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Price response functions and spread impact in correlated financial markets

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Abstract Recent research about the response of stock prices to trading activity revealed long lasting effects, even across stocks of different companies. These results implies non-markovian effects in price formation and reveal the consequences of trading many stocks at the same time, from trading costs to price correlations. How the price response is measured depends on the data set and research focus. However, it is still unclear, how the details of the price response definition modify the results. Here, we evaluate different price response implementations for the Trades and Quotes (TAQ) data set from the NASDAQ stock market and find that the results are qualitatevily the same for two different definitions of time scale, but the response can vary by up to a factor of two. Further, we confirm the dominating contribution of immediate price response directly after a trade, as we find that delayed responses are suppresed. Finally, we test the impact of the spread in the price response, spotting that large spreads have stronger impact.

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1 Introduction

Financial markets use order books to list the number of shares being bid or offered at each price point. An order book is an electronic list of buy and sell orders for a specific security or financial instrument organized by price level. Agents can place different types of instructions (orders) to buy or to sell a given number of shares.

In general, the dynamics of the prices follow a pure random walk. There are two extreme models that can describe this behavior: the Efficient Market Hypothesis (EMH) and the Zero Intelligence Trading (ZIT). The EMH declares that all available information is included in prices and price changes can only be the result of unanticipated news, which by definition are totally unpredictable [5,7,22,32]. On the other hand, the ZIT assumes that agents instead of being fully rational, have "zero intelligence" and randomly buy or sell. It is supposed that their actions are interpreted by other agents as potentially containing some information [5,7,32,40]. In both cases the outcome is the same, the prices follow a random walk. In real life, they will not behave as one of these extreme scenarios, but somewhere in between [7,32].

There are diverse studies focused on the price response [3,4,5,7,13,15,16,19,21,22,26,28,29,37,38,39,40], but they concentrate on a general definition and discuss the results without going deep in the specific details of the price response measurements and how these aspects of the price response definition affect the results.

Regarding price self-response functions [4,5,7], Bouchaud et al. found an increase to a maximum followed by a decrease as the time lag grows. In [16], the researchers found that larger sized transactions have a larger absolute impact than smaller sized transactions but a much smaller relative impact. In [29], the results show that the impact of small trades on the price is, in relative terms, much larger than that of large trades and the impact of trading on the price is quasi-permanent.

For price cross-responses functions, [3] found that the diagonal terms are on average larger than the off-diagonal ones by a factor ~ 5. The response at positive times is roughly constant, consistently with the hypothesis of a statistically efficient price. In other words, the current sign does not predict future returns. In [40] the results show the trends in the cross-responses does not depend on whether or not the stock pairs are in the same economic sector or extend over two sectors.

In this paper[1], we want to discuss, based on a series of detailed empirical results obtained on trade by trade data, that the variation in the details of the parameters used in the price response definition modify the characteristics of the results. Aspects like time scale, time shift, time lag and spread used in the price response calculation have an influence on the outcomes.

Here, we delve into the key details needed to compute the price response functions, and explore their corresponding roles. We perform a empirical study in different

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[1] To facilitate the reproduction of our results, the source code for the data analysis is available at ...

time scales using financial data and find that using different implementations of the price response, the results are qualitatively the same, but the response can vary up to a factor of two. We show that the order between the trade signs and the returns have a key importance in the price response signal and suggest an interval where the time shift have to be set. We split the time lag to understand the contribution of the immediate returns and the late returns. The price response is highly influenced by the instantaneous returns and as the time lag grows, the influence starts to decrease. We shed light on the spread impact in the response functions for single stocks. We check that when the spread is large, the price response tend to be large.

A direct comparison between the trade time scale and the physical time scale is not possible. To compare them directly we need to assume whether the midpoint price or the trade signs are in the same scale. In our case, in Sect. 4, we assume the midpoint prices in the trade time scale to be the same as the midpoint prices in physical time scale. Therefore, we have the time lag for both computations in seconds. This approximation allow us to directly compare both scales to have an idea of the difference and similarities they have. In the other sections, as we are not directly comparing the time scales, the corresponding quantities of each time scale are not mixed. Thus physical time scale is measured in seconds and trade time scale is measured in trades.

The paper is organized as follows: in Sect. 2 we present our data set of stocks. We then analyze the definition of the price response functions and describe the physical and trade time in Sect. 3. We implement different price responses for several stocks and pairs of stocks in Sect. 4. In Sect. 5 we show how the relative position between trade signs and returns has a huge influence in the results of the computation of the response functions. In Sect. 6 we explain in detail how the time lag τ behaves in the response functions. Finally, in Sect. 7 we analyze the spread impact in the price response functions. Our conclusions follows in Sect. 8.

2 Data set

In a modern financial market, there is a double continuous auction. To find possible buyers and sellers in the market, agents can place different types of orders to buy or to sell a given number of shares, that can be grouped into two categories: market orders and limit orders.

Market orders will go into market to execute at the best available buy or sell price. Limit orders allow to set a maximum purchase price for a buy order, or a minimum sale price for a sell order. If the market does not reach the limit price, the order will not be executed [13,14,20,32].

Limit orders often fail to result in an immediate transaction, and are stored in a queue called the limit order book [8,14,20,29]. An order book is an electronic list of buy and sell orders for a specific security or financial instrument organized by price level. An order book lists the number of shares being bid or offered at each price point. It also identifies the market participants behind the buy and sell orders, although some choose to remain anonymous. The order book is visible for all traders and its main purpose is to ensure that all traders have the information about what is offered on the market. The order book is the ultimate microscopic level of description of financial markets.

Buy limit orders are called “bids”, and sell limit orders are called “asks”. At any given time there is a best (lowest) offer to sell with price $a(t)$, and a best (highest) bid to buy with price $b(t)$ [5,9,11,29,32]. The price gap between them is called the spread $s(t) = a(t) - b(t)$ [5,6,7,9,13,32]. Spreads are significantly positively related to price and significantly negatively related to trading volume. Companies with more liquidity tend to have lower spreads [1,2,9,17].

In this study, we analyzed trades and quotes (TAQ) data from the NASDAQ stock market. We selected NASDAQ because it is an electronic exchange where stocks are traded through an automated network of computers instead of a trading floor, which makes trading more efficient, fast and accurate. Furthermore, NASDAQ is the second largest stock exchange based on market capitalization in the world.

In the TAQ data set, there are two data files for each stock. One gives the list of all successive quotes. Thus, we have the best bid price, best ask price, available volume and the time stamp accurate to the second. The other data file is the list of all successive trades, with the traded price, traded volume and time stamp accurate to the second. Despite the one second accuracy of the time stamps, in both files more than one quote or trade may be recorded in the same second.

Due to the the time stamp accuracy, it is not possible to match each trade with the directly preceding quote. Hence, we cannot determine the trade sign by comparing the traded price and the preceding midpoint price [40]. In this case we need to do a preprocessing of the data to relate the midpoint prices with the trade signs in trade time scale and in physical time scale.

To analyze the response functions across different liquid stocks in Sects. 4, 5 and 6, we select the six companies with the largest average market capitalization (AMC) (Alphabet Inc., Mastercard Inc., CME Group Inc., Goldman Sachs Group Inc., Transocean Ltd. and Apache Corp.) in three economic sectors (information technology, financials and energy) of the S&P index in 2008. Table 1 shows the companies analyzed with their corresponding symbol and sector, and three average values for a year.

To analyze the spread impact in response functions (Sect. 7), we select 524 stocks in the NASDAQ stock market for the year 2008. The selected stocks are listed in Appendix A.

In order to avoid overnight effects and any artifact due to the opening and closing of the market, we systematically discarded the first ten and the last ten minutes of trading in a given day [7,13,19,40]. Therefore, we only consider trades of the same day from 9:40:00 to 15:50:00 New York local time. We will refer to this interval of time

Table 1. Analyzed companies.

Company	Symbol	Sector	Quotes[1]	Trades[2]	Spread[3]
Alphabet Inc.	GOOG	Information Technology (IT)	164489	19029	0.40$
Mastercard Inc.	MA	Information Technology (IT)	98909	6977	0.38$
CME Group Inc.	CME	Financials (F)	98188	3032	1.08$
Goldman Sachs Group Inc.	GS	Financials (F)	160470	26227	0.11$
Transocean Ltd.	RIG	Energy (E)	107092	11641	0.12$
Apache Corp.	APA	Energy (E)	103074	8889	0.13$

[1] Average number of quotes from 9:40:00 to 15:50:00 New York time during 2008.
[2] Average number of trades from 9:40:00 to 15:50:00 New York time during 2008.
[3] Average spread from 9:40:00 to 15:50:00 New York time during 2008.

as the "market time". The year period 2008 corresponds to 253 bussiness days.

3 Price response function definitions

In Sect. 3.1 we establish the fundamental quantities used in the price response definitions. In Sect. 3.2 we describe the physical time scale and the trade time scale. We introduce the price response functions used in literature in Sect. 3.3.

3.1 Key concepts

The average of the best ask and the best bid is the midpoint price, which is defined as [5,7,13,29,32]

$$m\left(t\right)=\frac{a\left(t\right)+b\left(t\right)}{2} \tag{1}$$

As the midpoint price depends on the quotes, it changes if the quotes change. The midpoint price grows if the best ask or the best bid grow. On the other hand, the midpoint price decreases if the best ask or the best bid decrease.

Price changes are typically characterized as returns. If one denotes $S\left(t\right)$ the price of an asset at time t, the return $r^{g}\left(t,\tau\right)$, at time t and time lag τ is simply the relative variation of the price from t to $t+\tau$ [5,10,23,24,25,31],

$$r^{g}\left(t,\tau\right)=\frac{S\left(t+\tau\right)-S\left(t\right)}{S\left(t\right)} \tag{2}$$

It is also common to define the returns as [3,5,10,12,13,15,16,19,27,30]

$$r^{l}\left(t,\tau\right)=\ln S\left(t+\tau\right)-\ln S\left(t\right)=\ln\frac{S\left(t+\tau\right)}{S\left(t\right)} \tag{3}$$

Equation 2 and Eq. 3 agree if τ is small enough [5,10].

At longer timescales, midpoint prices and transaction prices rarely differ by more than half the spread. The midpoint price is more convenient to study because it avoids problems associated with the tendency of transaction prices to bounce back and forth between the best bid and ask [13].

We define the returns via the midpoint price as

$$r\left(t,\tau\right)=\frac{m\left(t+\tau\right)-m\left(t\right)}{m\left(t\right)} \tag{4}$$

The distribution of returns is strongly non-Gaussian and its shape continuously depends on the return period τ. Small τ values have fat tails return distributions [5].

The trade signs are defined for general cases as

$$\varepsilon\left(t\right)=\text{sign}\left(S\left(t\right)-m\left(t-\delta\right)\right) \tag{5}$$

where δ is a positive time increment. Hence we have

$$\varepsilon\left(t\right)=\begin{cases}+1, \text{ If } S\left(t\right) \text{ is higher than the last } m\left(t\right)\\ -1, \text{ If } S\left(t\right) \text{ is lower than the last } m\left(t\right)\end{cases} \tag{6}$$

$\varepsilon(t)=+1$ indicates that the trade was triggered by a market order to buy and a trade triggered by a market order to sell yields $\varepsilon(t)=-1$ [5,7,19,26,34].

It is well-known that the series of the trade signs on a given stock exhibit large autocorrelation. A very plausible explanation of this phenomenon relies on the execution strategies of some major brokers on given markets. These brokers have large transactions to execute on the account of some clients. In order to avoid market making move because of an inconsiderable large order, they tend to split large orders into small ones [10].

3.2 Time definition

A key concept in the analysis of the response functions is the time. Due to the nature of the data, they are several options to define it.

In general, the time series are indexed in calendar time (hours, minutes, seconds, milliseconds). Moreover, tick-by-tick data available on financial markets all over the world is time stamped up to the millisecond, but the order of magnitude of the guaranteed precision is much larger, usually one second or a few hundreds of milliseconds [6,10]. In several papers are used different time definitions (calendar time, physical time, event time, trade time, tick time) [10, 18,33]. The TAQ data used in the analysis has the characteristic that the trades and quotes can not be directly related due to the time stamp resolution of only one second [40]. Hence, it is impossible to match each trade with the directly preceding quote. However, using a classification for the trade signs, we can compute trade signs in two scales: trade time scale and physical time scale.

The trade time scale is increased by one unit each time a transaction happens. The advantage of this count is that limit orders far away in the order book do not increase the time by one unit. The main outcome of trade time scale is its "smoothing" of data and the aggregational normality [10].

The physical time scale is increased by one unit each time a second passes. This means that computing the responses in this scale involves sampling [18,40], which has to be done carefully when dealing for example with several stocks with different liquidity. This sampling is made in the trade signs and in the midpoint prices.

Facing the impossibility to relate midpoint prices and trade signs with the TAQ data in trade time scale, we will use the midpoint price of the previous second with all the trade signs of the current second. This will be our definition of trade time scale analysis for the response function analysis.

For physical time scale, as we can sampling, we relate the unique value of midpoint price of a previous second with the unique trade sign value of the current second.

3.2.1 Trade time scale

We use the trade sign classification in trade time scale proposed by S. Wang et al. in [40] and used in [35,36,39] that reads

$$\varepsilon^{t}\left(t,n\right)=\begin{cases}\operatorname{sgn}\left(S\left(t,n\right)-S\left(t,n-1\right)\right), & \text{if}\\ \quad S\left(t,n\right)\neq S\left(t,n-1\right) & \\ \quad \varepsilon\left(t,n-1\right), & \text{otherwise}\end{cases} \tag{7}$$

$\varepsilon^{t}(t,n)=+1$ implies a trade triggered by a market order to buy, and a value $\varepsilon^{t}(t,n)=-1$ indicates a trade triggered by a market order to sell.

In the second case of the classification, if two consecutive trades with the same trading direction did not exhaust all the available volume at the best quote, the trades would have the same price, and in consequence they will have the same trade sign.

With this classification we obtain trade signs for every single trade in the data set. According to [40], the average accuracy of the classification is 85% for the trade time scale.

TAQ time step is one second, and as it is impossible to find the correspondences between trades and midpoint prices values inside a second step, We used the last midpoint price of every second as the representative value of each second. This introduce an apparent shift between trade signs and returns. In fact, we set the last midpoint price from the previous second as the first midpoint price of the current second, as explained in [40].

As we know the second in which the trades were made, we can relate the trade signs and the midpoint prices as shown in Fig. 1. For the trade time scale, they are in general, several midpoint prices in a second. For each second we select the last midpoint price value, and we relate it to the next second trades. In Fig. 1, the last midpoint price (circle) between the second −1 and 0 is related with all the trades (squares and triangles) in the second 0 to 1, and so on. It is worth to note, in the seconds that there are no changes in the quotes, it is used the value of the previous second (vertical line over the physical time interval). Thus, all the seconds in the open market time have a midpoint price value, and in consequence returns values. We assume that as long as they were not changes in the quotes, the midpoint price remain the same as the one of the previous second.

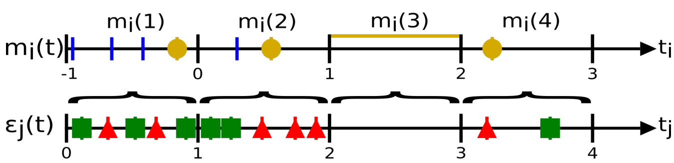

Figure 1. Sketch of data processing for trade time scale. In the midpoint price time line, the vertical lines represent the change in price of the quotes and the circles represent the last price change in a quote in a second. In the trade signs time line, the squares represent the buy market orders and the triangles represent the sell market orders. The midpoint price time line and the trade sign time line are shifted in one second.

The methodology described is an approximation to compute the response in the trade time scale. A drawback in the computation could come from the fact that the return of a given second is composed by the contribution of small returns corresponding to each change in the midpoint price during a second. As we are assuming only one value for the returns in each second, we are supposing all the returns in one second interval to be positive or negative with the same magnitude, which could not be the case. This could increase or decrease the response signal at the end of the computation.

Figure 2 illustrate with one example this point. Suppose one second interval, in which they are three different midpoint prices, and as result, three different returns for this three midpoint price values. Furthermore, consider that the volume of limit orders that have the corresponding midpoint prices are the same in the bid and in the ask (so the returns have the same magnitude). In the case of the top left (top right) sketch, all the changes are due to the rise (decrease) of the midpoint price, that means, consumption of the best ask (bid), so all the contributions of the individual returns in the second are positive (negative), and in consequence, the net return is positive (negative). In the case of the bottom, the changes are due to a combination of increase and decrease of the midpoint price, so in the end, the individual returns sum up to a

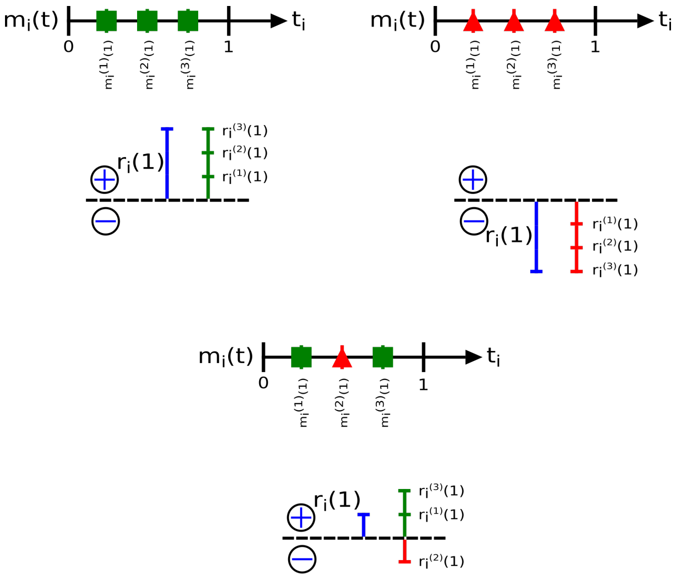

Figure 2. Sketch of the return contributions from every midpoint price change in a second. The squares represent the rise of the price of the midpoint price and the triangles represent the decrease of the price of the midpoint price. We illustrate three cases: (top left) the changes of the midpoint prices and return are due to the rise of the prices, (top right) the changes of the midpoint prices and return are due to the decrease of the prices, and (bottom) the changes of the midpoint prices and return are due to a combination of rise and decrease of the prices. The blue vertical line represents the net return in each case.

net return, which can be positive or negative, depending of the type of midpoint price values in the interval. Thus, in this case, we are assuming in the end that all the returns were positive or negative, what probably was not the case, and in consequence will increase or decrease the real value of the net return.

In all the cases, we choose the last change in the midpoint price in a second interval as described before (Fig. 1). We use this method knowing that the variation in one second of the midpoint price is not large (in average, the last midpoint price of a second differ with the average midpoint of that second in 0.007%), so it can give us representative information about the response functions.

3.2.2 Physical time scale

We use the trade sign definition in physical time scale proposed by S. Wang et al. in [40] and used in [35,39], that depends on the classification in Eq. 7 and reads

$$\varepsilon^p\left(t\right) = \begin{cases} \mathrm{sgn}\left(\sum_{n=1}^{N(t)} \varepsilon^t\left(t,n\right)\right), & \text{If } N\left(t\right) > 0 \\ 0, & \text{If } N\left(t\right) = 0 \end{cases} \tag{8}$$

Where $N\left(t\right)$ is the number of trades in a second interval. $\varepsilon^p\left(t\right) = +1$ implies that the majority of trades in second t were triggered by a market order to buy, and a value $\varepsilon^p\left(t\right) = -1$ indicates a majority of sell market orders. In this definition, they are two ways to obtain $\varepsilon^p\left(t\right) = 0$. One way is that in a particular second there is not trades, and then no trade sign. The other way is that the addition of the trade signs ($+1$ and -1) in a second be equal to zero. In this case, there is a balance of buy and sell market orders.

Market orders show opposite trade directions to limit order executed simultaneously. An executed sell limit order corresponds to a buyer-initiated market order. An executed buy limit order corresponds to a seller-initiated market order.

As in the trade time scale, in the physical time scale we use the same strategy to obtain the midpoint price for every second, so all the seconds in the open market time have a midpoint price value. It is worth to note again, that even if the second does not have a change in quotes, it will has still a midpoint price value and a return value.

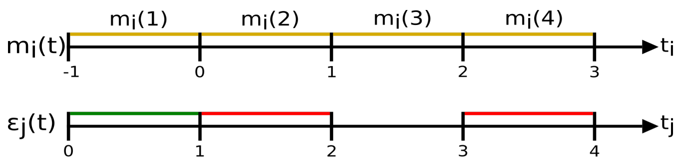

Figure 3. Sketch of data processing for physical time scale. In the midpoint price time line, the horizontal lines between seconds represent the midpoint prices. In the trade signs time line, the horizontal lines between seconds represent the trade sign values. The midpoint price time line and the trade sign time line are shifted in one second.

In this case we do not compare every single trade sign in a second, but the net trade sign obtained for every second with the definition (Eq. 8). This can be seen in Fig. 3, where we related the midpoint price of the previous second with the trade sign of the current second.

According to [40], this definition has an average accuracy up to 82% in the physical time scale.

3.3 Response function definitions

In general, the different works in price response functions use the returns and the trade signs with variations in the quantities and the time scale.

The response function is used to study the mutual dependence between stocks. In [4,5,7], Bouchaud et al. use a self-response function that only depends on the time lag τ

$$R\left(\tau\right) = \left\langle \left(S_{n+\tau} - S_n\right) \cdot \varepsilon_n \right\rangle_t \tag{9}$$

Where ε_n is the sign of the n^{th} trade and the price S_n is defined as the midpoint just before the n^{th} trade, all averaged in trade time scale. The quantity $R\left(\tau\right)$ measures how much, on average, the price moves up (down) at time τ conditioned to a buy (sell) order at time zero. They found for France Telecom that $R\left(\tau\right)$ increases by a factor 2 between $\tau = 1$ and $\tau \approx 1000$, before decreasing back.

Using larger τ, they also confirm that $R(\tau)$ decreases, and even becomes negative beyond $\tau \approx 5000$. However, in some cases the maximum is not observed and rather $R(\tau)$ keeps increasing mildly [5].

In [16], the price impact function, $R(\cdot)$, is defined as the average price response due to a transaction as a function of the transaction's volume v_i

$$R(v_i) \equiv \langle \varepsilon_i r_i^l | v_i \rangle \tag{10}$$

Empirically they found the function is highly concave. The curvature of $R(v_i)$ is entirely due to the probability that a transaction causes a nonzero impact - the larger the size of the transaction, the larger the probability. In [29], the response function is given by

$$R(V,\tau) = \langle \varepsilon(t) \cdot [m(t+\tau) - m(t)] \, | V \rangle \tag{11}$$
$$= R(\tau) \ln V \tag{12}$$

Where V is the volume. They found that $R(\tau)$ for three French stocks first increases from $\tau = 10s$ to a few hundred seconds, and then appears to decrease back to a finite value.

In [3] they use a market response defined as

$$R_{ij}(\tau) = \langle [\ln S_i(t+\tau) - \ln S_i(t)] \cdot \varepsilon_j(t) \rangle_p \tag{13}$$

The response measures the average price change of contract i at time $t+\tau$, after experiencing a sign imbalance $\varepsilon_j(t)$ in contract j at time t. In this work τ is used in units of five minutes.

In a later work [19,40], S. Wang et al. and Grimm et al. use the logarithmic return for stock i and time lag τ, defined via the midpoint price $m_i(t)$. The cross-response function is then defined as

$$R_{ij}(\tau) = \langle r_i(t-1,\tau) \cdot \varepsilon_j(t) \rangle \tag{14}$$

They found that in all cases, an increase to a maximum is followed by a decrease. The trend is eventually reversed.

Finally, in [37], S. Wang et al. define the response function as

$$R_{ij} = \left\langle \left[\ln m_i^{(f)}(t_j) - \ln m_i^{(p)}(t_j) \right] \cdot \varepsilon_j(t_j) \right\rangle_{t_j} \tag{15}$$

For the price change of stock i caused by a trade of stock j.

Here, $m_i^{(p)}(t_j)$ is the midpoint price of stock i previous to the trade of stock j at its event time t_j and $m_i^{(f)}(t_j)$ is the midpoint price of stock i following that trade.

The difference between the definition in [40] and in [37], is that [40] measures how a buy or sell order at time t influences on average the price at a later time $t+\tau$. The physical time scale was chosen since the trades in different stocks are not synchronous (TAQ data). In [37], it was used a response function on a trade time scale (Totalview data), as the interest is to analyze the immediate responses. In [37] the time lag τ is restricted to one, such that the price response quantifies the price impact of a single trade.

4 Price response function implementations

The main objective of this work is to analyze the price response functions. In general we define the self- and cross-response functions in a correlated financial market as

$$R_{ij}^{scale}(\tau) = \left\langle r_i^{scale}(t-1,\tau) \cdot \varepsilon_j^{scale}(t) \right\rangle_{scale} \tag{16}$$

where the index i and j correspond to stocks in the market, r_i^{scale} is the return of the stock i in a time lag τ in the corresponding scale and ε_j^{scale} is the trade sign of the stock j in the corresponding scale. The subscript and superscript *scale* refer to the time scale used, whether physical time scale ($scale = p$) or trade time scale ($scale = t$). Finally, we average the product over the physical time or trade time depending on the time scale.

We use the returns and the trade signs to define three response functions: trade time scale response, physical time scale response and activity response.

To compare the three response functions, we define the following quantities

$$E_{j,d}(t) = \sum_{n=1}^{N(t)} \varepsilon_{j,d}^{t}(t,n) = \mathrm{sgn}(E_{j,d}(t)) \cdot |E_{j,d}(t)| \tag{17}$$
$$\varepsilon_{j,d}^{p}(t) = \mathrm{sgn}(E_{j,d}(t)) \tag{18}$$

Where the subscript d refers to the days used in the response computation. We use Eq. 17 to make easier the comparison between the results, as all the defined responses use the trade sign term.

In Sect. 4.1 we analyze the responses functions in trade time scale, in Sect. 4.2 we analyze the responses functions in physical time scale and in Sect. 4.3 we define a response function to analyze the influence of the frequency of trades in a second.

4.1 Response functions in trade time scale

We define the self- and cross-response functions in trade time scale, using the trade signs in trade time scale and for the returns, we select the last midpoint price of every second and compute them. We use this strategy with the TAQ data set considering that the price response in trade time scale can not be directly compared with the price response in physical time scale. In this case we related each trade sign in one second with the midpoint price of the previous second. Then, to compute the returns, instead of using trades as the time lag (it would make no sense as all the midpoint price are the same in one second) we use seconds. Thus, we force the response in trade time scale to have a physical time lag, and then, be able to compare with the physical time scale response. This approximation is feasible considering the facts explained in Sect. 3. The price response function in trade time scaled is defined as

$$R_{ij}^{t}(\tau) = \left\langle r_i^{p}(t-1,\tau) \cdot \varepsilon_j^{t}(t,n) \right\rangle_t \tag{19}$$

where the superscript t refers to the trade time scale. However, to be explicit with the way the averaging is made, the function reads

$$R_{ij}^{t}\left(\tau\right)=\frac{1}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}\left(t\right)} \cdot\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}\sum_{n=1}^{N(t)}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\varepsilon_{j,d}^{t}\left(t,n\right) \tag{20}$$

$$=\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\frac{\sum_{n=1}^{N(t)}\varepsilon_{j,d}^{t}\left(t,n\right)}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}\left(t\right)} =\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\mathrm{sgn}\left(E_{j,d}\left(t\right)\right)\cdot w_{j,d}^{t}\left(t\right) \tag{21}$$

Where

$$w_{j,d}^{t}\left(t\right)=\frac{\left|E_{j,d}\left(t\right)\right|}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}\left(t\right)} \tag{22}$$

is a weight function that depends on the normalization of the response.

To compute the response functions in trade time scale, we used all the trade signs during a day in market time. As we can not associate an individual midpoint price with their corresponding trade signs, all the trade signs in one second are associated with the midpoint price of the previous second. As τ depends on the midpoint price, even if we are using trade signs in trade time scale, the value of τ is in seconds.

The results of Fig. 4 show the self- responses of the six stocks used in the analysis and the cross-responses for pairs of stocks representing three different economic sectors.

The self-response functions increase to a maximum and then slowly decrease. In some stocks this behavior is more pronounced than in others. For our selected tickers, a time lag of $\tau = 10^3 s$ is enough to see an increase to a maximum followed by a decrease. Thus, the trend in the self-response functions is eventually reversed. On the other hand, the cross-response functions have smaller signal strength than the self-response functions. For our cross-response functions of stocks in the same sectors, some couples exhibit the increase-decrease behavior inside a time lag of $\tau = 10^3 s$. Other couples seems to need a larger time lag to reach the decrease behavior.

4.2 Response functions in physical time scale

One important detail to compute the market response in physical time scale is to define how the averaging of the function will be made. This, because the response functions highly differ when we include or exclude $\varepsilon_j^p\left(t\right)=0$ [40]. The cross-responses including $\varepsilon_j^p\left(t\right)=0$ are weaker than the excluding ones due to the omission of direct influence of the lack of trades. However, either including or excluding $\varepsilon_j^p\left(t\right)=0$ does not change the trend of price reversion versus the time lag, but it does affect the response function strength [39].

Regarding the definition of the response functions in- and excluding $\varepsilon_j^p\left(t\right)=0$, the general averaging is

$$R_{ij}^{(\text{inc. }0)}\left(\tau\right)=\frac{\sum_{t=1}^{T_j+T_{j,n}}r_i^p\left(t-1,\tau\right)\cdot\varepsilon_j^p\left(t\right)}{T_j+T_{j,n}} \tag{23}$$

$$R_{ij}^{(\text{exc. }0)}\left(\tau\right)=\frac{\sum_{t=1}^{T_j}r_i^p\left(t-1,\tau\right)\cdot\varepsilon_j^p\left(t\right)}{T_j} \tag{24}$$

Where the superscript inc. and exc. refers to including and excluding $\varepsilon_j^p\left(t\right)=0$. For stock j, T_j is the total trading time of stock j and $T_{j,n}$ is the total time of lack of trading or buy sell balance. The numerators in Eqs. 23 and 24 are the same, while the denominators differ [39].

Hence,

$$R_{ij}^{(\text{inc. }0)}\left(\tau\right)=f_j\cdot R_{ij}^{(\text{exc. }0)}\left(\tau\right) \tag{25}$$

Where the relative trading frequency is defined as [39]

$$f_j=\frac{T_j}{T_j+T_{j,n}} \tag{26}$$

The most frequently traded stocks have $f_i = 1$, because the time $T_{j,n}$ is zero. According to Eq. 25, the cross-response including $\varepsilon_j^p\left(t\right)=0$ is the one excluding $\varepsilon_j^p\left(t\right)=0$ scaled by a proper probability.

Then, we will only take in account the cross-response function excluding $\varepsilon_j^p\left(t\right)=0$.

We define the self- and cross-response functions in physical time scale, using the trade signs and the returns in physical time scale. The price response function in physical time scale is defined as

$$R_{ij}^{p}\left(\tau\right)=\left\langle r_i^p\left(t-1,\tau\right)\cdot\varepsilon_j^p\left(t\right)\right\rangle_p \tag{27}$$

where the superscript p refers to the physical time scale. The corresponding explicit expression reads

$$R_{ij}^{p}\left(\tau\right)=\frac{1}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}\eta\left[\varepsilon_{j,d}^{p}\left(t\right)\right]} \cdot\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\varepsilon_{j,d}^{p}\left(t\right)\cdot\eta\left[\varepsilon_j^p\left(t\right)\right] \tag{28}$$

$$=\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\frac{\varepsilon_{j,d}^{p}\left(t\right)\cdot\eta\left[\varepsilon_{j,d}^{p}\left(t\right)\right]}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}\eta\left[\varepsilon_{j,d}^{p}\left(t\right)\right]} =\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}r_{i,d}^{p}\left(t-1,\tau\right)\cdot\mathrm{sgn}\left(E_{j,d}\left(t\right)\right)\cdot w_{j,d}^{p}\left(t\right) \tag{29}$$

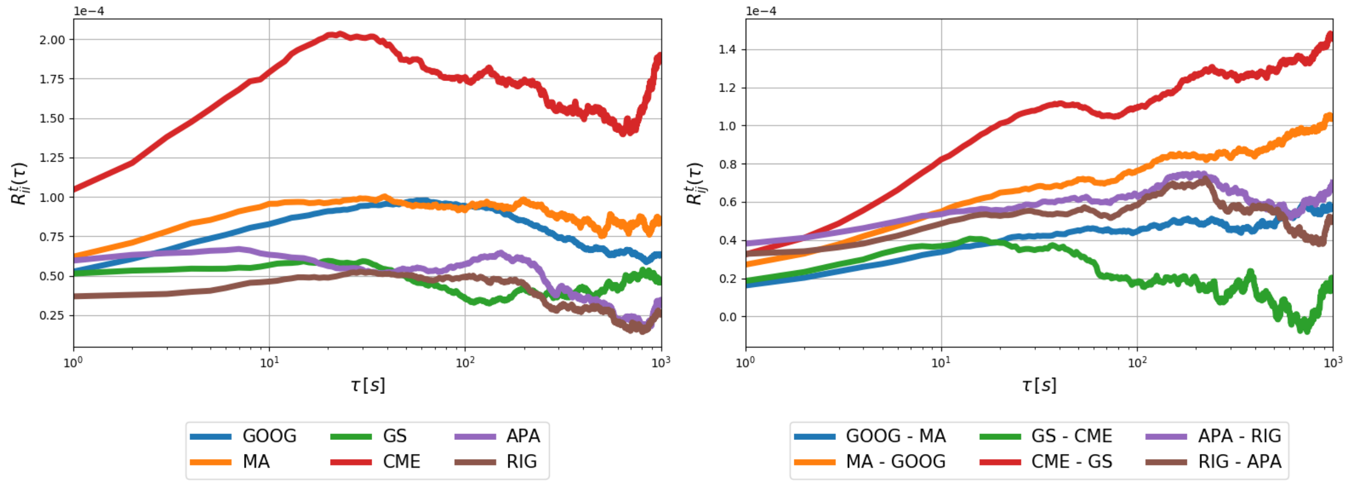

Figure 4. Self- and cross-response functions $R^t_{ij}(\tau)$ in 2008 versus time lag τ on a logarithmic scale in trade time scale. Self-response functions (left) of individual stocks and cross-response functions (right) of stock pairs from the same economic sector.

Where

$$\eta(x) = \begin{cases} 1, & \text{If } x \neq 0 \\ 0, & \text{otherwise} \end{cases} \tag{30}$$

take only in account the seconds with trades and

$$w^p_{j,d}(t) = \frac{\eta\left[\operatorname{sgn}\left(E_{j,d}(t)\right)\right]}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}\eta\left[\operatorname{sgn}\left(E_{j,d}(t)\right)\right]} \tag{31}$$

is a weight function that depends on the normalization of the response.

The results showed in Fig. 5 are the self- and cross-response functions in physical time scale. For the self-response functions we can say again that in almost all the cases, an increase to a maximum is followed by a decrease. Thus, the trend in the self- and cross-response is eventually reversed. In the cross-response functions, we have a similar behavior with the previous subsection, where the time lag in some pairs was not enough to see the decrease of the response.

Compared with the response functions in trade time scale, the response functions in physical time scale are stronger.

4.3 Activity response functions in physical time scale

Finally, we define the activity self- and cross-response functions in physical time scale, using the trade signs and the returns in physical time scale. We add a factor $N_{j,d}(t)$ to check the influence of the frequency of trades in a second in the response functions. The activity price response function is defined as

$$R^a_{ij}(\tau) = \left\langle r^p_i(t-1,\tau)\cdot\varepsilon^p_j(t)\cdot N(t)\right\rangle_p \tag{32}$$

where the superscript a refers to the activity response function. The corresponding explicit expression reads

$$\begin{aligned} R^a_{ij}(\tau) &= \frac{1}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}(t)} \\ &\quad\cdot\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f} r^p_{i,d}(t-1,\tau)\cdot\varepsilon^p_{j,d}(t)\cdot N_{j,d}(t) \qquad (33) \\ &= \sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f} r^p_{i,d}(t-1,\tau)\cdot\frac{\varepsilon^p_{j,d}(t)\cdot N_{j,d}(t)}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}(t)} \\ &= \sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f} r^p_{i,d}(t-1,\tau)\cdot\operatorname{sgn}\left(E_{j,d}(t)\right)\cdot w^a_{j,d}(t) \end{aligned} \tag{34}$$

Where

$$w^a_{j,d}(t) = \frac{N_{j,d}(t)}{\sum_{d=d_0}^{d_f}\sum_{t=t_0}^{t_f}N_{j,d}(t)} \tag{35}$$

is a weight function that depends on the normalization of the response.

As $E_{j,d}(t)$ is the sum of $+1$ and -1 in one second and $N_{j,d}(t)$ is the number of trades in a second, $N_{j,d}(t) \geq E_{j,d}(t)$. $N_{j,d}(t) = E_{j,d}(t)$ only when all the trades in a second are buys ($+1$).

The trade weight $w^t_{j,d}(t)$ reduces noises, The physical weight $w^p_{j,d}(t)$ gives every step the same weight, and the activity weight $w^a_{j,d}(t)$ emphasizes seconds with large activity.

In Fig. 6, we can see how the three responses have approximately the same shape, but the strength of the signal varies depending on the definition. The frequency of trades have a large influence in the responses.

As predicted by the weights, the event response is weaker than the physical response, and the activity response is the strongest response.

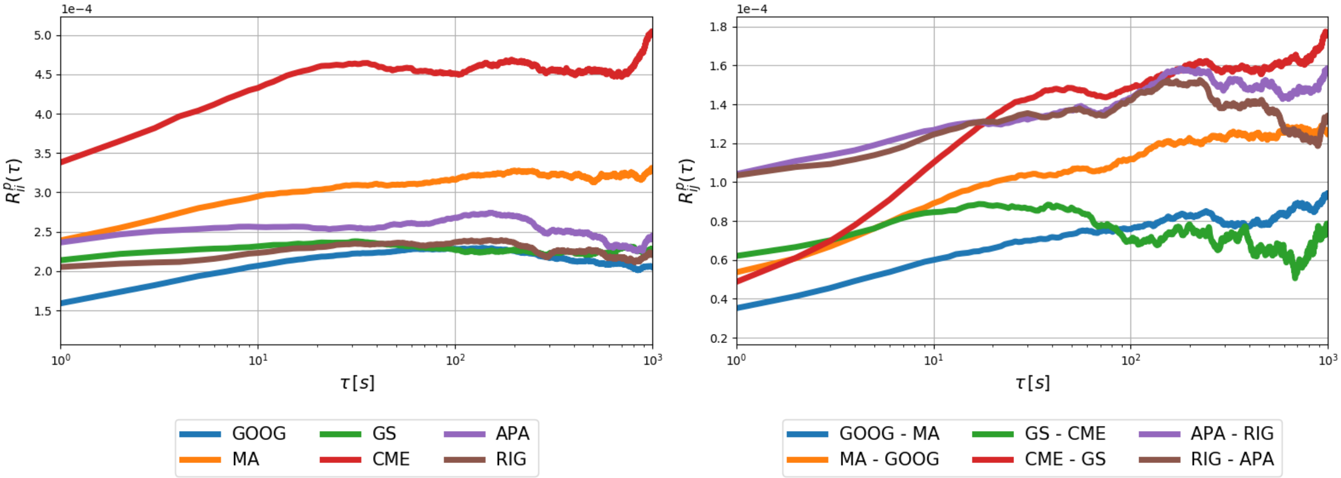

Figure 5. Self- and cross-response functions $R_{ij}^{p}(\tau)$ excluding $\varepsilon_j^p(t) = 0$ in 2008 versus time lag τ on a logarithmic scale in physical time scale. Self-response functions (left) of individual stocks and cross-response functions (right) of stock pairs from the same economic sector.

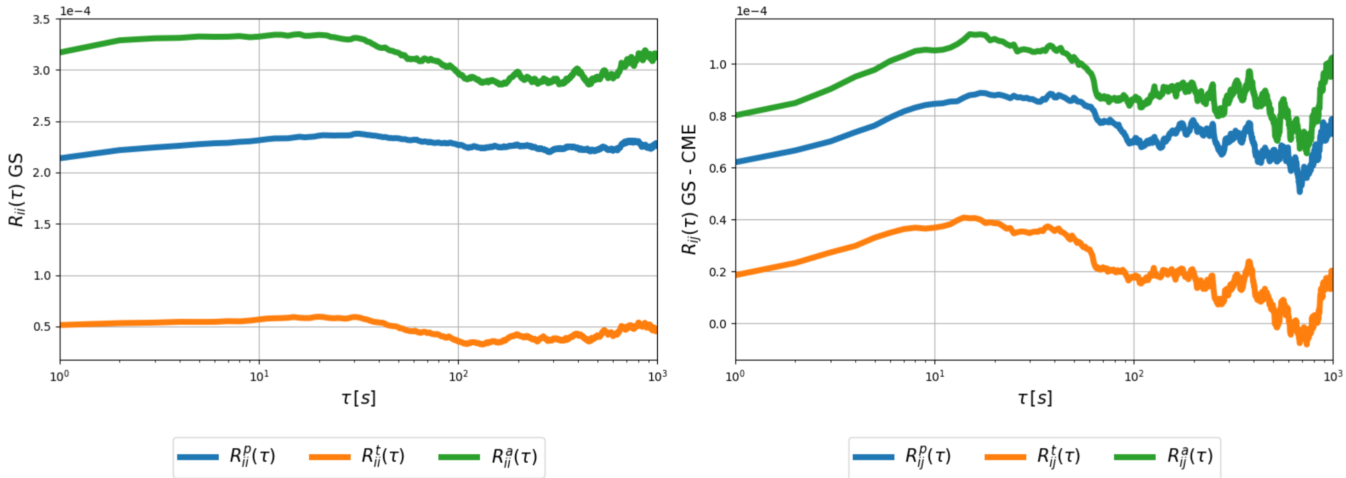

Figure 6. Self- and cross-response functions $R_{ij}^{scale}(\tau)$ excluding $\varepsilon_j^p(t) = 0$ in 2008 versus time lag τ on a logarithmic scale. Self-response functions (left) of Goldman Sachs Group Inc. stock and cross-response functions (right) of Goldman Sachs Group Inc.-CME Group Inc. stocks.

In the three curves in the figure can be seen the increase-decrease behavior of the response functions.

Our results are consistent with the current literature, where the results differ about a factor of two depending on the time scale. We must note that the activity implementation is only a test and was never defined in previous works. That is why the factor of two difference can not be seen in Fig. 6 for $R_{ij}^{a}(\tau)$.

5 Time shift response functions

The relative position between returns and trade signs directly impact the result of the response functions. Shifting the values to the right or to the left either in trade time scale or physical time scale have approximately the same effect.

To test this claim, we used the definition of the response function from [40] that is showed in Eq. 14 and added a parameter t_s that shifts the position between returns and trade signs. To see the impact of the time shift we analyzed the stocks showed in Table 1 in the year 2008. We used different time shifts in the response function

$$R_{ij}^{s,scale}(\tau) = \left\langle r_i^{scale}(t - t_s, \tau) \cdot \varepsilon_j^{scale}(t) \right\rangle_{scale} \qquad (36)$$

where the index i and j correspond to stocks in the market, r_i^{scale} is the return of the stock i in a time lag τ with a time shift t_{shift} in the corresponding scale and ε_j^{scale} is the trade sign of the stock j in the corresponding scale. The subscript and superscript *scale* refer to the time scale used, whether physical time scale or trade time scale and $R_{ij}^{s,scale}$ is the time shift price response function, where the superscript s refers to the time shift. Finally, we average the product over the physical time or trade time depending on the time scale.

We compute the response functions according to two cases. In one case we set τ to a constant value and vary t_s, and in the other case we set t_s to a constant value and vary τ.

In Sect. 5.1 we analyze the influence of the time shift between the trade signs and returns in trade time scale and in Sect. 5.2 we analyze the influence of the time shift between the trade signs and returns in physical time scale.

5.1 Trade time scale shift response functions

In the trade time scale we compute the response function

$$R_{ij}^{s,t}(\tau) = \left\langle r_i^t(t - t_s, \tau) \cdot \varepsilon_j^t(t) \right\rangle_t \tag{37}$$

In this case for r_i^t, we associate all the trade signs to a return value and create pseudo midpoint price values in trade time scale. Then, we shift the trade signs and the returns by trades. Hence, the time lag and time shift are in trade time scale.

In Fig. 7, it can be seen the response functions results for fixed τ values while t_s is variable. In the different τ values figures, the results are almost the same. The response functions are zero either if the time shift is larger than τ, or if the time shift is smaller than zero. However, related on the time lag, there is a zone where the signal is different to zero. For values between zero and τ there is a peak in a position related to τ. The response function grows and decrease relatively fast.

We tested the response function for fixed time shift values while τ is variable. In Fig. 8) we use a time shift of 10 trades (left) and 100 trades (right). In both, self- and cross-response results are qualitatively the same. It can be seen that the response functions have a zero signal before the time shift. After the returns and trade signs find their corresponding order the signals grow. In comparison with the values obtained in Fig. 4, it looks like the response function values with large time shift are stronger. However, this is an effect of the averaging of the functions. As the returns and trade signs are shifted, they are less values to average, and then the signals are stronger. Anyway, the figure shows the importance of the position order between the trade signs and returns to compute the response function.

5.2 Physical time scale shift response functions

In the physical time scale we compute the response function

$$R_{ij}^{s,p}(\tau) = \left\langle r_i^p(t - t_s, \tau) \cdot \varepsilon_j^p(t) \right\rangle_t \tag{38}$$

Similar to the results in Subsect. 5.1, Fig. 9 shows the responses functions for fixed τ values while t_s is variable. Again, the response functions are zero if the time shift is larger than the time lag, or if the time shift is smaller than zero. For every τ value, there is a peak. The peak grows and decay relatively fast. The response signal usually starts to grow in zero or a little bit earlier and grows to a value around to τ. In this zone the response functions are different to zero.

The results for fixed time shift values and variable time lag ($t_s = 10s$ and $t_s = 100s$) are shown in Fig. 10. The self- and cross-response results are qualitatively the same compared with the previous subsection. The response functions are zero before the time shift value. After the returns and the trade signs reach their order, the signals grow. The same effect of the apparent stronger signal can be seen here, and again, it is due to the averaging values.

The results in trade time scale and physical time scale can be explained understanding the dynamics of the market. A trade can or can not change the price of a ticker. Therefore, when a change in price happens, a change in midpoint price, and consequently in returns happens. Thus, it is extremely important to keep the order of the events and the relation between them. When we shift the trade signs and returns, this order is temporarily lost and as outcome the signal does not have any meaningful information. When the order is recovered during the shift, the signal grows again, showing response function values different to zero. In this section we were interested only in the order (shift) and not in the responses values, which were analyzed in Sect. 4. A time shift smaller than zero does not have any useful information about the response. If the time shift is equal to zero, the signal is weak, due to the time needed by the market to react to the new information. On the other hand, a time shift larger than two steps shows the information is lost and the signal only grows when the original order is resumed.

Then the question is what is the ideal time shift to compute the response functions. Our approach in Sect. 4 takes in account that the changes in the quotes are the ones that attract the agents to buy or sell their shares. Hence, they directly impact the trade signs. According to the results, the response can take up to two time steps in the corresponding scale to react to the change in quotes. Thus, a time shift larger than two time steps makes no sense. On the other hand, in the case of the physical time scale, where a sampling is used, to assure the selection of a midpoint price at the beginning of a second, it is a good strategy to use the last midpoint of the previous second as the first midpoint price of the current second. In this case an apparent one second shift is used between returns and trade signs.

6 Short and long response functions

Regarding Equation 2, we use a time lag τ in the returns to see the gains or loses in a future time. However, the strength of the return in the time lag should not be equal along its length. Then, we divide the full range time lag τ

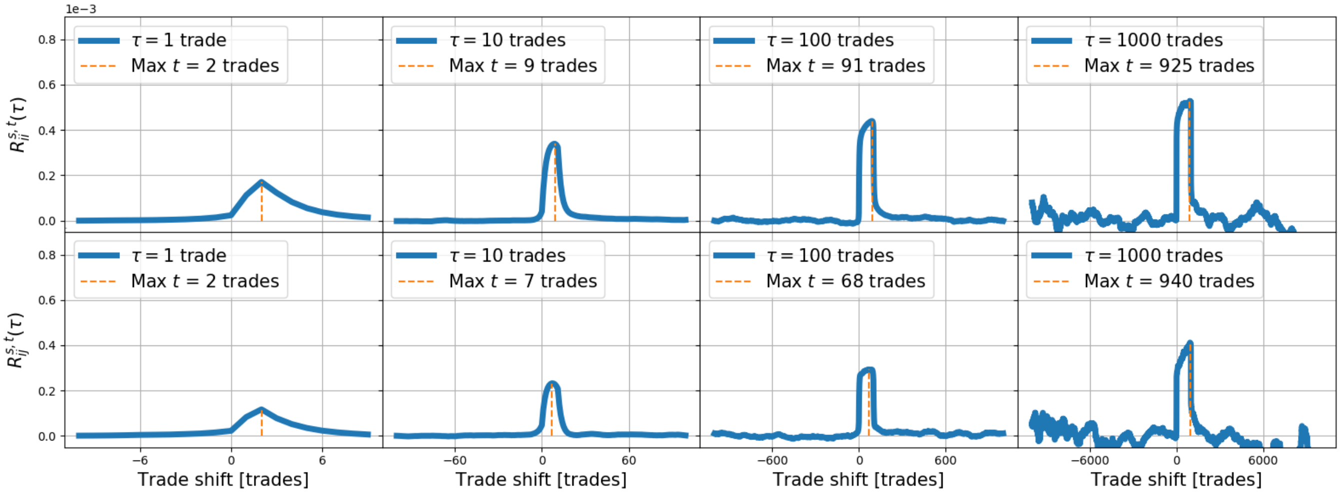

Figure 7. Self-response functions $R_{ii}^{t}(\tau)$ in 2008 versus shift for the Transocean Ltd. stock (top) and cross-response functions $R_{ij}^{t}(\tau)$ in 2008 versus shift for the Transocean Ltd.-Apache Corp. stocks (bottom) in trade time scale.

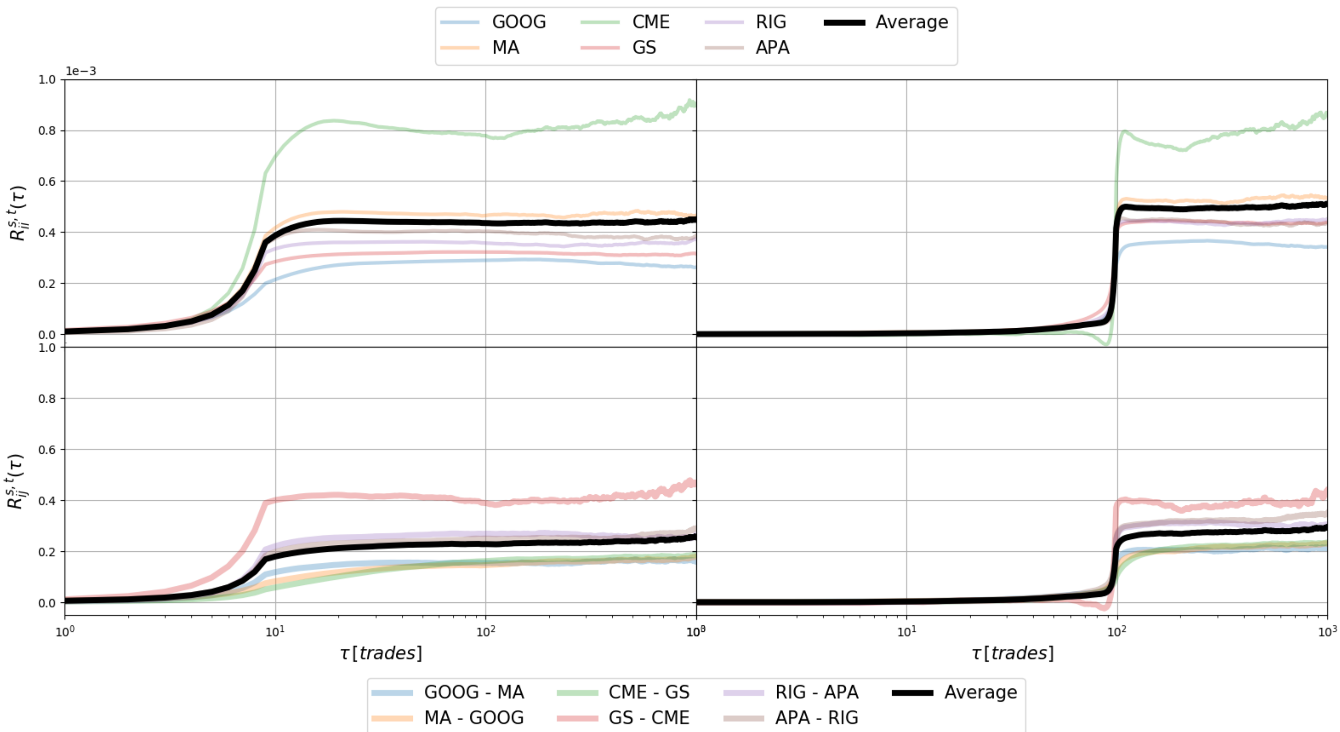

Figure 8. Self- and cross-response functions $R_{ij}^{t}(\tau)$ in 2008 versus time lag τ on a logarithmic scale for different shifts in trade time scale. Self-response functions (top) of individual stocks and cross-response functions (bottom) of stocks pairs from the same economic sector. We use time lag values $\tau = 10$ trades (left) and $\tau = 100$ (right).

in an immediate time lag and in a late time lag as show in Fig. 11, where

$$\tau = \tau' + (\tau - \tau') \tag{39}$$

for $\tau' < \tau$. This distinguish the returns depending in the time lag as the short (immediate) return τ' with the long return $\tau - \tau'$.

To use the short and long time lag, we rewrite the returns in physical time scale as

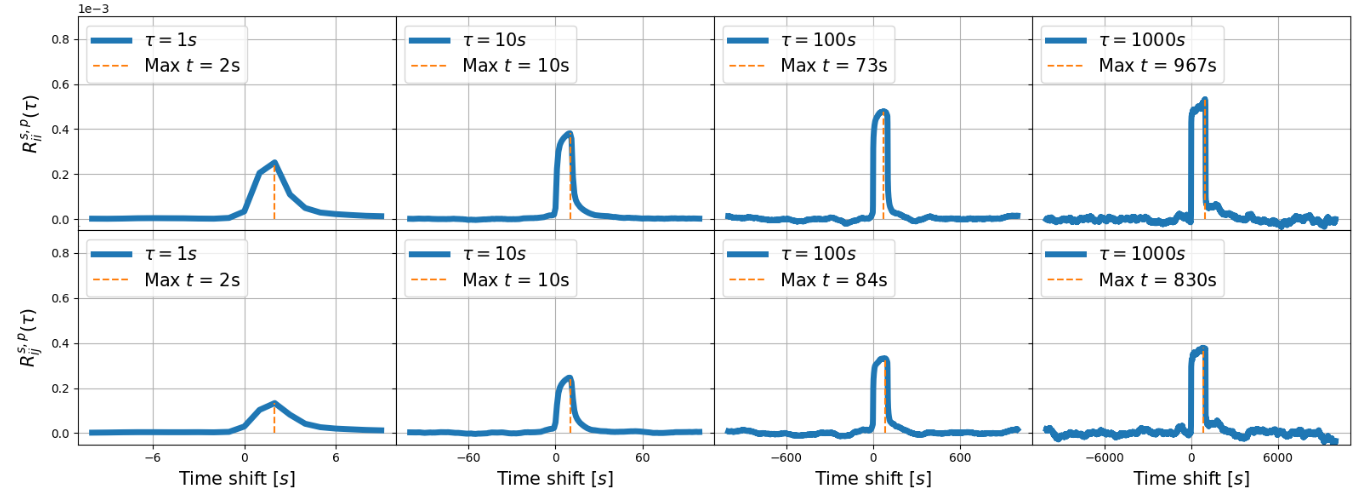

Figure 9. Self-response functions $R_{ii}^{p}(\tau)$ excluding $\varepsilon_{i}^{p}(t)=0$ in 2008 versus shift for the Transocean Ltd. stock (top) and cross-response functions $R_{ij}^{p}(\tau)$ excluding $\varepsilon_{j}^{p}(t)=0$ in 2008 versus shift for the Transocean Ltd.-Apache Corp. stocks (bottom) in physical time scale.

$$
\begin{aligned}
r_{i}^{sl,p}(t,\tau) &= \ln\left(\frac{m_i(t+\tau)}{m_i(t)}\right) \\
&= \ln\left(\frac{m_i(t+\tau)}{m_i(t+\tau')}\cdot\frac{m_i(t+\tau')}{m_i(t)}\right) \\
&= \ln\left(\frac{m_i(t+\tau)}{m_i(t+\tau')}\right)+\ln\left(\frac{m_i(t+\tau')}{m_i(t)}\right) \\
&\approx \frac{m_i(t+\tau)-m_i(t+\tau')}{m_i(t+\tau')}+\frac{m_i(t+\tau')-m_i(t)}{m_i(t)}
\end{aligned} \tag{40}
$$

where sl refers to short-long and the second term of the right part is constant with respect to τ. Replacing Equation 40 in the response function (Eq. 27) we have

$$
\begin{aligned}
R_{ij}^{sl,p}(\tau) &= \left\langle r_{i}^{sl,p}(t-1,\tau)\cdot\varepsilon_{j}^{p}(t)\right\rangle_{p} \\
&\approx \left\langle\frac{m_i(t-1+\tau)-m_i(t-1+\tau')}{m_i(t-1+\tau')}\cdot\varepsilon_j(t)\right\rangle_{p} \\
&+\left\langle\frac{m_i(t-1+\tau')-m_i(t-1)}{m_i(t-1)}\cdot\varepsilon_j(t)\right\rangle_{p}
\end{aligned} \tag{41}
$$

Where the first term in the right side of Equation 41 is the long response and the right term is the short response. Again, the right term of Equation 41 is independent of τ.

The results in Fig. 12 show the short response, the long response, the addition of the short response and long response (Sum), the original response, a random response and the value of τ'.

The main signal of the response function come from the short response. Depending on the stock and the value of τ' the long response can increase or decrease the short response signal, but in general the long response does not give a significant contribution to the complete response.

Before τ', the short response and long response are the same, as the self and cross-response definition do not define values smaller than τ', so it is computed as the original response. In the figure, the curves of the short and long response are under the curve of the original response. After τ', the short response is a strong constant signal. On the other hand, the long response immediately fades, showing the small contribution to the final response. To compare the significance of the long response, We added a random response made with the trade signs used to compute the response but with a shuffle order. The long response and the random response are comparable, and show how the long response is not that representative in the final response. If we add the short and long response, we obtain the original response. In Fig. 12, the original response (red line) has the same shape to the addition of the short and long response (green line).

For the response functions that show the increase-decrease behavior in between the time lag $\tau = 10^3$, the peak is usually between $\tau = 10^1$ and $\tau = 10^2$. In these cases the long response are always negative after the τ' value and is comparable in magnitude with the random signal.

On the other hand, the response functions that requires a bigger time lag to show the increase-decrease behavior, have non negative long responses, but still they are comparable in magnitude with the random signal.

7 Spread impact

When we calculate the price response functions, the signal of the response depends directly on the analyzed stock. Thus, even if the responses functions are in the same scale, their values differ from one to another. We choose the spread to group 530 stocks in the NASDAQ stock market for the year 2008, and check if the average strength of the price self-response functions in physical time scale were

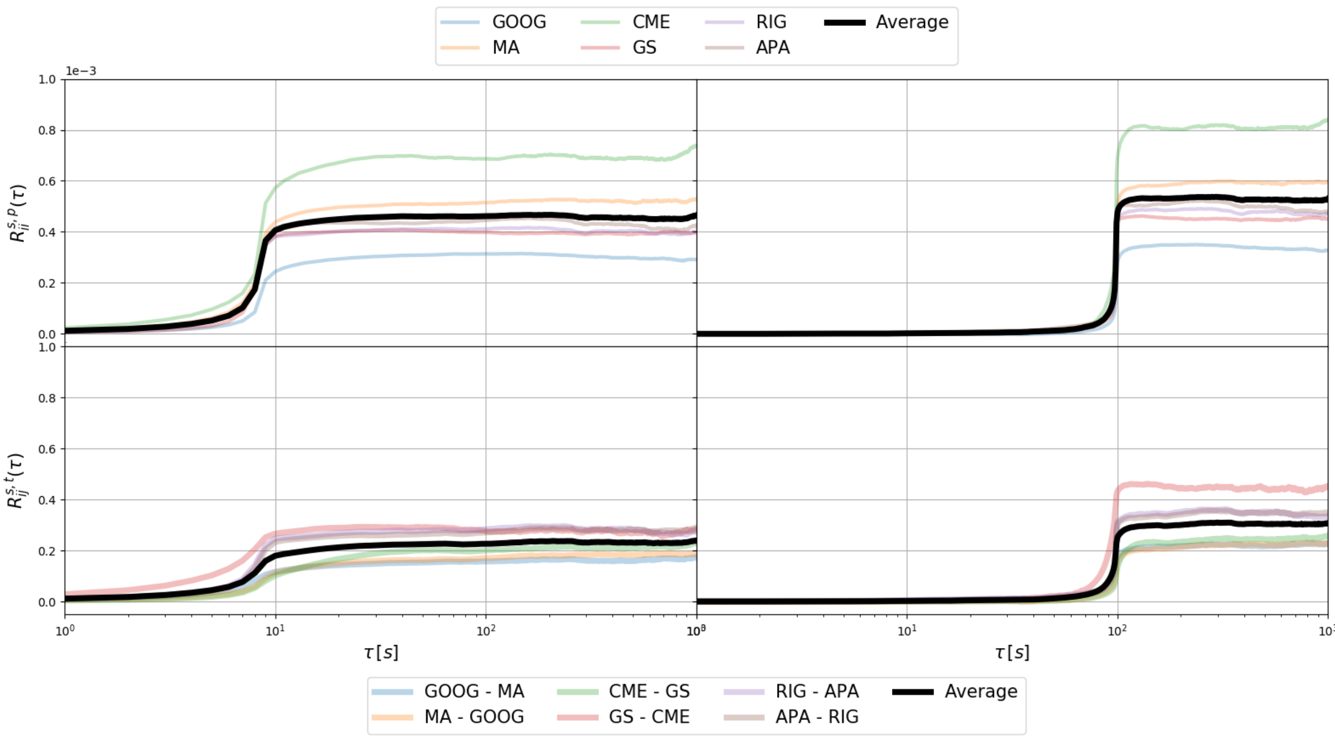

Figure 10. Self- and cross-response functions $R_{ij}^{p}(\tau)$ excluding $\varepsilon_j^p(t) = 0$ in 2008 versus time lag τ on a logarithmic scale for different shifts in physical time scale. Self-responses functions (top) of individual stocks and cross-response functions (bottom) of stocks pairs from the same economic sector. We use time lag values $\tau = 10$ trades (left) and $\tau = 100$ (right).

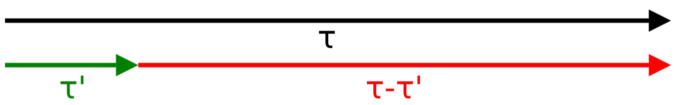

Figure 11. τ value divided in short and long time lag.

similar for this groups. For each stock we computed the average spread for a year, and using this value we classified the groups.

We used three intervals to select the stocks groups to average the response functions ($s < 0.05\$$, $0.05\$ \leq s < 0.10\$$ and $0.10\$ \leq s < 0.40\$$). The detailed information of the groups can be seen in Appendix A.

In Fig. 13 can be seen the average response functions for the three groups. The response functions start at the bottom with the average response for the stocks with the smaller spreads (more liquid) and grow to the larger average response functions for the stocks with the larger spreads (less liquid). All the average response functions follows the increment to a maximum followed by a decrease described in Sect. 3 and 4.

The strength of the self-response function signals grouped by spread can be explained knowing that the response functions directly depend on trade signs. As long as the stock is liquid, the number of trade signs grow. Thus, at the moment of the averaging, the large amount of trades, reduces the response function signal. Therefore, the response function decrease as long as the liquidity grows. And as stated in the introduction the spread is negatively related to trading volume, hence, firms with more liquidity tend to have lower spreads.

8 Conclusion

We went into detail about the response functions in correlated financial markets. We define the trade time scale and physical time scale to compute the self- and cross-response functions for six companies with the largest average market capitalization for three different economic sectors of the S&P index in 2008. Due to the characteristics of the data used, we had to classify and sampling values to obtain the corresponding quantities in different time scales. The classification and sampling of the data had impact on the results, making them smoother or stronger, but always keeping their shape and behavior.

The response functions were analyzed according to the time scales. To compare price response functions from different scales, we used the same midpoint prices in physical time scale with the corresponding trade signs in trade time scale or physical time scale. This assumption allowed us to get an idea of how representative was the behavior obtained in both cases. For trade time scale, the signal is weaker due to the large averaging values from all the trades in a year. In the physical time scale, the response functions had less noise and their signal were stronger. The activity in every second highly impact the responses. As the response functions can not grow indefinitely with

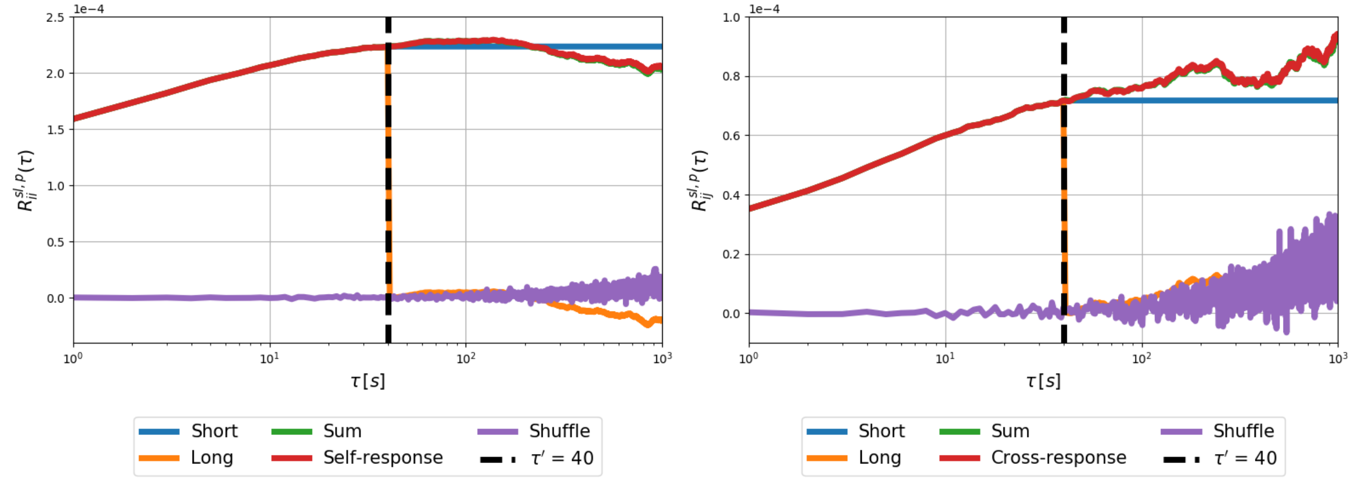

Figure 12. Self- and cross-response functions $R_{ij}^{sl,p}(\tau)$ excluding $\varepsilon_j^p(t) = 0$ in 2008 versus time lag τ on a logarithmic scale using a $\tau' = 40$ in physical time scale. Self-response functions (left) of Alphabet Inc. stock and cross-response functions (right) of Alphabet Inc.-Mastercard Inc. stocks.

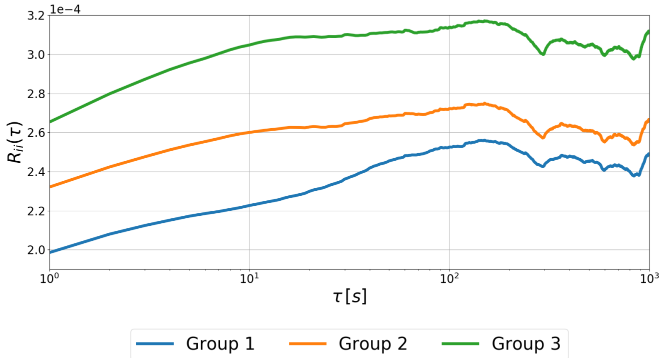

Figure 13. Average price self-response functions $R_{ii}^{p}(\tau)$ excluding $\varepsilon_i^p(t) = 0$ in 2008 versus time lag τ on a logarithmic scale in physical time scale for 530 stocks divided in three representative groups.

the time lag, they increase to a peak, to then decrease. It can be seen that the market needs time to react and revert the growing. In both time scale cases depending on the stocks, two characteristics behavior were shown. In one, the time lag was large enough to show the complete increase-decrease behavior. In the other case, the time lag was not enough, so some stocks only showed the growing behavior.

We modify the response function to add a time shift parameter. With this parameter we wanted to analyze the importance in the order of the relation between returns and trade signs. In trade time scale and physical time scale we found similar results. When we shift the order between returns and trade signs, the information from the relation between them is temporarily lost and as outcome the signal does not have any meaningful information. When the order is recovered, the response function grows again, showing the expected shape. We showed that this is not an isolated conduct, and that all the shares used in our analysis exhibit the same behavior. Thus, even if they are values of time shift that can give a response function signal, from the theory this time shift should be a value between $t_s = [0, 2]$.

We analyzed the impact of the time lag in the response functions. We divided the time lag in a short and long time lag. The response function that depended on the short time lag, showed a stronger response. The long response function depending on the stock could take negative and non-negative values. However, in general the influence were not intense.

Finally, we checked the spread impact in self-response functions. We divided 530 stocks from the NASDAQ stock market in five groups depending on the year average spread of every stock. The response functions signal were stronger for the group of stocks with the larger spreads and weaker for the group of stocks with the smaller spreads.

9 Author contribution statement

TG proposed the research. SMK and JCHL developed the method of analysis. The idea to look the time shift and to analyze the spread impact was due to JCHL, and the idea of the time lag analysis was due to SMK. JCHL carried out the analyisis. All the authors contributed equally to analyze the results and write the paper.

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Appendix A NASDAQ stocks used to analyze the spread impact

We analyzed the spread impact in the response functions for 524 stocks from the NASDAQ stock market for the year 2008. In Tables 2, 3, 4 and 5, we listed the stocks in their corresponding spread groups.

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Table 2. Information of the stocks in Group 1.

Group 1			Symbol	Company	Spread[1]	Symbol	Company	Spread[1]
Symbol	Company	Spread[1]	LUV	Southwest Airlines Company	0.01$	TSN	Tyson Foods Inc.	0.02$
F	Ford Motor Company	0.01$	BRCM	Broadcom Inc.	0.01$	CA	CA Inc.	0.02$
Q	Qwest Communications Int.	0.01$	TXN	Texas Instruments Inc.	0.01$	XEL	Xcel Energy Inc.	0.02$
ETFC	E-Trade Financial Corp.	0.01$	TER	Teradyne Inc.	0.01$	AA	Alcoa Corp.	0.02$
PFE	Pfizer Inc.	0.01$	MYL	Mylan N.V.	0.01$	KR	Kroger Company	0.02$
MOT	Motus GI Holdings Inc.	0.01$	HCBK	Hudson City Bancorp	0.01$	MRK	Merck & Company Inc.	0.02$
AMD	Advanced Micro Devices	0.01$	SPLS	Staples Inc.	0.01$	NSM	Nationstar Mortgage Holdings	0.02$
TLAB	Tellabs Inc.	0.01$	SGP	Siamgas and Petrochemicals	0.01$	WMT	Walmart Inc.	0.02$
INTC	Intel Corp.	0.01$	HST	Host Hotels & Resorts Inc.	0.01$	FITB	Fifth Third Bancorp	0.02$
TWX	Time Warner Inc.	0.01$	AES	Aes Corp.	0.01$	EK	Eastman Kodak Company	0.02$
CSCO	Cisco Systems Inc.	0.01$	KFT	Kraft Foods Inc.	0.01$	PHM	PulteGroup Inc.	0.02$
THC	Tenet Healthcare Corp.	0.01$	NTAP	NetApp Inc.	0.01$	JPM	JP Morgan Chase & Co.	0.02$
LSI	Life Storage Inc.	0.01$	BAC	Bank of America Corp.	0.01$	WAG	Walgreen Co.	0.02$
MU	Micron Technology Inc.	0.01$	HD	Home Depot Inc.	0.01$	SCHW	Charles Schwab Corp.	0.02$
EMC	EMC Corp.	0.01$	SOV	Life Storage Inc.	0.01$	RF	Regions Financial Corp.	0.02$
MSFT	Microsoft Corp.	0.01$	QLGC	QLogic Corp.	0.01$	ADBE	Adobe Inc.	0.02$
NOVL	Novell Inc.	0.01$	T	AT&T Inc.	0.01$	MAT	Mattel Inc.	0.02$
JAVA	Sun Microsystems Inc.	0.01$	GPS	Gap Inc.	0.01$	PAYX	Paychex Inc.	0.02$
ORCL	Oracle Corp.	0.01$	DIS	Walt Disney Company	0.01$	PGR	Progressive Corp.	0.02$
S	Sprint Nextel Corp.	0.01$	GM	General Motors Company	0.01$	HPQ	HP Inc.	0.02$
DELL	Dell Technologies Inc.	0.01$	JNPR	Juniper Networks Inc.	0.01$	DOW	Dow Inc.	0.02$
AMAT	Applied Material Inc.	0.01$	LOW	Lowe's Companies Inc.	0.01$	TE	TECO Energy Inc.	0.02$
SLE	Spark Energy Inc.	0.01$	CBS	CBS Corp.	0.01$	BBBY	Bed Bath & Beyond Inc.	0.02$
SBUX	Starbucks Corp.	0.01$	CAG	ConAgra Brands Inc.	0.01$	JNJ	Johnson & Johnson	0.02$
DYN	Dynergy Inc.	0.01$	LLTC	Linear Technology Corp.	0.01$	IP	International Paper Company	0.02$
DUK	Duke Energy Corp.	0.01$	DTV	DirecTV Group	0.01$	RSH	Respiri Ltd.	0.02$
CMCSA	Comcast Corp.	0.01$	HBAN	Huntington Bancshares Inc.	0.01$	MER	Mears Group PLC	0.02$
IPG	Interpublic Group of Co.	0.01$	KG	Kinross Gold Corp.	0.01$	HAL	Halliburton Company	0.02$
BSX	Boston Scientific Corp.	0.01$	CMS	CMS Energy Corp.	0.01$	KO	Coca-Cola Company	0.02$
GE	General Electric Company	0.01$	ODP	Office Depot Inc.	0.01$	PBCT	People's United Financial Inc.	0.02$
SYMC	Symantec Corp.	0.01$	NVLS	Nivalis Therapeutics Inc.	0.01$	WU	Western Union Company	0.02$
C	Citigroup Inc.	0.01$	WFC	Wells Fargo & Company	0.01$	USB	U.S. Bancorp	0.02$
CPWR	Ocean Thermal Energy	0.01$	MO	Altria Group Inc.	0.01$	MCHP	Microchip Technology Inc.	0.02$
NVDA	Nvidia Corp.	0.01$	VZ	Verizon Communications	0.01$	SO	Southern Company	0.02$
YHOO	Yahoo Inc.	0.01$	DHI	D. R. Horton Inc.	0.01$	BJS	BJ's Wholesale Club Holdings	0.02$
BMY	Bristol-Myers Squibb Co.	0.01$	SNDK	Sandisk Corp.	0.01$	MAS	Masco Corp.	0.02$
ALTR	Altair Engineering Inc.	0.01$	CCE	Coca-Cola Enterprises	0.01$	NWL	Newell Brands Inc.	0.02$
GLW	Corning Inc.	0.01$	NI	NiSource Inc.	0.01$	M	Macy's Inc.	0.02$
JDSU	JDS Uniphase Corp.	0.01$	EXPE	Expedia Group Inc.	0.01$	CVS	CVS Health Corp.	0.02$
NCC	NCC Group	0.01$	AIG	American International	0.01$	CTSH	Cognizant Tech. Solutions	0.02$
EBAY	eBay Inc.	0.01$	INTU	Intuit Inc.	0.01$	GNW	Genworth Financial Inc.	0.02$
EP	El Paso Corp.	0.01$	LTD	Limited Brands Inc.	0.01$	DFS	Discover Financial Services	0.02$
XRX	Xerox Holdings Corp.	0.01$	CNP	CenterPoint Energy Inc.	0.02$	KEY	KeyCorp	0.02$
WIN	Windstream Holdings Inc.	0.01$	QCOM	Qualcomm Inc.	0.02$	ADI	Analog Devices Inc.	0.02$
XLNX	Xilinx Inc.	0.01$	JBL	Jabil Inc.	0.02$	SYY	Sysco Corp.	0.02$

[1] Average spread from 9:40:00 to 15:50:00 New York time during 2008.

Table 3. Information of the stocks in Group 1.

Group 1

Symbol	Company	Spread[1]
WMB	Williams Companies Inc.	0.02$
SE	Sea Ltd American Dep.	0.02$
PG	Procter & Gamble Co.	0.02$
CIEN	Ciena Corp.	0.02$
LEN	Lennar Corp.	0.02$
AKAM	Akamai Technologies	0.02$
UNH	UnitedHealth Group Inc.	0.02$
DD	DuPont de Nemours Inc.	0.02$
KLAC	KLA Corp.	0.02$
CIT	CIT Group Inc.	0.02$
LEG	Leggett & Platt Inc.	0.02$
WFMI	Whole Foods Market Inc.	0.02$
MS	Morgan Stanley	0.02$
VRSN	VeriSign Inc.	0.02$
AN	AutoNation Inc.	0.02$
RHT	Red Hat Inc.	0.02$
CTXS	Citrix Systems Inc.	0.02$
MDT	Medtronic plc.	0.02$
NBR	Nabors Industries	0.02$
MOLX	Molex Inc.	0.02$
GILD	Gilead Sciences Inc.	0.02$
CHK	Chesapeake Energy	0.02$
TJX	TJX Companies Inc.	0.02$
AMGN	Amgen Inc.	0.02$
SWY	Safeway Inc.	0.02$
XOM	Exxon Mobil Corp.	0.02$
STZ	Constellation Brands	0.02$
ADSK	Autodesk Inc.	0.02$
LLY	Eli Lilly and Company	0.02$
CTAS	Cintas Corp.	0.02$
LIZ	Liz Claiborne Inc.	0.02$
GCI	Gannett Co. Inc.	0.02$
AXP	American Express Co.	0.02$
TIE	Titanium Metals Corp.	0.02$
SAI	SAIC Inc.	0.02$
PDCO	Patterson Companies	0.02$
WYE	Wyeth Inc.	0.02$
COH	Cochlear Ltd.	0.02$
CVG	Convergys Corp.	0.02$
WDC	Western Digital Corp.	0.02$
AVP	Avon Products Inc.	0.02$
A	Agilent Technologies	0.02$
JNY	Jones Apparel Group	0.02$
SLM	SLM Corp.	0.02$

Symbol	Company	Spread[1]
TGT	Target Corp.	0.02$
VLO	Valero Energy Corp.	0.02$
MMC	Marsh & McLennan Co.	0.02$
CPB	Campbell Soup Company	0.02$
TYC	Tyco International PLC	0.02$
MCD	McDonald's Corp.	0.02$
HON	Honeywell International Inc.	0.02$
DF	Dean Foods Company	0.02$
NWSA	News Corp.	0.02$
URBN	Urban Outfitters Inc.	0.02$
RX	Recylex	0.02$
BBY	Best Buy Inc.	0.02$
CCL	Carnival Corp.	0.02$
WMI	WMI Investment Corp.	0.02$
POM	Polymet Mining Corp.	0.02$
ERTS	Electronic Arts Inc.	0.03$
MBI	MBIA Inc.	0.03$
ADM	Archer-Daniels-Midland Co.	0.03$
PEP	PepsiCo Inc.	0.03$
CBG	CBRE Group Inc.	0.03$
IR	Ingersoll Rand Inc.	0.03$
HNZ	Heinz Company	0.03$
CTX	Qwest Corp.	0.03$
TSO	Tesoro Corp.	0.03$
IGT	International Game Tech.	0.03$
WYN	Wynnstay Group PLC	0.03$
GT	The Goodyear Tire & Rubber	0.03$
JCI	Johnson Controls Int.	0.03$
JWN	Nordstrom Inc.	0.03$
FRX	Fennec Pharmaceutical Inc.	0.03$
FHN	First Horizon National Corp.	0.03$
ABT	Abbott Laboratories	0.03$
ADP	Automatic Data Processing	0.03$
PBG	Pacific Brands Ltd.	0.03$
ROST	Ross Stores Inc.	0.03$
KBH	KB Home	0.03$
YUM	Yum! Brands Inc.	0.03$
MAR	Marriott International	0.03$
STJ	St Jude Medical Inc.	0.03$
FDO	Family Dollar Stores Inc.	0.03$
ED	Consolidated Edison Inc.	0.03$
UNM	Unum Group	0.03$
ORLY	O'Reilly Automotive Inc.	0.03$
SVU	SUPERVALU Inc.	0.03$
EMR	Emerson Electric Company	0.03$

Symbol	Company	Spread[1]
MI	Marshall and Lisley Corp.	0.03$
AEP	American Electric Power Co.	0.03$
NEM	Newmont Corp.	0.03$
MRO	Marathon Oil Corp.	0.03$
ITW	Illinois Tool Works Inc.	0.03$
FFIV	F5 Networks Inc.	0.03$
CVX	Chevron Corp.	0.03$
PCAR	PACCAR Inc.	0.03$
OMC	Omnicom Group Inc.	0.03$
XRAY	DENTSPLY SIRONA Inc.	0.03$
COP	ConocoPhillips	0.03$
PCS	MetroPCS Communications	0.03$
RHI	Robert Half International Inc.	0.03$
SEE	Sealed Air Corp.	0.03$
COST	Costco Wholesale Corp.	0.03$
FIS	Fidelity National Info. Services	0.03$
PKI	PerkinElmer Inc.	0.03$
BMC	BMC Software Inc.	0.03$
RRD	R.R. Donnelley & Sons Co.	0.03$
UTX	United Technologies Corp.	0.03$
D	Dominion Energy Inc.	0.03$
PBI	Pitney Bowes Inc.	0.03$
ACAS	American Capital Ltd.	0.03$
K	Kellogg Company	0.03$
JCP	J. C. Penney Company	0.03$
AMT	American Tower Corp.	0.03$
ALL	Allstate Corp.	0.03$
MWV	MeadWestvaco Corp.	0.03$
HRB	H&R Block Inc.	0.03$
NYT	New York times Company	0.04$
RDC	Redcape Hotel Group	0.04$
PTV	Pactiv Company	0.04$
FISV	Fiserv Inc.	0.04$
EXPD	Expeditors Int.of Washington	0.04$
BBT	BB&T Corp.	0.04$
PCG	Pacific Gas & Electric Co.	0.04$
BIG	Big Lots Inc.	0.04$
KMX	CarMax Inc.	0.04$
TSS	Total System Services Inc.	0.04$
BK	The Bank of N. Y. Mellon Corp.	0.04$
TEL	Tellurian Inc.	0.04$
KSS	Kohl's Corp.	0.04$
CAT	Caterpillar Inc.	0.04$
HSY	The Hershey Company	0.04$
GIS	General Mills Inc.	0.04$

[1] Average spread from 9:40:00 to 15:50:00 New York time during 2008.

Table 4. Information of the stocks in Group 1 and 2.

Group 1

Symbol	Company	Spread[1]
HAS	Hasbro Inc.	0.04$
XTO	XTO Energy Inc.	0.04$
PPL	PPL Corp.	0.04$
HOG	Harley-Davidson Inc.	0.04$
UPS	United Parcel Service	0.04$
HSP	Hospira Inc.	0.04$
CTL	CenturyLink Inc.	0.04$
TDC	Teradata Corp.	0.04$
BA	Boeing Company	0.04$
BAX	Baxter International Inc.	0.04$
PGN	Progress Energy Inc.	0.04$
DRI	Darden Restaurants Inc.	0.04$
JNS	Janus Capital Group Inc.	0.04$
BMS	Bristol-Myers Squibb	0.04$
RTN	Raytheon Company	0.04$
CVC	Cablevision Systems	0.04$
PWR	Quanta Services Inc.	0.04$
LXK	Lexmark International	0.04$
CELG	Celgene Corp.	0.04$
MMM	3M Company	0.04$
RSG	Republic Services Inc.	0.04$
DNR	Denbury Resources Inc.	0.04$
MTW	Manitowoc Company	0.04$
NE	Noble Corp.	0.04$
NDAQ	Nasdaq Inc.	0.04$
CINF	Cincinnati Financial	0.04$
BIIB	Biogen Inc.	0.04$
KMB	Kimberly-Clark Corp.	0.04$
EIX	Edison International	0.04$
NRG	NRG Energy Inc.	0.04$
IVZ	Invesco Ltd.	0.04$
AEE	Ameren Corp.	0.04$
AAPL	Apple Inc.	0.04$
CAH	Cardinal Health Inc.	0.04$
MCO	Moody's Corp.	0.04$

Group 2

Symbol	Company	Spread[1]
HOT	Hot Topic Inc.	0.05$
GPC	Genuine Parts Co	0.05$
PEG	Public Service Ent.	0.05$
EFX	Equifax Inc.	0.05$
COV	Covidien Ltd.	0.05$
AET	Aetna Inc.	0.05$
XL	XL Group Ltd.	0.05$

Symbol	Company	Spread[1]
DOV	Dover Corp.	0.05$
VIAB	Viacom Inc.	0.05$
ABC	AmerisourceBergen Corp.	0.05$
APC	Anadarko Petroleum Corp.	0.05$
CBE	Cooper Industries	0.05$
FAST	Fastenal Company	0.05$
MHP	McGraw-Hill Companies Inc.	0.05$
AMZN	Amazon.com Inc.	0.05$
EQR	Equity Residential	0.05$
CL	Colgate-Palmolive Company	0.05$
ECL	Ecolab Inc.	0.05$
MKC	McCormick & Company Inc.	0.05$
AOC	Aon Corp.	0.05$
TXT	Textron Inc.	0.05$
CSX	CSX Corp.	0.05$
PNW	Pinnacle West Capital Corp.	0.05$
KIM	Kimco Realty Corp.	0.05$
WPI	Watson Pharmaceuticals	0.05$
MET	Metlife Inc.	0.05$
UST	ProShares Ultra	0.05$
TMO	Thermo Fisher Scientific Inc.	0.05$
HCP	HCP Inc.	0.05$
SIAL	Sigma-Aldrich Corp.	0.05$
FLIR	FLIR Systems Inc.	0.05$
EL	Estee Lauder Companies	0.05$
AYE	Allegheny Energy Inc.	0.05$
CI	Cigna Corp.	0.05$
NFLX	Netflix Inc.	0.05$
CHRW	C.H. Robinson Worlwide Inc.	0.05$
GENZ	Genzyme Corp.	0.05$
DDR	DDR Corp.	0.05$
WFT	West Fraser Timber Co. Ltd.	0.05$
MHS	Medco Health Solutions Inc.	0.05$
DTE	DTE Energy Company	0.05$
TRV	The Travelers Companies	0.05$
NYX	NYSE Group IPO	0.05$
COF	Capital One Financial Corp.	0.06$
WLP	WellPoint Inc.	0.06$
TWC	TWC Entreprises Ltd.	0.06$
SWN	Southwestern Energy Co.	0.06$
NSC	Norfolk Southern Corp.	0.06$
CMA	Comerica Inc.	0.06$
NKE	Nike Inc.	0.06$
EQ	Equillium Inc.	0.06$
CLX	Clorox Company	0.06$

Symbol	Company	Spread[1]
CAM	Corporate Actions Middleware	0.06$
RAI	Reynolds American Inc.	0.06$
NOC	Northrop Grumman Corp.	0.06$
PLL	Piedmont Lithium Ltd.	0.06$
SYK	Stryker Corp.	0.06$
SRE	Sempra Energy	0.06$
TIF	Tiffany & Co.	0.06$
NUE	Nucor Corp.	0.06$
OXY	Occidental Petroleum Corp.	0.06$
FPL	First Trust New Opportunities	0.06$
ESRX	Express Scripts Holding Co.	0.06$
AIV	Apartment Investment Co.	0.06$
BHI	Boulevard Holdings Inc.	0.06$
FCX	Freeport-McMoRan Inc.	0.06$
APOL	Apollo Group Inc.	0.06$
SCG	Scentre Group Ltd.	0.06$
CB	Chubb Limited	0.06$
IBM	Int. Business Machines Corp.	0.06$
GME	GameStop Corp.	0.06$
DE	Deere & Company	0.06$
LNC	Lincoln National Corp.	0.06$
AVY	Avery Dennison Corp.	0.07$
MCK	McKesson Corp.	0.07$
PLD	Prologis Inc.	0.07$
DGX	Quest Diagnostics Inc.	0.07$
CERN	Cerner Corp.	0.07$
IRM	Irom Mountain Inc.	0.07$
COL	Rockwell Collins Inc.	0.07$
NU	NeutriSci International Inc.	0.07$
CVH	Coventry Health Care Inc.	0.07$
WFR	MEMC Electronic Materials	0.07$
AGN	Allergan PLC.	0.07$
GAS	GAS	0.07$
APH	Amphenol Corp.	0.07$
EXC	Exelon Corp.	0.07$
WEC	WEC Energy Group Inc.	0.07$
AFL	AFLAC Inc.	0.07$
NOV	National Oilwell Varco Inc.	0.07$
SUN	Sunoco LP	0.07$
BTU	Peabody Energy Corp.	0.07$
ROK	Rockwell Automation Inc.	0.07$
PCL	Plum Creek Timber Co. Inc.	0.07$
JOYG	Joy Global Inc.	0.07$
AMP	Ameriprise Financial Inc.	0.07$
TAP	Molson Coors Beverage Co.	0.07$

[1] Average spread from 9:40:00 to 15:50:00 New York time during 2008.

Table 5. Information of the stocks in Group 2 and 3.

Group 2

Symbol	Company	Spread[1]
GD	General Dynamics Corp.	0.07$
CSC	Computer Sciences Corp.	0.07$
FII	Federated Investors Inc.	0.07$
HIG	Hartford Fin. Services Group	0.07$
ITT	ITT Inc.	0.07$
TROW	T. Rowe Price Group Inc.	0.07$
MDP	Meredith Corp.	0.07$
GR	Goodrich Corp.	0.07$
AKS	AK Steel Holding Corp.	0.07$
PFG	Principal Financial Group	0.07$
HUM	Humana Inc.	0.07$
STI	SunTrust Banks Inc.	0.07$
CMI	Cummins Inc.	0.08$
STR	Questar Corp.	0.08$
ZMH	Zimmer Holdings Inc.	0.08$
FO	Fortune Brands Inc.	0.08$
SII	Sprott Inc.	0.08$
HRL	Hormel Foods Corp.	0.08$
OI	O-I Glass Inc.	0.08$
HCN	Welltower Inc.	0.08$
ANF	Abercrombie & Fitch Co.	0.08$
LM	Legg Mason Inc.	0.08$
FDX	FedEx Corp.	0.08$
PRU	Prudential Financial Inc.	0.08$
DHR	Danaher Corp.	0.08$
WY	Weyerhaeuser Company	0.08$
OKE	ONEOK Inc.	0.08$
SRCL	Stericycle Inc.	0.09$
LUK	Leucadia National Corp.	0.09$
BLL	Ball Corp.	0.09$
FE	FirstEnergy Corp.	0.09$
SWK	Stanley Black & Decker Inc.	0.09$
ESV	Ensco PLC	0.09$
SHW	Sherwin-Williams Company	0.09$
BDX	Becton, Dickinson and Co.	0.09$
VAR	Varian Medical Systems Inc.	0.09$
TEG	Ten Ent. Group PLC	0.09$
PH	Parker-Hannifin Corp.	0.09$
ACE	ACE Ltd.	0.09$
BNI	Burlington Northern Santa Fe	0.09$
ACS	Affiliated Computer Services	0.09$
PXD	Pioneer Natural Resources	0.09$
AIZ	Assurant Inc.	0.09$
WAT	Waters Corp.	0.09$
ATI	Allegheny Technologies Inc.	0.09$
ETN	Eaton Corp.	0.09$
VTR	Ventas Inc.	0.09$

Group 3

Symbol	Company	Spread[1]
HAR	Harman Int. Industries Inc.	0.10$
NFX	Nasdaq Futures	0.10$
EQT	EQT Corp.	0.10$
HRS	Harris Corp.	0.10$
COG	Cabot Oil & Gas Corp.	0.10$
PPG	PPG Industries Inc.	0.10$
MEE	Massey Energy Company	0.10$
HP	Helmerich & Payne Inc.	0.10$
DVN	Devon Energy Corp.	0.10$
IFF	Int. Flavors & Fragrances	0.10$
LH	Laboratory Corp.	0.10$
UNP	Union Pacific Corp.	0.10$
EW	Edwards Lifesciences Corp.	0.10$
STT	State Street Corp.	0.10$
RRC	Range Resources Corp.	0.10$
SJM	J. M. Smucker Company	0.10$
SNA	Snap-On Inc.	0.11$
PNC	PNC Fin. Services Group	0.11$
CEG	Constellation Energy Group	0.11$
LMT	Lockheed Martin Corp.	0.11$
FTI	TechnipFMC PLC	0.11$
CNX	CNX Resources Corp.	0.11$
ARG	Amerigo Resources Ltd.	0.11$
DVA	DaVita Inc.	0.11$
GS	Goldman Sachs Group Inc.	0.11$
TMK	Torchmark Corp.	0.12$
EMN	Eastman Chemical Co.	0.12$
RIG	Transocean Ltd.	0.12$
PX	Pelangio Exploration Inc.	0.12$
RL	Ralph Lauren Corp.	0.12$
R	Ryder System	0.12$
MIL	Millipore Corp.	0.12$
CRM	Salesforce.com Inc.	0.12$
NTRS	Northern Trust Corp.	0.12$
APD	Air Products and Chemicals	0.12$
ANR	Alpha Natural Resources	0.13$
NBL	Noble Energy Inc.	0.13$
APA	Apache Corp.	0.13$
FMC	FMC Corp.	0.13$
BDK	Black and Decker Corp.	0.13$
MUR	Murphy Oil Corp.	0.13$
ROP	Roper Technologies Inc.	0.13$
JEC	Jura Energy Corp.	0.13$
HES	Hess Corp.	0.13$
ETR	Entergy Corp.	0.13$
MON	Monsanto Company	0.14$
DV	Dolly Varden Silver Corp.	0.14$
VFC	V.F. Corp.	0.14$
GWW	W. W. Grainger Inc.	0.14$
WHR	Whirlpool Corp.	0.15$
MOS	Mosaic Company	0.15$
SPG	Simon Property Group	0.15$
EOG	EOG Resources Inc.	0.15$
VMC	Vulcan Materials Co	0.16$
X	United States Steel Corp.	0.16$
SHLD	Sears Holding	0.16$
LLL	L3 Technologies Inc.	0.17$
WYNN	Wynn Resorts Ltd.	0.17$
BXP	Boston Properties Inc.	0.17$
PSA	Public Storaga	0.17$
PCP	Precision Castparts Corp.	0.18$
VNO	Vornado Realty	0.18$
DNB	Dun & Bradstreet Corp.	0.19$
CLF	Cleveland-Cliffs Inc.	0.19$
FLR	Fluor Corp.	0.20$
PCLN	Priceline Group Inc.	0.20$
MTB	M&T Banc Corp.	0.20$
AVB	AvalonBay Comm. Inc.	0.21$
BEN	Franklin Resources Inc.	0.21$
DO	Diamond Offshore Drilling	0.23$
CF	CF Industries Holdings	0.24$
AZO	AutoZone Inc.	0.25$
FLS	Flowserve Corp.	0.27$
ICE	Intercontinental Exc. Inc.	0.28$
MA	Mastercard Inc.	0.38$
FSLR	First Solar Inc.	0.38$
CMG	Chipotle Mexican Grill	0.38$

[1] Average spread from 9:40:00 to 15:50:00 New York time during 2008.

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