AI1103 : Assignment 5

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Download all python codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%205/Assignment%205.py

and latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%205/Assignment%205.tex

1 GATE IN 2007 Question No. 27

Assume that the duration in minutes of a telephone conversation follows the exponential distribution f(x) = $\frac{1}{5}e^{-\frac{x}{5}}$, x $\geq$ 0. The probability that the conversation will exceed five minutes is...

1) $\frac{1}{e}$
2) 1 - $\frac{1}{e}$
3) $\frac{1}{e^2}$
4) 1 - $\frac{1}{e^2}$

2 Solution

Let X be a Random variable defined,that denotes the duration of a telephonic conversation in minutes.
So, X $\in$ [0,∞)
Using the probability in exponential distribution,

$$\text{CDF of X is F(y)} = \Pr(X \leq y) \tag{2.0.1}$$

$$\implies F(y) = 1 - \int_y^{\infty} \frac{1}{5}e^{-\frac{x}{5}}\,dx \tag{2.0.2}$$

$$\text{Required Probability is } \Pr(X > 5) = 1 - \Pr(X \leq 5) \tag{2.0.3}$$

From (2.0.1), (2.0.2) and (2.0.3)

$$\implies \Pr(X > 5) = 1 - \left(1 - \int_5^{\infty} \frac{1}{5}e^{-\frac{x}{5}}\,dx\right) \tag{2.0.4}$$

$$\implies \Pr(X > 5) = \int_5^{\infty} \frac{1}{5}e^{-\frac{x}{5}} \tag{2.0.5}$$

$$\implies \Pr(X > 5) = \left(-e^{-\frac{x}{5}}\right)\Big|_5^{\infty} \tag{2.0.6}$$

$$\implies \Pr(X > 5) = -e^{-\infty} + e^{-1} \tag{2.0.7}$$

$$\therefore \Pr(X > 5) = 0 + e^{-1} = \frac{1}{e} \tag{2.0.8}$$

$\therefore$ The correct answer is **Option 1**