JACOBS UNIVERSITY

Dynamics of Rotational Motion

Santosh Luitel

October 26-27 , 2020

CO-486-A
Group-7

Contents

Abstract

The main objective of this experiment was to determine the moment of inertia of a rotating gyroscope around its major inertial axis and further precession and nutation with 3 free axes are investigated. The average polar momentum of inertia with two fixed axis and by measurement of the gyro and precession frequency was determined to be $(11.51 \pm 0.11) \cdot 10^{-3} kg/m^2$ and $(11.86 \pm 0.11) \cdot 10^{-3} kg/m^2$. In the second part of the experiment, the coriolis effect in a accelerated reference system is observed for a water stream.

1 Introduction and theory

1.1 Moment of inertia of a gyroscope disk

A gyroscope is defined as a rigid rotating object, symmetric about one axis.The motion has three degrees of freedom and obeys the principle of angular momentum. The rotation in only stable around the two principal axes of inertia with the minimal and maximal moment of inertia. The moment of inertia I of gyroscope can be derived as:

$$I = \int r^2 \cdot dm = \int r^2 \cdot \rho_{gd} \cdot dV \tag{1}$$

where r is the distance from and axis and ρ_{gd} is the density of the gyroscope. When volume is expressed in cylindrical coordinate, the integrated polar moment of inertia I_z can be derived as:

$$I_z = \frac{\pi}{2} \cdot \rho_{gd} \cdot r_{gd}^4 \cdot d_{gd} \tag{2}$$

where, r_{gd} is the radius and d_{gd} is the thickness of the gyroscope disk.

1.2 Polar momentum of inertia of a gyroscope disk

The polar momentum of inertia can be determined when gyroscope is set up in such way that it would only rotate around its polar axis when two axes of gyroscope are fixed. In order to apply torque, the gyroscope was accelerated using the falling mass m which was fixed by the thread winded around drum. For a free falling mass the Newton's law can be written as:

$$ma = mg - F_s \tag{3}$$

where m is the mass of falling body, g is the acceleration due to gravity. Also, for the rotational motion of gyro:

$$I\alpha = rF_s \tag{4}$$

Where, I_z is the polar moment of inertia, α is its angualar acceleration and r is the half radius of small drum. From Eq.1 and Eq.2 , the following equation is obtained:

$$I_z = \frac{r\ m(g-a)}{\alpha} = \frac{r^2 m(g-a)}{a} \tag{5}$$

where $a = r\alpha$. Also, potential energy is converted to kinetic and rotational energy as following:

$$m\ g\ h = \frac{1}{2} m\ v_{max}^2 + \frac{1}{2} I_z \omega_{max}^2 \tag{6}$$

Which can be rearranged as:

$$h = \frac{m\ r^2 + I_z}{2\ m\ g} \omega_{max}^2 \tag{7}$$

Where, h is the height of falling mass and $\omega = v/r$. By determining the value of ω^2 in dependency of h, the value of I_z can be easily determined from the above equation.

1.3 Polar momentum of inertia by measurement of gyro and precession frequency

In this part of the experiment, all three axes of the gyroscope were free to move. The angular momentum is now given by:

$$\vec{L} = I\vec{\omega} \tag{8}$$

The direction of angular velocity vector is determined using the right hand rule. When the fingertips of the right hand follow the direction of linear velocity $\vec{v}$, the thumb points into the direction of the angular velocity vector $\vec{w}$ in the equation given by:

$$= \frac{1}{r^2}\vec{r} \times \vec{v} \tag{9}$$

Now, the angular momentum is given by:

$$I\ \vec{\alpha} = \vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} \tag{10}$$

Now, the gryroscope is balanced and aligned with its principle by means of counterweight, torque is applied by adding a mass $m\prime$ on the other end of the rigid axis at a distance $r\prime$ from the pivot point. The graviatational force $\vec{F} = m'\vec{g}$ would act due to the mass hanging in other end along vertical direction. The horizontal movement of the gyroscope can be seen. 'p' and 'r' are used to indicate precession and rotation respectively. from the above equation:

$$\vec{\tau}' = \vec{r'} \times \vec{F}' = \vec{r'} \times m'\vec{g} = \frac{d\vec{L}_r}{dt} \tag{11}$$

Gravitational acceleration $\vec{g}$ is perpendicular to $\vec{r}\prime$ so the absolute value is given by:

$$\vec{\tau}' = m'gr' = \frac{|d\vec{L}_r|}{dt} \tag{12}$$

Due to torque , the angular momentum will be rotated with angle $d\varphi_p$ from its initial position. Now, it can be written as :

$$d\varphi = \frac{|d\vec{L}_r|}{|\vec{L}_r|} \tag{13}$$

The magnitude of the angular velocity of the precession ω_p is given by:

$$\omega_p = \frac{m'gr'}{I_z\omega_r} \tag{14}$$

By replacing $\omega_p = 2\pi/T_p$ and $\omega_r = 2\pi/T_r$:

$$\frac{1}{T_r} = \frac{m'gr'}{4\pi^2}\frac{1}{I_z}T_p \tag{15}$$

By determining the value of $\frac{1}{T_r}$ and T_P,the moment of inertia I_p can be determined.

1.4 Coriolis-force

The effect of the Coriolis force is an apparent deflection of the path of an object that moves within a rotating coordinate system. The object does not actually deviate from its path, but it appears to do so because of the motion of the coordinate system.In this part of the experiment, a rotating apparatus is used to investigate the coriolis effect for path of water jet perpendicular to the system.

2 Experimental Setup and procedure

2.1 Determination of the moment of inertia of a gyroscope disc from geometric data

The radius r_{gd}, the thickness d_{gd} and the mass of the gyroscope disk were measured in order to determine the moment of inertia.

2.2 Determination of the momentum of inertia of gyroscope I_z by fixing two axes.

In order to determine the polar momentum of inertia, the axes of the gyroscope was fixed in the edge of the table in the such way that it will only rotate around its polar axis. The gyroscope axis was fixed with right angle clamp to a circular support rod. In order to determine the period of rotation, the plastic blades was placed on the gyroscope disk which would interact with fork-type light barrier.The arrangement was done in such a way that, the light beam would only cut the plastic blades.The light barrier was connected with cassy through time box. The necessary setting was adjusted with 100 ms measurement interval to determine the time period between two plastic blade $T_{R/8}$. After all arrangement, the working status of our setup was checked by rotating the gyro.Then, the string was wounded around the drum of the gyro and mass m was fastened to the free end of the string.The height from which the mass was falling was measured. When mass was freely suspended, it would accelerate the gyro by applying torque. The measurement in cassy was started from the moment the mass suspended freely to the few seconds after the mass hit the ground. The minimum value of $T_r/8$ was taken from the measured data. The average value was used for further calculations. The same procedure was followed for different height h and again using three different falling masses. The raw data obtained were taken for further calculation to conclude meaningful results.

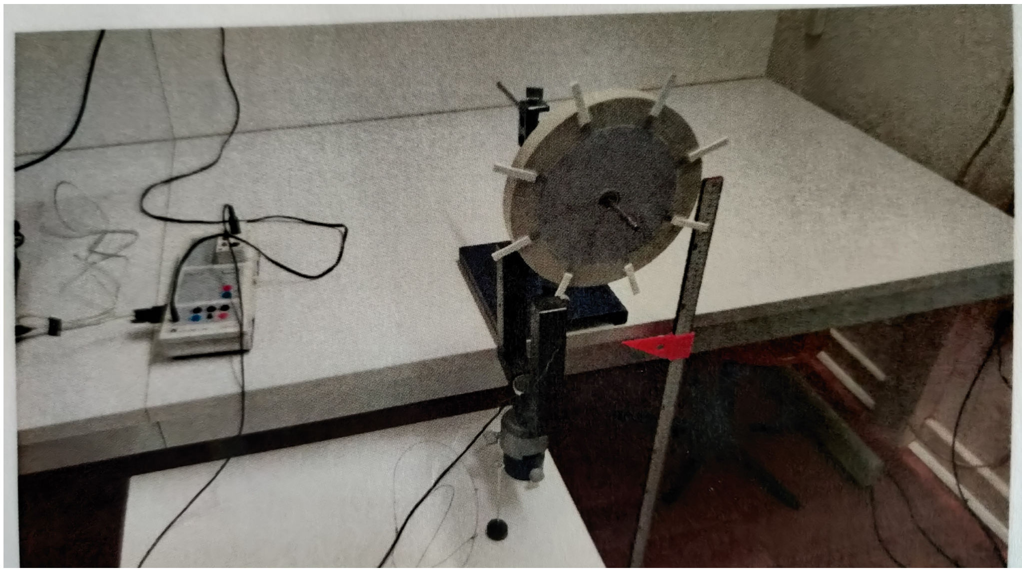


Figure 1: Experimental setu[for determination of I_z with two fixed axes of gyroscope disk

2.3 Change of angular velocity with time

In order to determine the angular velocity w_r, the gyroscope, the setup described in 2.2 was brought into fast rotation without hanging any mass by the help of string wounded around the gyroscope drum. Similarly, the time interval T_{r8} was measured using automatic recording in the cassy. The measurement of rotation was taken for at least 1 min. This data was taken in order to determine the reduction the reduction of angular velocity w_r due to friction.

2.4 Determination of the polar momentum of inertia by measurement of the rotational and precession frequency

In order to determine the rotational and precession frequency, the gycroscope was allowed to move freely in all three directions. The counter weight was mounted on the gyroscope in order to balance it on rest position. The period of rotation T_r was measured using the light barrier. The gyroscope was arranged in such a way that only plastic blade would cut the light barrier and it will pass freely without any physical obstruction to the hanging mass m so that, no false signal would be detected. In order to determine the precession frequency, second light barrier was also used. The setup is shown in Fig.2. As the fork light are placed at opposite to each other, it will only measure the $T_p/2$. The measurement interval of 100 ms without override for both time boxes. After all necessary arrangement, the gyroscope was brought into fast rotation using string wounded around the gyroscope in the horizontal axis. The measurement was started in the cassy and the small mass m' was hanged in other end of the rotating axis. The rotational time period $T_r/8$ was seen in the cassy screen. In order to determine the precession frequency, difference in time period $T_p/2$ when the results first show in first and 2nd light barrier was measured. The rotational time period T_r was determined from the average value from both light barrier. Same procedure was followed for another mass. The raw data data was taken for data analysis.

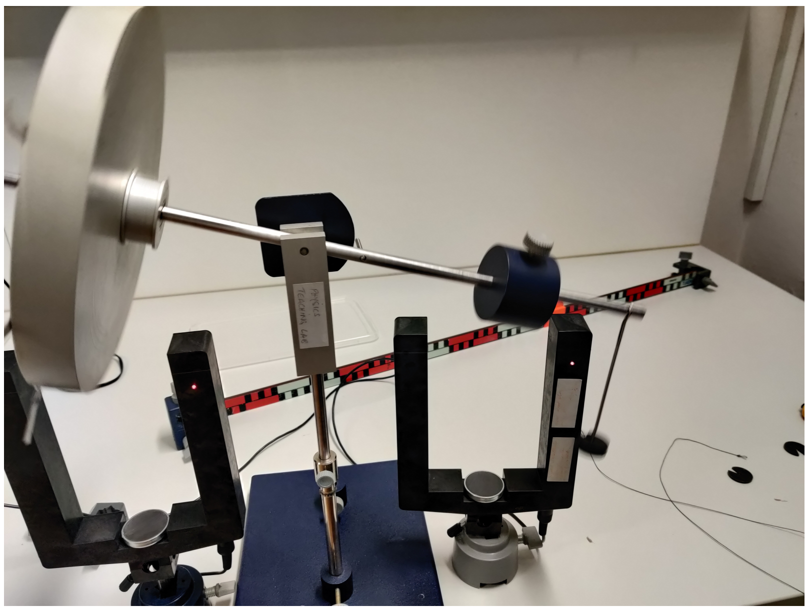


Figure 2: Rotational motion and Precession of gyroscope

2.5 Observation of coriolis effect

The necessary arrangement was done to in a such way that, the funnel along with the water pipette would come to rotational motion in a same reference frame. The rubber hose was fixed with right angle clamp in such way that the water jet coming out of the pipette nearly crosses the whole tube. The funnel was filled with water and the system was kept in rotational motion with a slight swing. The apparatus was rotated in both direction with different velocity and the resulting effect was observed. The experiment was repeated with the hose and pipette oriented such that water jet crosses tub in opposite direction.

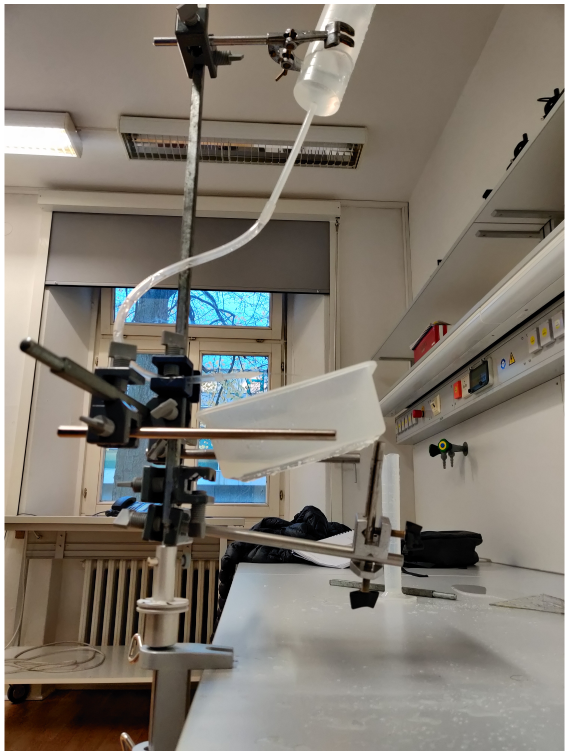


Figure 3: Rotational motion and Precession of gyroscope

3 Result data and error analysis

3.1 Moment of inertia from geometric data

The radius r_{gd} and thikcness d_{gd} of disk were measured to be $(122 \pm 0.05)mm$ and $(26 \pm 0.05)mm$ respectively. The mass of the gyroscope m_g was measured to be (1342.04 ± 0.02) gm. from this data the volume of the disk was determined to be 1176.212 cm^3.
From the above data the density ρ_{gd} was determined to be $1.1gm/cm^{-3}$. The theoretical value of density was given as $0.9gm/cm^3$. From Eq 2, moment of inertia of a gyroscope was determined to be $9.32 \cdot 10^{-3}kgm^2$.

3.2 Determination of polar moment of inertia of gyroscope with two fixed axes

After measuring the time period, the angular velocity was calculated and the square of angular velocity vs height was plotted in the graph for different masses. The following graph were obtained for mass of 30 gm, 60 gm, and 90 gm respectively.

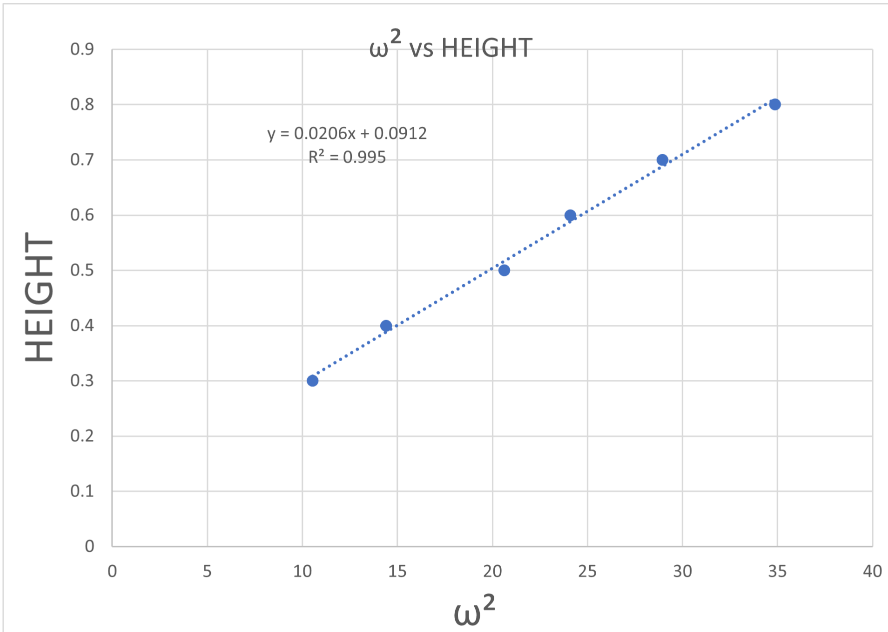


Figure 4: Height vs w^2 for 30 gm of mass

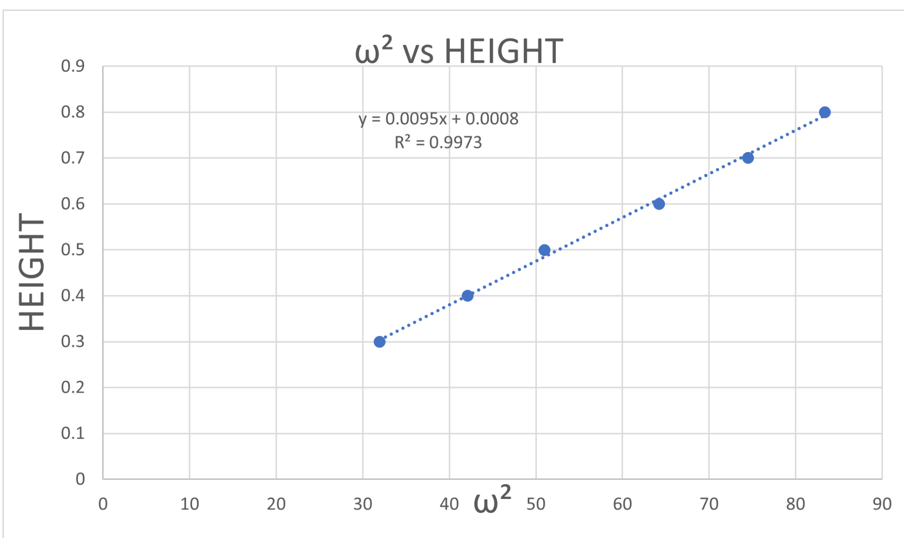


Figure 5: Height vs w^2 for 60 gm of mass

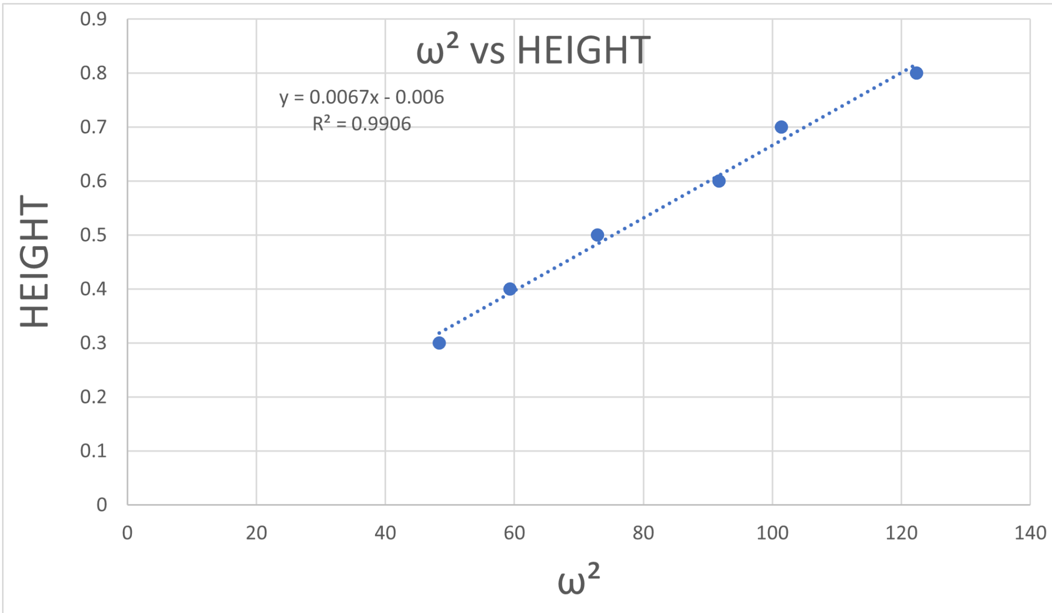
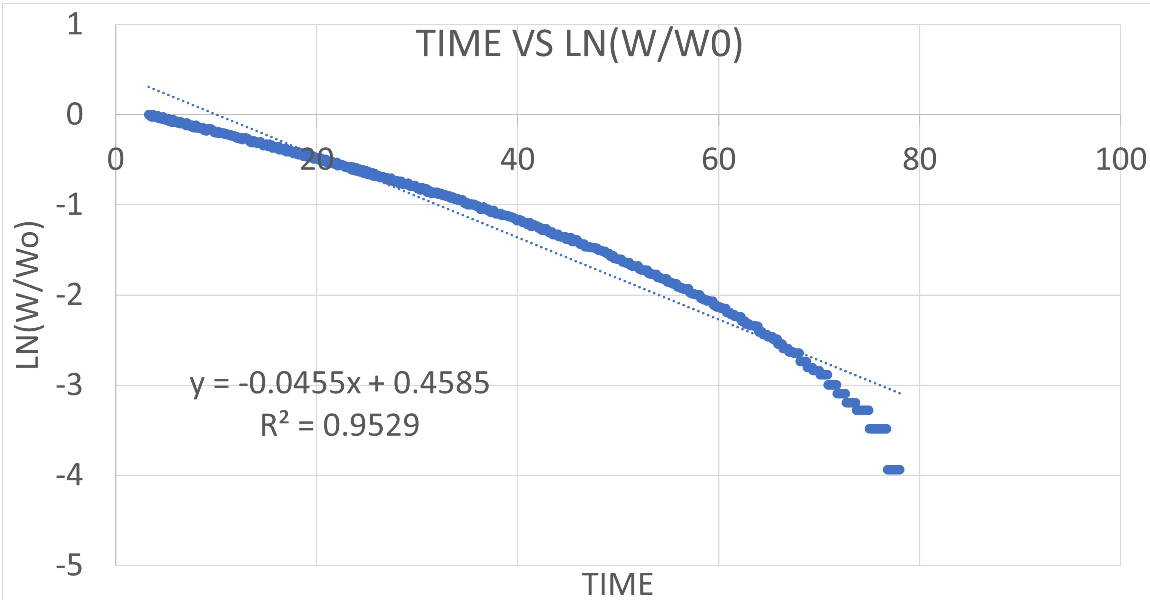


Figure 6: Height vs w^2 for 90 gm of mass

Accoring the graphs, the best fit line eqaution is given by: $y = 0.0206x + 0.0912$ for mass of 30 gm $y = 0.0095x + 0.006$ for mass of 60 gm
$y = 0.0067x - 0.006$ for mass of 90 gm
Using Eq , the Polar moment of inertia I_z was found to be $11.576 \cdot 10^{-3}$ kgm^2, $11.16 \cdot 10^{-3}$ kgm^2, $11.79 \cdot 10^{-3}$ kgm^2 respectively when mass of 30 gm, 60 gm, and 90 gm was used.

3.3 Change of angular veloctity with time

The experimental raw data were used to plot a graph of Time vs $Ln(\frac{w(t)}{w_0})$. The following graph was obtained.



w0

The equation of the best fit line was determined to be
$y = -0.0455x + 0.4585$
using Eq , the value of $\frac{k}{I}$ was determined to be 0.0455. Using the theoretical value of I_z, torque due to friction was determined to be $4.241 \cdot 10^{-4}$ Nm.

3.4 Determination of the polar momentum of inertia by measurement of the gyro and [recession frequency

After the necessary calutaion of data, Time period of rotation T_r and Time period of precession T_p was determined. The graph of T_p vs $1/T_r$ was plotted. the following graph was obtained for mass of 30 30 gm, and 50 gm respectively.

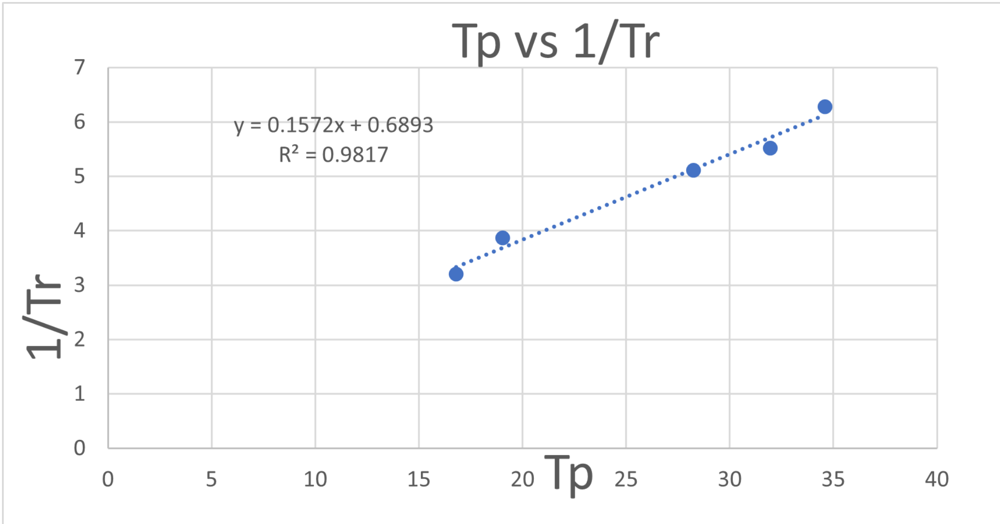


Figure 8: T_p vs $1/T_r$ for 30gm

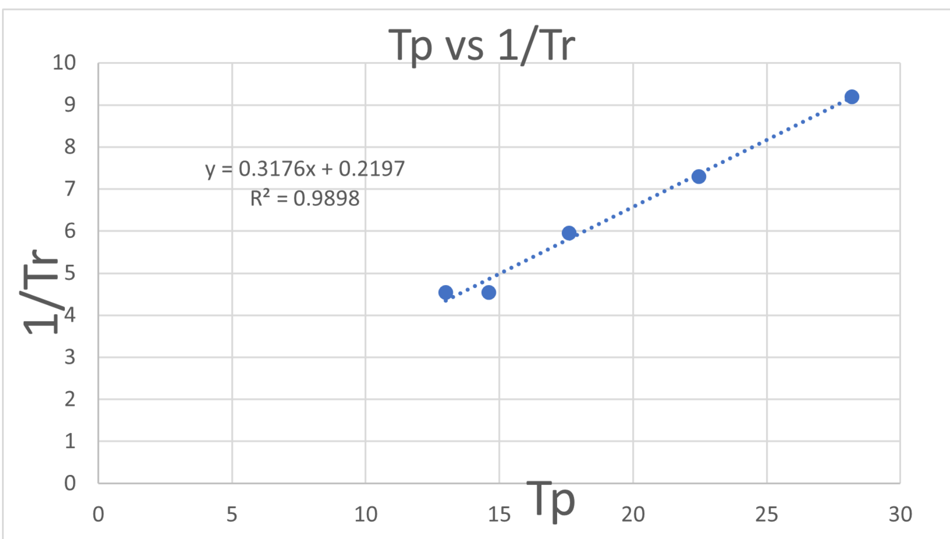


Figure 9: T_p vs $1/T_r$ for 50 gm

Accoring the graphs, the best fit line eqaution is given by: $y = 0.1572x+0.6893$ for mass of 30 gm $y = 0.3176x + 0.2197$ for mass of 50 gm Using Eq , the Polar moment of inertia I_z was found to be $13.16 \cdot 10^{-3}$ kgm^2 and $10.55 \cdot 10^{-3}$ kgm^2 respectively when mass of 30 gm and 50 gm were used.

3.5 coriolis effect

After necessary setup the system is kept in rotation and the motion water jet coming out at as well as going in right angel was observed. When the apparatus was rotated anti-clockwise, the curvature of water jet was seen in clockwise direction as shown as in figure 10. Again, when the apparatus was rotated in clockwise direction the curvature was seen in anti clockwise direction. Now, the water jet was attached to the outer edge and the direction of curvature was reversed this time. Both, rotation and deflection was seemed to be in same direction.

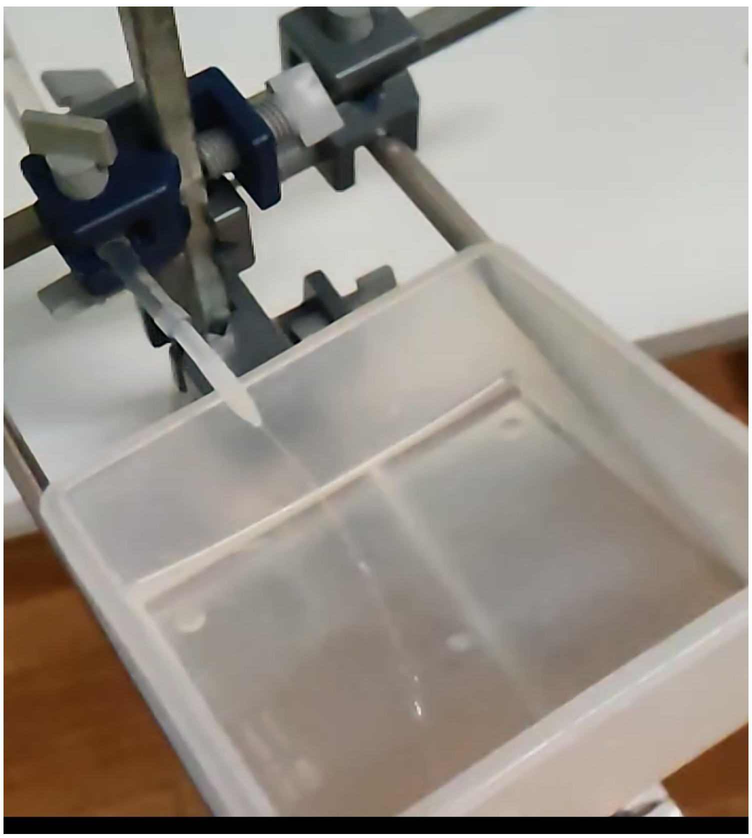


Figure 10: T_p vs $1/T_r$ for 50 gm

4 Error analysis

The error in the density ρ_{gd} was determined as:

$$\frac{\Delta\rho}{\rho} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + 2\left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta d}{d}\right)^2} \tag{16}$$

The error was calculated as 0.2 gm/cm^3.
Similarly, the error in measurement of I_z calculated from the error of geometric data by using following formula:

$$\frac{\Delta I_z}{I_z} = \sqrt{4\left(\frac{\Delta r_{gd}}{r_{gd}}\right)^2 + \left(\frac{\Delta d_{gd}}{dgd}\right)^2} \tag{17}$$

It was calculated to be $0.25 \cdot 10^{-3}$ kgm^2 The error in the slope can be determined by using the following formula:

$$\frac{\Delta m}{m} = \sqrt{\frac{1}{n-2} \cdot \frac{1-R^2}{R^2}} \tag{18}$$

where R is coffecient of regression, and n is number of data.
for section 3.2, the error in polar momentum of inertia for mass 30 gm ,60 gm, and 90 gm was determined to be $0.58 \cdot 10^{-3}$ kgm^2, $0.49 \cdot 10^{-3}$ kgm^2, $0.81 \cdot 10^{-3}$ kgm^2 respectively.
for section 3.4, the error in polar moment of inertia by measurement of the egyro and precession frequency was determined to be $0.13 \cdot 10^{-3}$ kgm^2 and $0.62 \cdot 10^{-3}$ kgm^2 respectively for mass of 30 gm and 50 gm.

5 Discussion and conclusion

The density $\rho_g d$ was determined to be 1.1 ± 0.2 gm/cm^3. It lies within the theoretical value. The moment of inertia of polar axis was determined to be $9.32 \pm 0.25 \cdot 10^{-3}$. The polar moment of inertia of gyro by accelerating from a free falling mass was determined to be $11.58 \pm 0.58 \cdot 10^{-3}$ kgm^2, $11.16 \pm 0.49 \cdot 10^{-3}$ kgm^2, $11.79 \pm 0.81 \cdot 10^{-3}$ kgm^2 respectively when mass of 30 gm, 60 gm, and 90 gm were used. From the measurement of gyro and precession frequency, the moment of inertia was determined to be to be $13.16 \pm 0.13 \cdot 10^{-3}$ kgm^2 and $10.55 \pm 0.62 \cdot 10^{-3}$ kgm^2 respectively when mass of 30 gm and 50 gm were used.When the angular frequency as a function of time was studied, friction was determined to be the main source of error.

The coriolis effect was observed by observing the curvature of water jet when the apparatus was rotated. When the water jet is moving away from the center, the curvature was seen in the reverse direction of the angular motion of apparatus. When the water jet was placed in such a way that the water jet would deviate to the same direction of angular rotation of apparatus.

6 References

- Advance physics Lab Manual C0-486 fall 2020 (Prof.Dr.Arnulf Materny and Dr. Vladislav Jovanov).
- Error analysis booklet for physics teaching lab at Jacobs university
- University Physics by Young and Freedman