LECTURE 30 - ELECTRIC FIELD POSTULATES

The property of air acting as a perfect dielectric is the reason that wireless communication is possible. Air can hold on to electric and magnetic fields perfectly. The explaination to this was given by Pauli's exclusion principle, or rather chemistry in general. This was followed by the formation of dipole and valence/conduction band in semiconductors. The parameter used for describing the medium's capability to hold on to electric field was permittivity, for which we defined electric flux.

> Lower the permittivity, the medium will hold on to electric field. Higher the permittivity, more and more free charges are created and the ability to hold on to electric field decreases.

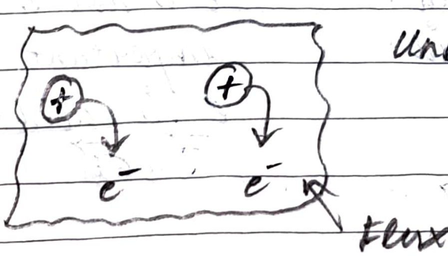
$$\vec{D}_{induced} = \varepsilon \vec{E}_{applied}$$

(Flux) ↳ Number of dipoles created

(or)

Number of free charges

Unbounded



$$\boxed{\text{Unit of } \vec{D} = \frac{C}{m^3}}$$

This is significant for lossy dielectric whereas for conductors and insulators, this is not a significant parameter.

In the same vein, for totally conducting medium where permittivity (ϵ) is meaningless, we have another parameter to describe the conducting media.

$$\vec{J} = \sigma \vec{E}$$

induced — applied

Conduction current density — conductivity (s/m)

The idea of conductivity, as mentioned above, was already in vogue, courtesy Ohm's law. Therefore, whenever we study EM, we should remember that lots of experiments were done between 1800 – 1950s in parallel by famous scientists such as Ohms, Faraday, Maxwell, Ampere, Hertz, etc. But when textbooks were written after 1950, they followed a sequential fashion so that the readers could understand the chronology.

	Insulators	Lossy Dielectrics	conductors
①	Field is maintained.	Field is maintained with loss.	Field is lost.
②	No J is created.	J is created.	Huge J is created.
③	ϵ is valid. σ is not much important.	ϵ is valid. σ is valid.	ϵ is not much important. σ is valid.
④	$\vec{D} = \epsilon\vec{E}$	$\vec{D} = \epsilon\vec{E}$ $\vec{J} = \sigma\vec{E}$	$\vec{J} = \sigma\vec{E}$

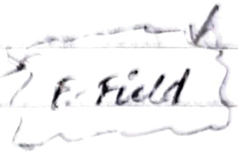
EM, alone, is a very big domain. We are studying it for communication (wireless) purposes only. It is the subject of interest for ECE engineers.

We are going to define postulates for electric field. These are some rules and functionalities that were observed while doing experiments.

If you have a source that is creating an E-field in a medium with permittivity ε, then -

Numerical Differentiation ← $\boxed{\begin{aligned} \nabla \times \vec{E} &= 0 \\ \nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon} \end{aligned}}$ (curl) (divergence) Source

Area of interest



may/may not include the source

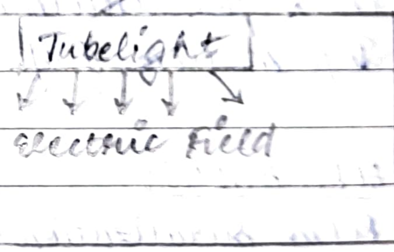
where ρ is any free charge density in the medium. If the source produces a static E-field (field that does not vary with time), then the field obeys the above postulates. The main aim is -

$$\boxed{\vec{E} = f(\text{sources, charges, } \varepsilon)}$$
$$= f(\text{sources, any geometry, medium})$$

★ **EXAMPLE:**

Let's say you are sitting in a room with tubelight (source). This source gives out intense E-field in the area. This E-field will satisfy the above postulates.

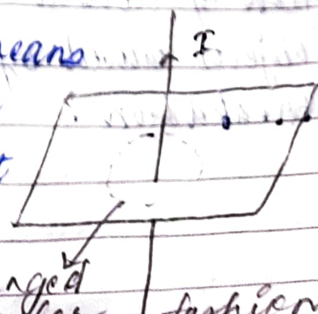
Source



Conference Room

$\nabla \times \vec{E} = 0$ implies that the rotational effect is absent. This happens only in the case of an electric field whereas, in a magnetic field, some conduction current is generated. This experiment was performed by Oersted.

The divergence not equal to zero means that there are sources and sinks to disturb the vector field, i.e., its not free-flowing.



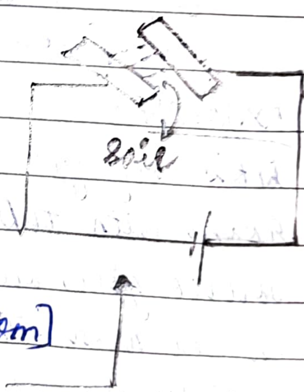
The filings arranged themselves in a circular fashion.

In the previous example, let us have some LED TVs and laptops present in the conference room. These have seperate charge densities and flux lines. This will tend to disturb the electric field. The capacity of disturbances to alter the $\vec{E}$-field in area is being described.

In case there are no disturbances in the room, just the tubelight (source) and a person standing inside. Therefore, the divergence becomes zero. The vector function can be characterized easily, therefore upon solving the differential equation, we can get the idea of charge density created by the tubelight.

★ **PROBLEM DEFINITION:**

1. Identify the source/charge.
2. Check for disturbances.
3. Information of field required.



★ **APPLICATIONS:**

1. Room steralization using UV [cleanroom]
2. Water-less Electric Field Agriculture
3. Electrostatic Precipitators for Pollution Control in Cement Plants
4. Remove Particles from Vehicle Emissions

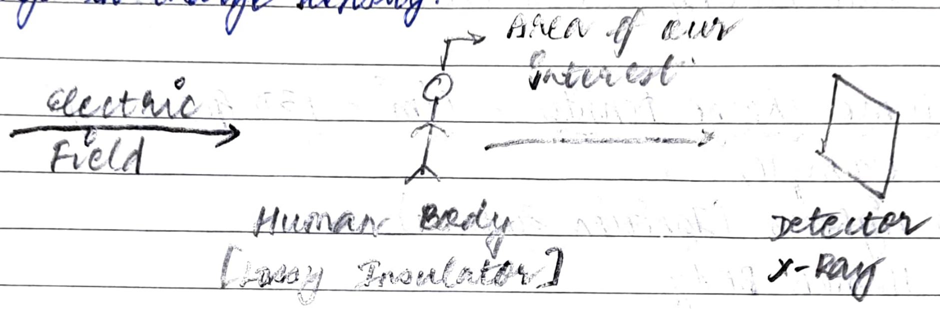
★ **ELECTROSTATIC DISCHARGE:**

Electrostatic discharge (ESD) is a sudden and momentary flow of electric current between two electrically charged objects caused by contact, an electrical short or dielectric breakdown.

LECTURE 31 - ELECTRIC FIELD EXPRESSIONS

Q. What is a static field?

A. Electric field due to a stationary (or) time-invariant charge (or) charge density. For example, in diagnostics, when we take an x-ray scan, we are monitoring the change in charge density.



In the body, J and D are induced. The variations in induced charge density in a healthy and diseased cases have been studied. The entire machine works on the this principle. During our time of diagnosis, we assume the field to be static for a short period of time.

Using the two postulates studied in the previous lecture, we may either create mathematical expressions for E or estimate the nature of source that created the known field.

* TYPES OF CHARGE DENSITIES:

① Line charge Density:- C/m - [1D Geometry]
For example,
(i) wires carrying data
(ii) wires carrying current (railway traction)
(iii) tube light
(iv) Levitation structures - Bullet Trains
PTO

② Surface Charge Density :- C/m^2 – [2D Geometry]
For example,
(i) Laptop / Touch Screens
(ii) LED / LCD Screen Display
(iii) Xerox (Repregraphic Technique)
(iv) Writing Pads / Tablets
(v) Skin heating due to excessive mobile exposure.

③ Volume Charge Density :- C/m^3 – [3D Geometry]
For example,
(i) Lighters (Ignition Systems)
(ii) Human Body
(iii) Projectors

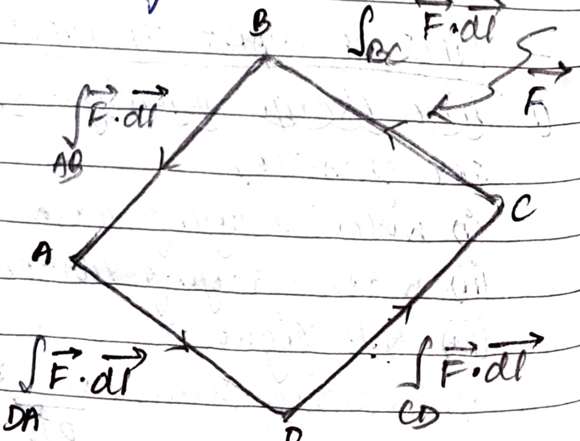
In many analysis, the source may be a combination of all these charge densities. It depends on the engineers' skill in identifying one and working on that, as per requirement.

* STOKES LAW: 1D → 2D
For any vector quantity, say force $\vec{F}$,

$$\boxed{\oint_c \vec{F}\cdot\vec{dl} = \int_s (\nabla\times\vec{F})\, ds}$$

closed curve enclosing a surface.

$$\int_{AB} \vec{F}\cdot\vec{dl} + \int_{BC} \vec{F}\cdot\vec{dl} + \int_{CD} \vec{F}\cdot\vec{dl} + \int_{DA} \vec{F}\cdot\vec{dl}$$

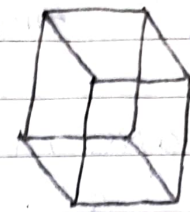


Four integrals reduced to one, if $\nabla\times\vec{F}$ exists. This law makes computation easier, provided $\vec{F}$ is differential.

* DIVERGENCE THEOREM : (GAUSS THEOREM) 2D → 3D

$$\boxed{\oint_S \vec{F}\cdot\vec{ds} = \int_V (\nabla\cdot F)\, dV}$$

surface enclosing a volume V

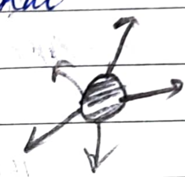


cube

6 sides
6 surface I
1 volume I

* RELATIONSHIP BETWEEN ELECTRIC FIELD AND VOLUME CHARGE DENSITY :

Consider a very small (relatively) volume charge c/m³. From experimental observations, we draw that the effect of charge is isotropic; i.e., the effect is equal over a ~~similar~~ spherical geometry similar to a candle.



This is an example of volume charge density, c/m³ on a three dimensional geometry.

Using surface integral,

$$\nabla\cdot E = \frac{\rho}{\varepsilon}$$

→ Integrating on both sides;

Applying divergence theorem,

$$\int_S \vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon}$$ [ρ (c/m³) when integrated over a volume yields q.]

Here, $\vec{E} = E_R a_R + E_\theta a_\theta + E_\phi a_\phi$

since a spherical surface is considered, $\vec{E}$ is taken in spherical coordinates. From observations, only E_R exists.

$\vec{E} = E_R a_R$; $\vec{ds} = R\sin\theta\, R d\theta\, d\phi\, a_R$

[a_θ and a_ϕ need not be considered]

$\vec{E}\cdot\vec{ds} = E_R R^2 \sin\theta\, d\theta\, d\phi$

$$\iint_{sphere} E_R R^2 \sin\theta\, d\theta\, d\phi = \frac{q}{\varepsilon}$$

$\theta \to 0 \to 180^\circ$ (π)

$\phi \to 0 \to 360^\circ$ (2π)

$E_R \to$ constant over sphere

Upon solving the integral, we find that -

$$\boxed{E_R = \frac{q}{4\pi\epsilon R^2} \Rightarrow \vec{E} = \frac{q}{4\pi\epsilon R^2}\, a_R}$$ Spherical System

Force experienced by a test charge 'q' -

$$\boxed{\vec{F} = q\vec{E} = \frac{qq}{4\pi\epsilon R^2}\, a_R}$$ [Coloumb's law between charges]

If the point charge q is very small,

$$\boxed{\vec{E} = \frac{q}{4\pi\epsilon R^2}\, a_R}$$

LECTURE 32 - LINE CHARGE DENSITY

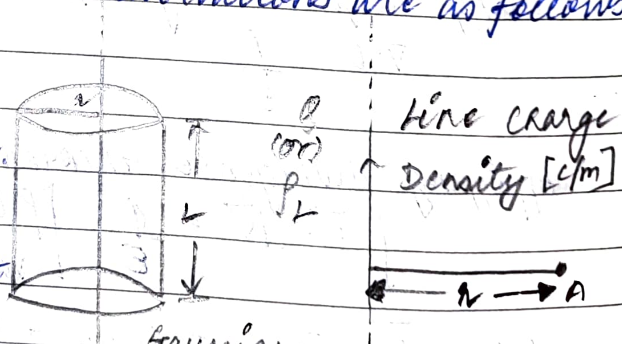
Let us consider a one-dimensional structure having a charge or charge density. Assuming that the thickness of the object is very small, the experimental observations are as follows -

(i) We have a point A at a distance r from the wire.

(ii) At that r along ϕ (0°-360°), the field does not change.

(iii) The field changes only with r.

(iv) The result is observed as a cylindrical symmetry.



Gaussian Cylinder
↓
Surface on which the field is uniform.

In point charge, we drew a gaussian sphere. Whereas, here we have a gaussian cylinder of length L and radius r.

$\overrightarrow{PTO}$

We have the postulate –

$$\nabla \cdot E = \frac{\rho}{\varepsilon}$$

where ρ is always the volume charge density in c/m^3.

Taking volume integral on both sides, we have –

$$\int_V (\nabla \cdot E)\, dV = \int_V \frac{\rho}{\varepsilon}$$

where V is the volume of the cylinder that we have drawn.

a_r is present;

along a_ϕ, the value is constant;

along a_z, we have no field present.

On applying divergence theorem, we find that –

$$\int_S \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon} = \frac{\rho_L L}{\varepsilon}$$

Here $\vec{E} = E_r a_r$

$\vec{dS} = dz\, r d\phi\, a_r + dz\, dr\, a_\phi + dr\, r d\phi\, a_z$

$\therefore \vec{E} \cdot \vec{dS} = E_r\, r d\phi\, dz$

Hence, $\displaystyle\int_0^L \int_0^{2\pi} E_r\, r d\phi\, dz = \frac{\rho_L L}{\varepsilon}$

Upon solving the integral, we find that –

$$\boxed{E_r = \frac{\rho_L}{2\pi r \varepsilon} \Rightarrow \vec{E} = \frac{\rho_L}{2\pi r \varepsilon} a_r \ V/m}$$

Cylindrical System

In all the expressions,

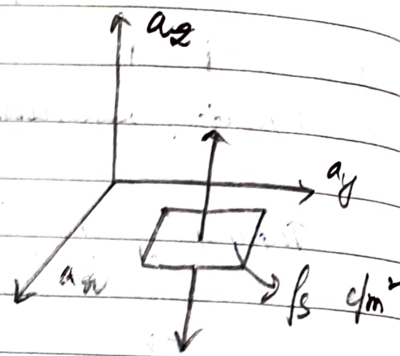
$\varepsilon = \varepsilon_0 \varepsilon_r$

For air, $\varepsilon_r = 1$.

LECTURE 33 - NUMERICALS ON ELECTRIC FIELD

* SURFACE CHARGE DENSITY :

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon} a_z}$$



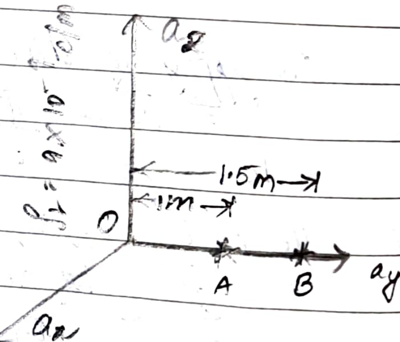
Here, the distance parameter (R/r) is missing. Electric field is always created perpendicular to the surface.

Q1. A line charge density $\rho_L = 9 \times 10^{-9}$ c/m is placed along a_z at the origin. Determine the electric field at a distance of 1m and 1.5m from the source along a_y direction. The medium is air by default.

A1. Line charge density,

$$\vec{E} = \frac{\rho_L}{2\pi r \varepsilon} a_r \text{ V/m}$$

Here $a_r = a_y$.



(i) At a distance of 1m,

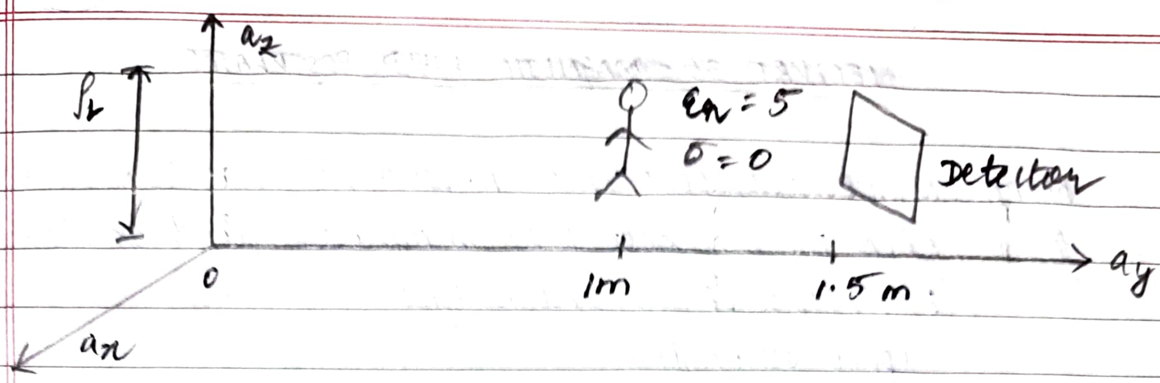
$$\vec{E} = \frac{9\times10^{-9} \cdot a_y}{2\pi\times1\times8.854\times10^{-12}} = 161.779\ a_y \text{ V/m}$$

(ii) At a distance of 1.5m,

$$\vec{E} = \frac{9\times10^{-9} \cdot a_y}{2\pi\times1.5\times8.854\times10^{-12}} = 107.852\ a_y \text{ V/m}$$

Q2. In the above question, we have a human body at 1m, which can be considered as a surface. Let $\varepsilon_r = 5$, $\sigma = 0$. An electric field detector is kept at 1.5m. Determine the total field received by the detector.

A2.



Without any disturbance (human body),

$$\vec{E}_{1m} = 161.779\ a_y\ V/m$$

$$\vec{E}_{1.5m} = 107.852\ a_y\ V/m$$

We need to find that with a disturbance at y=1m, what will be the field at 1.5m.

→ STEP 1:- Effect of Disturbance -

electric flux created in the new medium,

$$D_{\text{induced in human body}} = \epsilon \vec{E}_{\text{applied}}$$

$$= \epsilon_0 \epsilon_r \times \vec{E}_{\text{applied at 1m}}$$

$$= 5 \times 8.854 \times 10^{-12} \times 161.779$$

$$= 7.1619 \times 10^{-9}\ C/m^2$$

This can be considered as a surface charge density kept at y=1m, i.e., $\rho_s = 7.1619 \times 10^{-9}\ C/m^2$.

→ STEP 2:- Total Field -

$$\vec{E}_{\text{Total}} = \vec{E}_{\text{due to source at 1.5m}} + \vec{E}_{\text{due to disturbance at 1m}}$$

$$= \left[107.852 + \frac{\rho_s}{2\epsilon_0}\right] a_y\ V/m$$

$$= 512.2995\ a_y\ V/m.$$

LECTURE 35 - MAGNETIC FIELD POSTULATES

Bring an isolated pole into an area, if that experiences a force, then we can say that magnetic field exists.

$\vec{F} \cong q \times$ [Field Intensity]
(pole strength)

But, it is not possible to isolate north and south pole physically.

Therefore, magnetic field is studied in two ways -
1. Magnetic field due to a magnet.
2. Magnetic field due to a current carrying conductor.

Electromagnetism infers to the study of the second point above. Our source is a current-carrying conductor. This will involve changing the definitions slightly.

The force experienced by a moving charge is the indication of presence of a magnetic field.

In electric field, $\vec{F} = q\vec{E}$

In magnetic field, $\vec{F} = q\,[\vec{V} \times \vec{B}]$;

where q is the charge
$\vec{V}$ is the velocity of charge
$\vec{B}$ is the magnetic flux density.

This is known as the Lorentz force. We use this definition for defining magnetic field. Here,

$$F = \underbrace{q}_{\text{current}}(\vec{V} \times \vec{B})$$
$\hookrightarrow$ Flux Density

In electric field, $\vec{D}_{induced} = \varepsilon \vec{E}_{applied}$

In magnetic field, $\vec{B}_{induced} = \mu \vec{H}_{applied}$

By varying current, we could vary the magnetic field. A current carrying conductor provided more flexibility compared to a magnet.

* **DIMENSIONAL ANALYSIS :**

$$\vec{F} = q\vec{E}$$

$$N = C \cdot \frac{V}{m}$$

$$N = A \cdot s \cdot \frac{V}{m}$$

⇒ Force x Distance = Power x Time
(Energy) (Energy)

The aim of this study involves writing a function for magnetic field due to a current carrying conductor -

$$\vec{H} = f(I, \text{distance}, \text{permeability})$$

↳ Field Intensity

* **PERMEABILITY :**

$$\mu = \mu_r \mu_0 = \mu_r \times 4\pi \times 10^{-7} \ H/m$$

(Air / Vaccum)

Permeability is constant is either cases of study, due to a magnet or current carrying conductor.

* **MAGNETIC FLUX LINES :**

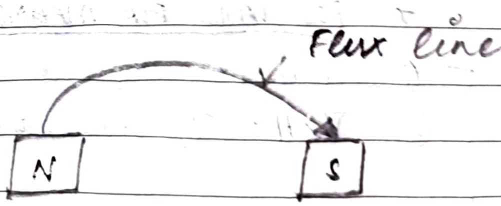
the number of lines per unit area.

Unit - Wb/m^2

↓

Flux Density (B)

(Imaginary Field Lines)



LECTURE 36 - MAGNETIC FIELD EXPRESSIONS

Q. Why did we use magnetic flux in Lorentz equation?

A. According to Faraday's law,

$$V = -\frac{\partial}{\partial t}\phi \; ; \; \phi \text{ - magnetic Flux}$$

$$V = -\frac{\partial}{\partial t}(BA) \; ; \; B \text{ - Flux Density}$$
A - Area Unit

Hence, the SI unit becomes, $B = \frac{Vs}{m^2} \left(\frac{Wb}{m^2}\right)$

Therefore, $\vec{F} = q(\vec{V} \times \vec{B})$ alone provides a correct dimensionality as opposed to $\vec{F} = q(\vec{V} \times \vec{H})$.

★ **UNIT OF FIELD INTENSITY:**

$$\vec{B} = \mu \vec{H} \Rightarrow \vec{H} = \frac{B}{\mu} = \frac{Vs/m^2}{Vs/Am} = \frac{A}{m}$$

★ **TYPES OF MEDIUM BASED ON MAGNETIC FIELD:**

① Paramagnetic ($\mu_r \gtrsim 1$)
② Diamagnetic ($\mu_r \lesssim 1$)
③ Ferromagnetic (Very High μ_r) - Material is magnetized Field is lost.

★ **POSTULATES FOR MAGNETIC FIELD:**

① $\nabla \cdot H = 0$ - Equivalent of charge density is absent.
② $\nabla \times H = J$, conduction current density.

→ If you break a magnet into two, it becomes two seperate magnets. All flux lines will be closed.

LECTURE 31 - DISCUSSION ON MAGNETIC FIELD

We defined the postulates to estimate the source to field expressions, atleast for geometrically regular sources.

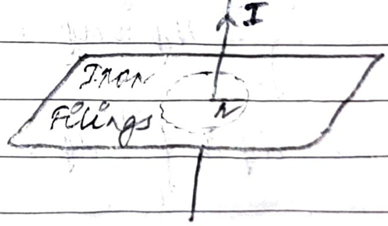
$\nabla \times H = J = I/A \rightarrow$ Ampere's law
($1/m$) (A/m) (A/m^2) (current per unit area) (A/m^2)

So, whenever we have a conductor carrying current, there is a magnetic field encircling it whose direction is given by right hand screw rule.

① Turning of screw - Direction of H
② Movement of screw - current flow.

Let us get an expression for H due to a current carrying conductor, having a small radius r. As per Oersted's experiment, the iron filings are arranged in circles. The density of filings reduces with increase in distance from the wire.

Using cylindrical systems,
I varies along a_z
H varies along a_ϕ and a_r



$\therefore \boxed{\vec{H} = H_\phi a_\phi}$ A/m   H - intensity varies with r.

$\boxed{I = a_z I_z}$ A.

LECTURE 38 - NUMERICALS ON MAGNETIC FIELD

In Oersted experiment, surface integral is taken to make J into I, i.e.,

$$\int_S (\nabla \times H) \cdot dS = \int_S J \cdot dS = I.$$

Take a surface perpendicular to the wire to bring I mathematically on the RHS.

$$\oint_c \vec{H} \cdot \vec{dl} = I$$

At any r along ϕ, H is constant.

$$\vec{H} = H_\phi a_\phi$$

$$\vec{dl} = dr\, a_r + r\, d\phi\, a_\phi + dz\, a_z$$

$$\therefore \vec{H} \cdot \vec{dl} = H_\phi r\, d\phi$$

$$\oint_c H_\phi r\, d\phi = I$$

ϕ varies from $0 \to 2\pi$ along c

$$\int_0^{2\pi} H_\phi r\, d\phi = I$$

$$\Rightarrow H_\phi r \int_0^{2\pi} d\phi = I$$

$$\Rightarrow \boxed{H_\phi = \frac{I}{2\pi r} \Rightarrow H = \frac{I}{2\pi r} a_\phi \text{ A/m}}$$

Q. The normal railway overhead electric line carries a current 0.25 kA. Determine the magnetic field intensity and flux density at a point 4m below the wire.

A1. Magnetic Field Intensity, $H = \frac{I}{2\pi r} a_\phi$ A/m

$$\Rightarrow H = \frac{0.25 \times 10^3}{2\pi \times 4} a_\phi \text{ A/m} = 9.94718\, a_\phi \text{ A/m}.$$

Flux density, $\vec{B} = \mu\vec{H}$
$= 4\pi \times 10^{-7} \times 9.94718$
$= 12.5 \times 10^{-6}\, T$ (or) $\frac{Wb}{m^2}$ (or) $\frac{Vs}{m^2}$.

Q2. In the above questions, a square loop of side 1m x 1m is placed parallel to the wire at 4m. Determine the induced emf.

A2. The loop is kept in such a position that magnetic field is going inside that loop. As per Faraday's law,

$$\text{Induced emf} = \frac{-d}{dt}\phi = \frac{-d}{dt}(BA)$$

$$= \frac{-d}{dt}(12.5 \times 10^{-6} \times 1 \times 1) = 0$$

In actual, the railway line is AC with 50 Hz.

$I = 0.25 \times 10^{3} (\sin 2\pi \times 50 \times t) = 0.25 \times 10^{3} \sin(100\pi t)$

$\vec{H} = 9.94118 (\sin 100\pi t)$; $\vec{B} = 12.5 \times 10^{-6} \sin(100\pi t)$

Now, Induced emf $= \frac{-d}{dt}(12.5 \times 10^{-6} (\sin 100\pi t))$

$= -12.5 \times 10^{-6} \cos(100\pi t) \times 100\pi$

$= -3.926 \times 10^{-3} \cos(100\pi t)$

LECTURE 39 - ELECTROMAGNETIC FIELD POSTULATES

Electric and magnetic fields were dependent on each other when experiments were conducted using a current carrying conductor. Hertz and JC Bose proved it and showed that it can travel. Later, Marconi thought of a business model and commercialised it.

Our basic idea was to create mathematical equations and postulates for describing these effects, using all previous works done in 17th/18th/19th century. To arrive at postulates combining E and H, we land in Maxwell's equations.

As per Faraday's law,

$$V_{emf} = -\frac{\partial}{\partial t}\phi = -\frac{\partial}{\partial t}(\vec{B}A)$$

Also, $V = \oint_C \vec{E}\cdot\vec{dl}$

$$\therefore \int_C \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}(BA)$$

Using stokes' law,

$$\int(\nabla\times\vec{E})\,ds = -\frac{\partial}{\partial t}\int\vec{B}\cdot\vec{ds}$$

$$\Rightarrow \boxed{\nabla\times\vec{E} = -\frac{\partial}{\partial t}B = -\frac{\partial}{\partial t}(\mu\vec{H})}$$

* **IMPLICATION:**

1. Ampere's law - $\nabla\times H = J$
2. Faraday's law - $\vec{H}$ is related to $\vec{E}$.

Therefore, a current can create EM fields.

* **SOME RESULTS:**

1. If J (or) I was time varying, i.e., an AC current, then $\vec{H}$ is also time varying. Then, we create an E-field.
2. If J (or) I was static, i.e., a DC current, then $\vec{H}$ is non-time varying and static. Therefore, E and H are not related; Independent.

therefore, only Interdependent E and H can travel and carry information. This is the aim of electromagnetism.

* POSTULATES FOR EM FIELD :

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\mu \vec{H}) \; ; \; \nabla \times \vec{H} = J$$

Faraday Ampere

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \; ; \; \nabla \cdot \vec{H} = 0$$

Gauss Gauss

LECTURE 40 - MAXWELL EQUATIONS

Maxwell provided an extra term in Ampere's law, which had such a great significance comercially, that the four postulates came to be known as Maxwell's equations. He corrected Ampere's equation which gave better clarity to study EM phenomena and formed the basis of wave propagation, especially wireless communication.

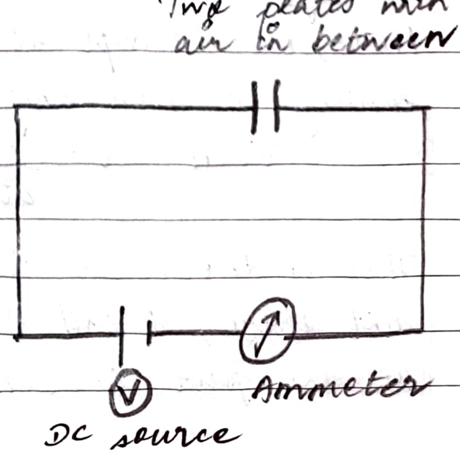
We know that,

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\mu \vec{H}) \text{ and } \nabla \times \vec{H} = J$$

Here, $\vec{H}$ is present in both the equations whereas $\vec{E}$ is only once. This raised maxwell's curiosity and he proposed that $\vec{E}$ should enter Ampere's law.

* MAXWELL's CAPACITOR EXPERIMENT:

① When V was fixed at a value, the ammeter flickered and settled at once. Here, the charging current alone flows and temporary magnetic field is created around the wire.



(2) When voltage (DC source) was changing / kept varying, ammeter kept on showing valid readings, i.e., there was current and magnetic field always.

(3) As long as voltage was varying, the current, and as a result, magnetic field was always maintained. This meant that something was taking place in the air gap. Now, this is Maxwell's contribution.

Inside a capacitor,

$q = CV$; q - charge
C - capacitance
V - applied voltage

When time variations took place,

$$\frac{dq}{dt} = C\frac{dV}{dt}$$

$$I_d = \frac{\varepsilon A}{d}\frac{dV}{dt}$$; I_d - current through air gap
A - Area of Metal Plates
d - Distance Between Plates

→ Empirically Electric Field

$$I_d = \varepsilon A \frac{d}{dt}\left(\frac{V}{d}\right) = \varepsilon A\left(\frac{d\vec{E}}{dt}\right)$$

$$\Rightarrow \frac{I_d}{A} = \frac{d}{dt}(\varepsilon\vec{E})$$; I_d - displacement current

Here, we attempt to model a current density term. Adding it with Ampere's law -

$$\nabla \times H = J + \frac{\partial}{\partial t}(\varepsilon\vec{E}) \quad ; \quad \nabla \times \vec{E} = -\frac{\partial}{\partial t}(\mu\vec{H})$$

This is the corrected Ampere's law. So, now even if J is not there, E and H can exist interdependently. Therefore, all the four equations came to be known as Maxwell's equation

LECTURE 41 - PLANE WAVE EQUATIONS

In wireless communications, Maxwell's equation provided a mathematical estimate for determining how much EM field should be created for a given distance R between transmitter and receiver.

Let us take two points on Earth where we need to communicate. Assume that there are no free charges, i.e., $\rho = \sigma = 0$, $J = 0$
Hence, the four equations become -

$$\nabla \times E = -\frac{\partial}{\partial t}(\mu H) \; ; \; \nabla \times H = J + \frac{\partial}{\partial t}(\varepsilon E) = \frac{\partial}{\partial t}(\varepsilon E)$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 0 \; ; \; \nabla \cdot H = 0$$

The first equation, $\nabla \times E = -\frac{\partial}{\partial t}(\mu H)$

Taking curl on both sides, we get -

$$\nabla \times (\nabla \times E) = -\nabla \times \frac{\partial}{\partial t}(\mu H)$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}\mu \left\{\frac{\partial}{\partial t}(\varepsilon E)\right\}$$

$$+\nabla^2 E = +\frac{\partial^2}{\partial t^2}(\mu \varepsilon E) \qquad [\because \nabla \cdot E = 0]$$

$$\boxed{\nabla^2 E = \mu \varepsilon \ddot{E}}$$

This is the basic time-space plane wave equation. Here, only E is present. Upon solving this differential equation,

$$\vec{E} = f(\text{time, distance})$$

$$\nabla^2 = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$$

Hence, we find -
① At transmission side, how much antenna power to be generated?
② At receiver side, how much sensitive should be our systems?
③ $\nabla \times \vec{E} = \mu \epsilon \ddot{E}$ is more preferred.
Similarly, $\nabla \times \vec{H} = \mu \epsilon \ddot{H}$

So, for antennas, design equations are available. We need to find how much field is generated for a given current input.

LECTURE 42 · PART I · NUMERICAL DIFFERENTIATION

Differenciation is a set of values that will represent the rate of change at each point in that interval. Whereas, integration gives a single value along with a constant.

In differenciation, the input will be ~~a set of values~~ f(x) at various points in [a, b] and the output of the algorithm will be a set of values. For the input, we need to choose points in [a, b] and have the f(x) values at all these points.

* ALGORITHM BASED ON TAYLOR SERIES:

If f(x) is known at a point x_i and the point is inside an [a, b], f(x) is continuous in the interval. The Taylor series provides an estimate of f(x) at x_{i+1}, another point in interval [a, b].

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \dots$$

Let $h = x_{i+1} - x_i$

$$f(x_{i+1}) = f(x_i) + \frac{f'(x_i)(h)}{1!} + \dots + \frac{f^n(x_i)(h^n)}{n!} + R^n$$

[remainder after assuming n terms.]

$$R^n = \frac{f^{(n+1)}(x_i)}{(n+1)!} h^{(n+1)}$$

For example, when $n=3$, we find that -

$$f(x_{i+1}) = f(x_i) + \frac{f'(x_i)}{1!} h + \frac{f''(x_i)}{2!} h^2 + \frac{f'''(x_i)}{3!} h^3$$

Remainder, $R = \frac{f^4(x_i)}{4!} h^4$

As value of n increases, the value of R decreases. Whether to add the remainder term to the estimate can be decided.

To estimate $f'(x)$ at a point,

$$f(x_{i+1}) = f(x_i) + f'(x_i) h + R$$

$$\Rightarrow f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} - \frac{R}{h}$$

This is similar to the basic concept of differentiation.

i.e, $\frac{d}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

The only addition is the error term, in this case, $R = \frac{f''(x_i)}{2!} h^2$

If h is very small, we get $R \sim 10^{-4}$ (or) 10^{-6} which can be ignored in many scenarios. The error is described in the order of h, i.e, $O(h)$. Whatever values we have obtained are correct till the h^{th} position after decimal.

This is also called Taylor's forward difference, i.e, knowing the value at $x = x_i$, we estimate value at $x = x_{i+1}$

LECTURE 43 - PART II. NUMERICAL DIFFERENCIATION

In a similar sense, we can write Taylor's backward series, i.e., knowing value at $x = x_i$, we estimate value at $x = x_{i-1}$. So, the Taylor backward approx is given as -

$$f(x_{i-1}) = f(x_i) + \frac{f'(x_i)}{1!}(x_{i-1} - x_i) + \frac{f''(x_i)}{2!}(x_{i-1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i-1} - x_i)^3 + \dots$$

$$= f(x_i) - \frac{f'(x_i)}{1!}(h) + \frac{f''(x_i)}{2!}h^2 - \frac{f'''(x_i)}{3!}h^3 + \dots$$

The basic idea of numerical differenciation starts here. In practice, we use both forward and backward series.

$$FD: f(x_{i+1}) = f(x_i) + \frac{f'(x_i)(h)}{1!} + \frac{f''(x_i)}{2!}(h)^2 + \frac{f'''(x_i)}{3!}(h)^3 + \dots \quad -(1)$$

$$BD: f(x_{i-1}) = f(x_i) - \frac{f'(x_i)(h)}{1!} + \frac{f''(x_i)}{2!}(h)^2 \quad \frac{f'''(x_i)}{3!}(h)^3 + \dots \quad -(2)$$

Subtracting (2) from (1), we get -

$$f(x_{i+1}) - f(x_{i-1}) = 2\frac{f'(x_i)}{1!}(h) + \frac{2f'''(x_i)}{3!}(h)^3 + \dots$$

$$\Rightarrow f(x_{i+1}) - f(x_{i-1}) = 2\frac{f'(x_i)}{1}(h) + R$$

$$\Rightarrow f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} - \frac{R}{2h}, \text{ where } R = \frac{f'''(x_i)}{3!}(h)^3$$

$$\Rightarrow \frac{R}{h} = \frac{f'''(x_i)}{3!}(h)^2$$

This is known as centred differences, which has an error of the order, $O(h^2)$.

So, if $h = 10^{-3}$, i.e., if we take centred differences, the error is of the order of $O(10^{-6})$, i.e., we can see with 100% confidence that the error is upto six digits after decimal point.

$$\therefore f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$$

This is the most used numerical differenciation formula.

Similarly, upon adding the two expressions, we get-

$$f(x_{i+1}) + f(x_{i-1}) = 2f(x_i) + 2\frac{f''(x_i)}{2!}(h)^2 + R$$

$$\Rightarrow f''(x_i) = \frac{f(x_{i+1}) + f(x_{i-1})}{h^2} - \frac{2f(x_i)}{h^2} - \frac{R}{h^2}$$

Again, the order of error is $O(h^2)$.