① a - k

c d f h i

ⓒ $f(x) = 2x^4 + x^3 - 2x - 1$

$f'(x) = 2 \cdot 4x^3 + 3 \cdot x^2 - 2 \cdot$

$= 8x^3 + 3x^2 - 2$

d) $-x^{-2} = -\frac{1}{x^2}$

$f(x) = -x^{-2}$

$f'(x) = -(-2) \cdot x^{-2-1} = 2x^{-3} = \frac{2}{x^3}$

f) $f(x) = \frac{2}{x^2} + \frac{x^2}{2} = 2 \cdot \underbrace{\frac{1}{x^2}} + \frac{1}{2} \cdot x^2$

$= 2 \cdot x^{-2} + \frac{1}{2}x^2$

$f'(x) = -4x^{-3} + 1x = -4x^{-3} + x$

$= -\frac{4}{x^3} + x$

$-\frac{4}{x^3} + {}^{x^3)}x = -\frac{4}{x^3} + \frac{x^4}{x^3} = \frac{-4 + x^4}{x^3} = \frac{x^4 - 4}{x^3}$

h) $x^2\sqrt{x} = x^2 x^{\frac{1}{2}} = x^{2+\frac{1}{2}} = x^{\frac{5}{2}}$

$$f(x) = x^{\frac{5}{2}}$$

$$f'(x) = \frac{5}{2} \cdot x^{\frac{5}{2}-1} = \frac{5}{2} x^{\frac{3}{2}}$$

II tapa tulon derivaatta

$$f(x) = x^2$$

$$g(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$Dfg = f'g + g'f$$

$$= 2x \cdot x^{\frac{1}{2}} + \frac{1}{2} \underbrace{x^{-\frac{1}{2}} \cdot x^2}$$

$$= 2x^{1+\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}+2}$$

$$= 2x^{\frac{3}{2}} + \frac{1}{2} x^{\frac{3}{2}}$$

$$= \left(2 + \frac{1}{2}\right) x^{\frac{3}{2}} = \frac{5}{2} x^{\frac{3}{2}} = \frac{5}{2}\sqrt{x^3}$$

$$= 2.5\sqrt{x^3}$$

$$\cancel{\sqrt[2]{x}} = \sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$x^{\frac{3}{2}} = \sqrt{x^3} = \left(x^3\right)^{\frac{1}{2}} = x^{3 \cdot \frac{1}{2}} = x^{\frac{3}{2}}$$

$$\left(\sqrt{x}\right)^3 = \left(x^{\frac{1}{2}}\right)^3 = x^{\frac{1}{2} \cdot 3} = x^{\frac{3}{2}}$$

i) $(x-5)(x^3+2x^2)$

I tapa: tulon derivaatta

II tapa: avata sulkeet ...

$$f(x) = (\overset{f}{x-5})(\overset{g}{x^3+2x^2})$$

$$f'(x) = 1\cdot(x^3+2x^2) + (3x^2+4x)(x-5)$$

$$= x^3+2x^2+3x^3-15x^2+4x^2-20x$$

$$= 4x^3-9x^2-20x$$

II tapa

$$(x-5)(x^3+2x^2)$$

$$x^4+\underbrace{2x^3-5x^3}-10x^2$$

$$x^4-3x^3-10x^2$$

$$f(x) = x^4-3x^3-10x^2$$

$$f'(x) = 4x^3-9x^2-20x$$

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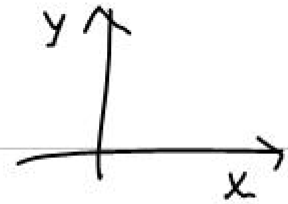


a) $f(x) = x + \underbrace{2t}_{k}$ $\quad Dk=0$

x muuttuja

x:n funktio

$f'(x) = 1$

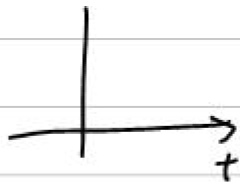
derivoidaan lauseke x:n suhteen

b) $f(t) = \underbrace{x}_{k} + 2t$

$Dk\,0$

t muuttuja

t:n funktio



$f'(t) = 2$

derivoidaan lauseke t:n suhteen

c) $\frac{d}{dx}(x+2t) = 1$

derivoidaan x:n suhteen

d) $\frac{d}{dt}(x+2t) = 2$

derivoidaan t:n suhteen

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g)

$f(x) = \sqrt{x}(2-x)$

$x^{\frac{1}{2}}$

tulon derivaatta

$f'g + g'f$

$= g'f + f'g$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}(2-x) + (-1)\cdot\sqrt{x} = \frac{(2-x)}{2\sqrt{x}} - \sqrt{x}$

(jos sieventää eteenpäin

$\frac{-3x+2}{2\sqrt{x}}$)

h) $f(x) = \frac{3x+3}{1-x}$ $\left(D\frac{f(x)}{g(x)}\right)$

$f'(x) = \frac{\overset{f'}{3}\overset{g}{(1-x)} - \overset{f}{(3x+3)}\cdot\overset{g'}{(-1)}}{\underset{g^2}{(1-x)^2}}$

$= \frac{3-\cancel{3x}+\cancel{3x}+3}{(1-x)^2} = \frac{6}{(1-x)^2}$

i) $f(x) = (9-2x^3)^3$

FUNKTION POTENSSIN D.

$f'(x) = 3(9-2x^3)^2\cdot\underset{f'}{(-6x^2)}$

$= -18x^2(9-2x^3)^2$

j) $f(x) = \sqrt{3x} = (3x)^{\frac{1}{2}}$ $= \underbrace{3^{\frac{1}{2}}}x^{2} \Rightarrow D$

$f'(x) = \frac{1}{2}(3x)^{-\frac{1}{2}} \cdot 3$ FUNKTION POTENSSIN D.

$= \frac{3}{2} \cdot \frac{1}{\sqrt{3x}} = \frac{3}{2\sqrt{3x}}$

k) $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \left(= \frac{1}{3x^{\frac{2}{3}}} = \frac{1}{3\sqrt[3]{x^2}} \right)$

l) $f(x) = \sin x$

$f'(x) = \cos x$

m) $f(x) = \cos x$

$f'(x) = -\sin x$

n) $f(x) = \tan x$

$f'(x) = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

o) $f(x) = 3\cos x$

$f'(x) = -3\sin x$

p) $f(x) = \cos^3 x = (\cos x)^3$ FUNKTION POTENSSIN D.

$f'(x) = 3(\cos x)^2 \cdot (-\sin x) = -3\cos^2 x \sin x$

q) $f(x) = \frac{1}{2}\cos \underbrace{6x}_{g[f(x)]}$

HUOM! sisäfunktio

$D \cos f(x) = -\sin f(x) \cdot f'(x)$

$D \sin f(x) = \cos f(x) \cdot f'(x)$

YHDISTETYN FUNKTION D.

$f'(x) = \frac{1}{2}(-\sin 6x) \cdot 6$

$= -3\sin 6x$

r) $f(x) = 2x + \sin \underbrace{2x}_{g[f(x)]}$

$f'(x) = 2 + \cos 2x \cdot 2 = 2 + 2\cos 2x$

s) $f(x) = \frac{1}{\sin^2 x} = \sin^{-2} x = (\sin x)^{-2}$

FUNKTION POTENSSIN D.

$f'(x) = -2(\sin x)^{-3} \cdot \cos x$

$= -\frac{2\cos x}{\sin^3 x}$

t) $f(x) = e^x$

$f'(x) = e^x$

u) $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

v) $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

w) $f(x) = \ln(2x+2)$

$f'(x) = \frac{1}{2x+2} \cdot 2$

$= \frac{2}{2x+2} \overset{(2}{=} \frac{1}{x+1}$

$$\boxed{\begin{array}{l} D \ln f(x) \\ = \frac{1}{f(x)} \cdot f'(x) \\ \text{YHDISTETYN} \\ \text{FUNKTION D.} \end{array}}$$

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$f(x) = (9-2x)^3$

$f'(x) = 3 \cdot (9-2x)^2 \cdot (-2) = -6(9-2x)^2$

$f'(2) = -6(9-2\cdot 2)^2 = -6 \cdot 5^2 = -150$

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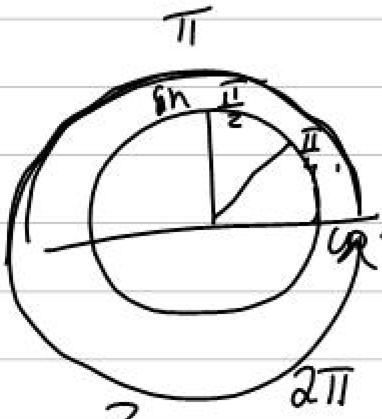
V: -3

$f(x) = \cos 2x - \sin^2 x = \cos \underbrace{2x}_{f(x)} - \underbrace{(\sin x)}_{f(x)}^2$



$f'(x) = -\sin 2x \cdot 2 - 2 \sin x \cos x$

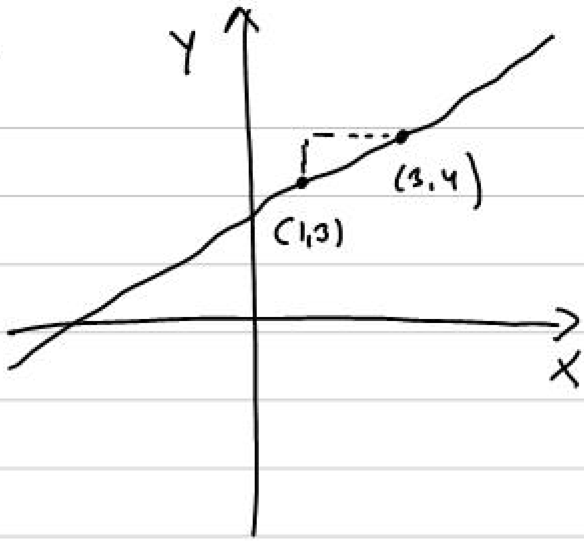
$= -2 \sin 2x - 2 \sin x \cos x$

$f'\left(\frac{\pi}{4}\right) = -2 \cdot \underbrace{\sin \frac{\pi}{2}}_{=1} - 2 \underbrace{\sin \frac{\pi}{4}}_{\frac{1}{\sqrt{2}}} \underbrace{\cos \frac{\pi}{4}}_{\frac{1}{\sqrt{2}}}$

$= -2 \cdot 1 - 2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = -2 - \frac{2}{\sqrt{2} \cdot \sqrt{2}}$

$= -2 - \frac{2}{2} = -2 - 1 = -3$

(63) Kulmakerroin $k = \frac{\Delta y}{\Delta x}$



$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(1,3) = (x_1, y_1)$$
$$(3,4) = (x_2, y_2)$$

$$k = \frac{4-3}{3-1} = \frac{1}{2}$$

$$\boxed{y = ax + b}$$

k

$$\boxed{y - y_0 = k(x - x_0)}$$

$k = \frac{1}{2}$

(x_0, y_0) valitaan $(1,3)$

$$y - 3 = \frac{1}{2}(x - 1)$$

$$y = \frac{1}{2}x - \frac{1}{2} + 3$$

$$y = \frac{1}{2}x + \frac{5}{2} =$$

y-akselin leikkauspiste

$x = 0 \Rightarrow \quad y = \frac{1}{2} \cdot 0 + \frac{5}{2} = \frac{5}{2}$

x-akselin leikkauspiste

$y = 0 \Rightarrow \quad 0 = \frac{1}{2}x + \frac{5}{2}$

$\frac{1}{2}x = -\frac{5}{2} \Rightarrow x = -5$

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$(1,1)$ (x_0, y_0)

$k=2$

$$\boxed{y - y_0 = k(x - x_0)}$$

$$y - 1 = 2(x-1)$$

$$y = 2x - 2 + 1$$

$$y = 2x - 1$$

65 $f(x) = -x^2 + 2x + 4$

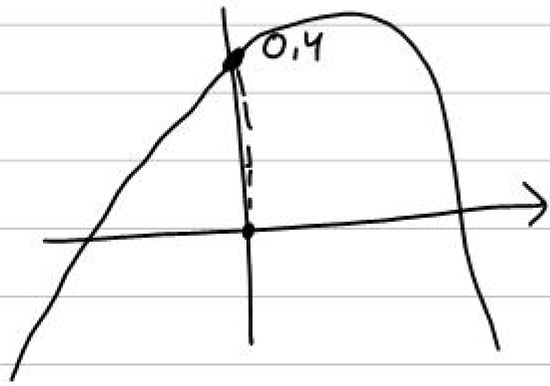
$f'(x) = -2x + 2$

$f'(0) = -2 \cdot 0 + 2 = 2$

Kulmakerroin on derivaatan arvo kohdassa 0.

$\Rightarrow$ tangentin kulmakerroin on 2!

piste $(0,4)$



$$\boxed{y - y_0 = k(x - x_0)}$$

$k = 2$

$(0,4)$

$y - 4 = 2(x - 0)$

$y = 2x + 4$

KOTITEHT 66,

Teht. paketti: 1–8, 10