

(68) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$3x = 0$ tai $x - 2 = 0$
 $x = 0$ t. $x = 2$

ÄÄRIARVOKOHDAT!

(69) $f(x) = x^2 + x - 6$

$f'(x) = 2x + 1$

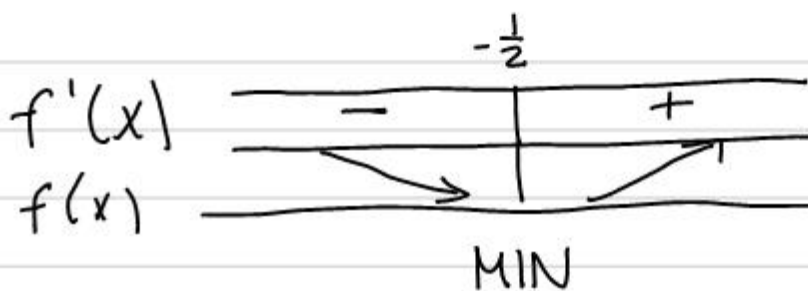
$f'(x) = 0$

$2x + 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$

deriv. 0-kohta



TAPA 1. laske määle

Valitaan vas. -1

$f'(-1) = 2 \cdot (-1) + 1 = -1 < 0$

Valitaan oik. 0

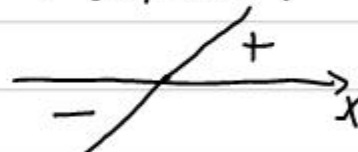
$f'(0) = 2 \cdot 0 + 1 = 1 > 0$

TAPA 2. tarkast.

deriv.funktion

Kuvaa ylös

$f'(x) = 2x + 1$



70)

$$f(x) = x^3 - 3x^2 - 24x - 1$$

$$f'(x) = 3x^2 - 6x - 24 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 3 \cdot (-24)}}{2 \cdot 3} = \frac{6 \pm \sqrt{324}}{6} = \frac{6 \pm 18}{6} = \begin{cases} 4 \\ -2 \end{cases}$$

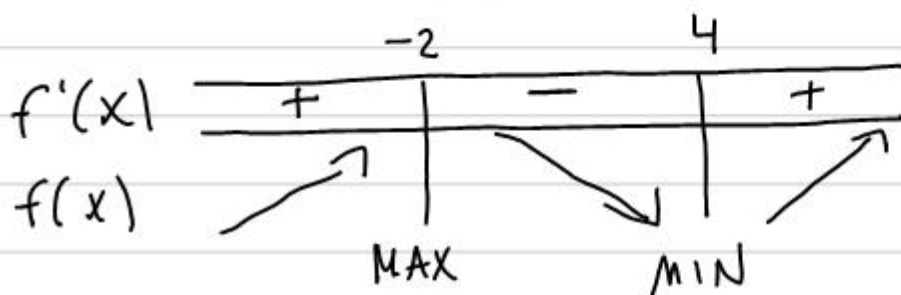


Табл. 1.

$$f'(-3) = 3 \cdot (-3)^2 - 6 \cdot (-3) - 24 = 21 > 0$$

$$f'(0) = 3 \cdot 0^2 - 6 \cdot 0 - 24 = -24 < 0$$

71)

$$f(x) = -x^2 + 9$$

$$f'(x) = -2x = 0$$

$$x = 0$$

$$f(0) = -0^2 + 9 = 9$$

писте (0, 9)



$$f'(5) = 3 \cdot 5^2 - 6 \cdot 5 - 24 = 21 > 0$$

Табл. 2.

$$f'(x) = 3x^2 - 6x - 24$$



72)

(1, 0)

73

$$f(x) = -\frac{1}{4}x^4 + \frac{1}{3}x^3 + 3x^2$$

$$f'(x) = -x^3 + x^2 + 6x = 0$$

$$x(-x^2 + x + 6) = 0$$

$$x=0 \quad \text{tai} \quad -x^2 + x + 6 = 0$$

$$x = -2 \text{ t. } x = 3$$

$f'(x)$	+	-	+	-
$f(x)$				
	MAX	MIN	MAX	

$$f'(-10) = 1040 > 0$$

$$f'(-1) = -4 < 0$$

$$f'(1) = 6 > 0$$

$$f(10) = -840 < 0$$

kasvava $] -\infty, -2[$, $] 0, 3[$

vähenevä $] -2, 0[$, $] 3, \infty[$

ääriarvokohtat: max: -2, 3
min: 0

$$\begin{aligned} \text{ääriarvot} \quad f(-2) &= -\frac{1}{4}(-2)^4 + \frac{1}{3}(-2)^3 + 3(-2)^2 \\ &= 5\frac{1}{3} = 5.33.. \end{aligned}$$

$$\begin{aligned} f(3) &= -\frac{1}{4}3^4 + \frac{1}{3}3^3 + 33^2 \\ &= 15\frac{3}{4} = 15.75 \end{aligned}$$

$$f(0) = -\frac{1}{4}0^4 + \frac{1}{3}0^3 + 30^2 = 0$$

Suurin arvo $f(3) = 15.75$

pienin arvo \nexists (ei ole olemassa)

74

pienin arvo 4

suurin \nexists



75

suurin \nexists

pieni \nexists



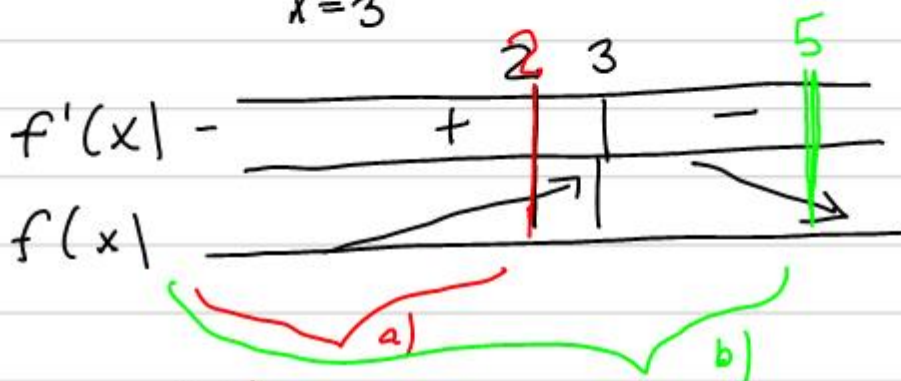
(76)

$$f(x) = 6x - x^2$$

$$f'(x) = 6 - 2x = 0$$

$$2x = 6 \parallel :2$$

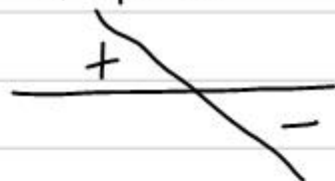
$$x = 3$$



$$f'(0) = 6 > 0$$

$$f'(4) = 6 - 2 \cdot 4 = -2 < 0$$

topo 2.



$$a)]-\infty, 2]$$

$$f(2) = 6 \cdot 2 - 2^2 = 12 - 4 = 8$$

suurin arvo

$$b)]-\infty, 5]$$

$$f(3) = 6 \cdot 3 - 3^2 = 18 - 9 = 9 \quad \text{SUURIN}$$

$$f(5) = 6 \cdot 5 - 5^2 = 30 - 25 = 5$$

$$3^2 = 9$$

$$(-3)^2 = 9$$

$$-3^2 = 9$$

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$$a+b+a+b = \underbrace{2a+2b}_{60}$$

pinta-ala $A = ab$

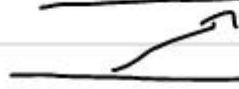

$$A = (30-b) \cdot b$$

$$f(b) = (30-b)b$$
$$= 30b - b^2$$

$$f'(b) = 30 - 2b = 0$$

$$2b = 30$$

$$b = \frac{30}{2} = 15$$

	15	
$f'(b)$	+	-
$f(b)$		
	MAX	

$$f'(0) = 30 - 2 \cdot 0 = 30$$

$$> 0$$

$$f(20) = 30 - 2 \cdot 20 = -10$$

$$< 0$$

Aitauksen mitet

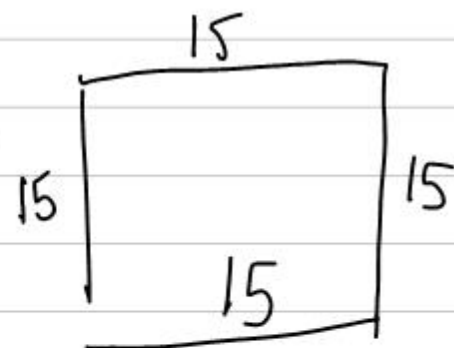
$$b = 15$$

$$a = ?$$

$$a = 30 - b = 30 - 15 = 15$$

$$A = ab = 15 \cdot 15 = 225$$

\Rightarrow



Ratk. a.

$$2a + 2b = 60$$

$$2a = 60 - 2b$$

$$a = \frac{60 - 2b}{2} =$$

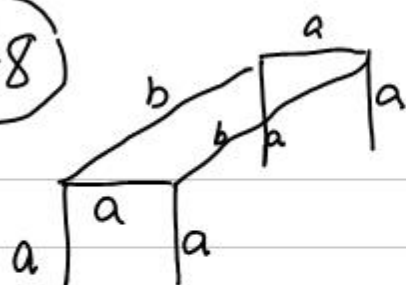
$$a = 30 - b$$

Sijoitetaan

pinta-ala lauseke!

"pinta-ala maks. suuri"

78



punkt
 $6a + 2b = 36$

$$2b = 36 - 6a$$

$$b = \frac{36 - 6a}{2} = 18 - 3a$$

VASTAVUSET

a) $a = 4, b = 6$

b) $a = 3, b = 9$

a) tihavus $V = a^2 b$

$$V = a^2 \cdot (18 - 3a)$$

$$f(a) = a^2(18 - 3a) = 18a^2 - 3a^3$$

$$f'(a) = 36a - 9a^2 = 0$$

$$9a(4 - a) = 0$$

$$9a = 0 \text{ t. } 4 - a = 0$$

$$a = 0 \text{ t. } a = 4$$

	0	4	
$f'(a)$	-	+	-
$f(a)$			
	(min)	Max	

$$a = 4 \Rightarrow b = 18 - 3a = 18 - 3 \cdot 4 = 6$$

tihavus $V = a^2 b = 4 \cdot 4 \cdot 6 = 96$

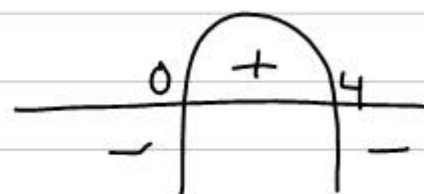
$$f(4) = 18 \cdot 4^2 - 3 \cdot 4^3 = 96$$

põõdyt 4m
 h m 6m

$$f'(-1) = 36 \cdot (-1) - 9 \cdot (-1)^2 = -36 - 9 = -45$$

$$f'(1) = 36 \cdot 1 - 9 \cdot 1^2 = 27$$

$$f'(10) = 36 \cdot 10 - 9 \cdot 10^2 = 360 - 900 = -540$$



b) At $A = ab$
 $A = a \cdot (18 - 3a) = 18a - 3a^2$

$$f(a) = 18a - 3a^2$$

$$f'(a) = 18 - 6a = 0$$

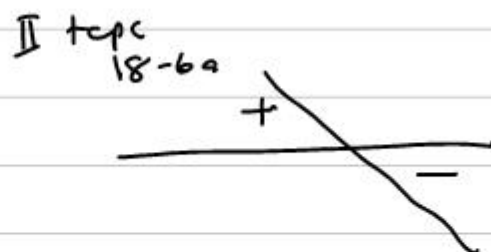
$$6a = 18$$

$$a = 3$$



$$f'(0) = 18 - 6 \cdot 0 = 18 > 0$$

$$f'(5) = 18 - 6 \cdot 5 = -12 < 0$$



$$a = 3$$

$$b = 18 - 3a = 18 - 3 \cdot 3 = 9$$

$$A = ab = 3 \cdot 9 = 27$$

$$f(3) = 18 \cdot 3 - 3 \cdot 3^2 = 27$$

КОТТЕЖИ.

12-15
17-21

3)
a. 1-8, 10

$$D \frac{f}{g} = \frac{f'g - fg'}{g^2}$$

① l. $D \frac{x+2}{x} = \frac{1x - (x+2) \cdot 1}{x^2} = \frac{x - x - 2}{x^2} = -\frac{2}{x^2} = -2x^{-2}$

o. $\frac{3x-1}{3x} + \frac{x}{3} = \frac{3x-1}{3x} + \frac{x^2}{3x} = \frac{3x-1+x^2}{3x}$

$$D \frac{3x-1+x^2}{3x} = \frac{(3+2x) \cdot 3x - (3x-1+x^2) \cdot 3}{(3x)^2}$$

$$\left(D \frac{3x-1}{3x} + D \frac{x}{3} \right) = \frac{9x+6x^2 - (9x-3+3x^2)}{9x^2}$$

$$= \frac{\cancel{9x} + 6x^2 - \cancel{9x} + 3 - 3x^2}{9x^2} = \frac{3x^2+3}{9x^2}$$

$$= \frac{\cancel{3}(x^2+1)}{\cancel{3}(3x^2)} = \frac{x^2+1}{3x^2} = \frac{1}{3} + \frac{1}{3x^2}$$

② a. $D \frac{1}{\sin x} = D (\sin x)^{-1}$

$$= -1 \cdot (\sin x)^{-2} \cdot \cos x = -\cos x \sin^{-2} x = -\frac{\cos x}{\sin^2 x}$$

$$D f(x)^r = r \cdot f(x)^{r-1} \cdot f'(x)$$

c. $D \frac{1}{2} \sin 2x = \frac{1}{2} D \sin 2x$

$$= \frac{1}{2} \left(\cos 2x \cdot \underset{\uparrow}{2} \right)$$

$$= \cos 2x \quad \begin{matrix} f(x)=2x \\ f'(x)=2 \end{matrix}$$

$$(\sin x)^{-2}$$

$$D k f(x) = k D f(x)$$

Yhd. funktion deriv.

$$D \tan x = \frac{1}{\cos^2 x}$$

i. $D \tan 2x = \frac{1}{\cos^2(2x)} \cdot 2 = \frac{2}{\cos^2(2x)}$

$$\begin{matrix} f(x) \\ f'(x)=2 \end{matrix}$$

(38)

$$a. (2+3x)^3$$

funktion potenzieren
derivieren

$$Df(x)^r = r \cdot f(x)^{r-1} \cdot f'(x)$$

$$D(2+3x)^3$$

$$= 3 \cdot (2+3x)^2 \cdot 3 = 9(2+3x)^2$$

$$d. D\sqrt{5-x} = D(5-x)^{\frac{1}{2}}$$

$$= \frac{1}{2} (5-x)^{-\frac{1}{2}} \cdot (-1)$$

$$= -\frac{1}{2} (5-x)^{-\frac{1}{2}} = -\frac{1}{2\sqrt{5-x}}$$

$$(4) De^{x^2} \leftarrow \begin{matrix} f(x)=x^2 \\ f'(x)=2x \end{matrix}$$

$$De^{ax} = ae^{ax}$$

$$= e^{x^2} \cdot 2x = 2xe^{x^2}$$

$$De^x = e^x$$

$$d. D(e^x)^3 = De^{3x} \overset{\substack{f(x)=3x \\ f'(x)=3}}{=} e^{3x} \cdot 3 = 3e^{3x}$$

(7) Tangenten kulmannen : derivatan arvo kohdassa 1

$$f(x) = \sqrt{x^2+3} = (x^2+3)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2} \cdot (x^2+3)^{-\frac{1}{2}} \cdot 2x$$

$$= \frac{1}{2} \cdot 2x \cdot (x^2+3)^{-\frac{1}{2}} = x(x^2+3)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2+3}}$$

$$f'(1) = \frac{1}{\sqrt{1^2+3}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

8) $f(x) = x^2$

Onko piste $(2, 4)$ käyrällä?

$$f(2) = 2^2 = 4$$

\Rightarrow ON!

$$f'(x) = 2x$$

tangentin kulmakerrin kohdassa 2

$$f'(2) = 2 \cdot 2 = 4$$

$$y - y_0 = k(x - x_0)$$

$$k = 4$$

$$(x_0, y_0) = (2, 4)$$

$$y - 4 = 4(x - 2)$$

$$y = 4x - 8 + 4$$

$$y = 4x - 4$$

$$\frac{1}{2} + \frac{2}{11} + \frac{3}{7} =$$

