Circuit Theory and Components

- Circuit analysing and solving methods
  - Ohm's law, KCL, KVL
  - Mesh current, node voltage, superposition and Thevenin methods
- Passive Components
  - Resistors, Capacitors, Inductors, Transformers and Electromechanical components
  - Standard component values

Basics of Electricity

- Electric current is movement of charges
- Every known charge is a multiple of electron's charge
- According to conductivity materials can be divided in three different categories
  - Conductor (metals like gold, copper, silver,…)
  - Dielectric (plastics, ceramics, …)
  - Semiconductor (silicon, germanium,…)

Some Basic Laws

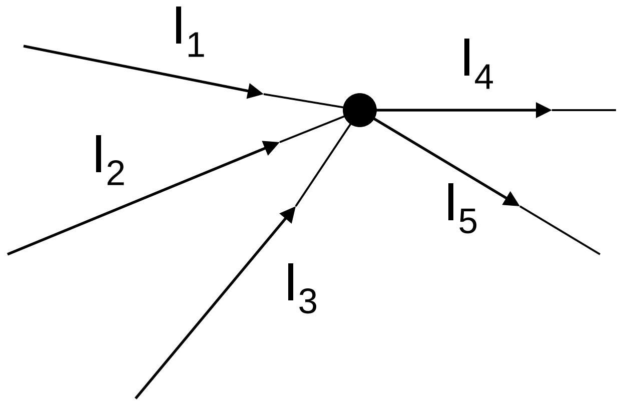
- Resistance $\mathbf{R=\rho l/A}$, R=resistance, ρ=resistivity, l=length
- TC $\mathbf{R_T=R_o(1+\alpha\Delta T)}$, R_T=Resistance in a new temperature, R_o=resistance in the nominal temp., α=TC, ΔT=temp. difference
- Ohms law **U=RI**, U=voltage, I=current
- Current **I=Q/t**, Q=charge, t=time
- Current density **J=I/A**, J=density, A=cross-section area
- Power **P=UI**, P=power, U=voltage, I=current
- Energy **W=Pt**, W=energy, t=time

DC Circuits

- We don´t have any complex impedances (like in AC circuits where inductors and capacitors cause a phase shift). At DC capacitors are like opens and inductors are like short-circuits
- Resistances form real impedances (means no phase shift) and dissipate the power supplied to the circuit.

Kirchhoff's Current Law (KCL)

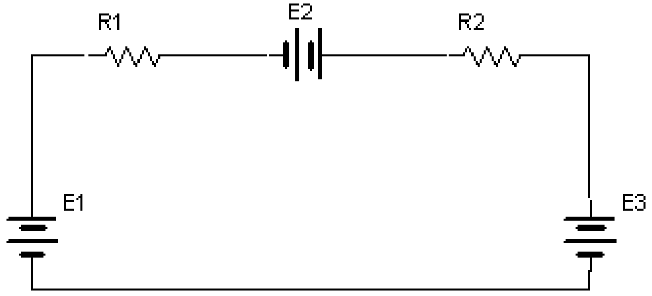
- The sum of incoming currents in a node is equal to the sum of the outcoming currents
- Law can also be formulated so that the sum of currents is zero in a node (if the direction of the currents is the same)



$$I_1 + I_2 + I_3 = I_4 + I_5$$

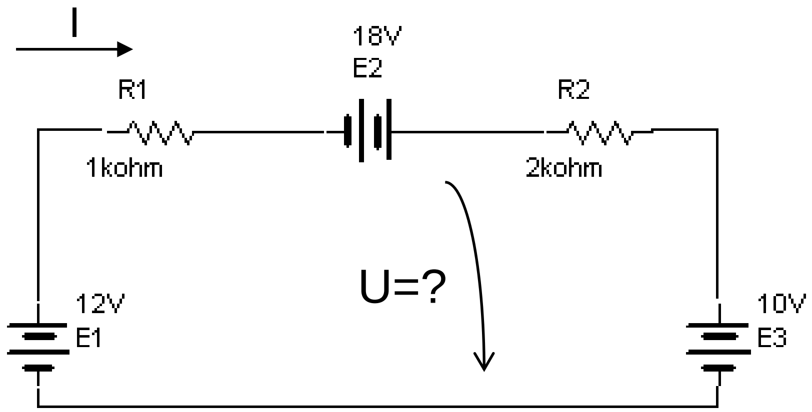
Kirchhoff's Voltage Law (KVL)

•<u>The sum of voltage sources (accumulators, batteries) is equal to voltages losses</u> (Sum of generated voltages is equal to voltage losses across components which don't generate voltages, like resistor, capacitors, inductors etc.)



$$E_1+E_2-E_3=U_{R1}+U_{R2}$$

An Example

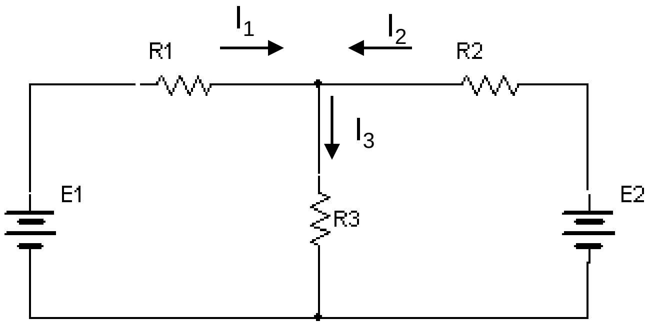


$$E_1+E_2-E_3=(R_1+R_2)I \quad => I=6{,}67mA$$

$$E_3= -R_2I+U \text{ or } E_1+E_2=R_1I+U \quad => U=23{,}3V$$

Circuit Analysis by Using Basic Laws

- Mark branch currents
- Write current equations by applying KCL
- Write loop equations by applying KVL
- Solve currents
- Solve desired voltages (etc.) by using branch currents



$$I_3 = I_1 + I_2$$

$$\begin{cases} E_1 = R_1 I_1 + R_3 I_3 \\ E_2 = R_2 I_2 + R_3 I_3 \end{cases}$$

$$\begin{cases} E_1 = (R_1 + R_3) I_1 + R_3 I_2 \\ E_2 = R_3 I_1 + (R_2 + R_3) I_2 \end{cases}$$

$$\begin{cases} I_1 = \ldots \\ I_2 = \ldots \end{cases}$$

Current Division

$$I = I_{R1} + I_{R2}$$

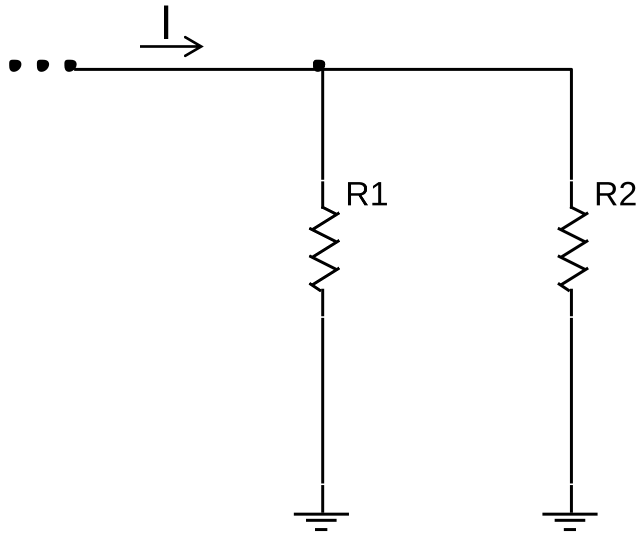
$$I_{R2} = I - I_{R1}$$

$$R_1 I_{R1} = R_2 I_{R2}$$

$$R_1 I_{R1} = R_2(I - I_{R1})$$

$$I_{R1} = IR_2/(R_1+R_2)$$

$$I_{R2} = IR_1/(R_1+R_2)$$



Voltage Division

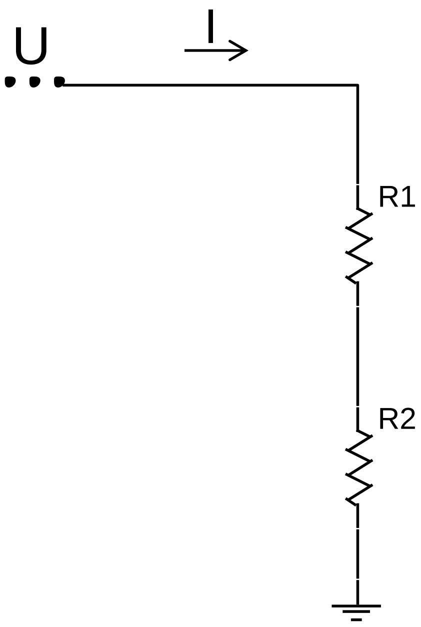
$$U = R_1 I + R_2 I$$

$$U = (R_1 + R_2) I$$

$$I = U/(R_1 + R_2)$$

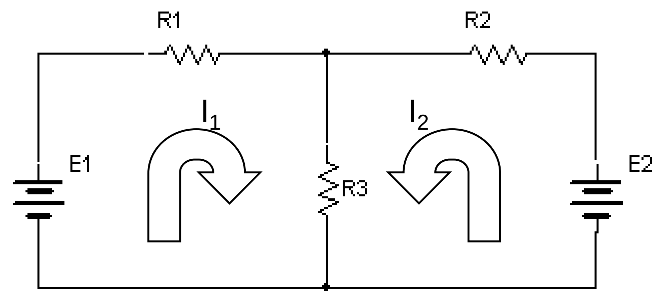
$$U_{R1} = R_1 U/(R_1 + R_2)$$

$$U_{R2} = R_2 U/(R_1 + R_2)$$



Mesh Current Method

- Mark loop currents, one current in each loop
- Write loop equations by using loop currents and applying KVL
- Solve loop currents
- Solve desired voltages (etc.) by using loop currents



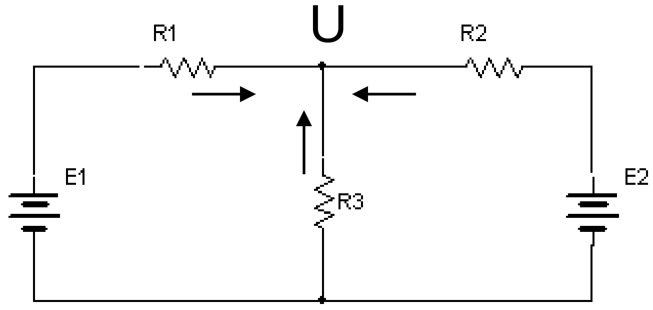
$$\begin{cases} E_1 = R_1 I_1 + R_3(I_1+I_2) \\ E_2 = R_2 I_2 + R_3(I_1+I_2) \end{cases}$$

$$\begin{cases} E_1 = (R_1 + R_3) I_1 + R_3 I_2 \\ E_2 = R_3 I_1 + (R_2 + R_3) I_2 \end{cases}$$

$$\begin{cases} I_1 = \ldots \\ I_2 = \ldots \end{cases}$$

Node Voltage Method

- Mark node voltages
- Write node equations by using marked node voltages (sum of branch currents is zero)
- Solve node voltages
- Find desired currents (etc.) by using solved node voltages



$(E_1- U)/R_1+(0-U)/R_3+ (E_2- U)/R_2=0$

$U=(E_1/R_1 +E_2/R_2)/(1/R_1 +1/R_2 +1/R_3)$

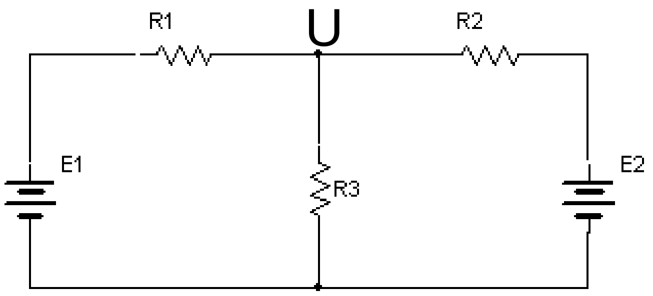
$I_{R1}=(E_1- U)/R_1$

$I_{R2}=(E_2- U)/R_2$

$I_{R2}=(0-U)/R_2$

Superposition Method

- The circuit is analysed source by source. Other current sources are opened and other voltage sources are short-circuited.
- Find the effect of each source
- Combine partial solutions to get a final value for desired current or voltage



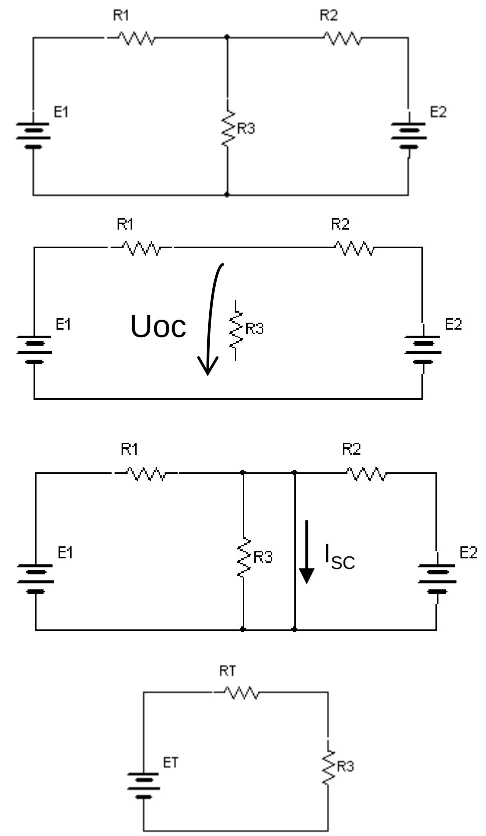
$U' = E_1 (R_2 // R_3) / (R_1 + R_2 // R_3)$

$U'' = E_2 (R_1 // R_3) / (R_2 + R_1 // R_3)$

$U = U' + U''$

Thevenin Method

- Find value for the Thevenin source by disconnecting the circuit element under study (E_T=Uoc)
- Find short-circuit current across the element under study (Isc)
- Find R_T=Uoc/Isc
- Draw simplified circuit including the Thevenin source with its internal resistance
- Find desired voltage or current



$U_{OC}=E_1-R_1(E_1-E_2)/(R_1+R_2)$

$I_{SC}=E_1/R_1+ E_2//R_2$, $E_T=U_{OC}$, $R_T=U_{OC}/I_{SC}$

$I_{R3}=E_T/(R_T+R_3)$

Resistances in the Series Connection

$U_1=R_1I$, $U_2=R_2I$, $U_3=R_3I$

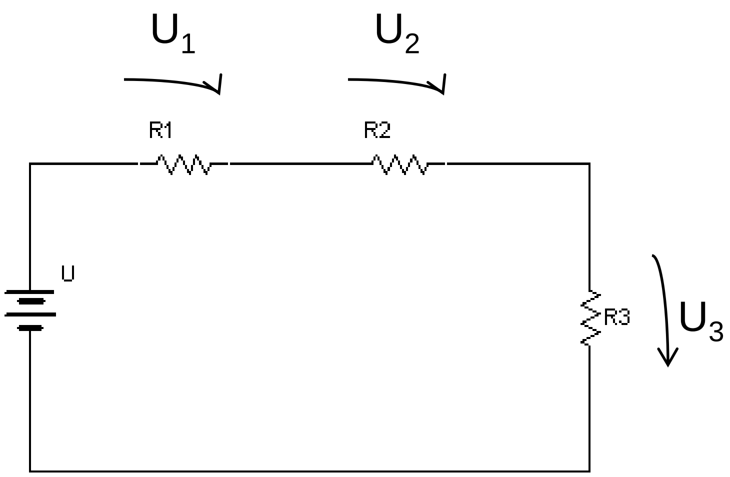
$U=U_1+U_2+U_3$

$U=(R_1+R_2+R_3)I$

$U=RI$,

$R=R_1+R_2+R_3$

If N resistances $$R=\sum_{i=1}^{N} R_i$$



Resistances in the Parallel Connection

$I_{R1}=U/R_1$, $I_{R2}=U/R_2$, $I_{R3}=U/R_3$

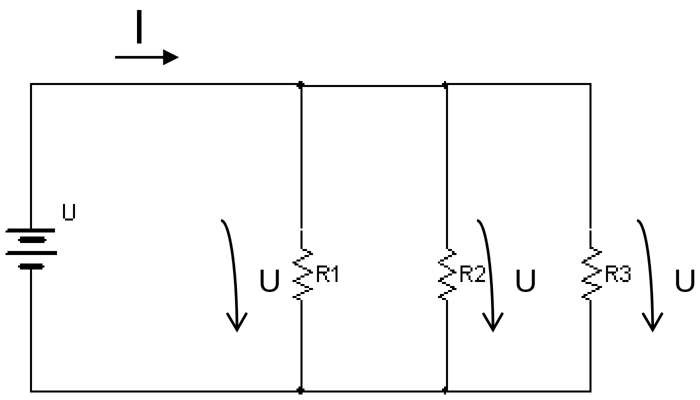
$I = I_{R1}+I_{R2}+I_{R3}$

$I= U/R_1+ U/R_2 + U/R_3$

$I=(1/R_1+ 1/R_2 + 1/R_3)U$

$R= (1/R_1+ 1/R_2 + 1/R_3)^{-1}$

If N resistances $$R = (\sum_{i=1}^{N} \frac{1}{R_i})^{-1}$$



Triangle-to-Star Conversion

$R_{AB}=R_1+R_2=R_B//(R_A+R_C)$

$R_{BC}=R_2+R_3=R_C//(R_A+R_B)$

$R_{CA}=R_1+R_3=R_A//(R_B+R_C)$

Triangle-to-star

$R_1=R_AR_B/(R_A+R_B+R_C)$

$R_2=R_BR_C/(R_A+R_B+R_C)$

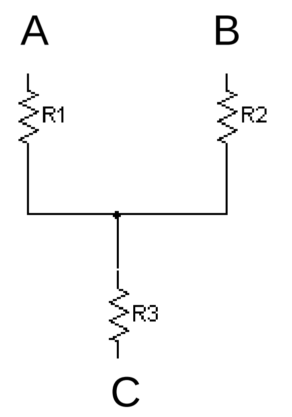
$R_3=R_AR_C/(R_A+R_B+R_C)$

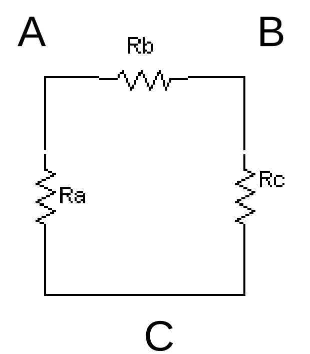
Star-to-triangle

$R_A=(R_1R_2+R_1R_3+R_2R_3)/R_2$

$R_B= (R_1R_2+R_1R_3+R_2R_3)/R_3$

$R_C= (R_1R_2+R_1R_3+R_2R_3)/R_1$





Gain and Attenuation

Voltage gain $A_u = u_o/u_i$

Current gain $A_i = i_o/i_i$

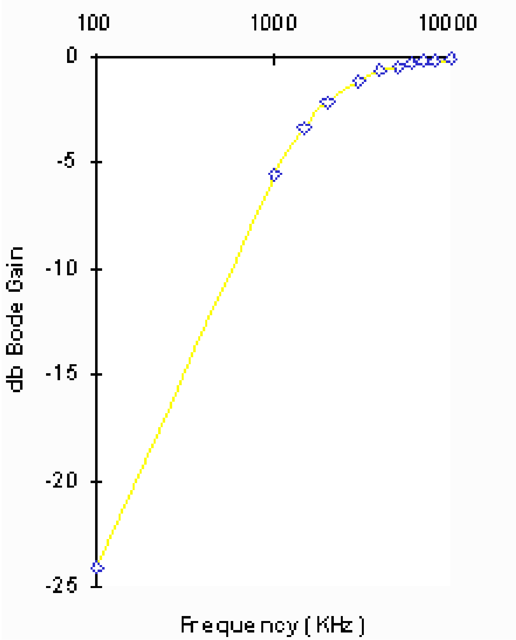
Power gain $A_p = P_o/P_i$

<u>In decibels</u>

$A_u = 20\log(u_o/u_i)$

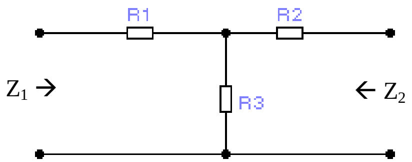
$A_i = 20\log(i_o/i_i)$

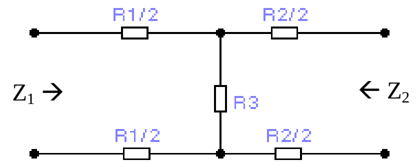
$A_p = 10\log(P_o/P_i)$!!!!!



Attenuators

$K = P_1 / P_2$ (attenuation !)

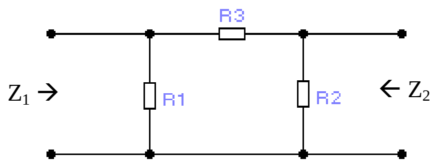


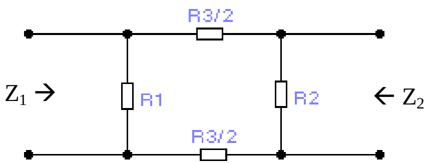


$R1 = [Z_1 (K + 1) - 2 (KZ_1Z_2)^{1/2}] / (K - 1)$

$R2 = [Z_2 (K + 1) - 2 (KZ_1Z_2)^{1/2}] / (K - 1)$

$R3 = [2(KZ_1Z_2)^{1/2}] / (K - 1)$





$R1 = [(K - 1) Z_1 (Z_2)^{1/2}] / [(K + 1) (Z_2)^{1/2} - 2 (K Z_1)^{1/2}]$

$R2 = [(K - 1) Z_2 (Z_1)^{1/2}] / [(K + 1) (Z_1)^{1/2} - 2 (K Z_2)^{1/2}]$

$R3 = [(K - 1)(Z_1Z_2/K)^{1/2}] / 2$

Resistor Classification by Function

Fixed resistors (resistor has a fixed value, can’t be adjusted)

Variable resistors (resistor value can be adjusted, trimmers, potentiometers)

Resistor Classification by Material

Carbon

- –Carbon film resistors
- –Solid resistors

Metal

- –Metal film resistors
- –Metal oxide film resistors

Metal glazed types *

- –Chip resistors
- –Chip network resistors
- –Network resistors

* These are made by sintering on alumina or other substrates mixtures of metal or metal oxide with glass.)

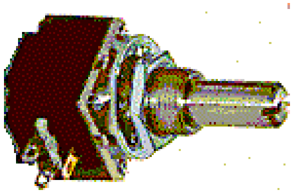
Variable Resistors

Others

- –Thermistors (NTCs and PTCs)
- –Varistors (Voltage Dependent Resistors)
- –LDRs (Light Dependent Resistors)
- –Wire wound resistors (power and accurate)
- –Enamel resistors
- –Cement resistors

Variable Resistors

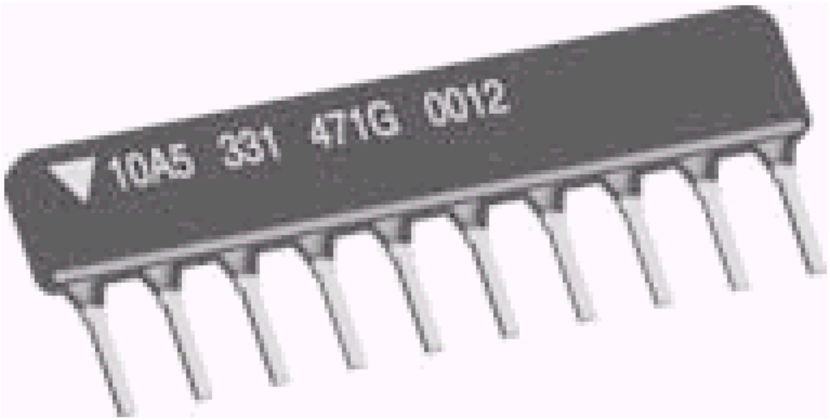
- Potentiometers (designed for heavy use)



- Trimmers (usually for factory settings)

Resistor Networks

- Used where several resistors are needed. For example in computer buses are needed lots of pull-up resistances.
- There are available several kinds of packages and resistor configurations on the market

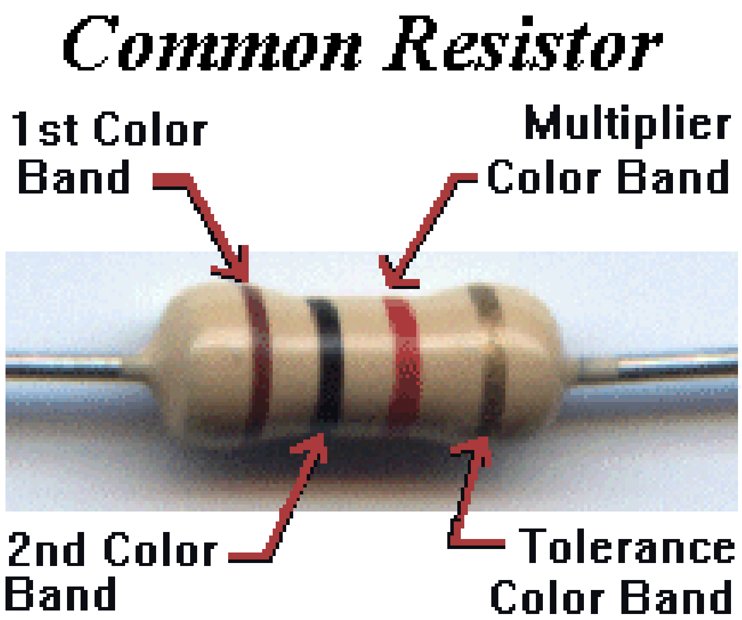


Standard Resistor Values

RESISTOR STANDARD DECADE VALUES

E192	E96	E192	E96	E192	E96	E192	E96	E192	E96
.1% .25% .5%	1%	.1% .25% .5%	1%	.1% .25% .5%	1%	.1% .25% .5%	1%	.1% .25% .5%	1%
10.0	10.0	16.9	16.9	28.7	28.7	48.1		81.6	
10.1		17.2		29.1		48.7	48.7	82.5	82.5
10.2	10.2	17.4	17.4	29.4	29.4	49.3		83.5	
10.4		17.6		29.8		49.9	49.9	84.5	84.5
10.5	10.5	17.8	17.8	30.1	30.1	50.5		85.6	
10.6		18.0		30.5		51.1	51.1	86.6	86.6
10.7	10.7	18.2	18.2	30.9	30.9	51.7		87.6	
10.9		18.4		31.2		52.3	52.3	88.7	88.7
11.0	11.0	18.7	18.7	31.6	31.6	53.0		89.8	
11.1		18.9		32.0		53.6	53.6	90.9	90.9
11.3	11.3	19.1	19.1	32.4	32.4	54.2		92.0	
11.4		19.3		32.8		54.9	54.9	93.1	93.1
11.5	11.5	19.6	19.6	33.2	33.2	55.6		94.2	
11.7		19.8		33.6		56.2	56.2	95.3	95.3
11.8	11.8	20.0	20.0	34.0	34.0	56.9		96.5	
12.0		20.3		34.4		57.6	57.6	97.6	97.6
12.1	12.1	20.5	20.5	34.8	34.8	58.3		98.8	
12.3		20.8		35.2		59.0	59.0		
12.4	12.4	21.0	21.0	35.7	35.7	59.7		E24	E12
12.6		21.3		36.1		60.4	60.4	2%, 5%	10%
12.7	12.7	21.5	21.5	36.5	36.5	61.2		10.0	10.0
12.9		21.8		37.0		61.9	61.9	11.0	
13.0	13.0	22.1	22.1	37.4	37.4	62.6		12.0	12.0
13.2		22.3		37.9		63.4	63.4	13.0	
13.3	13.3	22.6	22.6	38.3	38.3	64.2		15.0	15.0
13.5		22.9		38.8		64.9	64.9	16.0	
13.7	13.7	23.2	23.2	39.2	39.2	65.7		18.0	18.0
13.8		23.4		39.7		66.5	66.5	20.0	
14.0	14.0	23.7	23.7	40.2	40.2	67.3		22.0	22.0
14.2		24.0		40.7		68.1	68.1	24.0	
14.3	14.3	24.3	24.3	41.2	41.2	69.0		27.0	27.0
14.5		24.5		41.7		69.8	69.8	30.0	
14.7	14.7	24.9	24.9	42.2	42.2	70.6		33.0	33.0
14.9		25.2		42.7		71.5	71.5	36.0	
15.0	15.0	25.5	25.5	43.2	43.2	72.3		39.0	39.0
15.2		25.8		43.7		73.2	73.2	43.0	
15.4	15.4	26.1	26.1	44.2	44.2	74.1		47.0	47.0
15.6		26.4		44.8		75.0	75.0	51.0	
15.8	15.8	26.7	26.7	45.3	45.3	75.9		56.0	56.0
16.0		27.1		45.9		76.8	76.8	62.0	
16.2	16.2	27.4	27.4	46.4	46.4	77.7		68.0	68.0
16.4		27.7		47.0		78.7	78.7	75.0	
16.5	16.5	28.0	28.0	47.5	47.5	79.6		82.0	82.0
16.7		28.4				80.6	80.6	91.0	

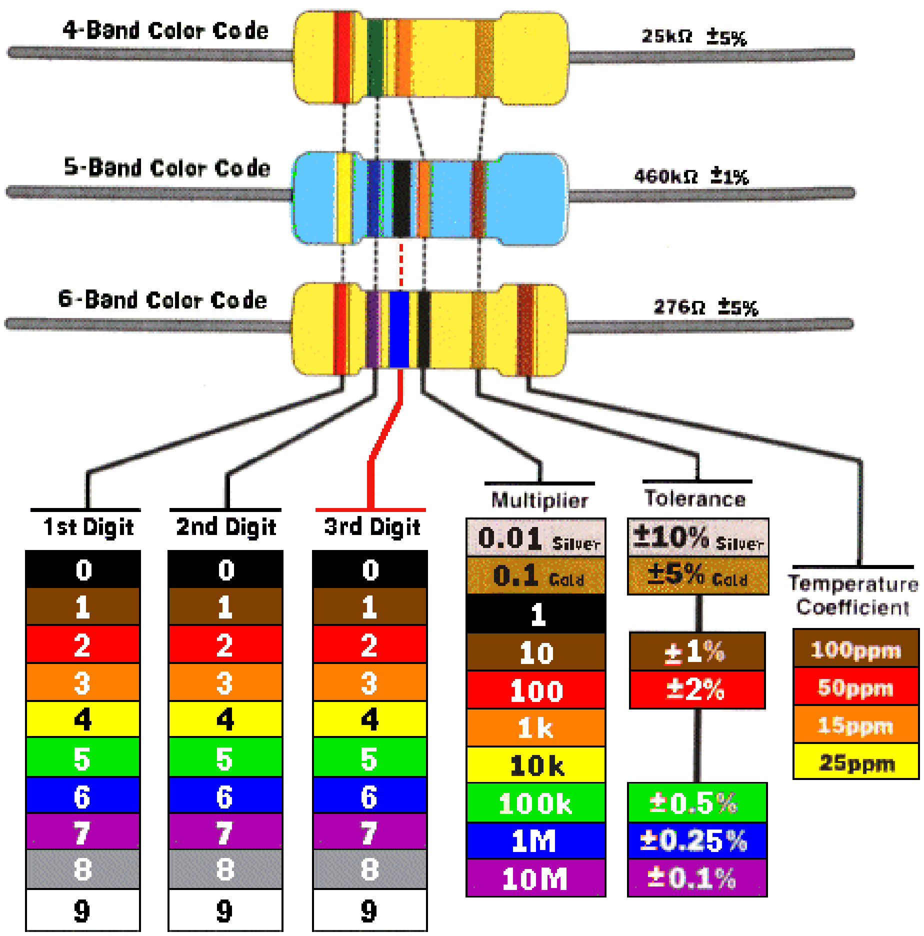
Resistor Colour Codes



Resistor Color Code Chart

1st. & 2nd Color Band	Digit it Represents	-----Multiplier-----
BLACK	0	X1
BROWN	1	X10
RED	2	X100
ORANGE	3	X1,000 or 1K
YELLOW	4	X10,000 or 10K
GREEN	5	X100,000 or 100K
BLUE	6	X1,000,000 or 1M
VIOLET	7	Silver is divide by 100
GRAY	8	Gold is divide by 10
WHITE	• 9	• Tolerances • Gold= 5% • Silver=10% • None=20%

Resistor Colour Codes

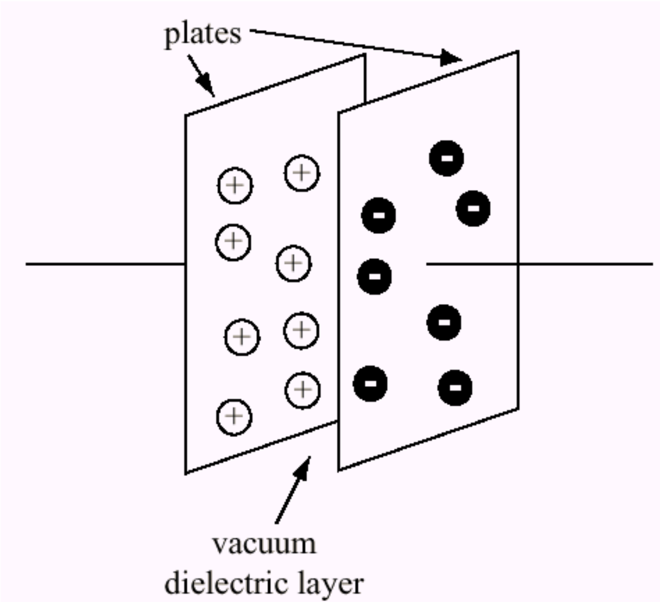


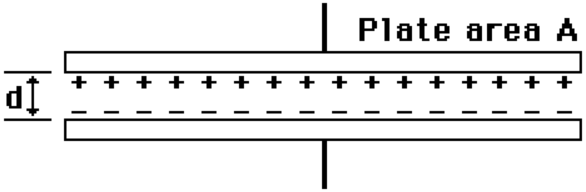
Resistor Colour Codes

Color	1st & 2nd Significant Figures	Multiplier	Tolerance
Black	0	1	--
Brown	1	10	±1%
Red	2	100	±2%
Orange	3	1,000	±3%
Yellow	4	10,000	±4%
Green	5	100,000	--
Blue	6	1,000,000	--
Violet	7	10,000,000	--
Gray	8	100,000,000	--
White	9	--	--
Gold	--	0.1	±5%
Silver	--	0.01	±10%
No Color	--	--	±20%

Capacitor Structure

- $C=\varepsilon_0\varepsilon_r A/d$,
  - C=capacitance
  - ε_0=vacuum permittivity
  - ε_r=relative permittivity
  - A=plate area
  - d =distance of plates
- Q=CU,
  - Q=charge
  - U=voltage
- Q=It,
  - t=time
  - I=current





Capacitors in the Series Connection

$U_1=Q/C_1$, $U_2=Q/C_2$, $U_3=Q/C_3$

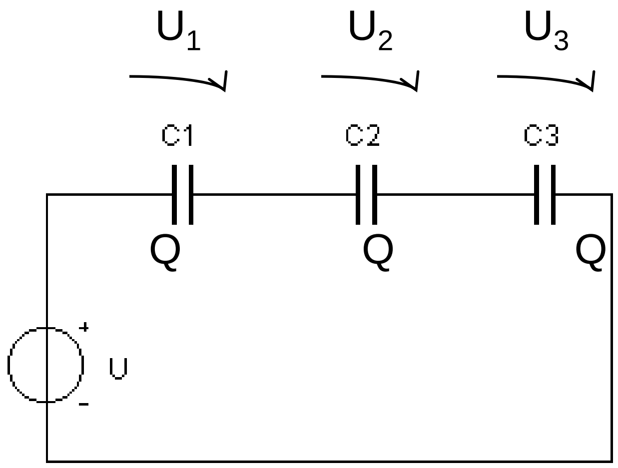
$U = U_1+U_2+U_3$

$U= Q/C_1+ Q/C_2 + Q/C_3$

$U=(1/C_1+ 1/C_2 + 1/C_3)Q$

$C= (1/C_1+ 1/C_2 + 1/C_3)^{-1}$

If N capacitances $$C = (\sum_{i=1}^{N} \frac{1}{C_i})^{-1}$$



Capacitors in the Parallel Connection

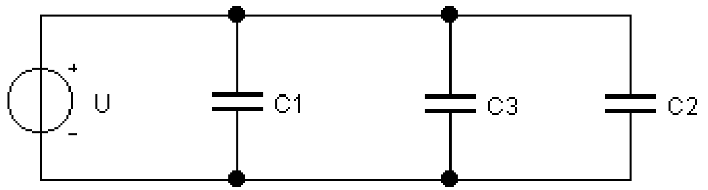
$Q_1=C_1U$, $Q_2=C_2U$, $Q_3=C_3U$

$Q=Q_1+Q_2+Q_3$

$Q=(C_1+C_2+C_3)U$

$Q=CU$,

$C=C_1+C_2+C_3$



If N capacitances

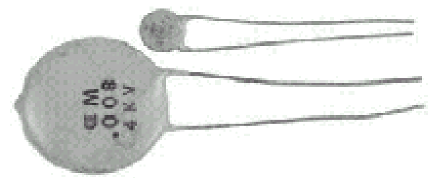
$$C=\sum_{i=1}^{N} C_i$$

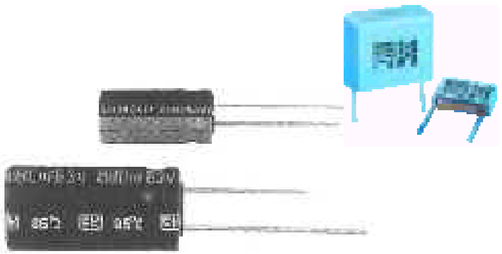
Capacitor Classification

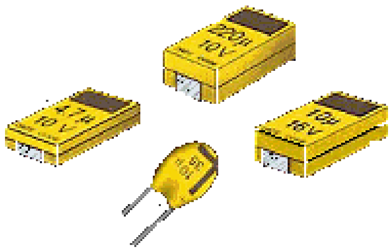
Non-polarized fixed capacitors

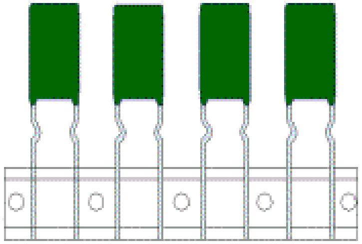
Polarized fixed capacitors

Variable capacitors









Capacitor Classification by Dielectric Material

Film Capacitors

- –Polyester (Mylar*)
- –Polycarbonate
- –Polystyrene
- –Polypropylene

* Dupont trade mark

Property	Polypropylene	Polyester (Mylar)	Polycarbonate	Polystyrene
Dielectric Constant	2.3	3.2	3.0	2.5
Density	0.905	1.395	1.20	0.95
Area Factor $in^2/lb/mil$	30.600	19.800	23.100	26.500
Temperature Rating (for capacitors) °C	105	125	125	85
Dissipation Factor % at 10^6 Hz	0.02	1.6	1.0	0.01
Flatness of Electrical Properties	Excellent	Fair	Good	Excellent
Dielectric Strength V/mil at 1 mil	7.000	7.000	4.000	5.000
Tensile Strength lb/in^2	28.000	30.000	8.000	5.000
Moisture Sensitivity	None	Small	Moderate	None

Capacitor Classification by Dielectric Material

Electrolytic Capacitors

–Aluminum

–Tantalum

•An aluminum electrolytic capacitor consist of cathode aluminum foil, capacitor paper (electrolytic paper), electrolyte and an aluminum oxide layer, which acts as dielectric. These capacitors have excellent withstand voltage per thickness.

•Tantalum electrolytic capacitors are suitable for applications where volumetric efficiency, stable electrical parameters and high reliability is needed

COMPARISON OF CAPACITOR DIELECTRIC CONSTANTS	
DIELECTRIC	**K DIELECTRIC CONSTANT**
Air or Vacuum	1.0
Paper	2.0 - 6.0
Plastic	2.1 - 6.0
Mineral Oil	2.2 - 2.3
Silicone Oil	2.7 - 2.8
Quartz	3.8 - 4.4
Glass	4.8 - 8.0
Porcelain	5.1 - 5.9
Mica	5.4 - 8.7
Aluminum Oxide	8.4
Tantalum Pentoxide	**26**
Ceramic	12 - 400,000

Capacitor Classification by Dielectric Material

Others

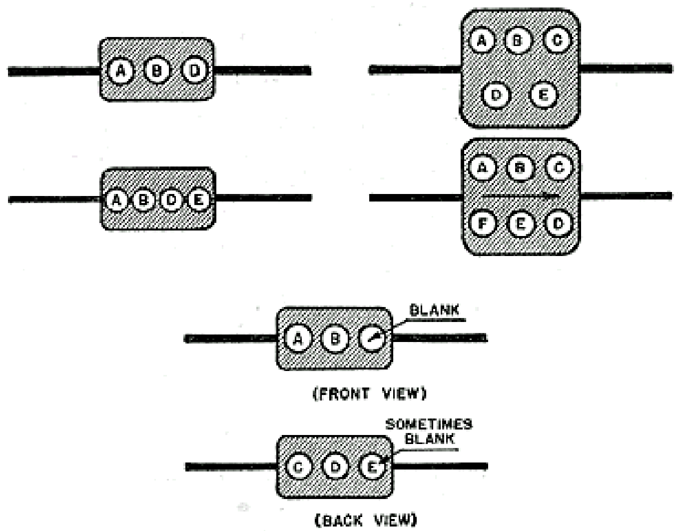
- Paper
- Glass
- Porcelain
- Mica

Dielectric	Construction	Capacitance Range
Air	Meshed plates	10 - 400 pF
Mica	Stacked Sheets	10 - 5000 pF
Paper	Rolled foil	0.001 - 1 μF
Ceramic	Tubular	0.5 - 1600 pF
Disk	Tubular	0.002 - 0.1 μF
Electrolytic	Aluminum	5 - 1000 μF
Tantalum	Aluminum	0.01 - 300 μF

Ceramic Capacitor Markings

VALUE	MARKING	VALUE	MARKING	VALUE	MARKING
1pf; 3pf; 5pf	1; 3; 5	2.7 , 3 or 3.3 pF can be interchanged with each other. 4.7 or 5 pF can be interchanged with each other.			
10 pf	10 or 100	0.001 uF	102	0.10 uF	104
12 pf	12 or 120	0.0012uF (1200pf)	122	0.12 uF	124
15 pf	15 or 150	0.0015uF	152	0.15 uf	154
18 pf	18 or 180	0.0018 uF (1800pf)	182	0.18 uF	184
22 pf	22 or 220	0.0022uF	222	0.22 uF	224
27 pf	27 or 270	0.0027uF	272	0.27 uF	274
33 pf	33 or 330	0.0033 uF	332	0.33 uF	334
39 pf	39 or 390	0.0039uF	392	0.39 uF	394
47 pf	47 or 470	0.0047uF	472	0.47 uF	474
58 pf	58 or 580	0.0056uF	562	0.56 uF	564
68 pf	68 or 680	0.0068uF	682	0.68 uF	684
82 pf	82 or 820	0.0082uF	822	0.82 uF	824

Mica Capacitor Colour codes



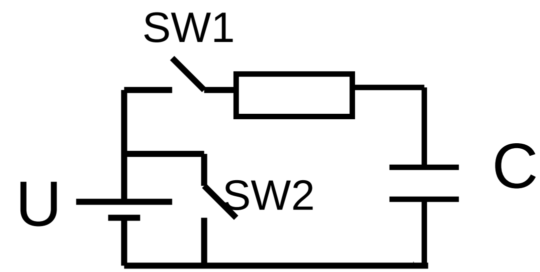
Dot Color	Significant Figures			Decimal Multiplier	Capacitive Tolerance	DC Test Voltage	Dot Color
	A	B	C	D	E	F	
Black	0	0	0	—	—	—	Black
Brown	1	1	1	10	± 1%	100	Brown
Red	2	2	2	100	± 2%	200	Red
Orange	3	3	3	1,000	± 3%	300	Orange
Yellow	4	4	4	10,000	± 4%	400	Yellow
Green	5	5	5	100,000	± 5%	500	Green
Blue	6	6	6	1,000,000	± 6%	600	Blue
Violet	7	7	7	10,000,000	± 7%	700	Violet
Gray	8	8	8	100,000,000	± 8%	800	Gray
White	9	9	9	1,000,000,000	± 9%	900	White
Gold	—	—	—	0.1	± 5%	1,000	Gold
Silver	—	—	—	0.01	± 10%	2,000	Silver
No Color	—	—	—	—	± 20%	500	No Color

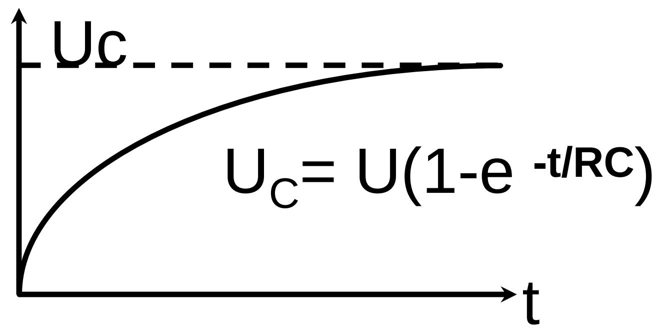
Charging of a Capacitor

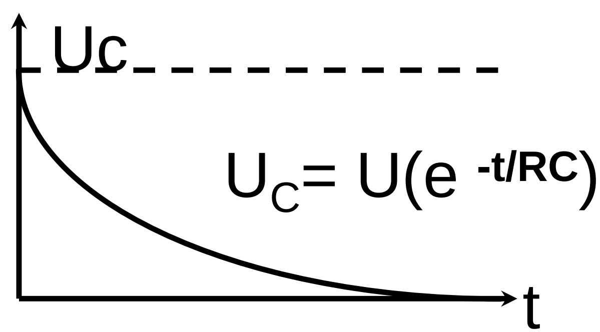
C has initially no charge and SW1 and SW2 are open

If SW1 is then closed C will be charged according to the curve (just below the circuit)

After that when C has been charged and if SW2 is closed, C will be discharged according to the curve on the bottom corner of the this slide.

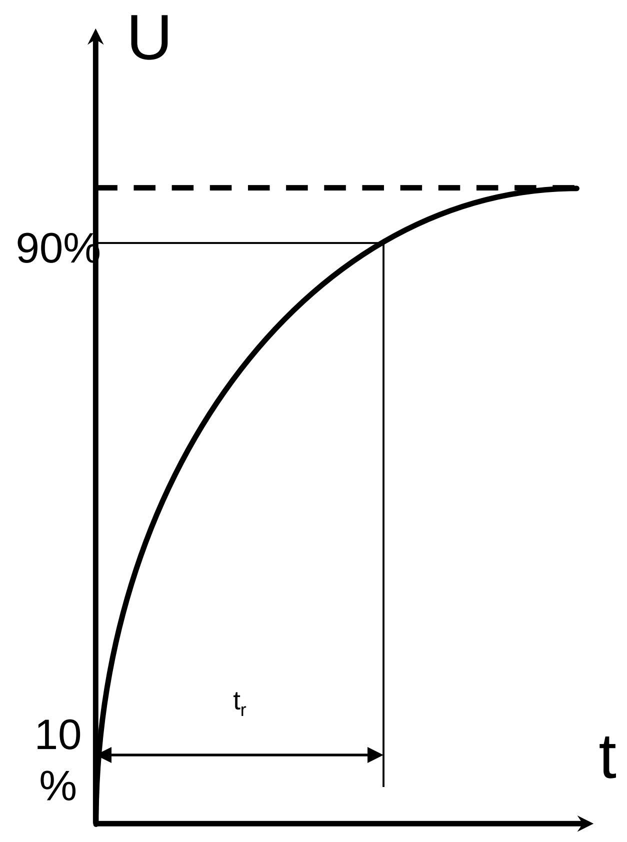






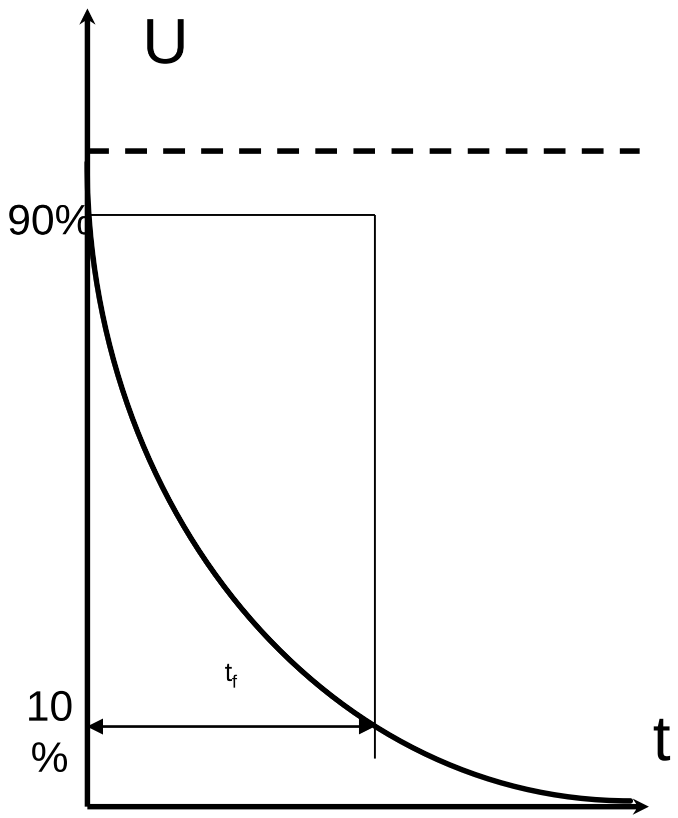
Signal Rise Time

- Usually input and stray capacitances cause that a signal has a certain rise time
- Rise time t_r is the time needed from the 10% point to the 90 % point of signal (during the state change from low to high)



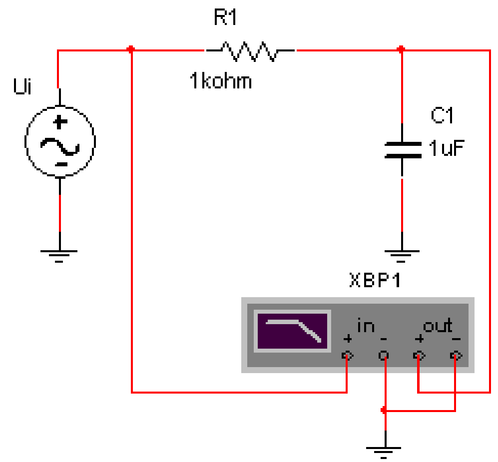
Signal Fall Time

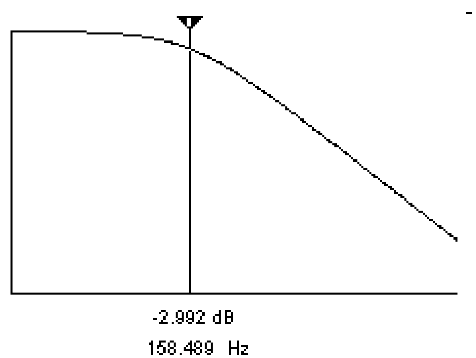
- Usually input and stray capacitances cause that a signal has a certain fall time
- Fall time t_f is the time needed from the 10% point to the 90 % point of signal (during the state change from low to high)



Simple RC Low Pass Filter

- $f=1/(2\pi RC)$
- Can be used in applications where a simple, low cost filter is needed.

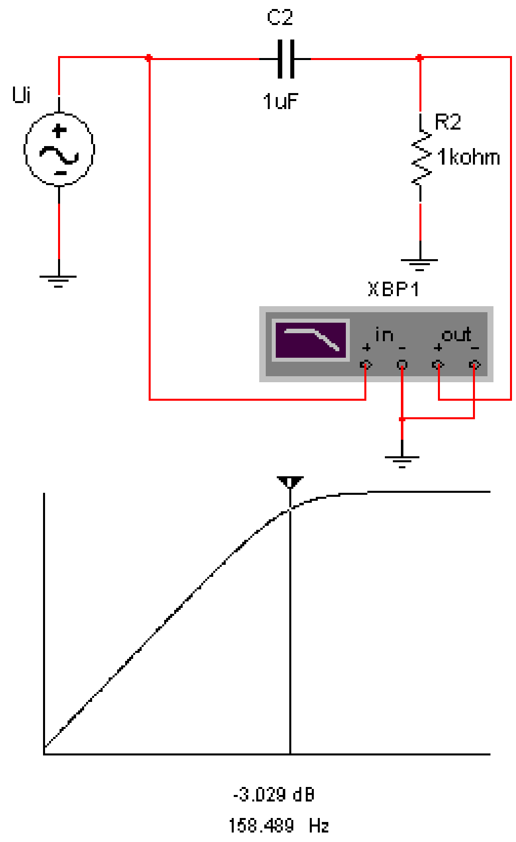




Simple RC High Pass Filter

$f=1/(2\pi RC)$

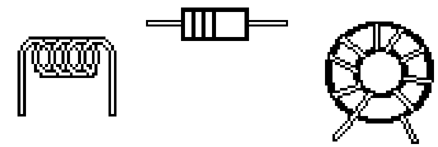
- Can be used in applications where a simple, low cost filter is needed.



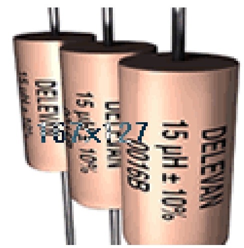
Inductors

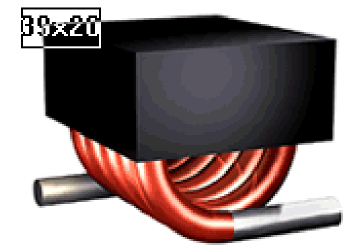
- Coil

$$L = \frac{\mu_0 \mu_r N^2 A}{l} = \frac{\mu_0 \mu_r N^2 \pi d^2}{4l}$$



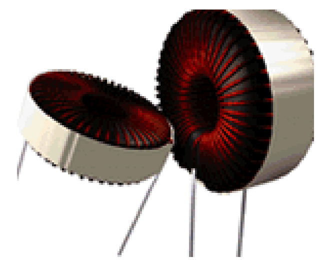
  - L=Inductance
  - A=Area
  - N=Number of turns
  - μ_0=Permeability of vacuum
  - μ_r=Relative permeability (of core)
  - l=length of coil
  - d=diameter of coil





- Toroid

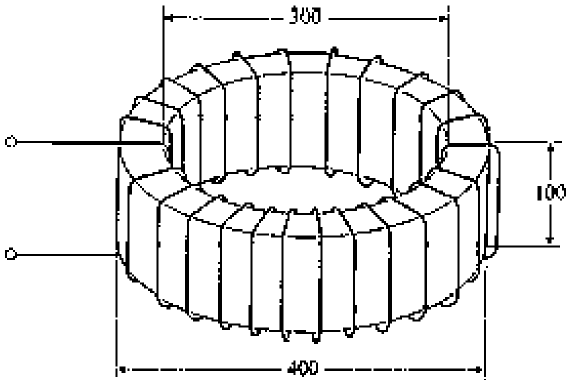
$$L = \frac{\mu_0 \mu_r N^2 r^2}{2R}$$



  - r=radius of winding
  - R=average radius of toroid

Reluctance

- Reluctance =magnetic resistance $=R_m= l/(\mu_0\mu_r A)$, l=length of magnetic circuit
- Reluctances in the series connection $R_m=R_{m1}+R_{m2}$
- In air-gapped magnetic circuits reluctances can be used to make analysis easier



$$L = \frac{N^2}{R_m}$$

Use of Commercial Cores

$$N = \sqrt{\frac{L}{A_L}}$$

N=Number of turns

A_L=Design parameter of a core

L=Inductance

$$d_w = \sqrt{\frac{A_E}{N}}$$

d_w=diameter of wire

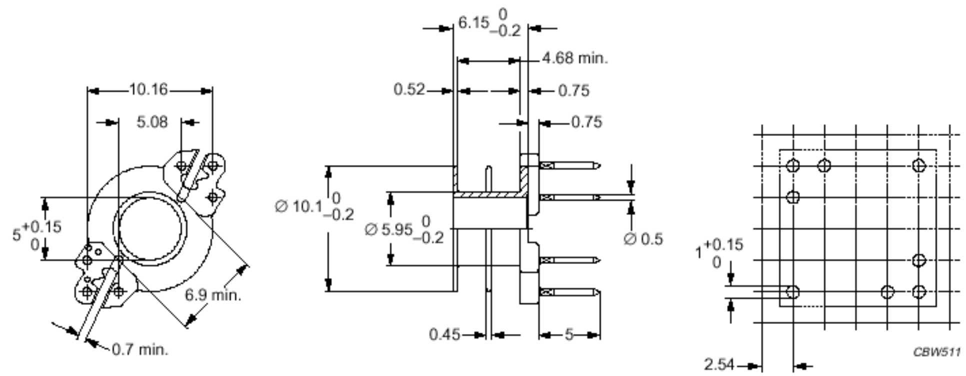
A_E=effective winding area

$$l_w = N l_N$$

l_w=Length of wire needed

l_N=average length of turn

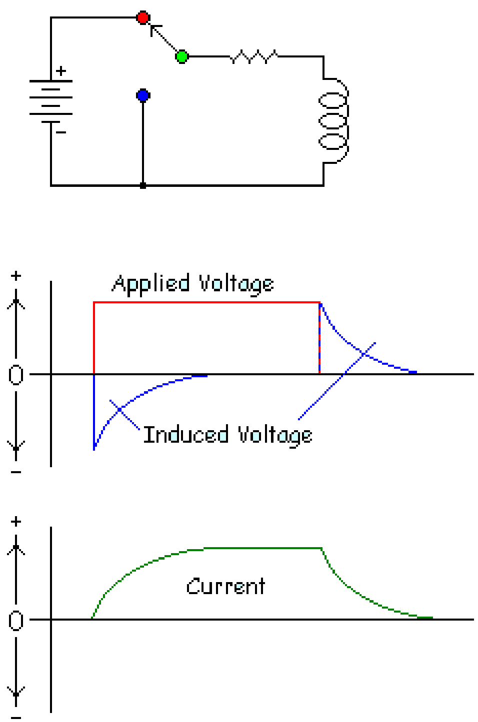
GRADE	A_L (nH)	μ_e	AIR GAP (μm)	TYPE NUMBER (WITH NUT)	TYPE NUMBER (WITHOUT NUT)
3D3	40 ±3%	≈32	≈990	RM5-3D3-E40/N	RM5-3D3-E40
	63 ±3%	≈51	≈540	RM5-3D3-E63/N	RM5-3D3-E63
	100 ±3%	≈80	≈300	RM5-3D3-E100/N	RM5-3D3-E100
	800 ±25%	≈640	≈0	–	RM5-3D3
3H3	160 ±3%	≈129	≈180	RM5-3H3-A160/N	RM5-3H3-A160
	250 ±3%	≈201	≈110	RM5-3H3-A250/N	RM5-3H3-A250
	315 ±3%	≈253	≈80	RM5-3H3-A315/N	RM5-3H3-A315
	400 ±5%	≈321	≈60	RM5-3H3-A400/N	RM5-3H3-A400
	1650 ±25%	≈1310	≈0	–	RM5-3H3



NUMBER OF SECTIONS	NUMBER OF PINS	PIN POSITIONS USED	WINDING AREA (mm^2)	WINDING WIDTH (mm)	AVERAGE LENGTH OF TURN (mm)	TYPE NUMBER
1	4	all	9.5	4.8	25	CSV-RM5-1S-4P
2	4	all	2 × 4.35	2 × 2.2	25	CSV-RM5-2S-4P

Effect of Inductance

- Induced voltage U_L=LdI/dt
- Inductance tries to oppose the changes caused by switching voltage on and off
- Constant DC voltage sees an inductor like “normal” wire

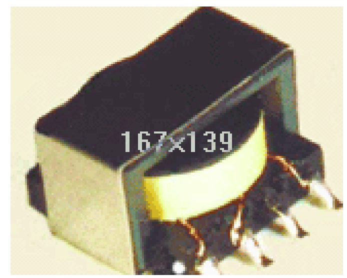


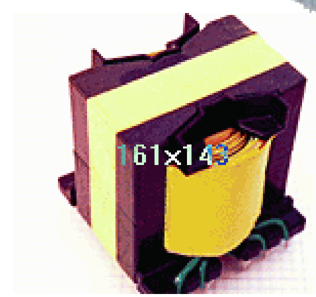
Transformers

•Transformers can be used for isolation

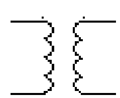
•Transformers are used for voltage amplitude reduction in power supplies

•Matching is also one application for transformers





$$\frac{U_1}{U_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

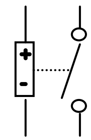
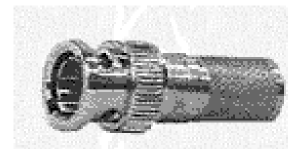
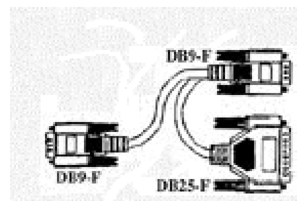


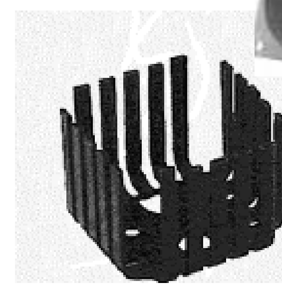
$$\frac{Z_1}{Z_2} = \left(\frac{N_1}{N_2}\right)^2$$

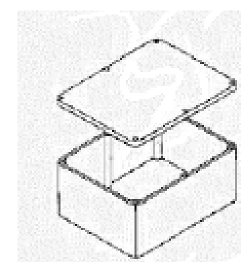
Electromechanical and Mechanical Components

- Relays
- Connectors (BNC…)
- Cables (Coaxial…)
- Boxes
- Heat sink
- Fans
- Racks
- PCBs

…

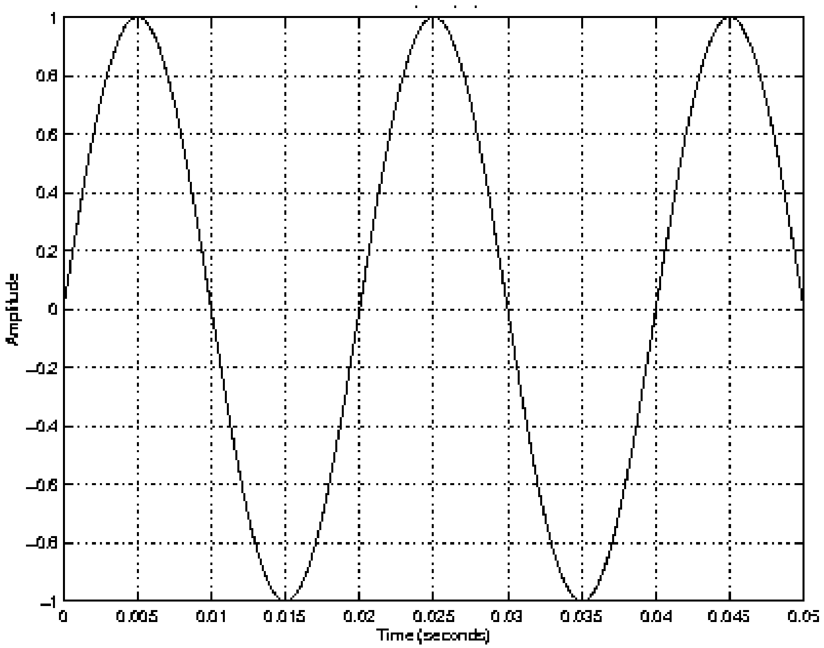




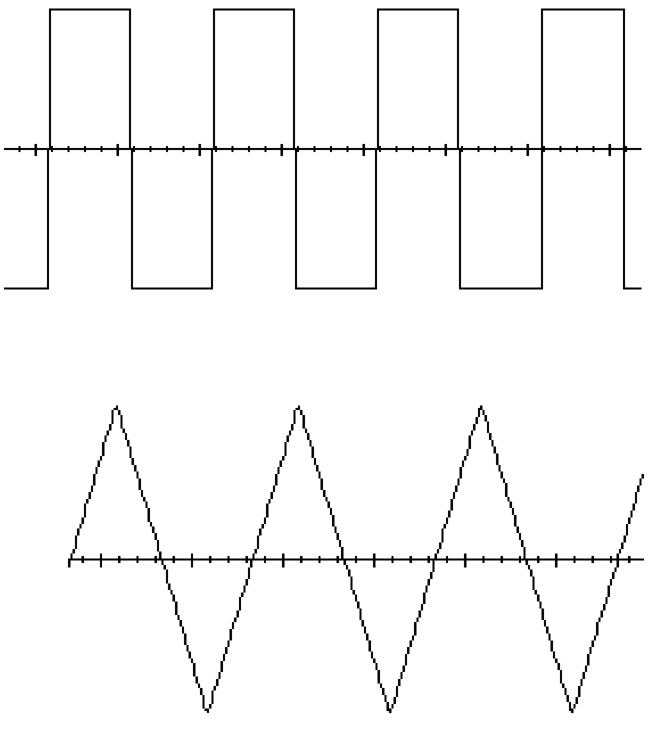

Sinusoidal Signal

- $u(t)=A\sin\omega t$,
  - A=amplitude
  - t=time
  - ω=angular frequency
- $\omega=2\pi f=2\pi/T$,
  - f=frequency
  - T=period of signal
- RMS-value=A/sqrt(2) (for sinusoidal signal)



Other Common AC Signals

- In a pure sinusoidal signal we have only a single frequency
- Pure triangle and rectangular AC signals have infinite number of frequencies



AC Circuits

- DC analysis methods are valid for AC circuits, too. KCL and KVL can be used. Main difference is that instead of resistance we have a more general term, <u>impedance</u>, and instead of real numbers, we use <u>complex numbers</u> in order to find out phase shifts between different signals
- Power is dissipated in resistors. Inductors and capacitors store power but do not dissipate that.

A Little Bit Complex Arithmetic

$$Z = a + jb$$

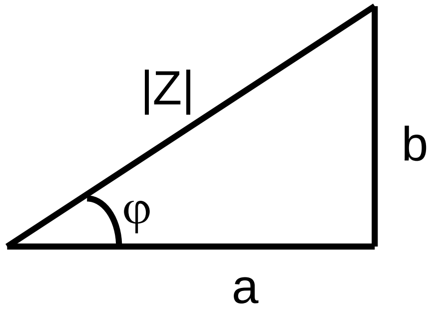
$$|Z| = \sqrt{a^2 + b^2}$$

$$\varphi = \arctan\frac{b}{a}$$

$$Z = \frac{a + jb}{c + jd}$$

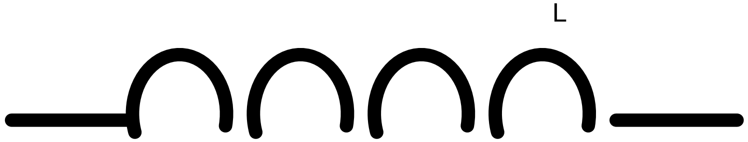
$$|Z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{c^2 + d^2}}$$

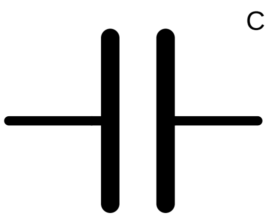
$$\varphi = \arctan\frac{b}{a} - \arctan\frac{d}{c}$$



AC Impedances

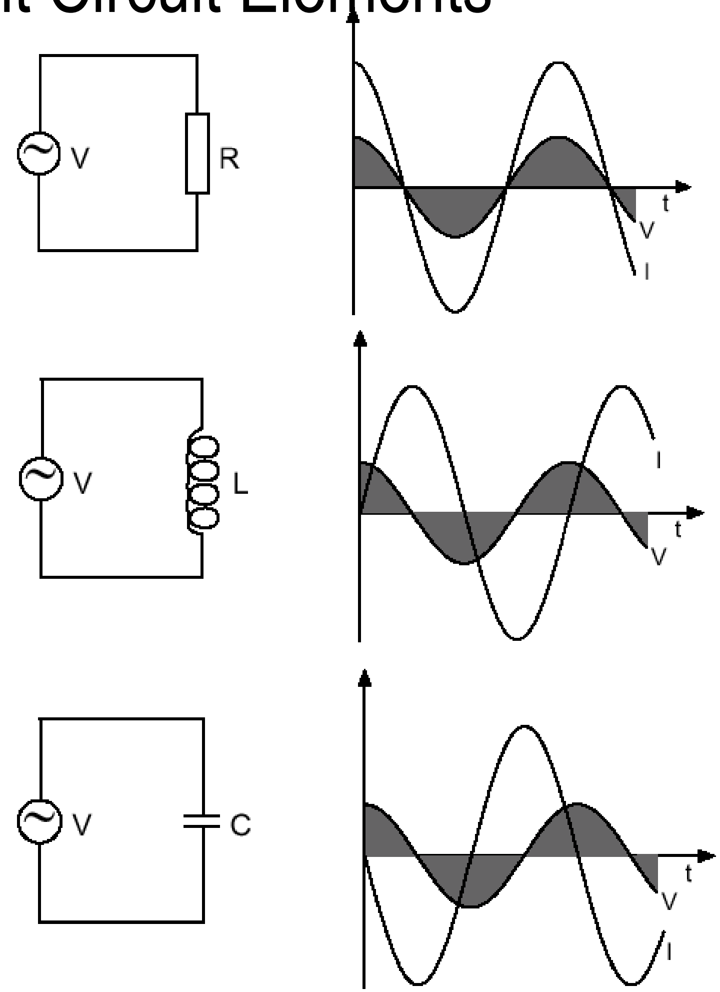
- $Z_L = j\omega L$,
  - $\omega = 2\pi f = 2\pi / T$





- $Z_C = 1/j\omega C = -j/\omega C$

Current and Voltage in different Circuit Elements



- Do phase difference between the current and the voltage in a resistor
- In an inductor the voltage leads current
- In a capacitor the voltage lags the current

Power in AC Circuits

- Apparent Power, $S=UI^*=(ZI)I^*=ZI^2$
- Reactive Power, $Q=XI^2$
- True Power, $P=RI^2$
- Apparent power is the power supplied from the source. True power is the power dissipated in the resistances of the circuit. Reactances can store energy, but they don't dissipate power.