AI1110 Assignment 4

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Question: Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

Solution: Consider first n positive integers.
Number of ways to select two random numbers(without replacement) from the first n positive positive integers = ${}^nP_2 = n(n-1)$.
Let X represent the larger of the two numbers obtained. Therefore, X can take the values of

$$2, 3, 4, 5 \ldots, n.$$

i.e. $\Pr(X = i)$ where i $\in (2, n)$

$$\therefore \Pr(X = i) = \frac{2(i-1)}{n(n-1)}$$

$$E(X) = \sum_{i=2}^{i=n} X_i P(X_i) \tag{1}$$

$$= \sum_{i=2}^{i=n} i \times \frac{2(i-1)}{n(n-1)} \tag{2}$$

$$= \frac{2}{n(n-1)} \sum_{i=1}^{i=n} i^2 - \frac{2}{n(n-1)} \sum_{i=1}^{i=n} i \tag{3}$$

$$= \frac{2}{n(n-1)} \times \frac{n(n+1)(2n+1)}{6} - \frac{2}{n(n-1)} \times \frac{n(n+1)}{2} \tag{4}$$

$$= \frac{2(n+1)}{3} \tag{5}$$

For n = 6, substituting value of 6 in (5), we get

$$\boxed{E(X) = \frac{14}{3}}$$