

APL(EE2703): Laplace Equation(Assignment 5)

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1 Aim of the Assignment:

- To find the variation of potential at each point in the given problem statement.
- Plot the potential variation, errors and the current flow

This shall be achieved by assigning initial values and iterating continuously to reduce the error.

2 Introduction

The variation in potential can be represented in the following equation:

$$\nabla^2 \phi = \frac{1}{\sigma} \frac{\partial \rho}{\partial t} \quad (1)$$

as we are dealing with DC, the RHS would become Zero. Hence the equation would finally be:

$$\nabla^2 \phi = 0 \quad (2)$$

3 Solving

- We are trying to solve for a plate in 2 Dimension hence the above differential equation when considered in 2 dimensions, say x and y, transforms as shown below which can be expanded based of the definitions

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \quad (3)$$

$$\frac{\partial \phi}{\partial x}(x_i, y_j) = \frac{\phi(x_{i+1/2}, y_j) - \phi(x_{i-1/2}, y_j)}{\Delta x} \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2}(x_i, y_j) = \frac{\phi(x_{i+1}, y_j) - 2\phi(x_i, y_j) + \phi(x_{i-1}, y_j)}{(\Delta x)^2} \quad (5)$$

- Using above equations we get

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4} \quad (6)$$

- It is clear that the potential at point can be approximated to the average of potentials at points in the proximity. Hence we start of with values, zero in this condition and keep iterating till the error is less than the tolerance, which happens as the solutions converge. $Error_k$ denotes the maximum change in elements of ϕ where 'k' is the number of iteration, the error is supposed to be less than some tolerance which is taken as 10^{-8}).
- With the given info of potentials and location of ground we calculate and plot the current flow direction, magnitude etc.

4 Questions

4.1 Contour Plot

Plotting the contour plot of the potential.

The following is the contour plot of the potentials:

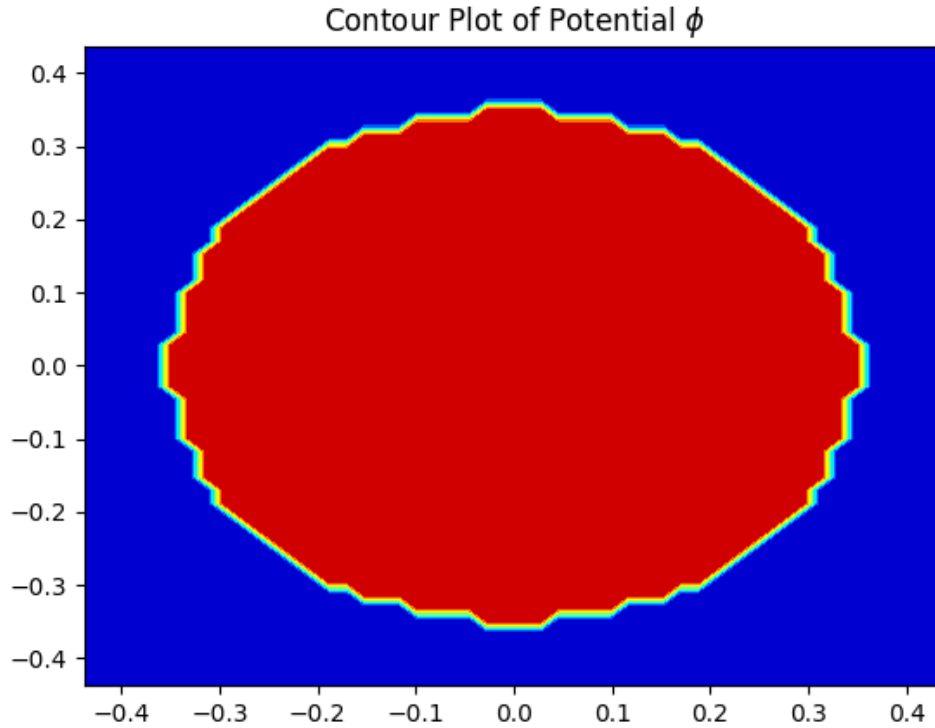


Figure 1: Contour plot of Potential (Plot for Q1)

- The smoothness increases with increase in N_x and N_y as there are more points to smoothen the potential gradient.

4.2 Iterations and Estimations

- To perform iterations keeping boundary conditions in check
- Boundary Conditions: Due to the absence of electrodes on three sides the gradient of potential should be tangential and the normal component should be zero as the current doesn't have any path to flow there after.

The iterations are performed and the error is calculated. The semilog and loglog plots are plotted as shown below:

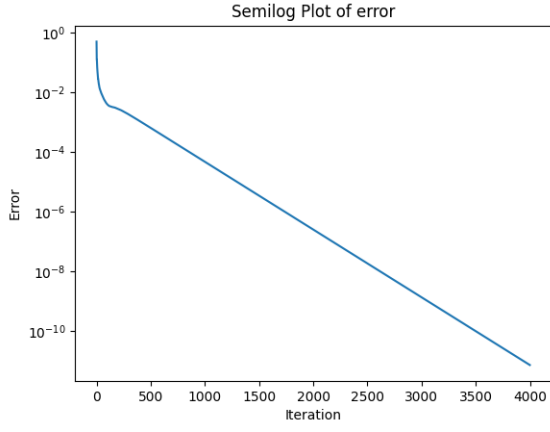


Figure 2: Semilog Plot of Error vs Iterations (Left)

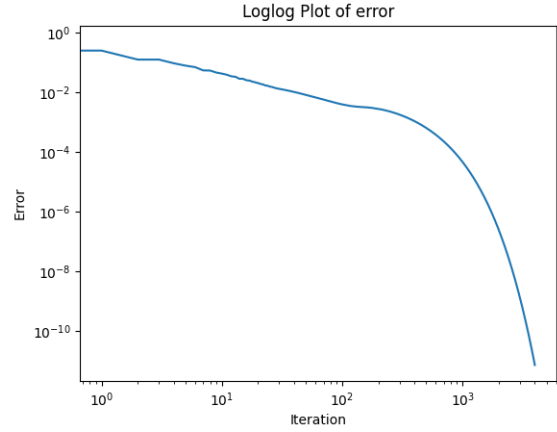


Figure 3: Loglog plot of Error vs Iterations (Right)

For small number of iterations(1-1000) it is clear from the LogLog plot that $\log(\text{error})$ is linearly related to logarithm of number of iterations, hence we can conclude the error in this is exponentially varying i.e Error is of the form a^x where "a" is a constant and "x" corresponds to the number of iterations.

For large number of iterations(2000-6000) the linear relation between the logarithm of error and number of iterations is very clear, hence we can conclude the Error to be of the form Ae^{Bx} , where "A, B" are suitable constants and "x" corresponds to the number of iterations.

$$\log(\text{Error}) = \log(A) + Bx \quad (7)$$

From the above hypothesis, the coefficients are calculated and the deviation(error in estimation) is plotted against number of iterations as shown below.

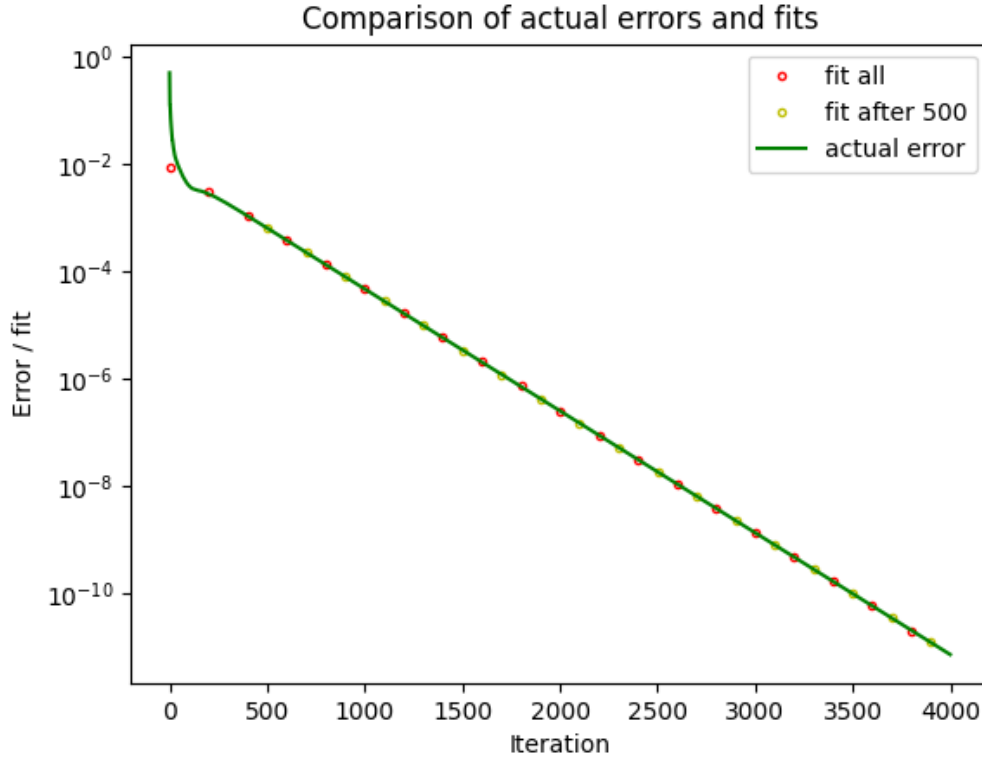


Figure 4: Semilog plot of errors vs Iterations

On solving the following coefficients have been achieved:

- Fit1 : $A = 0.00631664$, $B = -0.00371103$
Fit2 : $A = 0.00629419$, $B = -0.0037099$

- Fit1 is using least squares for all the iterations whereas Fit2 is using least squares fitting for iterations after the first 500 iterations.
- It is clear that coefficients in method 1 are larger compared to those in method 2, the deviation between these Fits would increase with number of iterations and it clear Fit2 gives a better approximation at larger number iterations where as both of them would almost be the same at lower number of iterations. This can also be attributed to the fact that we are ignoring the first set of iterations while calculating Fit2.

4.3 Maximum Error

The total error is being calculate and the magnitudes are summed up as that would result in the maximum error possible, based on this error, the iterations are stopped as there would not be any significant improvement, by still carrying out the iterations.

$$MaxError = \sum_{N+1}^{\infty} error_k \quad (8)$$

which can be approximated as follows:

$$MaxError \approx -\frac{A}{B} \exp(B(N + 0.5)) \quad (9)$$

N corresponds to the iteration Number.

- On calculating, $N = 3179$ is the result obtained when the total is error is around 9.98×10^{-8}
- The last observed change after iteration is 5.25×10^{-10}
- Though the change per iteration is small, the iterations are continuous summing to a significantly larger error and would also increase the time, hence being known as one of the worst possible ways to solve it.

4.4 Surface plot and Contour plot of potential

We need to plot the surface potential and the contour plot of the potentials, Based on the diagrams, necessary hypothesis is to be made.

The diagrams are plotted as shown below:

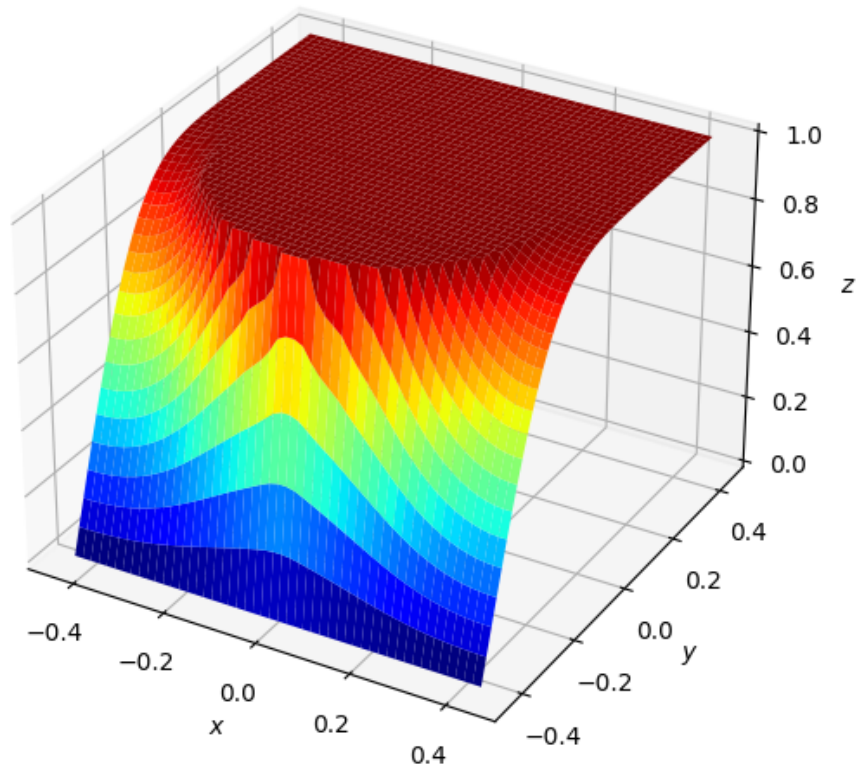


Figure 5: 3-D surface plot of potential

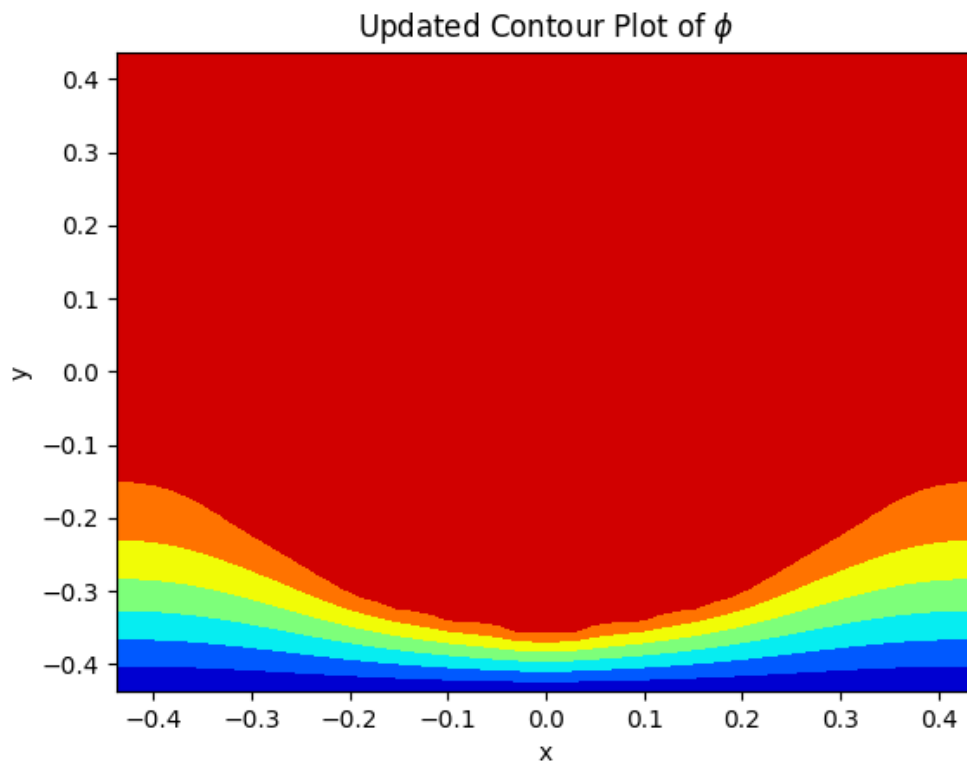


Figure 6: Contour plot of potential

- From Figure 6 it is very clear that there is very little potential on the top and sides of the plate as there is no electrode attached and hence current has no path to flow. As the current cannot flow, the potential wouldn't change and almost remains constant.

- The bottom side of the plate is attached to the ground and clearly provides a path for the current to flow hence there is clear drop in potential.
- As most of the current flows through the bottom part of the plate and flowing of current causes heating, the bottom part would heat up the most and hence would be at the highest temperature throughout the plate.

4.5 Vector plot of Currents

Based on the potential and their gradient calculated we can measure the current, both in terms of magnitude and direction, those obtained results are being plotted in the diagrams below:

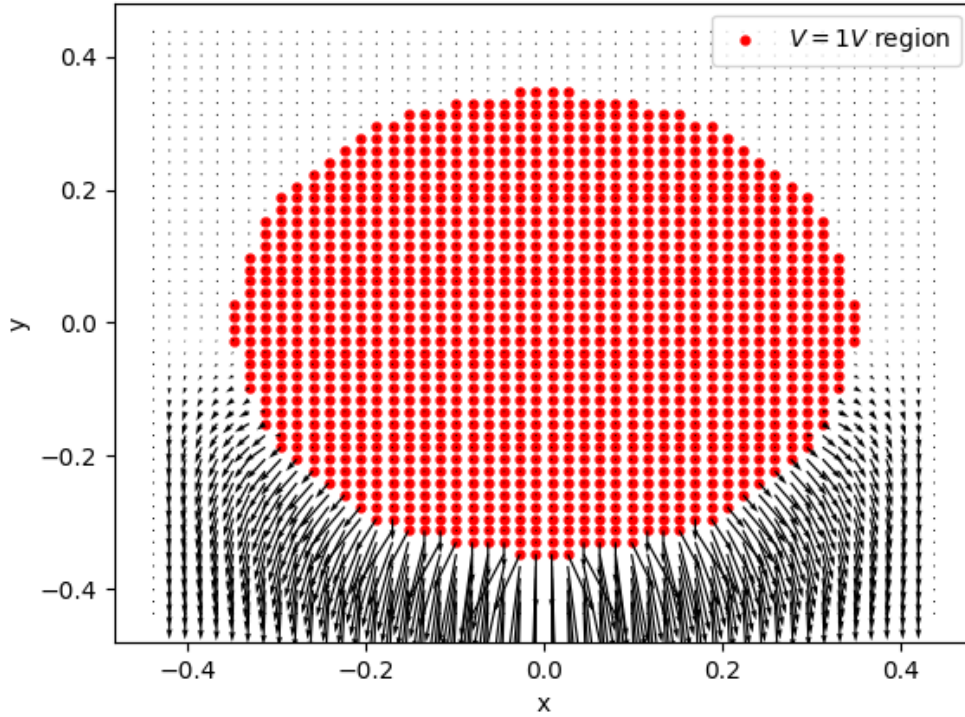


Figure 7: Vector Plot of currents

The red dots corresponds to places where the potential is 1V, i.e corresponds to the source wire and points of plate in close proximity to the wire.

- As discussed previously. most of the current flows through the bottom side of the plate and hence we can see large current density flowing through the bottom side and very little(almost zero) current density on the othe other sides.

5 Conclusion

- Increasing number of iteration while calculations increases the accuracy of the fit by reducing the error, increasing the points considered(N_x and N_y) would also improve the efficiency.
- By increasing the number of iterations would increase the time for calculations, hence there is a trade-off between the time taken and accuracy of the fit
- Most of the current passes through bottom side of the plate as there is complete path for current to flow unlike the other sides of the plate.
- The current flow causes heating of the bottom side and hence resulting in higher temperature on the bottom side of the