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FLUID MECHANICS

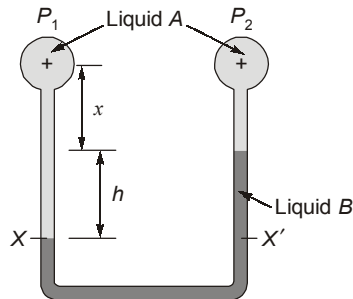
MECHANICAL ENGINEERING

Date of Test : 13/07/2023**ANSWER KEY >**

| | | | | |
|--------|---------|---------|---------|---------|
| 1. (c) | 7. (c) | 13. (c) | 19. (b) | 25. (b) |
| 2. (b) | 8. (c) | 14. (d) | 20. (d) | 26. (c) |
| 3. (a) | 9. (b) | 15. (c) | 21. (b) | 27. (b) |
| 4. (c) | 10. (d) | 16. (b) | 22. (a) | 28. (c) |
| 5. (c) | 11. (b) | 17. (d) | 23. (a) | 29. (a) |
| 6. (c) | 12. (d) | 18. (d) | 24. (a) | 30. (c) |

DETAILED EXPLANATIONS

1. (c)



Taking point X and X' and equating the pressure on both sides

$$\begin{aligned}
 P_1 + (h + x) \times 0.88 \times 9.81 \times 10^3 &= P_2 + x \times 0.88 \times 9.81 \times 10^3 + h \times 2.95 \times 9.81 \times 10^3 \\
 \Rightarrow P_1 + h \times 0.88 \times 9.81 \times 10^3 + x \times 0.88 \times 9.81 &= x \times 0.88 \times 9.81 + P_2 + h \times 2.95 \times 9.81 \times 10^3 \\
 \Rightarrow P_1 - P_2 &= h(2.95 \times 9.81 - 0.88 \times 9.81) \times 10^3 \\
 860 &= h(2.3067 \times 10^3) \\
 42.35 \times 10^{-3} &= h \\
 h &= 42.35 \text{ mm}
 \end{aligned}$$

2. (b)

For geometrically similar model and prototype

$$\left(\frac{P}{N^3 D^5} \right)_{\text{model}} = \left(\frac{P}{N^3 D^5} \right)_{\text{prototype}}$$

Given,

$$\begin{aligned}
 N_m &= N_p \\
 \Rightarrow \frac{P_m}{N_m^3 D_m^5} &= \frac{P_p}{N_p^3 D_p^5} \\
 \frac{P_m}{P_p} &= \frac{N_m^3 D_m^5}{N_p^3 D_p^5} \\
 \frac{P_m}{P_p} &= \frac{2^3 N_p^3}{N_p^3} \times \frac{D_m^5}{16^5 D_m^5} \\
 P_m &= \frac{10 \times 10^6 \times 2^3}{16^5} W = 76.29 \text{ W}
 \end{aligned}$$

3. (a)

$$\begin{aligned}
 P_A - H \times 9.81 \times 1 - 0.18 \times 9.81 \times 0.827 &= P_B - 13.6 \times 9.81 \times (H + 0.53) \\
 - H \times 9.81 - 1.4603 &= 97 - 13.6 \times 9.81 \times H - 13.6 \times 9.81 \times 0.53 \\
 \Rightarrow H &= 0.2245 \text{ m} \\
 \therefore H &= 22.45 \text{ cm}
 \end{aligned}$$

4. (c)

$$\therefore \bar{V} = \frac{Q}{A} = \frac{Q}{\frac{\pi d^2}{4}} \Rightarrow \frac{4Q}{\pi d^2} = \frac{4 \times 880 \times 10^{-9}}{\pi \times 0.50^2 \times 10^{-6}} = 4.48 \text{ m/s}$$

We know, $Q = \frac{\pi \Delta p D^4}{128 \mu L}$

$$\Rightarrow \mu = \frac{\pi \Delta p D^4}{128 Q L} = \frac{\pi \times 10^6 \times (0.5)^4 \times 10^{-12}}{128 \times 880 \times 10^{-9} \times 1}$$

$$\mu = 1.74 \times 10^{-3}$$

5. (c)

Newton's law of viscosity

$$\tau = \mu \frac{du}{dy}$$

Where

 τ = shear stress

$$\frac{du}{dy} = \text{Rate of strain}$$

6. (c)

$$\tau A = mg \sin \theta$$

$$\mu \left(\frac{v}{t} \right) A = mg \sin \theta$$

$$v = \frac{mgt \sin \theta}{\mu A} = \frac{15 \times 9.81 \times 0.1 \times 10^{-3} \times \sin 30^\circ}{8.14 \times 10^{-2} \times 0.25} = 0.36 \text{ m/s}$$

7. (c)

8. (c)

$$u = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 20} = 19.81 \text{ m/s}$$

$$v^2 - u^2 = 2as \quad (a = -g)$$

$$v^2 - 19.81^2 = -2 \times 9.81 \times 10$$

or

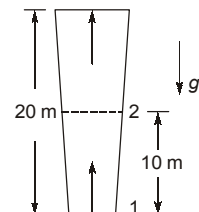
$$v = 14 \text{ m/s}$$

$$A_1 v_1 = A_2 v_2$$

$$D_2^2 = \frac{12^2 \times 19.81}{14}$$

or

$$D_2 = 14.27 \text{ cm}$$



9. (b)

Surface tension is due to cohesion between liquid particles at the surface.

10. (d)

$$P = \rho_{\text{Hg}} \times g \times H$$

$$6.8 \times 10^4 = 13.6 \times 10^3 \times 9.81 \times H$$

$$H_{\text{Hg}} = 0.5096 \text{ m}$$

$$H_{\text{water}} = \frac{13.6}{1} \times 0.5096$$

$$H_{\text{water}} = 6.931 \text{ m}$$

11. (b)

For just equilibrium condition,

$$\dot{m}[V \cos \theta] = \mu Mg$$

$$1000 \times \pi \times 0.25 \times 0.05^2 \times V^2 \times 0.5 = 0.55 \times M \times 9.81$$

$$1000 \times \pi \times 0.25 \times 0.05^2 \times 2 \times 9.81 \times 2 \times 0.5 = 0.55 \times M \times 9.81$$

$$\Rightarrow = 7.1399 \text{ kg}$$

12. (d)

$$\text{Volume of cube} = a^3$$

$$a^3 = 125 \times 10^{-3} \times 10^{-3} \text{ m}^3$$

$$\Rightarrow a = 5 \times 10^{-2}$$

$$\Rightarrow a = 0.05 \text{ m}$$

$$F = p \times A$$

$$P_{\text{bottom}} = p_{\text{atm}} + h_1 \rho_{\text{oil}} + h_2 \rho_{\text{water}} g$$

$$= 101325 + 0.5 \times 0.8 \times 1000 \times 9.81 + 0.3 \times 1000 \times 9.81$$

$$P_{\text{bottom}} = 108192$$

$$F = P_{\text{bottom}} \times A = 108192 \times 0.05^2 = 270.48 \text{ N}$$

$$T = \text{Upthrust} - W$$

$$= 125 \times 10^{-6} \times 1000 \times 9.81 - 125 \times 10^{-6} \times 0.77 \times 1000 \times 9.81$$

$$= 0.282 \text{ N}$$

13. (c)

$$u = \frac{\partial \psi}{\partial y}$$

$$u = 2x^2 + (x + t) 2y$$

\therefore for face OB ,

$$x \Rightarrow 0$$

$$u_{OB} = 2ty$$

Discharge through AB

$$\therefore Q_{AB} = \int_0^2 u_{OB} \cdot 5 dy = \int_0^2 2ty \cdot 5 dy$$

At

$$t = 1$$

$$Q_{OB} = 20 \text{ units}$$

$$\therefore V = -\frac{\partial \psi}{\partial x} = -[4xy + y^2]$$

$$\text{At } y = 0$$

$$V = 0$$

$$\therefore Q_{AO} = 0$$

$$\begin{aligned} \therefore Q_{AB} &= Q_{OB} + Q_{OA} \\ &= 20 + 0 \\ &= 20 \text{ units} \end{aligned}$$

14. (d)

$$V_2 = \frac{Q}{A_2} = \frac{1.13 \times 10^{-6}}{\frac{\pi}{4} \times (0.0012)^2} \simeq 1 \text{ m/s}$$

$$\frac{P_1}{\rho g} + \frac{\alpha_1 V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_f$$

$$h_f = z_1 - z_2 - \frac{\alpha_2 V_2^2}{2g}$$

$$\Rightarrow h_f = 0.6 - 0 - \frac{(2)(1)^2}{2 \times 9.81} = 0.5 \text{ m}$$

$$h_f = \frac{32 \mu V L}{\rho g D^2}$$

$$\Rightarrow 0.5 = \frac{32 \times \mu \times 0.3 \times 1}{9000 \times 0.0012}$$

$$\Rightarrow \mu = 6.75 \times 10^{-4} \text{ Pa-s}$$

15. (c)

$$v = \frac{\beta}{4\mu} \left(\frac{D^2}{4} - r^2 \right)$$

$$\left. \frac{\delta v}{\delta r} \right|_{r=D/2} = \left. \frac{\beta}{4\mu} (-2r) \right|_{r=D/2} = -\frac{\beta r}{2\mu} \Big|_{r=D/2} = -\frac{\beta r}{2\mu} \frac{D}{2} = \frac{-\beta D}{4\mu}$$

$$\tau = -\mu \left. \frac{\delta v}{\delta r} \right|_{r=D/2} = \frac{\beta D}{4}$$

16. (b)

$$Re_L = \frac{UL}{\nu} = \frac{1.75 \times 5}{1.475 \times 10^{-5}}$$

$$Re_L = 5.932 \times 10^5$$

$$C_f = \frac{0.074}{Re_L^{1/5}} = \frac{0.074}{(5.932 \times 10^5)^{1/5}} = 5.183 \times 10^{-3}$$

Drag force on one side of the plate,

$$F_d = C_f \times \text{area} \times \frac{1}{2} \rho U^2$$

$$= 5.183 \times 10^{-3} \times (1.8 \times 5) \times 1.22 \times \frac{(1.75)^2}{2}$$

$$F_d = 0.0871 \text{ N}$$

17. (d)

∴ Continuity equation holds,

$$\therefore \frac{\pi}{4} \times (5)^2 \times 2 = \frac{\pi}{4} \times 3^2 \times x$$

$$x = 5.55 \text{ m/s}$$

Mars flow rate

$$\Rightarrow \dot{m} = \int A_1 V_1 = 100 \times \frac{\pi}{4} \times 0.05^2 \times 2 = 3.9269 \text{ kg/s}$$

Let f_x and f_y be the force in Right and vertically upward diversion respectively to hold the box in position.

∴ Now, $\Sigma f_x = 0$ [Box is stationary after applying force]

$$-\dot{m} \times V_1 \cos 65^\circ + f_x = -\dot{m} \times V_2 \cos 0^\circ$$

$$-3.9269 \times 2 \times \cos 65^\circ + f_x = -3.9269 \times 5.55 \times 1$$

$$f_x = -18.475 \text{ N.}$$

f_x must be in left as f_x comes out to be negative.

Similarly for vertical direction $\Sigma f_y = 0$

$$f_y - 3.9269 \times 2 \times \sin 65^\circ = 0$$

$$f_y = 7.11 \text{ N}$$

∴ It is towards vertically upward direction.

18. (d)

$$Q = a_2 \sqrt{\frac{2(p_1 - p_2)}{\rho \left[1 - \left(\frac{a_2}{a_1} \right)^2 \right]}}$$

for a venturimeter

$$\therefore p_1 - p_2 = \frac{Q^2 \rho \left[1 - \left(\frac{a_2}{a_1} \right)^2 \right]}{2a_2^2}$$

$$= \frac{0.05^2 \times 850}{2 \times \left[\frac{\pi}{4} 0.06^2 \right]^2} \left(1 - \frac{0.06^4}{0.1^4} \right) = 1.16 \times 10^5 \text{ N/m}^2$$

$$= 116 \text{ kPa}$$

19. (b)

$$H_m = H + \frac{fL V_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

 H_m = Net head required

$$H_m = 30 + \frac{4f'LV_d^2}{2gd} + 1.3 \times \frac{V_d^2}{2g} + \frac{V_d^2}{2g}$$

$$f' = 0.02$$

$$A_d V_d = 50 \times 10^{-3} \text{ m}^3/\text{s}$$

$$d = 200 \text{ mm}$$

$$\frac{\pi}{4} \times 0.2^2 \times V_d = 50 \times 10^{-3}$$

$$V_d = 1.59 \text{ m/s}$$

$$\Rightarrow H_m = 30 + \frac{4 \times 0.02 \times 100 \times 1.59^2}{2 \times 9.81 \times 0.2} + \frac{1.3 \times 1.59^2}{2 \times 9.81} + \frac{1.59^2}{2 \times 9.81}$$

$$= 35.45 \text{ m}$$

$$\Rightarrow \text{Power} = \rho Q g H_m = 17388.225 \text{ W} = 17.388 \text{ kW}$$

20. (d)

 τ_1 = shear stress at bottom

$$= \mu_1 \times \frac{V}{x}$$

$$\tau_2 = \text{shear stress at top} = \mu_2 \frac{V}{a-x}$$

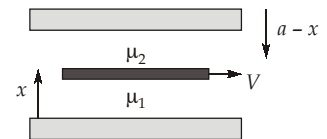
$$\text{drag force} = (\tau_1 + \tau_2) \times A = F_D$$

$$= F_D = A \times \left[\frac{\mu_1 V}{x} + \frac{\mu_2 V}{a-x} \right]$$

$$\frac{dF_D}{dx} = 0 = \frac{-\mu_1 V}{x^2} + \frac{\mu_2 V}{(a-x)^2} \Rightarrow \frac{\mu_1}{x^2} = \frac{\mu_2}{(a-x)^2}$$

$$a-x = \sqrt{\frac{\mu_2}{\mu_1}} x$$

$$\Rightarrow \frac{a\sqrt{\mu_1}}{\sqrt{\mu_1} + \sqrt{\mu_2}} = x$$



21. (b)

Given data:

Stream function,

$$\psi = x^2 - y^2$$

The velocity components u and v in the direction of x and y are given by

$$u = -\frac{\partial \psi}{\partial y} = -\frac{\partial}{\partial y}(x^2 - y^2) = 2y$$

or

$$v = \frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2) = 2x$$

The velocity at point (1, 1),

$$\begin{aligned}\vec{V} &= u\hat{i} + v\hat{j} = 2y\hat{i} + 2x\hat{j} \\ &= 2 \times 1\hat{i} + 2 \times 1\hat{j} = 2\hat{i} + 2\hat{j}\end{aligned}$$

The magnitude of the velocity,

$$V = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

22. (a)

(1)

$$\begin{aligned}u &= x^2 \cos y \\ v &= -2x \sin y\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(x^2 \cos y) + \frac{\partial}{\partial y}(-2x \sin y) \\ (2x)\cos y - (2x)\cos y &= 0\end{aligned}$$

hence satisfy the continuity equation.

(2)

$$\begin{aligned}u &= x + 2 \\ v &= 1 - y\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(x+2) + \frac{\partial}{\partial y}(1-y) = 1 - 1 = 0\end{aligned}$$

hence satisfy the continuity equation.

(3)

$$\begin{aligned}u &= xyt \\ v &= x^3 - y^2 \frac{t}{2}\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x}(xyt) + \frac{\partial}{\partial y}\left(x^3 - \frac{y^2 t}{2}\right) = yt - yt = 0\end{aligned}$$

(4)

$$\begin{aligned}u &= \ln(x+y) \\ v &= xy - \frac{y}{x}\end{aligned}$$

Continuity equation

$$\begin{aligned}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ &= \frac{\partial}{\partial x} \ln(x+y) + \frac{\partial}{\partial y} \left(xy - \frac{y}{x}\right) = \frac{1}{x+y} + x - \frac{1}{x} \neq 0\end{aligned}$$

Hence, does not satisfy continuity equation.

23. (a)

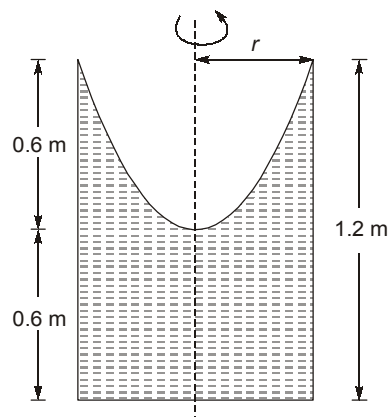
$$\text{Pressure head} = \frac{p}{\rho g} = \frac{19.62 \times 10^3}{1000 \times 9.81 \times 0.8} = 2.5 \text{ m of oil}$$

$$\text{Velocity head} = \frac{V^2}{2g} = \frac{Q^2}{2A^2g} = \frac{(0.12)^2}{2 \times \left(\frac{\pi}{4} \times 0.25^2\right)^2 \times 9.81} = 0.3 \text{ m of oil}$$

$$\text{Datum head} = 2.7 \text{ m}$$

$$\begin{aligned} \text{Total head} &= \text{Pressure head} + \text{Velocity head} + \text{Datum head} \\ &= 2.5 + 0.3 + 2.7 = 5.5 \text{ m} \end{aligned}$$

24. (a)



Original volume of

$$\text{Cylinder} = \pi r^2 h$$

$$V_1 = \pi r^2 \times 1.2$$

Volume of liquid spilled out

$$= \frac{1}{2} \pi r^2 \times h$$

$$V_2 = \frac{1}{2} \pi r^2 \times 0.6$$

 \therefore

$$\frac{V_2}{V_1} = \frac{\frac{1}{2} \times 0.6 \pi r^2}{\pi r^2 \times 1.2} = \frac{1}{4}$$

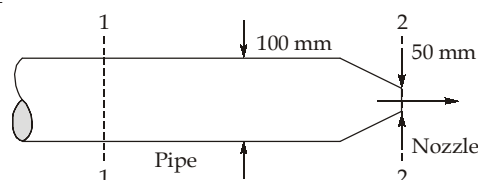
25. (b)

$$V_1 A_1 = V_2 A_2$$

$$5 \times \frac{\pi}{4} (0.1)^2 = V_2 \times \frac{\pi}{4} (0.05)^2$$

 \Rightarrow

$$V_2 = 20 \text{ m/s}$$



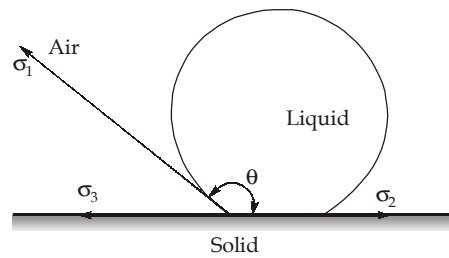
From force balance in x-diagram

$$\begin{aligned}
 P_1 A_1 + R_x - P_2 A_2 &= \rho Q (V_2 - V_1) \\
 \Rightarrow R_x &= \rho Q (V_2 - V_1) + P_2 A_2 - P_1 A_1 \\
 &= 1000 \times \frac{\pi}{4} (0.1)^2 \times 5 (20 - 5) + 100 \times 10^3 \times \frac{\pi}{4} (0.05)^2 - 500 \times 10^3 \times \frac{\pi}{4} (0.1)^2
 \end{aligned}$$

Force required to hold the nozzle, $R_x = -3141.6 \text{ N}$

26. (c)

From force balance at point of contact,



$$\begin{aligned}
 \sigma_1 \cos(180 - \theta) + \sigma_3 &= \sigma_2 \\
 \text{or } \cos(180 - \theta) &= \frac{\sigma_2 - \sigma_3}{\sigma_1} = -\cos \theta \\
 \therefore \sigma_1 &= 0.0720 \text{ N/m (liquid and air)} \\
 \sigma_2 &= 0.0418 \text{ N/m (liquid and solid)} \\
 \sigma_3 &= 0.0008 \text{ N/m (air and solid)} \\
 \cos \theta &= \frac{0.0008 - 0.0418}{0.072} = -0.56944 \\
 \theta &= 124.7^\circ
 \end{aligned}$$

27. (b)

$$\therefore \text{Re} = \frac{16}{f} = \frac{16}{0.04} = 400$$

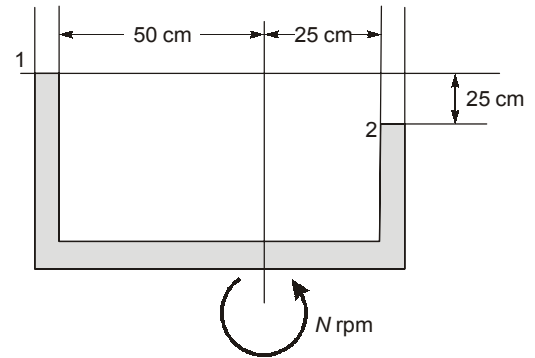
\therefore The flow is viscous.

The shear stress in case of viscous flow through a pipe is given by

$$\begin{aligned}
 \tau &= \frac{-\partial p}{\partial x} \left(\frac{r}{2} \right) \\
 \therefore \frac{\partial p}{\partial x} &\text{ is constant across a section.} \\
 \therefore \tau &\propto r \\
 \therefore \frac{\tau}{r} &= \frac{\tau_0}{R} = \frac{0.00981}{40} = \frac{\tau_0}{100} \\
 \therefore \tau_0 &= \frac{100 \times 0.00981}{40} = 0.0245 \text{ N/cm}^2
 \end{aligned}$$

28. (c)

$$\begin{aligned} \frac{P_1}{\rho g} - \frac{V_1^2}{2g} + Z_1 &= \frac{P_2}{\rho g} - \frac{V_2^2}{2g} + Z_2 \\ Z_1 - Z_2 &= \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \\ 0.25 &= \frac{1}{2g} \{r_1^2 \omega^2 - r_2^2 \omega^2\} \\ 0.25 \times 2 \times 9.81 &= \omega^2 \left\{ \left(\frac{50}{100} \right)^2 - \left(\frac{25}{100} \right)^2 \right\} \\ \omega &= 5.115 \text{ rad/s} \\ N &= \frac{60\omega}{2\pi} = \frac{60 \times 5.115}{2 \times \pi} \\ &= 48.8 \text{ rpm} \end{aligned}$$



29. (a)

$$\begin{aligned} Re_L &\leq 2000 \\ \Rightarrow \frac{\rho V D}{\mu} &\leq 2000 \\ \Rightarrow \frac{(0.92 \times 1000) \times \frac{Q}{A} \times (10 \times 10^{-2})}{0.9 \times 10^{-1}} &\leq 2000 \\ \Rightarrow Q &\leq \frac{2000 \times 0.9 \times 10^{-1} \times A}{(0.92 \times 1000) \times (10 \times 10^{-2})} \\ \Rightarrow Q &\leq 1.95652 \times \frac{\pi}{4} \times \left\{ \frac{10}{100} \right\}^2 \\ \Rightarrow Q &\leq 0.015366 \text{ m}^3/\text{s} \\ \Rightarrow Q &\leq 15.36 \simeq 15.4 \text{ L/s} \end{aligned}$$

30. (c)

$$\begin{aligned} f &= \frac{64}{Re} \\ Re &= \frac{UD}{\nu} = \frac{0.1 \times 0.1}{10^{-5}} = 1000 \\ f &= \frac{64}{1000} = 0.064 \end{aligned}$$

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