Punto 6 (Teórico):

Mostrar los parametros para minimizar $\chi^2(a_0, a_1)$

$$\chi^2(a_0, a_1) = \sum_{i=1}^{n} [y_i - (a_0 + a_1 x_i)]^2$$

1. Vamos a derivar respecto a a_0 y a_1 e igualar a 0:

$$\frac{\partial \chi^2}{\partial a_0} = -2 \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i)) = 0 \quad (1)$$

$$\frac{\partial \chi^2}{\partial a_1} = -2 \sum_{i=1}^{n} x_i (y_i - (a_0 + a_1 x_i)) = 0 \quad (2)$$

Definamos:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \quad \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i$$

☆ entonces: $a_0 \cdot n = \sum_{i=1}^{n} y_i + a_1 \sum_{i=1}^{n} x_i$

$\hookrightarrow a_0 = \bar{y} - a_1 \bar{x}$

☆ Simplificamos:

(1) $-2 \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i)) = n a_0 + a_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$

(2), $-2 \sum_{i=1}^{n} x_i (y_i - (a_0 + a_1 x_i)) = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2$

$\hookrightarrow \sum_{i=1}^{n} x_i y_i = a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2$

(Por lo que era una ecuacion igualada a 0, ahora esta igualada a $\sum_{i=1}^{n} x_i y_i$

Sustituimos a_0 en (2):

$$(\bar{y}-a_1\bar{x})\sum_{i=1}^{n} x_i + a_1\sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i$$

$$=\left(\bar{y}\sum_{i=1}^{n} x_i\right)-\left[\left(a_1\bar{x}\sum_{i=1}^{n} x_i\right)+\left(a_1\sum_{i=1}^{n} x_i^2\right)\right]=\sum_{i=1}^{n} x_i y_i$$

$$a_1\left(-\bar{x}\sum_{i=1}^{n} x_i+\sum_{i=1}^{n} x_i^2\right)=\sum_{i=1}^{n} x_i y_i-\bar{y}\sum_{i=1}^{n} x_i$$

▷ $\bar{y}\sum_{i=1}^{n} x_i = \dfrac{\sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i}{n} = \dfrac{\sum y \sum x}{n}$

▷ $\bar{x}\sum_{i=1}^{n} x_i = \left(\sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i\right)\cdot\dfrac{1}{n} = \dfrac{(\sum x)^2}{n}$

▷ $a_1 = \dfrac{\sum xy - \dfrac{\sum x \sum y}{n}}{\sum x^2 - \dfrac{(\sum x)^2}{2}}$

Hemos encontrado los parametros a_0 y a_1 para n parametros

En el caso de n puntos y un modelo cuadrático:

$$\chi^2(a_0, a_1, a_2) = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

Minimicemos igualando a 0 las derivadas parciales de χ^2 en a_0, a_1 y a_2:

▷ $$\frac{\partial \chi^2}{\partial a_0} = -\cancel{2}\sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2)) = 0$$

$$0 = -\sum_{i=1}^{n} y_i + n a_0 + a_1 \sum_{i=1}^{n} x_i + a_2 \sum_{i=1}^{n} x_i^2$$

$$\sum_{i=1}^{n} \left[y_i = a_0 + a_1 x_i + a_2 x_i^2 \right] \quad (1)$$

▷ $$\frac{\partial \chi^2}{\partial a_1} = -\cancel{2}\sum_{i=1}^{n} x_i (y_i - (a_0 + a_1 x_i + a_2 x_i^2)) = 0$$

$$\sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} x_i a_0 + \sum_{i=1}^{n} x_i^2 a_1 + \sum_{i=1}^{n} x_i^3 a_2$$

$$(2) \quad \sum_{i=1}^{n} \left[x_i y_i = x_i a_0 + x_i^2 a_1 + x_i^3 a_2 \right]$$

▷ $$\frac{\partial \chi^2}{\partial a_2} = -2 \sum_{i=1}^{n} x_i^2 (y_i - (a_0 + a_1 x_i + a_2 x_i^2)$$

$$\sum_{i=1}^{n} x_i^2 y_i = \sum_{i=1}^{n} x_i^2 a_0 + \sum_{i=1}^{n} x_i^3 a_1 + \sum_{i=1}^{n} x_i^4 a_2$$

$$(3) \quad \sum_{i=1}^{n} \left[x_i^2 y_i = a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4 \right]$$

Hemos encontrado el sistema de ecuaciones que relaciona los parametros, la regularidad se encuentra en el grado de χ^2, ya que al derivar frente a los parámetros vemos como se aumenta el grado de x_i mientras derivamos y hallamos las ecuaciones