Simulation of Covariance Analysis Describing Equation Technique(CADET) in Missile Hit Probability Calculation

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Abstract

1. Monte Carlo method is always used in calculating the probability of a missile hit, but it usually cost a lot of time to get accurate statistics.
2. This paper makes use of covariance analysis describing equation technique (CADET) by performance analysis of rapid and comprehensive of missile system.
3. First we elaborate on the primary coverage of CADET. Then we show the application of CADET in the assessment of missile hit probability in combination with a non-linear model of missile. At last, we analyze the pros and cons of CADET by comparing with the result of Monte Carlo method.

Theory of CADET

Equation for General Non - Linear Systems is

$$\dot{x}(t) = f(x, t) + G(t)w(t) \tag{1}$$

Where $x(t)$ is a n-dimensional state vector; $w(t)$ is a p-dimensional random force vector effected on system interference and control input. $G(t)$ is the corresponding dimension matrix.

Random State vector $x(t)$ is composed of a mean $m(t)$ and a random component $r(t)$, that is

$$\begin{cases} x(t) = m(t) + r(t) \\ m(t) = E[x(t)] \\ P(t) = E[r(t).r^T(t)] \end{cases} \tag{2}$$

Random Force vector $w(t)$ is also composed of a mean $b(t)$ and a random component $u(t)$, that is

$$\begin{cases} w(t) = b(t) + u(t) \\ b(t) = E[w(t)] \\ E[u(t).u^T(t)] = Q(t).\delta(t-\tau) \end{cases} \tag{3}$$

Where , $\delta(t-\tau)$ is the δ function. $Q(t)$ is the white noise spectral density matrix.

Statistical Linearisation of Non-Linear Systems

Given a function,

$$y = f(x)$$

a linearised function would look like,

$$y = A.x + b$$

where A can be a matrix and b is a vector. The objective of this technique is to find the best 'A' and 'b' that can approximate this function.
We will apply this technique for using CADET.

According to the Statistical Linearization principle,

$$f(x,t) = \hat{f} + N.r(t) \tag{4}$$

Substitute into equation (1), we get,

$$\dot{x}(t) = \hat{f} + N.r(t) + G(t)w(t) \tag{5}$$

$$\dot{x}(t) = N.x(t) + G(t)w(t) + R(t) \tag{6}$$

where,

$$R(t) = \hat{f} - N.m(t) \tag{7}$$

Since there is no random component,equation (6) is an obvious Linear Differential Equation.

According to covariance analysis theory of linear system, under the action of the white noise disturbance, the same has been a mean vector $m(t)$ of random state vector $x(t)$ and the propagation equation of covariance matrix $P(t)$.

$$\dot{m}(t) = \hat{f} + G(t)w(t) \tag{8}$$

$$\dot{P}(t) = N(t)P(t) + P(t)N^T(t) + G(t)Q(t)G^T(t) \tag{9}$$

As $\hat{f}$ and N generally contain the mean $m(t)$ and covariance $P(t)$ of $x(t)$, so equation (8) and equation (9) are generally nonlinear.

So long as given the probability density function $p(x)$ of random variable $x(t)$, we can compute corresponding describing function. In particular, when $x(t)$ is of joint normal distribution, then

$$\hat{f} = E[f(x,t)] = \int_{-\infty}^{\infty} f(x.t)p(x)dx \tag{10}$$

$$N = \frac{df}{dm} \tag{11}$$

the short-cycle perturbation equations of missiles lateral channel are:

$$\begin{cases}\ddot{\psi} + b_1\dot{\psi} + b_2\beta = -b_3\delta \\ \dot{\psi}_c - b_4\beta = 0 \\ \psi = \psi_c + \beta\end{cases} \tag{12}$$

Where ψ is missile yaw angle, ψ_c is velocity declination. Missile kinematics equation:

$$\begin{cases}\dot{x} = V.cos\psi_c \\ \dot{z} = -V.sin\psi_c\end{cases} \tag{13}$$

where V is the missile velocity.
Proportional navigation control equation

$$\delta = K.\dot{q} \tag{14}$$

Where $\dot{q}$ is rate of change of the sight angle high and low; K is proportional navigation coefficient

According Equations (12) and (13), the missile lateral channel is a nonlinear system. When only considering the measurement noise, we may impose noise e on $\dot{q}$, where e is a zero-mean Gaussian white noise corresponded with the CADET hypothesis, and its spectral density value is Q .

Selecting the system state variables:
$x_1 = \dot{\psi}$, $x_2 = \psi$, $x_1 = \psi_c$, $x_1 = x$, $x_1 = z$

State variables above are random variables, then the mean and covariance propagation equation are following:

$$\begin{cases} \dot{m}_1 = -b_1 m_1 - b_2 m_2 + b_1 m_3 - b_3 m_\delta \\ \dot{m}_2 = m_1 \\ \dot{m}_3 = b_4 m_2 - b_4 m_3 \\ \dot{m}_4 = V.e^{-\frac{1}{2}P_{33}} cos m_3 \\ \dot{m}_5 = -V.e^{-\frac{1}{2}P_{33}} sin m_3 \end{cases} \tag{15}$$

Now finally we use equation (9), where N is the following matrix,

$$N = \begin{bmatrix} -b_1 & -b_2 & b_2 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & b_4 & -b_4 & 0 & 0 \\ 0 & 0 & -V.e^{-\frac{1}{2}P_{33}} sin m_3 & 0 & 0 \\ 0 & 0 & -V.e^{-\frac{1}{2}P_{33}} cos m_3 & 0 & 0 \end{bmatrix}$$

This will be the mean and covariance equations of the system state variables. If the initial conditions both $m(t_o)$ and $P(t_o)$ are known, we can get the control systems' mean and covariance of the state variables at any time under the effect of random interference and noise. x_4 and x_5 are missile coordinates, through analyzing their value in the end of simulation moment in line for the mean miss distance.

Missile-Target Collision mean Curve

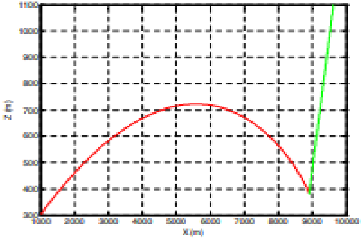

Fig. 1. The missile - target collision curve of CADET

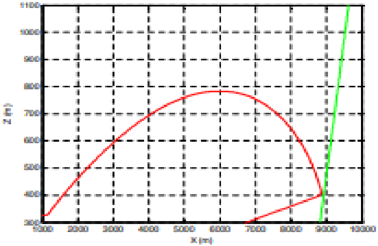

Fig. 2. The missile - target collision mean curve of Monte-Carlo method

Simulations and Analysis

mean of missiles in z coordinates

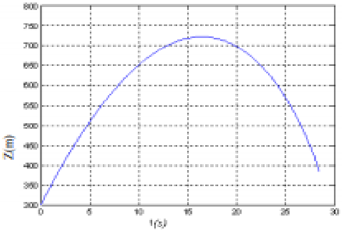

Fig. 3. CADET Method - the mean of missile in z coordinates

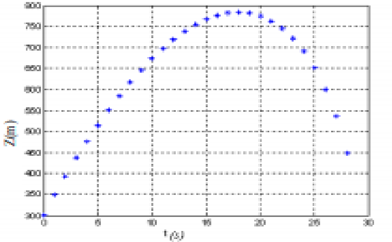

Fig. 4. Monte-Carlo Method - the missile mean of z coordinates

Simulations and Analysis

MSE of missiles in z coordinates

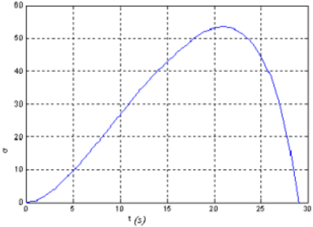

Fig. 5. CADET Method - the mean square error (MSE) of missile in z coordinates

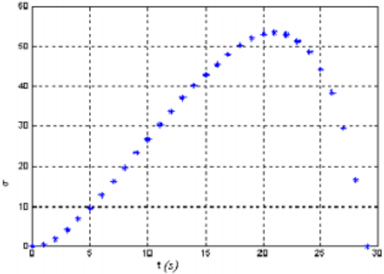

Fig. 6. Monte-Carlo Method - the missile mean square error of z coordinates

Simulations and Analysis

Inferences

1. This two kinds of methods are very close at the result of mean area, this is because the analysis of covariance for linear systems analysis is accurate, and the non-linearity of this example is not serious,the effect is not big.
2. In the mean-variance, although the results were different, they were very close. Therefore, the computed results of CADET also reflect the performance of the system.
3. The CADET is based on the equations of motion, so CADET method has the same ability of shown the effect of navigation ratio and other parameters to hit probability as Monte-Carlo method. Thus CADET method provides a way for the actual use of high efficiency and scientific accurate method of calculating the hit probability.