Punto 8 - Derivación

a) $P(x) = \sum_{i=0}^{2} y_i \prod_{\substack{0 \le j \le n \\ i \ne j}} \frac{x - x_j}{x_i - x_j} = y_0\left[\left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)\right] + y_1\left[\left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)\right]$

$+ y_2\left[\left(\frac{x-x_0}{x_2-x_0}\right)\left(\frac{x-x_1}{x_2-x_1}\right)\right]$

$$\boxed{P(x) = \frac{f(x_0)(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + \frac{f(x_1)(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + \frac{f(x_2)(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}}$$

Polinomio interpolador del conjunto soporte

b) $P'(x) = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)} \cdot [2x - (x_1+x_2)] + \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)} \cdot [2x - (x_0+x_2)]$

$+ \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)} \cdot [2x - (x_0+x_1)]$

Sea $h = x_2 - x_1 = x_1 - x_0$

$\Rightarrow P'(x) = \frac{f(x_0)}{h(x_2-x_0)}[2x - (x_1+x_2)] + -\frac{f(x_1)}{h^2}[2x - (x_0+x_2)]$

$+ \frac{f(x_2)}{h(x_2-x_0)}[2x - (x_0+x_1)]$

Evaluando $P'(x)\Big|_{x=x_0}$

$\Rightarrow P'(x_0) = \frac{f(x_0)}{h(x_2-x_0)}[2x_0 - x_1 - x_2] - \frac{f(x_1)}{h^2}[x_0 - x_2] + \frac{f(x_2)}{h(x_2-x_0)}[x_0 - x_1]$

$x_2 = x_0 + 2h$ y $x_1 = x_0 + h$

$\Rightarrow P'(x_0) = \frac{f(x_0)}{h(x_0+2h-x_0)}[2x_0 - x_0 - h - x_0 - 2h] - \frac{f(x_1)}{h^2}[x_0 - x_0 - 2h]$

$+ \frac{f(x_2)}{h(x_0+2h-x_0)}[x_0 - x_0 - h]$

$\Rightarrow P'(x_0) = \frac{f(x_0)}{2h^2}(-3\cancel{h}) - \frac{f(x_1)}{h^2}(-2\cancel{h}) + \frac{f(x_2)}{2h^2}(-\cancel{h})$

$\Rightarrow P'(x_0) = -\frac{3}{2}\frac{f(x_0)}{h} + 2\frac{f(x_1)}{h} - \frac{1}{2}\frac{f(x_2)}{h} \boxed{= \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h}}$

Derivada en el punto x_0

e) $f(x) = \sqrt{\tan(x)}$

$$f'(x) = \frac{1}{2\sqrt{\tan(x)}} \cdot \frac{d}{dx}\tan(x) = \frac{\sec^2(x)}{2\sqrt{\tan(x)}}$$

Powered by CamScanner

Interpolación

2. Demostrar $L_i(x) = \delta_{ij}$ $\underline{\forall j \in \{0,1,2,\dots,n\}}$ ①

- Recordemos que $L_i(x) = \prod_{\substack{0\le k\le n \\ k\neq i}} \frac{x - x_k}{x_i - x_k}$ y $\delta_{ij} = \begin{cases} 1 & \text{si } i=j \\ 0 & \text{si } i\neq j \end{cases}$

Es decir, basta demostrar que $L_i(x_j) = \begin{cases} 1 & \text{si } i=j \\ 0 & \text{si } i\neq j \end{cases}$

- Primer caso: $i=j$

Si $i=j$: $L_i(x_j) = \prod_{\substack{0\le k\le n \\ k\neq i}} \frac{\overset{j=i}{x_j} - x_k}{x_i - x_k} = \prod_{\substack{0\le k\le n \\ k\neq i}} \frac{x_i - x_k}{x_i - x_k} = \prod_{\substack{0\le k\le n \\ k\neq i}} 1 \boxed{= 1}$

$\boxed{\text{Conclusión 1: } L_i(x_j) = 1 \text{ si } i=j}$

- Segundo caso: $i\neq j$

$L_i(x_j) = \prod_{\substack{② \, 0\le k\le n, \\ k\neq i}} \frac{x_j - x_k}{x_i - x_k}$ $\Rightarrow$ $j \in \{0,1,2,\dots,n\}$ por ① $\Rightarrow$ j entra como un valor de k en la productoria.
$k \in \{0,1,2,\dots,n\}$ por ②

Entonces: $L_i(x_j) = \prod_{\substack{0\le k\le n \\ k\neq i}} \frac{x_j - x_k}{x_i - x_k} = \left(\frac{x_j - x_0}{x_i - x_0}\right)\left(\frac{x_j - x_1}{x_1 - x_1}\right)\cdots\left(\frac{x_j - x_j}{x_i - x_j}\right)\cdots\left(\frac{x_j - x_n}{x_i - x_n}\right)$

$= \left(\frac{x_j - x_0}{x_i - x_0}\right)\left(\frac{x_j - x_1}{x_i - x_1}\right)\cdots 0 \cdots \left(\frac{x_j - x_n}{x_i - x_n}\right) \boxed{= 0}$

$\boxed{\text{Conclusión 2: } L_i(x_j) = 0 \text{ si } i\neq j}$

$\boxed{\text{Por conclusión 1 y 2, } L_i(x_j) = \begin{cases} 1 & \text{si } i=j \\ 0 & \text{si } i \neq j \end{cases} \text{. Por lo tanto, } \forall j \in \{0,1,2,\ldots,n\} \ L_i(x) = \delta_{ij}}$ Q.E.D.

Powered by CamScanner