Pre-requisite - Multi Variable Calculus

This subject has two parts:
1. study of Electromagnetics
2. study of Computational Techniques

* COURSE OBJECTIVES:

1. To provide the basic skills required to understand, develop, and design various engineering applications involving electromagnetic fields.
2. To lay the foundations of electromagnetism and its practice in modern communications such as wireless, guided wave principles such as fiber optics and electronic electromagnetic structures.

* COURSE OUTCOME:

CO1- To understand the concept of electric and magnetic field in terms of mathematical descriptions.
CO2- To understand the phenomena of electromagnetic wave and its various parameters.
CO3- To apply the mathematical concepts for deriving the wave parameters and applications.
CO4- To apply and understand the electromagnetic concepts with numerical formulation.

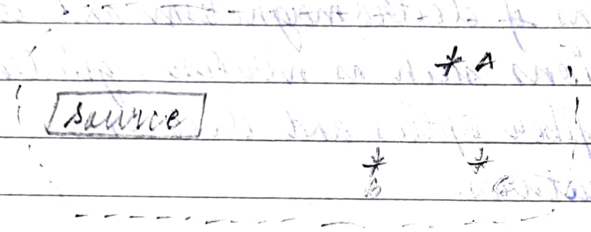
* ELECTROMAGNETICS - TERMS ASSOCIATED:

1. Electric Field
2. Magnetic Field
3. Statics
4. Electrodynamics
5. Solenoid
6. Charge / Current / Flux / Dipole
7. Induction / Hysteresis
8. Permittivity / Permeability
9. Para / Dia / Ferri / Ferro
10. Generator / Motor

11. Gauss (Marconi)
12. Oersted / Faraday / Ampere / Maxwell / Hertz / Lenz

* UNDERSTANDING TERMINOLOGY FIELD :

1. Sound field, Heat Field, Light Field, Electric Field, Magnetic Field, Gravitational Field, Electromagnetic Field. (Cricket field, paddy field !!!)
2. Heat, Light, Sound, Electric, Magnet - are all Natural elements or forces.
3. Field - means LARGE AREA (as per dictionary)
4. Why is the term field attached to it?

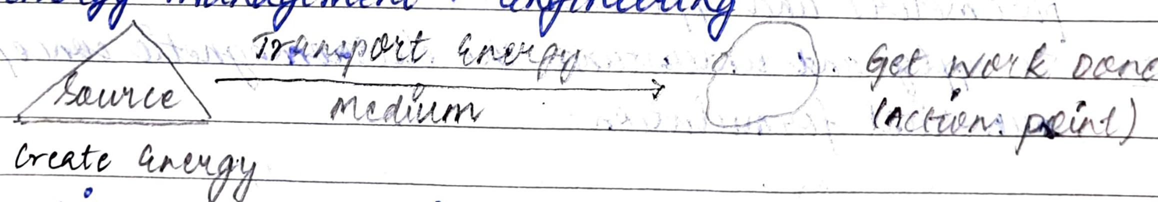


Force over an area is called as field.

1. Force that is felt over an area.
2. At any point, we experience ....
3. The source provides an effect.

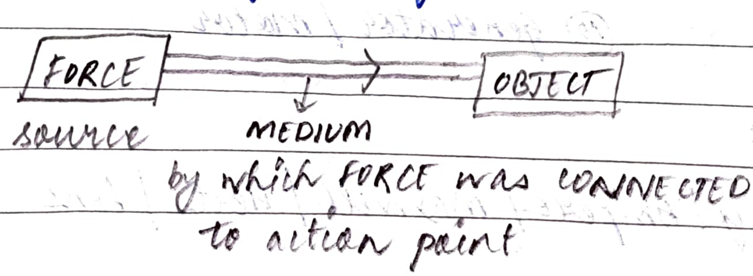
Q. What is engineering?

A. Energy management - Engineering



Action - Can be physical deformation, Heating, Lighting, Internal damage, Movement of the object

From this, Newton understood that $\int F \cdot dl$ is work done
Birth of engineering - Newton's Classical Mechanics.



* CLASSICAL MECHANICS:

① Newton's 'FORCE' Definition - Basic Idea
Anything that tends to change the state of rest or uniform motion of a body is called force.
change → Work Done → Energy :- Fundamental concept
② Point of origin - Transmission Medium - Physically observable by Human Senses
③ Mathematical Tool - Definition for 'Derivative' or Calculus - Field of mathematics ($\rightarrow$ rate of change)
④ Equations of Motion - Describe any physical activity in nature
⑤ Base for mechanical engineering

Using a "Force" to do some "Work" for "us" -
① Create a force (source).
② Able to control the force.
③ Make the force do something useful to people.

Forces were already available in nature. Engineering created a controlled source.

* GRAVITATION:

① Newton's most significant contribution
② Contradictory to Newton's Mechanics !!!!
③ No need for Physical Transmission Medium
④ Point of origin and action can be at any distance!
⑤ Forces which act like this - Fields - Forces that expand over an area. But earlier, Newton had defined it as "a force that was applied correctly in one direction".
⑥ Realization of Existence of Natural Forces which can act at large Distances !!!!

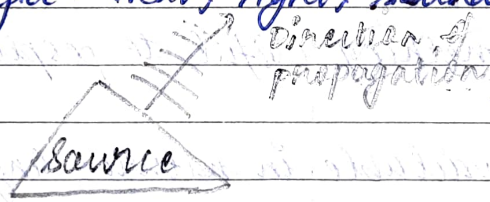
$\boxed{\text{Source}} \xrightarrow{\text{medium (?)}} \boxed{\text{Object}}$ All of us (living / non-living)
Earth — Prevented Newton from considering gravitation

* ELECTRIC - MAGNETIC FORCES/FIELDS:

1. Columb's law - Existence of 'invisible' Electric Force - Similar to Gravitation (Both laws follow inverse law)
2. Magnet - Discovered in Turkey - Could disturb similar magnets kept far apart
3. The magnitude of these Natural forces - Extremely high - Ability to spread out
4. Realization of the ultimate Forces of Nature !!!!!!!

Q. What is a FIELD?

A. 1. A force that spread over an area/volume.
2. Intensity decreases over distance
3. Area/volume between A and B - space of Influence or MEDIUM - defined by application
4. Nature's forces are all fields.
Example - Heat, Light, Sound, Wind, Electric, Magnetic



Engineering is all about knowledge and usage of force for our purpose.
History of Engineering ==== History of FIELD studies

Q. How do we study fields?

A. Mathematically Vector Calculus
- Space description of the area where force effect is present.
- "Realized" forces can spread over an area.
- "Fields" ⇒ Force that spreads

Q. Why should Engineers study about EM Fields?

A. Heat, Light, Sound

Next Generation Applications

Electric, Magnetic

Need Basic EM at physical level

Present

5G, IoT, Cloud Services

Industrial Revolution

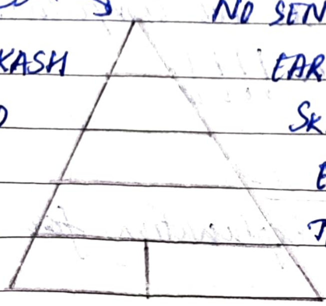
Present Day Mechanical Systems - Technologies: Auto, Defence, Bio, Space, Nuclear		ICT Revolution
	2000	Electronics - Semiconductor Industry
Thermal Engineering: Laws of Thermodynamics, Mathematical Models	1900	(Communication - Wireless) Marconi Experiment - Transmission
	1800	Hertz Experiment - Field Creation
Newton's Classical Mechanics		MAXWELL EQUATIONS
	1700	Faraday, Lorentz, Lenz, Volta
Laws of Optics, Reflection, Refraction, Lens, Mirror, Convex, Concave		Oersted Experiment
	1600	Concept of current is born. Gauss - Lines of Force - Laws. Knowledge of Existence of magnetic Fields
	1500	

+ ELEMENTS OF NATURE - HUMAN SENSING:

Abstraction (subtle)

PERCEPTIONS

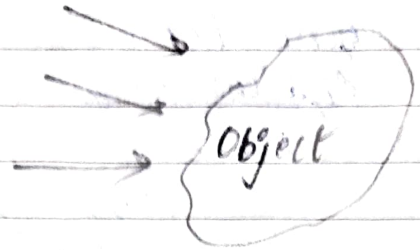
EM



Sound	AKASH	NO SENSOR
Vision	WIND	EAR
Touch	FIRE	SKIN
Smell	WATER	EYE
Taste	EARTH	TONGUE
		NOSE

Materialism (physical)

Gross



Heated Up
OR
Lights Up
OR
Makes Sound
OR
ELONGATES

We can study fields by observing the effects on an object.
This effects should be PHYSICAL.

In EM, ELECTRICAL CHARGE REDISTRIBUTION occurs -
When exceeds a limit - BURNS

Q. What makes EM tough?
A. ① Abstract nature
② Not observable by Human Body senses - Biggest Problem
③ Not a Logical Subject
④ EM Study - Thinking process - Teacher & Student should co-operate in study.

* EM EFFECT:
① Present Age EM - Based on Maxwell's Equations - JC Bose - Marconi - Hertz - Tesla
② Realized in various ways - In various ages
③ Indians, Chinese, Greeks - Astronomical Calculations
④ EM Effect - Most Significant "effect" Realized by Humans
⑤ EM - Complex but not at all complicated
⑥ Physical Idea Discussion!!!!

* SIGNIFICANCE OF EM STUDIES:
Provides Background Explanation for many applications.

* PHENOMENA / THINGS EXPLAINED :

1. Cell phone Basic operation, Radio, TV Broadcast
2. All wireless Applications (communication)
3. Car Lock - Automatic Entry / exit
4. Medical Diagnostics - X Ray, CT, MRI
5. Microwave oven
6. Airport scanner
7. And many more

Applications which we can feel directly.

8. Remote Sensing, Oil Exploration
9. Weather Monitoring, Navigation
10. Relief and Rescue operations - Cell communication during natural tragedies
11. Elon Musk - Hyper Loop
12. Gene Mutation - Biological effects of mobile phones
13. .... goes on
14. Completely UNAVOIDABLE STUDY - Required for complete understanding of Engineering

Applications Not felt Directly.

* ACADEMIC UNDERSTANDING OF ELECTRICAL ENGINEERING PHENOMENA :

Q. Can we design CE Amplifier using Maxwell's equations?

A. No, Definitely NOT

KCL, KVL - Approximations of Maxwell equations

Q. VLSI Algorithm - Does it Need EM study?

A. No, Definitely NOT

EMI / EMC compatibility - Requires Field Studies

Example of scalar quantities - temperature, mass, energy, distance, speed, density

Example of vector quantities - force, displacement, velocity, acceleration, momentum

LECTURE 2 - INTRODUCTION TO COMPUTATIONAL ELECTROMAGNETICS

Engineering is modelling a physical phenomena, i.e writing a equation $y = f(x)$, where y is the dependent variable and x is the independent variable and then, solve it for y. For example, let's take Newton's second law of motion, $s = ut + \frac{1}{2}at^2$. Here, s is the dependent variable and rest are independent.

Therefore, $s = f(u, t, a)$

In most engineering concepts, we get differential equations, i.e. rate of change of some quantity with respect to another.

Q. When do you get a complete solution?

A. We get a complete solution only if the following two conditions are satisfied -

① Differential equation is having a solution.

② Boundary / Initial conditions are known.

If the above two are not satisfied, we cannot get a CLOSED FORM expression as solution. When there is no scope for a closed form expression, we go for computational techniques. Here, the solution will be numbers where we get all 'y' values for every 'x'.

★ EXAMPLE - TO STUDY THE EFFECT OF LIGHTNING ON AIRCRAFT:

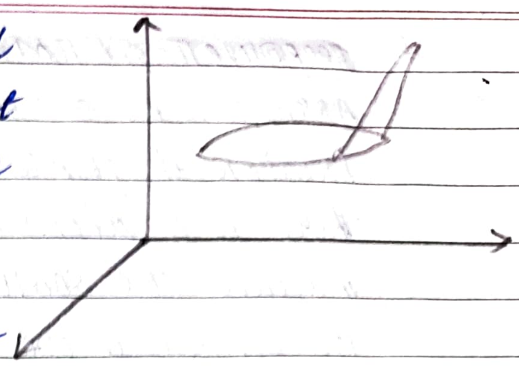
Here, we need to know charge created at each point on the aircraft. Gauss Law MAY be applied -

$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$, where ρ - charge density;

E - Applied Electric Field;

∇ - operation; ε_0 - Permittivity

In this way, you never get a closed form solution. It is very difficult to apply boundary conditions here because geometry is not perfect.



Another example is the EM effect on human body.

★ CIRCUIT THEORY:
The main cause of the theory is current, flowing through a copper wire has no spatial problems whereas fields expand in space. So, defining geometry is significant. This is the "requirement" of computational electromagnetics.

LECTURE 3- MATHEMATICS FOR FIELD DESCRIPTION

Previously, we learnt that current passes only through a wire and impact is felt at a point, whereas field spreads out in space.

★ MATHEMATICAL TECHNIQUES FOR DESCRIBING FORCES: (VECTOR CALCULUS)
The first step here is, to deal with forces. Let us start with an example. An antenna creates an EM field. What will be the requirements?

1. How much is the field at each point around the antenna? (spatial variation) → Differentiation
2. $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} = ?$
3. How does the field vary as we move in space? → disturbances [Buildings / Trees]
4. Describing "obstructions" to field in the space? → Integration

The combined effects of these points give out the engineering aspect.

★ COORDINATE SYSTEM:

Arrangement of reference lines or curves used to identify the location of points in space, determining each point uniquely. There are different types of coordinate systems to indicate the SAME point in specific applications. In EM, we generally use the following coordinate systems -

① Cartesian coordinate system - In the plane, two perpendicular lines are chosen and the coordinates of a point are taken to be the signed distances to the lines. In three dimensions, three mutually orthogonal planes are chosen and the three coordinates of a point are the signed distances to each of the planes. This can be generalized to create n coordinates for any point in n-dimensional Euclidean space.

② Cylindrical and spherical coordinate systems - There are two common methods for extending the polar coordinate system to three dimensions. In the cylindrical coordinate system, a z-coordinate with the same meaning as in Cartesian coordinates is added to the r and θ polar coordinates giving a triple (r, θ, z). Spherical coordinates take this a step further by converting the pair of cylindrical coordinates (r, z) to polar coordinates (ρ, ϕ) giving a triple (ρ, θ, ϕ).

★ VECTOR:

Unit vectors in each system describe the direction in space. Magnitude of force is described by the coordinate systems whereas the vector representation (unit vector, in particular) give out the direction. Together, these parameters brief out the "entity", force in this case.

In the next sessions, we'll learn about curl, gradient and divergence laying the foundation for surface, line and volume integrals. Solving these differentials and integrals numerically gives us "Computational EM".

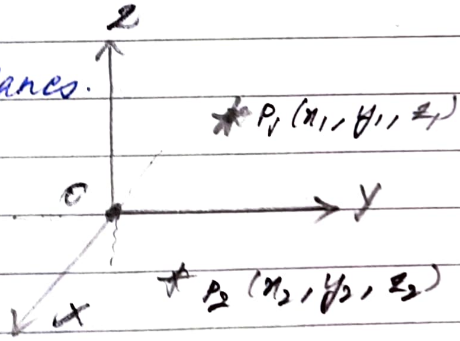
LECTURE 4 - CO-ORDINATE SYSTEMS

Space is described numerically through coordinate systems. In order to form a coordinate system -

1. Select atleast three geometrical shapes.
2. Intersect them.
3. Create a zero/reference/source starting point.

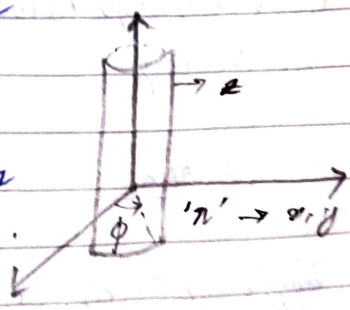
* CARTESIAN SYSTEM : [Rectangular]

→ Select three plane surfaces and intersect them.
→ Any point is referred by the perpendicular distance from the planes.
→ Linear distances.
→ Describes rectangular geometrical perfectly.



* CYLINDRICAL SYSTEM : [Polar]

→ One angle and Two linear distances
Two surfaces and cylinder
→ Able to represent ~~cyli~~ curvilinear geometry perfectly.
→ For example, we wish to analyze fluid flow through pipes. The aim is to identify the effect on various points on pipe. We need to create a coordinate system to describe 'points' on the pipe comfortably.
→ Optical fibre is another example.



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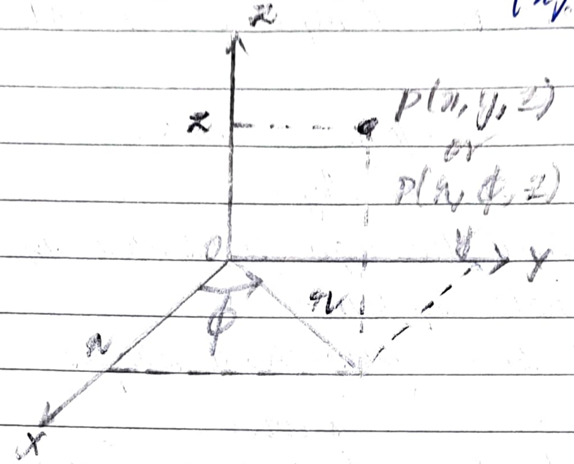
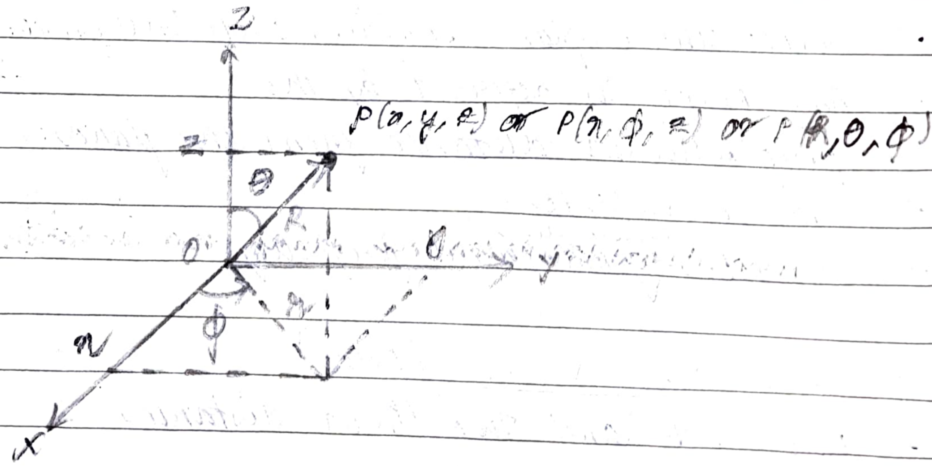
To represent a point from the cartesian to cylindrical system, we have $P(x, y, z)$ to $P(r, \phi, z)$,
where $r = \sqrt{x^2+y^2}$ and $\phi = \tan^{-1}\left(\frac{y}{x}\right)$.



* SPHERICAL SYSTEM:

Two angles and one linear distance.



Electromagnetics sources/waves follow spherical geometry.

LECTURE 5. COORDINATE SYSTEM APPLICATIONS

Different Applications
↳ Different Geometries
↳ Different co-ordinate systems

* UNIQUENESS :
Cartesian is the only unique system. Any curvilinear systems are not unique. Here, unique is meant that "a point is only one name.

For example, let us take a point $P(3,4,5)$. When put in cylindrical systems, the representation will be -
$x=3,\ y=4,\ z=5$
$r=\sqrt{3^2+4^2}=5,\ \phi=\tan^{-1}(4/3)=53^\circ$
Hence, $P(5, 53^\circ, 5)$ is the same point in cylindrical system.
Again, let us take the point $P(-3,-4,5)$. What do you get?
The same, $P(5, 53^\circ, 5)$.

Cartesian system, as explained before, is made up of linear distances. All the three relations, x, y, z vary from 0 to ∞ (or) / ($-\infty$ to ∞). Whereas, an angle can be expressed only from the range 0 to 2π (in radians) or 0 to 360° (in degrees).
[One representation has many nomenclatures.]
This is the problem of uniqueness in curvilinear systems.

Q. How uniqueness is solved/managed?
A. (1) Define the problem space correctly. For the above example, define $r: 0 \to 10$; $\phi: 0 \to 45$; $z: 0 \to 5$
(2) Define correct initial conditions. [boundaries]

Else:
Convert to cartesian and solve.
→ In cartesian and cylindrical coordinates systems, we have z as common to both.
→ In cylindrical and spherical coordinate systems, we have ϕ as common to both.

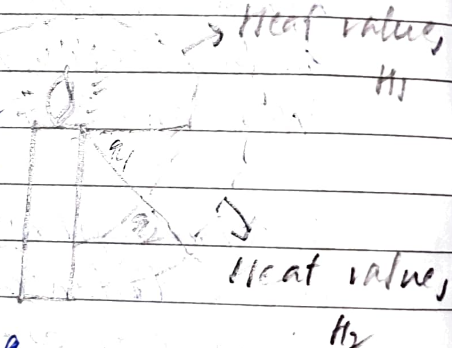
★ APPLICATIONS OF COORDINATE SYSTEM :

① CAD [Computer Aided Design] Based Software
② QR Code Scanner
③ Image processing applications
④ Graphical User Interfaces [GUI]

Now, that we have described the magnitude of a vector correctly, let us dive into directions.

LECTURE 6 - UNIT VECTORS

The magnitude of heat is of circular symmetry. If we insist on cartesian system, describing the system will be challenging. Rather, cylindrical will make it easier. Therefore, performing integration and differenciation operations will be easier.

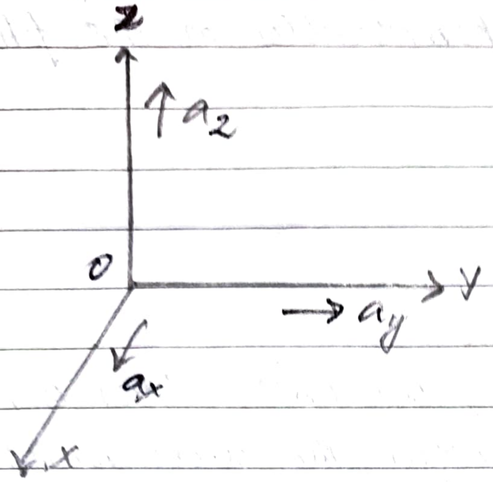


Choosing between cylindrical and spherical system depends on the given problem scenerio. If it demands a three dimensional (3D) analysis, we go ahead with spherical, or else stick to cartesian form. For example, in certain EM cases, a given 3D can be reduced 2D scenario for easier/faster computation. Therefore, in such cases 'r' in cylindrical system will have a greater preference over 'R' in spherical.

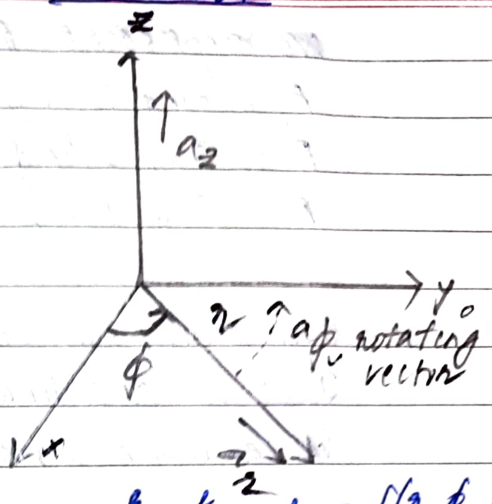
The unit vector is a direction in which a co-ordinate expands/increases. It is the mathematical representation of direction.

$\overrightarrow{PTO}$

CARTESIAN



CYLINDRICAL



$k_1, k_2, k_3 = f(x, y, z)$

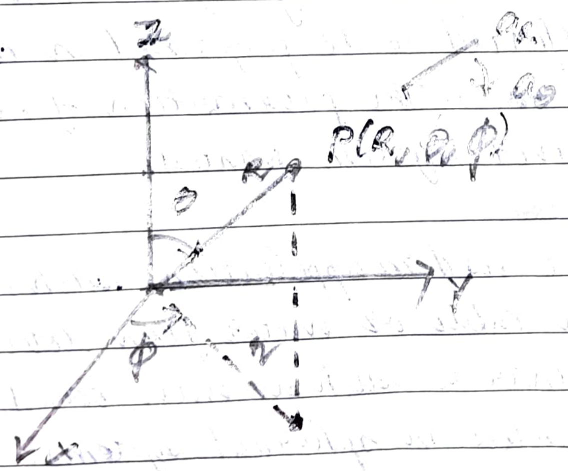
$\therefore \vec{F} = k_1 a_x + k_2 a_y + k_3 a_z$

$k_{11}, k_{12}, k_{13} = f(r, \phi, z)$

$\therefore \vec{P} = k_{11} a_r + k_{12} a_\phi + k_{13} a_z$

LECTURE 7 - VECTOR REPRESENTATION

Vector = Magnitude + [Unit Vectors]
(Eg. Force) [Co-ordinates] Direction



ϕ – present in XY
θ – z-axis and 'R'
a_ϕ – XY plane
a_θ – depends on a_R

SPHERICAL

→ In cartesian and cylindrical coordinate systems, we have a_z as common to both.

→ In cylindrical and spherical coordinate systems, we have a_ϕ as common to both.

We represent forces mathematically in the following way

$\vec{F} = 2a_x + 3a_y + 4a_z$

$\vec{F} = 3a_r + 5a_\phi + 7a_z$

$\vec{F} = 5a_r + 7a_\theta + 8a_\phi$

→ Directions never change.

* UNIQUENESS IN UNIT VECTORS:

Just as co-ordinate systems, the unit vectors a_x, a_y and a_z in cartesian system are unique. The directions are same anytime. But, the same cannot be told for curvilinear systems. In cylindrical coordinate system, a_ϕ is dependent on a_r and in spherical coordinate system, both a_θ and a_ϕ are dependent on how a_r is oriented.

The lack of uniqueness led to the development of formulae for converting from one system to another. In real life situations, modelling can be in any system. But, the processing part is preferred to be performed in cartesian system owing to uniqueness. The presentation of result can be given back in whichever system wanted.

Cylindrical systems are used in applications such as flow of energy through optical cable or water flow detection in pipes. Whereas, EM scenarios such as wave-based phenomenas are represented in spherical systems.

LECTURE 8- VECTOR CONVERSION

Now that we have fixed the representation, what are the mathematical operations that are possible using this representation which will help us understand what happens in real time, is what we gonna see next.

- There are mainly two physical possibilities taking place with the field.

1. There are two fields, whether they are opposing or adding.
2. Field interaction with disturbances.

What are the mathematics meant for it ?

Let $\vec{F} = 3a_x + 4a_y + 5a_z$

Therefore, the scalar projection of $\vec{F}$ on the unit vectors are

$\vec{F} \cdot a_x = 3$; $\vec{F} \cdot a_y = 4$; $\vec{F} \cdot a_z = 5$

The $\vec{F}$ represents the sum of all possible vector projections.

Again, $\vec{F} = k_1 a_x + k_2 a_y + k_3 a_z$ — ①

To convert this to cylindrical system, project $\vec{F}$ to a_r, a_ϕ, a_z. This means that:

$\vec{F} \cdot a_r = k_4$; $\vec{F} \cdot a_\phi = k_5$; $\vec{F} \cdot a_z = k_6$

This implies $\vec{F} = k_4 a_r + k_5 a_\phi + k_6 a_z$ — ②

[Here, equations ① and ②, LHS is the same physical quantity whereas RHS is different.]

To get $\vec{F}$ in cylindrical representation,

$\vec{F} \cdot a_r = k_1 (a_x \cdot a_r) + k_2 (a_y \cdot a_r) + k_3 (a_z \cdot a_r)$

$\vec{F} \cdot a_\phi = k_1 (a_x \cdot a_\phi) + k_2 (a_y \cdot a_\phi) + k_3 (a_z \cdot a_\phi)$

$\vec{F} \cdot a_z = k_1 (a_x \cdot a_z) + k_2 (a_y \cdot a_z) + k_3 (a_z \cdot a_z)$

So, to convert from one system to another, we need to know dot products between unit vectors of different systems.

To find k_4,

$k_4 = (k_1 a_x + k_2 a_y + k_3 a_z) \cdot a_r$

$= k_1 (a_x \cdot a_r) + k_2 (a_y \cdot a_r) + k_3 (a_z \cdot a_r)$

$a_x \cdot a_r = |a_x| |a_r| \cos\phi = \cos\phi$
$a_y \cdot a_r = |a_y| |a_r| \cos(90° - \phi) = \sin\phi$
$a_z \cdot a_r = |a_z| |a_r| \cos(90°) = 0$

$\Rightarrow k_4 = k_1 \cos\phi + k_2 \sin\phi + k_3 (0)$
Similarly, we need to find k_5 and k_6.

At any point, the unit vectors will be perpendicular to each other.

LECTURE 9 - UNIT VECTOR DOT PRODUCTS

Q. A vector $\vec{A} = 5.3\,a_r + 0.98\,a_\phi + 3a_z$ is acting at $(5.3, 79°, 2)$. Determine the cartesian vector.

A. $r = 5.3,\ \phi = 79°,\ z = 2$
$A_r = 5.3,\ A_\phi = 0.98,\ A_z = 3$

Given, $\vec{A} = A_r a_r + A_\phi a_\phi + A_z a_z$
To find, $\vec{A} = A_x a_x + A_y a_y + A_z a_z$
$\therefore A_x = ?\ ;\ A_y = ?$

$A_x = \vec{A} \cdot a_x$
$= A_r (a_r \cdot a_x) + A_\phi (a_\phi \cdot a_x) + A_z (0)$
$= 5.3 (\cos\phi) + 0.98 (-\sin\phi) + 0$
$= 0.0429$

$A_y = \vec{A} \cdot a_y$
$= A_r (a_r \cdot a_y) + A_\phi (a_\phi \cdot a_y) + A_z (a_z \cdot a_z)$
$= 5.3 (\sin\phi) + 0.98 (\cos\phi) + 0$
$= 5.3896$

$\vec{A} = 0.0429\, a_x + 5.3896\, a_y + 3a_z$

At $(5.3, 79°, 2)$,

$x = r\cos\phi = 1.01128$

$y = r\sin\phi = 5.2026$

$z = z = 2$

LECTURE 10 - VECTOR CONVERSION - CARTESIAN - CYLINDER

Vector calculus deals with mathematical operation involving vectors. Vector is the basic mathematical representation for a force that spreads over an area. Vector can be represented in any of the three co-ordinate systems. But due to uniqueness concept, mathematical operations are preferred to be executed in Cartesian co-ordinate system. It is in this requirement, that we require conversion formulae. Fundamentally, while analysing applications, we may have to choose the correct co-ordinate system to model the forces (uniqueness is not a big requirement there, modelling should be perfect !!). But after the modelling, for doing mathematics and associated operations, it is preferred to do in Cartesian co-ordinate system, which is unique in representation.

* BASIC CONCEPT INVOLVED IN CONVERSION:

A vector represented in any co-ordinate system is "addition of the projections of the vector on the unit vectors of that co-ordinate system". When a vector is represented as follows-

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

The coefficient become useful if we understand in the following sense-

Take the dot product of vector A with a_x direction.

$$\vec{A} \cdot a_x = A_x (a_x \cdot a_x) + A_y (a_x \cdot a_y) + A_z (a_x \cdot a_z)$$

PTO

Since, in any co-ordinate system, the unit vectors are all perpendicular to each other, we get -

$\vec{A} \cdot a_x = A_x(1) + A_y(0) + A_z(0)$

Thus, $A_x = \vec{A} \cdot a_x$

and similarly, $A_y = \vec{A} \cdot a_y$

$A_z = \vec{A} \cdot a_z$

Thus, we can view A_x, A_y, A_z as the dot product co-efficient when the vector A is projected in cartesian co-ordinate systems. The net representation is the sum of all the vector projections in the respective co-ordinate system.

* **CYLINDRICAL TO CARTESIAN CONVERSION :**

$r = \sqrt{x^2+y^2}$; $\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix}$$

* **CARTESIAN TO CYLINDRICAL CONVERSION :**

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$x = r\cos\phi$, $y = r\sin\phi$, $z = z$

LECTURE 11 - VECTOR CONVERSION - CARTESIAN - SPHERICAL

$R = \sqrt{x^2+y^2+z^2}$; $\phi = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) = \tan^{-1}\left(\frac{r}{z}\right)$

* **SPHERICAL TO CARTESIAN CONVERSION :**

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\theta \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_R \\ A_\theta \\ A_\phi \end{bmatrix}$$

* **CARTESIAN TO SPHERICAL CONVERSION :**

$$\begin{bmatrix} A_R \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

LECTURE 12 - NUMERICAL VECTOR CONVERSION

Q. Consider a vector expressed in cylindrical co-ordinate system as follows -

$\vec{A} = 2.236\, a_r + 1.93136 \times 10^{-4}\, a_\phi + 3a_z$

The vector is acting at a point $(r, \phi, z) = (2.236, 63.43^\circ, 3)$.
Express the vector in cartesian and spherical systems.

A. $A_r = 2.236$; $A_\phi = 1.93136 \times 10^{-4}$; $A_z = 3$
The cylindrical co-ordinates are $(r = 2.236,\ \phi = 63.43^\circ,\ z = 3)$

① **CARTESIAN REPRESENTATION :**
Required representation,

$\vec{A} = A_x a_x + A_y a_y + A_z a_z$

The conversion formulae are -

$A_x = A_r\cos\phi - A_\phi\sin\phi$; $A_y = A_r\sin(\phi) + A_\phi\cos\phi$

$A_x = 2.236 \cos(63.43^\circ) - 1.93136 \times 10^{-4} \sin(63.43^\circ)$
$= 1.0001 - 1.7273 \times 10^{-4}$
$= 0.999$

$A_y = 2.23 \sin(63.43^\circ) + 1.93136 \times 10^{-4} \cos(63.43^\circ)$
$= 1.9999$

A_z is same for cylindrical and cartesian system.
Hence, the cartesian vector is-
$\vec{A} = 0.999\, a_x + 1.999\, a_y + 3a_z$

Point conversion to cartesian provides-
$(r = 2.236, \phi = 63.43^\circ, z = 3)$
$= (x = r\cos\phi = 1.001, y = r\sin\phi = 1.9999, z = 3)$
$= (1.001, 1.9999, 3)$

① The above cartesian vector can represent the position vector at a point $(1,2,3)$.
② The point conversion also provides a same meaning.
③ To understand - That this need not be the case always. students need to have this clarity. It may happen sometimes, it may not.

② SPHERICAL REPRESENTATION :
First we will get the point conversion done to get an idea of "θ".

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right) ; \theta = \tan^{-1}\left(\frac{r}{z}\right)$$

Therefore the elevation angle is $\theta = 36.69^\circ$
Radial vector, $R = \sqrt{x^2+y^2+z^2} = \sqrt{14} = 3.741$
The spherical representation of the point-
$R = 3.741, \theta = 36.69^\circ, \phi = 63.43^\circ$

Now, the required representation is -

$$\vec{A} = A_R a_R + A_\theta a_\theta + A_\phi a_\phi$$

The conversion formulae are -

$$A_R = A_x \sin\theta\cos\phi + A_y \sin\theta \sin\phi + A_z \cos\theta$$

$$A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta \sin\phi + A_z \sin\theta$$

$$A_\phi = A_\phi \text{ in cylindrical system}$$

$$\sin\theta\cos\phi = \sin(36.69^\circ)\cos(63.43^\circ) = 0.26274$$

$$\sin\theta\sin\phi = \sin(36.69^\circ)\sin(63.43^\circ) = 0.53438$$

$$\cos\theta\cos\phi = \cos(36.69^\circ)\cos(63.43^\circ) = 0.35861$$

$$\cos\theta\sin\phi = \cos(36.69^\circ)\sin(63.43^\circ) = 0.71719$$

$$A_R = (0.999)(0.26274) + (1.999)(0.53438) + 3\cos(36.69^\circ) = 3.7408$$

$$A_\theta = (0.999)(0.35861) + (1.999)(0.71719) - 3\sin(36.69^\circ) = -4.6586 \times 10^{-4}$$

$$A_\phi = 1.93136 \times 10^{-4}$$

$$\therefore \boxed{\vec{A} = 3.7408\, a_R - 4.6586 \times 10^{-4}\, a_\theta + 1.93136 \times 10^{-4}\, a_\phi}$$

LECTURE 13 - DEL VECTOR FORMATION

With differenciation and integration, there is no aim in converting from one coordinate system to another computationally.

* VECTOR DIFFERENCIATION :

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\text{change in function value}}{\text{displacement encountered by the independent variable}}$$

In the above equation, when the "function" term is replaced with "vector", change in vector arises, i.e. change in magnitude and direction.

+ DEL (∇) :- Differential operator for vector
The main aim of creating ∇ is to track the change in direction. As a result, we define the ∇ operator.

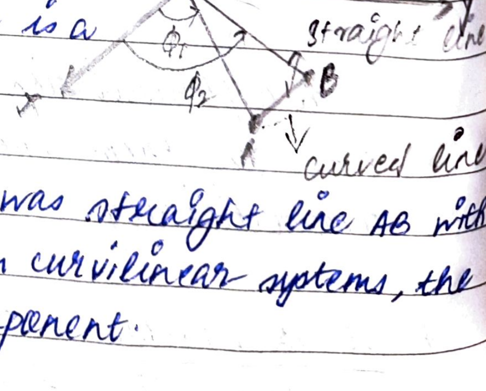
→ In cartesian systems,

$$\nabla = \underbrace{\frac{\partial}{\partial x}}_{\text{change in } x} a_x + \underbrace{\frac{\partial}{\partial y}}_{\text{change in } y} a_y + \underbrace{\frac{\partial}{\partial z}}_{\text{change in } z} a_z$$

We add the change in all direction to conclude at the end.

→ In cylindrical systems,

$$\nabla = \frac{\partial}{\partial r} a_r + \frac{1}{r}\frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z$$



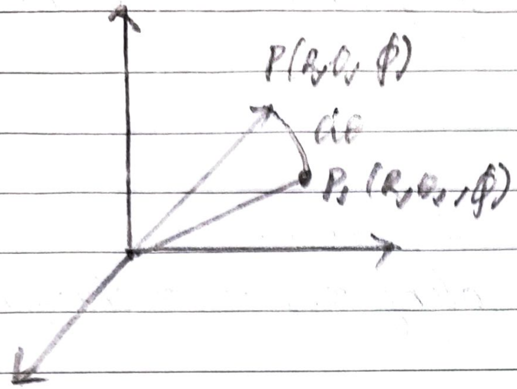
As ϕ changes from ϕ_1 to ϕ_2, a differential changes occurs which is a curved line, whose length is $r\,d\phi$.
In case of cartesian system, it was straight line AB with no uniqueness problems. But, in curvilinear systems, the curved surface has a linear component.

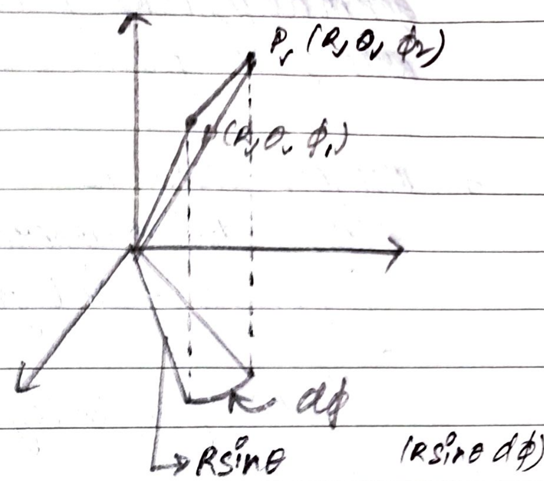
→ In spherical systems,

$$\nabla = \frac{\partial}{\partial R} a_R + \frac{1}{R}\frac{\partial}{\partial \theta} a_\theta + \underbrace{\frac{1}{R\sin\theta}}_{\text{displacement along the curved surface}}\frac{\partial}{\partial \phi} a_\phi$$

→ upon differentiating we get $a_\phi R\sin\theta$

Projection of the R vector on XY plane





When the directions change, displacement is unique. We have formed the differential operator for vectors. Now, we have two possible operations with ∇ -

① Dot Product :- $\nabla \cdot$ _ (Divergence)
② Cross Product :- $\nabla \times$ _ (Curl)

} Differenciation (rate of change)

For scalar functions, say $f(x)$, we just have the one possibility of $\frac{d}{dx} f(x)$.

LECTURE 14 - INTEGRATION OPERATORS

Upon differentiating a scalar, we get a function or constant. But in vector, either it is scalar or vector. Whether to do dot or cross product depends upon the given application scenario.

We know that performing dot product gives a scalar quantity whereas the cross product gives a vector quantity. For example-

$\nabla \cdot \vec{E}$ = charge accumulation

$\nabla \times \vec{E} = 0$ (static)

$= \frac{\partial \vec{H}}{\partial t}$ (time varying)

Generally, cross product is peculiar to EM. In all other field areas, dot product is used.

* INTEGRAL OPERATORS :

$f(x)$, "dx" → operator

$\int_a^b f(x)\,dx$ - integrate along x, as x changes

$\int_a^b f(x)\,dy$ - integrate along y, as y changes

* VECTOR INTEGRATION :

This is done to study the effect of force on structures.

1. Line - 1D $\vec{dl}$ operator
2. Surface - 2D $\vec{ds}$ operator
3. Volume - 3D $\vec{dv}$ operator

} Similar to how we formed '∇'.

LECTURE 15 - LINE INTEGRAL EXPLANATION

To integrate a vector, we need to incorporate direction into the operator. Therefore -

Increase in co-ordinate value

$$\vec{dl} = dx\, a_x + dy\, a_y + dz\, a_z$$
$$\vec{dl} = dr\, a_r + r\,d\phi\, a_\phi + dz\, a_z$$
$$\vec{dl} = dR\, a_R + R\,d\theta\, a_\theta + R\sin\theta\, d\phi\, a_\phi$$

To perform integration, we take dot / cross product.

$\int \vec{F} \cdot \vec{dl}$ or $\int \vec{F} \times \vec{dl}$

scalar

In most occassions, we take the dot product. Cross product is rarely observed. This is because upon performing cross, the perpendicular vector formed does not lie on the plane

The dimension of ∇ is $\frac{1}{m}\left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial r}, \dots\right]$

The displacement is in m.
Similarly, the dimension of $\vec{dl}$ is m.
All the surface integrals become meaningful once dimension is taken into account.

★ APPLICATIONS :
① Crack Analysis
② Pattern lock in Smart phone.

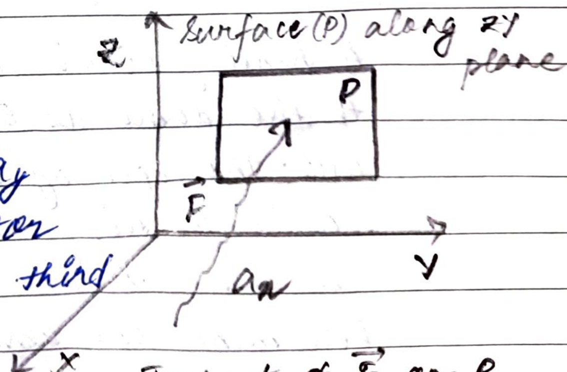
$\int_a^b \vec{F}\cdot\vec{dl}$ = meaning decided by application

LECTURE 16 - SURFACE INTEGRAL EXPLANATION
(2D vector integration)

For $f(x,y)$, $k = \int_c^d \int_a^b f(x,y)\, dx\, dy$

This calculates effect of force or vector on a surface.

$$\boxed{k = \int \vec{F}\cdot\vec{ds}}$$



Impact of $\vec{F}$ on P

$\vec{ds} = dx\,dy\, a_z + dy\,dz\, a_x + dz\,dx\, a_y$
$= \begin{bmatrix}\text{variations along} \\ \text{two directions}\end{bmatrix}$ unit vector along the third

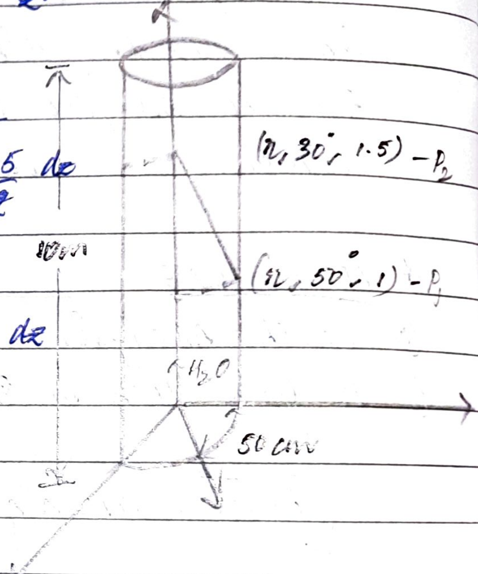
$\vec{ds} = dr\, r d\phi\, a_z + r d\phi\, dz\, a_r + dr\, dz\, a_\phi$

Q. Consider a rubber tube of length 10 m and radius 50 cm. Tube is along z axis. Water is pumped from the origin with velocity $\vec{F} = 0.025\, a_r + 0.05\, a_\phi + \frac{5}{z}\, a_z$. There is a crack from $z = 1m$, $\phi = 50°$ to $z = 1.5m$, $\phi = 30°$. Determine the amount of water coming out through crack.

A. $\int \vec{F} \cdot \vec{dl} = \int \vec{F} \cdot (dr\, a_r + r\,d\phi\, a_\phi + dz\, a_z)$

$= \int 0.025\, dr + r(0.05)\, d\phi + \frac{5}{z}\, dz$

$= \int_{r_1}^{r_2} \underbrace{0.025\, dr}_{(0)} + 0.05 \int_{50°}^{30°} r\, d\phi + \int_{1}^{1.5} \frac{5}{z}\, dz$



$= 0 + 0.05 \times 50 \times 10^{-2} \int_{50°}^{30°} d\phi + 5 \int_{1}^{1.5} \frac{1}{z}\, dz$

$= 0.05 \times 50 \times 10^{-2} [-20°] + 5 [\ln z]_1^{1.5}$

$\downarrow$ convert to radians

$= 2.01859\ m^3/s$

LECTURE 17 - SURFACE INTEGRAL NUMERICAL

Q. A gas source provides gas at a velocity,

$$V = \frac{5}{R} a_R + 5 a_\theta + 6 a_\phi \ m/s$$

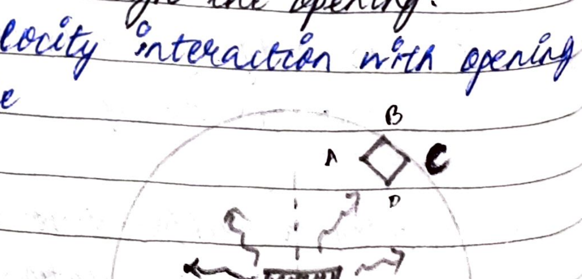
A hemispherical dome is placed over the source. The radius of the dome is 5m. An opening is provided for gas to come out. The opening is described by four points -

A $(5, 45°, 100°)$ C $(5, 55°, 110°)$
B $(5, 45°, 110°)$ D $(5, 55°, 100°)$ } (R, θ, ϕ)

Determine the amount of gas through the opening.

A. We need to perform gas velocity interaction with opening

$k = \int_S \vec{F} \cdot \vec{ds}$, where F is the velocity vector



$\vec{ds} = R\sin\theta\, d\phi\, R\,d\theta\, a_R +$
$R\sin\theta\, d\phi\, dR\, a_\theta +$
$R\,d\theta\, dR\, a_\phi$

$$\vec{F} \cdot ds = \underbrace{\frac{5}{R} R^2 \sin\theta \, d\theta \, d\phi}_{(A)} + \underbrace{5R \sin\theta \, d\phi \, dR}_{(B)} + \underbrace{6R \, dR \, d\phi}_{(C)}$$

Terms B and C will be zero because $dR = 0$

$$\therefore \int \vec{F} \cdot \vec{ds} = \int_{100^\circ}^{110^\circ} \int_{45^\circ}^{55^\circ} \frac{5}{R} R^2 \sin\theta \, d\theta \, d\phi \quad [R = 5m]$$

$$= 0.5820 \frac{m}{s} \cdot m^2$$

LECTURE 18 - INTRODUCTION TO NUMERICAL TECHNIQUES...

Q. How will you evaluate $\cos x$ for any x, computationally?

A. $x: 0 \rightarrow 90^\circ$ Taylor Series,
$0 \rightarrow 360^\circ$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
$0 \rightarrow 2\pi$

Hardware available - Basic gates, universal gates (NAND/NOR), Adder, Subtractors, circuits, etc ---

The number of terms depends on hardware "bit" representation, i.e. certain number of bits, 8 bit (or) 16 bit (or) 64 bit. In recent times, the processor is 64 bit, redundant [Representation Error]

* ROUND OFF ERROR:
"How much error" can be calculated only if we know the final result. Generally, we don't know what is the final answer.

Representation in hardware → Round-off errors → accumulate when we do many "+", "-" → Finally, a BIG error

How to fix this problem?

* TECHNIQUES TO FIX ERROR :

Iteration (k+1) − Iteration (k)

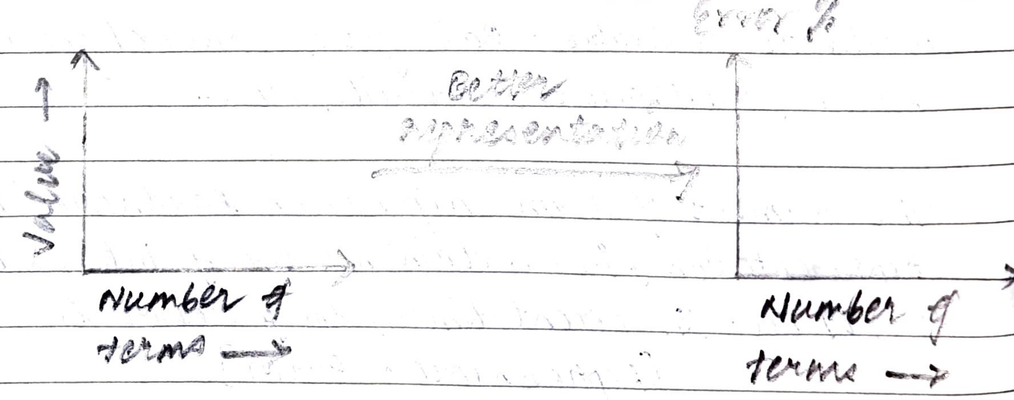
⇒ value is below a certain "ε" → 0 → 0.0001
or
0 → 0.5

Then, we stop "convergence point".
Works well for "MANY" maths functions

LECTURE 10 - ERROR CONVERGENCE COMPUTATION

If the correct value is not known, how much error can be tolerated. We will define -

$$\underset{\text{(error limit)}}{\varepsilon} = \frac{\text{Current value} - \text{Previous value}}{\text{current value}}$$



* CONVERGENCE :

We need to identify when a computational process converged. We cannot keep on doing computation, the breakpoint is defined through error representation. Finalising this conclusion is called as convergence.

How do we know our algorithm has converged ?

LECTURE 20 - GRADIENT

If we had a scalar function f, then the gradient was given by - ∇f, where the gradient of a scalar provided a vector.
$f = f(x, y, z)$ or $f(r, \phi, z)$ or $f(R, \theta, \phi)$

The source of f is from physical experiments/phenomena.
$\therefore T = f(x, y, z)$
$\Rightarrow \nabla T$ gives us the "direction" in which the maximum rate of change occurs. If we know this direction, the rate of change along any other direction can be easily obtained.

LECTURE 21 - NUMERICALS ON GRADIENT

It is very difficult to create a function $f(x, y, z)$ from numerical readings. It can be done only for specific applications. It cannot be learnt at a general level. It was this inability to form expressions that became the main motivation for computational studies. This gives rise to the following two cases-

1. Readings → expressions → evaluate
   ↓
   when it is complex → computational
2. Readings → unable to form expressions → Direct evaluation through computational methods.

LECTURE 22 - MORE NUMERICALS ON GRADIENT

Consider a heater present inside a room. The position of the heater is taken as the origin of a rectangular co-ordinate system. The temperature in the room is modelled as a scalar function-

$$T = 240 + z^2 - 2xy \text{ Kelvin}$$

Q1. Determine the gradient for this scalar function, and provide all interpretation for the obtained expressions:

A1. Let gradient be represented as $\vec{K}$.

$$\vec{K} = \nabla T$$

$$\vec{K} = \frac{\delta}{\delta x} T a_x + \frac{\delta}{\delta y} T a_y + \frac{\delta}{\delta z} T a_z \text{ Kelvin/metre}$$

$$\vec{K} = -2y\, a_x - 2x\, a_y + 2z\, a_z \text{ Kelvin/metre}$$

Interpretation for the obtained expression - The obtained gradient is a vector and at any point, this vector will provide the direction and amplitude of maximum rate of change of temperature at that point in the region of concern.

Q2. Consider a point P(1,2,3) in the region of concern, calculate the gradient and provide suitable interpretation.

A2. We will calculate the gradient vector at the point P(1,2,3). Let $\vec{K_1}$ be the gradient computed at the point.

$$\vec{K_1} = -4a_x - 2a_y + 6a_z \text{ Kelvin/metre}$$

We will calculate two quantities -

(a) Magnitude of $\vec{K_1}$ -

$|\vec{K_1}| = \sqrt{(-4)^2 + (-2)^2 + (6)^2}$ Kelvin/metre $= \sqrt{56}$ Kelvin/metre

(b) Unit vector along the direction of $\vec{K_1}$ -

Let this be represented as $\hat{K_1}$.

$$\hat{K_1} = \frac{\vec{K_1}}{|\vec{K_1}|}$$

This provides the unit vector as -

$$\hat{K_1} = \frac{-4a_x - 2a_y + 6a_z}{\sqrt{56}}$$

(c) Interpretation -

If we are at point P(1,2,3), then the direction provided by the unit vector $\hat{K_1}$ is the direction, from the point P(1,2,3), along which maximum rate of change of temperature would be observed, and that maximum value of rate of change is $\sqrt{56}$ Kelvin/metre.

This completes the first section on mathematically finding gradient and providing a meaningful interpretation to it. In the next session, we will discuss on application employing gradient, and the final session will be a concluding session.

* SECTION - II :
A sample application of Gradient concept is attempted:
Consider the gradient vector computed in the general form, from the above section.

$$\vec{K} = -2y\,a_x - 2x\,a_y + 2z\,a_z \text{ kelvin/metre}$$

Now consider two points A(1,1,1) and B(2,2,2). Let G represent the line integral of the gradient along the straight line connecting the points A and B.

$$G = \int_A^B \vec{K}\cdot\vec{dl}$$

Note the point that we have decided to take the line integral along the straight line connecting points A and B. The equation of the line connecting A and B is $x = y = z$.

$$G = \int_1^2 -2y\,dx + \int_1^2 -2x\,dy + \int_1^2 2z\,dz$$

The value is -3 kelvin.
This should be understood as the difference of temperature between the points A and B.
This value can be easily verified at by computing $T(2,2,2) - T(1,1,1)$, where T stands for the basic Temperature function with which we started off.

* APPLICATION BASED INTERPRETATION :
Now, consider that it is required to install a electrical wire between these two points, and in many cases, it may be required to find out the temperature difference across the

two points of device, to determine whether the equipment (wire in this case) can tolerate this temperature difference and can still work properly.

This is one sample application of the gradient function. Now depending on the physical parameter, sometimes the line integral of the gradient over a 2D area may provide more useful observations, specifically in weather monitoring, over an area.

* SIGNIFICANCE OF GRADIENT FOR COMPUTATIONAL ENGINEERS:

In present times, where the society and industry is expecting software for each and every small application, the knowledge of the "gradient" is the actual core concept that you need to know. The efficiency of your software when compared to others depends on the exact identification of the gradient vector. Particularly, this is evident in the immense number of freeware available for predicting stock market performance and associated financial activities.

Gradient description is your actual "patent" or the intellectual property right. The issue of providing intellectual property rights for software is still an issue