AI1110 Assignment II (ICSE Class 12 2018)

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Question 9b: The population of a town grows at the rate of 10% per year. Using differential equation, find how long will it take for the population to grow 4 times.

Solution: Let the initial population be P_o which increases at a rate of 10% and the population after t years be P i.e.

$$\frac{dP}{dt} = 10\% \text{ of } P \tag{1}$$

$$dP = \frac{10P}{100}dt \tag{2}$$

$$\frac{dP}{P} = \frac{dt}{10} \tag{3}$$

On integrating both sides, we have

$$\int_{P_o}^{P} \frac{1}{P}dp = \frac{1}{10}\int_{0}^{t} dt \tag{4}$$

$$\left(\because \int dx = x \text{ and } \int \frac{1}{x}dx = \log x\right)$$

$$\log P - \log P_o = \frac{1}{10}t \tag{5}$$

$$\log \frac{P}{P_o} = \frac{1}{10}t \tag{6}$$

Since it is given that the population grows 4 times,

$$P = 4P_o \tag{7}$$

Substituting this, we get

$$\log 4 = \frac{1}{10}t \tag{8}$$

$$\implies t = 10\log 4 \tag{9}$$

$$\implies t = 20\log 2 \tag{10}$$

($\because$ The value of $\log 2 = 0.3010$)

$$\implies t = 6.02 \tag{11}$$

Hence, the time required for the population to grow by four times is $\underline{6.02}$ years.