Velocity of Sound

Group 8
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Abstract

The report presents the measurements carried out to determine the velocity of sound in different media, the calculation of adiabatic index of CO_2 and He after the determination of the velocity of sound in the respective gases, and the calculation for the modulus of elasticity of Aluminum and Copper after the measurement of velocity of sound in respective metals. In the first experiment, the velocity of sound in gases were calculated from distance and time measurement. The measured values for the velocity of sound at $21°C$ were observed to be $(335.2 \pm 4.5)ms^{-1}$ (Air), $(258.1 \pm 3.5)ms^{-1}$ (CO_2), and $(1507 \pm 21)ms^{-1}$ (He). Hence, the calculated values for the adiabatic index of CO_2 and He are (1.20 ± 0.02) and (3.72 ± 0.09) respectively. In the final experiment, the velocity of sound in metals were studied from the longitudinal standing waves formed in the metal bar. The observed values for the velocity of sound in Aluminum and Copper were found to be $(4982 \pm 66)ms^{-1}$, and $(3810 \pm 24)ms^{-1}$ respectively. The measured values for the modulus of elasticity of Aluminum and Copper are, hence determined $(6.67 \pm 0.16) \cdot 10^{10} Pa$, and $(1.2 \pm 0.2) \cdot 10^{11} Pa$ respectively. The possible error sources in the measurements were also studied.

1 Introduction and Theory

Sound is disturbance travelling through a medium in space and time. Sound waves are the periodic disturbance in the medium, and can be described mathematically as

$$y(x,t) = A \cdot cos(kx - \omega t)$$

where y is the deflection of a particle in x-direction from its original position x at time t. A is the amplitude of particle motion, $(kx - \omega t)$ the phase of the wave, $k = \frac{2\pi}{\lambda}$ the wave number with wavelength λ, and the angular frequency $\omega = 2\pi f$ with frequency f. If v be the phase velocity, then

$$v = \lambda \cdot f \tag{1}$$

When waves travelling in opposite directions interfere, a standing wave is formed with certain nodes (minimum amplitude), and antinodes (maximum amplitude). For longitudinal wave in solid rods, when rods are clamped exactly at their center, the basic mode of the standing wave has one node at the center and antinodes at each free end. Accordingly this, a rod of length l has wavelength $\lambda = 2 \cdot l$. Then, from equation 1,

$$v = 2 \cdot l \cdot f \tag{2}$$

For longitudinal waves in solid homogeneous rods,

$$v = \sqrt{\frac{E}{\rho}} \tag{3}$$

with density ρ and elasticity modulus E.
For the speed of sound in gases, E can be replaced by the pressure p. Thus,

$$v = \sqrt{\frac{p}{\rho}} \tag{4}$$

However, sound oscillations are fast and is adiabatic process. Thus, p needs to be replaced by $p \cdot \gamma$, where γ is the adiabatic index of the gas.

$$v = \sqrt{\frac{p \cdot \gamma}{\rho}} \tag{5}$$

From ideal gas equation, $pV = nRT$ (Volume V, number of moles of molecules n, gas constant $R = 8.3143 JK^{-1}mol^{-1}$, and temperature T), equation 5 can be written as

$$v = \sqrt{\frac{\gamma \cdot R \cdot T}{M}} \tag{6}$$

For dry air, equation 6 can be simplified to

$$v_{air} = (331.3 + 0.6 \cdot \frac{\nu}{°C})ms^{-1} \tag{7}$$

using the ideal gas law, standard values of p and T. The value $0.6\frac{\nu}{°C}$ is called temperature coefficient, and ν the temperature in degree Celsius.

2 Experimental and Procedure

2.1 Setup and Procedure for Gases

A plastic tube with one end closed by a small loudspeaker and the other end with two hose nipples that allow the filling of the gases and a hole to connect to a microphone was used as the container in which the velocity of sound in Air, CO_2, and He was measured.

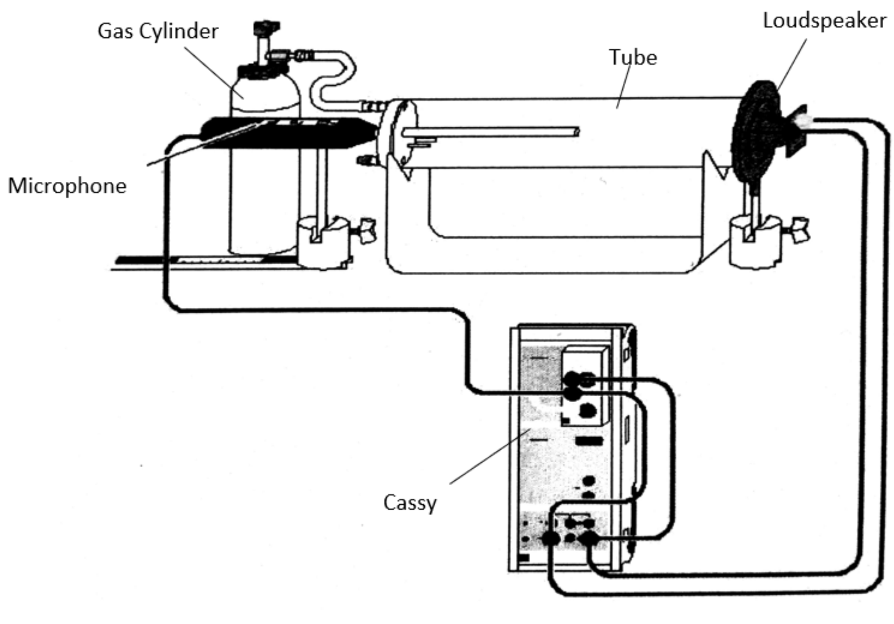

Figure 1: Setup of the experiment

Ref: Picture taken from Jacobs University Bremen, September 2019, F2019.1 Lab Manual

In the figure, loudspeaker and microphone are connected to the Cassy, which measures the transit time t between the generation of a pulse at the loudspeaker and its detection at the microphone. In the beginning, the microphone was positioned at $P_1, 100\ mm$ from the tube, and the transit time shown on the screen of the computer was noted down. The microphone was re-positioned at $P_2, 250\ mm$ from the tube, and the measured transit time was noted again. The procedure was repeated 5 times. The velocity of sound was then calculated from the path difference and the difference in transit time. The distance was then calculated from the known velocity for any time t to begin the experiment with gases.

The microphone was fixed at the calculated distance D from the tube, and the results for the velocity of sound in the air was verified again. The tube was then filled by carbon dioxide from the lower hole for a minute, and the velocities measured by Cassy were recorded within time interval of 15 sec. The same procedure was repeated for Helium, which was filled from the upper hole, being lighter than carbon dioxide and air. Helium, being highly volatile, the measurements were carried out within few seconds.

2.2 Setup and Procedure for Metals

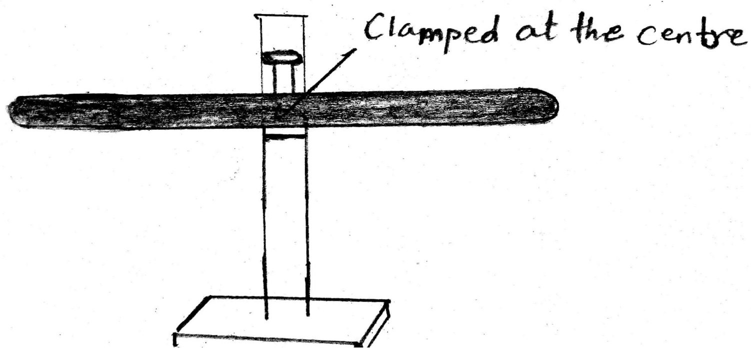

Figure 2: Setup for metal rods

In this experiment, the length and the diameter of the experimental rods (Aluminum and Copper) were measured using the meter scale and Vernier calipers respectively. The mass of the rods was then measured. The rod was then clamped at its center and was gently hit at its one end with a wooden block. A microphone, connected to Cassy, was placed at one end of the rod to measure the resulting sound frequency produced in the rod. Cassy settings were adjusted to measuring range $-3V$ to $+3V$, measuring interval $50\mu s$, and 2000 measuring points. The sinusoidal oscillation was then observed on the computer screen, from which the frequency was calculated, taken at least 20 periods for the calculation. The procedure was repeated 3 times.

3 Results and Data Analysis

3.1 Determination of velocity of sound in air at ambient temperature

See Error Analysis Section for the calculation of the errors

The measured room temperature is $(294 \pm 1)K$ with humidity 66%.
The transit time of the sound from the loudspeaker to the microphone for two different positions of the microphone P_1 and P_2, $150mm$ apart, is tabulated below.

Transit time	At position P_1 $[10^{-4}s]$	At position P_2 $[10^{-4}s]$
t_1	5.415	9.885
t_2	5.415	9.885
t_3	5.422	9.916
t_4	5.422	9.887
t_5	5.418	9.893
Average, $\bar{t}$	5.4184	9.8932
Error of the mean, $\Delta\bar{t}$	0.0016	0.0059

Table 1: Transit time measured at two different positions of the microphone.

The measured distance between two positions P_1 and P_2, $\delta s = (0.150 \pm 0.002)m$.
The difference in the measured transit time, $\delta t = (4.4748 \pm 0.0061) \cdot 10^{-4}s$.
Therefore,

$$\text{velocity}, v = \frac{\delta s}{\delta t} = \frac{0.150m}{4.4748 \cdot 10^{-4}s} = 335.2ms^{-1}$$

Thus, the velocity of sound in air is measured to be $(335.2 \pm 4.5)ms^{-1}$.
For velocity v, the distance D for time $t = 8.428 \cdot 10^{-4}s$ is

$$D = v \cdot t = (335.2ms^{-1}) \times (8.428 \cdot 10^{-4}s) = 0.2825m \tag{8}$$

The value of D with uncertainty is $(282.5 \pm 3.8) \cdot 10^{-3}m$.

3.2 Measurement of velocity of sound in different gases

The velocity of sound measured in CO_2 for distance $d = 0.425m$ *(set in Cassy)* is tabulated below.

Velocity $[ms^{-1}]$	Average, $\bar{v}$ $[ms^{-1}]$	Error of the mean, $\Delta\bar{v}$ $[ms^{-1}]$
387.9	388.22	0.15
388.1		
388.4		
388.7		
388.0		

Table 2: Velocity of sound measured in CO_2 for distance $d = 0.425m$

The corrected velocity of sound in CO_2 for distance D is

$$\begin{aligned} v_{CO_2} &= \bar{v} \cdot \frac{D}{d} \\ &= (388.22ms^{-1}) \cdot \frac{0.2825m}{0.425m} \\ &= 258.1ms^{-1} \end{aligned}$$

Therefore, the velocity of sound in CO_2 is measured to be $(258.1 \pm 3.5)ms^{-1}$.
From equation 6,

$$\begin{aligned}\gamma &= \frac{v^2 M}{RT} \\ &= \frac{(258.1^2 ms^{-1}) \cdot (44.01 \cdot 10^{-3} kg)}{(8.3143 JK^{-1} mol^{-1}) \cdot (294K)} \\ &= 1.2\end{aligned}$$

The measured value of adiabatic index for CO_2 is 1.20 ± 0.02.
The velocity of sound measured in He for distance $d = 0.425m$ *(set in Cassy)* is tabulated below.

Velocity $[ms^{-1}]$	Average, $\bar{v}$ $[ms^{-1}]$	Error of the mean, $\Delta\bar{v}$ $[ms^{-1}]$
2297.3		
2263.6		
2263.6	2267.4	8.4
2266.7		
2245.7		

Table 3: Velocity of sound measured in He for distance $d = 0.425m$

Similarly, the corrected velocity of sound in He for distance D is $(1507 \pm 21)ms^{-1}$, and the adiabatic index for He is observed to be 3.72 ± 0.09.

3.3 Determination of velocity of sound in metals

The measurements of different parameters for aluminium rod and copper rod is tabulated below.

	Aluminium	Copper
Length, l $[m]$	1.000 ± 0.001	1.000 ± 0.001
Diameter, d $[m]$	0.0100 ± 0.0001	0.0103 ± 0.0001
Mass, m $[10^{-3}kg]$	210.95 ± 0.01	697.28 ± 0.01
Volume, $V = \frac{\pi \cdot d^2}{4} \cdot l$ $[10^{-5}m^3]$	7.85 ± 0.11	8.33 ± 0.11
Density, $\rho = \frac{m}{V}$ $[kgm^{-3}]$	2687 ± 38	8371 ± 110

Table 4: Measurements of different parameters of aluminium rod and copper rod

The measurements of the frequencies of the respective rods are tabulated below.

Frequency	Aluminium $[10^3 s^{-1}]$	Copper $[10^3 s^{-1}]$
f_1	2.4545	1.9090
f_2	2.4615	1.9230
f_3	2.5556	1.8823
Average, $\bar{f}$	2.491	1.905
Error of the mean, $\Delta\bar{f}$	0.033	0.012

Table 5: Measured frequencies for Aluminum and Copper rod using Cassy

Therefore knowing the frequency and length of the rod, the velocity of sound in the respective rods can be calculated. From equation 2, the velocity of sound in Aluminium is

$$\begin{aligned} v_{Al} &= 2 \cdot l_{Al} \cdot f_{Al} \\ &= 2 \cdot (1m) \cdot (2.491 \cdot 10^3 s^{-1}) \\ &= 4982ms^{-1} \end{aligned}$$

Thus, the velocity of sound in Aluminium is measured to be $(4982 \pm 66)ms^{-1}$.
From equation 3, the elasticity modulus E of Aluminium can be calculated as,

$$\begin{aligned} E_{Al} &= v_{Al}^2 \rho_{Al} \\ &= (4982ms^{-1})^2 \cdot (2687kgm^{-3}) \\ &= 6.67 \cdot 10^{10} Pa \end{aligned}$$

The elasticity modulus of Aluminium is measured to be $(6.67 \pm 0.16) \cdot 10^{10} Pa$.
Similarly, the velocity of sound in copper, $v_{Cu} = (3810 \pm 24)ms^{-1}$, and the elasticity of modulus $E_{Cu} = (1.2 \pm 0.2) \cdot 10^{11} Pa$.

4 Error Analysis

The least count of the meter scale used in the experiment is $0.001m$. Since the difference of two positions is taken, the uncertainty in δs would be $2 \cdot (0.001m) = 0.002m$. The least count of the Cassy used to measure the transit time is $10^{-7}s$, so uncertainty in δt would be $2 \cdot 10^{-7}s$, which is the instrumental error in measuring the difference in transit time. The least count of the vernier calipers used is $0.0001m$.

4.1 Statistical Treatment

From Table 1, The error of the mean in transit time is calculated as

$$\Delta \bar{t} = \sqrt{\frac{1}{4} \sum_{i=1}^{5} (t_i - \bar{t})^2}$$

From Table 2 and 3, the error of the mean $\Delta \bar{v}$ is calculated as

$$\Delta \bar{v} = \sqrt{\frac{1}{4} \sum_{i=1}^{5} (v_i - \bar{v})^2}$$

Similarly, from Table 5, the error of the mean in frequency is calculated as

$$\Delta \bar{f} = \sqrt{\frac{1}{2} \sum_{i=1}^{3} (f_i - \bar{f})^2}$$

4.2 Propagated Error

The uncertainty in measuring transit time for different positions of the microphone has resulted in uncertainty in measuring the difference in transit time δt. If $\Delta\delta t$ be the uncertainty in the difference in transit time,

$$\begin{aligned}\Delta\delta t_{avg} &= \sqrt{\Delta t_{P_1}^2 + \Delta t_{P_2}^2}\\ &= 6.1 \cdot 10^{-7} s\end{aligned}$$

Similarly, error in the measurement of distance and transit time has produced uncertainty in measuring the velocity of sound in air, v. If Δv be the uncertainty in v,

$$\begin{aligned}\frac{\Delta v_{avg}}{v} &= \sqrt{\left(\frac{\Delta\delta s}{\delta s}\right)^2 + \left(\frac{\Delta\delta t}{\delta t}\right)^2}\\ \therefore \Delta v_{avg} &= 4.5 ms^{-1}\end{aligned}$$

The uncertainty in measuring v has uncertainty in the calculation of D in equation 8. If ΔD be the uncertainty in D.

$$\begin{aligned}\frac{\Delta D_{avg}}{D} &= \sqrt{\left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}\\ \therefore \Delta D_{avg} &= 3.8 \cdot 10^{-3} m\end{aligned}$$

The uncertainty in D and the statistical error in v, from Table 2, yields uncertainty in the corrected value for the velocity of sound in CO_2. If Δv_{CO_2} be the uncertainty in v_{CO_2},

$$\begin{aligned}\frac{\Delta v_{CO_{2avg}}}{v_{CO_2}} &= \sqrt{\left(\frac{\Delta\bar{v}}{\bar{v}}\right)^2 + \left(\frac{\Delta D}{D}\right)^2}\\ \therefore v_{CO_{2avg}} &= 3.5 ms^{-1}\end{aligned}$$

Similarly, if Δv_{He} be the uncertainty in v_{He}, $\Delta v_{He} = 21 ms^{-1}$.
The uncertainty in the measurement of velocity of sound has resulted the uncertainty in determining the value for the adiabatic index. If $\Delta\gamma$ be the uncertainty in adiabatic index γ, the uncertainty in adiabatic index of CO_2 is calculated as

$$\begin{aligned}\frac{\Delta\gamma_{CO_{2avg}}}{\gamma_{CO_2}} &= \sqrt{2\left(\frac{\Delta v_{CO_2}}{v_{CO_2}}\right)^2 + \left(\frac{\Delta T}{T}\right)^2}\\ \therefore \Delta\gamma_{CO_{2avg}} &= 0.02\end{aligned}$$

Similarly, $\Delta\gamma_{He_{avg}} = 0.09$.
The uncertainty in measuring the length, mass, and diameter of the metal rods has resulted the uncertainty in the calculation of volume and density. The uncertainty in volume ΔV and the uncertainty in density $\Delta\rho$ are calculated as

$$\frac{\Delta V}{V} = \sqrt{2\left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta l}{l}\right)^2}$$

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta V}{V}\right)^2}$$

The statistical error in the measurements of the frequency using Cassy, and the least count of the meter scale used to measure the length of metal rods yields uncertainty in the calculation of the velocity of sound in metal rods. If Δv be the uncertainty in v, uncertainty in the velocity of sound in Aluminium is calculated as

$$\frac{\Delta v_{Al}}{v_{Al}} = \sqrt{\left(\frac{\Delta l_{Al}}{l_{Al}}\right)^2 + \left(\frac{\Delta \bar{f}_{Al}}{\bar{f}_{Al}}\right)^2}$$
$$\therefore \Delta v_{Al} = 66ms^{-1}$$

And the uncertainty in elastic modulus ΔE is given as
For Aluminium,

$$\frac{\Delta E_{Al}}{E_{Al}} = \sqrt{2\left(\frac{\Delta v_{Al}}{v_{Al}}\right)^2 + \left(\frac{\Delta \rho_{Al}}{\rho_{Al}}\right)^2}$$
$$\therefore \Delta E_{Al} = 0.16 \cdot 10^{10} Pa.$$

Similarly, the uncertainty in the velocity of sound in Copper, $\Delta v_{Cu} = 24ms^{-1}$, and the uncertainty in the modulus of elasticity of Copper, $\Delta E_{Cu} = 0.2 \cdot 10^{11} Pa$.

5 Discussion and Conclusion

The velocity of sound in air is observed to be $(335.2 \pm 4.5)ms^{-1}$ at room temperature $294K$ (or $21°C$); however, the theoretical value for the velocity of sound in air at $21°C$ is $343.9ms^{-1}$. The observed value differs from the literature value by ca. 2.5%. It should be because of the inaccuracy in measuring the distance between the loudspeaker and the microphone as the microphone can't be perfectly aligned parallel to the meter scale and vice versa in the setup of the experiment. In addition, the meter scale has high least count.

The velocity of sound in CO_2 is observed to be $(258 \pm 3.5)ms^{-1}$, which differs from the literature value i.e. $267ms^{-1}$ by ca. 3.7%. This is because, the carbon dioxide was released from a cylinder at high pressure causing adiabatic expansion inside the tube, because of which the temperature of the gas molecules is lower than the measured room temperature. Thus, the measured velocity is lesser than the literature value. The literature value for the adiabatic index of CO_2 is 1.196, which agrees within the error range of the observed value, (1.20 ± 0.02). The measured value for the velocity of sound in He is $(1507 \pm 21)ms^{-1}$, which disagrees with the literature value, $1010ms^{-1}$ at $21°C$. The Table 3 shows that there is steep decrease in the measured velocity of sound in He. This hints that the stable value was not measured during the experiment, and it was also not possible to wait until the velocity reaches its stable value as He is highly volatile gas, and the measurements need to be carried out within few seconds. In addition, the consequence of performing the experiment quickly for the safety purpose, the microphone moved away from its fixed position, which causes inaccuracy in measurements. This is also the reason for the measured value of the adiabatic index of He (3.72 ± 0.09) to be higher than the literature value i.e. 1.67.

The observed value for the velocity of sound in Aluminum is $(4982 \pm 66)ms^{-1}$, and the theoretical value for the velocity of sound in Aluminum is $5000ms^{-1}$, which agrees within the error range of the measured value. The calculated modulus of elasticity of Aluminum is $(6.67 \pm 0.16) \cdot 10^{10}$ Pa, and the literature value is $6.89 \cdot 10^{10} Pa$, which differs from the measured value by ca. 3%. This is because of high least count of the meter scale used to measure the length of the rod, and the high least count of the analytical balance used to measure the mass of the rod. Similarly, the measured value for the velocity of sound in Copper is $(3810 \pm 24) ms^{-1}$, which agrees with the literature value $3810 ms^{-1}$. The observed value for the elasticity modulus of copper is $(1.2 \pm 0.2) \cdot 10^{11} Pa$, and the literature value is $1.17 \cdot 10^{11} Pa$, which agrees within the error range of the measured value.

References

[1] Prof. Dr. Jürgen Fritz and Faezeh Mohaghegh, Classical Physics Lab (CH-140-B) Fall 2019