

AI1103 : Assignment 3

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Download all latex codes from

<https://github.com/Santosh-Dhaladhuli2003/AI1103/blob/main/Assignment%203/Assignment%203.tex>

1 GATE MA 2005 QUESTION No. 25

Let A_1, A_2, \dots, A_n be n independent events in which the Probability of occurrence of the event A_i is given by $P(A_i) = 1 - \frac{1}{\alpha^i}$, $\alpha > 1$, $i = 1, 2, 3, \dots, n$. Then the probability that atleast one of the events occurs is

- (a) $1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}}$ (b) $\frac{1}{\alpha^{\frac{n(n+1)}{2}}}$
 (c) $\frac{1}{\alpha^n}$ (d) $1 - \frac{1}{\alpha^n}$

2 SOLUTION

Let $A_1 + A_2 + A_3 + \dots + A_n = S$,

$\Pr(S)$ = Probability of atleast one event occurring

De morgan's law states that $(A + B)' = A' B'$

$$\implies \Pr(S) = 1 - \Pr(S') \quad (1.1)$$

$$1 - \Pr(S') = 1 - \Pr(A'_1 A'_2 A'_3 \dots A'_n) \quad (1.2)$$

Since, A_1, A_2, \dots, A_n are independent. Complements of A_1, A_2, \dots, A_n are also independent.

\implies

$$\Pr(A'_1 A'_2 A'_3 \dots A'_n) = \prod_{i=1}^n \Pr(A'_i) \quad (2.1)$$

$$\Pr(A_i) = 1 - \frac{1}{\alpha^i} \implies \Pr(A'_i) = \frac{1}{\alpha^i} \quad (2.2)$$

$$\text{Let, } x = \sum_{i=1}^n i \implies x = \frac{n(n+1)}{2} \quad (2.3)$$

from equations (1.1), (1.2), (2.1) and (2.2)

$$\implies \Pr(S) = 1 - \frac{1}{\alpha^x} = 1 - \frac{1}{\alpha^{\frac{n(n+1)}{2}}} \quad (2.4)$$

\therefore The correct option is (a)