APL(EE2703): The Laplace Transform(Assignment 6B)

Santosh G (EE19B055)

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1 Aim of the Assignment:

- Analyse LTI systems using Laplace Tranformations
- Use SciPy Signals toolkit in Python for the above
- Plot various graphs and draw conclusions from them.

2 Introduction

Signals Toolkit of the Scientific Python(SciPy) is very useful to anayse signals, Linear Time Invariant Systems, Circuits and multiple other things, in this assignment we shall use the toolbox to do the the laplace transofrmation and solve the questions. Laplace Transformation changes equation etc from time domain to laplace domain simplyfing the questions that are to be solved and analysis of the same becomes easier.

3 Questions

3.1 Questions 1, 2 : Varying Decay constant

We use the Laplace transform to solve a simple spring system. The system is characterized by the given differential equation.

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

when considered in the laplace domain, the equation transforms into

$$X(s) = \frac{F(s)}{s^2 + 2.25} \tag{2}$$

The F(s) is of the input signal f(t) which is of the form "$\cos(\omega t)\exp(-at)u(t)$", where a is the decay factor and ω is the frequency of the cosine.

The Laplace Transform of the such input signal f(t) is given by

$$F(s) = \frac{s + a}{(s + a)^2 + \omega^2}$$

These polynomials are defined using "poly1d" which are then multiplied to get the output laplace transform. Finally we take the Inverse Laplace Transform of the function using "sp.impulse" to get the time domain sequences and those are plotted. We do this for the natural frequency of the system ω=1.5, and decay values of "a" as 0.5 and 0.05.

The following plots are obtained after solving the above.

Plots:

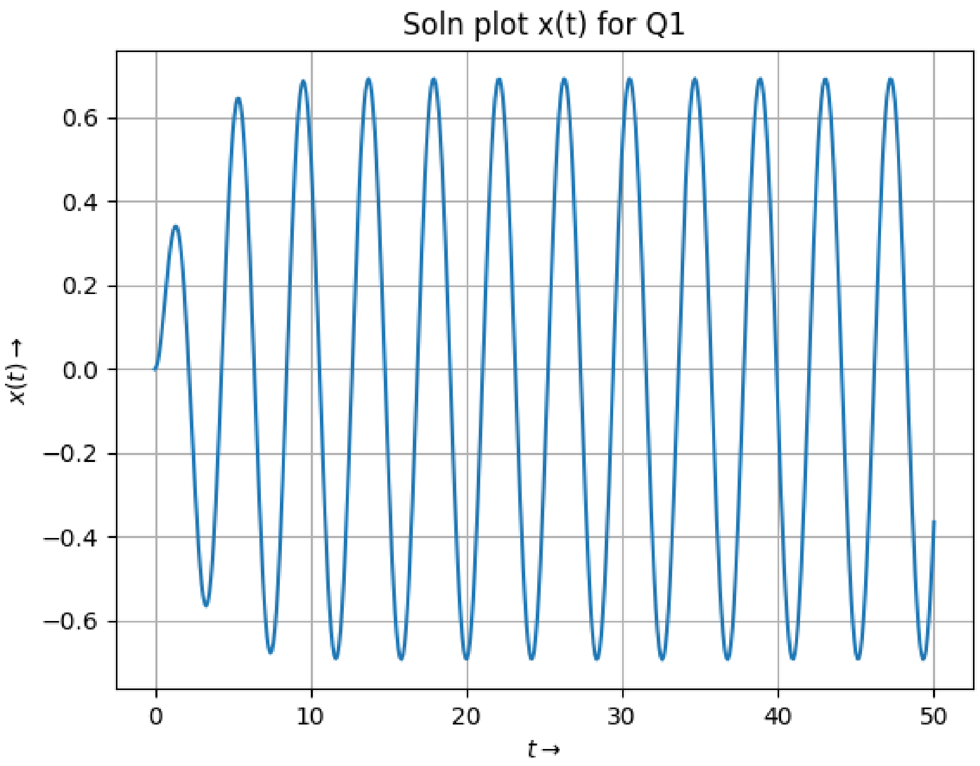

Figure 1: x(t) with decay constant "a" as 0.5

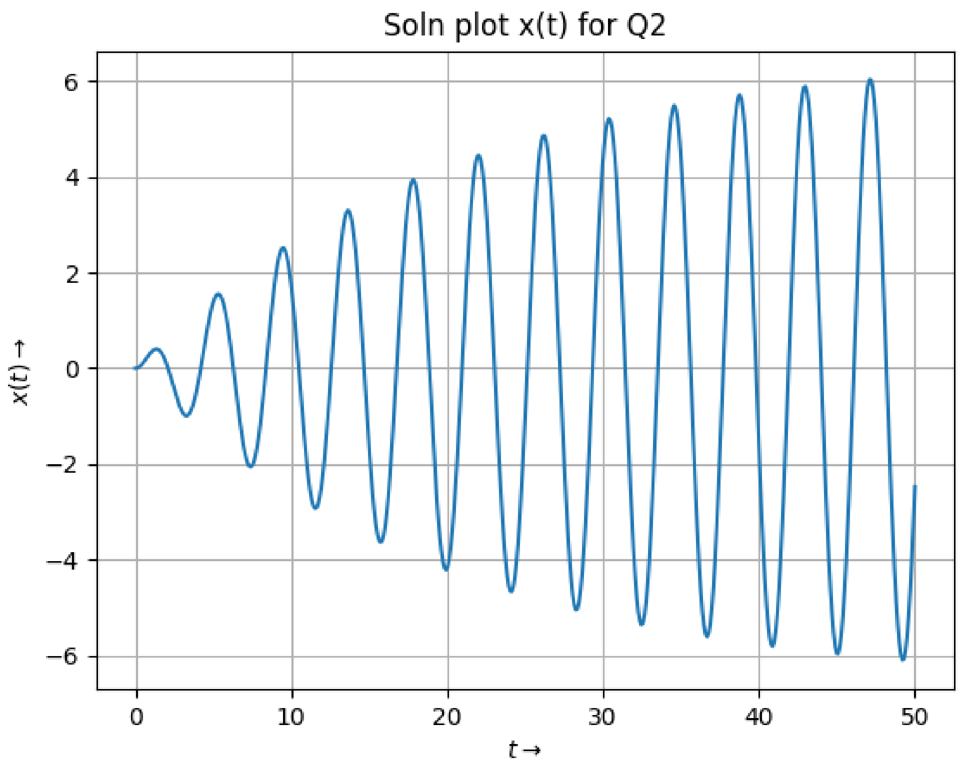

Figure 2: x(t) with decay constant "a" as 0.05

Following observations are made:

- x(t) has a higher amplitude when f(t) has lower decay contstant
- x(t) takes more time to saturate in case of a lower decay constant, as the resistance offered is less
- Amplitude of x(t) converges to a fixed value in both the cases

3.2 Question 3 : Varying Input Frequency

Let the Transfer function H(s) be of the for as shown below:

$$H(s) = \frac{X(s)}{F(s)} = \frac{((s+a)^2 + \omega^2)}{(s^2 + 2.25)(s+a)} \tag{3}$$

where "a" varies from 1.4 to 1.6. The following plot is obtained after considering the above frequencies.

Plots:

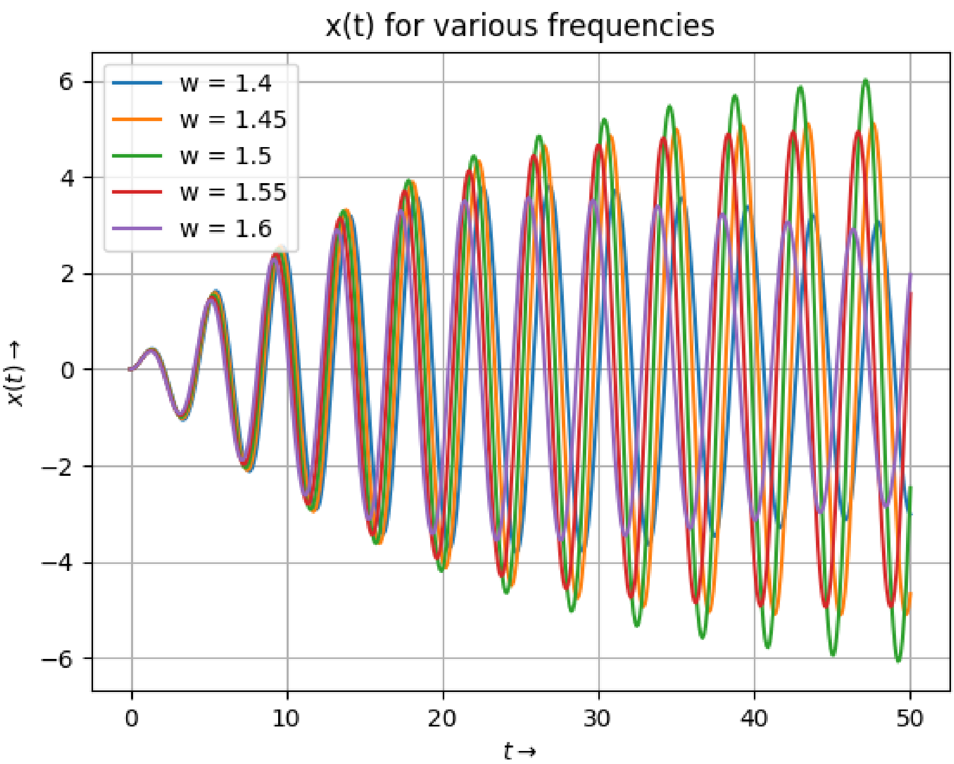

Figure 3: x(t) with varying input frequencies

By virtue of resonance, when input frequency matches the natural frequency of the system the particular output has the highest amplitude among the other cases.

3.3 Question 4 : Coupled Mass system

In this problem we have the two following differential equations and two variables to solve for:

$$\ddot{x} + (x - y) = 0$$

$$\ddot{y} + 2(y - x) = 0$$

On trying to solve the equations we get a fourth order differential equation in terms of x. Simplifying by assuming the initial condition x(0) = 1; and substituting to find the equation we would get the following results:

$$X(s) = \frac{s^2+2}{s^3+3s}$$

$$Y(s) = \frac{2}{s^3+3s}$$

We can hence find the time domain series by taking the Inverse Laplace Transform of the above. By solving the above we obatin the following results which are plotted as shown.

Plots:

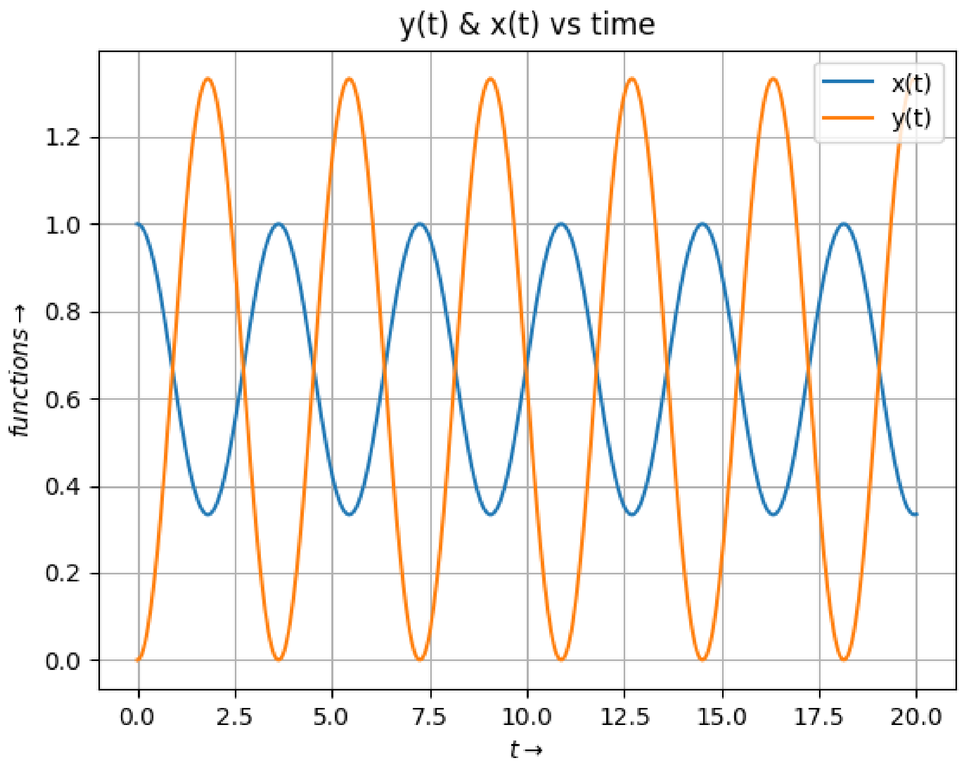

Figure 4: Plots of x(t) and y(t) vs time

- The functions are opposite in phase(i.e have a phase difference of 180 degrees)
- This model can be compared to coupled spring mass system.

3.4 Questions 5, 6 : LCR Filter

On finding the transfer function in the given circuit, we find the following result:

$$\frac{V_o(s)}{V_i(s)} = \mathcal{H}(s) = \frac{10^6}{s^2+100s+10^6} \tag{4}$$

and the input is of the form

$$x(t) = \cos(10^3 t) + \cos(10^6 t)$$

It is clear that the input is a superposition of two sinusoids with large variance in frequencies.

Firstly the bode plots are plotted, in which magnitude and phase are shown in the following:

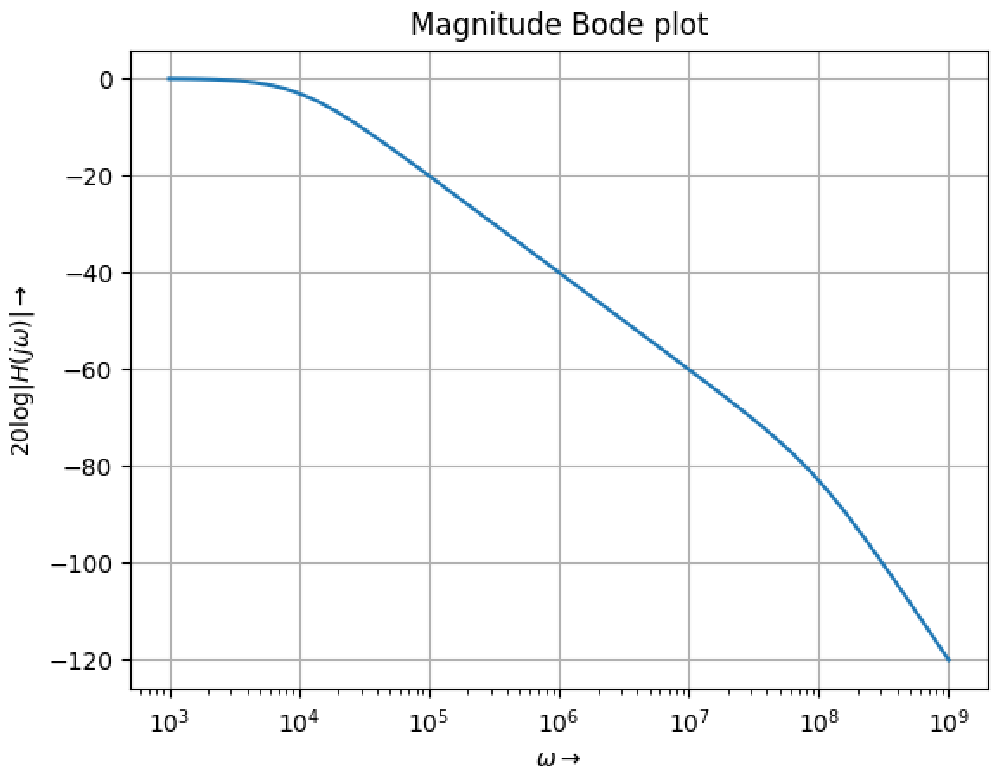

Figure 5: Magnitude Bode Plot of the Transfer Function

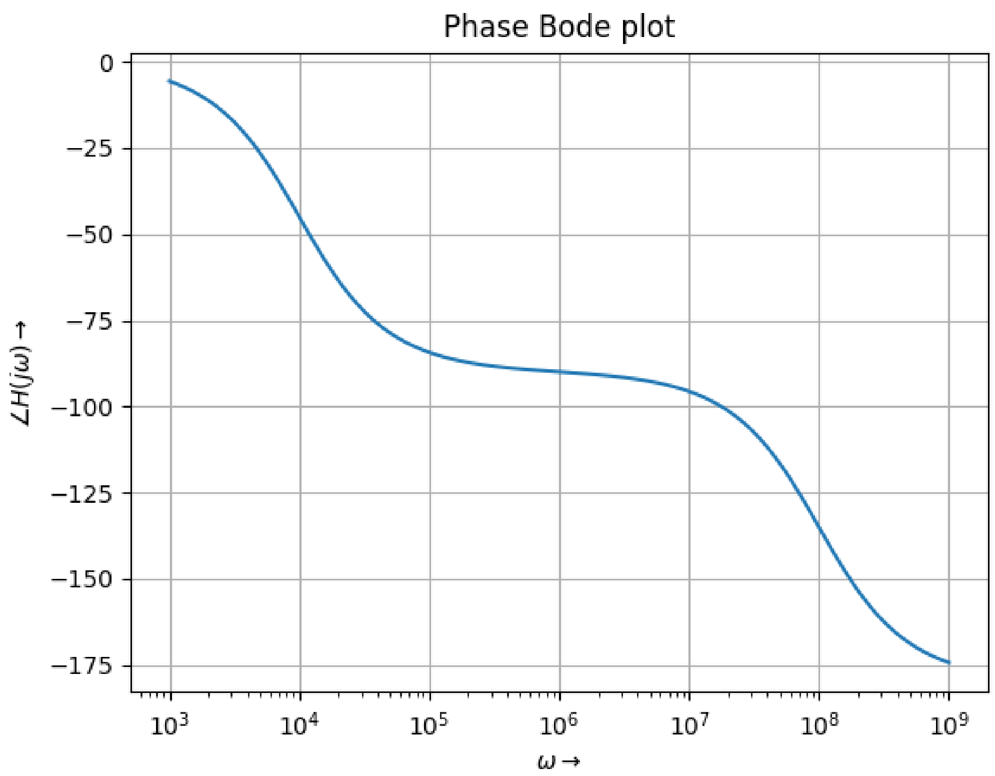

Figure 6: Phase Bode Plot of the Transfer Function

Now the ouputs are plotted over various time intervals. One plot shall be plotted in the interval 0 to 30μs(i.e short term response). Second plot shall be plotted in the interval 0 to 25 msec(i.e long term response). As the frequencies in the order 10^3 and 10^6 , considering intervals of microseconds and milliseconds would be useful to analyse the circuit and the transfer function.

The plot of output voltage in long time interval is as shown below:

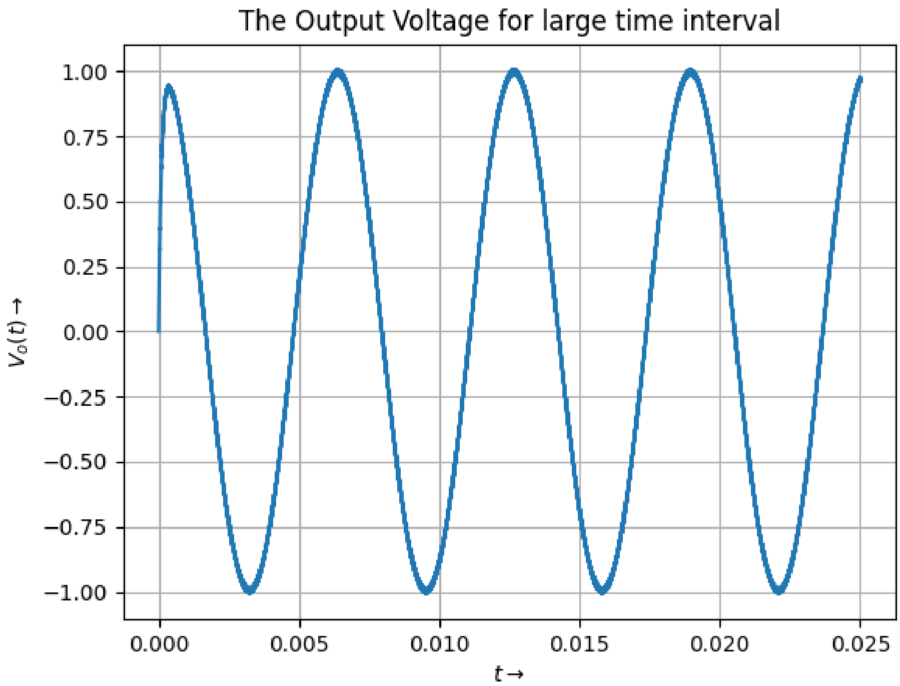

Figure 7: Output voltage plot over long interval

The plot of output voltage in short time interval is as shown below:

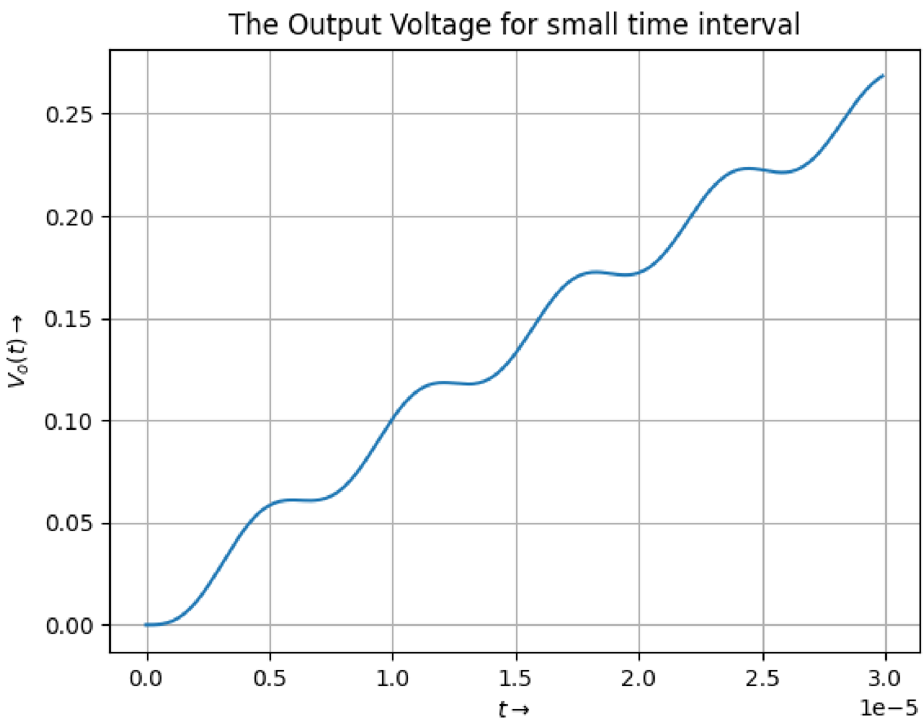

Figure 8: Output voltage plot over short interval

- As the magnitude bode plot is monotonically decreasing as frequency increases, it is a low pass filter and we can see two poles from the phase plot, hence the transfer is a 2nd Order Low Pass filter
- In the short interval plot, the high frequency term(10^6) can be seen as ripple terms, due to its very high frequency it fails to appear in the long interval plot.
- The low frequency component(10^3) passes almost as it is, whereas the high frequency component(10^6) is attenuated and fails to appear significantly in the final plot, making it clear, that this system is a low pass filter and also because the lower frequency lies within the 3dB bandwidth of the system.

4 Conclusion

- We have analyzed Laplace Transformation using Signals Toolbox of Python, which has quite a many inbuilt functions to make the processing much easier, to sketch Bode Plots, find Impulse response, Compute ILTs etc
- RLC Circuit can be used to realise a Low Pass filter, by considering the voltage across the capacitor