# AI1103 : Assignment 7

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Download all latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%207/Assignment%207.tex

## 1 CSIR - UGC 2014 Dec Q.103

Suppose X is a Random Variable such that E(X) = 0, E($X^2$) = 2 and E($X^4$)=4.Then

1) E($X^3$)=0
2) $\Pr(X \geq 0)$= $\frac{1}{2}$
3) X ∼ N(0,2)
4) X is bounded with Probability 1.

## 2 Solution

Let X be a Random variable.
Compute Variance of $X^2$, Let Y = $X^2$

$$\sigma(Y) = E(Y^2) - (E(Y))^2$$
$$= 4 - 2^2$$
$$= 0$$
$$\implies \sigma^2(X^2) = 0 \tag{1}$$
$$\sigma^2(Y) = E((Y - E(Y))^2) = 0$$

We Know Z = $(Y - E(Y))^2 \geq 0$
If E(Z) = 0 then it means Z = 0

$$\implies Y = E(Y)$$
$$X^2 = E(X^2) = 2 \tag{*}$$

$$\boxed{X^2 \textbf{ is constant.}}$$

Given E($X^2$) = 2,

$$E(X^2) = \Sigma X^2 \Pr(X)$$
$$= X^2 \Sigma \Pr(X)$$
$$= X^2 (\because \Sigma \Pr(X) = 1)$$
$$X^2 = 2$$
$$\implies X = \pm\sqrt{2} \tag{2}$$

Given E(X) = 0,

$$E(X) = \Sigma X \Pr(X) = 0$$
$$\sqrt{2} \Pr\left(X = \sqrt{2}\right) - \sqrt{2} \Pr\left(X = -\sqrt{2}\right) = 0$$
$$\implies \Pr\left(X = \sqrt{2}\right) = \Pr\left(X = -\sqrt{2}\right) \tag{3}$$

Also, Sum of Probabilities is 1,

$$\implies \Pr\left(X = \sqrt{2}\right) + \Pr\left(X = -\sqrt{2}\right) = 1$$
$$\implies \Pr\left(X = \sqrt{2}\right) = \frac{1}{2} \tag{4}$$
$$\implies \Pr\left(X = -\sqrt{2}\right) = \frac{1}{2} \tag{5}$$

1) **Option 1**: E($X^3$) = 0,

$$E(X^3) = \Sigma X^3 \Pr(X)$$
$$= X^2.\Sigma X \Pr(X) \ (\because X^2 \text{ is constant})$$
$$= X^2 E(X)$$
$$\implies E(X^3) = 0$$

$$\boxed{\textbf{Option 1} \text{ is a } \textbf{correct} \text{ answer}}$$

2) **Option 2**: $\Pr(X \geq 0) = \frac{1}{2}$,

$$\Pr(X \geq 0) = \Pr\left(X = \sqrt{2}\right) = \frac{1}{2}$$
$$\implies \Pr(X \geq 0) = \frac{1}{2}$$

$$\boxed{\textbf{Option 2} \text{ is a } \textbf{correct} \text{ answer}}$$

3) **Option 3**: X ∼ N(0,2)
Let $\mu$ be the mean of X

$$\mu = E(X)$$
$$\implies \mu = 0 \tag{6}$$
$$\sigma^2(X) = E(X^2) - (E(X))^2$$
$$= 2 - 0^2$$
$$\implies \sigma^2(X) = 2 \tag{7}$$

But Random Variable X is defined for $\pm\sqrt{2}$ only.

Distribution of X is not continuous,but discrete.

$\boxed{\textbf{Option 3} \text{ is a } \textbf{WRONG} \text{ answer}}$

4) **Option 4**: X is bounded with probability 1,
Eq. (4) and (5) show that $X \in \{-\sqrt{2}, \sqrt{2}\}$ with Probability 1.

$\boxed{\textbf{Option 4} \text{ is a } \textbf{correct} \text{ answer}}$

$\boxed{\text{So, only Options } \textbf{1, 2 and 4} \text{ are correct}}$