b) Derivada progresiva de orden $\mathcal{O}(h^2)$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2 f''(x)}{2} + \frac{h^3 f'''(x)}{6} + \dots$$

$$\frac{f(x+h) - f(x)}{h} - \underbrace{\frac{h}{2} f''(x)}_{\mathcal{O}(h)} - \underbrace{\frac{h^2 f'''(x)}{6}}_{\mathcal{O}(h^2)} = f'(x)$$

$$f'(x_0) \approx p'(x_0)$$

$$p'(x_0) = \frac{p(x_0+h) - p(x_0)}{h} - \frac{h}{2} p''(x_0)$$

$$p'(x) = f(x_0)\left[\left(\frac{1}{x_0 - x_1}\right)\left(\frac{x - x_2}{x_0 - x_2}\right) + \left(\frac{1}{x_0 - x_2}\right)\left(\frac{x - x_1}{x_0 - x_1}\right)\right]$$

$$+ f(x_1)\left[\left(\frac{1}{x_1 - x_0}\right)\left(\frac{x - x_2}{x_1 - x_2}\right) + \left(\left(\frac{1}{x_1 - x_2}\right)\left(\frac{x - x_0}{x_1 - x_0}\right)\right)\right]$$

$$+ f(x_2)\left[\left(\frac{1}{x_2 - x_0}\right)\left(\frac{x - x_1}{x_2 - x_1}\right) + \left(\frac{1}{x_2 - x_1}\right)\left(\frac{x - x_0}{x_2 - x_0}\right)\right]$$

$$p''(x_0) = \frac{p(x_0+h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

$$p'(x) = \frac{p(x_0+h) - p(x_0)}{h} - \frac{h}{2}\left(\frac{p(x_0+h) - 2p(x_0) + p(x_0 - h)}{h^2}\right)$$

$$P(x_0+h) = f(x_0)\left(\frac{x_0+h-x_1}{x_0-x_1}\right)\left(\frac{x_0+h-x_2}{x_0-x_2}\right) + f(x_1)\left(\frac{x_0+h-x_0}{x_1-x_0}\right)\left(\frac{x_0+h-x_2}{x_1-x_2}\right) + f(x_2)\left(\frac{x_0+h-x_0}{x_2-x_0}\right)\left(\frac{x_0+h-x_1}{x_2-x_1}\right)$$

$$= f(x_0)\left(\frac{x_0+h-x_1}{x_0-x_1}\right)\left(\frac{x_0+h-x_2}{x_0-x_2}\right) + f(x_1)\left(\frac{h}{x_1-x_0}\right)\left(\frac{x_0+h-x_2}{x_1-x_2}\right) + f(x_2)\left(\frac{h}{x_2-x_0}\right)\left(\frac{x_0+h-x_1}{x_2-x_1}\right)$$

$$P(x_0) = f(x_0)$$

$$P(x_0-h) = f(x_0)\left(\frac{x_0-h-x_1}{x_0-x_1}\right)\left(\frac{x_0-h-x_2}{x_0-x_2}\right) + f(x_1)\left(\frac{-h}{x_1-x_2}\right)\left(\frac{x_0+h-x_2}{x_1-x_2}\right) + f(x_2)\left(\frac{h}{x_2-x_0}\right)\left(\frac{x_0+h-x_1}{x_2-x_1}\right)$$

$$P'(x_1) = \frac{P(x_0+h)}{h} - \frac{P(x_0)}{h} + \frac{1}{2}\left(\frac{P(x_0+h)}{h} - \frac{2P(x_0)}{h} + \frac{P(x_0-h)}{h}\right)$$