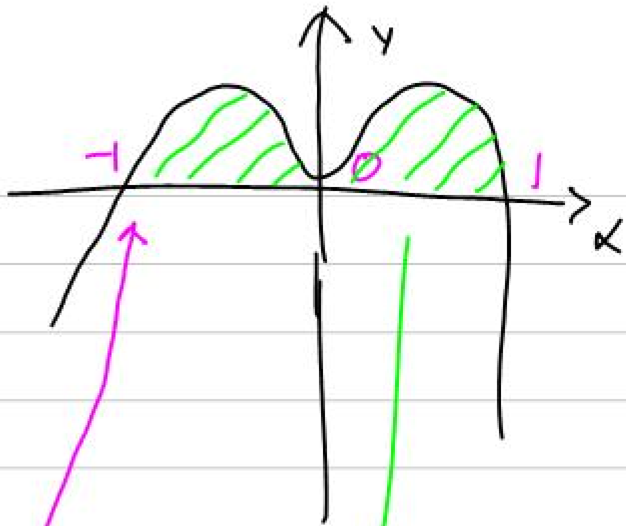
39) $f(x) = x^2 - x^4$



Ratk. 0-kohdat

(missä pisteissä kuvaaja leikkaa y-akselin)

$$f(x) = x^2 - x^4 = 0$$

$$x^2(1 - x^2) = 0$$

$x^2 = 0$ tai $(1 - x^2) = 0$

$x = 0$ $\qquad$ $x^2 = 1$

$\qquad\qquad x = \pm 1$

VÄLI $[-1, 1]$

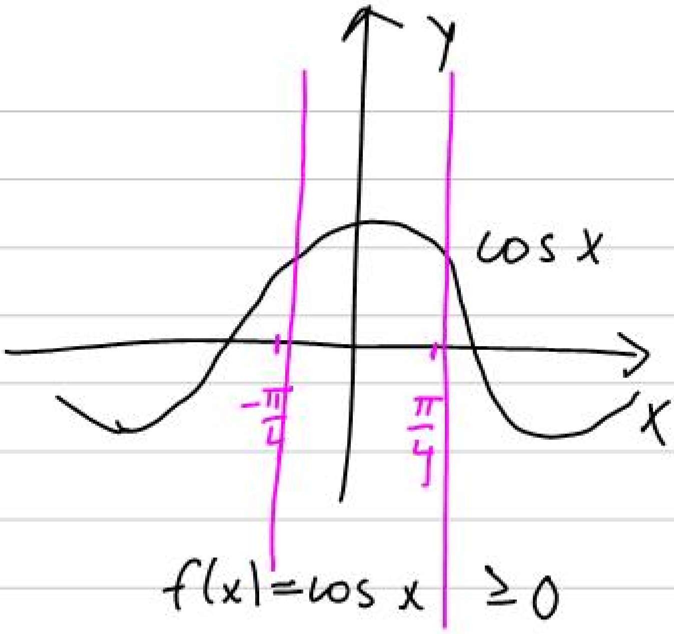
$$\int_{-1}^{1} (x^2 - x^4)\,dx = \Big/_{-1}^{1} \frac{x^3}{3} - \frac{x^5}{5} = \Big/_{-1}^{1} \frac{1}{3}x^3 - \frac{1}{5}x^5$$

$$= \left[\underbrace{\frac{1}{3}\cdot 1^3}_{\frac{1}{3}} - \underbrace{\frac{1}{5}\cdot 1^5}_{\frac{1}{5}}\right] - \left[\underbrace{\frac{1}{3}\cdot(-1)^3}_{-\frac{1}{3}} - \underbrace{\frac{1}{5}\cdot(-1)^5}_{-\frac{1}{5}}\right]$$

$$= \frac{1}{3} - \frac{1}{5} + \frac{1}{3} - \frac{1}{5} = {}^{5)}\frac{2}{3} - {}^{3)}\frac{2}{5} = \frac{10}{15} - \frac{6}{15} = \frac{4}{15}$$

(40) $f(x) = \cos x$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos x \, dx = \Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin x$$



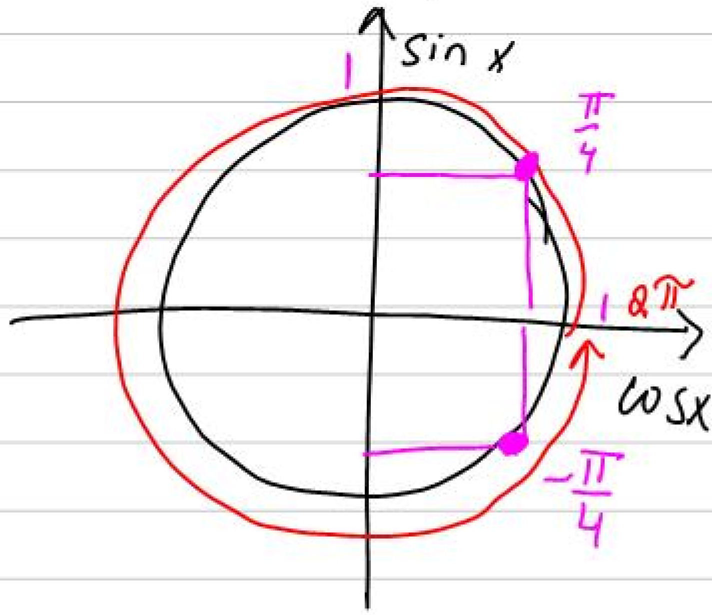
$= \sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right)$

$f(x) = \cos x \geq 0$

value $[-\frac{\pi}{4}, \frac{\pi}{4}]$

$= \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \approx 1.414$

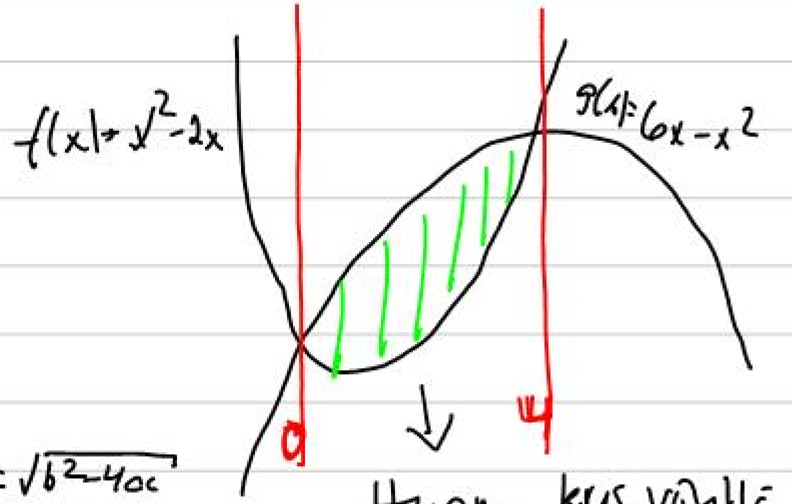
$-\frac{\pi}{4} = \frac{7\pi}{4}$



43 $f(x) = x^2 - 2x \quad g(x) = 6x - x^2$

Missä pisteissä kuvaajat leikkaa?

ratk. $f(x) = g(x)$



Huom. kys. välillä $g(x) > f(x)$

$$x^2 - 2x = 6x - x^2$$

$$2x^2 - 8x = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2x(x-4) = 0$$

$2x = 0$ tai $x - 4 = 0$

$x = 0 \qquad x = 4$

VÄLI [0, 4]

$$\int_0^4 (g(x) - f(x))\,dx = \int_0^4 ((6x - x^2) - (x^2 - 2x))\,dx = \int_0^4 (-2x^2 + 8x)\,dx$$

$$= \Big/_0^4 -\frac{2}{3}x^3 + 4x^2 = \left[-\frac{2}{3}\cdot 4^3 + 4\cdot 4^2\right] - \underbrace{\left[-\frac{2}{3}\cdot 0^3 + 4\cdot 0^2\right]}_{=0}$$

$$= -\frac{2}{3}\cdot 64 + 64 = \frac{64}{3} \approx 21.33$$

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2 OSASSA?

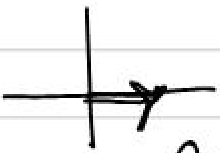
$$\int_{-\sqrt{3}}^{0}(x^3-3x)\,dx - \int_{0}^{\sqrt{3}}(x^3-3x)\,dx$$

$$\downarrow \qquad\qquad \downarrow$$

$f(x) \geq 0$ $\qquad$ $f(x) < 0$

$z = 2$

$r = \sqrt{2^2 + 0^2} = 2$



$\theta = 0°$

$\theta = 0$ t. 2π

$2 \angle 0$

$2 \angle 2\pi$

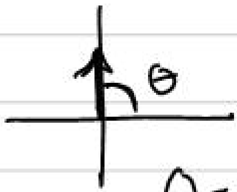
$\Downarrow$

$2(\underbrace{\cos 2\pi}_{1} + i \underbrace{\sin 2\pi}_{0})$

$= 2$

$z = 2i$

$r = \sqrt{2^2 + 0^2} = 2$



$90°$

$\theta = 90°$

$\theta = \frac{\pi}{2}$

$\left(\tan\theta = \frac{b}{a} \nearrow a = 0\right)$

$2 \angle \frac{\pi}{2}$

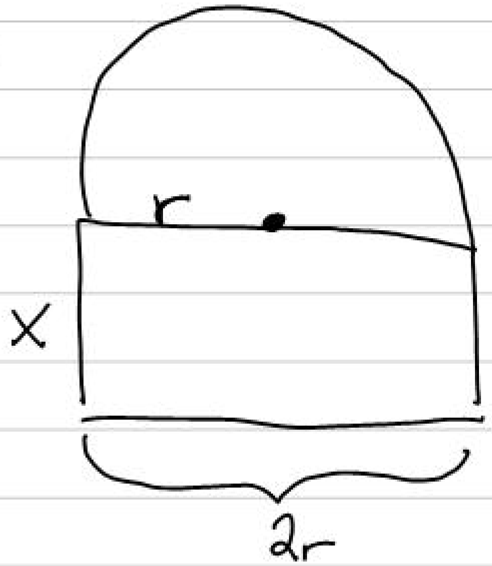
$\Downarrow$

$2(\underbrace{\cos \frac{\pi}{2}}_{=0} + i \underbrace{\sin \frac{\pi}{2}}_{1})$

$= 2i$

Kertaus 1

11



$\leftarrow A = \frac{1}{2}\pi r^2$

$\leftarrow A = 2rx$

(ratkaisu
$r = 1.4$
$x = 1.4$)

$$\sqrt{-8} = \sqrt{\underset{2^2}{4}\cdot -2} = \sqrt{2^2\cdot(-2)} = 2\sqrt{-2}$$

$$\sqrt{-8} = \sqrt{-1\cdot 8} = \sqrt{(i)^2\, 8} = \underset{\substack{\downarrow \\ 2\sqrt{2}}}{\sqrt{8}}\, i$$

$$\boxed{\frac{\cancel{2}\sqrt{2}}{\cancel{2}} = \sqrt{2}}$$

$$\sqrt{4} = 2 \Rightarrow 2^2 = 4$$

$$\boxed{i^2 = -1}$$

$$\sqrt{-4} = 2i \Rightarrow (2i)^2 = -4$$

Kertaus 1.

11

$f(r) = 10r - (2 + \frac{1}{2}\pi)r^2$

$f'(r) = \ldots$

$f'(r) = 0 \quad \ldots \quad r = \frac{5}{2 + \frac{1}{2}\pi} \approx 1.4$

$$\boxed{\begin{aligned} A &= 2rx + \tfrac{1}{2}\pi r^2 \\ 2x + 2r + \pi r &= 10 \end{aligned}}$$