AI1103 : Assignment 7

Santosh Dhaladhuli MS20BTECH11007

Download all latex codes from

https://github.com/Santosh−Dhaladhuli2003/AI1103/blob/main/Assignment%207/Assignment%207.tex

1 CSIR - UGC 2014 Dec Q.103

Suppose X is a Random Variable such that E(X) = 0, E(X^2) = 2 and E(X^4)=4.Then

1) E(X^3)=0
2) $\Pr(X \geq 0)$= $\frac{1}{2}$
3) X ∼ N(0,2)
4) X is bounded with Probability 1.

2 Solution

Let X be a Random variable.
Compute Variance of X^2

$$\begin{aligned} Var(X^2) &= E(X^4) - (E(X^2))^2 \\ &= 4 - 2^2 \\ &= 0 \\ \implies Var(X^2) &= 0 \end{aligned} \tag{1}$$

∴ X is a random variable such that X^2 is constant.
Given E(X^2) = 2,

$$\begin{aligned} E(X^2) &= \Sigma X^2 \Pr(X) \\ &= X^2 \Sigma \Pr(X) \\ &= X^2 (\because \Sigma \Pr(X) = 1) \\ X^2 &= 2 \\ \implies X &= \pm\sqrt{2} \end{aligned} \tag{2}$$

Given E(X) = 0,

$$E(X) = \Sigma X \Pr(X) = 0$$

$$\sqrt{2}\Pr\left(X = \sqrt{2}\right) - \sqrt{2}\Pr\left(X = -\sqrt{2}\right) = 0$$

$$\implies \Pr\left(X = \sqrt{2}\right) = \Pr\left(X = -\sqrt{2}\right) \tag{3}$$

Also, Sum of Probabilities is 1,

$$\implies \Pr\left(X = \sqrt{2}\right) + \Pr\left(X = -\sqrt{2}\right) = 1$$

$$\implies \Pr\left(X = \sqrt{2}\right) = \frac{1}{2} \tag{4}$$

$$\implies \Pr\left(X = -\sqrt{2}\right) = \frac{1}{2} \tag{5}$$

Option 1 says E(X^3) = 0,

$$\begin{aligned} E(X^3) &= \Sigma X^3 \Pr(X) \\ &= X^2.\Sigma X \Pr(X) \\ &= X^2 E(X) \\ \implies E(X^3) &= 0 \end{aligned}$$

$\boxed{\textbf{Option 1} \text{ is a } \textbf{correct} \text{ answer}}$

Option 2 says $\Pr(X \geq 0) = \frac{1}{2}$,

$$\Pr(X \geq 0) = \Pr\left(X = \sqrt{2}\right) = \frac{1}{2}$$

$$\implies \Pr(X \geq 0) = \frac{1}{2}$$

$\boxed{\textbf{Option 2} \text{ is a } \textbf{correct} \text{ answer}}$

Option 3 says X ∼ N(0,2),
Let μ be the mean of X

$$\mu = E(X)$$

$$\implies \mu = 0 \tag{6}$$

$$\begin{aligned} \sigma^2 = Var(X) &= E(X^2) - (E(X))^2 \\ &= 2 - (0)^2 \end{aligned}$$

$$\implies Var(X) = 2 \implies N(\mu, \sigma^2) = N(0, 2) \tag{7}$$

$$\implies X \sim N(0, 2)$$

$\boxed{\textbf{Option 3} \text{ is a } \textbf{correct} \text{ answer}}$

Option 4 says X is bounded with probability 1,
Equations (4) and (5) show that $X \in (-\sqrt{2}, \sqrt{2})$
with Probability 1.

$\boxed{\textbf{Option 4} \text{ is a } \textbf{correct} \text{ answer}}$