AI1103 : Assignment 7

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Download all latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%207/Assignment%207.tex

1 CSIR - UGC 2014 Dec Q.103

Suppose X is a Random Variable such that E(X) = 0, E(X^2) = 2 and E(X^4)=4.Then

1) E(X^3)=0
2) $\Pr(X \geq 0)$= $\frac{1}{2}$
3) X ∼ N(0,2)
4) X is bounded with Probability 1.

2 Solution

Let X be a Random variable.
Compute Variance of X^2, Let Y = X^2

$$\begin{aligned}
\sigma(Y) &= E(Y^2) - (E(Y))^2 \\
&= 4 - 2^2 \\
&= 0 \\
\implies \sigma^2(X^2) &= 0 \quad (1)\\
\sigma^2(Y) &= E((Y - E(Y))^2) = 0 \\
&= \Sigma(Y - E(Y))^2 \Pr(X) \\
(Y - E(Y))^2 &\geq 0 \text{ and} \Pr(X) \geq 0
\end{aligned}$$

Since each term inside summation are non-negative For summation to be 0,term in it must be 0,

$$\begin{aligned}
\implies (Y - E(Y))^2 = 0 &\implies Y = E(Y) \\
X^2 = E(X^2) &= 2 \quad (*)
\end{aligned}$$

$\boxed{X^2 \textbf{ is constant.}}$

Given E(X^2) = 2,

$$\begin{aligned}
E(X^2) &= \Sigma X^2 \Pr(X) \\
&= X^2 \Sigma \Pr(X) \\
&= X^2 (\because \Sigma \Pr(X) = 1) \\
X^2 &= 2 \\
\implies X &= \pm\sqrt{2} \quad (2)
\end{aligned}$$

Given E(X) = 0,

$$\begin{aligned}
E(X) &= \Sigma X \Pr(X) = 0 \\
\sqrt{2}\Pr\left(X = \sqrt{2}\right) - \sqrt{2}\Pr\left(X = -\sqrt{2}\right) &= 0 \\
\implies \Pr\left(X = \sqrt{2}\right) &= \Pr\left(X = -\sqrt{2}\right) \quad (3)
\end{aligned}$$

Also, Sum of Probabilities is 1,

$$\begin{aligned}
\implies \Pr\left(X = \sqrt{2}\right) + \Pr\left(X = -\sqrt{2}\right) &= 1 \\
\implies \Pr\left(X = \sqrt{2}\right) &= \frac{1}{2} \quad (4)\\
\implies \Pr\left(X = -\sqrt{2}\right) &= \frac{1}{2} \quad (5)
\end{aligned}$$

1) **Option 1**: E(X^3) = 0,

$$\begin{aligned}
E(X^3) &= \Sigma X^3 \Pr(X) \\
&= X^2.\Sigma X \Pr(X) (\because X^2 \text{ is constant}) \\
&= X^2 E(X) \\
\implies E(X^3) &= 0
\end{aligned}$$

$\boxed{\textbf{Option 1} \text{ is a } \textbf{correct} \text{ answer}}$

2) **Option 2**: $\Pr(X \geq 0) = \frac{1}{2}$,

$$\begin{aligned}
\Pr(X \geq 0) &= \Pr\left(X = \sqrt{2}\right) = \frac{1}{2} \\
\implies \Pr(X \geq 0) &= \frac{1}{2}
\end{aligned}$$

$\boxed{\textbf{Option 2} \text{ is a } \textbf{correct} \text{ answer}}$

3) **Option 3**: X ∼ N(0,2)
Let μ be the mean of X

$$\begin{aligned}
\mu &= E(X) \\
\implies \mu &= 0 \quad (6)\\
\sigma^2(X) &= E(X^2) - (E(X))^2 \\
&= 2 - 0^2 \\
\implies \sigma^2(X) &= 2 \quad (7)
\end{aligned}$$

But Random Variable X is defined for $\pm\sqrt{2}$ only.

Distribution of X is not continuous,but discrete.

$\boxed{\textbf{Option 3} \text{ is a } \textbf{WRONG} \text{ answer}}$

4) **Option 4**: X is bounded with probability 1,
Eq. (4) and (5) show that $X \in \{-\sqrt{2}, \sqrt{2}\}$ with
Probability 1.

$\boxed{\textbf{Option 4} \text{ is a } \textbf{correct} \text{ answer}}$

$\boxed{\text{So, only Options } \textbf{1, 2 and 4} \text{ are correct}}$