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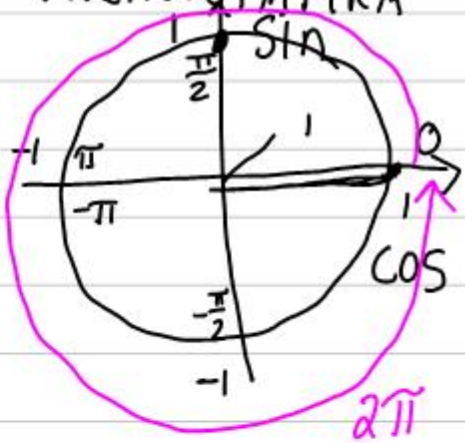
Taulukko. cos-funktion arvoja (lisäys $\frac{\pi}{2}$)

yksikköympyrän kehän pituus

$\cos 0$	1
$\cos \frac{\pi}{2}$	0
$\cos \pi$	−1
$\cos \frac{3\pi}{2}$	0
$\cos 2\pi$	1
$\cos \frac{5\pi}{2}$	0
$\cos 3\pi$	−1
.....	
$\cos 0$	1
$\cos -\frac{\pi}{2}$	0
$\cos -\pi$	−1
$\cos -\frac{3\pi}{2}$	0

YKSIKKÖYMPYRÄ



cos x

Sininen kuvaaja

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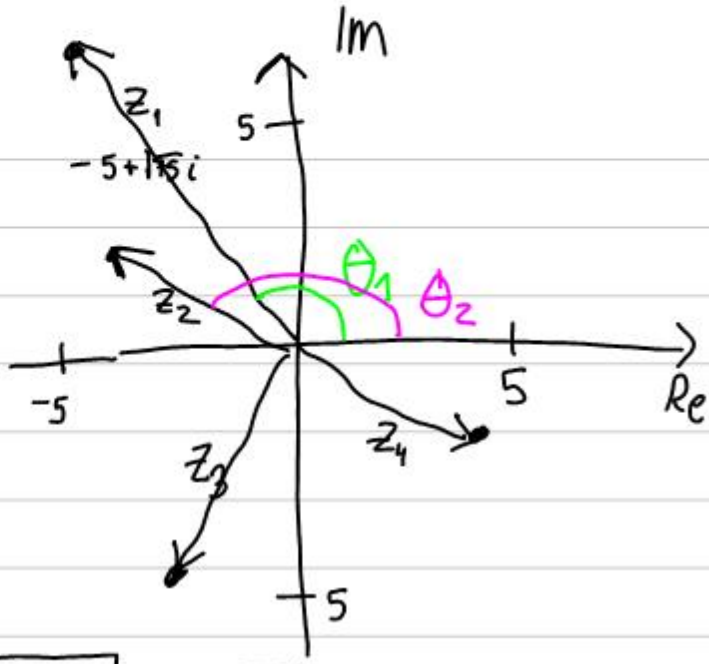
$z = a + bi$ $\quad r \angle \theta$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \frac{b}{a}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$



$z_1 = -5 + \sqrt{75}\,i$

$r = \sqrt{(-5)^2 + (\sqrt{75})^2} = \sqrt{25+75} = \sqrt{100} = 10$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$= \tan^{-1}\left(\frac{\sqrt{75}}{-5}\right) =$ ↑ a on negat

DEG-tila (°)

$-60° + 180° = 120°$

RAD (rad)

$-1.047 + \pi = 2.0945$ rad

LISÄYS a negat. lisää $\begin{cases} 180° \\ \pi \end{cases}$

$10 \angle 120°$

$10 \angle 2.0945$

b) $r = \sqrt{(-4)^2 + (2)^2} = \sqrt{20} \approx 4.47$

$\theta = \tan^{-1}\left(\frac{2}{-4}\right) = \begin{cases} -26.6° + 180° = 153.4° \\ -0.46 + \pi = 2.68 \end{cases}$

$4.47 \angle 153.4°$

$4.47 \angle 2.68$ rad

$z_3 = -2-5i$

$r = \sqrt{(-2)^2+(-5)^2} = \sqrt{29} \approx 5.39$

$\theta = \begin{cases} 248.20^\circ \\ 4.33 - 2\pi = -1.95 \end{cases}$

↑ 2π-monikerta

$5.39 \angle 248.2^\circ$

$5.39 \angle 4.33 \text{ rad}$

$5,39 \angle -1.95 \text{ rad}$

d) $z_4 = 4-2i$ (a positiivinen)

$r = \sqrt{20} \approx 4.47$

$\theta = \begin{cases} -26.6^\circ \\ -0.46 \text{ rad} \end{cases}$

(33) VAST.

$\cos 180^\circ = -1$ $\sin 180^\circ = 0$ $\cos 110^\circ = -0.34$ $\sin 110^\circ = 0.94$

$\cos 355^\circ = 0.996$ $\sin 355^\circ = -0.087$

(34) $270^\circ = \frac{3\pi}{2} \text{ rad}$

$80^\circ = \frac{4\pi}{9} \text{ rad} \approx 1.396 \text{ rad}$

(35) $\frac{\pi}{4} \text{ rad} = 45^\circ$

$1.5 \text{ rad} = 85.94^\circ = \frac{270}{\pi}^\circ$

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$$z = a + bi$$
$$= r\cos\theta + r\sin\theta\, i$$
$$= r(\cos\theta + \sin\theta\, i)$$

a. $3\angle\frac{\pi}{6}$

$r = 3 \quad \theta = \frac{\pi}{6}$

$\Rightarrow r(\cos\theta + i\sin\theta)$

$= 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

RAD-Mode

$\frac{\sqrt{3}}{2}$

$= 3(0.866 + 0.5i)$

$= \begin{cases} 2.598 + 1.5i \\ \frac{3\sqrt{3}}{2} + 1.5i \end{cases}$

b) $2\angle\frac{\pi}{4}$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = 1.414 + 1.414i$

$= 2\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}i = \sqrt{2} + \sqrt{2}i$

c) $2\angle 1\,(\text{rad})$

$2(\cos 1 + i\sin 1) = 2\cos 1 + 2i\sin 1 = 1.081 + 1.683i$

32/31

$e^{z=i\pi}$ $\qquad re^{ix}$

$r = 1$

$\theta = \pi$

$$re^{i\theta} = 1 \cdot e^{i\pi} = e^{i\pi} = 1(\cos\pi + i\sin\pi)$$

Euler

$$= \cos\pi + \sin\pi\, i \longrightarrow \text{taulukko}$$

$\cos\pi = -1$

$\sin\pi = 0$

$$= -1 + 0 \cdot i$$

$$= -1$$

b) $e^{i3} = 1(\cos 3 + i\sin 3) = \cos 3 + i\sin 3$

RAD-tila

$r = 1$

$\theta = 3$

$$= -0.99 + 0.14\,i$$

c) $5e^{i\frac{\pi}{3}} = 5\left(\cos\frac{\pi}{3} + i\cdot\sin\frac{\pi}{3}\right) \longrightarrow$ taulukko

$\cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$r = 5$

$\theta = \frac{\pi}{3}$

$$= 5\left(\frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}\right)$$

$$= \frac{5}{2} + \frac{5\sqrt{3}}{2}\,i \quad \left(= 2.5 + 4.33\,i\right)$$

d) $3e^{i\frac{\pi}{2}} = 3\left(\underbrace{\cos\frac{\pi}{2}}_{=0} + i\underbrace{\sin\frac{\pi}{2}}_{1}\right) = 3i$

(32) $z = 7+5i$ $\qquad re^{i\theta}$

$r = ?$

$\theta = ?$

$r = \sqrt{a^2+b^2} = \sqrt{7^2+5^2} = \sqrt{74}$

$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\theta = \tan^{-1}\left(\frac{5}{7}\right) = 0.62$

$\Rightarrow \sqrt{74}\, e^{i\,0.62}$

b) $z = 3+4i$

$r = \sqrt{3^2+4^2} = \sqrt{25} = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.927$

$\Rightarrow 5e^{i\,0.927}$

c) $z = 1-i$

$r = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\theta = \tan^{-1}(-1) \Rightarrow$

$\theta = \begin{cases} -0.785 \\ -\frac{\pi}{4} \\ \frac{3\pi}{4} \end{cases}$

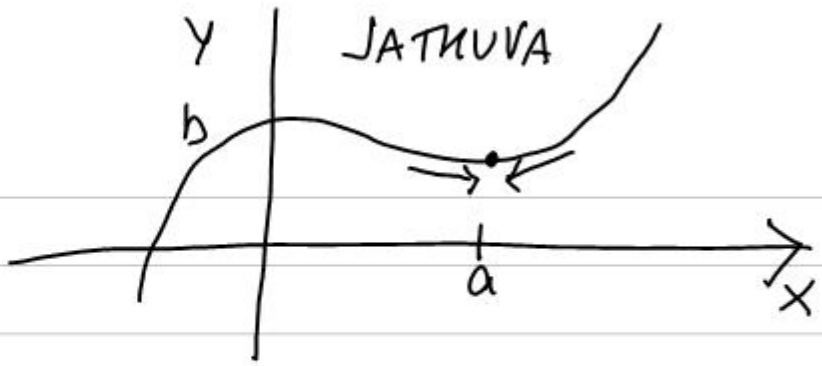
$\begin{cases} \sqrt{2}\, e^{i\cdot -0.785} \\ \sqrt{2}\, e^{-\frac{\pi}{4}i} \\ \sqrt{2}\, e^{\frac{3\pi}{4}i} \end{cases}$

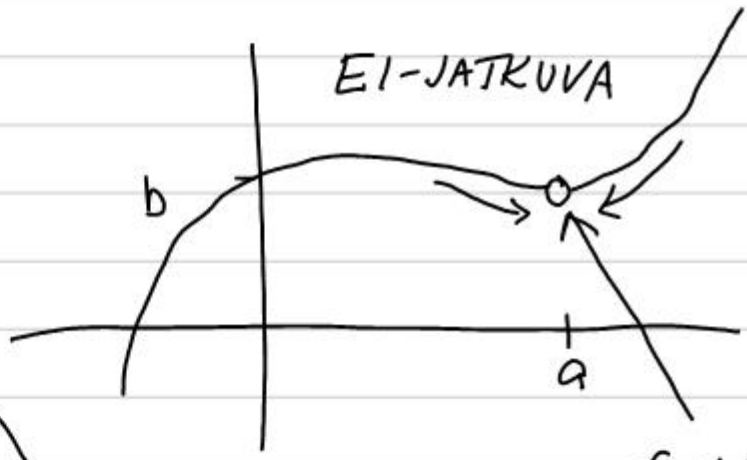
КОПТЕНТ:
37-42

(43) $f(x) = 2x + 5$

x	y
1,5	$2 \cdot 1,5 + 5 = 8$
1,9	8,8
1,99	8,98
1,999	8,998

x	y
2,5	$2 \cdot 2,5 + 5 = 10$
2,1	9,2
2,01	9,02
2,001	9,002





funktiota ei määritelty

lähestyy lukua 9!

$\lim_{x \to 2} 2x + 5 \overset{\text{eli } x=2}{=} 2 \cdot 2 + 5 = 9$

$\Rightarrow$ Funktion raja-arvo on 9.

Onko funktio jatkuva?

$f(2) = 2 \cdot 2 + 5 = 9$ $\Leftarrow$

$\Rightarrow$ Funktion arvo on 9.

ELI $\lim_{x \to 2} f(x) = f(x)$

Funktio on jatkuva.

44)

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{(x-2)}$$

ei: $x=2$

$$= \lim_{x\to 2} x+2 = 2+2=4$$

x	y
1,5	3,5
1,9	3,9
1,99	3,99
1,999	3,999

Funktiolla on raja-arvo kohdassa 2.

Funktion raja-arvo on 4.

x	y
2,5	4,5
2,1	4,1
2,01	4,01
2,001	4,001

Funktion arvo kohdassa 2?

$f(2) = ?$

Funktiota ei ole määritelty kohdassa 2.

(f(2) ei voi laskea!)

Funktio ei ole jatkuva kohdassa 2.

Määrittelyjoukko $\mathbb{R} \setminus \{2\}$

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x	y
1,5	9
1,9	-57
1,99	-597
1,999	-5997

x	y
2,5	15
2,1	63
2,01	603
2,001	6003

ei ~~lähestyy mitään~~ tiettyä lukua!

$\Rightarrow$ Raja-arvoa ei ole olemassa.

KOTITEHT. 46-47