37

a. $\sqrt{8} \angle 45^\circ$

$$\boxed{\begin{array}{l} a+bi = r(\cos\theta + i\sin\theta) \\ r\angle\theta \end{array}}$$

$\Rightarrow \sqrt{8}\left(\cos 45^\circ + i\sin 45^\circ\right)$ $\boxed{\text{DEG-tila}}$

$= \sqrt{8}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

$= \frac{2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} + i\frac{2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = 2+2i$

38 $3\angle\frac{\pi}{6}$ $\boxed{\text{RAD-tila}}$

$\Rightarrow 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

$= 3\left(\frac{\sqrt{3}}{2} + i\cdot\frac{1}{2}\right) = \frac{3\sqrt{3}}{2} + i\frac{3}{2} \approx 2.6 + 1.5i$

39

$$\boxed{a+bi + r(\cos\theta + i\sin\theta) = re^{i\theta}}$$

$2e^{i\frac{\pi}{3}} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(0.5 + i\cdot\frac{\sqrt{3}}{2}\right)$

$= 1 + i\sqrt{3} \approx 1 + 1.7i$

40 $2+2i$

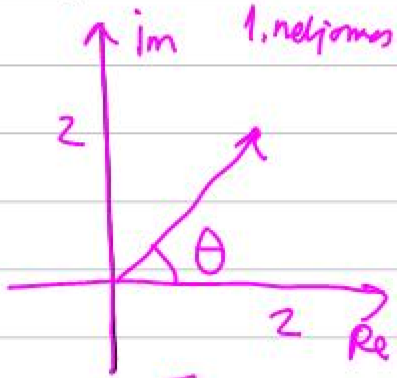
$\sqrt{8}e^{i\frac{\pi}{4}}$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{2^2+2^2} = \sqrt{8}$

$\tan\theta = \frac{b}{a}$

$\tan\theta = \frac{2}{2} = 1 \Rightarrow \theta \Rightarrow \frac{\pi}{4}$



41 vrt. tehd. 38

c. $z_1: 3\angle\frac{\pi}{6}$ $z_2: 2\angle\frac{\pi}{4}$

$r_1\angle\theta_1 \quad r_2\angle\theta_2 \qquad z_1\cdot z_2 = r_1\cdot r_2\angle\theta_1+\theta_2$

$$z_1\cdot z_2 = 3\cdot 2\angle \frac{\pi}{6}+\frac{\pi}{4}$$

$$= 6\angle\frac{2\pi}{12}+\frac{3\pi}{12}$$

$$= 6\angle\frac{5\pi}{12}$$

d. $\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\theta_1-\theta_2$

$$= \frac{3}{2}\angle\frac{\pi}{6}-\frac{\pi}{4}$$

$$= 1.5\angle\frac{2\pi}{12}-\frac{3\pi}{12}$$

$$= 1.5\angle\frac{-\pi}{12}$$

42 $z_1 = 3e^{i\frac{\pi}{6}}$ $z_2 = 2e^{i\frac{\pi}{4}}$

$$z_1\cdot z_2 = 3e^{i\frac{\pi}{6}}\cdot 2e^{i\frac{\pi}{4}} = 3\cdot 2\cdot \underbrace{e^{i\frac{\pi}{6}}\cdot e^{i\frac{\pi}{4}}}$$

$$= 6e^{i\frac{\pi}{6}+i\frac{\pi}{4}} = 6e^{i\frac{2\pi}{12}+i\frac{3\pi}{12}} = 6e^{i\left(\frac{2\pi}{12}+\frac{3\pi}{12}\right)}$$

$$= 6e^{i\frac{5\pi}{12}}$$

42

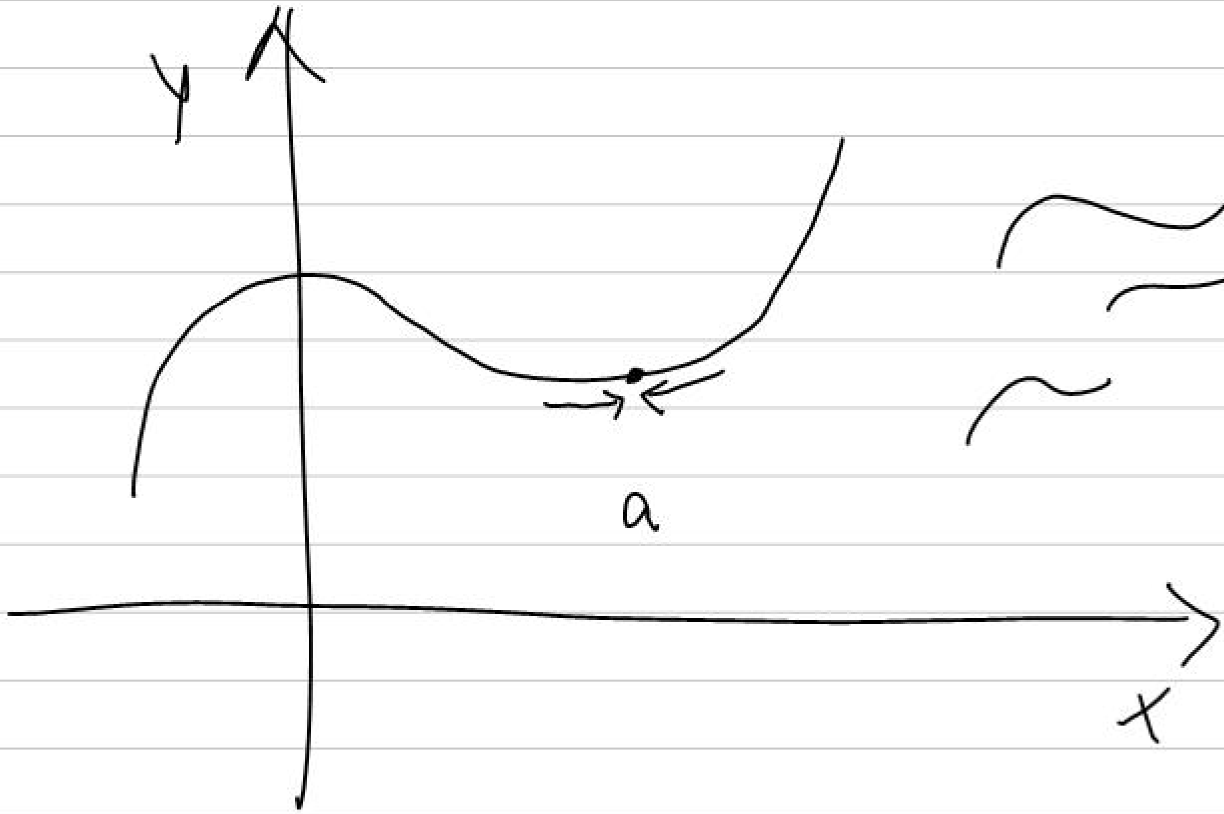
d.

$$\frac{z_1}{z_2} = \frac{3e^{i\frac{\pi}{6}}}{2e^{i\frac{\pi}{4}}} = \frac{3}{2}\cdot\frac{e^{i\frac{\pi}{6}}}{e^{i\frac{\pi}{4}}} = 1.5\,e^{i\frac{\pi}{6}-i\frac{\pi}{4}}$$

$$\boxed{\frac{a^m}{a^n} = a^{m-n}}$$

$$= 1.5\,e^{i\left(\frac{2\pi}{12}-\frac{3\pi}{12}\right)}$$

$$= 1.5\,e^{-\frac{\pi}{12}i}$$



(46) $\begin{cases} \sin x \\ \cos x \end{cases}$ määrittelyjoukko $\mathbb{R}$

arvojoukko $[-1, 1]$

$-1 \leq f(x) \leq 1$

(47)

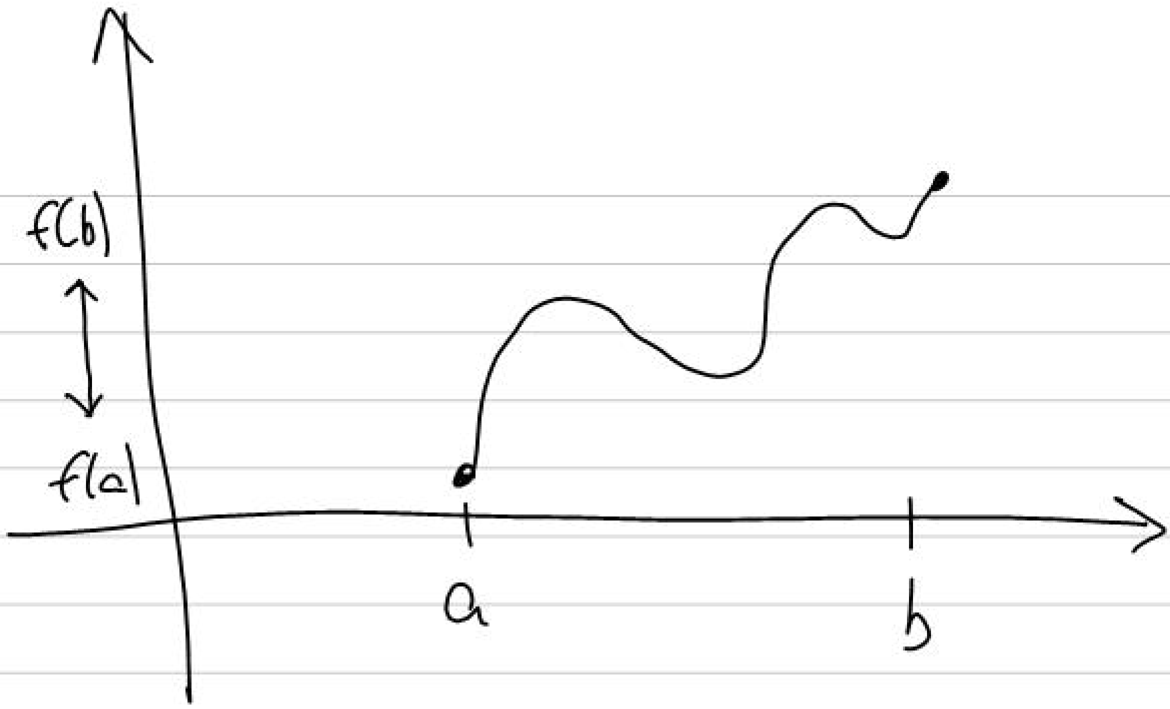
$\tan x$ ei määritelty kohdassa $\frac{\pi}{2}$

$\tan x$ määrjoukko

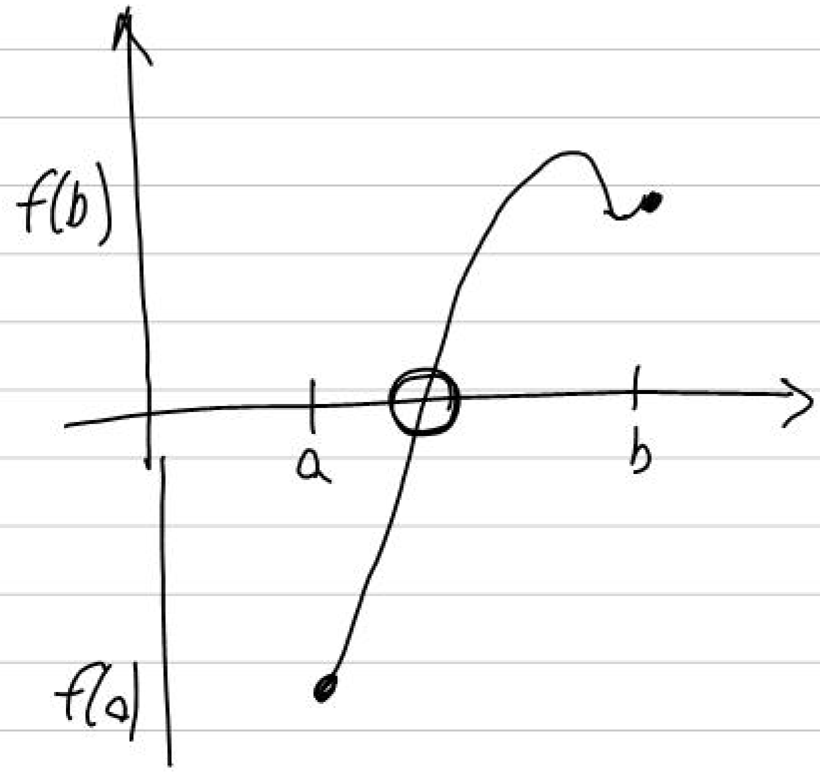
$\mathbb{R} \setminus \{ \frac{\pi}{2} + n\pi$, kun n on kok.luku $\}$

arvojoukko $\mathbb{R}$

$\tan x = \frac{\sin x}{\cos x}$ $\to$ $\cos x \neq 0$?



f(b)
f(a)
a
b



f(b)
f(a)
a
b

(48) $f(x) = x^3 - 5x^2 + 11$

$$\left.\begin{array}{l} f(4) = 4^3 - 5 \cdot 4^2 + 11 = -5 \\ f(5) = 5^3 - 5 \cdot 5^2 + 11 = 56 \end{array}\right\} \text{erimerkkiset!}$$

$\Rightarrow$ yksi 0-kohta välillä $]4,5[$

$$\left.\begin{array}{l} f(-2) = -17 \\ f(0) = 11 \end{array}\right\} \text{0-kohta }]-2,0[$$

$$\left.\begin{array}{l} f(0) = 11 \\ f(2) = -1 \end{array}\right\} \text{0-kohta }]0,2[$$

(49) tarkastele pisteet

$(-2,5)$ $(-1,9)$

$$\underset{\text{kohta } (-2)}{\text{kasvunopeus}} = \frac{\Delta y}{\Delta x} = \frac{4}{1} = 4$$

tangentin kulmakerroin on 4

$(1,8)$ $(0,10)$

$$\frac{\Delta y}{\Delta x} = \frac{2}{-1} = -2$$

pisteeseen $(1,8)$ piirretyn tangentin kulmakerroin on -2

50 $f(x) = 2x + 1$

tangentti piirtyy kuvaajan päälle

eli

sivuaa kuvaajaa!

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

Kulmakerroin 2.

$(a-b)^2 = a^2 - 2ab + b^2$

$$\frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$$

51 $f(x) = x^2$

a. $$\frac{f(x) - f(2)}{x - 2} = \frac{x^2 - 2^2}{x - 2} = \frac{x^2 - 4}{x - 2}$$

raja-arvo $$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} \overset{\text{sij. } x=2}{=} 2 + 2 = 4$$

$f'(2) = 4$

$f(x) = x^2$

erotusosamäärä kohdassa a

$$\frac{f(x)-f(a)}{x-a}$$

erotusosamäärän raja-arvo kohdassa a

$$\lim_{x \to a} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a} \frac{x^2-a^2}{x-a} = \lim_{x \to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}}$$

sij. $x=a$

$$= a + a = 2a$$

$$\boxed{f'(a) = 2a}$$

c. $f'(0) = 2 \cdot 0 = 0$

$f'(1) = 2 \cdot 1 = 2$

$f'(-1) = 2 \cdot (-1) = -2$

DESMOS!

52 $f(x) = -2x^2 + x + 2$

$f(1) = -2(1)^2 + 1 + 2$
$= -2 + 1 + 2$
$= 1$

$$f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1}$$

$$= \lim_{x \to 1} \frac{-2x^2 + x + 2 - 1}{x - 1} = \lim_{x \to 1} \frac{-2x^2 + x + 1}{x - 1}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$-2x^2 + x + 1$ 0-kohdat $\begin{cases} -\frac{1}{2} \\ 1 \end{cases}$

$$= \lim_{x \to 1} \frac{-2(x + \frac{1}{2})\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 1} -2x - 1$$

$\overset{\text{sij. } x=1}{=} -2 \cdot 1 - 1 = -3$

$$f'(1) = -3$$

54 $f(x) = x^3$

$$\boxed{D x^n = n x^{n-1}}$$

$f'(x) = 3x^{3-1} = 3x^2$

55 $f(x) = 2x^2 + 4x - 5$

Summan d.

$$\boxed{D\big(f(x) + g(x)\big) = f'(x) + g'(x)}$$

$f'(x) = \underbrace{2 \cdot 2x^{2-1}} + \underbrace{4} + \underbrace{0}$ $\quad f(x) = 2x^2$

$2 D x^2$

$= 4x + 4$

$$\boxed{D\,kf(x) = kD\,f(x)}$$

56 $f(x) = \frac{1}{2}x^4 - 5x^2$

$f'(x) = \frac{1}{2} \cdot 4x^3 - 5 \cdot 2 \cdot x = 2x^3 - 10x$

57 $f(x) = x^2$

$f'(x) = 2x \qquad f'(1) = 2 \cdot 1 = 2$

58 $f(x) = -2x^2 + x + 2$

$f'(x) = -2 \cdot 2x + 1 = -4x + 1$

$f'(1) = -4 \cdot 1 + 1 = -3$ (Vrt. tehT.52)

60 a. $7x^6$

b. 4

c. $-2x$

d. $x^{-3} \Rightarrow -3x^{-3-1} = -3x^{-4}$

$\frac{1}{x^3} \Rightarrow -\frac{3}{x^4}$

e. $\frac{5}{x^4} = 5\underbrace{x^{-4}} \Rightarrow 5 \cdot (-4) \cdot x^{-4-1} = -20x^{-5}$

$= -\frac{20}{x^5}$

f. Tulon derivaatta!

$(4x-3)(9-x^2)$

$f(x) = 4x - 3$

$g(x) = 9 - x^2$

$D(f(x) \cdot g(x)) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$

$\Rightarrow 4(9-x^2) + (-2x)(4x-3)$

$= 36 - 4x^2 - 8x^2 + 6x = -12x^2 + 6x + 36$

9. $\sqrt{x} \cdot (2-x)$

I тәсіл

$2\sqrt{x} - x\sqrt{x}$

II тәсіл : [illegible] [illegible]

$$\boxed{\sqrt{x} = x^{\frac{1}{2}}}$$

$$\boxed{\sqrt[3]{x} = x^{\frac{1}{3}}}$$

КОНТЕНТ.

[illegible]

тест 1 а) – к)