

AI1103 : Assignment 7

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Download all latex codes from

<https://github.com/Santosh-Dhaladhuli2003/AI1103/blob/main/7th%20Assignment/7th%20Assignment.tex>

1 GOV/STATS/2015/STATISTICS-I(1), Q.1(D)

Let X be a Random Variable with $E[X] = 3$, $E[X^2] = 13$. Use Chebyshev's Inequality to obtain $\Pr(-2 < X < 8)$

2 SOLUTION

Chebyshev's Inequality:

Let X be a random variable with finite expected value $E[X]$ and finite non-zero variance σ^2 . Then for any real number $k > 0$,

$$\Pr(|X - E[X]| \geq k\sigma) \leq \frac{1}{k^2} \quad (1)$$

computing the variance,

$$\begin{aligned} \sigma^2 &= E[X^2] - E[X]^2 \\ \implies \sigma^2 &= 13 - 9 = 4 \end{aligned} \quad (2)$$

$$\sigma = 2 \quad (3)$$

using (3),

$$\Pr(-2 < X < 8) = 1 - \Pr(|X - 3| > 5) \quad (4)$$

$$\Pr(|X - 3| > 5) = \Pr(|X - E[X]| > k\sigma) \quad (5)$$

$$k\sigma = 5$$

$$\implies 2k = 5$$

$$\therefore k = \frac{5}{2} \quad (6)$$

Using (1), (5) and (6) in (4),

$$\begin{aligned} \Pr(-2 < X < 8) &\geq 1 - \left(\frac{2}{5}\right)^2 \\ \implies \Pr(-2 < X < 8) &\geq \frac{21}{25} \end{aligned} \quad (7)$$