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# How to measure response functions in correlated financial markets

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**Abstract** Insert your abstract here. Hoki aaajaja

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## 1 Possible paper names (I will erase this when we choose a name)

The following are possible names for the paper:

- Details that influence the response functions results.
- Influence of the details in the response function measurement.
- Response function measurement in correlated financial markets.
- Response function calculation in correlated financial markets.
- Influence of the methodology in response functions results.

Or we can use another.

## 2 Introduction

Shares are the equal parts in which the capital of a company is divided. A share is an indivisible unit of capital, expressing the ownership relationship betwen a company and a shareholder. The shareholder owns a percentage of the company depending on the amount of shares he has. Shares are issued in two moments: when the company is created and when the company want to raise funds.

The shares can be taken as an invesment, and receive dividends from them, or they can be traded at any time. This second possibility is the want that concerns us.

To trade the stocks exist markets where the buyers and sellers meet. sellers transfer (in exchange for money) the ownership of equities to buyers. This requires these two parties to agree on a price.

In a modern financial market, there is a double continous auction. To find possible buyers and sellers in the market, agents can place different types of instructions (known as orders) to buy or to sell a given number of shares, that can be grouped into two categories: market orders and limit orders.

Market orders will go into market to execute at the best available buy or sell price, they are executed as fast as possible and only after the purchase of the stock is possible to know the exact price [7].

Limit orders allow to set a maximum purchase price for a buy order, or a minimum sale price for a sell order. If the market does not reach the limit price, the order will not be executed [7].

Limit orders often fail to result in an immediate transaction, and are stored in a queue called the limit order book. An order book is an electronic list of buy and sell orders for a specific security or financial instrument organized by price level. An order book lists the number of shares being bid or offered at each price point. It also identifies the market participants behind the buy and sell orders, although some choose to remain anonymous. The order book is visible for all traders, its main purpose is to ensure that all traders have the information about what is offered on the market.

Buy limit orders are called “bids”, and sell limit orders are called “asks”. At any given time there is a best (lowest) offer to sell with price $a(t)$, and a best (highest) bid to buy with price $b(t)$ [3]. These are also called the inside quotes or the best prices. The price gap between them is called the spread $s(t) = a(t) - b(t)$ [7,4,3].

The average of the best ask and the best bid is the midpoint price, which is defined as [4,7,3]

$$m(t) = \frac{a(t) + b(t)}{2} \tag{1}$$

As the midpoint price depends on the quotes, it changes if the quotes change. The midpoint price grows if the best ask or the best bid grow. This happen if someone buys and consumes all the volume of the sell limit order with the price of the best ask, or someone sets a buy limit order with a bigger price than the previous best bid, or there is a cancellation of the best ask.

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On the other hand, the midpoint price decreases if the best ask or the best bid decrease. This happen if someone sells and consumes all the volume of the buy limit order with the price of the best bid, or someone sets a sell limit order with a lower price than the previous best bid, or there is a cancellation of the best bid.

The midpoint price will not change if there is no activity in the market.

Price changes are typically characterized as returns. If one denotes $S(t)$ the price of an asset at time $t$, the return $r(t)$, at time $t$ and time lag $\tau$ is simply the relative variation of the price from $t$ to $t+\tau$ [3,5],

$$r^{general}(t,\tau) = \frac{S(t+\tau) - S(t)}{S(t)} \tag{2}$$

It is also common to define the returns as [6,11,7,3,8,5,1,9]

$$r^{log}(t,\tau) = \ln S(t+\tau) - \ln S(t) = \ln \frac{S(t+\tau)}{S(t)} \tag{3}$$

Equation 2 and Eq. 3 coincide if $\tau$ is small enough [3,5].

At longer timescales, midpoint prices and transaction prices rarely differ by more than half the spread. The midpoint price is more convenient to study because it avoids problems associated with the tendency of transaction prices to bounce back and forth between the best bid and ask [7].

We define the returns via the midpoint price as

$$r(t,\tau) = \frac{m(t+\tau) - m(t)}{m(t)} \tag{4}$$

The distribution of returns is strongly non-Gaussian and its shape continuously depends on the return period $\tau$. Small $\tau$ values have fat tails return distributions [3].

Then we can expect three kind of values of the returns. The returns are positive values, when the midpoint price $m(t+\tau) > m(t)$, hence, there is a buy in the market or there is a cancellation of the best ask or an addition in the best bid during the time lag $\tau$. The returns are negative values, when the midpoint price $m(t+\tau) < m(t)$, thus, there is a sell in the market, or there is a cancellation of the best bid or an addition in the best ask during the time lag $\tau$. The returns are zero when there is no activity during the time lag $\tau$.

The trade signs are defined for general cases as

$$\varepsilon(t) = \text{sign}(S(t) - m(t-\delta)) \tag{5}$$

where $\delta$ is a positive time increment. Hence we have

$$\varepsilon(t) = \begin{cases} +1, \text{ If } S(t) \text{ is higher than the last } m(t) \\ -1, \text{ If } S(t) \text{ is lower than the last } m(t) \end{cases} \tag{6}$$

$\varepsilon(t) = +1$ indicates that the trade was triggered by a market order to buy and a trade triggered by a market order to sell yields $\varepsilon(t) = -1$ [10,4,3,12,9].

It is well-known that the series of the trade signs on a given stock exhibit large autocorrelation. A very plausible explanation of this phenomenon relies on the execution strategies of some major brokers on a given markets. These brokers have large transaction to execute on the account of some clients. In order to avoid market making move because of an inconsiderable large order, they tend to split large orders into small ones [5].

The response function is used to study the mutual dependence between stocks. In [4,2], Bouchaud et al. use the response function that only depends on the time lag $\tau$

$$R(\tau) = \langle (S_{n+\tau} - S_n) \cdot \varepsilon_n \rangle_{trades} \tag{7}$$

Where $\varepsilon_n$ is the sign of the $n^{th}$ trade and the price $S_n$ is defined as the midpoint just before the $n^{th}$ trade ($S_n \equiv m_{n^-}$). The quantity $R(\tau)$ measures how much, on average, the price moves up (down) at time $\tau$ conditioned to a buy (sell) order at time zero.

In a later work [14], S. Wang et al. use the logarithmic return for stock $i$ and time lag $\tau$, defined via the midpoint price $m_i(t)$. The cross-response function is then defined as

$$R_{ij}(\tau) = \langle r_i(t-1,\tau) \cdot \varepsilon_j(t) \rangle_t \tag{8}$$

Finally, in [13], S. Wang et al. define the response function as

$$R_{ij} = \left\langle \left( \ln m_i^{(f)}(t_j) - \ln m_i^{(p)}(t_j) \right) \cdot \varepsilon_j(t_j) \right\rangle_{t_j} \tag{9}$$

For the price change of stock $i$ caused by a trade of stock $j$.

Here, $m_i^{(p)}(t_j)$ is the midpoint price of stock $i$ previous to the trade of stock $j$ at its event time $t_j$ and $m_i^{(f)}(t_j)$ is the midpoint price of stock $i$ following that trade.

The difference between the definition in [14] and in [13], is that [14] measures how a buy or sell order at time $t$ influences on average the price at a later time $t+\tau$. The physical time scale was chosen since the trades in different stocks are not synchronous (TAQ data). In [13], it was used a response function on a trade time scale (Totalview data), as the interest is to analyze the immediate responses. In [13] the time lag $\tau$ is restricted to one, such that the price response quantifies the price impact of a single trade.

The paper is organized as follows: in Sect. 3 we present our data set of stocks and describe the physical and trade time. We then analyze the definition of the response functions in Sect. 4, and compute them for several stocks and pairs of stocks. In Sect. 5 we show how the relative position between trade signs and returns has a huge impact in the results of the computation of the response functions. Finally, in Sect. 6 we explain in detail how the time lag $\tau$ behaves in the response functions. Our conclusions follows in Sect. 7.

## 3 Data and time definition

In this study, we have analyzed trades and quotes (TAQ) data from the NASDAQ Stock Market in the year 2008.

## 4 Response functions

## 5 Time shift

## 6 Short- and long-response

## 7 Conclusion

## 8 Author contribution statement

All the authors were involved in the preparation of the manuscript. All the authors have read and approved the final manuscript.

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