AI1103 : Assignment 1

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Download all python codes from

```
https://github.com/Santosh−Dhaladhuli2003/
    AI1103−Assignment−1/blob/main/Assignment
    %201.py
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and latex codes from

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https://github.com/Santosh−Dhaladhuli2003/
    AI1103−Assignment−1/blob/main/Assignment
    %201.tex
```

PROBLEM 5.12

Random Variable X has the following Probability Distribution

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$ + k

Determine:

1) k
2) $\Pr(X < 3)$
3) $\Pr(X > 6)$
4) $\Pr(0 < X < 3)$

Solution

PMF of X:

$$\Pr(X) = \begin{cases} 0, & \text{for } X = 0 \\ k, & \text{for } X = 1 \\ 2k, & \text{for } X = 2 \\ 2k, & \text{for } X = 3 \\ 3k, & \text{for } X = 4 \\ k^2, & \text{for } X = 5 \\ 2k^2, & \text{for } X = 6 \\ 7k^2 + k, & \text{for } X = 7 \end{cases} \tag{5.12.1}$$

1) It is known that the sum of probabilities of a probability distribution is always one.

$$\therefore 0 + k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1 \tag{5.12.2}$$

$$\implies 10k^2 + 9k - 1 = 0 \implies (10k - 1)(k + 1) = 0 \tag{5.12.3}$$

$$\implies k = -1, \frac{1}{10} \tag{5.12.4}$$

$$\therefore k = \frac{1}{10} (\because k \geq 0) \tag{1}$$

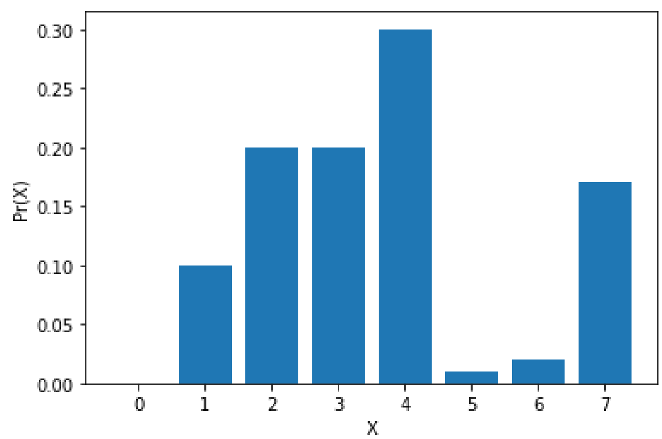

Fig. 1: Probability Mass Function(PMF)

CDF of X:

X	0	1	2	3	4	5	6	7
F(X)	0	1/10	3/10	5/10	8/10	81/100	83/100	1

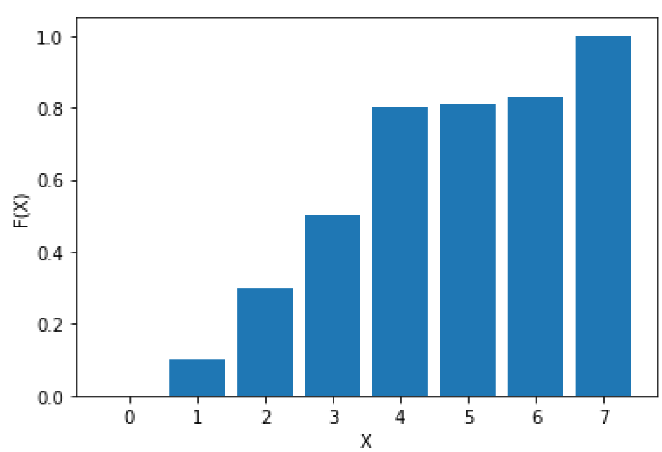

Fig. 1: Cumulative Distribution Function(CDF)

We know that $\Pr(X \le x) = \mathrm{F(x)}$
and $\Pr(x < X \le y) = \mathrm{F(y)} - \mathrm{F(x)}$

2) $\Pr(X < 3) = \Pr(X \le 3) - \Pr(X = 3)$

$$\implies \Pr(X < 3) = F(3) - \Pr(X = 3) \quad (5.12.5)$$

$$\implies \Pr(X < 3) = \frac{5}{10} - \frac{2}{10} \quad (5.12.6)$$

$$\therefore \Pr(X < 3) = \frac{3}{10} \quad (2)$$

3) $\Pr(X > 6) = 1 - \Pr(X \le 6) = 1 - \mathrm{F(6)}$

$$\implies \Pr(X > 6) = 1 - \frac{83}{100} \quad (5.12.7)$$

$$\therefore \Pr(X > 6) = \frac{17}{100} \quad (3)$$

4) $\Pr(0 < X < 3) = \Pr(0 < X \le 3) - \Pr(X = 3)$

$$\implies \Pr(0 < X < 3) = F(3) - F(0) - \Pr(X = 3) \quad (5.12.8)$$

$$\implies \Pr(0 < X < 3) = \frac{5}{10} - 0 - \frac{2}{10} \quad (5.12.9)$$

$$\therefore \Pr(0 < X < 3) = \frac{3}{10} \quad (4)$$