategies employed in areas like The Red Zone in Amsterdam.

Additionally, implementing a comprehensive registration system would allow monitoring of short-term rentals in the market, facilitating effective taxation. These measures collectively aim to regulate the Airbnb market without eliminating its presence. The goal is to strike a balance, offering accommodation options for demand while safeguarding residents from escalating rental and housing prices. Given that Airbnb is widely accepted as a substitute for hotels and hostels, such regulations allow for flexibility in choices for visitors while ensuring the protection of local residents in the designated zones, such as Santa Monica.