Glossary of Z notation

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Glossary of Z notation

Names

a,b	identifiers
d,e	declarations (e.g., $a : A; b, ... : B ...$)
f,g	functions
m,n	numbers
p,q	predicates
s,t	sequences
x,y	expressions
A,B	sets
C,D	bags
Q,R	relations
S,T	schemas
X	schema text (e.g., d, $d \mid p$ or S)

Definitions

$a == x$	Abbreviated definition
$a ::= b \mid \ldots$	Data type definition (or $a ::= b\langle\!\langle x \rangle\!\rangle \mid \ldots$)
$[a]$	Introduction of a given set (or $[a, \ldots]$)
$a\_$	Prefix operator
$\_a$	Postfix operator
$\_a\_$	Infix operator

Logic

$true$	Logical true constant
$false$	Logical false constant
$\neg\ p$	Logical negation
$p \land q$	Logical conjunction
$p \lor q$	Logical disjunction
$p \Rightarrow q$	Logical implication ($\neg\ p \lor q$)
$p \Leftrightarrow q$	Logical equivalence ($p \Rightarrow q \ \land\ q \Rightarrow p$)
$\forall\ X \bullet q$	Universal quantification
$\exists\ X \bullet q$	Existential quantification
$\exists_1\ X \bullet q$	Unique existential quantification
$\mathbf{let}\ a == x; \ldots \bullet p$	Local definition

Sets and expressions

$x = y$	Equality of expressions
$x \neq y$	Inequality ($\neg\ (x = y)$)
$x \in A$	Set membership
$x \notin A$	Non-membership ($\neg\ (x \in A)$)
$\varnothing$	Empty set
$A \subseteq B$	Set inclusion
$A \subset B$	Strict set inclusion ($A \subseteq B \land A \neq B$)
$\{x, y, \ldots\}$	Set of elements
$\{X \bullet x\}$	Set comprehension
$\lambda\ X \bullet x$	Lambda-expression – function
$\mu\ X \bullet x$	Mu-expression – unique value
$\mathbf{let}\ a == x; \ldots \bullet y$	Local definition
$\mathbf{if}\ p\ \mathbf{then}\ x\ \mathbf{else}\ y$	Conditional expression
$(x, y, \ldots)$	Ordered tuple
$A \times B \times \ldots$	Cartesian product
$\mathbb{P}\ A$	Power set (set of subsets)
$\mathbb{P}_1\ A$	Non-empty power set
$\mathbb{F}\ A$	Set of finite subsets
$\mathbb{F}_1\ A$	Non-empty set of finite subsets
$A \cap B$	Set intersection
$A \cup B$	Set union
$A \setminus B$	Set difference
$\bigcup A$	Generalized union of a set of sets
$\bigcap A$	Generalized intersection of a set of sets
$first\ x$	First element of an ordered pair
$second\ x$	Second element of an ordered pair
$\#A$	Size of a finite set

Relations

$A \leftrightarrow B$	Relation ($\mathbb{P}(A \times B)$)
$a \mapsto b$	Maplet ((a, b))
$\mathrm{dom}\ R$	Domain of a relation
$\mathrm{ran}\ R$	Range of a relation
$\mathrm{id}\ A$	Identity relation
$Q \fatsemi R$	Forward relational composition
$Q \circ R$	Backward relational composition ($R \fatsemi Q$)
$A \lhd R$	Domain restriction
$A \ntriangleleft R$	Domain anti-restriction
$A \rhd R$	Range restriction
$A \ntriangleright R$	Range anti-restriction
$R(\!\vert A \vert\!)$	Relational image
$iter\ n\ R$	Relation composed n times
R^n	Same as $iter\ n\ R$
$R^\sim$	Inverse of relation (R^{-1})
R^*	Reflexive-transitive closure
R^+	Irreflexive-transitive closure
$Q \oplus R$	Relational overriding ($(\mathrm{dom}\ R \ntriangleleft Q) \cup R$)
$a\ \underline{R}\ b$	Infix relation

Functions

$A \nrightarrow B$	Partial functions
$A \longrightarrow B$	Total functions
$A \rightarrowtail\!\!\!\!\!\nrightarrow B$	Partial injections
$A \rightarrowtail B$	Total injections
$A \twoheadrightarrow\!\!\!\!\!\nrightarrow B$	Partial surjections
$A \twoheadrightarrow B$	Total surjections
$A \rightarrowtail\!\!\!\!\!\twoheadrightarrow B$	Bijective functions
$A \mathrel{+\!\!\!\!+\!\!\!\!\rightarrow} B$	Finite partial functions
$A \mathrel{\rightarrowtail\!\!\!\!+\!\!\!\!+} B$	Finite partial injections
$f\ x$	Function application (or $f(x)$)

Numbers

$\mathbb{Z}$	Set of integers
$\mathbb{N}$	Set of natural numbers $\{0, 1, 2, ...\}$
$\mathbb{N}_1$	Set of non-zero natural numbers ($\mathbb{N} \setminus \{0\}$)
$m + n$	Addition
$m - n$	Subtraction
$m * n$	Multiplication
$m \operatorname{div} n$	Division
$m \operatorname{mod} n$	Modulo arithmetic
$m \leq n$	Less than or equal
$m < n$	Less than
$m \geq n$	Greater than or equal
$m > n$	Greater than
$succ\ n$	Successor function $\{0 \mapsto 1, 1 \mapsto 2, ...\}$
$m \mathinner{..} n$	Number range
$min\ A$	Minimum of a set of numbers
$max\ A$	Maximum of a set of numbers

Sequences

$\mathrm{seq}\ A$	Set of finite sequences
$\mathrm{seq}_1\ A$	Set of non-empty finite sequences
$\mathrm{iseq}\ A$	Set of finite injective sequences
$\langle\rangle$	Empty sequence
$\langle x, y, ...\rangle$	Sequence $\{1 \mapsto x, 2 \mapsto y, ...\}$
$s \frown t$	Sequence concatenation
$\frown / s$	Distributed sequence concatenation
$head\ s$	First element of sequence ($s(1)$)
$tail\ s$	All but the head element of a sequence
$last\ s$	Last element of sequence ($s(\#s)$)
$front\ s$	All but the last element of a sequence
$rev\ s$	Reverse a sequence
$squash\ f$	Compact a function to a sequence
$A \upharpoonleft s$	Sequence extraction ($squash(A \lhd s)$)
$s \upharpoonright A$	Sequence filtering ($squash(s \rhd A)$)
$s \operatorname{prefix} t$	Sequence prefix relation ($s \frown v = t$)
$s \operatorname{suffix} t$	Sequence suffix relation ($u \frown s = t$)
$s \operatorname{in} t$	Sequence segment relation ($u \frown s \frown v = t$)
$\operatorname{disjoint} A$	Disjointness of an indexed family of sets
$A \operatorname{partition} B$	Partition an indexed family of sets

Bags

$\mathrm{bag}\ A$	Set of bags or multisets ($A \nrightarrow \mathbb{N}_1$)
$[\![\,]\!]$	Empty bag
$[\![x, y, ...]\!]$	Bag $\{x \mapsto 1, y \mapsto 1, ...\}$
$count\ C\ x$	Multiplicity of an element in a bag
$C \sharp x$	Same as $count\ C\ x$
$n \otimes C$	Bag scaling of multiplicity
$x \sqsubset C$	Bag membership
$C \sqsubseteq D$	Sub-bag relation
$C \uplus D$	Bag union
$C \uminus D$	Bag difference
$items\ s$	Bag of elements in a sequence

Schema notation

```
┌ S ────────
│ d
├──────
│ p
└───────────
```

Vertical schema.

New lines denote '$;$' and '$\wedge$'. The schema name and predicate part are optional. The schema may subsequently be referenced by name in the document.

```
│ d
├──────
│ p
```

Axiomatic definition.

The definitions may be non-unique. The predicate part is optional. The definitions apply globally in the document.

```
┌═[a, ...]══
│ d
├──────
│ p
└───────────
```

Generic definition.

The generic parameters are optional. The definitions must be unique. The definitions apply globally in the document.

$S \mathrel{\widehat{=}} [X]$	Horizontal schema
$[T; ... \mid ...]$	Schema inclusion
$z.a$	Component selection (given $z : S$)
θS	Tuple of components
$\neg\ S$	Schema negation
$\mathrm{pre}\ S$	Schema precondition
$S \wedge T$	Schema conjunction
$S \vee T$	Schema disjunction
$S \Rightarrow T$	Schema implication
$S \Leftrightarrow T$	Schema equivalence
$S \setminus (a, ...)$	Hiding of component(s)
$S \upharpoonright T$	Projection of components
$S \fatsemi T$	Schema composition (S then T)
$S \gg T$	Schema piping (S outputs to T inputs)
$S[a/b, ...]$	Schema component renaming (b becomes a, etc.)
$\forall X \bullet S$	Schema universal quantification
$\exists X \bullet S$	Schema existential quantification
$\exists_1 X \bullet S$	Schema unique existential quantification

Conventions

$a?$	Input to an operation
$a!$	Output from an operation
a	State component before an operation
a'	State component after an operation
S	State schema before an operation
S'	State schema after an operation
ΔS	Change of state (normally $S \wedge S'$)
ΞS	No change of state (normally $[S \wedge S' \mid \theta S = \theta S']$)

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