

# AI1103 : Assignment 3

Santosh Dhaladhuli MS20BTECH11007

Download all latex codes from

<https://github.com/Santosh-Dhaladhuli2003/AI1103/blob/main/Assignment%203/Assignment%203.tex>

## 1 GATE MA 2005 QUESTION No. 25

Let  $A_1, A_2, \dots, A_n$  be  $n$  independent events in which the Probability of occurrence of the event  $A_i$  is given by  $P(A_i) = 1 - \frac{1}{\alpha^i}$ ,  $\alpha > 1$ ,  $i = 1, 2, 3, \dots, n$ . Then the probability that atleast one of the events occurs is

- (a)  $1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}}$       (b)  $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$   
 (c)  $\frac{1}{\alpha^n}$       (d)  $1 - \frac{1}{\alpha^n}$

## 2 SOLUTION

Let  $A_1 \cup A_2 \cup A_3 \dots \cup A_n = S$ ,

$\Pr(S)$  = Probability of atleast one event occuring

De morgan's law states that  $(A \cup B)^C = A^C \cap B^C$

$$\implies \Pr(S) = 1 - \Pr(S^C) \quad (1.1)$$

$$1 - \Pr(S^C) = 1 - \Pr(A_1^C \cap A_2^C \cap A_3^C \cap \dots \cap A_n^C) \quad (1.2)$$

Since,  $A_1, A_2, \dots, A_n$  are independent. Complements of  $A_1, A_2, \dots, A_n$  are also independent.

$\implies$

$$\Pr(A_1^C \cap A_2^C \cap \dots \cap A_n^C) = \prod_{i=1}^n \Pr(A_i^C) \quad (2.1)$$

$$\Pr(A_i) = 1 - \frac{1}{\alpha^i} \implies \Pr(A_i^C) = \frac{1}{\alpha^i} \quad (2.2)$$

$$\text{Let, } x = \sum_{i=1}^n i \implies x = \frac{n(n+1)}{2} \quad (2.3)$$

from equations (1.1), (1.2), (2.1) and (2.2)

$$\implies \Pr(S) = 1 - \frac{1}{\alpha^x} = 1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \quad (2.4)$$

$\therefore$  The correct option is (a)