SERIES DE FOURIER

Integrantes:

Santiago Ramírez Arenas

Brandon Ávila Vargas

Luis Fernando Bastidas

UNIVERSIDAD TECNOLÓGICA DE PEREIRA

COMUNICACI0NES I

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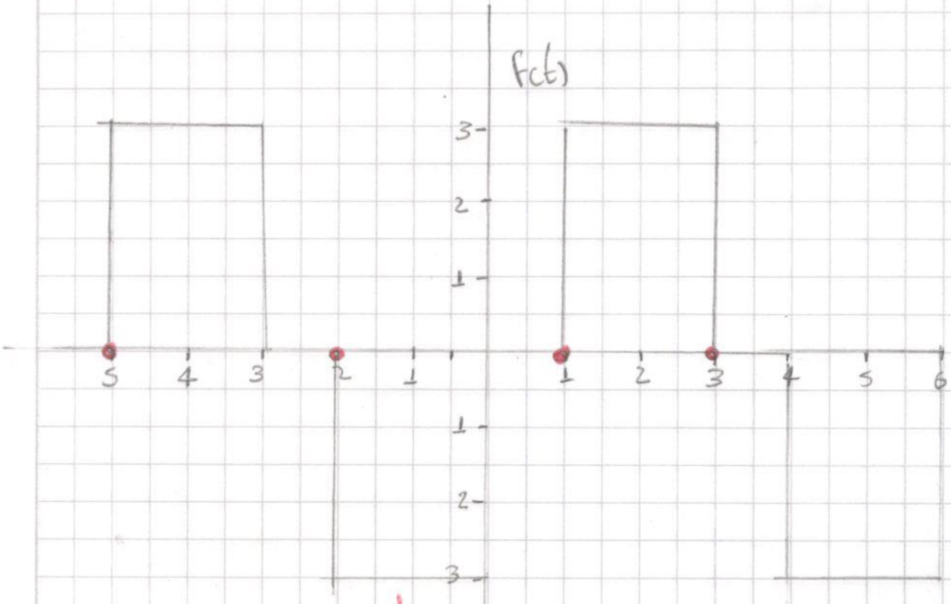
Taller #1 Comunicaciones.



$T = 6$

$\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

Serie Trigonométrica

$$f(t) = \begin{cases} 3 & 1 \le t \le 3 \\ 0 & 3 \le t \le 4 \rightarrow \text{No!} \\ -3 & 4 \le t \le 6 \end{cases}$$

- $a_0 = \frac{2}{T}\int_0^T f(t)\,dt \Rightarrow a_0 = \frac{2}{6}\left[\int_4^6 -3\,dt + 0 + \int_1^3 3\,dt\right] =$

$a_0 = \frac{1}{3}\left[-3t\Big|_4^6 + 3t\Big|_1^3\right] = \frac{1}{3}(3)\left[(6-4) - (3-1)\right] =$

$a_0 = (1)\ (2-2) = 0$ $\quad a_0 = 0$ ✓

- Para a_n

$a_n = \frac{2}{T}\int f(t)\cos(n\omega_0 t)\,dt$

$a_n = \frac{2}{6}\left[-3\int_{-2}^{0}\cos(n\omega_0 t)\,dt + 3\int_1^3 \cos(n\omega_0 t)\,dt\right]$

2

$$a_n = \frac{1}{3}(-3)\left[\int_{-2}^{0} \cos(n\omega_0 t)\,dt + 3\int_{1}^{3} \cos(n\omega_0 t)\,dt\right]$$

$$a_n = -\left[\frac{\operatorname{Sen}(n\omega_0 t)}{n\omega_0}\Big|_{-2}^{0} - \frac{\operatorname{Sen}(n\omega_0 t)}{n\omega_0}\Big|_{1}^{3}\right]$$

$$a_n = -\left[\frac{\operatorname{Sen}(0)}{n\omega_0} - \frac{\operatorname{Sen}(-2n\omega_0)}{n\omega_0} - \frac{\operatorname{Sen}(3n\omega_0)}{n\omega_0} + \frac{\operatorname{Sen}(n\omega_0)}{n\omega_0}\right]$$

$$a_n = \frac{-1}{n\omega_0}\left[-\operatorname{Sen}\left(-2n\frac{\pi}{3}\right) - \underbrace{\operatorname{Sen}\left(3n\frac{\pi}{3}\right)}_{\to 0} + \operatorname{Sen}\left(n\frac{\pi}{3}\right)\right]$$

$$a_n = \frac{-3}{n\pi}\left[\operatorname{Sen}\left(2n\frac{\pi}{3}\right) + \operatorname{Sen}\left(n\frac{\pi}{3}\right)\right]$$

- Para Bn

$$b_n = \frac{2}{T}\int_{0}^{T} f(t)\operatorname{Sen}(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\left[-3\int_{-2}^{0} \operatorname{Sen}(n\omega_0 t)\,dt + 3\int_{1}^{3} \operatorname{Sen}(n\omega_0 t)\,dt\right]$$

$$b_n = \frac{2}{6}(-3)\left[\int_{-2}^{0} \operatorname{Sen}(n\omega_0 t)\,dt - \int_{1}^{3} \operatorname{Sen}(n\omega_0 t)\,dt\right]$$

$$b_n = -\left[-\frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_{-2}^{0} + \frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_{1}^{3}\right]$$

$$b_n = -\left[-\frac{\cos(0)}{n\omega_0} + \frac{\cos(-2n\omega_0)}{n\omega_0} + \frac{\cos(3n\omega_0)}{n\omega_0} - \frac{\cos(n\omega_0)}{n\omega_0}\right]$$

$$b_n = \frac{-1}{n\omega_0}\left[-1 + \cos(-2n\omega_0) + \underbrace{\cos(3n\omega_0)}_{\to(-1)^n} - \cos(n\omega_0)\right]$$

$$B_n = \frac{-1}{n\omega_0}\left[-1 + (-1)^n + \cos\left(-2n\frac{\pi}{3}\right) - \cos\left(n\frac{\pi}{3}\right)\right]$$

$$B_n = -\frac{3}{n\pi}\left[1 - (-1)^n - \cos\left(2n\frac{\pi}{3}\right) + \cos\left(n\frac{\pi}{3}\right)\right]$$

$$f(t) = \sum_{n=1}^{\infty} \left(\frac{-3}{n\pi}\left(\operatorname{Sen}\left(\frac{2n\pi}{3}\right) + \operatorname{Sen}\left(\frac{n\pi}{3}\right)\right] \cdot \cos\left(\frac{n\pi t}{3}\right) + \frac{3}{n\pi}\left(1 - (-1)^n - \cos\left(\frac{2n\pi}{3}\right) + \cos\left(\frac{n\pi}{3}\right)\right] \cdot \operatorname{Sen}\left(\frac{n\pi t}{3}\right)\right)$$

- Complejo

$$f(t)\begin{cases} 3 & 1\leq t\leq 3 \\ -3 & 4\leq t\leq 6 \end{cases} \qquad T=6 \qquad \omega_0=\frac{\pi}{3}$$

$$C_n=\frac{1}{T}\int_0^{1} f(t)\, e^{-in\omega_0 t}\, dt$$

$$C_n=\frac{1}{6}\left[3\int_1^3 e^{-in\omega_0 t}\, dt -3\int_4^6 e^{-in\omega_0 t}\, dt\right]$$

$$C_n=\frac{1}{6}\cdot(3)\left[\left.\frac{e^{-in\omega_0 t}}{-in\omega_0}\right|_1^3-\left.\frac{e^{-in\omega_0 t}}{+in\omega_0}\right|_4^6\right]$$

$$C_n=\frac{1}{2in\omega_0}\left[-e^{-3in\omega_0}+e^{-in\omega_0}+e^{-6in\omega_0}-e^{-4in\pi/3}\right]$$

$$C_n=\frac{1}{2in\omega_0}\left[-e^{-in\pi}+e^{-in\pi/3}+e^{-2in\pi}-e^{-in\pi/3}\right]$$

$$C_n=\frac{1}{2in\omega_0}\left[-e^{-4in\pi/6}\left(e^{2in\pi/3}-e^{-2in\pi/3}\right)+e^{-\frac{5in\pi}{6}}\left(e^{in\pi/3}-e^{-in\pi/5}\right)\right]$$

$$C_n=\frac{1}{2in\omega_0}\left[-e^{-4in\pi/6}\left(2\,Sen\left(\frac{2n\pi}{3}\right)\right)+e^{-\frac{5en\pi}{6}}\left(2\,Sen\left(\frac{n\pi}{3}\right)\right)\right]$$

$$C_n=\frac{3}{in\pi}\left[e^{-\frac{5en\pi}{6}}\left(Sen(2n\pi/3)\right)-e^{-4in\pi/6}\left(Sen(2\pi/3)\right)\right]$$

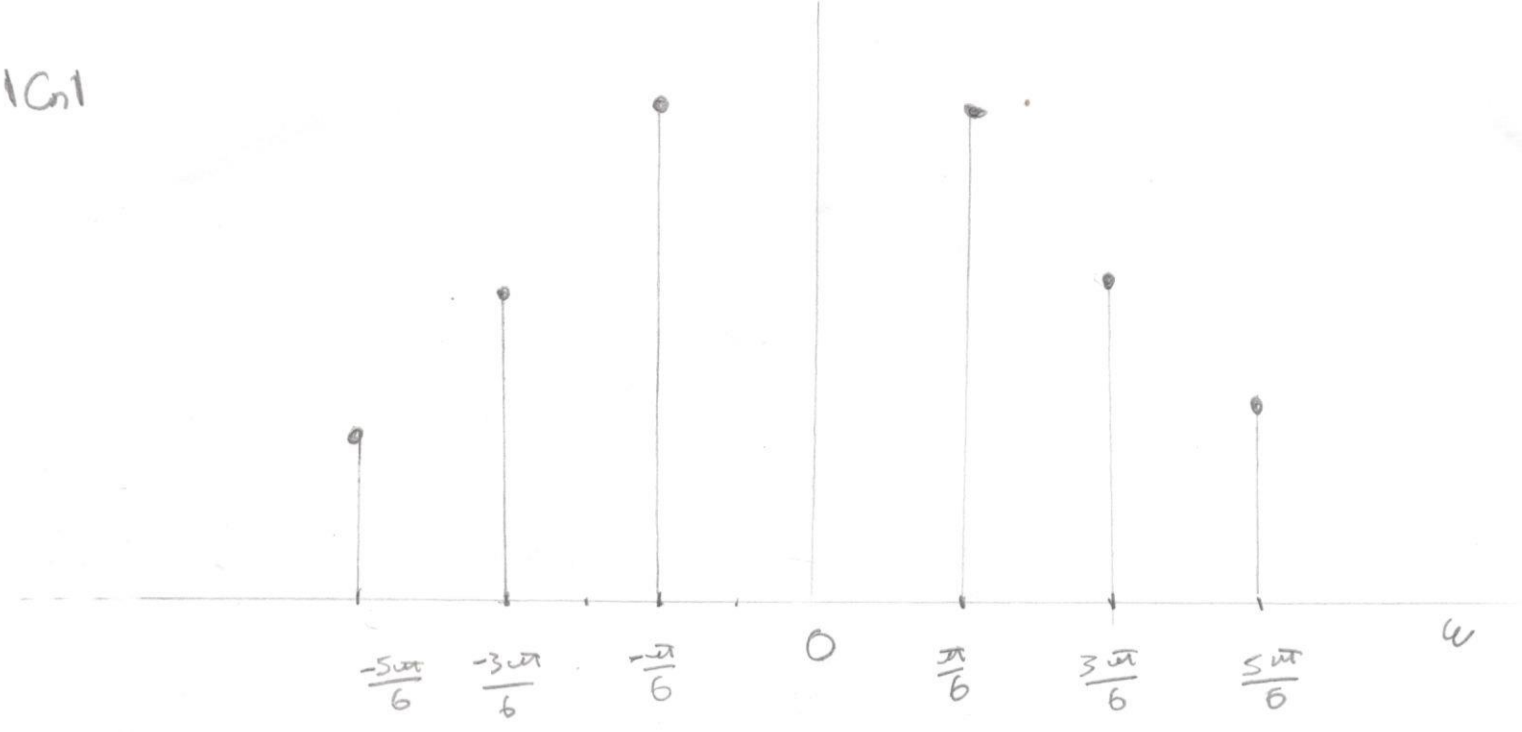
$C_1=0{,}636\ e^{-5\pi/6}$

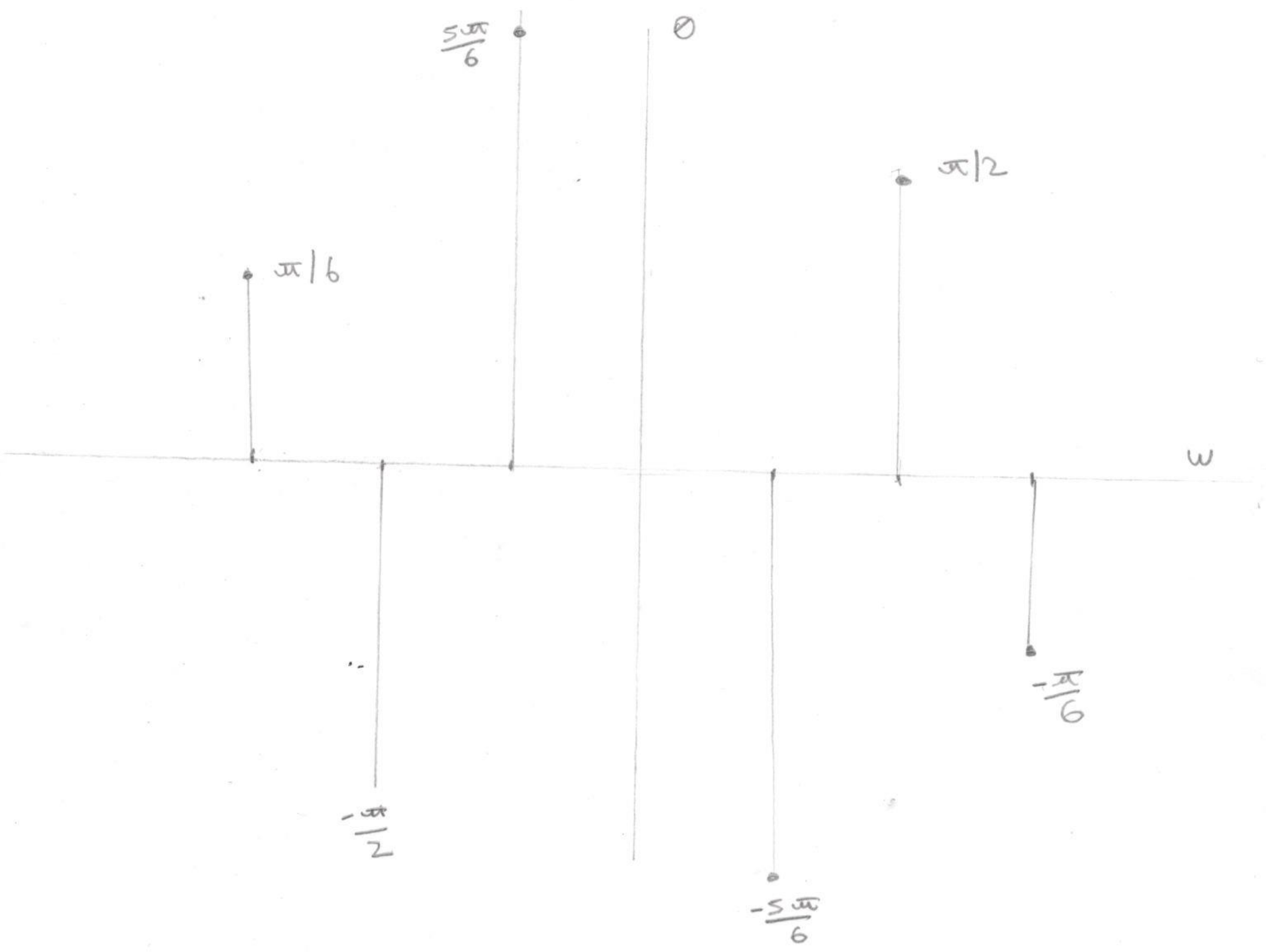
$C_2=0$

$C_3=0{,}424\ e^{-7\pi/2}$

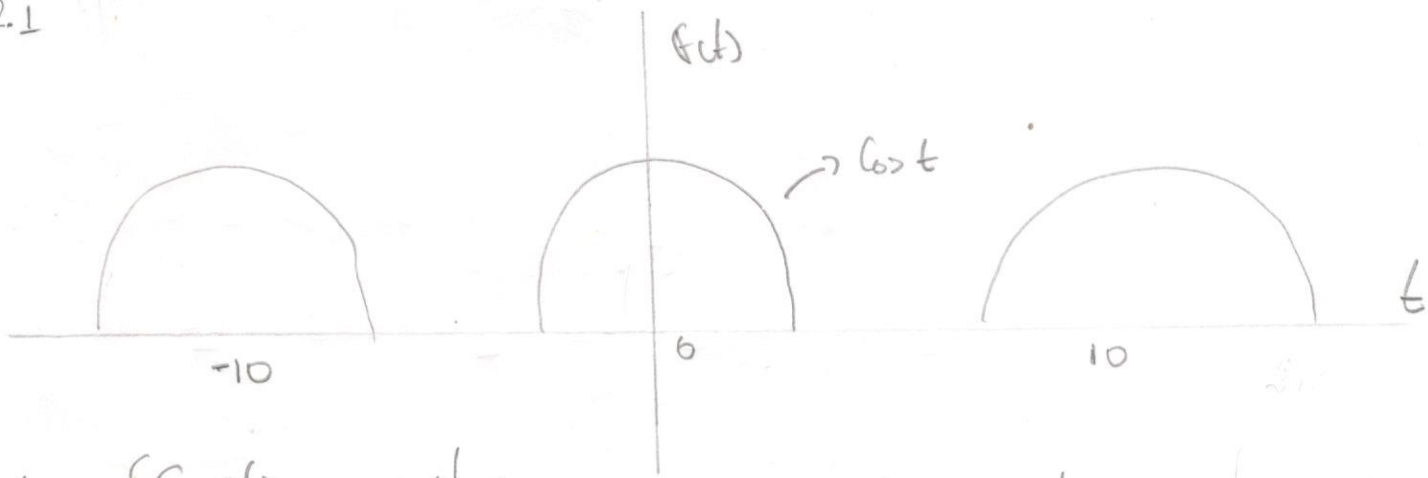
$C_4=0$

$C_5=0{,}127\ e^{-37\pi/6}$


|Cn|
-5π/6
-3π/6
-π/6
0
π/6
3π/6
5π/6
ω


5π/6
0
π/2
π/6
ω
-π/6
-π/2
-5π/6

2.1



$$f(t) = \begin{cases} \cos(t) & -\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \\ 0 = & \frac{\pi}{2} \leq t \leq -\frac{\pi}{2} \end{cases} \quad \Big| \quad T = 10 \quad f(t) = \cos(t) \quad \omega_0 = \frac{\pi}{5}$$

Trigonométrica.

$$a_0 = \frac{1}{5}\int_{-\pi/2}^{\pi/2} \cos t \, dt \Rightarrow a_0 = \frac{1}{5}\left(\operatorname{Sen}(t)\right)\Big|_{-\pi/2}^{\pi/2} = \frac{1}{5}\operatorname{Sen}\left(\frac{\pi}{2}\right) - \frac{1}{5}\operatorname{Sen}\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{5}\operatorname{Sen}\left(\frac{\pi}{2}\right) + \frac{1}{5}\operatorname{Sen}\left(\frac{\pi}{2}\right) = \frac{2}{5} \Rightarrow \boxed{a_0 = \frac{2}{5}} \qquad a_0 = \frac{2}{5}$$

$$a_n = \frac{1}{5}\int_{-\pi/2}^{\pi/2} \cos(t)\cos(n\omega_0 t)$$

$$a_n = \frac{1}{10}\int_{-\pi/2}^{\pi/2} (\cos(n\omega_0 t - t) + \cos(n\omega_0 t + t))\, dt$$

$$a_n = \frac{1}{10}\int_{-\pi/2}^{\pi/2} \cos((n\omega_0 - 1)t)\, dt + \int_{-\pi/2}^{\pi/2} \cos((n\omega_0 + 1)t)\, dt$$

$$a_n = \frac{1}{10}\left[\left(\frac{\operatorname{Sen}((n\omega_0 - 1)t)}{n\omega_0 - 1}\right)\Big|_{-\pi/2}^{\pi/2} + \left(\operatorname{Sen}((n\omega_0 + 1)t)\right)\Big|_{-\pi/2}^{\pi/2}\right] =$$

$$a_n = \frac{1}{10}\left[\frac{(n\omega_0 + 1)\operatorname{Sen}((n\omega_0 - 1)t) + (n\omega_0 - 1)\operatorname{Sen}((n\omega_0 + 1)t)}{n^2\omega_0^2 - 1}\right]\Big|_{-\pi/2}^{\pi/2}$$

$$a_n = \frac{1}{5}\left[\frac{n\omega_0 \operatorname{Sen}(n\omega_0 t)\cos(t) - \operatorname{Sen}(t)\cos(n\omega_0 t)}{n^2\omega_0^2 - 1}\Big|_{-\pi/2}^{\pi/2}\right]$$

2.2

$$a_n = \frac{1}{5}\left[\frac{-\cos\left(\frac{\pi n w_0}{2}\right) - \cos\left(-\frac{\pi n w_0}{2}\right)}{n^2 w_0^2 - 1}\right]$$

$$a_n = \frac{2}{5}\left[\frac{-\cos\left(\frac{\pi n w_0}{2}\right)}{n^2 w_0^2 - 1}\right] = \frac{2}{5}\left[\frac{-\cos(\pi 2n/10)}{\frac{n^2 \pi^2}{25} - 1}\right] =$$

$$= 10\left[\frac{-\cos(\pi 2n/10)}{\pi^2 n^2 - 25}\right] \Rightarrow a_n$$

$$b_n = \frac{1}{5}\int_{-\pi/2}^{\pi/2} \cos(t)\,\mathrm{Sen}(n w_0 t)\,dt = \frac{1}{10}\int_{-\pi/2}^{\pi/2} (\mathrm{Sen}((n w_0 - 1)t) + \mathrm{Sen}((n w_0 + 1)t))\,dt$$

$$= \frac{1}{10}\left[\int_{-\pi/2}^{\pi/2} \mathrm{Sen}((n w_0 t - 1)t)\,dt + \int_{-\pi/2}^{\pi/2} \mathrm{Sen}((n w_0 + 1)t)\,dt\right]$$

$$= \frac{1}{10}\left[\left(\frac{-\cos((n w_0 - 1)t)}{n w_0 - 1}\right)\Bigg|_{-\pi/2}^{\pi/2} + \left(\frac{-\cos((n w_0 + 1)t)}{n w_0 + 1}\right)\Bigg|_{-\pi/2}^{\pi/2}\right]$$

$$= \frac{1}{10}\left[\frac{-(n w_0 + 1).\cos((n w_0 - 1)t) - (n w_0 - 1)\cos((n w_0 + 1)t)}{n^2 w_0^2 - 1}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{10}\left[\frac{(n w_0 + 1)\cos((n w_0 - 1)t) + (n w_0 - 1)\cos((n w_0 + 1)t)}{1 - n^2 w_0^2}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{5}\left[\frac{\mathrm{Sen}(t)\,\mathrm{Sen}(n w_0 t) + n w_0 \cos(t)\cos(n w_0 t)}{1 - n^2 w_0^2}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$b_n = \frac{1}{5}\left(\frac{\mathrm{Sen}(\pi/2)\,\mathrm{Sen}\left(\frac{\pi n w_0}{2}\right) - \mathrm{Sen}(-\pi/2)\,\mathrm{Sen}\left(-\frac{\pi n w_0}{2}\right)}{1 - n^2 w_0^2}\right)$$

$$= \frac{1}{5}\left[\frac{\mathrm{Sen}(\pi/2)\,\mathrm{Sen}\left(\frac{\pi n w_0}{2}\right) - \mathrm{Sen}(\pi/2)\,\mathrm{Sen}\left(\frac{\pi n w_0}{2}\right)}{1 - n^2 w_0^2}\right]$$

$\hookrightarrow 0$ $\hookrightarrow 0$

$$b_n = \frac{1}{5}[0] = 0 \Rightarrow b_n$$

2.3

$$f(t) = \frac{1}{5} + \sum_{n=1}^{\infty} 10 \left(\frac{-\cos\left(\frac{\pi^2 n}{10}\right)}{\pi^2 n^2 - 25} \right) \cos\left(\frac{\pi n t}{5}\right)$$ ⇒ Serie Trigonométrica

• Compleja

$$C_n = \frac{1}{10} \int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-in\omega_0 t}\, dt$$

Por Partes.

$u = \cos(t)$ $\quad du = -\operatorname{Sen}(t)\, dt$

$dv = e^{-in\omega_0 t}\, dt$ $\quad v = \frac{-e^{-in\omega_0 t}}{in\omega_0}$

$$= \frac{1}{10} \int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-in\omega_0 t}\, dt = \frac{1}{10}\left[\frac{-\cos(t)\, e^{-in\omega_0 t}}{in\omega_0} \Bigg|_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} \frac{\operatorname{Sen}(t)\, e^{in\omega_0 t}}{n\omega_0} \right]$$

Integral

$u = \operatorname{Sen}(t)$ $\quad du = -\cos(t)\, dt$

$dv = \frac{e^{-in\omega_0 t}}{in\omega_0}\, dt$ $\quad v = \frac{-e^{-in\omega_0 t}}{i^2 n^2 \omega_0^2}$

$$= \frac{1}{10} \int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-in\omega_0 t}\, dt$$

$$= \frac{1}{10}\left[- \frac{\cos(t)\, e^{-in\omega_0 t}}{in\omega_0} \Bigg|_{-\pi/2}^{\pi/2} \frac{-\operatorname{Sen}(t)\, e^{-in\omega_0 t}}{n^2 \omega_0^2} \Bigg|_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} \frac{\cos(t)\, e^{-in\omega_0 t}}{i^2 n^2 \omega_0^2} \right]$$

$$= \frac{1}{10} + \frac{1}{10\, i^2 n^2 \omega_0^2} \int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-in\omega_0}\, dt$$

$$\frac{1}{10}\left[-\frac{\cos(t)\, e^{-in\omega_0 t}}{in\omega_0} - \frac{\operatorname{Sen}(t)\, e^{-in\omega_0 t}}{n^2 \omega_0^2} \Bigg|_{-\pi/2}^{\pi/2} \right]$$

$$\frac{1}{10} \int_{-\pi/2}^{\pi/2} \cos(t)\, e^{-in\omega_0}\, dt$$

2.4

$$= \frac{1}{10\,(1-1/n^2\omega_0^2)}\left[\frac{-\cos(t)\,e^{-in\omega_0 t}}{in\omega_0} - \frac{\mathrm{Sen}(t)\,e^{-in\omega_0 t}}{n^2\omega_0^2}\right]\Bigg|_{-\pi/2}^{\pi/2}$$

$$C_n = \frac{1}{10\,(1-1/n^2\omega_0^2)}\left[\frac{-e^{-\frac{\pi i n\omega_0}{2}}}{n^2\omega_0^2} - \frac{e^{\frac{\pi i n\omega_0}{n}}}{n^2\omega_0^2}\right]$$

$$C_n = \frac{-1}{10\left(\frac{n^2\omega_0^2-1}{n^2\omega_0^2}\right)}\left[\frac{e^{\frac{\pi i n\omega_0}{2}} + e^{-\frac{\pi i n\omega_0}{2}}}{n^2\omega_0^2}\right]$$

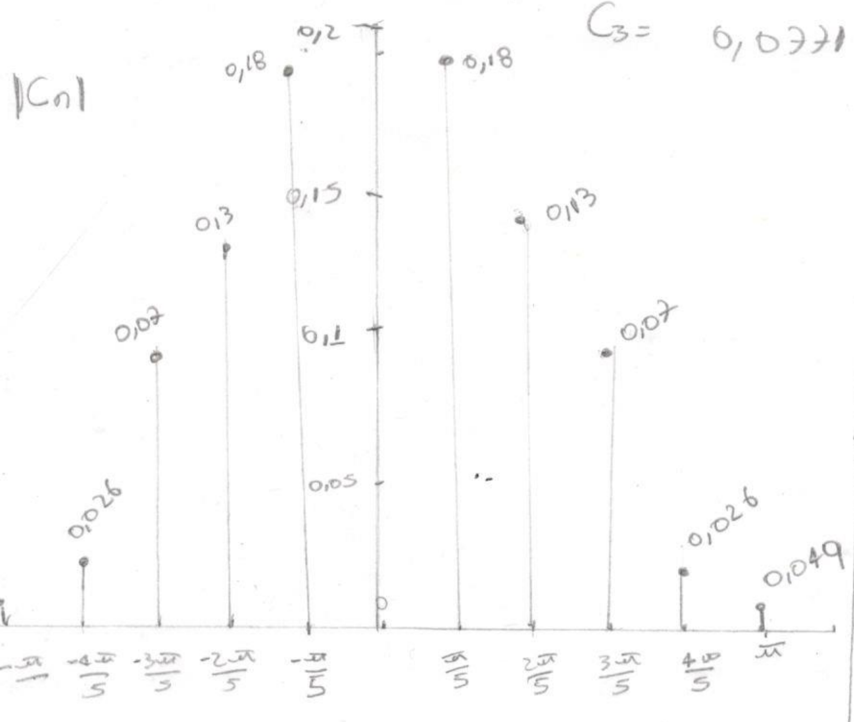
$$C_n = -\frac{1}{5\,(n^2\omega_0^2-1)}\left[\cos\left(\frac{\pi n\omega_0}{2}\right)\right]$$

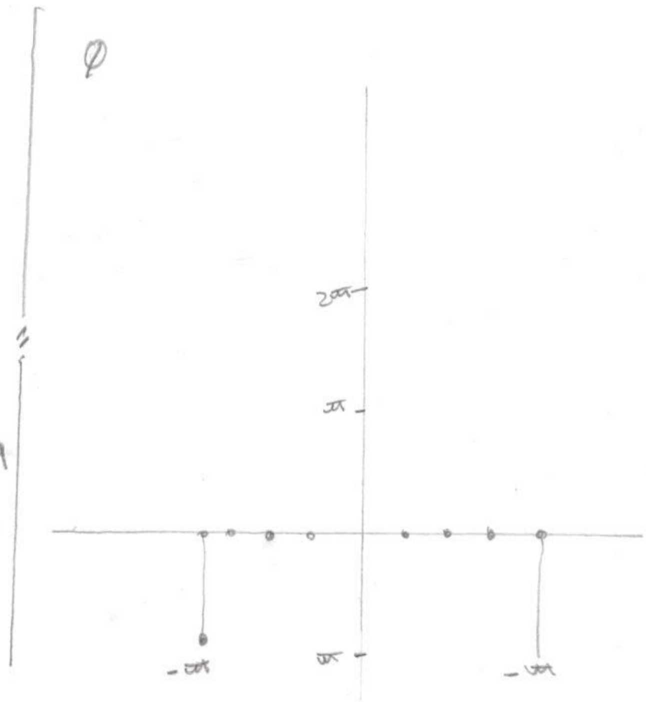
$$C_n = 5\left[\frac{-\cos(\pi^2 n/10)}{\pi^2 n^2 - 25}\right]$$

$C_1 = 0{,}1821$ $\quad C_4 = 0{,}022604$

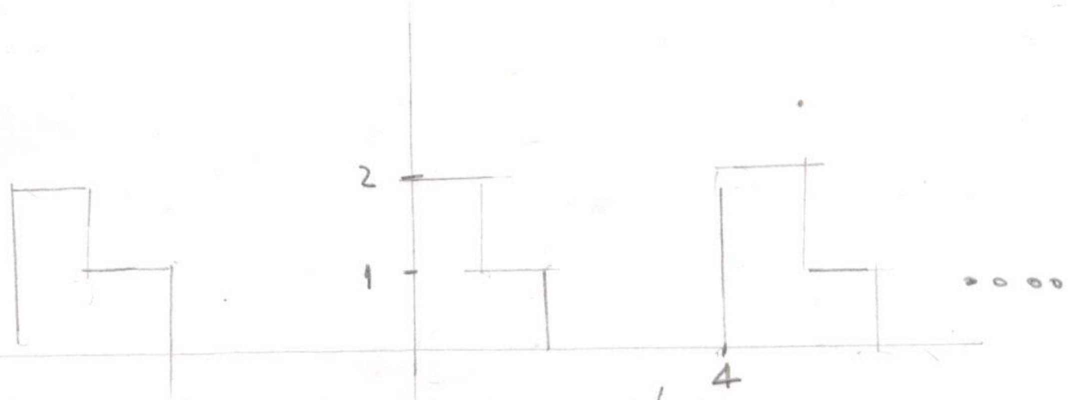
$C_2 = 0{,}1355$ $\quad C_5 = -0{,}00497$

$C_3 = 0{,}0771$





3.1



$T=4 \quad \omega_0 = 2\pi/T = \pi/2 \quad f(t) = \begin{cases} 2 & 0 \le t \le 1 \\ 1 & 1 \le t \le 2. \end{cases}$

$$a_0 = \frac{2}{T}\left[\int_0^1 2\,dt + \int_1^2 dt\right] = \frac{2}{4}\left[2t\Big|_0^1 + t\Big|_1^2\right] = \frac{1}{2}\left[2(1-0) + (2-1)\right] =$$

$$\frac{1}{2}(3) = \frac{3}{2} \checkmark$$

$$\bullet\; a_n = \frac{2}{T}\left[2\int_0^1 \mathrm{Cos}(n\omega_0 t)\,dt + \int_1^2 \mathrm{Cos}(n\omega_0 t)\,dt\right] = \frac{2}{4}\left[\frac{2\,\mathrm{Sen}(n\omega_0 t)}{n\omega_0}\Big|_0^1\right.$$

$$\left. + \frac{\mathrm{Sen}(n\omega_0 t)}{n\omega_0}\Big|_1^2\right] = \frac{1}{2n\omega_0}\left[2\,\mathrm{Sen}(n\pi/2) - \overset{0}{\mathrm{Sen}(0)} + \overset{0}{\mathrm{Sen}(n\pi)} - \mathrm{Sen}(n\pi/2)\right]$$

$$a_n = \frac{1}{n\pi}\left[\mathrm{Sen}(n\pi/2)\right] \Rightarrow a_n$$

$$b_n = \frac{2}{T}\left[2\int_0^1 \mathrm{Sen}(n\omega_0 t)\,dt + \int_1^2 \mathrm{Sen}(n\omega_0 t)\,dt\right] = \frac{2}{4}\left[\frac{-2\,\mathrm{Cos}(n\omega_0 t)}{n\omega_0}\Big|_0^1\right.$$

$$\left. - \frac{\mathrm{Cos}(n\omega_0 t)}{n\omega_0}\Big|_1^2\right] = \frac{1}{2n\omega_0}\left[-2\left(\mathrm{Cos}(n\pi/2) - \overset{1}{\mathrm{Cos}(0)}\right) - \left(\mathrm{Cos}(n\pi) - \mathrm{Cos}(n\pi/2)\right)\right]$$

$$b_n = \frac{1}{n\pi}\left[-\mathrm{Cos}(n\pi/2) - (-1)^n + 2\right] \Rightarrow b_n$$

$$f(t) = \frac{3}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{n\pi}\left[\mathrm{Sen}(n\pi/2)\right]\mathrm{Cos}\left(\frac{n\pi t}{2}\right) + \frac{1}{n\pi}\left[-\mathrm{Cos}\left(\frac{n\pi}{2}\right) - (-1)^n + 2\right] \mathrm{Sen}\left(\frac{n\pi t}{2}\right)\right]$$

3.2

- Compleja => Serie de Fourier.

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-inw_0 t}\, dt$$

$$C_0 = \frac{1}{4}\int_0^1 2\,dt + \frac{1}{4}\int_1^2 dt$$

$$C_0 = \frac{1}{2} + \frac{1}{4}(2-1) = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

$$C_n = \frac{1}{2}\int_0^1 e^{-inw_0 t}\, dt \; \frac{1}{4}\int_1^2 e^{-jnw_0 t}\, dt$$

$$C_n = \frac{e^{-jnw_0 t}}{-2jnw_0}\Big|_0^1 - \frac{e^{-jnw_0 t}}{4jnw_0}\Big|_1^2$$

$$C_n = -\frac{e^{-jnw_0}}{2jw_0} + \frac{1}{2jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} + \frac{e^{-jnw_0}}{4jnw_0} =$$

$$C_n = \frac{e^{-jnw_0}}{2jnw_0}\left(-\frac{2}{2} + \frac{1}{2}\right) + \frac{1}{2jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} =$$

$$C_n = -\frac{e^{-jnw_0}}{4jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} + \frac{1}{2jnw_0} =$$

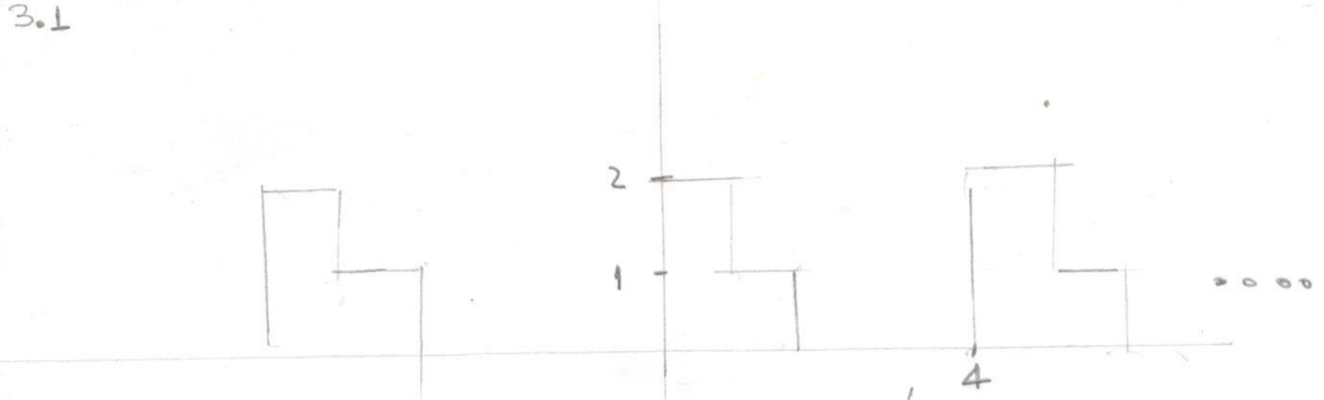
$$C_n = \frac{je^{-jnw_0}}{4nw_0} + \frac{je^{-2jw_0}}{4nw_0} - \frac{1}{2nw_0}$$

$$C_n = \frac{1}{n\pi}\left(\frac{e^{-jnw_0}}{2} + \frac{e^{-jn\pi}}{2} - 1\right) =$$

$$C_n = \frac{j}{2\pi n}\left(e^{-jn\pi/2} + (-1)^n - 2\right) =$$

$$C_1 = \frac{j}{2\pi}\left(e^{-1\pi/2} - 3\right) = \frac{j}{2\pi}(-j - 3)$$

3.1



$T=4 \quad \omega_0 = 2\pi / T = \pi/2 \quad f(t) = \begin{cases} 2 & 0 \le t \le 1 \\ 1 & 1 \le t \le 2. \end{cases}$

$$a_0 = \frac{2}{T}\left[\int_0^1 2\,dt + \int_1^2 dt\right] = \frac{2}{4}\left[2t\Big|_0^1 + t\Big|_1^2\right] = \frac{1}{2}\left[2(1-0)+(2-1)\right] =$$

$$\frac{1}{2}(3) = \frac{3}{2} \checkmark$$

$$a_n = \frac{2}{T}\left[2\int_0^1 \cos(n\omega_0 t)\,dt + \int_1^2 \cos(n\omega_0 t)\,dt\right] = \frac{2}{4}\left[\frac{2\,\mathrm{Sen}(n\omega_0 t)}{n\omega_0}\Big|_0^1\right.$$

$$\left. + \frac{\mathrm{Sen}(n\omega_0 t)}{n\omega_0}\Big|_1^2\right] = \frac{1}{2n\omega_0}\left[2\,\mathrm{Sen}(n\pi/2) - \overset{0}{\mathrm{Sen}(0)} + \overset{0}{\mathrm{Sen}(n\pi)} - \mathrm{Sen}(n\pi/2)\right]$$

$$a_n = \frac{1}{n\pi}\left[\mathrm{Sen}(n\pi/2)\right] \Rightarrow a_n$$

$$b_n = \frac{2}{T}\left[2\int_0^1 \mathrm{Sen}(n\omega_0 t)\,dt + \int_1^2 \mathrm{Sen}(n\omega_0 t)\,dt\right] = \frac{2}{4}\left[\frac{-2\cos(n\omega_0 t)}{n\omega_0}\Big|_0^1\right.$$

$$\left. - \frac{\cos(n\omega_0 t)}{n\omega_0}\Big|_1^2\right] = \frac{1}{2n\omega_0}\left[-2(\cos(n\pi/2) - \overset{1}{\cos(0)}) - (\cos(n\pi) - \cos(n\pi/2))\right]$$

$$b_n = \frac{1}{n\pi}\left[-\cos(n\pi/2) - (-1)^n + 2\right] \Rightarrow b_n$$

$$f(t) = \frac{3}{4} + \sum_{n=1}^{\infty}\left[\frac{1}{n\pi}\left[\mathrm{Sen}(n\pi/2)\right]\cos\left(\frac{n\pi t}{2}\right) + \frac{1}{n\pi}\left[-\cos\left(\frac{n\pi}{2}\right) - (-1)^n + 2\right]\mathrm{Sen}\left(\frac{n\pi t}{2}\right)\right]$$

3.2

- Compleja $\Rightarrow$ Serie de Fourier.

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-inw_0t}\, dt$$

$$C_0 = \frac{1}{4}\int_0^1 2\, dt + \frac{1}{4}\int_1^2 dt$$

$$C_0 = \frac{1}{2} + \frac{1}{4}(2-1) = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

$$C_n = \frac{1}{2}\int_0^1 e^{-inw_0t}\, dt \ \frac{1}{4}\int_1^2 e^{-jnw_0t}\, dt$$

$$C_n = \frac{e^{-jnw_0t}}{-2jnw_0}\Bigg|_0^1 - \frac{e^{-jnw_0t}}{4jnw_0}\Bigg|_1^2$$

$$C_n = -\frac{e^{-jnw_0}}{2jw_0} + \frac{1}{2jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} + \frac{e^{-jnw_0}}{4jnw_0} =$$

$$C_n = \frac{e^{-jnw_0}}{2jnw_0}\left(-\frac{2}{2} + \frac{1}{2}\right) + \frac{1}{2jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} =$$

$$C_n = -\frac{e^{-jnw_0}}{4jnw_0} - \frac{e^{-2jnw_0}}{4jnw_0} + \frac{1}{2jnw_0} =$$

$$C_n = \frac{je^{-jnw_0}}{4nw_0} + \frac{je^{-2jw_0}}{4nw_0} - \frac{1}{2nw_0}$$

$$C_n = \frac{1}{n\pi}\left(\frac{e^{-jnw_0}}{2} + \frac{e^{-jn\pi}}{2} - 1\right) =$$

$$C_n = \frac{j}{2\pi n}\left(e^{-jn\pi/2} + (-1)^n - 2\right) =$$

$$C_1 = \frac{j}{2\pi}\left(e^{-1\pi/2} - 3\right) = \frac{j}{2\pi}(-j - 3)$$

3.b

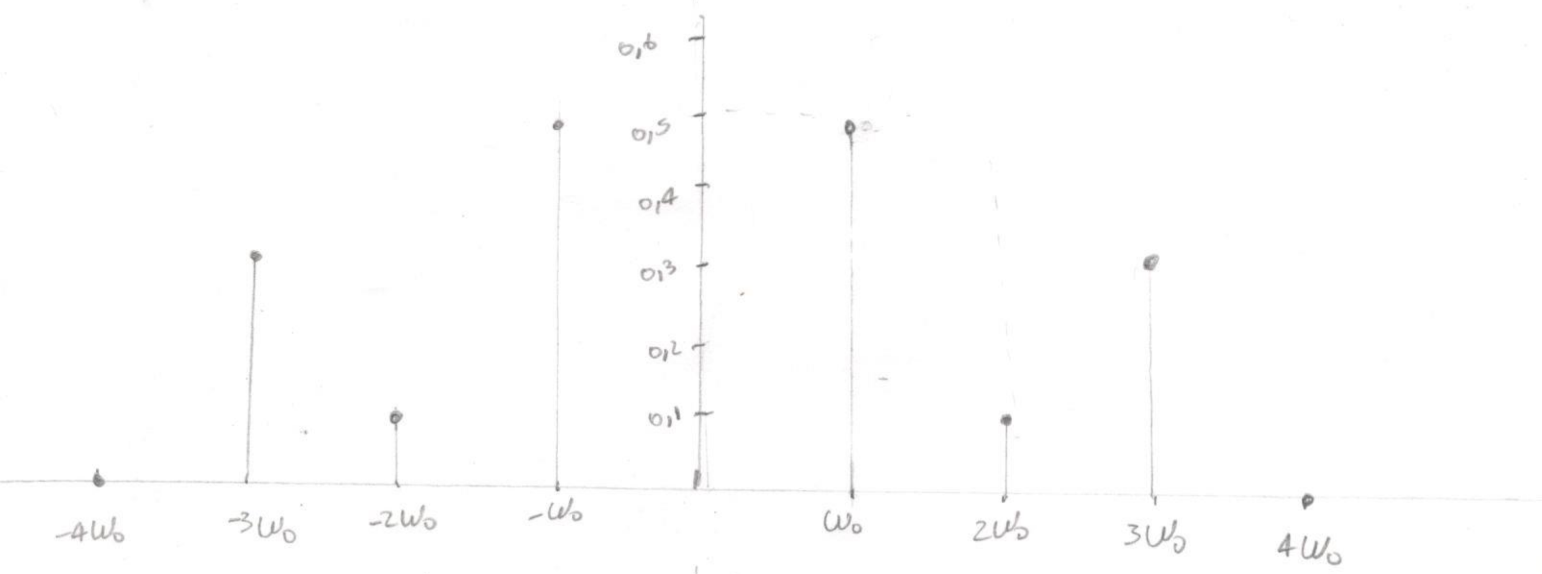
$C_1 = \frac{1}{2\pi} - \frac{3j}{2\pi} \Rightarrow |C_1| = 0{,}30 \; ; \; \theta = -1{,}24$

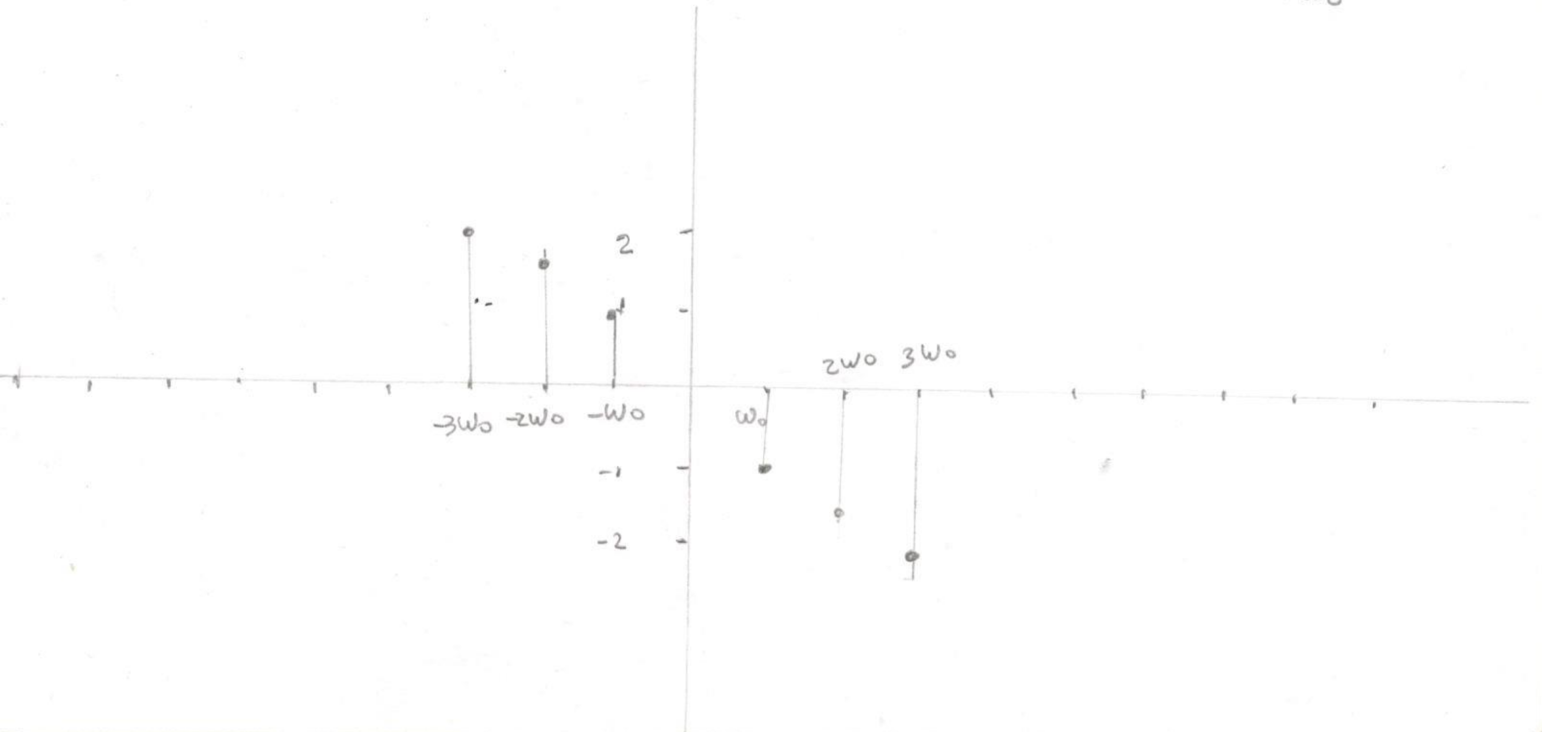
$C_2 = \frac{-j}{3\pi} \Rightarrow |C_2| = 0{,}15 \; ; \; \theta = -\frac{\pi}{2}$

$C_3 = \frac{-1}{6\pi} \frac{-j}{2\pi} \Rightarrow |C_3| = 0{,}16 \; ; \; \theta = -1{,}33$

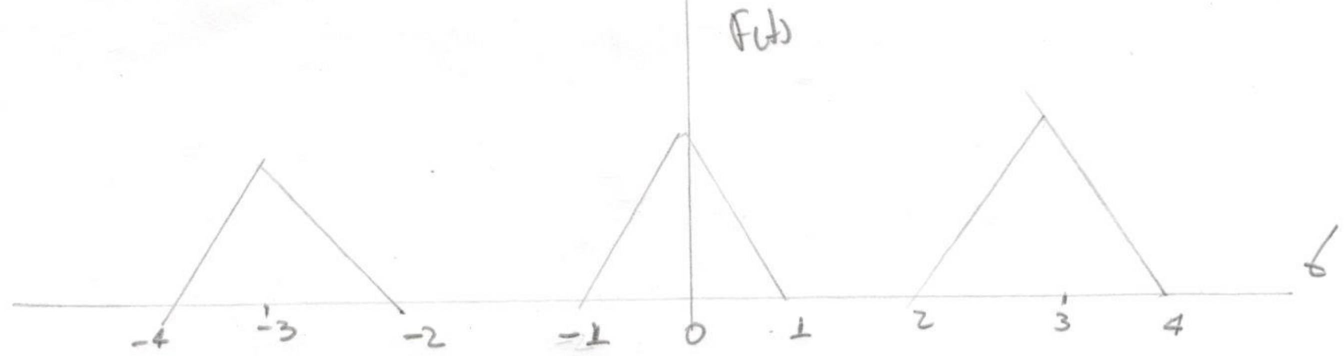
$C_4 = 0$

$C_5 = \frac{1}{10\pi} - \frac{3j}{10\pi} \Rightarrow |C_5| = 0{,}10 \; ; \; \theta = -1{,}24$





4.1



$T=3 \quad \omega = \frac{2\pi}{T} \qquad m = \frac{1+0}{0+1} = \frac{1}{1} = 1 \Rightarrow \quad y-0 = 1(x+1) \Rightarrow y = x+1$

$m = \frac{1+0}{0-1} = \frac{1}{-1} = -1 \quad y-0 = -1(x-1) \Rightarrow y = -x+1$

$a_0 = \frac{2}{T}\left[\int_{-1}^{0}(t+1)\,dt + \int_{0}^{1}(-t+1)\,dt\right] =$

$a_0 = \frac{2}{T}\left[\frac{t^2}{2} + t\,\Big|_{-1}^{0} + \left(-\frac{t^2}{2} + t\right)\Big|_{0}^{1}\right] =$

$a_0 = \frac{2}{T}\left[0+0-\left(\frac{1}{2}-1\right) + \left(-\frac{1}{2}+1\right) - (0+0)\right]$

$a_0 = \frac{2}{T}\left[-1+1+1\right] = \frac{2}{3}(1) = a_0 = \frac{2}{3}$

$a_n = \frac{2}{T}\left[\overset{1}{\int_{-1}^{0}(t+1)\cos(n\omega_0 t)\,dt} + \overset{2}{\int_{0}^{1}(-t+1)\cos(n\omega_0 t)\,dt}\right]$

Integral 1

$\int_{-1}^{0} t\cos(n\omega_0 t)\,dt + \int_{0}^{1}(-t+1)\cos(n\omega_0 t)\,dt$

$u = t \qquad du = dt$

$dv = \cos(n\omega_0 t) \quad v = \frac{\operatorname{Sen}(n\omega_0 t)}{n\omega_0 t}$

$\frac{t\operatorname{Sen}(n\omega_0 t)}{n\omega_0 t} - \int \frac{\operatorname{sen}(n\omega_0 t)}{n\omega_0}\,dt + \frac{\operatorname{Sen}(n\omega_0 t)}{n\omega_0}\Big|_{-1}^{0}$

$\frac{t\operatorname{Sen}(n\omega_0 t)}{n\omega_0 t} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2} + \frac{\operatorname{Sen}(n\omega_0 t)}{n\omega_0}\Big|_{-1}^{0}$

4.2

$$= \frac{1}{n\omega_0}\left[t\, \text{Sen}(n\omega_0 t) + \frac{\text{Cos}(n\omega_0 t)}{n\omega_0} + \text{Sen}(n\omega_0 t) \right] \Big|_{-1}^{0}$$

$$= \frac{1}{n\omega_0}\left[0+0-1-\left(\text{Cos}(-n\omega_0) + \frac{\text{Sen}(-n\omega_0)}{n\omega_0} - \text{Cos}(-n\omega_0) \right) \right]$$

$$= \frac{1}{n\omega_0}\left[-1-\text{Cos}(-n\omega_0) - \frac{\text{Sen}(-n\omega_0)}{n\omega_0} + \text{Cos}(-n\omega_0) \right]$$

$$a_n = \frac{1}{n\omega_0}\left[-1 - \frac{\text{Sen}(-n\omega_0)}{n\omega_0} \right]$$

Integral 2

$$\int_0^1 (-t+1)\, \text{Sen}(n\omega_0 t)\, dt$$

$$\int_0^1 t\, \text{Sen}(n\omega_0 t)\, dt + \int_0^1 \text{Sen}(n\omega_0 t)\, dt$$

$$= -\left[-\frac{t\,\text{Cos}(n\omega_0 t)}{n\omega_0} - \int -\frac{\text{Cos}(n\omega_0 t)}{n\omega_0 t}\, dt \right] - \frac{\text{Cos}(n\omega_0 t)}{n\omega_0} \Big|_0^1$$

$$= -\left[-\frac{t\,\text{Cos}(n\omega_0 t)}{n\omega_0} + \frac{\text{Sen}(n\omega_0 t)}{(n\omega_0)^2} \right] - \frac{\text{Cos}(n\omega_0 t)}{n\omega_0} \Big|_0^1$$

$$= \frac{-1}{n\omega_0}\left[-t\,\text{Cos}(n\omega_0 t) + \frac{\text{Sen}(n\omega_0 t)}{n\omega_0} + \text{Cos}(n\omega_0 t) \right] \Big|_0^1$$

$$= \frac{-1}{n\omega_0}\left[-\text{Cos}(n\omega_0) + \frac{\text{Sen}(n\omega_0)}{n\omega_0} + \text{Cos}(n\omega_0) - 1 \right]$$

$$= \frac{1}{n\omega_0}\left[\frac{\text{Sen}(n\omega_0)}{n\omega_0} - 1 \right]$$

4.3

- $b_n = \frac{2}{T}\left[\cancel{\frac{-1}{(n\omega_0)^2}} - \cancel{\frac{Sen(-n\omega_0)}{(n\omega_0)^2}} - \cancel{\frac{Sen(n\omega_0)}{(n\omega_0)^2}} + \cancel{\frac{1}{(n\omega_0)^2}} \right]$

$b_n = \frac{2}{T}(0) \quad b_n = 0$

- Serie Compleja

$$\underbrace{\int_0^1 -t\, e^{-jn\omega t}\, dt}_{0} + \underbrace{\int_0^1 e^{-jn\omega_0 t}\, dt}_{2}$$

- $\int_0^1 t\, e^{-jn\omega t}\, dt = -\left[\frac{t e^{-jn\omega t}}{-jn\omega} - \frac{e^{-jn\omega t}}{(-jn\omega)^2} \right]$

- $\frac{e^{-jn\omega t}}{-jn\omega} = \frac{e^{-jn\omega t}}{-jn\omega} - \frac{t e^{-jn\omega t}}{-jn\omega} + \frac{e^{-jn\omega t}}{-jn\omega} \Big|_0^1$

$= \left(\frac{e^{-jn\omega t}}{(-jn\omega)^2} - \frac{e^{jn\omega}}{-jn\omega} + \frac{e^{-jn\omega}}{-jn\omega} \right) - \left(\frac{1}{(-jn\omega)^2} - 0 + \frac{1}{-jn\omega} \right)$

$= \frac{1}{3jn\omega}\left[\frac{-1}{jn\omega} + 2e^{jn\omega} + \frac{e^{jn\omega}}{jn\omega} \cancel{-1} + \frac{e^{-jn\omega}}{jn\omega} - \frac{1}{jn\omega} \cancel{+1} \right]$

$= \frac{1}{3jn\omega}\left[-\frac{2}{jn\omega} + 2e^{jn\omega} + 2\frac{e^{jn\omega}}{jn\omega} \right]$

4.4

$$= \frac{2}{3jn\omega}\left[-\frac{1}{jn\omega} + e^{jn\omega} + \frac{e^{jn\omega}}{jn\omega}\right]$$

$$\frac{1}{3}\underbrace{\int_{-1}^{0} t\, e^{-jn\omega t}}_{①} dt + \underbrace{\int_{-1}^{0} e^{-jn\omega t}}_{2}$$

$$\int_{-1}^{0} t\, e^{-jn\omega t}$$

$u = t \quad du = dt$

$dv = e^{-jn\omega t} \quad V = \frac{e^{-jn\omega t}}{jn\omega}$

$$= \frac{t\,e^{-jn\omega t}}{-jn\omega} - \int_{-1}^{0} \frac{e^{-jn\omega t}}{-jn\omega}\, dt$$

$$= \frac{t\,e^{-jn\omega t}}{-jn\omega} - \frac{e^{-jn\omega t}}{(-jn\omega)^2} =$$

$$= \frac{t\,e^{-jn\omega t}}{-jn\omega} - \frac{e^{-jn\omega t}}{(-jn\omega)^2} + \frac{e^{-jn\omega t}}{-jn\omega}\Big|_{-1}^{0}$$

$$\left[\left(0 - \frac{1}{(-jn\omega)^2}\right) - \left(-\frac{e^{jn\omega}}{jn\omega} - \frac{e^{jn\omega}}{(-jn\omega)^2}\right)\right] + \left[\frac{1}{-jn\omega} - \frac{e^{jn\omega}}{-jn\omega}\right]$$

$$= \left[-\frac{1}{(-jn\omega)^2} + \frac{e^{jn\omega}}{jn\omega} + \frac{e^{jn\omega}}{(-jn\omega)^2} - \frac{1}{jn\omega} + \frac{e^{jn\omega}}{jn\omega}\right.$$

$$= \frac{1}{jn\omega}\left[-\frac{1}{jn\omega} + e^{jn\omega} + e^{jn\omega} - 1 + e^{jn\omega}\right]$$

$$= \frac{1}{jn\omega}\left[2e^{jn\omega} + \frac{e^{jn\omega}}{jn\omega} - \frac{1}{jn\omega} - 1\right]$$

$$a_0 = \frac{2}{3} \qquad \omega_0 = \frac{2\pi}{3} \qquad T = 3$$

$$a_n = -\frac{3}{2\pi n}\left[1 + \frac{3\,\mathrm{sen}\left(-\frac{2\pi n}{3}\right)}{2\pi}\right]$$

$$P_{media} = \frac{1}{2}\left(\frac{a_0^2}{2} + \sum_{n=1}^{+\infty}\left(a_n^2 + b_n^2\right)\right)$$

$$P_{media} = \frac{1}{2}\left(\frac{\frac{2^2}{3^2}}{2} + \sum_{n=1}^{5}\left[\left(-\frac{3}{2\pi n}\left(1 + \frac{3\,\mathrm{sen}\left(-\frac{2\pi n}{3}\right)}{2\pi}\right)\right)^2 + 0^2\right]\right)$$

$$P_{media} = \frac{1}{2}\left(\frac{4}{18_9} + \sum_{n=1}^{5}\frac{36\pi^2 - 108\pi\,\mathrm{sen}\left(\frac{2\pi n}{3}\right) + 81\,\mathrm{sen}\left(\frac{2\pi n}{3}\right)^2}{16\pi^4 n^4}\right)$$

$$P_{media} = \frac{1}{2}\left(\frac{2}{9} + \frac{24606864\pi^2 - 32479704\pi\sqrt{3} + 47524083}{10240000\pi^4} + \frac{1}{36\pi^2}\right)$$

$$P_{media} = \frac{1}{2}\left(0{,}3329589376\right)$$

$$P_{media} = 0{,}1664794688$$

GRÁFICAS SERIES DE FOURIER CON N=10

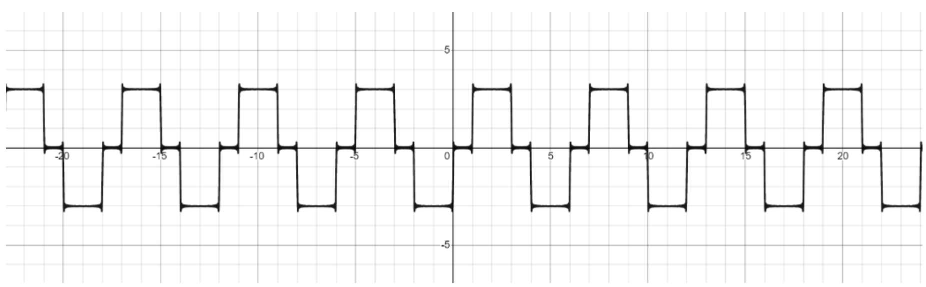
April 17, 2021

1 Gráficas Series de Fourier

Serie Trigonmétrica de Fourier Uno

$$y = \sum_{n=1}^{a} \left(\left(-\frac{3}{n\pi} \left(\sin\left(\frac{2n\pi}{3} \right) + \sin\left(\frac{n\pi}{3} \right) \right) \cos\left(\frac{n\pi x}{3} \right) \right) + \frac{3}{n\pi} \left(1 - (-1)^{n} - \cos\left(\frac{2n\pi}{3} \right) + \cos\left(\frac{n\pi}{3} \right) \right) \sin\left(\frac{n\pi x}{3} \right) \right)$$

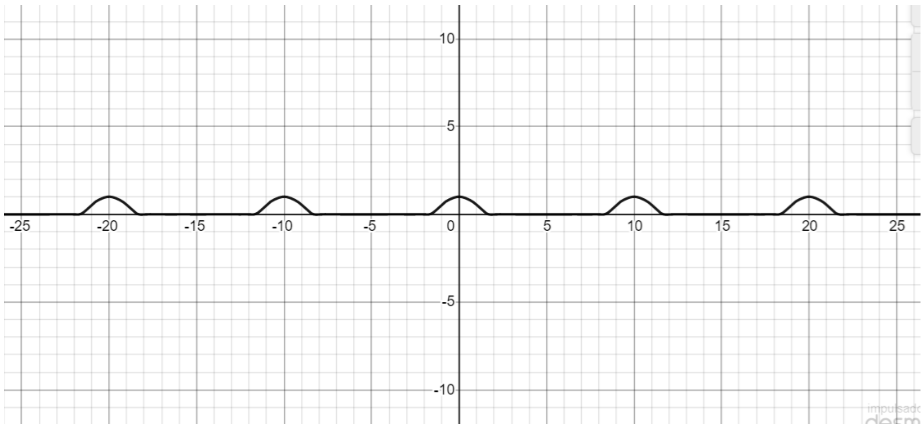
Gráfica Serie Trgonométrica de Fourier Uno



Serie Trigonométrica de Fourier Dos

$$y = \frac{1}{5} + \sum_{n=1}^{a} 10 \left(-\frac{\cos\left(\frac{\pi^2 n}{10} \right)}{\pi^2 n^2 - 25} \right) \cos\left(\frac{\pi n x}{5} \right)$$

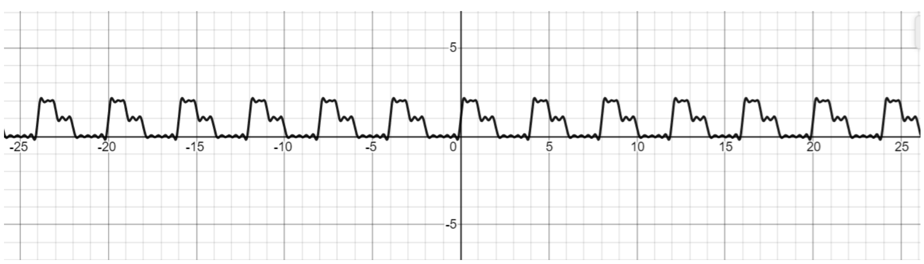
Gráfica Serie Trgonométrica de Fourier Dos



Serie Trigonométrica de Fourier Tres

$$y = \frac{3}{4} + \sum_{n=1}^{a} \left(\frac{1}{n\pi} \left(\sin \left(\frac{n\pi}{2} \right) \right) \cos\left(\frac{n\pi x}{2} \right) + \frac{1}{n\pi} \left(-\cos\left(\frac{n\pi}{2} \right) - (-1)^{n} + 2 \right) \sin\left(\frac{n\pi x}{2} \right) \right)$$

Gráfica Serie Trgonométrica de Fourier Tres



Serie Trigonométrica de Fourier Cuatro

$$y = \frac{2}{3} + \sum_{n=1}^{a} \frac{1}{\frac{n2\pi}{3}} \left(\left(1 - \frac{\sin\left(\left(-\frac{n\pi}{3} \right) \right)}{\frac{n\pi}{3}} \right) \cos\left(\frac{2\pi}{3} nx \right) \right)$$

Gráfica Serie Trgonométrica de Fourier Cuatro