

AI1103 : Assignment 7

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Download all latex codes from

<https://github.com/Santosh-Dhaladhuli2003/AI1103/blob/main/Assignment%207/Assignment%207.tex>

1 CSIR - UGC 2014 DEC Q.103

Suppose X is a Random Variable such that $E(X) = 0$, $E(X^2) = 2$ and $E(X^4) = 4$. Then

- 1) $E(X^3) = 0$
- 2) $\Pr(X \geq 0) = \frac{1}{2}$
- 3) $X \sim N(0, 2)$
- 4) X is bounded with Probability 1.

2 SOLUTION

Let X be a Random variable.

Compute Variance of X^2 , Let $Y = X^2$

$$\begin{aligned}\sigma(Y) &= E(Y^2) - (E(Y))^2 \\ &= 4 - 2^2 \\ &= 0 \\ \Rightarrow \sigma^2(X^2) &= 0 \\ \sigma^2(Y) &= E((Y - E(Y))^2) = 0\end{aligned}\quad (1)$$

We Know $Z = (Y - E(Y))^2 \geq 0$

If $E(Z) = 0$ then it means $Z = 0$

$$\begin{aligned}\Rightarrow Y &= E(Y) \\ X^2 &= E(X^2) = 2\end{aligned}\quad (*)$$

$\therefore X$ is a random variable such that X^2 is constant.
Given $E(X^2) = 2$,

$$\begin{aligned}E(X^2) &= \sum X^2 \Pr(X) \\ &= X^2 \sum \Pr(X) \\ &= X^2 (\because \sum \Pr(X) = 1) \\ X^2 &= 2 \\ \Rightarrow X &= \pm \sqrt{2}\end{aligned}\quad (2)$$

Given $E(X) = 0$,

$$\begin{aligned}E(X) &= \sum X \Pr(X) = 0 \\ \sqrt{2} \Pr(X = \sqrt{2}) - \sqrt{2} \Pr(X = -\sqrt{2}) &= 0 \\ \Rightarrow \Pr(X = \sqrt{2}) &= \Pr(X = -\sqrt{2})\end{aligned}\quad (3)$$

Also, Sum of Probabilities is 1,

$$\begin{aligned}\Rightarrow \Pr(X = \sqrt{2}) + \Pr(X = -\sqrt{2}) &= 1 \\ \Rightarrow \Pr(X = \sqrt{2}) &= \frac{1}{2}\end{aligned}\quad (4)$$

$$\Rightarrow \Pr(X = -\sqrt{2}) = \frac{1}{2}\quad (5)$$

1) **Option 1:** $E(X^3) = 0$,

$$\begin{aligned}E(X^3) &= \sum X^3 \Pr(X) \\ &= X^2 \cdot \sum X \Pr(X) \\ &= X^2 E(X) \\ \Rightarrow E(X^3) &= 0\end{aligned}$$

Option 1 is a correct answer

2) **Option 2:** $\Pr(X \geq 0) = \frac{1}{2}$,

$$\begin{aligned}\Pr(X \geq 0) &= \Pr(X = \sqrt{2}) = \frac{1}{2} \\ \Rightarrow \Pr(X \geq 0) &= \frac{1}{2}\end{aligned}$$

Option 2 is a correct answer

3) **Option 3:** $X \sim N(0, 2)$
Let μ be the mean of X

$$\begin{aligned}\mu &= E(X) \\ \Rightarrow \mu &= 0\end{aligned}\quad (6)$$

$$\begin{aligned}\sigma^2(X) &= E(X^2) - (E(X))^2 \\ &= 2 - 0^2 \\ \Rightarrow \sigma^2(X) &= 2\end{aligned}\quad (7)$$

But Random Variable X is defined for $\pm \sqrt{2}$ only.

This means that distribution of X is not continuous, but discrete.

Option 3 is a **WRONG** answer

- 4) **Option 4:** X is bounded with probability 1,
Eq. (4) and (5) show that $X \in (-\sqrt{2}, \sqrt{2})$
with Probability 1.

Option 4 is a **correct** answer

So, only Options **1, 2 and 4** are correct