AI1103 : Assignment 3

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Download all latex codes from

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https://github.com/Santosh−Dhaladhuli2003/
    AI1103/blob/main/Assignment%203/
    Assignment%203.tex
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1 GATE MA 2005 Question No. 25

Let $A_1, A_2, ...A_n$ be n independent events in which the Probability of occurence of the event A_i is given by P(A_i) = 1 - $\frac{1}{\alpha^i}$, $\alpha > 1$, i = 1,2,3,....n.Then the probability that atleast one of the events occurs is

(a) 1 - $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$ (b) $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$

(c) $\frac{1}{\alpha^n}$ (d) 1 - $\frac{1}{\alpha^n}$

2 Solution

Let $A_1 \cup A_2 \cup A_3 .... \cup A_n$ = S,
$\Pr(S)$ = Probability of atleast one event occuring
De morgan's law states that $(A \cup B)^{\complement} = A^{\complement} \cap B^{\complement}$

$$\implies \Pr(S) = 1 - \Pr\left(S^{\complement}\right) \tag{1.1}$$

$$1 - \Pr\left(S^{\complement}\right) = 1 - \Pr\left(A_1^{\complement} \cap A_2^{\complement} \cap A_3^{\complement} \cap ....A_n^{\complement}\right) \tag{1.2}$$

Since,$A_1, A_2, ...A_n$ are independent.Complements of $A_1, A_2, ...A_n$ are also independent.
$\implies$

$$\Pr\left(A_1^{\complement} \cap A_2^{\complement} \cap ....A_n^{\complement}\right) = \Pr\left(A_1^{\complement}\right)\times\Pr\left(A_2^{\complement}\right)...\times\Pr\left(A_n^{\complement}\right) \tag{2.1}$$

$$\Pr(A_i) = 1 - \frac{1}{\alpha^i} \implies \Pr\left(A_i^{\complement}\right) = \frac{1}{\alpha^i} \tag{2.2}$$

$$\text{Let,}x = \sum_{i=1}^{n} i \implies x = \frac{n(n+1)}{2} \tag{2.3}$$

from equations (1.1), (1.2) and (2.2)

$$\implies \Pr(S) = 1 - \frac{1}{\alpha^x} = 1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \tag{2.4}$$

$\therefore$ The correct option is **(a)**