

AI1110 Assignment II (ICSE Class 12 2018)

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Question 9b: The population of a town grows at the rate of 10% per year. Using differential equation, find how long will it take for the population to grow 4 times.

Solution: Let the initial population be P_o which increases at a rate of 10% and the population after t years be P i.e.

$$\frac{dP}{dt} = 10\% \text{ of } P \quad (1)$$

$$dP = \frac{10P}{100} dt \quad (2)$$

$$\frac{dP}{P} = \frac{dt}{10} \quad (3)$$

On integrating both sides, we have

$$\int_{P_o}^P \frac{1}{P} dp = \frac{1}{10} \int_0^t dt \quad (4)$$

$$\left[\because \int \frac{1}{x} dx = \log x \right]$$

$$\log P - \log P_o = \frac{1}{10} t \quad (5)$$

$$\log \frac{P}{P_o} = \frac{1}{10} t \quad (6)$$

Since it is given that the population grows 4 times,

$$P = 4P_o \quad (7)$$

Substituting this, we get

$$\log 4 = \frac{1}{10} t \quad (8)$$

$$\implies t = 10 \log 4 \quad (9)$$

$$\implies t = 20 \log 2 \quad (10)$$

$$\left[\because \text{The value of } \log 2 = 0.3010 \right]$$

$$\implies t = 6.02 \quad (11)$$

Hence, the time required for the population to grow by four times is 6.02 years.