AI1103 : Assignment 3

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Download all latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%203/Assignment%203.tex

1 GATE MA 2005 Question No. 25

Let $A_1, A_2, ...A_n$ be n independent events in which the Probability of occurence of the event A_i is given by P(A_i) = 1 - $\frac{1}{\alpha^i}$, $\alpha > 1$, i = 1,2,3,....n.Then the probability that atleast one of the events occurs is

(a) 1 - $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$ (b) $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$

(c) $\frac{1}{\alpha^n}$ (d) 1 - $\frac{1}{\alpha^n}$

2 Solution

Let $A_1 + A_2 + A_3.... + A_n$ = S,
$\Pr(S)$ = Probability of atleast one event occuring
De morgan's law states that $(A + B)' = A'B'$

$$\implies \Pr(S) = 1 - \Pr(S') \tag{1.1}$$

$$1 - \Pr(S') = 1 - \Pr(A_1'A_2'A_3'....A_n') \tag{1.2}$$

$\forall$ i $\in$ 1,2,....n
Since, A_i are independent.
$\therefore$ Complements of A_i are also independent.
$\implies$

$$\Pr(A_1'A_2'A_3'....A_n') = \prod_{i=1}^{n} \Pr(A_i') \tag{2.1}$$

$$\Pr(A_i) = 1 - \frac{1}{\alpha^i} \implies \Pr(A_i') = \frac{1}{\alpha^i} \tag{2.2}$$

substituting (2.2) in (2.1),

$$\Pr(A_1'A_2'A_3'....A_n') = \prod_{i=1}^{n} \frac{1}{\alpha^i} \tag{2.3}$$

$$\prod_{i=1}^{n} \frac{1}{\alpha^i} = \frac{1}{\alpha^{\sum_i^n i}} = \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \tag{2.4}$$

$$\therefore \Pr(A_1'A_2'A_3'....A_n') = \Pr(S') = \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \tag{2.5}$$

from equations (1.2) and (2.5)

$$\implies \Pr(S) = 1 - \Pr(S') = 1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \tag{2.6}$$

$\therefore$ The correct option is **(a)**