AI1103 : Assignment 5

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Download all python codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%205/Assignment%205.py

and latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%205/Assignment%205.tex

1 GATE IN 2007 Question No. 27

Assume that the duration in minutes of a telephone conversation follows the exponential distribution f(x) = $\frac{1}{5}e^{-\frac{x}{5}}$, x $\geq$ 0. The probability that the conversation will exceed five minutes is...

1) $\frac{1}{e}$
2) 1 - $\frac{1}{e}$
3) $\frac{1}{e^2}$
4) 1 - $\frac{1}{e^2}$

2 Solution

Let X be a Random variable defined,that denotes the duration of a telephonic conversation in minutes.
So, X $\in$ [0,∞)
Given, f_X(x) = $\frac{1}{5}e^{-\frac{x}{5}}$
Let CDF of X be F_X(x)

$$\begin{aligned} F_X(x) &= \int_{-\infty}^{x} f_X(t)\,dt \\ &= \int_{-\infty}^{0} f_X(t)\,dt + \int_{0}^{x} f_X(t)\,dt \\ F_X(x) &= \int_{0}^{x} f_X(t)\,dt \because f_X(x) = 0 \forall x < 0 \\ \therefore F_X(x) &= \int_{0}^{x} \frac{1}{5}e^{-\frac{t}{5}}\,dt \\ \implies F_X(x) &= 1 - e^{-\frac{x}{5}} \end{aligned} \tag{1}$$

$$\begin{aligned} F_X(x) &= \Pr(X \leq x) \\ \Pr(X > 5) &= 1 - \Pr(X \leq 5) \\ \implies \Pr(X > 5) &= 1 - F_X(5) \\ &= 1 - (1 - e^{-\frac{5}{5}}) \\ &= e^{-\frac{5}{5}} \\ &= e^{-1} = \frac{1}{e} \end{aligned}$$

$\therefore$ The correct answer is **Option 1**