79

$$
\begin{tikzcd}
\frac{1}{2}x^2 \arrow[r, bend left, "\text{derivi}"] & x \arrow[l, bend left, "\text{integroi}"]
\end{tikzcd}
$$

a. $f(x) = \frac{1}{2}x^2 \Rightarrow f'(x) = x$

b. $f(x) = x$

$$\int x\,dx = \frac{1}{1+1}x^{1+1} = \frac{1}{2}x^2$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} = \frac{1}{r+1} \cdot x^{r+1}$$

80 $f(x) = x^4$

$$\int x^4\,dx = \frac{1}{4+1}x^{4+1} = \frac{1}{5}x^5$$

(81) $f(x) = 3x^2 - 2$

$F(x) = x^3 - 2x$

$\hookrightarrow F'(x) = 3x^2 - 2 = f(x)$

a. $F(x) = x^3 - 2x + 1$

$\hookrightarrow F'(x) = 3x^2 - 2$

integral

b. $F(x) = x^3 - 2x - 4$

$\hookrightarrow F'(x) = 3x^2 - 2$

integral

c. $f(x) = 3x^2 - 2$

$$\int (3x^2 - 2)\,dx = \int 3x^2\,dx - \int 2\,dx$$

$$= 3\int x^2\,dx - 2\int dx$$

$$= 3 \cdot \frac{1}{2+1} \cdot x^{2+1} - 2x + C$$

$$= x^3 - 2x + C$$

82

$$F(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6x - 4$$

$$\hookrightarrow F'(x) = \frac{2}{\cancel{3}} \cdot \cancel{3}x^2 - \frac{1}{2} \cdot 2x - 6$$

$$= 2x^2 - x - 6 \quad = f(x)$$

Tarkastelu toisinpäin

$$f(x) = 2x^2 - x - 6$$

$$\int (2x^2 - x - 6)\,dx = 2 \cdot \frac{1}{2+1}x^{2+1} - \frac{1}{1+1}x^{1+1} - 6x + C$$

$$= \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6x + C \quad = F(x)$$

83

$$f(x) = x^2 + 3x - 2$$

$$\int (x^2 + 3x - 2)\,dx = \frac{1}{3}x^3 + 3 \cdot \frac{1}{2}x^2 - 2x + C$$

$$= \frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + C \quad = F(x)$$

$F(2) = 1$

$$\frac{1}{3} \cdot 2^3 + \frac{3}{2} \cdot 2^2 - 2 \cdot 2 + C = 1$$

$$\frac{8}{3} + 6 - 4 + C = 1$$

$$C = 1 - \frac{8}{3} - 6 + 4 = -\frac{11}{3}$$

V: $\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x - \frac{11}{3}$

(84)

$$\int x^8 dx = \frac{1}{9}x^9 + C$$

$$\int x^{-4} dx = \frac{1}{-4+1}x^{-4+1} + C = \frac{1}{-3}x^{-3} + C$$

$$= -\frac{1}{3}x^{-3} + C$$

$$\int \frac{1}{5}x^{-4} dx = \frac{1}{5}\int x^{-4} dx = \frac{1}{5}\cdot\frac{1}{-3}x^{-3} + C$$

$$= -\frac{1}{15}x^{-3} + C$$

$$\int \frac{4}{x^3} dx = \int 4x^{-3} dx = 4\cdot\frac{1}{-2}x^{-2} + C = -2x^{-2} + C$$

$$\int \frac{x^3}{4} dx = \int \frac{1}{4}x^3 dx = \frac{1}{4}\cdot\frac{1}{4}x^4 + C = \frac{1}{16}x^4 + C$$

$$\frac{1}{2} + \frac{2}{2} = \frac{3}{2}$$

$$\int \sqrt{x}\, dx = \int (x)^{\frac{1}{2}} dx = \frac{1}{\frac{1}{2}+1}x^{\frac{1}{2}+1} + C$$

$$\boxed{\frac{a}{b} : \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c}}$$

$$= \frac{1}{\frac{3}{2}}x^{\frac{3}{2}} + C = \frac{2}{3}x^{\frac{3}{2}} + C$$

$$\frac{\frac{1}{1}}{\frac{3}{2}} = \frac{1}{1}\cdot\frac{2}{3} = \frac{2}{3} \qquad \left(= \frac{2}{3}\sqrt{x^3} + C\right)$$

$$\int (x^3 + 2x^2 - \frac{1}{4}x + 5)\, dx$$

$$= \frac{1}{4}x^4 + 2\cdot\frac{1}{3}x^3 - \frac{1}{4}\cdot\frac{1}{2}x^2 + 5x + C$$

$$= \frac{1}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{8}x^2 + 5x + C$$

(85)

$$\int \underbrace{3x^2}_{f'(x)} \underbrace{(x^3+2)^2}_{f(x)^n} dx$$

$$\int f(x)^r f'(x)\, dx = \frac{1}{r+1} f(x)^{r+1} + C$$

$f(x) = (x^3+2) \quad r=2$

$f'(x) = 3x^2$

$$= \frac{1}{3}(x^3+2)^3 + C$$

b. $$\int (2x-5)^3 dx = \int \frac{1}{2} \cdot 2 \cdot (2x-5)^3 dx$$

($\frac{1}{2} \cdot 2 = 1$; $2 \cdot (2x-5)^3$ is $f'(x) f(x)^r = f(x)^r f'(x)$)

$f(x) = (2x-5) \quad r=3$

$f'(x) = 2$

$$= \frac{1}{2}\int 2(2x-5)^3 dx = \frac{1}{2} \cdot \frac{1}{4}(2x-5)^4 + C$$

$$= \frac{1}{8}(2x-5)^4 + C$$

c. $$\int \frac{2x}{x^2} dx = \ln|x^2| + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$= \ln x^2 + C$$

$f(x) = x^2$

$f'(x) = 2x$

$(= 2\ln x + C)$

d. $$\int \frac{4x}{x^2-9} dx = \int 2 \cdot \frac{2x}{x^2-9} dx = 2\int \frac{2x}{x^2-9} dx$$

$f(x) = x^2-9$

$f'(x) = 2x$

$$= 2\ln|x^2-9| + C$$

(86)

$$\int e^x dx = e^x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

$$\int \tan x\, dx = -\ln|\cos x| + C, \quad x \neq \frac{\pi}{2} + n\pi$$

(87)

$$\int \frac{1}{2} e^x dx = \frac{1}{2}\int e^x dx = \frac{1}{2}e^x + C$$

$$\int e^{2x} dx = \frac{1}{2}e^{2x} + C$$

(Yhdistetyn funktion)
integrointi

$$\int \cos 4x\, dx$$

$f(x) = \cos(\ )$

$g(x) = 4x$

$g'(x) = 4$

$$\int f(g(x)) \cdot g'(x)\, dx$$

$$F(g(x)) + C$$

$$F(x) = \int f(x)\, dx$$

$$= \int \left(\overbrace{\cos 4x \cdot 4}\cdot \frac{1}{4}\right) dx \qquad \underbrace{4 \cdot \frac{1}{4}}_{=1}$$

$$= \frac{1}{4}\int \cos 4x \cdot 4\, dx = \frac{1}{4}\cdot \sin 4x + C$$

d. $$\int 4 \sin 2x\, dx = \int 2 \cdot \underbrace{2 \cdot \sin 2x}\, dx$$

$f(x) = \sin(\ )$

$g(x) = 2x$

$g'(x) = 2$

$$= 2\int \sin 2x \cdot 2\, dx = 2 \cdot (-\cos 2x) + C$$

$$= -2\cos 2x + C$$

$$\int fg' = fg - \int f'g$$

(88) $\int xe^x\,dx$

$f(x) = x \qquad g'(x) = e^x$

$f'(x) = 1 \qquad g(x) = e^x$

$$= x \cdot e^x - \int 1 \cdot e^x\,dx = xe^x - \int e^x\,dx$$

$$= xe^x - e^x + C$$

$$= e^x(x-1) + C$$

b. $\int x\cos x\,dx = x \cdot \sin x - \int \sin x$

$f(x) = x \qquad g'(x) = \cos x$ integrointi

$f'(x) = 1 \quad$ dif. $\quad g(x) = \sin x$

$$= x\sin x - (-\cos x) + C = x\sin x + \cos x + C$$

C. $\int (x+1)(2x^2-5)\,dx$

VAIHTOEHTO

1) osittaisintegrointi

$f(x) = x+1$

$g'(x) = 2x^2-5$

$\vdots$

$(x+1)\left(\frac{2}{3}x^3-5x\right) - \int \left(\frac{2}{3}x^3-5\right)dx$

$\vdots$

$\frac{1}{2}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 5x + C$

2) AVAA SULUT JA SIEVENNÄ FUNKTIO !!

$\int (x+1)(2x^2-5)\,dx$

$= \int (2x^3 + 2x^2 - 5x - 5)\,dx$

$= \frac{1}{2}x^4 + \frac{2}{3}x^3 - \frac{5}{2}x^2 - 5x + C$

89

$$\int_1^3 x^2\,dx = \Big|_1^3 \tfrac{1}{3}x^3 = \underbrace{\tfrac{1}{3}\cdot 3^3}_{F(3)} - \underbrace{\tfrac{1}{3}\cdot 1^3}_{F(1)}$$

$$= 9 - \tfrac{1}{3} = 8\tfrac{2}{3}$$

$f(x) = x^2$

$F(x) = \frac{1}{3}x^3$

$$\int_{\frac{\pi}{2}}^{\pi} \sin x\,dx = \Big|_{\frac{\pi}{2}}^{\pi} -\cos x = \underbrace{-\cos\pi}_{F(\pi)} - \underbrace{\left(-\cos\tfrac{\pi}{2}\right)}_{F(\frac{\pi}{2})}$$

$$= -\cos\pi + \cos\tfrac{\pi}{2}$$

$f(x) = \sin x$

$F(x) = -\cos x$

$$= -(-1) + 0 = 1$$

c. $$\int_{-2}^{3} (t+t^2)\,dt = \Big|_{-2}^{3} \tfrac{1}{2}t^2 + \tfrac{1}{3}t^3$$ $14\frac{1}{6}$

$$= \underbrace{\left(\tfrac{1}{2}\cdot 3^2 + \tfrac{1}{3}\cdot 3^3\right)}_{F(3)} - \underbrace{\left(\tfrac{1}{2}\cdot(-2)^2 + \tfrac{1}{3}\cdot(-2)^3\right)}_{F(-2)}$$

$$= \frac{9}{2} + 9 - \left(2 - \frac{8}{3}\right) = \frac{^{3/}9}{2} + \underbrace{9 - 2}_{=7} + \frac{^{2/}8}{3}$$

$$= 7 + \underbrace{\frac{27}{6} + \frac{16}{6}}_{} = 7 + \underbrace{\frac{43}{6}}_{7\frac{1}{6}} = 14\frac{1}{6} \approx 14.166\ldots$$

d. $\int_{2}^{6} (3x^2-2x+4)\,dx$

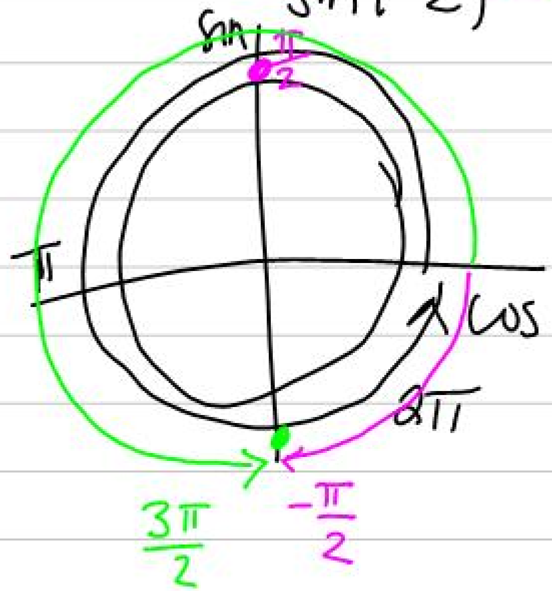
$$= \Big|_{2}^{6} 3\cdot\tfrac{1}{3}x^3 - 2\cdot\tfrac{1}{2}x^2 + 4x = \Big|_{2}^{6} x^3 - x^2 + 4x$$

$$= 6^3 - 6^2 + 4\cdot 6 - \left(2^3 - 2^2 + 4\cdot 2\right) = 192$$

e. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx = \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x = \sin\frac{\pi}{2} - \left(\sin\left(-\frac{\pi}{2}\right)\right) = 1-(-1)$

$= 2$

$\sin\left(-\frac{\pi}{2}\right) =$



$\sin\left(\frac{\pi}{2}\right) = 1$

$\sin\left(-\frac{\pi}{2}\right)$

$= \sin\left(\frac{3\pi}{2}\right) = -1$

f. $\int_1^4 \frac{1}{x}\,dx = \int_1^4 x^{-1}\,dx$

$\int x^r = \frac{1}{r+1} x^{r+1}$ ....

EI VOI KÄYTTÄÄ TÄSSÄ TEHTÄVÄSSÄ KOSKA $r = -1$

$x \in \mathbb{R}_+$

$r \in \mathbb{R} \setminus \{-1\}$

$$= \int_1^4 \frac{1}{x}\,dx = \Big/_1^4 \ln|x| = \ln 4 - \underbrace{\ln 1}_{=0} = \ln 4$$

$$= \ln 2^2 = 2\ln 2$$

$$\boxed{\begin{array}{l}\text{KOTITEHT.} \\ \quad 29-30 \\ \quad 31-33 \\ \quad 35-36\end{array}}$$