Lenses and Optical Instruments

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Abstract

The series of four experiments: a.**Determination of focal length of two lenses**, b.**Determination of focal length and image distance of a combined lens system**, c.**Construction of a telescope after Kepler**, d.**Construction of a microscope** performed, showed the findings that the combination of the lens system is a lens of focal length whose reciprocal equals the sum of the reciprocals of individual focal lengths. Optical instruments like telescope and microscope can be constructed with a certain arrangement of lenses.

1 Introduction and Theory

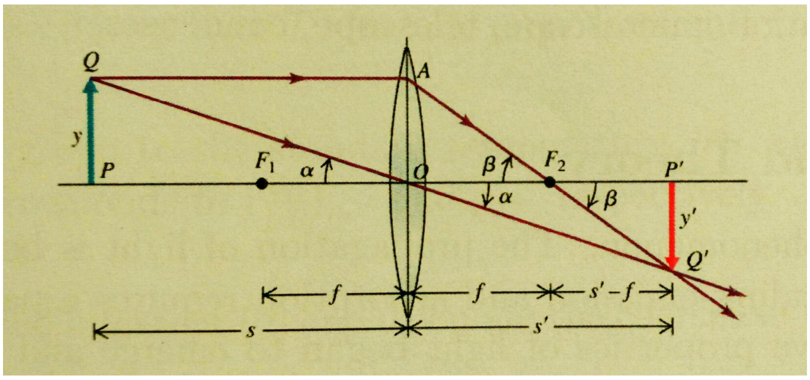

Figure 1: Image construction by two principal rays

The focal ray, the parallel ray, and the central ray shown in the Figure 1 forms an image y' at P' of an object y at P. The relationship between the focal length of a lens, the object distance s and the image distance s' is obtained from geometrical optics and is given by the so-called lens formula:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$$

In case two, lenses are combined, and their distance is much smaller than their focal length. Their total focal length is given by

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the distance between the lenses could not be maintained negligibly very small, the distance d between the lenses is taken into account, and the corrected combined lens formula is

$$\frac{1}{f_{tot}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

These basic formulae are applied to investigate the properties of lenses and setup different optical instruments.

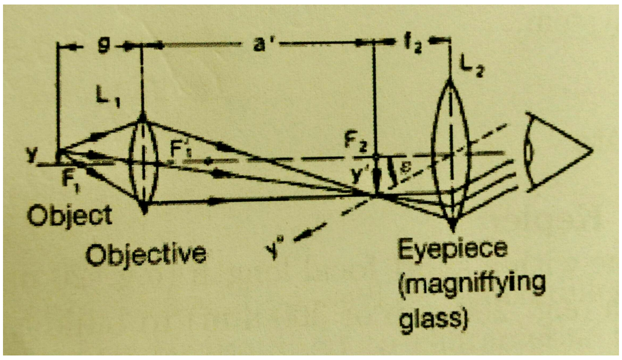

Figure 2: Rays in Microscope

1.1 Microscope

A simple microscope consists of two lenses: an objective lens that forms a real, magnified image, and an ocular lens that forms a virtual image at infinity. Its magnification is the product of the magnification of the objective and the magnification of the ocular.

$$M = M_{ob} \cdot M_{oc} = \frac{s'_{ob}}{f_{ob}} \cdot \frac{s'_{oc}}{f_{oc}}$$

However, the final image being virtual can't be cast into the screen. Thus, shifting the image formed by objective outside of the focal length of the eyepiece would enable to cast the final image on a screen, so the distance of the screen from the eyepiece is also used to calculate the magnification, which is given as

$$M = M_{ob} \cdot M_{oc} = \frac{s'_{ob}}{f_{ob} + \delta} \cdot \frac{s'_{oc}}{f_{oc} + \delta}$$

where δ indicates that the actual position has to be only a bit outside of the respective focal length.

1.2 Telescope

A telescope consists of two lenses: an objective lens that forms a real image and an ocular lens which leads to an angular magnification. The Kepler type telescope, consisting of two convex lenses has magnification of

$$M = -\frac{f_{ob}}{f_{oc}}$$

where the negative sign denotes that the image formed is inverted.

2 Experimental and Procedure

2.1 Equipment

- Optical bench with holders
- Set of converging and diverging lenses

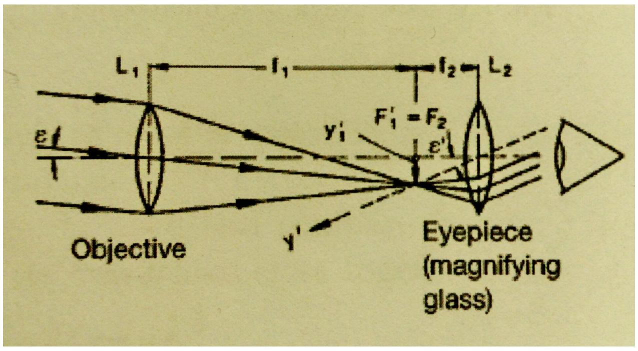

Figure 3: Rays in Kepler Telescope

- Screen
- Different objects
- Experimental lamp
- Power supply 0-12V DC

2.2 Setup and Procedure

In the following experiments, the source of light, lenses, and screen was mounted on an optical bench. The source of light was installed on one side of the optical bench and fixed such that the lamp filament was focused to infinity. All the equipment was positioned at the same height.

2.2.1 Determination of the focal length of lenses

In this experiment, an object (arrow slit) was positioned close to the source of light, and the lens of unknown focal length was positioned close to the arrow slit. The screen was positioned on the other side of the optical bench. After installing all the elements for the experiment, the room was darkened to improve the observations. The beam of light was passed through the lens to hit the screen. Initially, the blurred image was obtained on the screen, and the screen was gradually shifted to obtain a "sharp" image. The position of all the elements was noted down. The procedure was repeated for five different object distances and with the new lens.

2.2.2 Determination of the focal length of combined lens system

The lens of +100 mm was positioned at 20 cm from the object (arrow slit), and the screen was adjusted to obtain a "sharp" image. The position of all the elements was recorded. Again +200 mm lens was placed as close as possible behind the first lens, and the screen was adjusted to obtain a sharp image. The new position of the screen was noted. Now, the +200mm lens was replaced by -200mm lens and positioned behind the first lens. The screen was glided to obtain a sharp image; however, the image of the same characteristics (blurriness, contrast) was obtained on the optical bench and the opposite wall.

2.2.3 Build a telescope after Kepler

Two lenses objective (+200 mm) and eyepiece (+20 mm) were used for the construction of the Kepler telescope. The lenses were positioned on each end of the optical bench. The distant object was viewed through the eyepiece and was slid until a "sharp" image appeared. The magnified and inverted image was observed.

2.2.4 Setup a microscope

During this experiment, an object (flea), attached in the cover glass, was positioned close to the filament lamp. Two lenses objective (+200 mm) and eyepiece (+20 mm) were used for the setup of the microscope. The screen was positioned behind the eyepiece on the opposite side of the optical bench. Initially, the blurred image was viewed on the screen. The object was adjusted to get a "sharp" and magnified image, and the position of all elements was noted down.

3 Results and Data Analysis

3.1 Determination of the focal length of lenses

From observations, the positions of the object, unknown lens(lens A) and screen of the setup, where the sharp image was obtained is recorded in the following table:

Position of			Object Distance (cm)	Image Distance (cm)	Focal length (cm)
lens(P_l)	arrow(P_a)	image(P_i)	$s = P_l - P_a$	$s' = P_i - P_l$	$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$
29.0	17.5	96.6	11.5	67.6	9.83
30.0	17.5	73.5	12.5	43.5	9.71
31.0	17.5	65.1	13.5	34.1	9.67
32.0	17.5	61.7	14.5	29.7	9.74
33.0	17.5	58.9	15.5	25.9	9.70

Table 1: Object and Image distances for Lens A

From table 1 the average value $\bar{f_A}$ is calculated as

$$\bar{f_A} = 9.7cm$$

The similar measurement for the unknown lens "lens B" is noted in the following table:

Position of			Object Distance (cm)	Image Distance (cm)	Focal length (cm)
lens(P_l)	arrow(P_a)	image(P_i)	$s = P_l - P_a$	$s' = P_i - P_l$	$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$
43.0	17.5	78.0	25.5	35.0	14.7
44.0	17.5	77.6	26.5	33.6	14.8
45.0	17.5	76.2	27.5	31.2	14.6
46.0	17.5	76.9	28.5	30.9	14.8
47.0	17.5	76.1	29.5	29.1	14.6

Table 2: Object and Image distances for Lens B

From Table 2, the average value $\bar{f_B}$ is calculated as

$$\bar{f_B} = 14.7cm$$

3.2 Determination of the focal length of combined lens system

For +100 mm lens fixed, the following measurements were noted,
object distance, s= 20 cm
image distance, s' = 19.6 cm
From lens formula,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{s} + \frac{1}{s'} \\ &= \frac{1}{20} + \frac{1}{19.6} \\ \therefore f = 9.90cm \end{aligned}$$

When +200 mm lens was placed just behind the +100 mm lens, the new position of the screen, where the sharp image formed, was found to be 50.4 cm, and the image distance was measured from the midpoint between two lenses to the position of the screen. Thus, from observations
object distance, s = 20 + $\frac{3.9}{2}$ = 21.9 cm
new image distance, s' = 10.9 cm
From lens formula,

$$\begin{aligned} \frac{1}{f} &= \frac{1}{s} + \frac{1}{s'} \\ &= \frac{1}{21.9} + \frac{1}{10.9} \qquad (1) \\ \therefore f = 7.2cm \end{aligned}$$

Also, from combined focal length formula

$$\begin{aligned} \frac{1}{f_{tot}} &= \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \\ &= \frac{1}{10} + \frac{1}{20} - \frac{3.9}{10 \times 20} \qquad (2) \\ \therefore f_{tot} = 7.70cm \end{aligned}$$

When -200 mm lens was placed just behind the +100 mm lens, we were not able to obtain the sharp image. The image of same characteristics i.e, blurriness, contrast was obtained at the end of the optical bench and on the opposite side of the wall.
Theoretically, the combined focal length f_{tot} is

$$\begin{aligned} \frac{1}{f_{tot}} &= \frac{1}{f_1} + \frac{1}{f_2} \\ &= \frac{1}{10} - \frac{1}{20} \\ \therefore f_{tot} = 20cm \end{aligned}$$

3.3 Build a telescope after Kepler

From observations, the actual size of the image couldn't be measured by the instruments we had. The estimated height of image is 1.5 cm for the object when viewed from distant appeared to be approximately 0.3 cm. Thus

$$\text{Magnification} = \frac{size\,of\,image}{size\,of\,object} = \frac{1.5}{0.3} = 5$$

Theoretically, when the objective lens of +200 mm and ocular lens of +20 mm are used.

$$\text{Magnification} = -\frac{f_{ob}}{f_{oc}} = \frac{-20}{2} = -10$$

3.4 Setup a microscope

From observations, for the sharp and magnified image obtained, the positions for the elements on the optical bench were noted. The image distance for objective lens s'_{ob} is measured as the difference of the distance between objective and ocular lens and the focal length of ocular lens.

i.e, image distance for objective lens = (8 - 2) cm = 6 cm

The image distance of ocular lens is measured to be 31.1 cm from the observation. Thus,

$$\text{Magnification, M} = \frac{s'_{ob}}{f_{ob}} \cdot \frac{s'_{oc}}{f_{oc}} = \frac{6}{5} \cdot \frac{31.1}{2} = 18.66$$

From direct observations,

image height = 5.4 cm

object height = 0.3 cm

$$\text{Magnification} = \frac{image\ height}{object\ height} = \frac{5.4}{0.3} = 18$$

4 Error Analysis

The least count of the measuring ruler attached to an optical bench is $\pm 0.1cm$. Therefore, the instrumental error is $\pm 0.1cm$.

4.1 Statistical Treatment

From Table 1, the standard deviation σ_{f_A} and the error of the mean value $\Delta\bar{f}_A$ are calculated as

$$\begin{aligned}\sigma_{f_A} &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(f_i - \bar{f}_A)^2} \\ &= \sqrt{\frac{1}{4}\sum_{i=1}^{5}(f_i - 9.73)^2} \\ &= 0.06cm\end{aligned}$$

$$\begin{aligned}\Delta\bar{f}_A &= \frac{\sigma_{f_A}}{\sqrt{n}} \\ &= \frac{0.06}{\sqrt{5}} \\ &= 0.03cm\end{aligned}$$

Similarly, from Table 2, the standard deviation σ_{f_B} and the error of the mean value $\Delta\bar{f}_B$ are calculated as

$$\begin{aligned}\sigma_{f_B} &= 0.09cm \\ \Delta\bar{f}_B &= 0.04cm\end{aligned}$$

4.2 Propagation of Error

The instrumental error produces uncertainty in measuring object and image distance, so there would be uncertainty in measuring the focal length of the lenses. Let, Δs be the uncertainty in object distance s, $\Delta s'$ be the uncertainty in image distance s', and Δf be the uncertainty in f. Thus,

$$\begin{aligned}\frac{1}{\Delta f_{avg}} &= \sqrt{\left(\frac{1}{\Delta s}\right)^2 + \left(\frac{1}{\Delta s'}\right)^2} \\ &= \sqrt{\left(\frac{1}{0.1}\right)^2 + \left(\frac{1}{0.1}\right)^2} \\ \therefore \Delta f_{avg} &= 0.07cm\end{aligned}$$

Therefore, the calculated propagated instrumental error i.e, 0.07cm is much larger than the statistical error. Thus, the dominant error source i.e, instrumental error is taken, and the focal length of lens A is $(9.7 \pm 0.1)cm$; taken the fact that the correct significant digits should be used. Similarly, the focal length of lens B is $(14.7 \pm 0.1)cm$.

The propagated error for magnification of the microscope for image height i and object height h is

For i = 5.4 cm, and h = 0.3 cm,

$$\begin{aligned}\Delta M &= \sqrt{\left(\frac{\Delta i}{i}\right)^2 + \left(\frac{\Delta h}{h}\right)^2} \\ &= \sqrt{\left(\frac{0.1}{5.4}\right)^2 + \left(\frac{0.1}{0.3}\right)^2} \\ &= 0.4\end{aligned}$$

- **Determination of the focal length of a combined lens system**: From results and data analysis section 3.2, the observed value of the focal length of +100 mm lens is 9.90 cm, and when the calculated propagated instrumental error with proper significant digits are taken in account, its focal length is observed to be $(9.9 \pm 0.1) cm$, which lies in the range of the literature value i.e, 10.0 cm.
  Similarly, from equation (1) the observed combined focal length for +100 mm and +200 mm lens with uncertainty is $(7.66 \pm 0.1) cm.$, which is in close approximation with the theoretical value, derived in equation (2).

- **Build a telescope after Kepler**: The high uncertainty in the observed magnification of the telescope is due to the incapability in measuring the height of image with the available instruments.

- **Setup a microscope**: From Results and Data Analysis Section 3.4, the magnification of the microscope with uncertainty is (18.0 ± 0.4), which lies in the range of theoretical value calculated in Results and Data Analysis Section 3.4.

5 Discussion

From Error Analysis Subsection: Statistical Treatment and Propagation of Error, it is observed that the instrumental error is higher than the statistical error. The small statistical error is not because of precise measurement but because of limited data. The precise measurement can be obtained with a higher number of observations. Otherwise, the instrument of very small least count should be used.
The result from the combined focal length of +100mm and +200mm lens is reduced, which shows that this combination can be used to treat the shortsightedness of the eye. However, in the case of the combination of +100 mm lens and -200 mm lens, the sharp image could not be produced within the optical bench, so this combination can't be used to treat the defects of eye.

6 Conclusion

From the observations of the determination of the focal length of lenses, the focal length of lens A is found to be $(9.7 \pm 0.1) cm$, and the focal length of lens B is found to be $(14.7 \pm 0.1) cm$.
The combination of two lenses systems is found to be the remedy for the defects in eye. Construction of the Kepler telescope from the combination of two lenses produces a magnified, inverted image of distant objects. The image can be obtained upright if concave lens is used instead of convex.

The magnification of the telescope couldn't be measured accurately because of incapability to measure the height of image with the available instrument.
Finally, a microscope was setup from the combination of two convex lens of smaller focal length, and a magnified image of flea of high sharpness was obtained on the screen. The magnified image of the flea is shown in the figure. The observed magnification was found to be (18.0 ± 0.4), which agrees with the literature value.

Figure 4: Magnified image of flea on screen

7 References

[1] Prof. Dr. Jürgen Fritz and Faezeh Mohaghegh, Classical Physics Lab (CH-140-B) Fall 2019