Punto 1 (Teórico) Algebra lineal:

Demostrar recursividades

$$P(\theta)\begin{cases} x_0 = 4\sin^2\theta \\ x_n = 4x_{n-1} - (x_{n-1})^2 \end{cases}$$

Supongamos que para algun n $x_n = 4\sin^2(2^n\theta)$
demostrar que $x_{n+1} = 4\sin^2(2^{n+1}\theta)$

$x_{n+1} = 4x_n - x_n^2$ entonces

$$\begin{aligned} x_{n+1} &= 4(4\sin^2(2^n\cdot\theta)) - (4\sin^2(2^n\cdot\theta))^2 \\ &= 16\sin^2(2^n\cdot\theta) - 16\sin^4(2^n\cdot\theta) \\ &= 16\sin^2(2^n\theta)(1-\sin^2(2^n\theta)) \\ &= 16\sin^2(2^n\theta)(\cos^2(2^n\theta)) \end{aligned}$$

Angulo doble : $\sin(2\cdot 2^n\theta) = 2\sin(2^n\theta)\cos(2^n\theta)$

$(\phi = 2^n\theta)$

$$4\sin^2(2^{n+1}\theta) = 4(\sin^2(2^n\theta)(\cos^2(2^n\theta)))$$

Caso base) $n=0$: $x_0 = 4\sin^2(2^0\theta)$
$= 4\sin^2(\theta)$

(Funciona)! $n=1$: $x_1 = 4\sin^2(2\theta)$

$$P(\theta) = \begin{cases} x_0 = \sin^2\theta \\ x_{n+1} = 4x_n - 4x_n^2 \end{cases}$$

- entonces, $x_n = \sin^2(2^n\theta)$, demostremos $x_{n+1} = \sin^2(2^{n+1}\theta)$

$x_n = \sin^2(2^n\theta) \rightarrow x_{n+1} = 4x_n - 4(x_n)^2$

$$\begin{aligned} x_{n+1} &= 4\sin^2(2^n\theta) - 4\sin^4(2^n\theta) \\ &= 4\sin^2(2^n\theta)(1 - \sin^2(2^n\theta)) \\ &= 4\sin^2(2^n\theta)(\cos^2(2^n\theta)) \end{aligned}$$

Angulo doble $\sin(2\phi) = 2\sin(\phi)\cos(\phi)$

$x_{n+1} = \sin^2(2^{n+1}\theta)$

Caso base: $n=0$, $x_0 = \sin^2(2^0\theta) = \sin^2(\theta)$

(Funciona)! $n=1$, $x_1 = \sin(2\theta)$