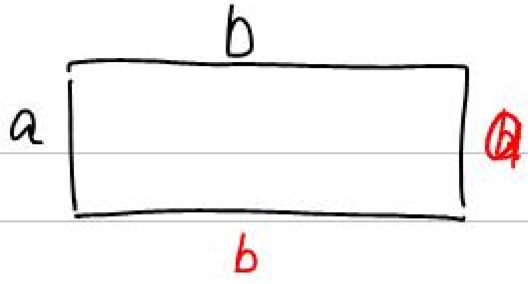
Kotiteht.
17 - 21

29 - 30

31 - 33

35 - 36

17



100 m

$3a + 2b = 100$

alue $A = ab$ mahd. suuri

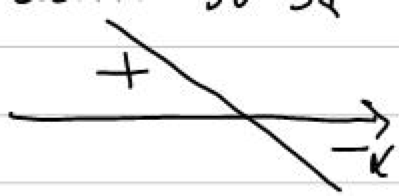
$$3a + 2b = 100$$
$$2b = 100 - 3a$$
$$b = \frac{100-3a}{2}$$

$$A = ab \overset{\text{sij.}}{=} a \cdot \frac{100-3a}{2} = 50a - \frac{3}{2}a^2$$

$$f(a) = 50a - \frac{3}{2}a^2$$
$$f'(a) = 50 - \frac{3}{2} \cdot 2a = 50 - 3a$$
$$f'(a) = 0 \qquad 50 - 3a = 0$$
$$3a = 50$$
$$a = \frac{50}{3} \approx 16.67$$

		$\frac{50}{3}$	
$f'(a)$	+		−
$f(a)$	↗	MAX	↘

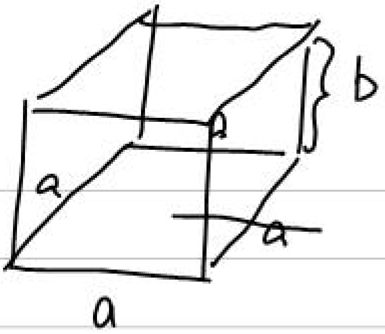
derv. $50 - 3a$



$$a = \frac{50}{3} \Rightarrow b = \frac{100 - 3 \cdot \frac{50}{3}}{2} = 25$$

$$A = ab = \frac{50}{3} \cdot 25 = 416.67\, m^2$$

(18)



$$\boxed{V = a \cdot a \cdot b = 25\,m^3}$$

$$a^2 b = 25$$

$$b = \frac{25}{a^2}$$

Kaakelin määrä mahd. pieni

pohja a^2

sivuseinät $4 \times ab$

$$\boxed{\text{Kaakelia: } a^2 + 4ab}$$

Sij. $b = \frac{25}{a^2}$

$$f(a) = a^2 + 4a \cdot \frac{25}{a^2} = a^2 + \frac{100}{a} = a^2 + 100a^{-1}$$

$$f'(a) = 2a + 100 \cdot (-a^{-2}) = 2a - 100a^{-2}$$

$f'(a) = 0$ $\quad 2a - 100a^{-2} = 0$

$$2a = 100a^{-2}$$

$$2a = \frac{100}{a^2}$$

$$2a \cdot a^2 = 100$$

$$2a^3 = 100$$

$$a^3 = 50$$

$$a = \sqrt[3]{50} \approx 3.68$$

		$\sqrt[3]{50}$	
$f'(a)$	$-$		$+$
$f(a)$	↘	MIN	↗

$f'(1) = 2 \cdot 1 - 100 \cdot (1)^{-2} = -98 < 0$

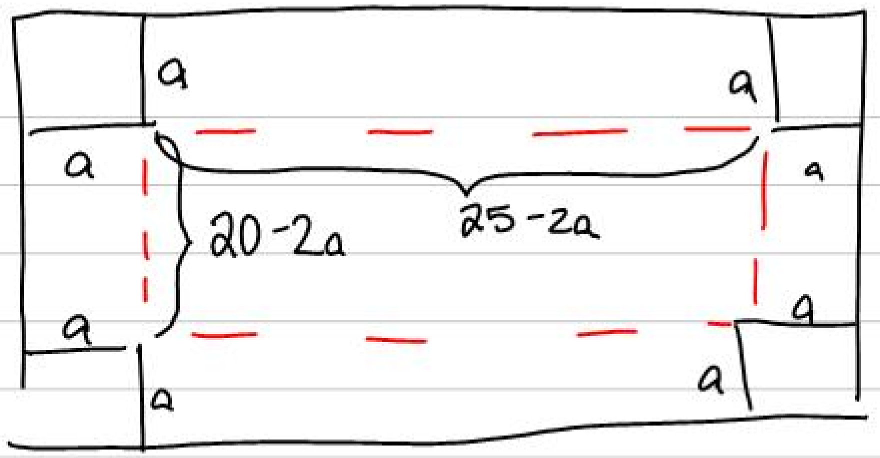
$f'(10) = 2 \cdot 10 - 100 \cdot (10)^{-2}$

$= 20 - \frac{100}{10^2} = 19 > 0$

V. $\begin{cases} a = \sqrt[3]{50} \approx 3.68 \\ b = \frac{25}{(\sqrt[3]{50})^2} = 1.84 \end{cases}$

19

20



laatikon korkeus a

25

$$V = (25-2a)(20-2a)\cdot a = 4a^3 - 90a^2 + 500a$$

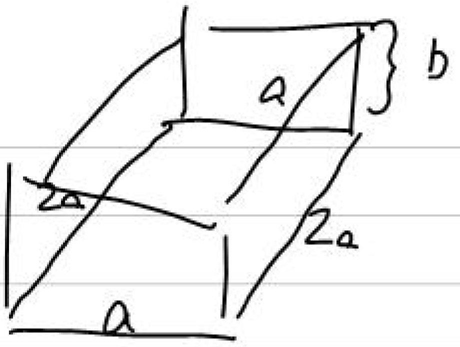
$$V'(a) = \ldots = 12a^2 - 180a + 500$$

0-kohtia: $a = \dfrac{180 \pm \sqrt{8400}}{24} = \begin{cases} 11{,}32 \\ 3{,}68 \end{cases}$

		3,68		11,32	
$V'(a)$	+		−		+
$V(a)$	↗	MAX	↘	(MIN)	↗

$a = 3{,}68 \ldots\ldots$

(20)



pohvinkul

$4a^2+6ab$

tilavuus $a \cdot 2a \cdot b = 2a^2b = 72$

$\hookrightarrow b = \frac{36}{a^2}$

$f(a) = 4a^2 + \frac{216}{a}$

$f'(a) = 8a - \frac{216}{a^2}$

$f'(a) = 0$

0-kohta: 3

		3	
$f'(a)$	$-$	|	$+$

MIN

$a = 3$

$\Rightarrow b = \ldots = 4$

(21) $V = \frac{1}{3}Ah$

. . .

31)

a) $\int x^{20}dx = \frac{x^{20+1}}{20+1} = \frac{x^{21}}{21}$

$\Downarrow$

$\frac{1}{21}\cdot x^{21} + C$

$$\boxed{x^r \Rightarrow \frac{x^{r+1}}{r+1} = \frac{1}{r+1}\cdot x^{r+1}}$$

tark.

$D\frac{1}{21}x^{21} = \frac{1}{\cancel{21}}\cdot \cancel{21}\cdot x^{20}$

$= x^{20}$

b) $\int \frac{1}{4}x^{-3}dx =$

$\frac{1}{4}\cdot\frac{1}{-2}\cdot x^{-2} = -\frac{1}{8}x^{-2} + C$

c) $\int \sqrt{x}\,dx =$

c) $\int \sqrt{x}\,dx = \int (x)^{\frac{1}{2}}dx = \frac{2}{3}x^{\frac{3}{2}} + C$

h) $\int \left(2 + 3x^2 - \frac{x^4}{2}\right)dx = \int \left(2 + 3x^2 - \frac{1}{2}x^4\right)dx$

$= 2x + 3\cdot\underbrace{\frac{1}{3}x^3} - \frac{1}{2}\cdot\underbrace{\frac{1}{5}x^5}$

$= 2x + x^3 - \frac{1}{10}x^5 + C$

j) $\int (2-2x)^3 dx$ s. 53 k3

$f(x) = (2-2x)$ $(f(x))^3$ $\int f(x)^r f'(x)\,dx$

$f'(x) = -2$ $= \frac{1}{r+1}(f(x))^{r+1} + C$

$= \int \overbrace{(2-2x)^3\cdot(-2)}^{f(x)^3 f'(x)}\cdot\underbrace{(-2)\cdot\left(-\frac{1}{2}\right)}_{=1}\,dx$

$= -\frac{1}{2}\int (2-2x)^3\cdot(-2)\,dx = -\frac{1}{2}\cdot\frac{1}{4}\cdot(2-2x)^4$

$= -\frac{1}{8}(2-2x)^4 + C$

k) $\int (4x-2)^4 dx$

$f(x) = 4x - 2$

$f'(x) = 4$

$= \frac{1}{4}\int (4x-2)^4 \cdot 4\,dx = \frac{1}{20}(4x-2)^5 + C$

32 a) $\int (\sin x + x)\,dx = -\cos x + \frac{1}{2}x^2 + C$

S. 53 K6

$\int f(g(x))\,g'(x)\,dx$

$= F(g(x)) + C$

c) $\int \sin \underbrace{4x}\,dx$

$f(x) = \sin(\quad)$

$g(x) = 4x$

$g'(x) = 4$

$= \int \left(\overbrace{\sin 4x \cdot 4}^{f(g(x))\,g'(x)} \cdot \frac{1}{4}\right) dx = \frac{1}{4}\int \sin 4x \cdot 4\,dx$

$= \frac{1}{4} \cdot (-\cos 4x) = -\frac{1}{4}\cos 4x + C$

g) $\int 3\sin 3x\,dx = \int \sin 3x \cdot 3\,dx = -\cos 3x + C$

$D\,3x = 3$

(29) $f(x) = -3x + 2$ $\qquad F(0) = -2$

$$\int (-3x+2)\,dx = \underbrace{-\frac{3}{2}x^2 + 2x + C}_{F(x)}$$

$$F(0) = -\frac{3}{2}\cdot 0^2 + 2\cdot 0 + C = C = -2$$

$$\Rightarrow C = -2$$

$$F(x) = -\frac{3}{2}x^2 + 2x - 2$$

5.55 K6

(30) $\int (e^{-x} + 2x + 3)\,dx$ $\qquad F(2) = 10$

$$= -e^{-x} + \cancel{2}\cdot\frac{1}{\cancel{2}}x^2 + 3x + C = -e^{-x} + x^2 + 3x + C$$

$$F(2) = -e^{-2} + 2^2 + 3\cdot 2 + C = 10$$

$$-e^{-2} + \cancel{4+6} + C = \cancel{10}$$

$$-e^{-2} + C = 0$$

$$C = e^{-2}$$

(35) $\displaystyle\int_0^2 x^3\,dx = \Big|_0^2 \tfrac{1}{4}x^4 = \underset{\substack{\uparrow\\ F(2)}}{\tfrac{1}{4}\cdot 2^4} - \underset{\substack{\uparrow\\ F(0)}}{\tfrac{1}{4}\cdot 0^4}$

$f(x) = x^3 \qquad F(x) = \tfrac{1}{4}x^4$

$= 4$

$\displaystyle\int_{-2}^{3} (x+x^2)\,dx = \Big|_{-2}^{3} \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 = \underset{F(3)}{\left[\tfrac{1}{2}\cdot 3^2 + \tfrac{1}{3}\cdot 3^3\right]} - \underset{F(-2)}{\left[\tfrac{1}{2}\cdot(-2)^2 + \tfrac{1}{3}(-2)^3\right]}$

$f(x) = x + x^2 \qquad F(x) = \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3$

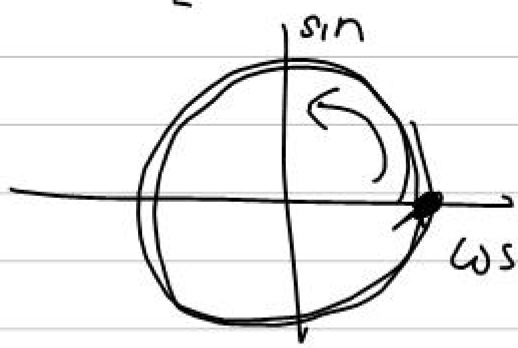
$= \left[\tfrac{1}{2}\cdot 9 + \tfrac{1}{3}\cdot 27\right] - \left[\tfrac{1}{2}\cdot 4 + \tfrac{1}{3}\cdot(-8)\right]$

$11\tfrac{1}{2} + \tfrac{8}{3}$

$= \left[4\tfrac{1}{2} + 9\right] - \left[2 - \tfrac{8}{3}\right] = 13\tfrac{1}{2} - 2 + \tfrac{8}{3}$

$= 14\tfrac{1}{6}$

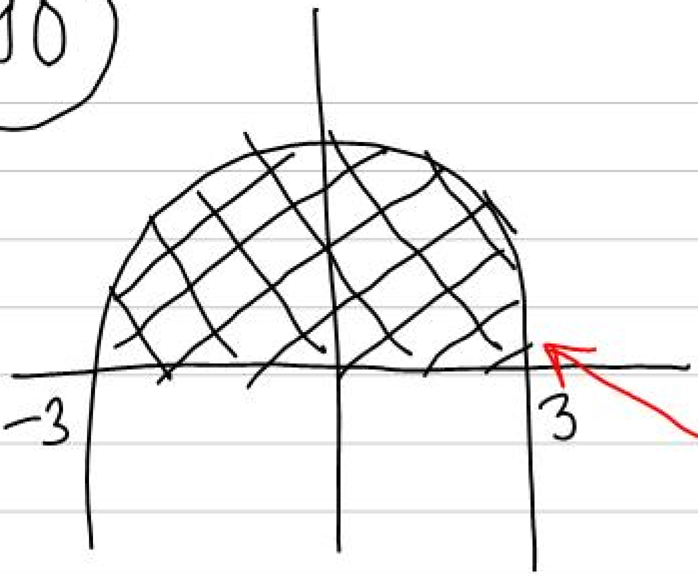
d) $\displaystyle\int_0^{2\pi} \sin x\,dx = \Big|_0^{2\pi} -\cos x = \left[-\cos 2\pi\right] - \left[-\cos 0\right]$



$= -1 + 1 = 0$

$-\cos 2\pi + \cos 2\pi = 0$

90



$$\int_{-3}^{3}(-x^2+9)\,dx$$

laskee käyrän ja x-akselin välien alueen pinta-alan

(pinta-ala kohtien -3 ja 3 välissä)

---

90 $f(x) = -x^2+9$

x-akselin leikkauspisteet:

$$-x^2+9=0$$

$$x^2=9$$

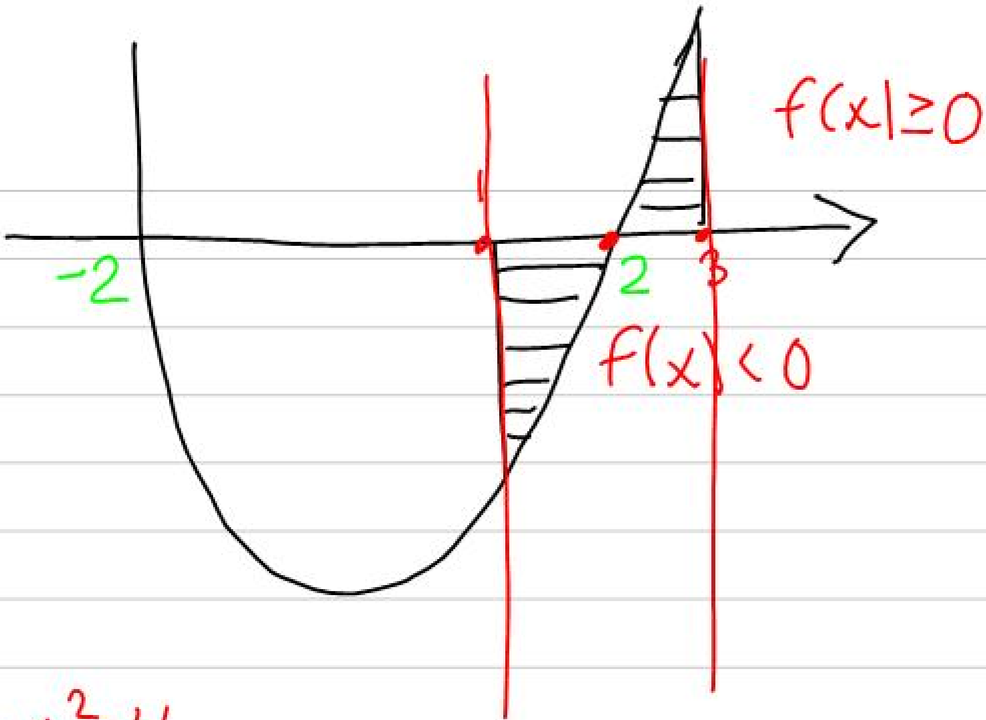
$$x=\pm 3$$

$$\int_{-3}^{3}(-x^2+9)\,dx = \Big/_{-3}^{3} -\frac{1}{3}x^3+9x = \left[-\frac{1}{3}\cdot 3^3+9\cdot 3\right]-\left[-\frac{1}{3}\cdot(-3)^3+9(-3)\right]$$

$$=\left[-9+27\right]-\left[9-27\right]$$

$$=18-(-18)=18+18=36$$

91



$f(x) = x^2 - 4$

X-akselin leikkauspisteet:

$f(x) = 0 \qquad x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

Jaetaan osiin

Väli $[1,2]$ $\quad f(x) < 0$

Väli $[2,3]$ $\quad f(x) \geq 0$

$$A = -\int_a^b f(x)\,dx = \int_b^a f(x)\,dx$$

$$-\int_1^2 (x^2-4)\,dx = -\Big/_1^2 \tfrac{1}{3}x^3 - 4x$$

$$= -\left[\left(\tfrac{1}{3}\cdot 2^3 - 4\cdot 2\right) - \left(\tfrac{1}{3}\cdot 1^3 - 4\cdot 1\right)\right] = \left[\tfrac{8}{3} - 8 - \tfrac{1}{3} + 4\right]$$

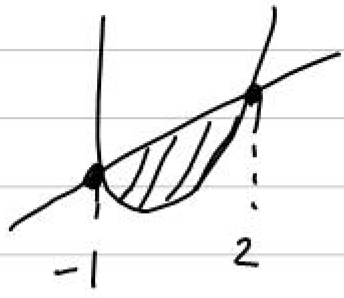
$$= 1\tfrac{2}{3}$$

$$\int_2^3 (x^2-4)\,dx = \Big/_2^3 \tfrac{1}{3}x^3 - 4x = \underbrace{\left(\tfrac{1}{3}\cdot 3^3 - 4\cdot 3\right)}_{-3} - \underbrace{\left(\tfrac{1}{3}\cdot 2^3 - 4\cdot 2\right)}_{-5\frac{1}{3}} = 2\tfrac{1}{3}$$

V: $1\tfrac{2}{3} + 2\tfrac{1}{3} = 4$

(93) $f(x) = x^2 \qquad g(x) = x+2$

käyrien leikkauspisteet



$$f(x) = g(x)$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$x = \frac{1 \pm \sqrt{1 - 4 \cdot (-2)}}{2} = \frac{1 \pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$$

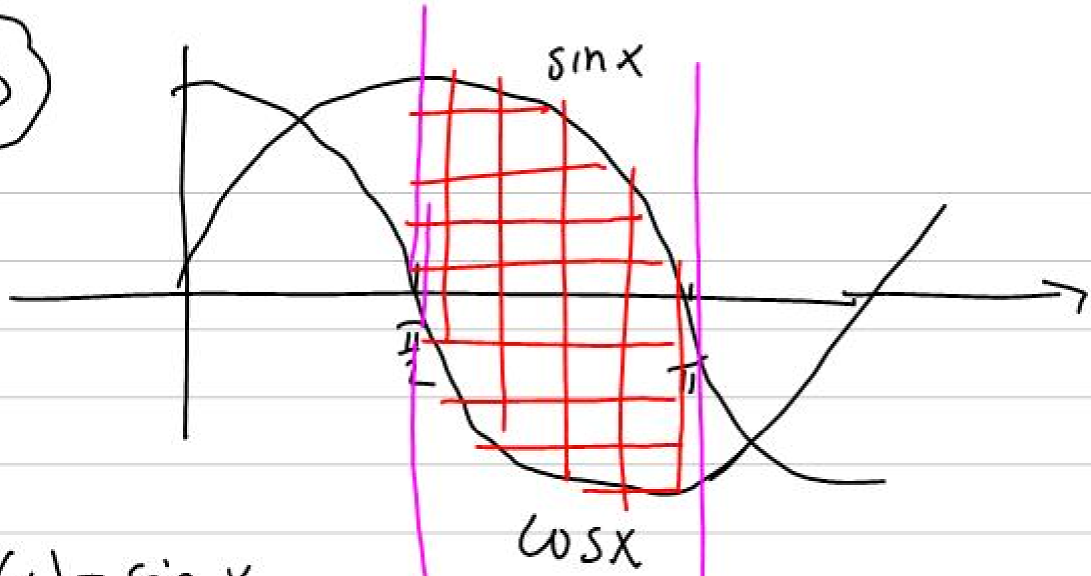
Tässä $g(x) \geq f(x)$

$$\int_{-1}^{2} (g(x) - f(x))\,dx = \int_{-1}^{2} (x+2) - x^2\,dx = \int_{-1}^{2} (x + 2 - x^2)\,dx$$

$$= \Big/_{-1}^{2} \frac{1}{2}x^2 + 2x - \frac{1}{3}x^3 = \Big[\underbrace{\frac{1}{2} \cdot 2^2}_{2} + \underbrace{2 \cdot 2}_{4} - \underbrace{\frac{1}{3} \cdot 2^3}_{\frac{8}{3}}\Big] - \Big[\underbrace{\frac{1}{2} \cdot (-1)^2}_{\frac{1}{2}} + \underbrace{2(-1)}_{-2} - \underbrace{\frac{1}{3} \cdot (-1)^3}_{-\frac{1}{3}}\Big]$$

$$= 4\frac{1}{2}$$

93



$f(x) = \sin x$

$g(x) = \cos x$

Tässä: $f(x) > g(x)$

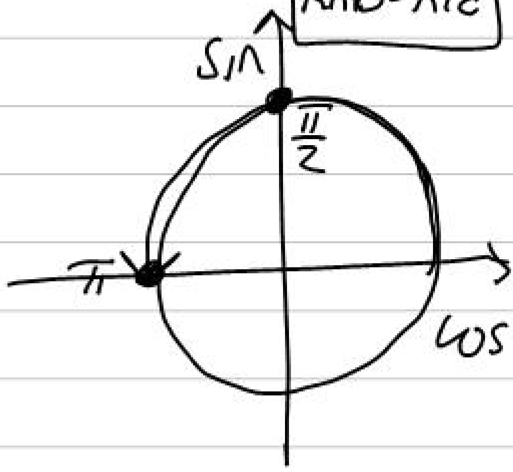
$$\int_{\frac{\pi}{2}}^{\pi} (\sin x - \cos x)\,dx = \Big/_{\frac{\pi}{2}}^{\pi} -\cos x - \sin x$$

$$= \left[-\cos \pi - \sin \pi\right] - \left[-\cos \frac{\pi}{2} - \sin \frac{\pi}{2}\right]$$

$$= \left[-(-1) - 0\right] - \left[-0 - 1\right]$$

$$= 2$$

RAD-tila



$\boxed{\text{KOTITEHT: 39-43}}$