AI1103 : Assignment 6

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https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%206/Assignment%206.tex

1 GATE ST 2021 Q.1 st. section

Let X be a non-constant positive Random Variable such that $E(X) = 9$.
Then which of the following statements is True?

1) $E\left(\frac{1}{X+1}\right) > 0.1$ and $\Pr(X \geq 10) \leq 0.9$
2) $E\left(\frac{1}{X+1}\right) < 0.1$ and $\Pr(X \geq 10) \leq 0.9$
3) $E\left(\frac{1}{X+1}\right) > 0.1$ and $\Pr(X \geq 10) > 0.9$
4) $E\left(\frac{1}{X+1}\right) < 0.1$ and $\Pr(X \geq 10) > 0.9$

2 Solution

Given, for X > 0 ,$E(X) = 9$, $E\left(\frac{1}{X+1}\right)$ can be estimated by Jensens's Inequality.
pre - requisites:
In general, $\phi(X)$ is a convex function iff:

$$\frac{d^2\phi}{dX^2} \geq 0$$

Jensen's Inequality:
In the context of probability theory, it is generally stated in the following form: if X is a random variable and ϕ is a convex function, then

$$\phi(E(X)) \leq E(\phi(X)) \tag{1}$$

So for $\phi(X) = \dfrac{1}{X+1}$,

$$\frac{d\phi}{dX} = -\frac{1}{(X+1)^2}$$

$$\frac{d^2\phi}{dX^2} = \frac{2}{(X+1)^3} \implies \frac{d^2\phi}{dX^2} \geq 0, (\because X > 0) \tag{2}$$

by eq (1) and (2)

$$E\left(\frac{1}{X+1}\right) \geq \frac{1}{E(X)+1}$$

$$\implies E\left(\frac{1}{X+1}\right) \geq \frac{1}{9+1}$$

$$\implies E\left(\frac{1}{X+1}\right) \geq 0.1 \tag{3}$$

$\Pr(X \geq 10)$ can be estimated by Markov's Inequality.
Markov's Inequality: If X is a non-negative random variable and a > 0, then the probability that X is at least a is at most the expectation of X divided by a.
Mathematically,

$$\Pr(X \geq a) \leq \frac{E(X)}{a} \tag{4}$$

by (4) for a = 10

$$\Pr(X \geq 10) \leq \frac{E(X)}{10}$$

$$\implies \Pr(X \geq 10) \leq \frac{9}{10}$$

$$\therefore \Pr(X \geq 10) \leq 0.9 \tag{5}$$

So, from (3) and (5)

$$\boxed{\textbf{Option 1 is the Correct Answer}}$$