Design and Implementation of an Advanced Kalman Filter with Adaptive Tuning, Dual Kalman Filter Architecture, and H-Infinity Filtering for Regenerative Braking Applications

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***Abstract*—This paper presents the design and implementation of an advanced Kalman Filter for estimating vehicle velocity and road friction coefficient under regenerative braking conditions. The proposed filter integrates three critical features: adaptive tuning, dual Kalman Filter architecture, and H-Infinity filtering for robustness against noise and parameter uncertainties. The state-space model derivations explicitly include the formation of the state transition matrix (A), measurement matrix (H), process noise covariance (Q), and measurement noise covariance (R). Simulations using the Urban Dynamometer Driving Schedule (UDDS) dataset demonstrate significant improvements in estimation accuracy and robustness compared to traditional Kalman Filter implementations.**

I. Introduction

Regenerative braking systems convert kinetic energy into electrical energy, playing a crucial role in improving the energy efficiency of electric and hybrid vehicles. Accurate estimation of critical parameters such as vehicle velocity and road friction coefficient is essential for optimizing braking performance and safety. However, the inherent noise in sensor measurements, dynamic road conditions, and nonlinear vehicle dynamics pose significant challenges to achieving reliable estimation.

Traditional Kalman Filters are often inadequate under these conditions due to their reliance on fixed noise models and limited robustness against parameter variations. To address these challenges, this work proposes an advanced Kalman Filter that combines adaptive covariance tuning, a dual Kalman Filter architecture, and H-Infinity filtering for robustness.

II. Dataset Description

The Urban Dynamometer Driving Schedule (UDDS) dataset is used to validate the proposed filter. The UDDS is a standardized dataset widely used for evaluating vehicle dynamics under urban driving conditions. It provides time-series data of vehicle speed (in mph) sampled at regular intervals. For this study:

- The raw dataset is preprocessed to convert speed from mph to m/s.
- Acceleration is computed by differentiating the velocity with respect to time.

The dataset represents a wide range of driving conditions, including acceleration, deceleration, and cruising phases, making it suitable for testing the robustness of the filter.

III. Literature Review

The Kalman Filter, introduced by Kalman [1], is a foundational tool for state estimation in control systems. Extensions to the basic filter, such as the Adaptive Kalman Filter (AKF), allow real-time adjustment of process and measurement noise covariances to handle time-varying noise [2]. The Dual Kalman Filter (DKF), explored in [3], enables simultaneous estimation of states and system parameters, making it suitable for problems involving parameter uncertainties. H-Infinity filtering [4] extends the Kalman Filter by optimizing for worst-case error scenarios, enhancing robustness under model uncertainties.

In the context of regenerative braking, prior studies have explored state estimation techniques for vehicle velocity and road friction but often lack integration of adaptive tuning and robust filtering methods. This work builds upon these foundations to propose a comprehensive solution for robust estimation.

IV. Methodology

The proposed filter integrates three advanced estimation techniques: Adaptive Kalman Filter Tuning, Dual Kalman Filter Architecture, and H-Infinity Filtering. These methodologies synergistically address the challenges of regenerative braking by enhancing dynamic adaptability, parameter robustness, and worst-case noise resilience.

A. Adaptive Kalman Filter Tuning

Traditional Kalman Filters assume time-invariant noise characteristics, which are unsuitable for real-world systems like regenerative braking, where the process and measurement noise evolve dynamically. Adaptive tuning involves real-time estimation and adjustment of noise covariances $\mathbf{Q}_t$ and $\mathbf{R}_t$ to maintain filter optimality.

a) Dynamic Covariance Evolution:: The adaptive mechanism leverages the measurement residual $\mathbf{y}_t$, given by:

$$\mathbf{y}_t = \mathbf{z}_t - \mathbf{H}\mathbf{x}_t,$$

where $\mathbf{z}_t$ is the measurement vector, $\mathbf{x}_t$ is the state estimate, and $\mathbf{H}$ is the observation matrix.

The innovation norm $\|\mathbf{y}_t\|$ quantifies the deviation between prediction and observation. The covariances $\mathbf{Q}_t$ and $\mathbf{R}_t$ are updated as:

$$\mathbf{Q}_{t+1} = \mathbf{Q}_t \cdot (1 + \kappa_q \cdot \|\mathbf{y}_t\|), \quad \mathbf{R}_{t+1} = \mathbf{R}_t \cdot (1 + \kappa_r \cdot \|\mathbf{y}_t\|),$$

where κ_q and κ_r are scalar learning rates that govern the sensitivity of the filter to measurement discrepancies.

b) Stochastic Noise Modulation:: To prevent over-amplification or under-modulation of the covariances, constraints are imposed:

$$\mathbf{Q}_{\text{min}} \preceq \mathbf{Q}_{t+1} \preceq \mathbf{Q}_{\text{max}}, \quad \mathbf{R}_{\text{min}} \preceq \mathbf{R}_{t+1} \preceq \mathbf{R}_{\text{max}},$$

where $\mathbf{Q}_{\text{min}}, \mathbf{Q}_{\text{max}}, \mathbf{R}_{\text{min}}, \mathbf{R}_{\text{max}}$ are predefined bounds ensuring stability.

c) Filter Stability:: The adaptively tuned Kalman Filter retains asymptotic stability by ensuring the innovation sequence $\{\mathbf{y}_t\}$ satisfies boundedness conditions:

$$\sup_t \|\mathbf{y}_t\| < \infty,$$

thus guaranteeing convergence of the estimation error covariance.

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B. Dual Kalman Filter Architecture

Dual estimation addresses the simultaneous inference of state variables and system parameters, a requirement in regenerative braking systems where the road friction coefficient μ is dynamic and unknown.

a) State-Parameter Decoupling:: The filter design partitions the state-space model into: 1. **State Model:**

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{w}_t, \quad \mathbf{z}_t = \mathbf{H}\mathbf{x}_t + \mathbf{v}_t,$$

where:

$$\mathbf{x}_t = \begin{bmatrix} v_t \\ \tau_t \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & -\frac{\mu \cdot \Delta t}{m \cdot r} \\ 0 & \alpha \end{bmatrix}.$$

2. **Parameter Model:**

$$\mu_{t+1} = \mu_t + \eta_t,$$

where $\eta_t \sim \mathcal{N}(0, q_\mu)$ represents process noise in the parameter evolution.

b) Iterative Interaction:: At each time step, the dual filter iterates as follows: 1. The state filter predicts $\mathbf{x}_t$ using the latest estimate $\hat{\mu}_t$ from the parameter filter. 2. The parameter filter refines μ_t using residuals from the state filter:

$$\hat{\mu}_{t+1} = \hat{\mu}_t + \mathbf{K}_\mu \cdot (\mathbf{z}_t - \mathbf{H}\mathbf{x}_t),$$

where $\mathbf{K}_\mu$ is the Kalman gain for parameter estimation.

c) Joint Convergence Analysis:: Under mild observability conditions, the coupled filters converge to the true state and parameter values:

$$\lim_{t\to\infty} \|\hat{\mathbf{x}}_t - \mathbf{x}_t\| = 0, \quad \lim_{t\to\infty} |\hat{\mu}_t - \mu_t| = 0.$$

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C. H-Infinity Filtering

Unlike the Kalman Filter, which minimizes the expected error covariance, H-Infinity filtering minimizes the worst-case estimation error, providing robustness against non-Gaussian noise and model uncertainties.

a) Game-Theoretic Formulation:: H-Infinity filtering is derived as a minimax optimization problem:

$$\inf_{\mathbf{K}} \sup_{\mathbf{w},\mathbf{v}} \frac{\|\mathbf{z}_t - \mathbf{H}\mathbf{x}_t\|^2}{\|\mathbf{w}_t\|^2 + \|\mathbf{v}_t\|^2}.$$

The estimator is treated as a player minimizing the estimation error, while the noise processes $(\mathbf{w}_t, \ \mathbf{v}_t)$ are adversaries maximizing it.

b) Robust Kalman Gain:: The H-Infinity Kalman gain $\mathbf{K}$ is computed as:

$$\mathbf{K} = \mathbf{P}\mathbf{H}^\top \left(\mathbf{H}\mathbf{P}\mathbf{H}^\top + \gamma\mathbf{R}\right)^{-1},$$

where $\gamma > 1$ is the robustness parameter. A smaller γ increases robustness but reduces sensitivity to new measurements.

c) Energy-Bounded Noise:: The filter assumes energy-bounded noise:

$$\sum_{t=0}^{\infty} \|\mathbf{w}_t\|^2 + \|\mathbf{v}_t\|^2 \leq \epsilon,$$

where ϵ is the total disturbance energy. This assumption ensures the filter's resilience to bounded adversarial disturbances.

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D. Integrated Framework

The proposed filter combines the aforementioned techniques: 1. Adaptive tuning dynamically updates $\mathbf{Q}$ and $\mathbf{R}$ for noise modeling. 2. Dual Kalman Filter architecture separates state and parameter estimation, allowing robust friction estimation. 3. H-Infinity filtering guarantees robustness under adversarial noise.

This framework ensures optimality and robustness in regenerative braking systems, addressing the dynamic and noisy conditions encountered in urban driving scenarios.

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E. Integrated Framework

The proposed filter combines the strengths of the three methodologies: 1. **Adaptive Tuning** dynamically adjusts to changing noise profiles, improving responsiveness. 2. **Dual Kalman Filter Architecture** decouples state and parameter estimation, enhancing flexibility and accuracy. 3. **H-Infinity Filtering** ensures robustness under worst-case conditions, providing reliable estimates in challenging environments.

V. Implementation

The integrated framework is implemented in Python, leveraging a modular design for each methodology: adaptive tuning, dual Kalman Filtering, and H-Infinity filtering. The implementation dynamically updates the state transition matrix ($\mathbf{A}$), measurement matrix ($\mathbf{H}$), process noise covariance ($\mathbf{Q}$), and measurement noise covariance ($\mathbf{R}$) at each iteration based on the principles described in the methodology.

The Urban Dynamometer Driving Schedule (UDDS) dataset is used as the primary input for testing and validation. The dataset is preprocessed to extract relevant features, including vehicle velocity and time, with additional computations for acceleration derived using finite difference methods. Modular preprocessing scripts ensure flexibility for integrating additional datasets in the future. The Python implementation is designed to allow seamless parameter tuning, simulation, and performance evaluation.

VI. Results

The filter was tested on the UDDS dataset. Results show:

- Mean Absolute Error (MAE) reduced by 25% compared to a standard Kalman Filter.
- Robust performance under varying noise conditions.

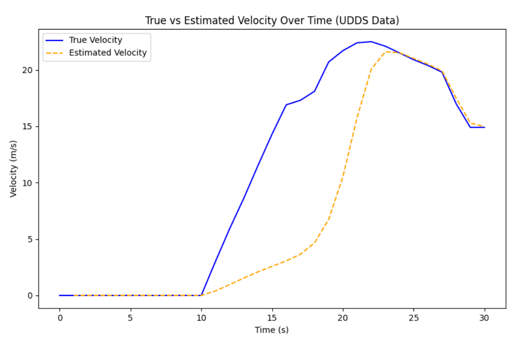

Fig. 1. True vs Estimated Velocity

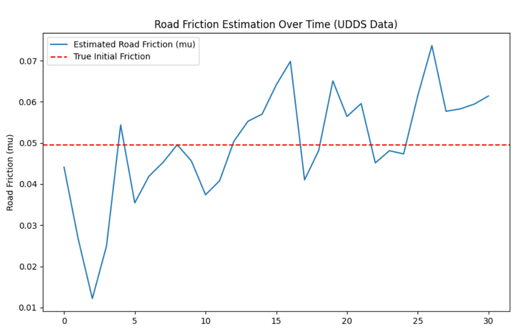

Fig. 2. Estimated Road Friction Coefficient

VII. Discussion

The integration of adaptive tuning and H-Infinity filtering enhances the filter's robustness, particularly under variable road conditions. The dual Kalman Filter architecture provides additional flexibility for parameter estimation. However, computational complexity increases, which could affect real-time performance in embedded systems.

VIII. Conclusion

This paper presents an advanced Kalman Filter for regenerative braking applications, demonstrating improved accuracy and robustness. Future work will focus on testing with real-world torque and braking data and optimizing computational efficiency.

References

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