Behzad Razavi - RF Microelectronics Solution (2012)

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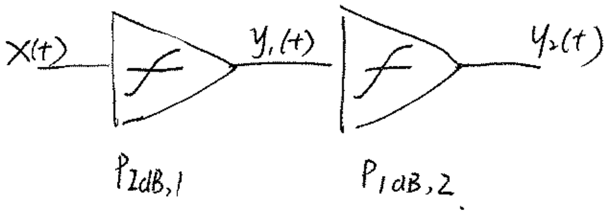
2.1 Solu:



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_2^2(t) + \beta_3 y_3^2(t)$$

then. $$y_2(t) = \beta_1 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3 [\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3$$

Considering only the first- and third-order terms,

$$y_2(t) = \alpha_1 \beta_1 x(t) + (\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) x^3(t) + \cdots$$

$$= [\alpha_1 \beta_1 A + \frac{3}{4}(\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) A^3] \cos(\omega t) + \cdots$$

$P_{1dB,1}$: $A_{in1,1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|}$; $P_{1dB,2}$: $A_{in2,1dB} = \sqrt{0.145 \left|\frac{\beta_1}{\beta_3}\right|}$

$P_{1dB} \Rightarrow$ $$20 \log \left| \alpha_1 \beta_1 + \frac{3}{4}(\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3) \cdot A_{in,1dB}^2 \right| = 20 \log |\alpha_1 \beta_1| - 1dB$$

$$A_{in,1dB} = \sqrt{0.145 \left| \frac{\alpha_1 \beta_1}{\alpha_3 \beta_1 + 2\alpha_1 \alpha_2 \beta_2 + \alpha_1^3 \beta_3} \right|}$$

Represented by the P_{1dB} of first and second stage.

$$\frac{1}{A_{in,1dB}^2} = \frac{1}{0.145} \left| \frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2 \beta_3}{\beta_1} \right|$$

$$= \left| \frac{1}{A_{in1,1dB}^2} + \frac{2}{0.145} \frac{\alpha_2 \beta_2}{\beta_1} + \frac{\alpha_1^2}{A_{in2,1dB}^2} \right|$$

2.2 Solu:

assuming $-3\,dBm$ A_1 at $2.42\,G$

$-35\,dBm$ A_2 at $2.43\,G$.

IM product : $\frac{3}{4}\gamma_3 A_1^2 \cdot A_2$

$-3\,dBm \Rightarrow \quad A_1 = \sqrt{2 \cdot 50 \cdot 10^{-0.3} \times 10^{-3}} = 223.9\,mV$

$-35\,dBm \Rightarrow \quad A_2 = \sqrt{2 \cdot 50 \times 10^{-3.5} \times 10^{-3}} = 5.6\,mV.$

We can write at LNA output :

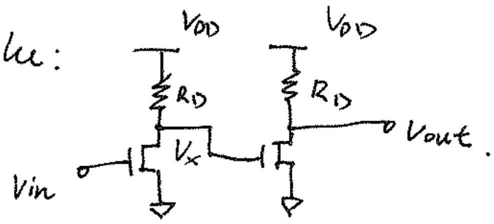
$$20\lg|\gamma_1 \cdot A_{sig}| - 20\,dB = 20\lg\left|\frac{3}{4}\gamma_3 A_1^2 \cdot A_2\right|.$$

$$\Rightarrow \quad \lg|\gamma_1 \cdot A_{sig}| = \lg\left|\frac{30}{4}\gamma_3 A_1^2 \cdot A_2\right|$$

$$IIP_3 = \sqrt{\frac{4}{3}\left|\frac{\gamma_1}{\gamma_3}\right|} = \sqrt{\frac{4}{3} \cdot \frac{30}{4} \cdot \frac{A_1^2 \cdot A_2}{A_{sig}}} = 9.43\,V_p$$

$$= 29.5\,dBm.$$

2.3 solu:



$$I_D = K \cdot (V_{GS} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D (V_x - V_T)^2$$

$$V_x = V_{DD} - K \cdot R_D (V_{in} - V_T)^2$$

$$V_{out} = V_{DD} - K \cdot R_D [\, V_{DD} - K \cdot R_D (V_{in} - V_T)^2 - V_T \,]^2$$

$$= V_{DD} - K \cdot R_D [\, (V_{DD} - V_T) - K \cdot R_D (V_{in} - V_T)^2 \,]^2$$

$$= V_{DD} - K \cdot R_D [\, (V_{DD} - V_T)^2 + K^2 R_D^2 (V_{in} - V_T)^4 - 2 K R_D (V_{DD} - V_T)(V_{in} - V_T)^2 \,]$$

1st order of V_{in}

$$\Rightarrow [\, 4 \cdot (V_{DD} - V_T) \cdot K \cdot R_D V_T - 4 K^2 R_D^2 \cdot V_T^3 \,] \cdot V_{in}$$

3rd order of V_{in}

$$\Rightarrow [\, -4 K^2 R_D^2 V_T \,] \cdot V_{in}^3$$

$$A_{IP3} = \sqrt{\frac{4}{3} \cdot \frac{4 \cdot (V_{DD} - V_T) K \cdot R_D V_T - 4 K^2 R_D^2 V_T^3}{4 K^2 R_D^2 V_T}}$$

2.4. Solu.

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \alpha_4 x^4(t) + \alpha_5 x^5(t).$$

1° $\cos^3\omega t = \frac{3}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t.$

2° $\cos^2\omega t = \frac{1+\cos 2\omega t}{2}$

3° $\cos^4\omega t = \frac{1+\cos^2 2x + 2\cos 2x}{2^2}$

4° $\cos^5\omega t = \left(\frac{3}{4}\cos\omega t + \frac{1}{4}\cos 3\omega t\right)\cdot\left(\frac{1+\cos 2\omega t}{2}\right).$

$$= \Big(\frac{3}{8}\cos\omega t + \frac{1}{8}\cos 3\omega t + \underbrace{\frac{3}{8}\cos\omega t\cdot\cos 2\omega t}_{=\frac{3}{16}[\cos\omega t + \cos 3\omega t]} + \underbrace{\frac{1}{8}\cos 3\omega t\cdot\cos 2\omega t}_{=\frac{1}{16}[\cos\omega t + \cos 5\omega t]}\Big)$$

1st order $\Rightarrow$. $\alpha_1 A + \frac{3}{4}\alpha_3 A^3 + \left(\frac{3}{8} + \frac{3}{16} + \frac{1}{16}\right)\cdot\alpha_5 A^5.$

3rd order

$\Rightarrow$ $\frac{1}{4}\alpha_3 A^3 + \left(\frac{1}{8} + \frac{3}{16}\right)\alpha_5 \cdot A^5.$

(1) P1dB $\Rightarrow$. $20\lg\left|\alpha_1 + \frac{3}{4}\alpha_3 A^2 + \frac{5}{8}\alpha_5 A^4\right| = 20\lg|\alpha_1| - 1\,dB.$

$$\Rightarrow A_{in,1dB} = \sqrt{\frac{0.8\cdot(0.5625\,\alpha_3^2 - 0.27175\,\alpha_1\cdot\alpha_5)^{\frac{1}{2}} - 0.6\cdot\alpha_3}{\alpha_5}}.$$

(2) IIP3 doesn't change.

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}.$$

2.5 (a) Solu: $A_{sig} = 0.1mV \times 0.7943 \Leftarrow -2dB = 10^{-0.1} = 0.7943$

$A_2 = 10mV \times 0.1413 \Leftarrow -17dB = 10^{-0.85} = 0.1413$

$A_3 = 10mV \times 0.0141 \Leftarrow -37dB = 10^{-1.85} = 0.0141$

at the output of amplifier:

$20\lg|\gamma_1 \cdot A_{sig}| - 20dB = 20\lg|\frac{3}{4}\gamma_3 \cdot A_2^3 \cdot A_3|$

$\Rightarrow |\gamma_1 \cdot 0.07943m| = |\frac{30}{4}\gamma_3\, 1.413m^2 \cdot 0.141m|$

$\therefore A_{IIP3} = \sqrt{\frac{30}{4}\frac{1.413m^2 \cdot 0.141}{0.07943}\cdot\frac{4}{3}} = 5.95mV_p = -34.5dBm.$

Neglect the nonlinearity of BPF.

(b). $y_1(t) = \gamma_1 x(t) + \gamma_3 x^3(t)$

$y_2(t) = \beta_1 y_1(t) + \beta_3 y_1^3(t)$.

Only considering the first and third order:

$y_2(t) = \gamma_1\beta_1 x(t) + (\gamma_3\beta_1 + \gamma_1^3\beta_3)x^3(t) + \cdots$

$\therefore A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\gamma_1\beta_1}{\gamma_3\beta_1 + \gamma_1^3\beta_3}\right|}$

$\frac{1}{A_{IP3}^2} = \frac{1}{A_{IP3,1}^2} + \frac{\gamma_1^2}{A_{IP3,2}^2}$

$\Rightarrow \frac{1}{A_{IP3}^2} = \frac{1}{500m^2} + \frac{10}{5.95m^2}$

$\Rightarrow A_{IP3} = 1.875mV_p$

2.6 Solu:

Let $x(t)$ be a random signal (Wide-sense stationary process)

Auto correlation function: $R_x(\tau) = E[x(t) \cdot x(t+\tau)]$

Let me prove that : $S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$.

Proof : $X_T(f) \triangleq \int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt.$

$$S_T(f) \triangleq E[\frac{1}{T} |X_T(f)|^2]$$

$$S_x(f) = \lim_{T\to\infty} S_T(f).$$

$$E[|X_T(f)|^2] = E[\int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt]^2$$

$$= E[\int_{-T/2}^{T/2} x(t) e^{-j2\pi ft} dt \cdot \int_{-T/2}^{T/2} x(\tau) e^{-j2\pi f\tau} d\tau]$$

$$= E[\int_{-T/2}^{T/2}\int_{-T/2}^{T/2} x(t) x(\tau) e^{-j2\pi f(t-\tau)} dt\, d\tau]$$

$$= \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} E[x(t) \cdot x(\tau)] e^{-j2\pi f(t-\tau)} dt\, d\tau.$$

$$= \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} R_x(t-\tau) e^{-j2\pi f(t-\tau)} dt\, d\tau.$$

$$= \int_{-T}^{T} (T-|\tau|) R_x(\tau) e^{-j\pi f\tau} d\tau.$$

$$E[\frac{1}{T}|X_T(f)|^2] = \int_{-T}^{T} (1-\frac{|\tau|}{T}) R_x(\tau) e^{-j2\pi f\tau} d\tau.$$

Therefore : $S_x(f) = \lim_{T\to+\infty} E[\frac{1}{T}|X_T(f)|^2] = \int_{-\infty}^{+\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau.$

2.7. Solu:

Assume $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$.

$x(t) = V_0 \cos w_0 t$.

3rd - harmonic : $\frac{\alpha_3 V_0^3}{4} = V_3$

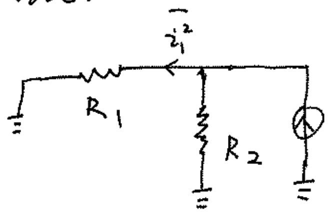
$\Rightarrow \alpha_3 = \frac{4V_3}{V_0^3}$

$$A_{in,1dB} = \sqrt{0.145 \left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= \sqrt{0.145 \cdot \left|\alpha_1 / 4V_3/V_0^3\right|}$$

$$= \sqrt{\frac{0.145}{4}\left|\frac{\alpha_1 V_0^3}{V_3}\right|}.$$

12.8 soln:



$\overline{v_n^2} = \frac{4KT}{R_2}$

$$P_{R_1} = \overline{i_1^2}^{\,2} \cdot R_1$$

$$= \left(\sqrt{\frac{4KT}{R_2}} \cdot \frac{R_2}{R_1+R_2}\right)^2 \cdot R_1$$

$$= \frac{4KT}{(R_1+R_2)^2} \cdot R_1 R_2 .$$

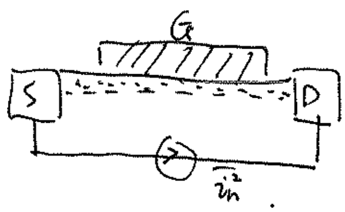
$\therefore P_{R_1} = P_{R_2}$

So it proves that the noise power delivered by R_1 to R_2 is equal to that delivered by R_2 to R_2 at the same temperature.

$\therefore$ If it is not the truth, the energy would not be conserved.

2.9 Solu:

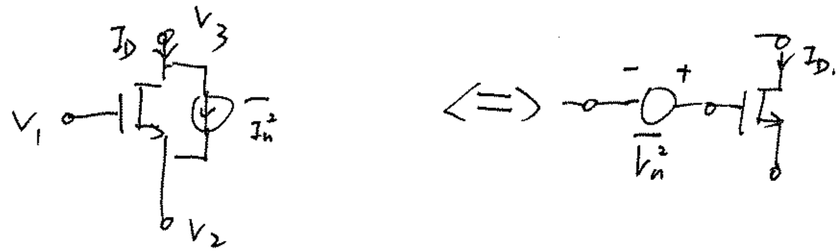
Why the channel thermal noise of a MOSFET is model by a current sourse bw. S & D. rather than G & D.



Firstly, from the figure we can find the channel resistor is between source & drain. As a result, it is reasonable to model the noise by a current source between source and drain.

Secondly, MOSFET has the function of transconductance. It's easy to transfer the current source ~~from~~ bewteen S & D to the voltage source at the gate.

2.10 Solu:



Proof: transconductance : g_m.

Assume the transistor is in saturation region.

For small-signal analysis, $V_1 = 0$

$$I_D = \sqrt{\overline{I_n^2}} = \sqrt{4KT\gamma g_m}.$$

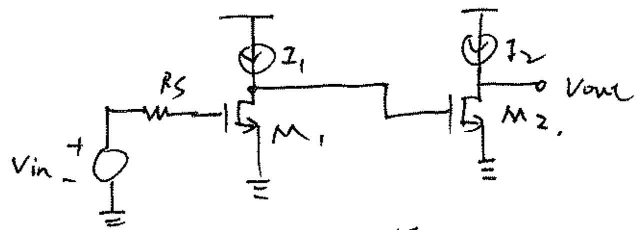
At the same time, for voltage source model.

$$I_D = V_{in} \cdot g_m = \sqrt{4KT\gamma \cdot g_m}$$

$$\Rightarrow \quad V_{in} = \sqrt{\frac{4KT\gamma}{g_m}}.$$

$$\Rightarrow \quad \overline{V_n^2} = \frac{4KT\gamma}{g_m}.$$

2.11 Solu:



$$NF_1 = 1 + \frac{\gamma}{g_{m1}R_S} \qquad (2.122)$$

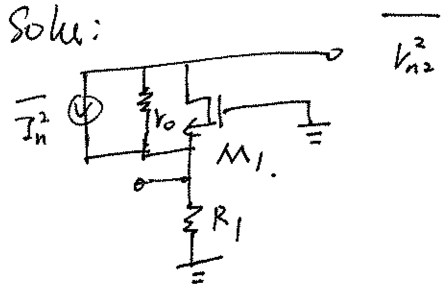
$$NF_2 = 1 + \frac{\gamma}{g_{m2}r_{o1}} \; ; \qquad A_{p1} = \frac{P_{out,av,1}}{P_{in,av,1}} = \frac{V_{in}^2 \cdot A_{V1}^2 \cdot \frac{1}{4r_{o1}}}{V_{in}^2 \cdot \frac{1}{4R_S}} = g_{m1}^2 \cdot r_{o1} \cdot R_S$$

$$\therefore NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}}$$

$$= 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m2} \cdot r_{o1}} \Big/ g_{m1}^2 r_{o1} R_S$$

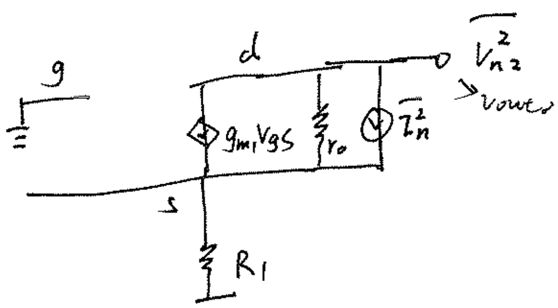
$$= 1 + \frac{\gamma}{g_{m1}R_S} + \frac{\gamma}{g_{m1}^2 r_{o1}^2 g_{m2} \cdot R_S}$$

12.12. Solu:



assume I_1 is ideal, and neglect the noise of R_1.

Proof:



For small-signal analysis,

$$g_m(-V_S) + \frac{V_{out} - V_S}{r_o} + I_n = \frac{V_S}{R_1}.$$

Because we cannot find any loop for the current through R_1

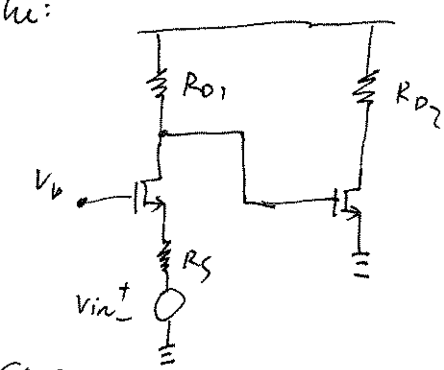
$$V_S = 0$$

$$\Rightarrow \quad \frac{V_{out}}{r_o} = -I_n$$

$$V_{out} = -I_n \cdot r_o$$

$$\therefore \quad \overline{V_{n2}^2} = \overline{I_{n2}^2} \cdot r_o^2.$$

2.13. Soln:

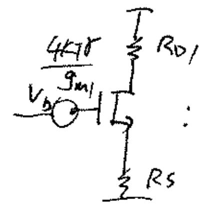


Neglect transistor cap.
flicker noise.
CLM.
body effect.

For 1st stage:

$R_{in1} = \frac{1}{g_{m1}}$, $R_{h2} = \infty$,

$$\overline{V_{n1}^2} = 4KTR_{D1} + \frac{4KT\gamma}{g_{m1}}\left(\frac{R_{D1}}{\frac{1}{g_{m1}}+R_S}\right)^2 \Leftarrow$$



: unloaded output noise.

For 2nd stage:

$$\overline{V_{n2}^2} = 4KTR_{D2} + 4KT\gamma \cdot g_{m2} \cdot R_{D2}^2 .$$

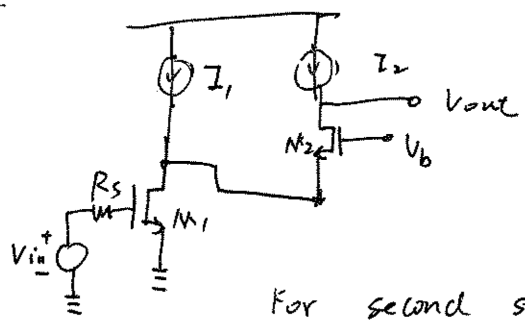
We now substitute these values in Eq. (2.126)

$$NF_{tot} = 1 + \frac{4KTR_{D1} + \frac{4KT\gamma}{g_{m1}}\left(\frac{R_{D1}}{\frac{1}{g_m}+R_S}\right)^2}{\left(\frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}}+R_S}\right)^2 (g_{m1} \cdot R_{D1})^2} \cdot \frac{1}{4KTR_S}$$

$$+ \frac{4KTR_D + 4KT\gamma \cdot g_{m2} \cdot R_{D2}^2}{\left(\frac{\frac{1}{g_{m1}}}{\frac{1}{g_{m1}}+R_S}\right)^2 (g_{m1} \cdot R_{D1})^2 \cdot (g_{m2} \cdot R_{D2})^2} \cdot \frac{1}{4KTR_S} .$$

This result is different from the CS + CG configuration because the first stage's NF and input impedance are different, which affect the NF_{tot}.

2.14. Solu:

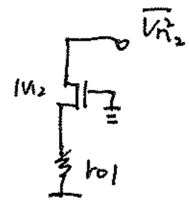


Consider CLM.

$$\overline{V_{n_1}^2} = \frac{4KT\gamma}{g_{m1}} \cdot (g_{m1} r_{o1})^2$$

$$= 4KT\gamma \cdot g_{m1} r_{o1}^2$$

For second stage:



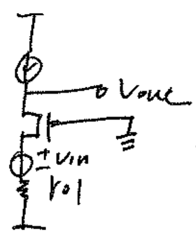
$$\overline{V_{n_2}^2} = 4KT\gamma \cdot g_{m2} \cdot r_{o2}^2$$

We now substitute these values in Eq. (2.126).

$$NF_{tot} = 1 + \frac{4KT\gamma \cdot g_{m1} r_{o1}^2}{(g_{m1} r_{o1})^2} \cdot \frac{1}{4KTR_S}$$

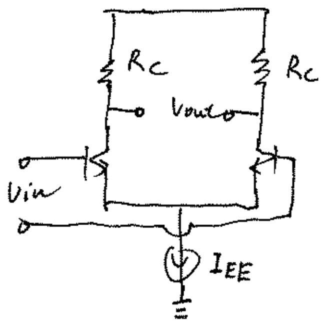
$$+ \frac{4KT\gamma\, g_{m2} r_{o2}^2}{(g_{m1} r_{o1})^2 \cdot \left(\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + r_{o1}}\right)^2 \cdot (g_{m2} r_{o2} + 1)^2} \cdot \frac{1}{4KTR_S}.$$

Note :



$\Rightarrow$ gain: $g_{m2} r_{o2} + 1$.

2.15 Soln:



IP3.

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \cdots$$

$$V_{out} = -2R_C I_{EE} \tanh\left[\frac{V_{in}}{2V_T}\right].$$

Only consider the first and third order.

$$V_{out} = -2R_C I_{EE} \cdot \left(\frac{V_{in}}{2V_T} - \frac{1}{3}\left(\frac{V_{in}}{2V_T}\right)^3\right)$$

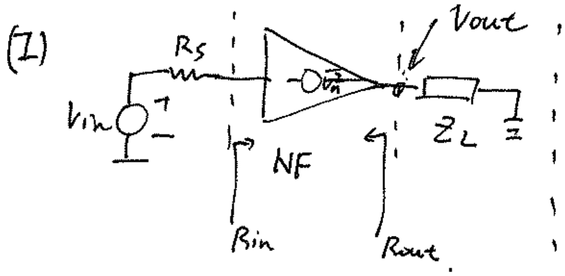
$$A_{in,IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{\frac{1}{2V_T}}{\frac{1}{3}\frac{1}{(2V_T)^3}}}$$

$$= 4V_T = 4\frac{KT}{q} = 4 \times 26mV = 104 mV$$

2.16 Solu:

(I)



$$\frac{V_{out}}{V_{in}} = A_o \frac{Z_L}{Z_L + R_{out}}$$

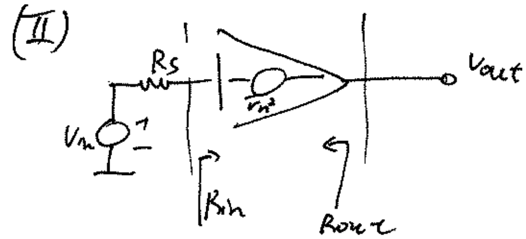
$$\overline{V_{n,out}^2} = \overline{V_n^2} \cdot \left(\frac{Z_L}{Z_L + R_{out}} \right)^2$$

$$NF = 1 + \frac{V_{n,out}^2}{\frac{V_{out}}{V_{in}}}$$

$$= 1 + \frac{\overline{V_n^2} \left(\frac{Z_L}{Z_L + R_{out}} \right)^2}{A_o^2 \left(\frac{Z_L}{Z_L + R_{out}} \right)^2} \cdot \frac{1}{4KTR_S}$$

$$= 1 + \frac{\overline{V_n^2}}{A_o^2} \cdot \frac{1}{4KTR_S}$$

(II)

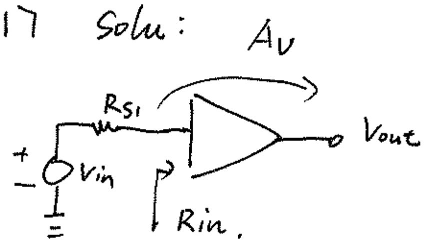


$$A_o = \frac{V_{out}}{V_{in}}.$$

$$NF = 1 + \underbrace{\frac{\overline{V_n^2}}{A_o^2}}_{\text{unloaded noise at output.}} \times \frac{1}{4KTR_S} \text{(unloaded gain)}$$

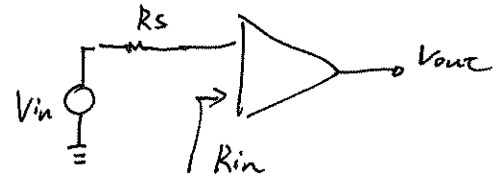
Compared with (I) and (II), we find noise figures of two situations are the same. i.e. output load doesn't affect the circuits noise figure.

2.17 Solu:



(I)

$\Rightarrow$



(II)

A_v is unloaded voltage gain from input to output of amplifier.

$\therefore$ In Figure (I),

$$NF_1 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_{in}}{R_{in}+R_{s1}} \cdot A_v\right)^2} \cdot \frac{1}{4KTR_{s1}}. \quad (1)$$

In Figure (II)

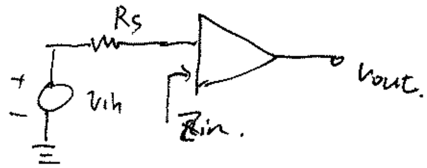
$$NF_2 = 1 + \frac{\overline{V_n^2}}{\left(\frac{R_{in}}{R_{in}+R_{s2}}\right)^2 \cdot A_v^2} \cdot \frac{1}{4KTR_{s2}}. \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{NF_1 - 1}{NF_2 - 1} = \frac{R_{s2}}{R_{s1}} \cdot \left(\frac{R_{in}+R_{s2}}{R_{in}+R_{s1}}\right)^2.$$

So if we know the input impedence and R_{s1}, R_{s2}, it's possible to compute the noise figure for another source impedance R_{s2}.

2.18 solu:

$$NF = \frac{\gamma}{g_m R_S} + 1 \qquad Eq. (2.122).$$



$$\alpha = \frac{Z_{in}}{Z_{in} + R_S} \quad ; \quad \overline{V_{RS}^2} = 4KTR_S.$$

$$SNR_{in} = \frac{V_{in}^2}{\overline{V_{RS}^2}}$$

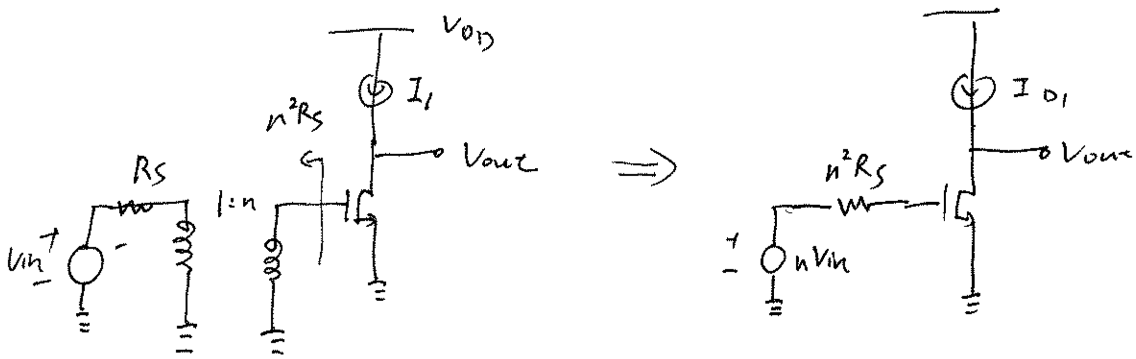
$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}.$$

So. if R_s increases SNR_{out} will fall.

It seems that the result is contradicted with. NF's falling.

Because SNR_{in} will also fall if R_s increases, the ratio of SNR_{in} and SNR_{out} is reasonable.

2.19 Solu:



$$\overline{V_n^2} = 4KT\gamma \cdot g_m r_o^2$$

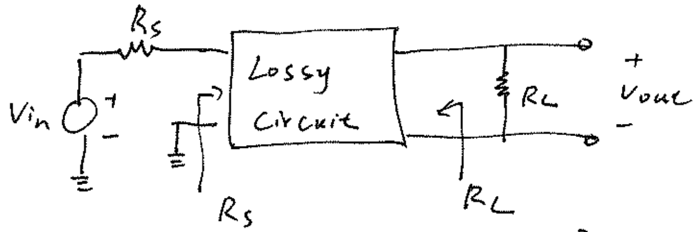
$$A = \frac{V_{out}}{V_{in}} = n \cdot g_m r_o$$

$$\therefore \; NF = 1 + \frac{\cancel{4KT}\gamma g_m \cancel{r_o^2}}{(n \cdot g_m \cancel{r_o})^{\cancel{2}}} \cdot \frac{1}{\cancel{4KT} n^2 R_S}.$$

$$= 1 + \frac{\gamma}{n^4 g_m R_S}.$$

From the result, we can find transformer improves the noise performance of Amplifier greatly.

2.20 Solu:



Proof:

$$L = \frac{P_{in}}{P_{out}} = \frac{\frac{V_{in}^2}{4R_s}}{\frac{V_{out}^2}{R_L}} = \frac{V_{in}^2}{4R_s} \cdot \frac{R_L}{V_{out}^2} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2}$$

Theorem: For a passive (reciprocal) network, the PSD of thermal noise is given by $\overline{V_n^2} = 4KT\,Re\{Z_{out}\}$.

$$\overline{V_{n,out}^2} = 4KT \cdot R_L \left(\frac{1}{2}\right)^2 \text{ — all the noise} \qquad ①$$

$$A_o = \frac{V_{out}}{V_{in}}. \qquad ②$$

$$\therefore NF = \frac{①}{②^2} = \frac{4KT \cdot R_L \cdot \frac{1}{4}}{\frac{V_{out}^2}{V_{in}^2}} \cdot \frac{1}{4KT R_s} = \frac{R_L}{4R_s} \cdot \frac{V_{in}^2}{V_{out}^2}$$

$$= L.$$

2.21. Solu: Neglect CLM & Body effect.

(a).
$$\overline{V_{n,out}^2} = \frac{4KT\gamma}{g_{m1}}\left(\frac{g_{m1}}{g_{m2}}\right)^2 + 4KT\gamma \cdot g_{m2}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}}{g_{m2}}.$$

$$NF = 1 + \frac{\overline{V_{n,out}^2}}{\left(\frac{V_{out}}{V_{in}}\right)^2} \cdot \frac{1}{4KTR_S}$$

$$= 1 + 4KT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}^3}{g_{m1}^2}\right) \cdot \frac{1}{4KTR_S}$$

(b)
$$\overline{V_{n,out}^2} = \frac{4KT\gamma}{g_{m1}} \cdot \left(\frac{g_{m1}}{g_{m3}}\right)^2 + 4KT\gamma \cdot g_{m3}$$

$$+ \frac{4KT\gamma}{g_{m2}} \cdot \left(\frac{g_{m2}}{g_{m3}}\right)^2.$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}}{g_{m3}}.$$

$$NF = 1 + 4KT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m3} \cdot g_{m2}^2}{g_{m3}^2}\right.$$

$$\left. + \frac{1}{g_{m2}} \cdot \frac{g_{m2}^2}{g_{m1}^2}\right) \cdot \frac{1}{4KTR_S}$$

(c)
$$\overline{V_{n,out}^2} = \frac{4KT\gamma}{g_{m1}}\left(\frac{g_{m1}}{g_{m3}}\right)^2 + \frac{4KT\gamma}{g_{m2}} \cdot \left(\frac{g_{m2}}{g_{m3}}\right)^2$$

$$+ 4KT\gamma \cdot g_{m3}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}}{g_{m3}}$$

$$NF = 1 + 4KT\gamma\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\frac{g_{m2}^2}{g_{m1}^2} + \frac{g_{m3}^3}{g_{m1}}\right) \cdot \frac{1}{4KTR_S}$$

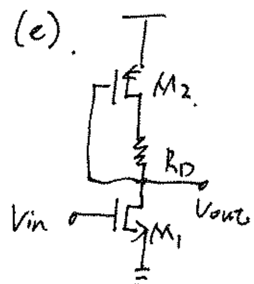
(d)
$$\overline{V_{n,out}^2} = \frac{4KT\gamma}{g_{m2}} \cdot \left(\frac{g_{m2}}{g_{m3}}\right)^2 + 4KT\gamma \cdot g_{m3}$$

$$+ \frac{4KT\gamma}{g_{m1}}\left(\frac{g_{m1}}{g_{m3}}\right)^2$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m2}}{g_{m3}}$$

$$NF = 1 + 4KT\gamma\left(\frac{1}{g_{m2}} + \frac{g_{m3}^3}{g_{m2}^2} + \frac{1}{g_{m1}} \cdot \frac{g_{m1}^2}{g_{m2}^2}\right) \cdot \frac{1}{4KTR_S}.$$

(e).



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}}{g_{m2}}.$$

$$\overline{V_{n,out}^2} = \frac{4KT\gamma}{g_{m1}} \cdot \left(\frac{g_{m1}}{g_{m2}}\right)^2 + 4KT \cdot R_D$$

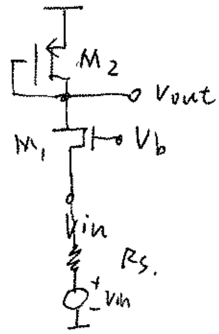
$$+ \frac{4KT\gamma}{g_{m2}} \Rightarrow \text{M2's contribution.}$$

$$\therefore NF = 1 + 4KT\gamma\left(\frac{1}{g_{m1}} + \frac{R_D}{\gamma} \cdot \frac{g_{m2}^2}{g_{m1}^2}\right.$$

$$\left. + \frac{1}{g_{m2}} \cdot \frac{g_{m2}^2}{g_{m1}^2}\right) \cdot \frac{1}{4KTR_S}$$

2.22 Part I Solu:

(a)



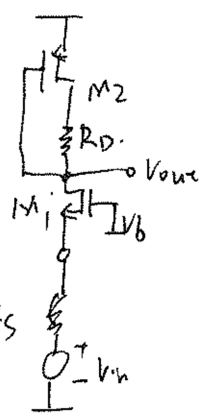
$$\overline{V^2_{n,out}} = 4KT\gamma \cdot g_{m2} + \frac{4KT\gamma}{g_{m1}}\left(\frac{1}{R_S g_{m2}}\right)^2$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{R_S g_{m2}}$$

$$NF = 1 + \cancel{4KT}\gamma\left(g_{m2} R_S^2 g_{m2}^2 + \frac{1}{g_{m1}}\right)\cdot\frac{1}{\cancel{4KT}R_S}$$

$$= 1 + \frac{\gamma}{R_S}\left(R_S^2 g_{m2}^3 + \frac{1}{g_{m1}}\right)$$

(b)



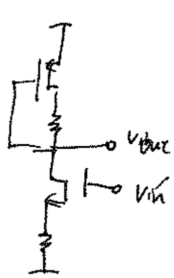
noise by M_2:

$$4KT\gamma/g_{m2}$$

noise by RD:

$$4KTR_D$$

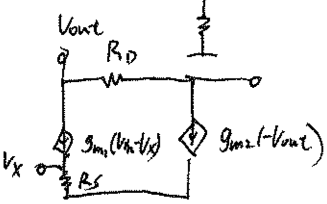
noise by M_1:



$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}}{g_{m2} + g_{m1}g_{m2}R_S}$$

$$\frac{4KT\gamma}{g_{m1}}\left(\frac{V_{out}}{V_{in}}\right)^2$$

$\Longleftrightarrow$

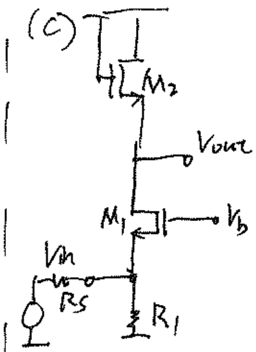


gain of circuit

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{m2}}$$

$$NF = 1 + 4KT\left[\gamma/g_{m2} + R_D + \frac{\gamma \cdot g_{m1}}{(g_{m2} + g_{m1}g_{m2}R_S)^2}\right] \times \frac{1}{4KTR_S} \times \frac{g_{m2}^2}{g_{m1}^2}.$$

(c)



noise of M2: $\frac{4KT\gamma}{g_{m2}}$ ①

noise of M1: $\frac{4KT\gamma}{g_{m1}}\cdot\left(\frac{1}{R_1 \| R_S \cdot g_{m2}}\right)^2$ ②

noise of R_1:

$$4KTR_1\left(\frac{g_{m1}^{-1}}{R_S + g_{m1}^{-1}}\right)^2\left(\frac{1}{R_S g_{m2}}\right)^2$$ ③

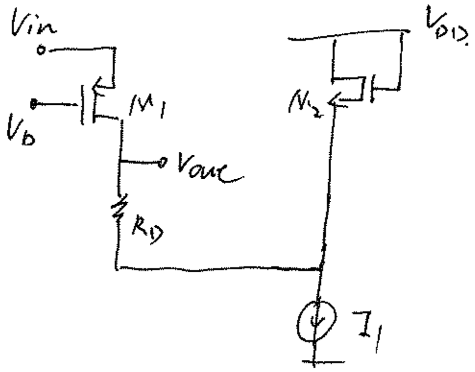
$$NF = 1 + \frac{① + ② + ③}{A_v}\,\frac{1}{4KTR_S}$$

Note:

$$A_v = \frac{R_1 \| \frac{1}{g_{m1}}}{R_S + R_1 \| \frac{1}{g_{m1}}}\cdot\frac{1}{R_1 \| R_S \cdot g_{m2}}$$

2.22 Part 2. Solu:

(d)

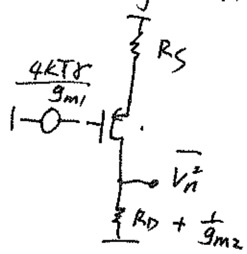


$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_{m1}}{R_D + \frac{1}{g_{m2}}}$$

noise of RD : $4KTR_D$. ①

noise of M2 : $\frac{4KT\gamma}{g_{m2}}$ ②

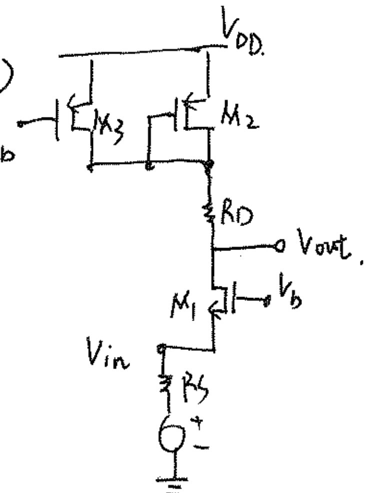
noise of M1 :



$$\frac{4KT\gamma}{g_{m1}} \cdot \left(\frac{R_D + \frac{1}{g_{m2}}}{R_S}\right)^2 \quad ③$$

$$NF = 1 + \frac{① + ② + ③}{\left|\frac{g_{m1}}{R_D + \frac{1}{g_{m2}}}\right|^2} \times \frac{1}{4KTR_S}.$$

(e)



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_D + \frac{1}{g_{m2}}}{R_S}$$

noise of RD : $4KTR_D$ ①

Noise of M1:

$$\frac{4KT\gamma}{g_m} \cdot \left(\frac{R_D + \frac{1}{g_{m2}}}{R_S}\right)^2 \quad ②$$

Noise of M2: $\frac{4KT\gamma}{g_{m2}}$ ③

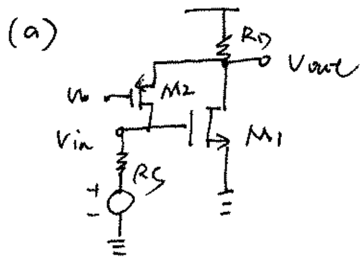
noise of M3 : $\frac{4KT\gamma}{g_{m3}} \left(\frac{g_{m3}}{g_{m2}}\right)^2$ ④.

$$NF = 1 + \frac{① + ② + ③ + ④}{\left(\frac{R_D + \frac{1}{g_{m2}}}{R_S}\right)^2} \cdot \frac{1}{4KTR_S}$$

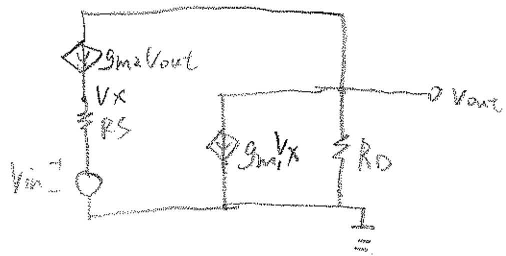
part ①

2.23 solu:

(a)



Through small-signal analysis,

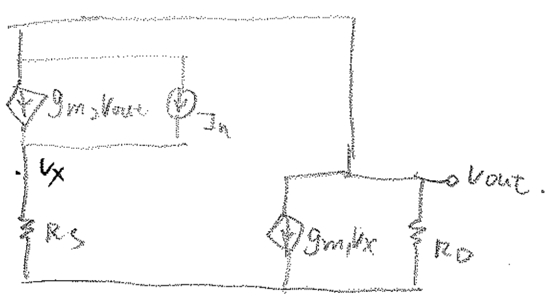


$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{g_{m1}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_S}$$

noise from RD : $4KTR_D$ ①

noise from M1 : $\dfrac{4KT\gamma \cdot g_{m1}}{g_{m2} + g_{m1}g_{m2}R_S + \frac{1}{R_D}}$ ②

noise from M2 :



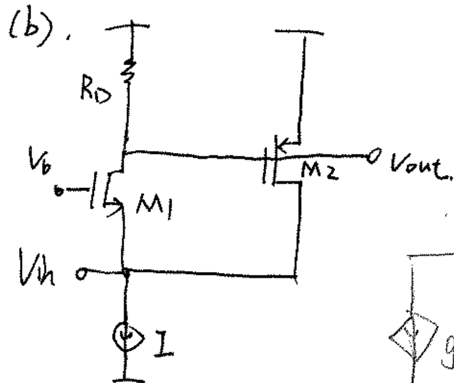
$$g_{m1}V_X + \frac{V_{out}}{R_D} = -\frac{V_X}{R_X} = -(I_n + g_{m2}V_{out})$$

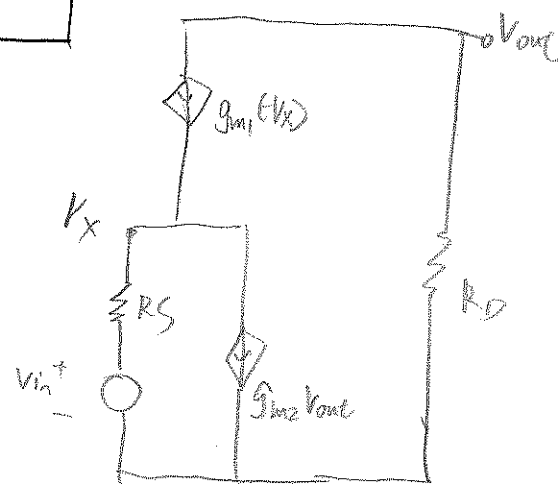
$$\Rightarrow \left|\frac{V_{out}}{I_n}\right| = \frac{g_{m1}R_D + 1}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_D}$$

$$\Rightarrow \left|\frac{V_{out}}{I_n}\right|^2 \cdot 4KT\gamma \cdot g_{m2}$$ ③

$$NF = 1 + \frac{① + ② + ③}{\left(\frac{g_{m1}}{g_{m2} + \frac{1}{R_D} + g_{m1}g_{m2}R_D}\right)^2} \cdot \frac{1}{4KT \cdot R_S}.$$

(b).

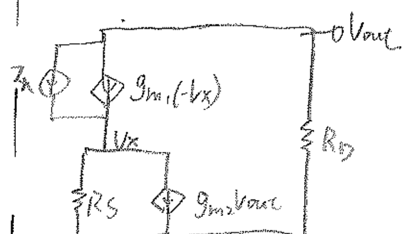




$$\frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + R_S}.$$

noise from RD : $4KTR_D$ ①

noise from M_1 :



$$4KT\gamma g_m \cdot \left[\frac{1}{g_{m1}R_S}\left(g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_DR_S}\right)\right]^2$$ ②

noise from M2 :

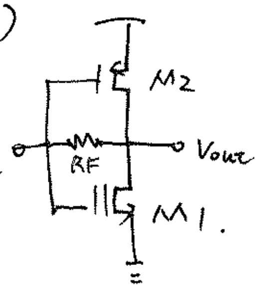
$$4KT\gamma g_{m2}\left[g_{m2} + \frac{1}{R_D} + \frac{1}{g_{m1}R_DR_S}\right]^2$$ ③

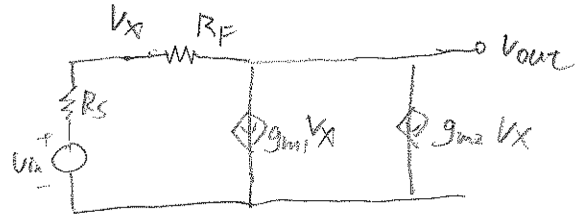
$$NF = 1 + \frac{① + ② + ③}{\left(\frac{R_D}{\frac{1}{g_{m1}} + R_S}\right)^2} \cdot \frac{1}{4KTR_S}$$

2.23 Part ②

Solu:

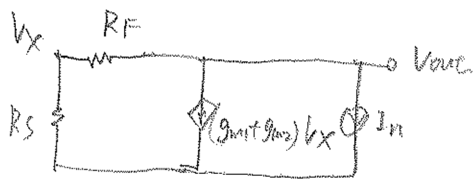
(c)





$$\frac{V_{out}}{V_{in}} = \frac{(g_{m1}+g_{m2})R_F - 1}{(g_{m1}+g_{m2})R_S + 1}$$

noise from M_1 $\Rightarrow$ $4KT\gamma \cdot g_{m1} \cdot \left(\frac{R_S + R_S}{(g_{m1}+g_{m2})R_S + 1}\right)^2$ ①



noise from M_2 $\Rightarrow$ $4KT\gamma \cdot g_{m2} \cdot \left(\frac{R_S + R_F}{(g_{m1}+g_{m2}) \cdot R_S + 1}\right)^2$. ②

noise from R_F $\Rightarrow$ $4KT\gamma R_F$. ③

$$\therefore NF = 1 + \frac{① + ② + ③}{\left(\frac{(g_{m1}+g_{m2})R_F - 1}{(g_{m1}+g_{m2})R_S + 1}\right)^2} \cdot \frac{1}{4KTR_S}.$$

3.1 Solu:

$$X_{16QAM}(t) = \gamma_1 A_c \cos(w_c t + \Delta\theta) - \gamma_2 A_c (1+\varepsilon) \sin w_c t$$

(a) $\Delta\theta \neq 0$, $\varepsilon = 0$

$$X_{16QAM}(t) = \gamma_1 A_c \cos(w_c t + \Delta\theta) - \gamma_2 A_c \sin w_c t$$

$$= \gamma_1 A_c [\cos w_c t \cos\Delta\theta - \sin w_c t \sin\Delta\theta] - \gamma_2 A_c \sin w_c t$$

$$= \gamma_1 A_c \cos\Delta\theta \cos w_c t - (\gamma_1 A_c \sin\Delta\theta + \gamma_2 A_c) \cdot \sin w_c t$$

nomalized coefficient: $\gamma_1 \cos\Delta\theta$, $-(\gamma_1 \sin\Delta\theta + \gamma_2)$

$\beta_1 = \cos\Delta\theta$, $\beta_2 = -\sin\Delta\theta + 1$; $\beta_1 = \cos\Delta\theta$, $\beta_2 = -\sin\Delta\theta + 2$;

$\beta_1 = \cos\Delta\theta$, $\beta_2 = -\sin\Delta\theta - 1$; $\beta_1 = \cos\Delta\theta$, $\beta_2 = -\sin\Delta\theta - 2$;

$\beta_1 = -\cos\Delta\theta$, $\beta_2 = \sin\Delta\theta + 1$; $\beta_1 = -\cos\Delta\theta$. $\beta_2 = \sin\Delta\theta + 2$;

$\beta_1 = -\cos\Delta\theta$, $\beta_2 = \sin\Delta\theta - 1$; $\beta_1 = -\cos\Delta\theta$, $\beta_2 = \sin\Delta\theta - 2$;

$\beta_1 = 2\cos\Delta\theta$, $\beta_2 = -2\sin\Delta\theta + 1$; $\beta_1 = 2\cos\Delta\theta$, $\beta_2 = -2\sin\Delta\theta + 2$;

$\beta_1 = 2\cos\Delta\theta$, $\beta_2 = -2\sin\Delta\theta - 1$; $\beta_1 = 2\cos\Delta\theta$, $\beta_2 = -2\sin\Delta\theta - 2$;

$\beta_1 = -2\cos\Delta\theta$, $\beta_2 = 2\sin\Delta\theta + 1$; $\beta_1 = -2\cos\theta$. $\beta_2 = 2\sin\Delta\theta + 2$;

$\beta_1 = -2\cos\Delta\theta$, $\beta_2 = 2\sin\Delta\theta - 1$; $\beta_1 = -2\cos\theta$, $\beta_2 = 2\sin\Delta\theta - 2$;

(b) $\Delta\theta = 0$, $\varepsilon \neq 0$.

$$X_{16QAM}(t) = \gamma_1 A_c \cos w_c t - \gamma_2 (A_c (1+\varepsilon) \sin w_c t$$

nomalized coefficient: $(\gamma_1, \; -\gamma_2(1+\varepsilon))$.

Similar to (a), there are 16 different combinations

3.2 Solu:

If $NF < 10dB$.

$$NF = \frac{Noise, out}{A_0^2 \cdot P_{RS}}$$

$$\frac{Noise, out}{A_0^2} = Noise, in = NF \cdot P_{RS}$$

$$\therefore \quad Noise, in \cdot B = NF/_{dB} - 174 dB/Hz + 10\lg B$$
$$= (10 - 174 + 53)\ dBm$$
$$= -111\ dBm.$$

IF $NF < 10dB \Rightarrow Noise, in \cdot B < -111 dBm$.

$$IIP_3 = P_{in} + \frac{P_{in} - P_{IM,in}}{2}$$

Since the maximum tolerable noise is $-108dBm$, the IM can contribute more than 3 dB.

So $P_{IM,in} > -111 dBm$

$$\Rightarrow IIP_3 < -18 dBm.$$

It demonsrrates that if the RX contributes less noise, the receiver's linearity requirement can be loose.

3.3 solu:

For WCDMA.

Receiver sensitivity : $-104\,dBm$.

β : 5MHz.

Assum: $NF = 3\,dB$

$$Noise_{in\,B} = -174\,dBm + 3 + 10\lg(384KHz)$$
$$= -115\,dBm.$$

For an acceptable BER, SNR of 9 dB is required.
i.e. the total noise in the desired channel must remain below $-113\,dBm$.

$\Rightarrow$ The intermodulation can contribute at most 2dB

i.e. $P_{IM,in} = -117\,dBm$.

$$IIP_3 = \frac{-46\,dBm - (-117\,dBm)}{2} + (-46\,dBm)$$

$$= -10.5\,dBm.$$

3.4 Solu:

IMX2000. maximum tolerable relative noise floor.

In DCS1800 RX Band 1805 ~ 1880 MHz.

TX Power remain below -71 dBm. in 100-KHz bandwidth.

of DCS1800.

$-71 dBm - 10 lg 10kHz = -121 dBm/Hz$.

Tx outpower : 24 dBm.

$\Rightarrow$ The Max Tolerable relative noise floor:

$-145 dBc/Hz$.

3.5

Soln:

This problem is the same as Problem 3.3.

3.6 Solu:

SNR = 17 dB

That means the total noise should reamain below -81 dBm.

Let me assume NF = 10 dB

B = 1 MHz.

Noise by Rx $= -174\ dBm/Hz + 10\lg B + NF$

$= -104$ dBm.

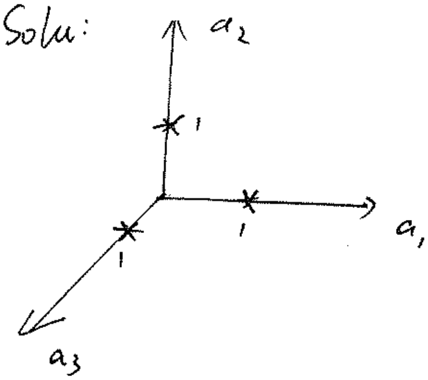
So IM can contribute maximum 23 dB.

$\Rightarrow -81.02$ dBm.

$$IIP_3 = \frac{(-39) - (-81.02)}{2} dB + (-39\ dBm)$$

$= -18$ dBm.

3.7 Solu:



the constellation of $X_{FSK}(t) = a_1 \cos w_1 t + a_2 \cos w_2 t + a_3 \cos w_3 t$.

3.8 solu:

Demodulation of AM

$$x_{AM}(t) = A_c [1 + m\, x_{BB}(t)] \cdot \cos w_c t$$

$$x_{AM}(t) \longrightarrow \otimes \longrightarrow y_1(t) \; \boxed{LPF} \longrightarrow y(t)$$

$$\uparrow$$

$$\cos w_c t$$

$$y_1(t) = A_c [1 + m\, x_{BB}(t)] \cos^2 w_c t$$

$$= A_c [1 + m\, x_{BB}(t)] \frac{1 + \cos 2\theta}{2}$$

$$y(t) = LBF [y_1(t)]$$

$$= \frac{1}{2} A_c [1 + m\, x_{BB}(t)]$$

From the equation of $y(t)$, we can easily find the original information $x_{BB}(t)$.

3.9 Solu:

Demodulation of PSK

$$x_{PSK}(t) = a_n \cos w_c t$$

$$\begin{tikzcd} x_{PSK}(t) \arrow[r] & \otimes \arrow[r, "y_1(t)"'] & \boxed{LPF} \arrow[r] & y(t) \\ & \cos w_c t \arrow[u] & & \end{tikzcd}$$

$$y_1(t) = a_n \cos w_c t \cdot \cos w_c t$$

$$= \frac{1}{2} a_n (1 + \cos 2 w_c t) .$$

$\Downarrow$

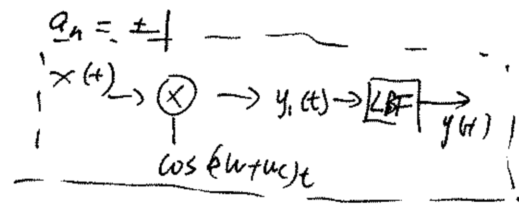
$$y(t) = LPF [y_1(t)]$$

$$= \frac{1}{2} a_n .$$

From the result of $y(t)$, we can easily find the original binary sequence a_n.

3.6 Solu:

$$x_{BPSK}(t) = a_n \cos w_c t$$



Proof:

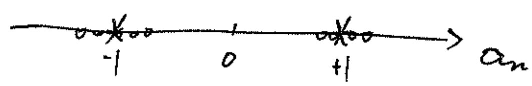
$$y_1(t) = x_{BPSK} \cdot \cos(w_c + \Delta w)t.$$

$$= a_n \cos w_c t \cdot \cos(w_c + \Delta w)t.$$

$$= \frac{1}{2} a_n [\cos(2w_c + \Delta w)t + \cos \Delta w t]$$

$$y(t) = [y_1(t)]_{LPF}$$

$$= \frac{1}{2} a_n \cdot \cos \Delta w t.$$



4.1 Soln:

(a). If input RF range is from f_1 to f_2.

$\therefore\ [\frac{4}{5}f_1,\ \frac{4}{5}f_2]$ — LO freq. range.

Suppose input band is partitioned into N channels

$\frac{f_2 - f_1}{N} = \Delta f$.

The first channel $\Rightarrow$ $f_{LO,} = \frac{4}{5}(f_1 + \frac{\Delta f}{2})$

The Second channel $\Rightarrow$ $f_{LO} = \frac{4}{5}(f_1 + \frac{3}{2}\Delta f)$.

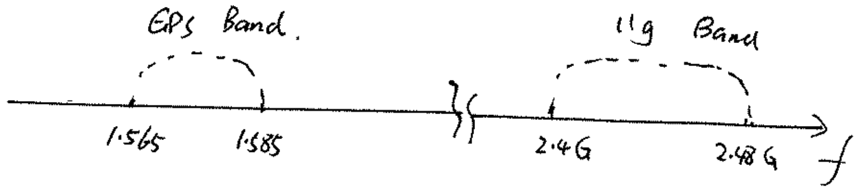
$\therefore$ LO increments in steps of $\frac{4}{5}\Delta f$.

(b). Image range.

For $f_{LO} = \frac{4}{5}f_1$, the image lies at $2f_{LO} - f_{in} = \frac{3}{5}f_1$

$\Rightarrow$ Image Freq. Range $= [\frac{3}{5}f_1,\ \frac{3}{5}f_2]$

4.2 Soln :



For Fig 4.26 Sliding-IF Receiver,

$$f_{LO1} = \frac{2}{3} f_{in}$$

Image $\quad 2f_{LO1} - f_{in}$

$$= \frac{1}{3} f_{in}$$

$$\in [0.8\,GHz,\ 0.827\,GHz]$$

Image Range : 27 MHz.

GPS Band : 20 MHz.

So. it's not possible to design an 11g receiver whose image is confined to GPS Band.

4.3 Soln:

$$f_{LO1} = \frac{2}{3} f_{in}$$

$$f_{LO2} = \frac{1}{3} f_{in}.$$

$$f_{in} - \frac{2}{3} f_{in} - \frac{1}{3} f_{in} = 0.$$

$$W_{int} \pm m W_{LO1} \pm n W_{LO2} = 0.$$

$$W_{int} = \pm m W_{LO} \pm n W_{LO2}$$

i.e. $f_{int} = \pm m f_{LO1} \pm n f_{LO2}$

$$= (\pm 2m \pm n) \frac{f_{in}}{3}.$$

$\Rightarrow$ mixing spurs. (m, n are integers).

4.4. Soln:

(a) assume the second IF is zero.

$$f_{in} - \frac{1}{2} f_{LO} - f_{LO} = 0.$$

$$f_{LO} = \frac{2}{3} f_{in}.$$

If the input RF Range is $[f_1, f_2]$, the LO freq. Range is $[\frac{2}{3} f_1, \frac{2}{3} f_2]$

(b) $2 \cdot (\frac{1}{2} f_{LO}) - f_{in} = f_{image}$.

$\Rightarrow f_{image} = -\frac{1}{3} f_{in}.$

The Range of image freq. is $[-\frac{1}{3} f_2, -\frac{1}{3} f_1]$.

(c) No.

Because the image freq range doesn't change.

And the I. Q mixer's operate at much higher frequency $\Rightarrow$ difficult to design.

4.5
Soln :

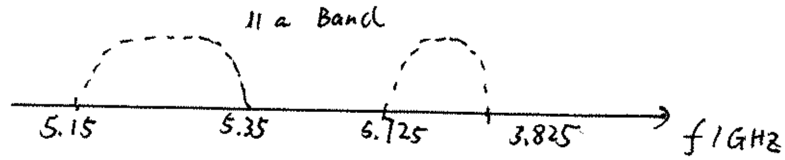
Mixing spurs :

$$f_{int} = \pm m f_{LO1} \pm n f_{LO2}$$

$$= \left(\pm \frac{m}{2} \pm n\right) \cdot \frac{2}{3} f_{in}$$

4.6.

(a).



assume the second IF is zero

$$f_{in} - f_{LO1} - \frac{1}{8} f_{LO1} = 0$$

$$f_{LO1} = \frac{8}{9} f_{in}.$$

$$f_{image} = 2 f_{LO1} - f_{in}$$

$$= \frac{7}{9} f_{in}. \Rightarrow \text{Image band:}$$

$$= [4.01\,GHz,\ 4.16\,GHz] \cup [4.45\,GHz,\ 4.53\,GHz]$$

(b). Mixed with 3rd harmonic of the first LO.

$$f_{int} - 3 \cdot f_{LO1} - m \frac{1}{8} f_{LO1} = 0$$

$$f_{int} = 3 f_{LO} + \frac{m}{8} f_{LO1}. = \left(3 + \frac{m}{8}\right) f_{LO1}$$

Mixed with 3rd harmonic of the second LO

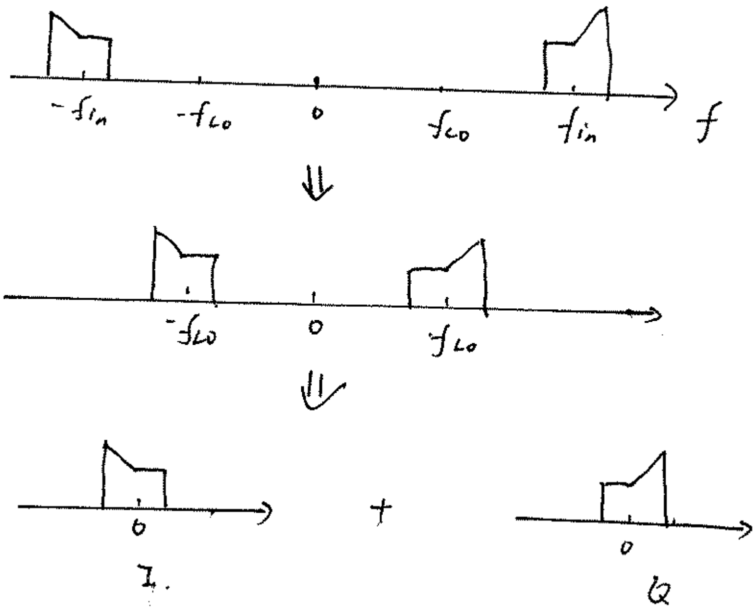
$$f_{int} - \cancel{3} n \cdot f_{LO1} - \frac{3}{8} f_{LO1} = 0$$

$$f_{int} = n \cdot f_{LO1} - \frac{3}{8} f_{LO1} = \left(n - \frac{3}{8}\right) f_{LO1}.$$

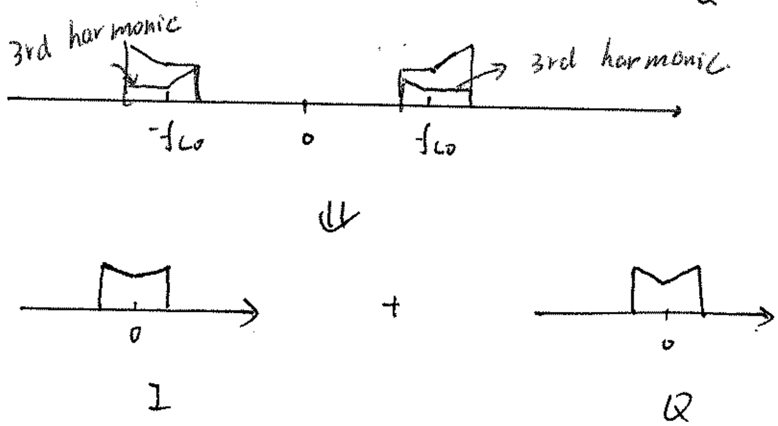
4.7. Soln:

$f_{LO} = \frac{f_{in}}{2}$

(a)



(b)



(c) Because the second IF is zero, while the flicker noise has a huge component at low frequency.

4.8 Soln:

$$A(t)\cos w_c t \rightarrow \otimes \rightarrow y(t)$$

$\uparrow$ LO

(a). $A(t)\cdot \cos w_c t \cdot \cos w_c t.$

$= A(t)\,\dfrac{(1+\cos 2w_c t)}{2},$

After LPF, baseband signal : $\dfrac{A(t)}{2}$.

(b) $A(t)\cdot \cos w_c t \cdot \sin(w_c t).$

$= \frac{1}{2} A(t)\, \sin(2w_c t).$

After LPF, base band signal is nothing.

A signal modulated by $\cos w_c t$ should be demodulated by $\cos w_c t$. Vice Versa. If a signal modulated by $\cos(w_c t + \phi)$, ϕ is phase mismatch, the quadrature downconversion is necessary.

4.9 Solu:

$V_0 \cos w_{LO} t + V_{int}(t) \cos w_{int} t$

(a). components near carrier.

assume LNA : $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$.

$$\frac{3}{4} \alpha_3 V_0^2 V_{int}(t) \cdot \cos(2w_{LO} - w_{int})$$

$$+ \frac{3}{4} \alpha_3 V_{int}^2(t) \cdot V_0 \cos(2w_{int} - w_{LO}).$$

(b). baseband componene.

$$(\alpha_1 V_0 + \frac{3}{4}\alpha_3 V_0^3 + \frac{3}{2}\alpha_3 V_0 \cdot V_{int}^2(t)) \cdot \cos w_{LO} t,$$

They will corrupt the desired signal surely.

4.10 Solu:

A higher front-end gain directly arise Sth. in Fig. 4.44.

$$\frac{P_{n1}}{P_{n2}} = \frac{8.2\, f_c}{100\, KHz}.$$

$$\frac{\gamma}{f_c} = S_{th}.$$

$$f_{c\,new} = \frac{\gamma}{S_{th}} \cdot \frac{1}{A} = f_c \cdot \frac{1}{A}.$$

$$\therefore \quad \frac{P_{n1}}{P_{n2}} = \frac{8.2}{100K} \cdot \frac{200K}{A} < 10$$

$$\Rightarrow \quad A > 1.64.$$

So that the penalty remains below 1 dB.

4.11 Solu:

$$\frac{P_{n1}}{P_{n1}} = \frac{8.2 f_c}{100K} < 10$$

$$f_c < 122 \text{ KHz}.$$

If the penalty must remain below 1 dB,

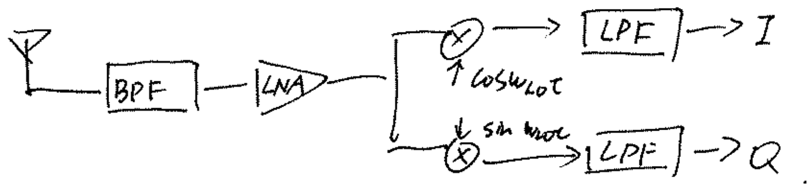
the flicker noise corner frequency should

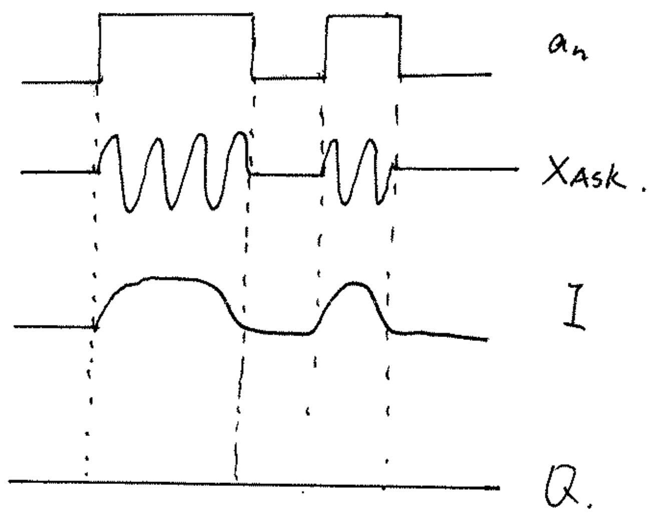
be smaller than 122 kHz.

4.12 Solu:

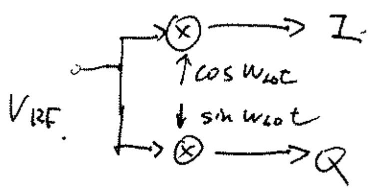
$$x_{ASK}(t) = a_n \cos w_c t$$

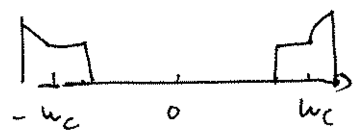
$(a_n = 1 \text{ or } 0)$.

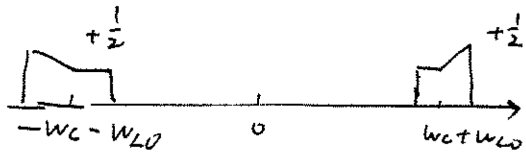




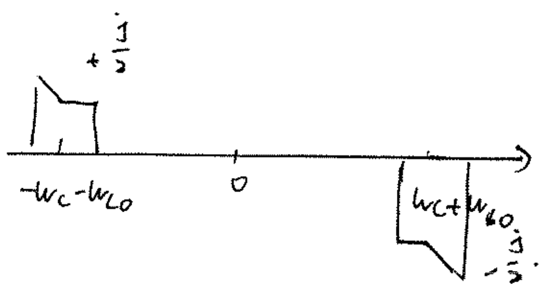
4.13 Solu:







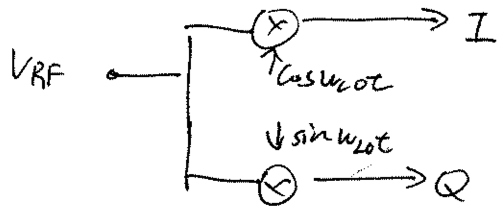
I : upconverted output

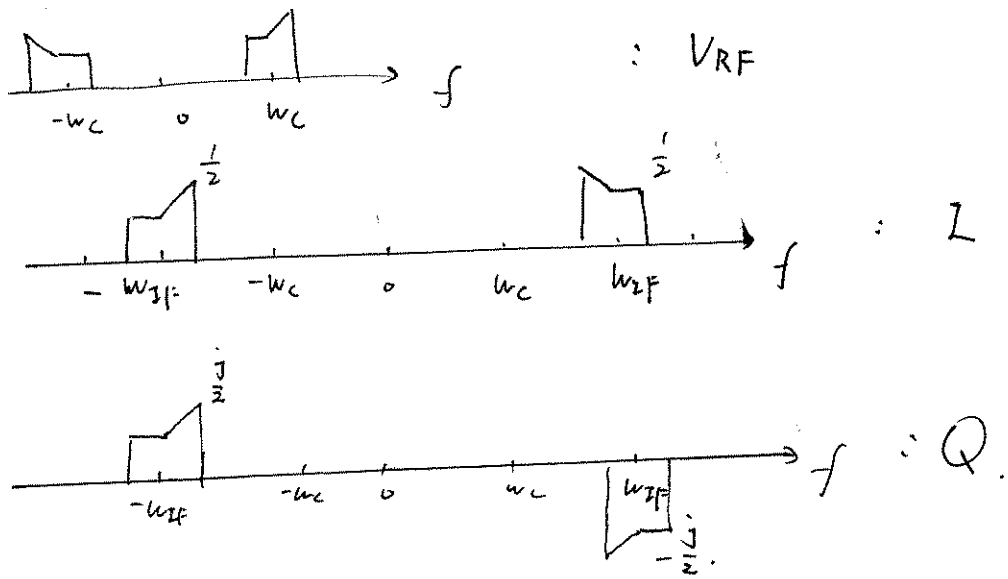


Q : upconverted output.

Fig 4.59(a) performs a Hilbert Transform even if the up converted are considered.

4.14. Solu: If $w_{IF} > w_c$.





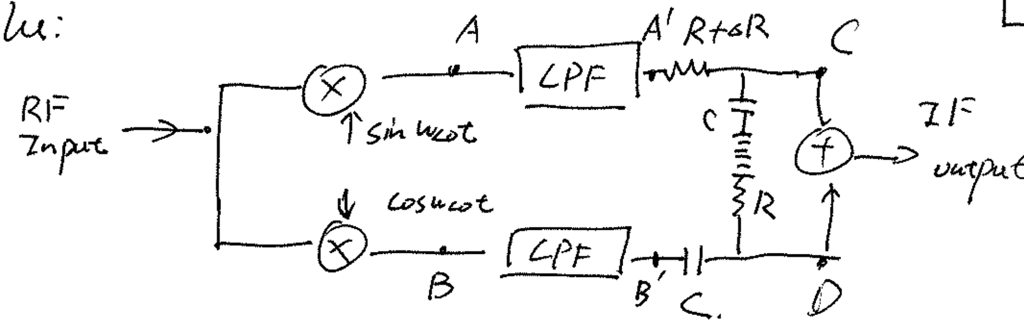
the result is the same as orignal analysis.

4.15 Solu:

Yes. Hartley architechture can cancel the image if the IF Low-pass filters are replaced with high-pass filters.

As the analysis of. problem 4.13, the upconverted components can be used successfully, just like the downconverted components.

4.16 Solu:



$$X_{A'}(t) = -\frac{A_{sig}}{2}\sin\left[(\omega_c-\omega_{LO})t+\phi_{sig}\right]-\frac{A_{im}}{2}\sin\left[(\omega_{im}-\omega_{LO})t+\phi_{im}\right].$$

$$X_{B'}(t) = \frac{A_{sig}}{2}\cos\left[(\omega_c-\omega_{LO})t+\phi_{sig}\right]+\frac{A_{im}}{2}\cos\left[(\omega_{im}-\omega_{LO})t+\phi_{im}\right].$$

$$\arctan\left(RC\cdot\frac{1}{RC}\right) = 45^\circ$$

$$\arctan\left((R+\Delta R)\cdot C\cdot\frac{1}{RC}\right) = \arctan\left(1+\frac{\Delta R}{R}\right)$$

$$\Delta\theta = \arctan\left(1+\frac{\Delta R}{R}\right) - \arctan(1)$$

$$= \arctan\left(\frac{\Delta R}{1+1+\frac{\Delta R}{R}}\right)$$

$$= \arctan\left(\frac{\Delta R}{2+\frac{\Delta R}{R}}\right).$$

$$\because \cos(\arctan(x)) = \frac{1}{\sqrt{x^2+1}}.$$

$$IRR = \frac{2+2\cos\Delta\theta}{2-2\cos\Delta\theta} = \frac{1+\cos\Delta\theta}{1-\cos\Delta\theta}.$$

$$= \frac{1+\dfrac{1}{\sqrt{\left(\frac{1}{\frac{2}{\Delta R}+\frac{1}{R}}\right)^2+1}}}{1-\dfrac{1}{\sqrt{\frac{1}{\left(\frac{2}{\Delta R}+\frac{1}{R}\right)^2}+1}}}$$

$$= \frac{2\sqrt{\left(\frac{\Delta R\cdot R}{2R+\Delta R}\right)^2+1}+2}{\left(\frac{\Delta R\cdot R}{2R+\Delta R}\right)^2}+1.$$

4.17 Solu:

$$\omega_{IF} = (R_1 C_1)^{-1}.$$

$$\Delta\theta = 2\arctan\left(R_1C_1(\omega_{IF} + \Delta\omega)\right) - \frac{\pi}{2}.$$

$$IRR = \frac{1+\cos\Delta\theta}{1-\cos\Delta\theta}$$

$$= \frac{1 + \sin\left(2\cdot\arctan\left(1+\frac{\Delta\omega}{\omega_{IF}}\right)\right)}{1 - \sin\left(2\cdot\arctan\left(1+\frac{\Delta\omega}{\omega_{IF}}\right)\right)}$$

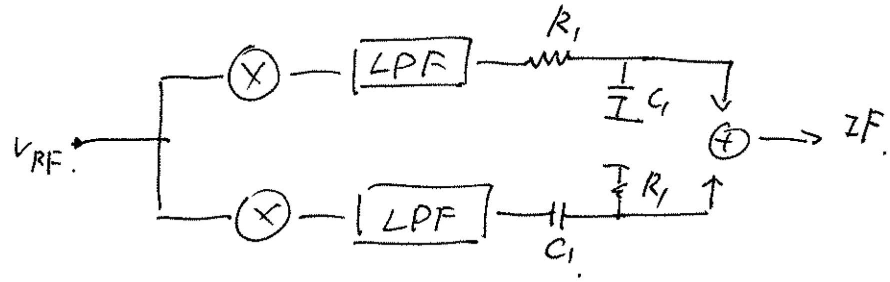
$$= \left(\frac{\frac{\Delta\omega}{\omega_{IF}}+1}{\frac{\Delta\omega}{\omega_{IF}}}\right)^2.$$

$$\boxed{\sin(\arctan(x)) = \frac{x}{\sqrt{x^2+1}}}$$

$\because \Delta\omega \ll \omega_{IF}$

$$\therefore IRR \approx \left(\frac{\omega_{IF}}{\Delta\omega}\right)^2.$$

4.18. Soln:



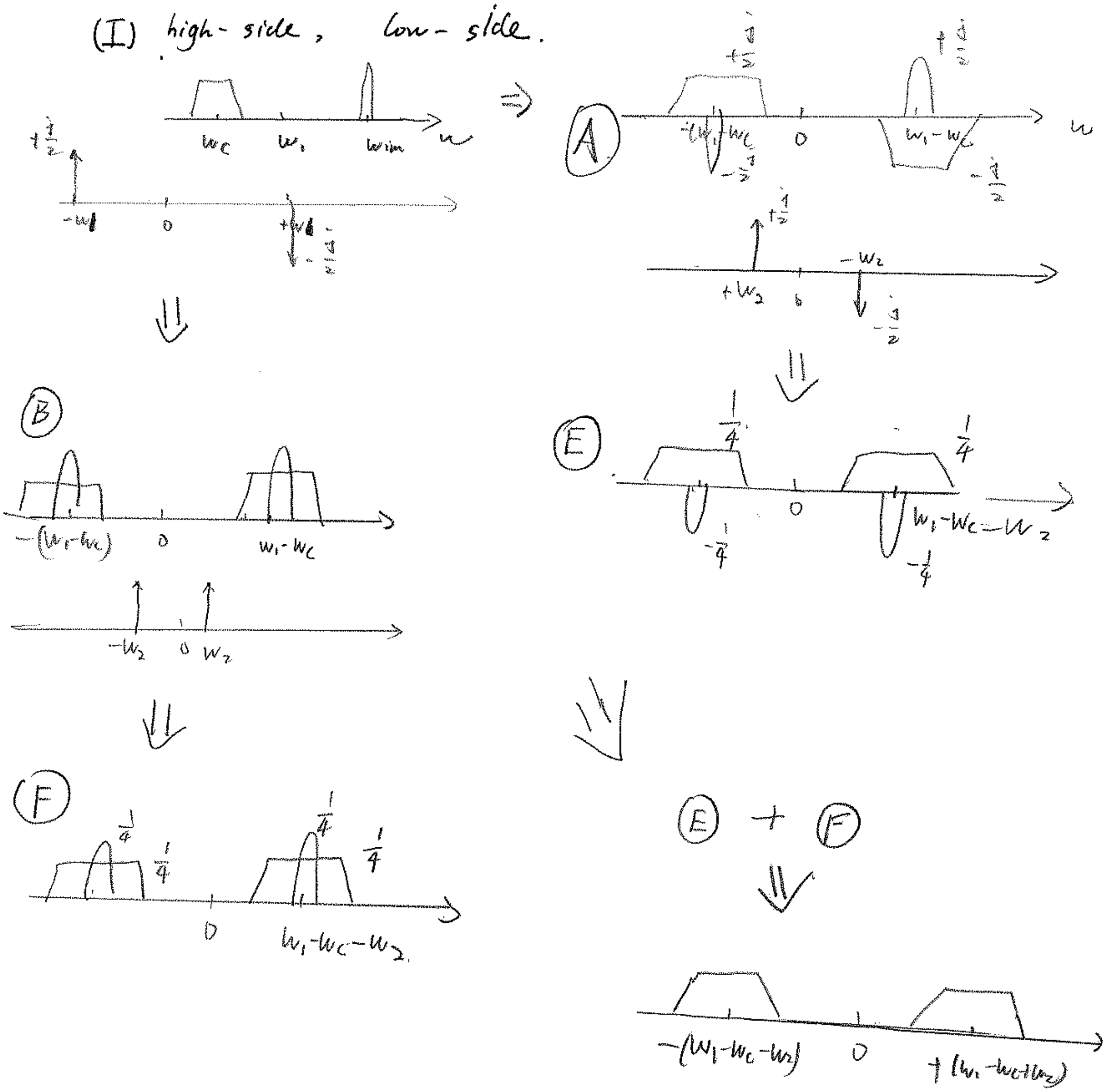
assume mixers, and LPF and adder are free of noise.

$$NF = 1 + \frac{2 \cdot 4KT \cdot R_1}{A_1^2} \cdot \frac{1}{4KT \cdot R_D}$$

$$= 1 + \frac{2R_1}{R_D} \cdot \frac{1}{A_1^2}.$$

4.19 Solu:

(I) high-side, low-side.



For high-side, low-side confugiration,

E should be added to F. In order to

Cancell the image component.

(II). high-side, high-side

similar analysis to (I).

(III) low-side, high-side

4.20 Solu:

(a). It cannot reject the image.

LO are not quadrature.

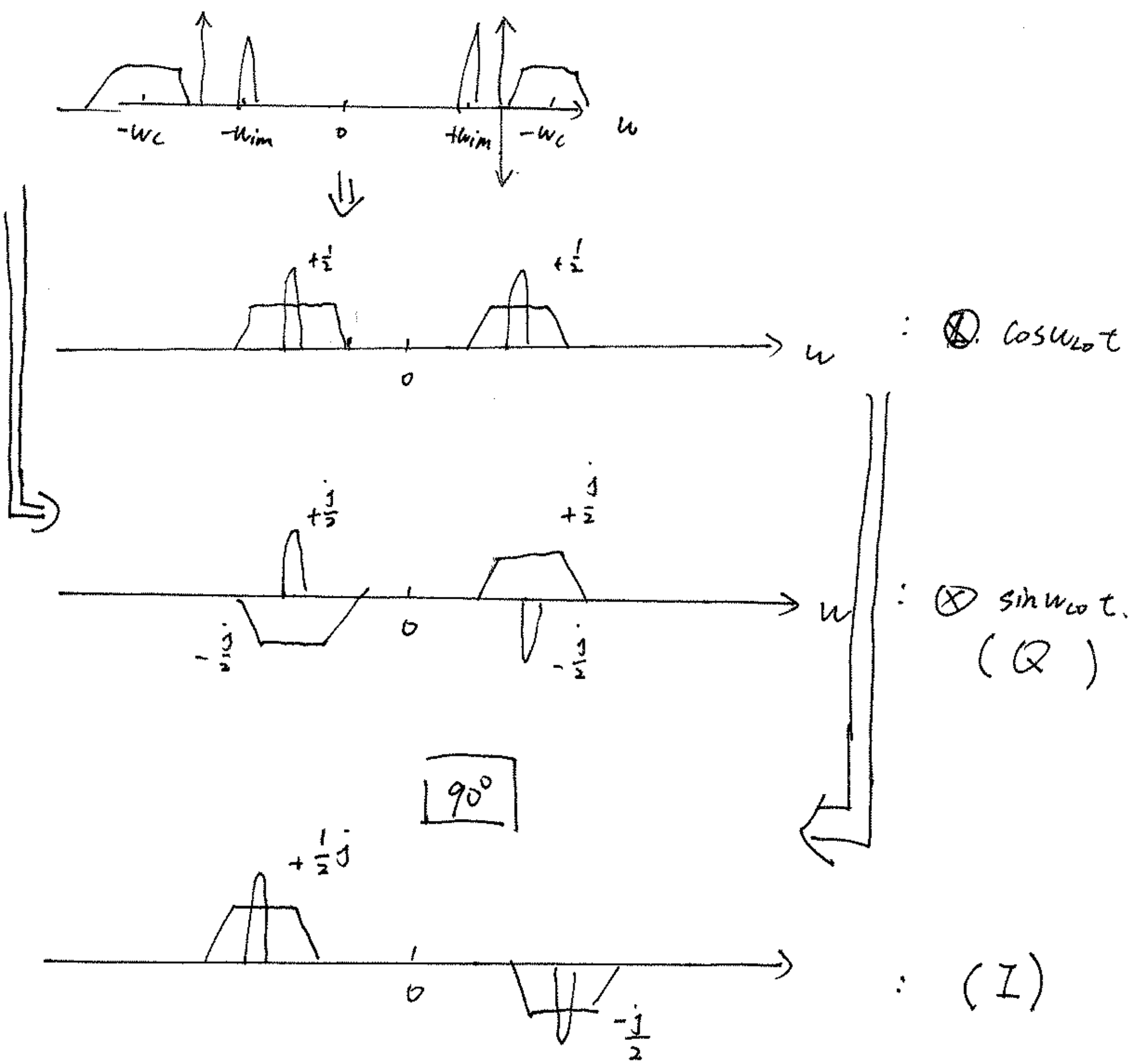
(b). It can reject the image.

The structure is similar to the original one.

(c) It cannot reject the image.

LO are not quadrature. So it is not possible to provide $\pm j$ factor, which can cancel the image by proper operations.

4.21. Solu:

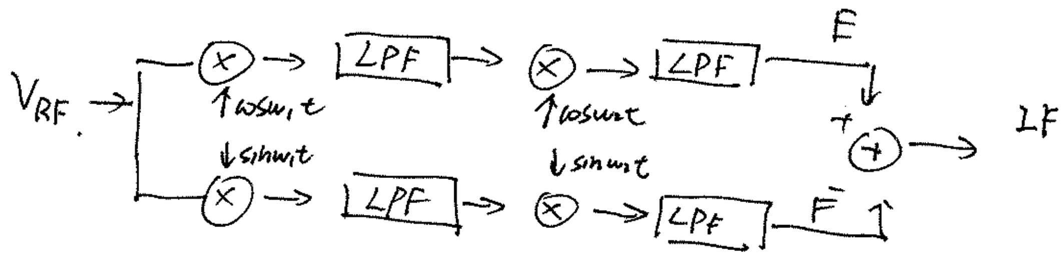


I − Q operation can cacell the image.

So the answer is Yes, $\sin w_{LO}t$ & $\cos w_{LO}t$ can be swapped. The only thing needs to be considered is that I should be subtracted by Q.

4.22 Solu:

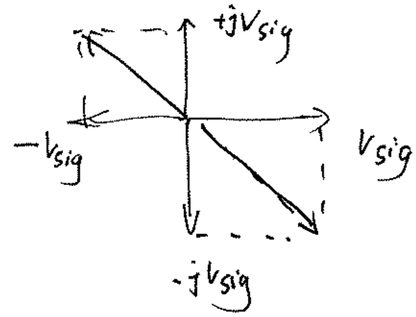
In Weaver architecture.



The answer is the same as the previous problem 4.21. Yes. It can cancel the image. Which need to be paid more attention is that the last operation on E and F.

4.23. Soln:

IRR at the output is.



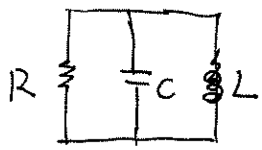
$$\frac{P_{im}}{P_{sig}} =$$

$$\simeq \frac{|V_{im}|^2 \cdot \left(\frac{RC\Delta W}{2}\right)^2}{|V_{sig}|^2 (2\sqrt{2})^2}$$

assum $|V_{im}| = |V_{sig}|$

$$IRR = \frac{(RC\Delta W)^2}{16}.$$

4.24.
Solu:



1° $3w_0 = \frac{1}{\sqrt{LC}}$.

2° $Q = R \cdot C \cdot 3w_0$.

Proof: $Y_{in} = \frac{1}{R} + \frac{1}{jwL} + jwC$.

when $w = w_0$

$$Y_{in,w_0} = \frac{1}{R} + \frac{1}{jw_0L} + jw_0C.$$

$$= \frac{1}{R} + j\left(w_0C - \frac{1}{w_0L}\right)$$

$$Y_{in,3w_0} = \frac{1}{R}.$$

$$\frac{|Y_{in,w_0}|}{|Y_{in,3w_0}|} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(w_0C - \frac{1}{w_0L}\right)^2} \cdot R.$$

$$= \sqrt{1 + w_0^2C^2 \cdot R^2 + \frac{R^2}{w_0L^2} - 2 \cdot \frac{C}{L} \cdot R^2}$$

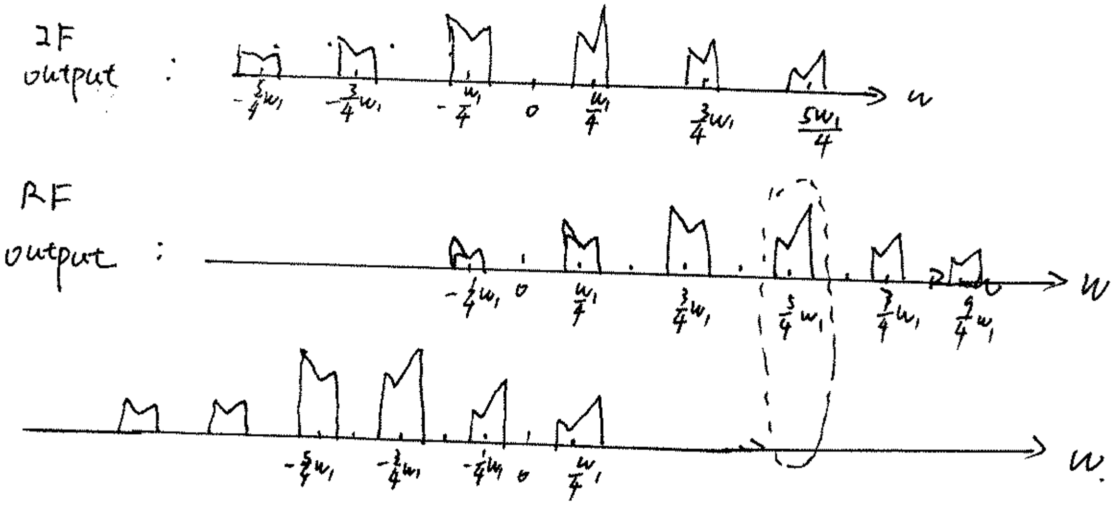
$$= \sqrt{1 + \frac{Q^2}{9} + 9 \cdot Q^2 - 2Q^2}$$

$$= \sqrt{1 + \frac{64}{9}Q^2} \quad (\text{assume } Q \gg 1).$$

$$\approx \frac{8}{3}Q.$$

4.25 Solu: $\frac{1}{4}\omega_1$ and ω_1. output $\frac{5}{4}\omega_1$

1° consider the first LO.



The unwanted signal at $\frac{1}{4}\omega_1$, $\frac{3}{4}\omega_1$, $\frac{7}{4}\omega_1$ $\frac{9}{4}\omega_1$ must be suppressed by RF bandpass filter.

2° consider the second LO.

IF outp should be mixed not only ω_1 but $3\omega_1$ and $5\omega_1$.

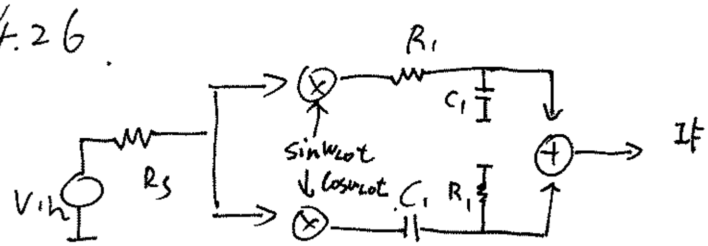
$\otimes 3\omega_1 \Rightarrow$ $-\frac{5}{4}\omega_1$ will be translated to $\frac{7}{4}\omega_1$

$-\frac{3}{4}\omega_1$ will be translated to $\frac{9}{4}\omega_1$

$\otimes 5\omega_1 \Rightarrow$ $-\frac{5}{4}\omega_1$ will be translated to $\frac{15}{4}\omega_1$.

So in the band of $+\frac{5}{4}\omega_1$ the wanted spectrum is alone.

4.26



1° Noise, out $= 4kTR_1 \cdot 2$.

2° find the gain.

only up-branch $\Rightarrow A_{mix} \times \frac{1}{2} \times \frac{1}{2}$

only up-branch $\Rightarrow A_{mix} \times \frac{1}{2} \times \frac{1}{2}$

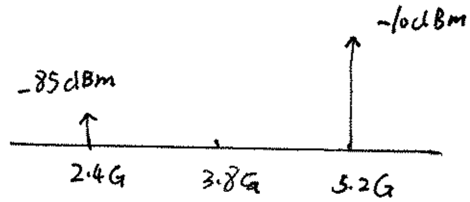
$\xRightarrow{\times 2} \frac{1}{2} \cdot A_{mix}$.

3° $NF = 1 + \dfrac{\cancel{4k}T \cdot R_1 \cdot 2}{(\frac{1}{2} \cdot A_{mix})^2} \cdot \dfrac{1}{\cancel{4kT}R_S}$

$= 1 + \dfrac{8 \cdot R_1}{R_S} \cdot \dfrac{1}{A_{mix}^2}$.

4.27 Solu:

(a).



IRR = 45dB

Neglect the noise. and nonlinearity of Rx.

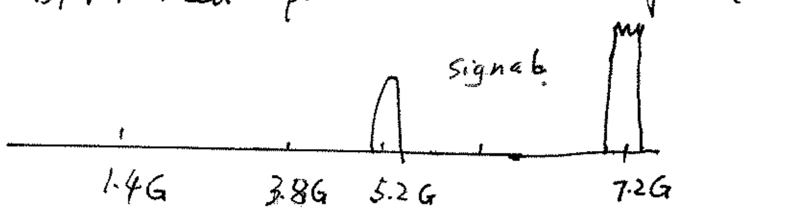
$SNR = 20\,dB \quad V_{RF} = -85\,dBm$

$\Rightarrow$ Noise, in $<$ $-65\,dBm$.

$-10\,dBm - 45\,dB = -55\,dBm$.

So BPF. need provide 10 dB rejection at 5.2GHz.

(b).



$\omega_{LO_1} = 3.8G$

$\omega_{LO_2} = 1.4G$.

$3\omega_{LO_1} = 11.4G$.

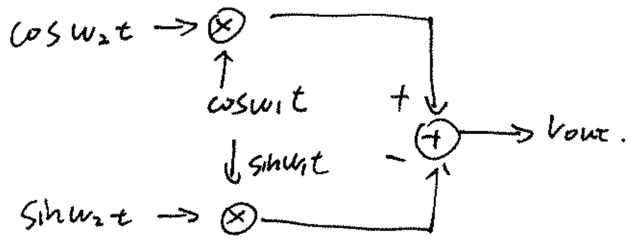
$3\omega_{LO_2} = 4.2G$

$\omega_{interferer} - 3\omega_{LO_1} - 3\omega_{LO_2} = 0$. baseband.

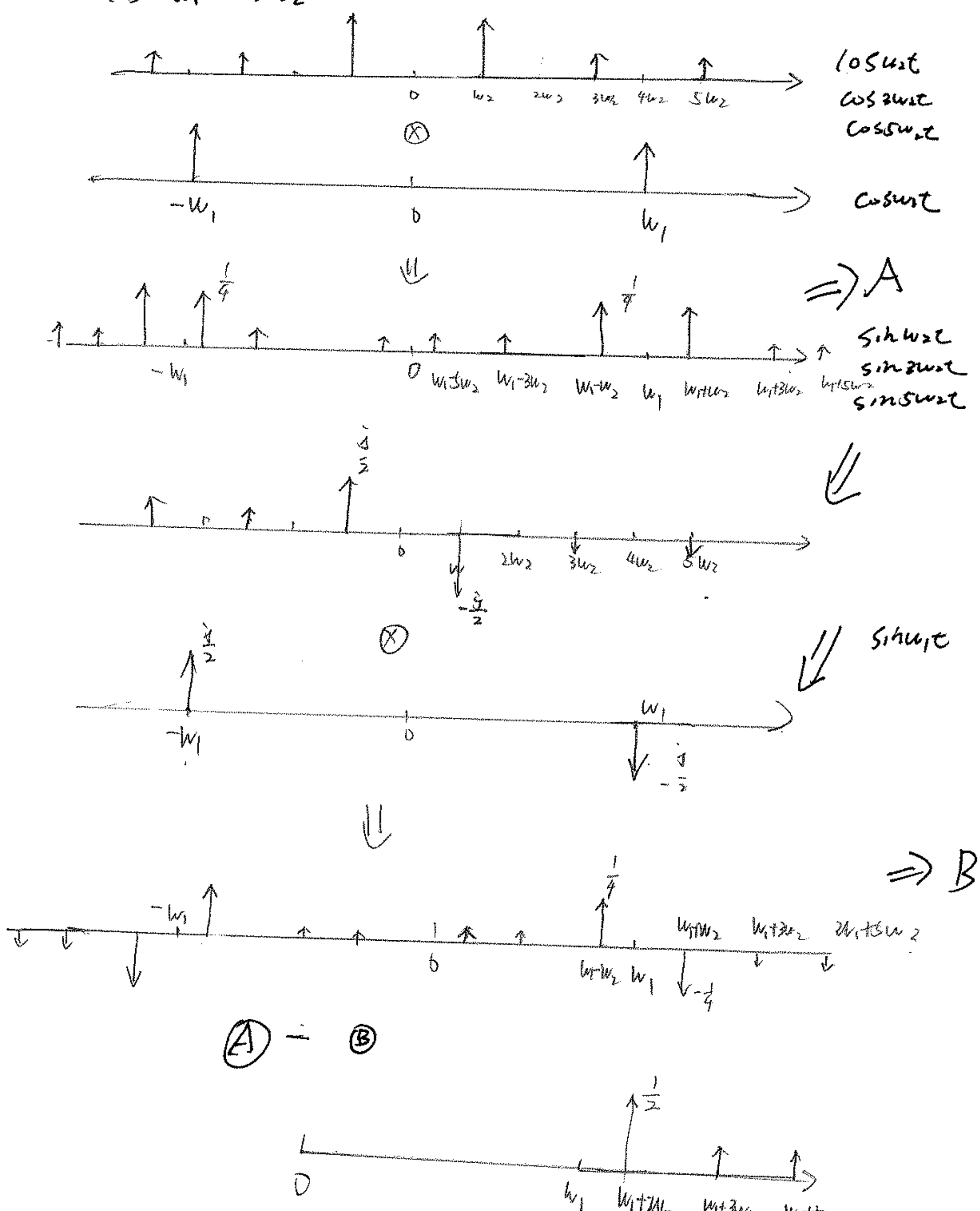
Yes. The Weaver Architure can prohibit this phenomenon. Because the 5.2GHz. band circuit is design for high-side injection. Ant the 7.2G Interferer can be mixed to baseband. This only happens for low-side injection.

Part ①.

4.28. Solu:

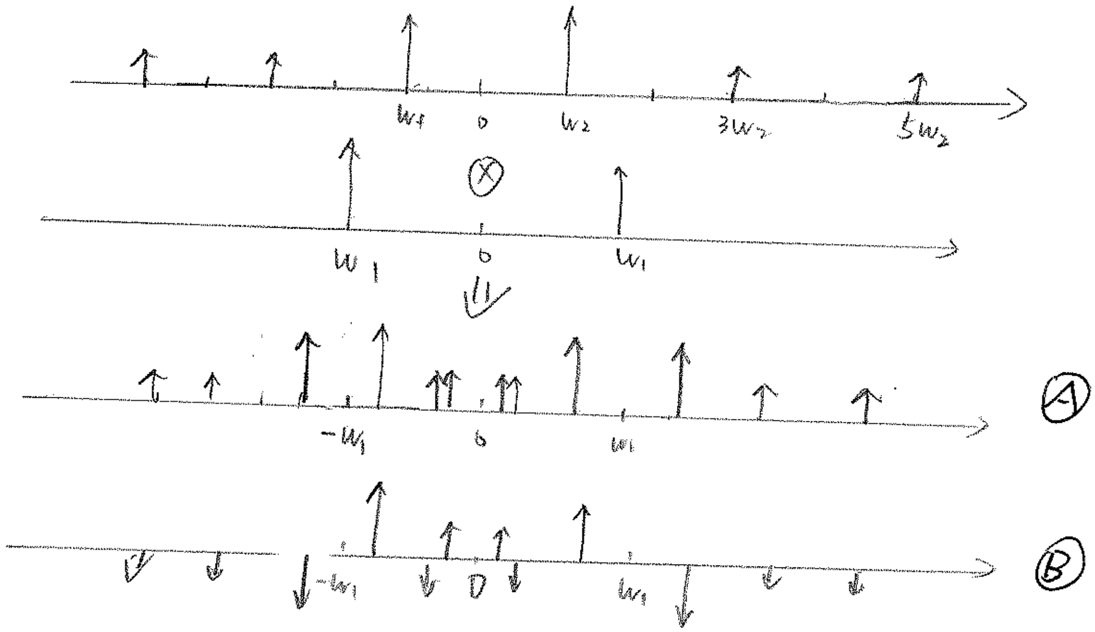


(a) $w_1 > 3w_2$

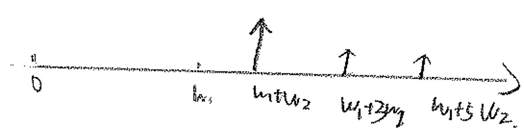


4.28 part 2

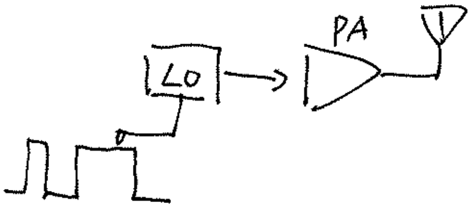
(b) $\omega_1 < 3\omega_2$.



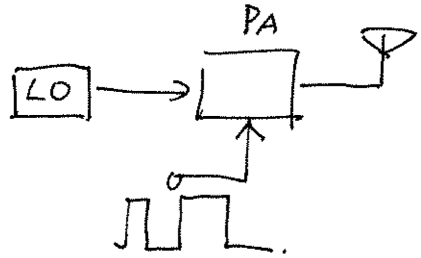
A - B.



4.29 Solu:



(a)



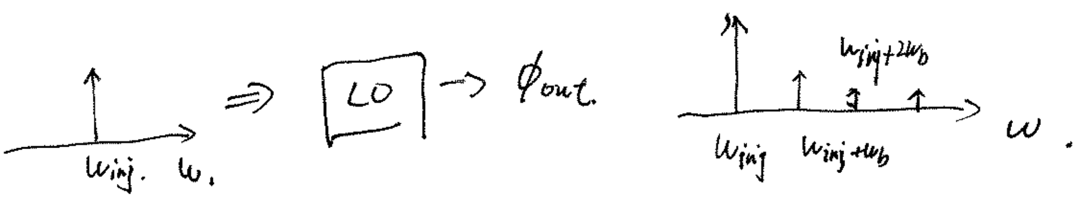
V.S.

study the injection pulling.

(b)

In The archeticture (b), PA is controlled to turn on & off.

The output spectrum of LO changes a lot in this situation.



5.1 Sol:

$$\Gamma = \left| \frac{x+jy-R_s}{x+jy+R_s} \right|^2$$

$$\Gamma = \frac{(x-R_s)^2+y^2}{(x+R_s)^2+y^2}$$

$$\Gamma(x^2+R_s^2+2R_s x)+\Gamma y^2 = (x+R_s^2-2R_s x)+y^2$$

$$(\Gamma-1)x^2+2R_s[\Gamma+1)x+(\Gamma-1)R_s^2+(\Gamma-1)y^2=0.$$

$$x^2+2R_s\frac{\Gamma+1}{\Gamma-1}x+R_s^2+y^2=0.$$

$$x^2+2R_s\frac{\Gamma+1}{\Gamma-1}x+\left(R_s\cdot\frac{\Gamma+1}{\Gamma-1}\right)^2+y^2=-R_s^2+R_s^2\left(\frac{\Gamma+1}{\Gamma-1}\right)^2$$

$$\left(x-\frac{1+\Gamma}{1-\Gamma}R_s\right)^2+y^2=4\Gamma\frac{\cdot R_s^2}{(\Gamma-1)^2}.$$

$\Rightarrow$ This is a circle with $\left(\frac{1+\Gamma}{1-\Gamma}R_s, 0\right)$ as center, with $\sqrt{4\Gamma\cdot\frac{R_s^2}{(\Gamma-1)^2}}$ as radius.

5.2 Solu:

Eq. (5.18)

$$NF = 1 + \frac{R_S}{R_P} + \frac{\gamma R_S}{g_m (R_S \| R_P)^2} + \boxed{\frac{R_S}{g_m^2 (R_S \| R_P)^2 \cdot R_D}}$$

neglect.

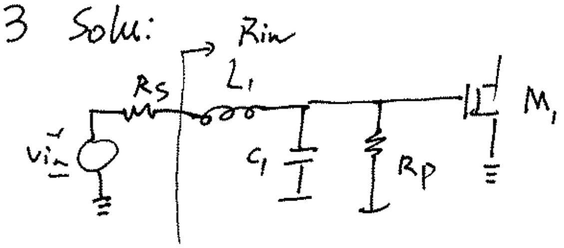
If $R_S = R_P$ $g_m R_S \approx 1$.

$$NF = 1 + 1 + \frac{\gamma}{g_m \cdot \frac{1}{4}} = 2 + \frac{4\gamma}{g_m} = 3.5\,dB$$

$$\Rightarrow \quad 2 + \frac{4\gamma}{g_m} = 10^{0.35}$$

$$\Rightarrow \quad g_m = 16.76\,\gamma.$$

5.3 Solu:



$$R_{in} = j\omega L_1 + \left(R_p \,//\, \frac{1}{j\omega C_1}\right)$$

$$= j\omega L_1 + \frac{R_p}{j\omega R_p C_1 + 1}.$$

$$= \underbrace{\frac{R_p}{1+(R_p C_1 \omega)^2}}_{=R_S} + j\underbrace{\left(\omega L_1 - \frac{R_p C_1 \omega}{1+(R_p C_1 \omega)^2}\right)}_{=0.}.$$

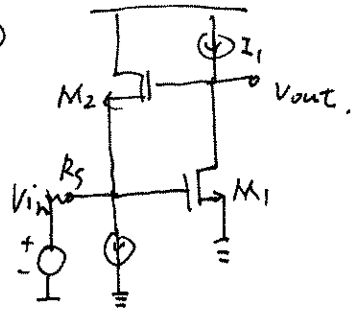
Rp cannot choose arbitary because w & C1 depend on system requirements and. technology.

So this topoloy is not difference from Fig. 5.9 (a).

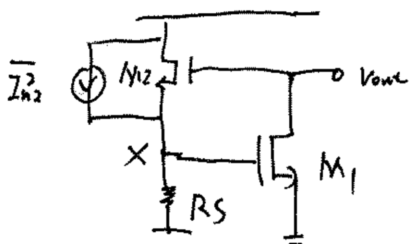
As a result, it's impossible to achieve a noise figure less than. 3 dB.

5.4. Solu. Part ①.

(a)



1° noise of M_2.



For M_1, there is no ac pass

$\Rightarrow V_X = 0.$

$$g_{m2} \cdot V_{out} = I_{n2}$$

$$V_{out} = \frac{I_{n2}}{g_{m2}}$$

$$\overline{V_{n,M2}^2} = \frac{4KT\gamma}{g_{m2}}$$

noise of M_1

$$\overline{V_{n,M1}^2} = 4KT\gamma \cdot g_{m1} \cdot \left(\frac{g_{m1} \cdot g_{m2}}{\frac{1}{R_S} + g_{m2}}\right)^2$$

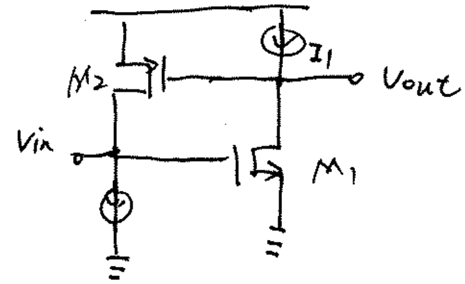
2° $\frac{V_{out}}{V_{in}}$

$$= -\frac{1}{R_S g_{m2}}$$

3° $$NF = 1 + \frac{\frac{\gamma}{g_{m2}} + \gamma g_{m1}\left(\frac{g_{m1} g_{m2}}{\frac{1}{R_S} + g_{m2}}\right)^2}{\frac{1}{R_S^2 g_{m2}^2}} \cdot \frac{1}{R_S}$$

$$= 1 + \gamma \cdot g_{m2} R_S + \gamma g_{m1} R_S g_{m2}^2 \left(\frac{g_{m1} g_{m2}}{\frac{1}{R_S} + g_{m2}}\right)^2$$

(b)



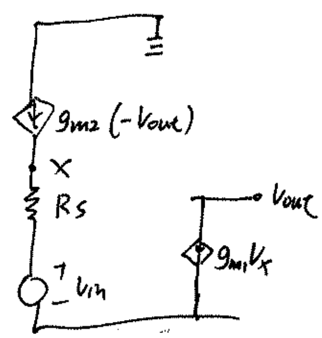
1° noise of M_2

$$\overline{V_{n,M2}^2} = \frac{4KT\gamma}{g_{m2}}$$

noise of M_1

$$\overline{V_{n,M1}^2} = \frac{4KT\gamma \cdot g_{m1}}{g_{m1}^2 R_S^2 \cdot g_{m2}^2}$$

2° $\frac{V_{out}}{V_{in}} = ?$



$$g_{m2}(-V_{out}) = \frac{V_X - V_{in}}{R_S}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{g_{m2} R_S}.$$

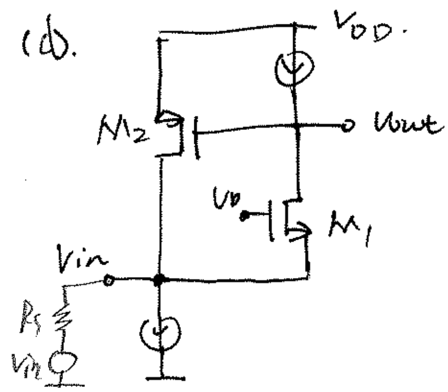
3° NF

$$= 1 + \frac{\frac{\gamma}{g_{m2}} + \frac{\gamma g_{m1}}{g_{m1}^2 R_S^2 g_{m2}^2}}{\frac{1}{g_{m2}^2 \cdot R_S^2}} \cdot \frac{1}{R_S}.$$

$$= 1 + \gamma R_S g_{m2} + \frac{\gamma}{g_{m1} R_S}$$

5.4 Solu: Part ②

(d).



1° noise of M2

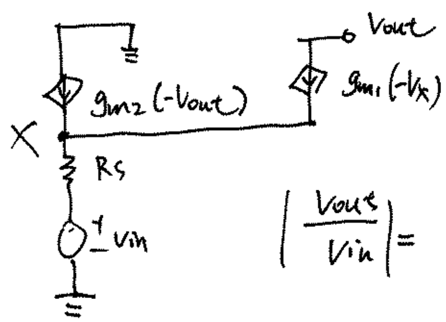
$$g_{m2}(-V_{out}) = I_{n2}$$

$$\overline{V_{n,M2}^2} = \frac{4kT\gamma}{g_{m2}}$$

noise of M1

$$\overline{V_{n,M1}^2} = \frac{4kT\gamma}{g_{m1}g_{m2}^2R_S^2}$$

2° $\frac{V_{out}}{V_{in}} = ?$



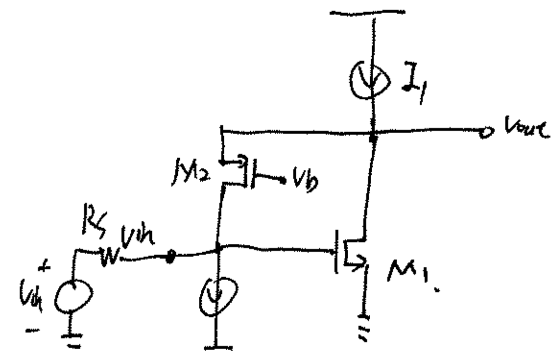
$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{g_{m2}R_S}.$$

3° NF

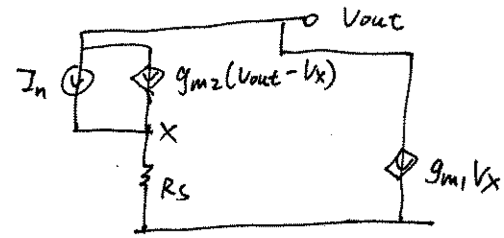
$$= 1 + \frac{\frac{\gamma}{g_{m2}} + \frac{\gamma}{g_{m1}g_{m2}^2R_S^2}}{\frac{1}{g_{m2}^2R_S^2}} \cdot \frac{1}{R_S}$$

$$= 1 + \gamma g_{m2}R_S + \frac{\gamma}{g_{m1}R_S}$$

(c)



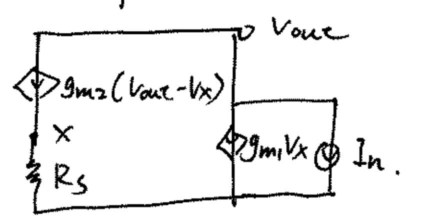
1° noise of M2



$V_X = 0$

$$\overline{V_{n,M2}^2} = \frac{4kT\gamma}{g_{m2}} \quad ①$$

noise of M1



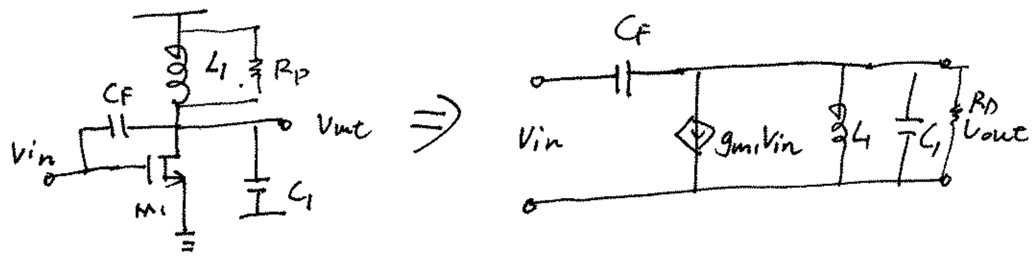
$$\begin{cases} g_{m1}V_X + I_n + g_{m2}(V_{out}-V_X) = 0 \\ g_{m2}(V_{out}-V_X) = \frac{V_X}{R_S} \end{cases}$$

$$\Rightarrow \overline{V_{n,M1}^2} = 4kT\gamma \cdot g_{m1}\left[\frac{g_{m2}+\frac{1}{R_S}}{g_{m2}(g_{m1}+2g_{m2}+\frac{1}{R_S})}\right]^2 \quad ②$$

2° $$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{g_{m2}R_S - (g_{m2}R_S+1)\cdot\frac{g_{m2}}{g_{m2}-g_{m1}}}$$

3° $$NF = 1 + \frac{① + ②}{\left|\frac{V_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4kTR_S}.$$

5.5 solu:



at $\omega_0 = \frac{1}{\sqrt{L_1(C_1+C_F)}}$ $\quad |jC_1\omega_0| \ll g_m$.

$$\frac{V_{in}-V_{out}}{C_F s} = g_{m1}V_{in} + \frac{V_{out}}{Ls} + \frac{V_{out}}{\frac{1}{C_1 s}} + \frac{V_{out}}{R_p}$$

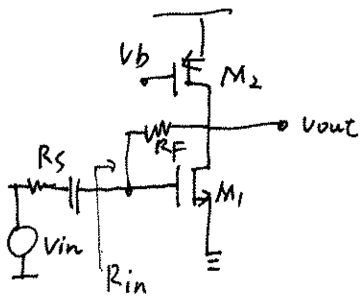
$$\frac{V_{out}}{V_{in}} = \frac{C_F s - g_{m1}}{(C_1+C_F)s + \frac{1}{L_1 s} + R_p}$$

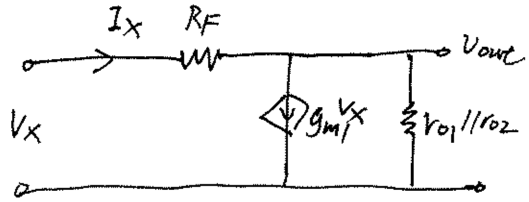
(R_p is the model of loss).

at ω_0

$$\frac{V_{out}}{V_{in}}(\omega_0) = \frac{j\omega_0 C_F - g_{m1}}{R_p} \approx -\frac{g_{m1}}{R_p}$$

5.6 Solu:





1° $R_{in} = ?$ $\quad \dfrac{V_X - I_X \cdot R_F}{r_{o1} \parallel r_{o2}} + g_{m1} V_X = I_X$

$$\Rightarrow \frac{V_X}{I_X} = \frac{R_F + r_{o1} \parallel r_{o2}}{1 + g_{m1} \cdot r_{o1} \parallel r_{o2}} = R_S$$

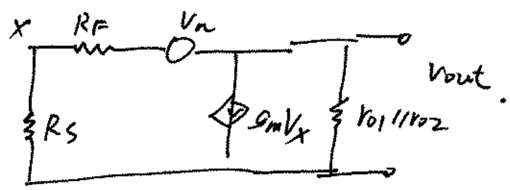
2°

$$\frac{V_{out}}{V_X} = 1 - \frac{1 + g_{m1}(r_{o1} \parallel r_{o2})}{R_F + r_{o1} \parallel r_{o2}} \cdot R_F.$$

$$= 1 - \frac{R_F}{R_S}.$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2}\left(1 - \frac{R_F}{R_S}\right).$$

3°. noise of R_F.



$$\overline{V_{n,RF}^2} = 4KTR_F \left(\frac{1 - g_m R_F}{1 - g_{m1} R_S - \frac{R_F + R_S}{r_o}}\right)^2 \approx 4KTR_F.$$

4° noise of M_1 M_2

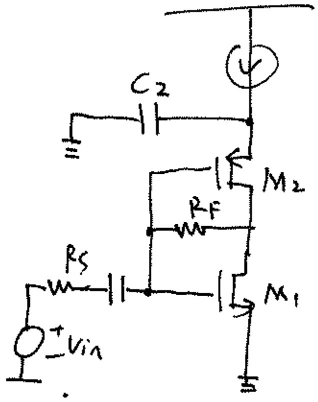
$$R_{out} = \frac{R_F + R_S}{\frac{R_F + R_S}{r_o} + R_F g_{m1} + 1}. \qquad (r_o = r_{o1} \parallel r_{o2})$$

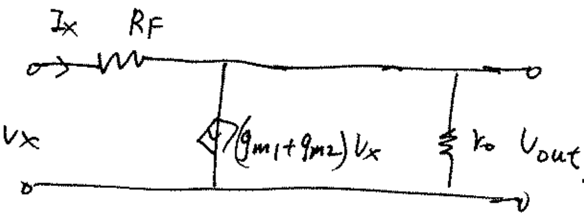
$$\overline{V_{n,M_1\&M_2}^2} = 4KT\gamma (g_{m1} + g_{m2}) R_{out}^2.$$

5° NF

$$= 1 + \frac{4KTR_F + 4KTR_{out}^2 (g_{m1} + g_{m2})}{\left(\frac{1}{2}\left(1 - \frac{R_F}{R_S}\right)\right)^2} \cdot \frac{1}{4KTR_S}.$$

5.7 Solu:





$r_o = r_{o1} \,//\, r_{o2}$

1° $R_{in} = ?$

$$I_X = (g_{m1}+g_{m2})V_X + \frac{V_X - I_X \cdot R_F}{r_o}$$

$$\Rightarrow \frac{V_X}{I_X} = R_{in} = \frac{r_o + R_F}{(g_{m1}+g_{m2})r_o + 1} = R_S.$$

2° $$\frac{V_{out}}{V_X} = 1 - \frac{R_F}{R_S}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2}\left(1 - \frac{R_F}{R_S}\right)$$

3°. noise of RF.

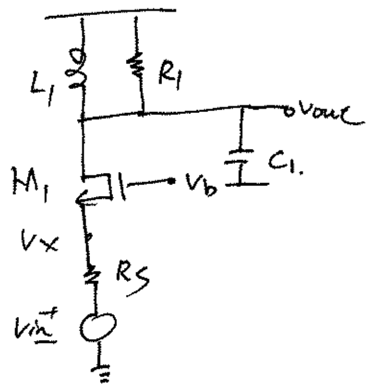
$$\overline{V_{n,RF}^2} = 4KTR_F\left(\frac{1-(g_{m1}+g_{m2})R_F}{1-(g_{m1}+g_{m2})R_F - \frac{R_F+R_S}{r_o}}\right)^2$$

4° noise of M1, M2

$$\overline{V_{n,M_1\&M_2}^2} = 4KT\gamma\,(g_{m1}+g_{m2})\cdot\frac{R_F+R_S}{\frac{R_F+R_S}{r_o} + R_F(g_{m1}+g_{m2}) + 1}.$$

5° $$NF = 1 + \frac{\overline{V_{n,RF}^2} + \overline{V_{n,M_1\&M_2}^2}}{\left[\frac{1}{2}\left(1-\frac{R_F}{R_S}\right)\right]^2}\cdot\frac{1}{4KTR_S}.$$

5.8 Solu:



1° $$\frac{V_{out}}{V_x} = g_m \cdot R_1$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{g_m}}{R_S + \frac{1}{g_m}} \cdot g_m \cdot R_1$$

$$= \frac{R_1}{R_S + \frac{1}{g_m}}$$

2° noise of M_1

$$\overline{V_{n,M_1}^2} = \frac{4KT\gamma}{g_m} \left(\frac{R_1}{R_S + \frac{1}{g_m}}\right)^2.$$

3° noise of R_1

$$\overline{V_{n,R_1}^2} = 4KTR_1.$$

4° $$NF = 1 + \frac{\gamma}{g_m R_S} + \frac{R_1}{R_S} \cdot \frac{\left(R_S + \frac{1}{g_m}\right)^2}{R_1^2}$$

$$= 1 + \frac{\gamma}{g_m R_S} + \frac{1}{R_1 R_S}\left(R_S + \frac{1}{g_m}\right)^2.$$

$$= 1 + \frac{\gamma}{g_m R_S} + \frac{R_S}{R_1}\left(1 + \frac{1}{g_m R_S}\right)^2$$

$$NF < 1 + \frac{\gamma}{g_m R_S} + 4\frac{R_S}{R_1}$$

$$\Rightarrow \quad g_m \cdot R_S > 1$$

i.e. $g_m > \frac{1}{R_S}$.

5.9 Solu:

(1) $NF = 1 + \frac{\overline{V^2_{n,out}}}{A_v^2} \cdot \frac{1}{4kTR_s}$

where $\overline{V^2_{n,out}}$ is not due to R_s.

$\therefore NF = 1 + \frac{\overline{V^2_{n,in}}}{4kTR_s} = 3\,dB.$

$\therefore \overline{V^2_{n,in}} = 4kTR_s$

$\Rightarrow$ 50 %

(2) $NF = 1\,dB = 10^{0.1} = 1.26.$

$NF = 1 + \frac{\overline{V^2_{n,in}}}{4kTR_s} = 1.26$

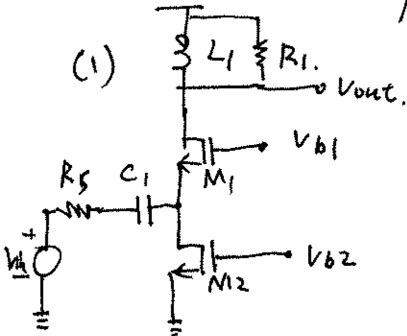
$\overline{V^2_{n,in}} = 0.26 \cdot 4kTR_s$

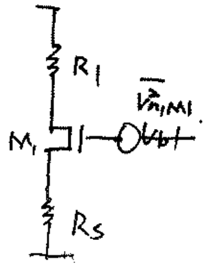
$\Rightarrow 1 - \frac{0.26}{1.26} = 1 - 20.6\% = 79.4\%$

5.10 Solu:

Neglect Channel-length Modulation & body effect.

(1)





$$\Rightarrow \overline{V^2_{n,out\,M_1}} = \frac{4KT\gamma}{g_m}\left(\frac{R_1}{R_S+\frac{1}{g_m}}\right)^2$$

$$= \frac{KT\gamma}{\cancel{g_m}}\frac{R_1}{R_S}^2$$

$$\Rightarrow \overline{V^2_{n,out\,M_2}} = 4KT\gamma \cdot g_{m2} \cdot R_1^2.$$

$$\Rightarrow \overline{V^2_{n,out\,R_1}} = 4KT R_1.$$

If input is matched. $\frac{V_{out}}{V_{in}} = \frac{R_1}{2R_S}.$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + 4\frac{R_S}{R_1} + 4R_S \cdot g_{m2}.$$

(2).

If. M_2 is replaced by R_B

$$\overline{V^2_{n,out,RB}} = \frac{4KT}{R_B} \cdot R_1^2.$$

$$NF = 1 + \frac{\gamma}{g_{m1}R_S} + 4\frac{R_S}{R_1} + 4\frac{R_S}{R_B}.$$

5.11 Soln:

$$(R_1 C_x)^{-1} < \omega << \frac{g_{m2}}{C_{GS2} + C_x}.$$

$$\frac{1}{R_1 C_x} << \frac{g_{m2}}{C_{GS2} + C_x}$$

$$\Rightarrow \frac{1}{R_1 C_x} << \frac{g_{m2}}{2 C_x}$$

$$\Rightarrow g_{m2} R_1 >> 2$$

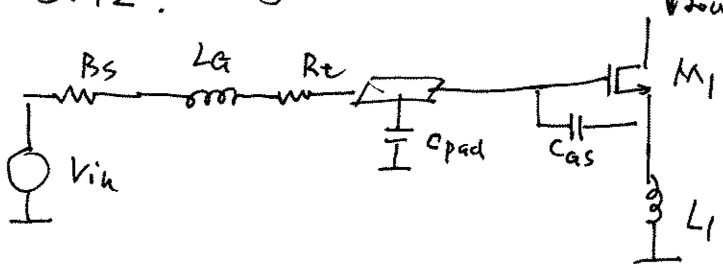
$$\Rightarrow g_{m1} R_1 >> 2.$$

gain of LNA is often 15 ~ 20dB

i.e. $g_{m1} R_1 = 32 \sim 100 >> 2.$

$\therefore$ Such a frequency range does exist.

Solu: 5.12. neglect. CLM, body-effect, C_{GD} & C_{pad}.



From. (5.89) Eq.

$$\Rightarrow V_{in} = I_{out}\left(jL_1 w_o + \frac{j(R_s + R_t) C_{GS1} w_o}{g_m}\right) - I_{n1}\frac{j(R_s + R_t) C_{GS1} w_o}{g_m}.$$

1° Transconductance gain:

$$\left|\frac{I_{out}}{V_{in}}\right| = \frac{1}{w_o\left(L_1 + \frac{(R_s+R_t) C_{GS}}{g_m}\right)} = \frac{w_T}{(2R_s + R_t) w_o}.$$

$$|I_{n,out}|_{M_1} = |I_n| \frac{(R_s+R_t) \cdot C_{GS1}}{g_m L_1 + (R_s + R_t) C_{GS1}} \approx \frac{|I_{n1}|}{2 + \frac{R_t}{R_s}}.$$

2° $$|\overline{I^2_{n,out}}|_{M_1} = 4KT\gamma g_m \cdot \frac{1}{(2+\frac{R_t}{R_s})^2}.$$

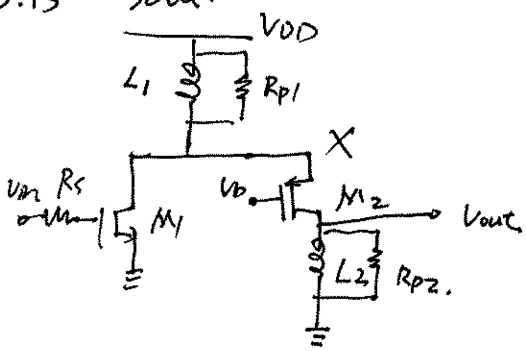
3° noise of R_t.

$$|I^2_{n,out}|_{R_t} = 4KTR_t\left[\frac{w_T}{(2R_s+R_t) w_o}\right]^2.$$

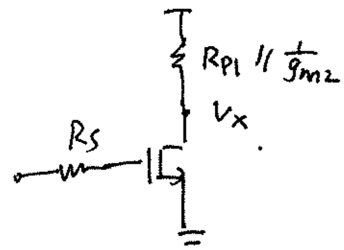
4° $$NF = 1 + \frac{|\overline{I^2_{n,out}}|_{M_1} + |\overline{I^2_{n,out}}|_{R_t}}{\left|\frac{I_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4KTR_s}.$$

$$= 1 + \frac{R_t}{R_s} + \gamma \cdot g_m R_s \left(\frac{w_o}{w_T}\right)^2.$$

5.13 Solu:



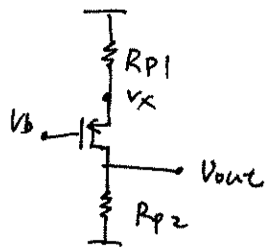
1°



⇓

$$\frac{V_X}{V_{in}} = g_{m1} \cdot \left(R_{P1} // \frac{1}{g_{m2}}\right)$$

2°



$$\frac{V_{out}}{V_X} = \frac{g_{m2}}{1+g_{m2} \cdot R_{P1}} \cdot R_{P2}.$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m1} \cdot g_{m2} R_{P1} \cdot R_{P2}}{(1+g_{m2} R_{P1})^2}.$$

3°. $\overline{V_{n,M1}^2} = \frac{4KT\gamma}{g_{m1}} \cdot \left|\frac{V_{out}}{V_{in}}\right|^2$

4° noise of M_2

$$\overline{V_{n,M_2}^2} = 4KT\gamma g_{m2} \cdot R_{P2}^2.$$

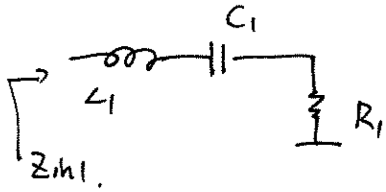
5° noise of R_{P1}.

$$\overline{V_{n,R_{P1}}^2} = 4KTR_{P1} \cdot \left|\frac{V_{out}}{V_X}\right|^2$$

6° $$NF = 1 + \frac{\frac{4KT\gamma}{g_{m1}}\left|\frac{V_{out}}{V_{in}}\right|^2 + 4KT\gamma g_{m2} R_{P2}^2 + 4KTR_{P1} \cdot \left|\frac{V_{out}}{V_X}\right|^2}{\left|\frac{V_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4KTR_S}.$$

$$= 1 + \frac{\gamma}{g_{m1} R_S} + \frac{R_{P1}}{R_S}\left(\frac{1+g_{m2}R_{P1}}{g_{m1}R_{P1}}\right)^2 + \frac{\gamma g_{m2} R_{P2}^2}{R_S} \cdot \left|\frac{V_{out}}{V_{in}}\right|^2$$

5.14 Soln:



$$Re\{Z_{in}\} = R_1$$

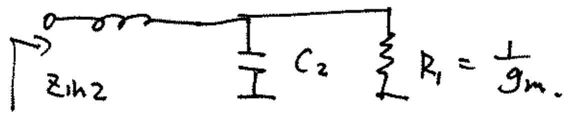
$$Im\{Z_{in}\} = \frac{L_1C_1w^2-1}{C_1w}$$

$$\approx 2L_1\Delta w \quad \frac{L_1\Delta w}{w_0}$$

$$S_{11} = \frac{j2L_1\Delta w\frac{L_1\Delta w}{w_0}}{2R_1 + j2L_1\Delta w\frac{L_1\Delta w}{w_0}}$$

$$\frac{2L_1\Delta w\frac{L_1\Delta w}{w_0}}{\sqrt{4R_1^2 + (2L_1\Delta w\frac{L_1\Delta w}{w_0})^2}} \leq 0.1$$

$\Rightarrow$ solve Δw_{max}



$$Re\{Z_{in2}\} \approx R_1$$

$$Im\{Z_{in2}\} = (L_2 - R_1^2C_2)w$$

$$\approx (L_2 - R_1^2C_2)(w_0+\Delta w)$$

$$S_{11} = \frac{j(L_2 - R_1^2C_2)(w_0+\Delta w)}{2R_1 + j(L_2-R_1^2C_2)(w_0+\Delta w)}.$$

$$\frac{(L_2 - R_1^2C_2)(w_0+\Delta w)}{\sqrt{4R_1^2 + (L_2-R_1^2C_2)(w_0+\Delta w)}} \leq 0.1.$$

$\Rightarrow$ Solve Δw_{max}.

5.15 Solu:

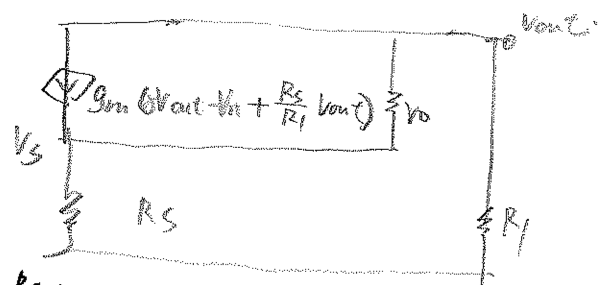
1° $$Z_{in} = \frac{\frac{1}{g_m} \parallel r_o}{(1+g_m(R_1 \parallel r_o)\gamma)^{-1}} = R_S$$

$$= \left(\frac{1}{g_m} \parallel r_o\right) \cdot \left(1 + g_m (R_1 \parallel r_o)\gamma\right)$$

$$= \frac{1}{g_m} \parallel r_o + \frac{r_o g_m^2 \gamma}{1 + r_o g_m}(R_1 \parallel r_o)$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2} \cdot \frac{g_m(R_1 \parallel r_o)}{1 + \gamma g_m (R_1 \parallel r_o)}$$

2° noise of M_1.



$$\frac{V_{out}}{R_1} + \frac{V_{out} + \frac{R_S}{R_1} V_{out}}{r_o} + g_m\left(\gamma V_{out} - V_{n1} + \frac{R_S}{R_1} V_{out}\right) = 0$$

$$\Rightarrow \left.\frac{V_{out}}{V_{n1}}\right|_{M1} = \frac{g_m}{\frac{1}{R_1} + g_m\left(\gamma + \frac{R_S}{R_1}\right) + \frac{1}{r_o} + \frac{R_S}{R_1 r_o}}$$

$$\overline{V^2_{n,M1}} = \frac{4KT\gamma}{g_m} \cdot \left|\left.\frac{V_{out}}{V_{n1}}\right|_{M}\right|^2.$$

3° noise of R_1

$$R_{out} = R_1 \parallel r_o \parallel (1 + g_m R_S / (\gamma g_m))$$

$$\overline{V^2_{n,R1}} = \frac{4KT}{R_1} \cdot R_{out}^2.$$

4° $$NF = 1 + \frac{\overline{V^2_{n,M1}} + \overline{V^2_{n,R1}}}{\left|\frac{V_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4KTR_S}.$$

5.16 Solu:

From 4.15 Problem, we can find the gain from the gate of M to vout is $\left|\frac{V_{out}}{V_{in}}\Big|_{M_1}\right|$. So is $\overline{V_{n}^2 F}$.

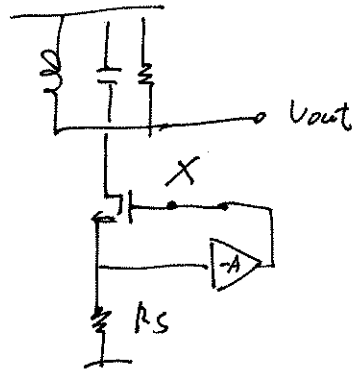
$$\overline{V_{n}^{2}F}\Big|_{out} = \overline{V_{n}^{2}F}\cdot\left|\frac{V_{out}}{V_{in}}\Big|_{M}\right|^2$$

$$\therefore NF = 1 + \frac{\overline{V_{n,F}^{2}} + \overline{V_{n,R1}^{2}} + \overline{V_{n,M1}^{2}}}{\left|\frac{V_{out}}{V_{in}}\right|^2}\cdot\frac{1}{4kTR_S}.$$

$$= NF_{\text{Problem 4.15}} + \frac{\overline{V_{n,F}^{2}}}{\left|\frac{V_{out}}{V_{in}}\right|^2}\cdot\frac{1}{4kTR_S}.$$

5.17. Soln:

Proof:



$$\frac{V_{out}}{V_x} = -\frac{g_m R_1}{(1+A)\, g_m R_s + 1} \qquad (\text{Eq. } 5.122).$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{2}(1+A)\, g_m R_1 \qquad (\text{matched}).$$

The forth term in Eq (5.124)

$$= \frac{\overline{V_{nA}^2} \cdot A^2 \cdot \left(\frac{V_{out}}{V_x}\right)^2}{\left|\frac{V_{out}}{V_{in}}\right|^2} \cdot \frac{1}{4KTR_s}$$

$$= \frac{\overline{V_{nA}^2} \cdot A^2 \cdot \frac{g_m^2 R_1^2}{((1+A) g_m R_s + 1)^2}}{\frac{1}{4}[(1+A) g_m R_1]^2} \cdot \frac{1}{4KTR_s}.$$

$R_s = R_{in}$ (matched

$$\simeq \frac{\overline{V_{nA}^2} \cdot A^2 \frac{g_m^2 R_1^2}{4}}{\frac{1}{4}((1+A) g_m R_1)^2} \frac{1}{4KTR_s}$$

$$= \frac{\overline{V_{nA}^2}}{4KTR_s} \cdot \frac{A^2}{(1+A)^2}$$

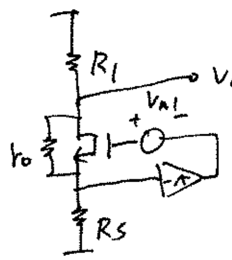
5.18 Solu:

1° $R_{in\text{-}original} = \dfrac{R_1 + r_o}{1+g_m r_o}$.

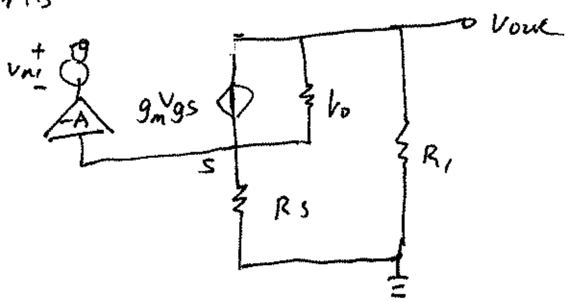
$$R_{in} = \frac{R_1 + r_o}{1+g_m(1+A)r_o} = R_s \quad (\text{matched})$$

$$A_v = \frac{V_{out}}{V_{in}} = \cancel{\tfrac{1}{2}} \cdot \frac{(1+A)g_m r_o + 1}{r_o + R_s + (1+A)g_m \cancel{r_o} R_s + R_1} \cdot R_1. \quad (\text{matched})$$
$$= \frac{R_1}{2\cancel{4}R_s}$$

2° noise of M_1.



$\Rightarrow$



$$\begin{cases} \dfrac{V_{out}}{R_1} + \dfrac{V_{out} - V_s}{r_o} + g_m(-A V_s + V_{n1} - V_s) = 0 \\ \dfrac{V_{out}}{R_1} + \dfrac{V_s}{R_s} = 0 \end{cases}$$

$$\Rightarrow \quad \frac{V_{n,out,M1}}{V_{n1}} = \frac{-g_m \cancel{V_{n1}}}{\left(\frac{1}{R_1} + \frac{1}{r_o} + \frac{R_s}{R_1 r_o} + g_m(A+1)\frac{R_s}{R_1}\right)}$$

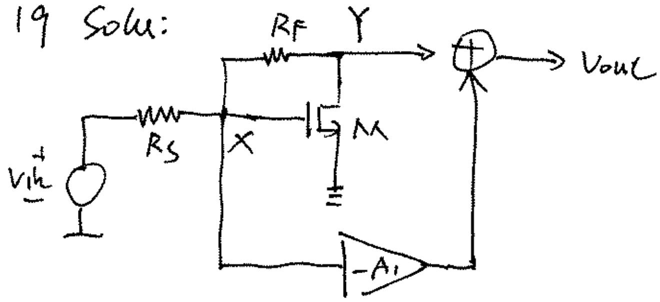
$$\overset{(\text{matched})}{=} -\frac{R_1 r_o g_m}{2(r_o + R_1)}$$

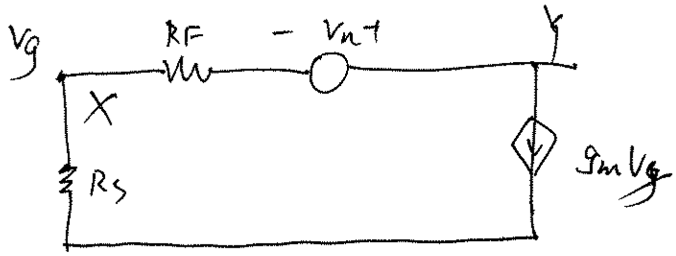
3° noise of R_1

$$R_{out} = \frac{2R_1 r_o + R_1^2}{2(r_o + R_1)} \qquad \overline{V_{n,R1}^2} = \frac{4kT}{R_1} \cdot R_{out}^2.$$

4° $$NF = 1 + \frac{\frac{4kT}{R_1} R_{out}^2 + \frac{4kT\gamma}{g_m}\left(\frac{R_1 r_o g_m}{2(r_o + R_1)}\right)^2}{\left(\frac{R_1}{2R_s}\right)^2} \cdot \frac{1}{4kTR_s}.$$

5.19 Solu:





$$\frac{V_Y - V_n}{R_F + R_S} = -g_m V_g = \frac{V_Y - V_X - V_n}{R_F}.$$

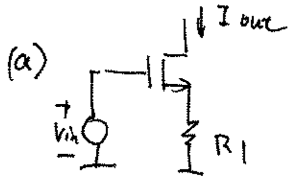
$$\Rightarrow \quad \begin{cases} V_Y = V_n \\ V_X = 0 \end{cases}$$

The noise votlage of R_F produces V_n at note Y, but produces zero at note X.

$\Rightarrow$ So the archetecture canot cancel the noise of R_F.

5.20 Solu:

(a)



$$I_{out} = K\left(V_{in} - I_{out}\cdot R_1 - V_{th}\right)^2$$

$$\frac{\partial I_{out}}{\partial V_{in}} = K\cdot 2\cdot\left(V_{in} - I_{out}\cdot R_1 - V_{th}\right)\left(1 - R_1\frac{\partial I_{out}}{\partial V_{in}}\right)$$

$$\Rightarrow \alpha_1 = \left.\frac{\partial I_{out}}{\partial V_{in}}\right|_{\substack{V_0\\ I_0}} = \frac{2K\left(V_0 - I_0R_1 - V_{th}\right)}{1 + 2R_1K\left(V_0 - I_0R_1 - V_{th}\right)} = \frac{g_m}{1 + g_mR_1}.$$

(b).

$$\frac{\partial^2 I_{out}}{\partial V_{in}^2} = 2K\left(1 - R_1\frac{\partial I_{out}}{\partial V_{in}}\right)^2 + 2K\left(V_{in} - R_s I_{out} - V_{th}\right) \times \left(-R_s\frac{\partial^2 I_D}{\partial V_{in}^2}\right)$$

$$\Rightarrow \left.\frac{\partial^2 I_{out}}{\partial V_{in}^2}\right|_{\substack{V_0\\ I_0}} = 2\alpha_2 = \frac{2K}{\left(1 + g_mR_1\right)^3} = \frac{g_m^2}{2I_0\left(1 + g_mR_1\right)^3}$$

(c).

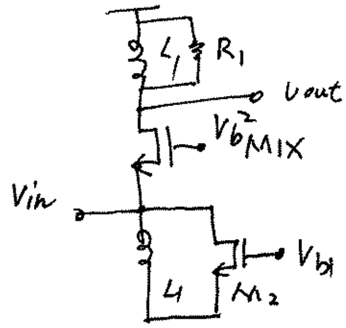
$$\begin{aligned} y &= a_1x + \alpha_2x^2 \\ &= a_1\cos\omega t + \alpha_2\frac{1 + \cancel{2}\cos 2\omega t}{2}. \\ &= a_1\cos\omega t + \frac{1}{2}\alpha_2\cos 2\omega t + \frac{\alpha_2}{2}. \end{aligned}$$

$$\left|\alpha_1 A_{IP2}\right| = \left|\frac{1}{2}\alpha_2\cdot A_{IP2}^2\right|$$

$$\therefore\ A_{IP2} = 2\cdot\frac{\alpha_1}{\alpha_2} = 2\cdot\frac{\cancel{\frac{g_m}{1 + g_mR_1}}}{\frac{g_m^{\cancel{2}}}{2\cdot 2I_0\left(1 + g_mR_1\right)^{\cancel{3}\,2}}}$$

$$= \frac{8I_0\left(1 + g_mR_1\right)^2}{g_m}.$$

5.21. Solu:



$\because g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th}) \propto W.$

$$g_{mix} = \frac{g_m}{\sqrt{2}}$$

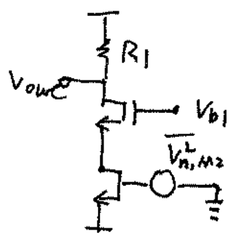
$$R_{ox2} = \frac{\sqrt{2}}{\sqrt{2}-1} R_S \quad \text{(input matched)}$$

1° $\frac{V_{out}}{V_{in}} = \frac{g_m R_1}{\sqrt{2}}$

2° noise of MIX.

$$\overline{V_{n\_out,mix}^2} = 4KT \frac{g_m}{\sqrt{2}} R_1^2$$

3° noise of M_2



$$\overline{V_{n,out,M2}^2} = \frac{4KT}{g_{m2}} \cdot \left(g_{m2} \cdot \frac{\sqrt{2}}{g_m}\right)^2$$

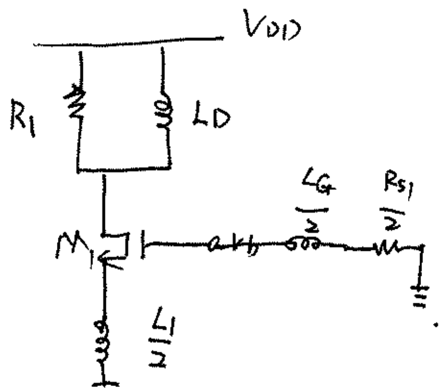
4° noise of R_1

$$\overline{V_{n\_out,R_1}^2} = 4KT \cdot R_1$$

5° $NF = 1 + \frac{\sqrt{2}}{g_m R_S} + \frac{4\left(\frac{g_{m2}}{g_m}\right)^2}{(g_m R_1)^2 \cdot R_S} + \frac{2R_1}{(g_m R_1)^2 R_S}.$

5.22 Soln:

(a).



If the input is matched.

$$\frac{L_1}{2}\cdot\frac{g_{m1}}{C_{GS1}} = \frac{R_{S1}}{2}$$

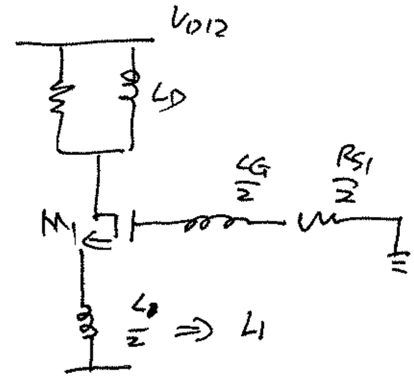
$$g_{m1} = \frac{R_{S1}}{L_1}C_{GS1} = \frac{2I_D}{V_{GS}-V_{th}}$$

$$= \sqrt{2I_D \underbrace{\mu_n C_{ox}\cdot\frac{W}{L}}_{K}}$$

$$Power\_diff = \frac{g_{m1}^2}{2K}\cdot V_{DD}\cdot 2 = \frac{V_{DD}\cdot g_{m1}^2}{K}.$$

$$Power\_sing = \frac{g_{m1}^2}{2K}\cdot V_{DD} = \frac{1}{2}Power\_diff.$$

(b)



If we need NF is the same as sigal-ended one.

$$\therefore A_{V} = \frac{\omega_T}{\omega_0}\frac{R_1}{2\cdot\frac{R_{S1}}{2}} \quad unchange$$

If only consider the noise of MOS.

$$\overline{V_{n,M1}^2} = 2\times KT\gamma g_{m1}\cdot R_1^2.$$

So if g_{m1} changes to $g_{m1}/2$.

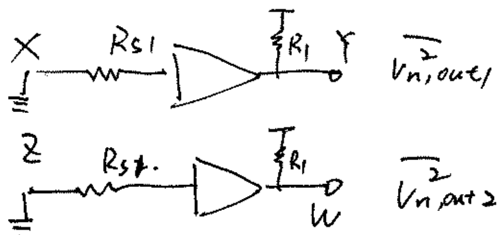
the M1's contribution in noise figure is unchanged.

$$Power\_diff = \frac{(\frac{g_{m1}}{2})^2}{2K}\cdot V_{DD}\cdot 2 = \frac{1}{4}\frac{V_{DD}\cdot g_{m1}^2}{K}$$

$$Power\_sing = \frac{g_{m1}^2}{2K}\cdot V_{DD} = 2\cdot Power\_diff.$$

5.23 Solu:

If 1 to 2 balun is used,



gain from X to Y is $\frac{R_1}{2R_{S1}}$

$$\overline{V_{n,out1}^2} = 4kTR_1 + 4kTR_{S1}\left(\frac{R_1}{2R_{S1}}\right)^2 + kT\gamma\cdot\frac{R_1^2}{R_{S1}}.$$

$$A_v = \frac{R_1}{2\cdot R_{S1}}\cdot 2$$

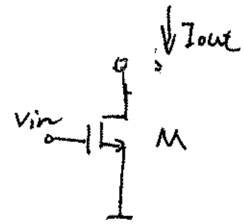
$$\therefore NF = \frac{2\cdot\overline{V_{n,out}^2}}{2^2\left(\frac{R_1}{2R_{S1}}\right)^2}\cdot\frac{1}{4kTR_{S1}}$$

$$= \frac{2}{4}\left(\frac{R_1}{R_{S1}}\cdot\frac{4R_{S1}^2}{R_1^2} + 1 + \frac{\gamma}{4}4\right)$$

$$= \frac{1}{2} + \frac{\gamma}{2} + \frac{2R_S}{R_1}.$$

From the result compared with 1 to 1 balun, NF is much smaller.

5.24 Soln:



(a) $I_{out} = K\,(V_{GS} - V_{th})^2$

$= K\,(V_{in} - V_{th})^2.$

$$\frac{\partial I_{out}}{\partial V_{in}} = 2K\,(V_{in} - V_{th}) = \alpha_1$$

$$\frac{\partial^2 I_{out}}{\partial V_{in}^2} = 2K. = 2\alpha_2 \quad \Rightarrow \quad \alpha_3 = 0$$

$$\frac{\partial^3 I_{out}}{\partial V_{in}^3} = 0 = 6\alpha_3 \quad \Rightarrow \quad \begin{cases} P_{1dB} = \infty \\ IP_3 = \infty. \end{cases}$$

(b).

$$I_{out} = \frac{1}{2}\mu_0 C_{ox}\frac{W}{L}\,\frac{(V_{in} - V_{th})^2}{1 + \left(\frac{\mu_0}{2V_{sat}L} + \theta\right)(V_{in} - V_{th})}.$$

From (5.186) (5.187) Eq.

$$\alpha_1 = K\,[2 - 3a\,(V_{GS0} - V_{th})]\,(V_{GS0} - V_{th})$$

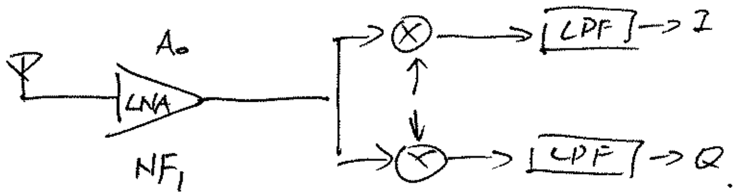
$$\alpha_3 = -Ka$$

where $K = \frac{1}{2}\mu_0 C_{ox}\frac{W}{L}$, $a = \mu_0/(2V_{sat}L) + \theta$.

$$A_{IIP_3} = \sqrt{\frac{4}{3} \times \frac{2 - 3a\,(V_{GS0} - V_{th})}{a}(V_{GS0} - V_{th})}.$$

$$A_{1dB} = \sqrt{0.145 \cdot \frac{2 - 3a(V_{GS} - V_{th})}{a}(V_{GS0} - V_{th})}.$$

6.1 Solu:



assuming the conversion gain of mixers is unity.

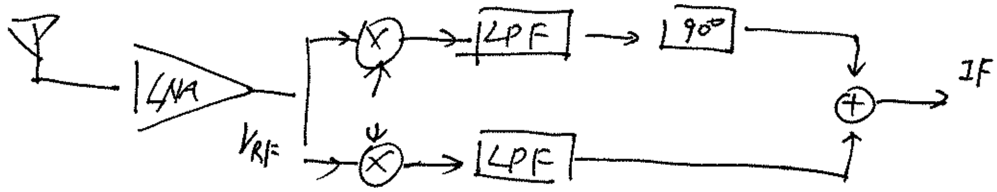
$$\because NF_1 = 1 + \frac{\overline{V_{n,out,LNA}^2}}{A_o^2} \cdot \frac{1}{4KTR_S}$$

$$\Rightarrow \overline{V_{n,out,LNA}^2} = (NF_1 - 1) \cdot A_o^2 \cdot 4kTR_S.$$

$$\overline{V_{n,out}^2} = \overline{V_{n,out,LNA}^2} + 1 \cdot \overline{V_{n,in,mixer\,I}^2} \cdot 2$$

$$\Rightarrow NF_{tot} = 1 + \frac{(NF_1 - 1) A_o^2 \cdot 4KTR_S + 2 \cdot \overline{V_{n,in,mixer\,I\,or\,Q}^2}}{A_o^2} \cdot \frac{1}{4KTR_S}$$

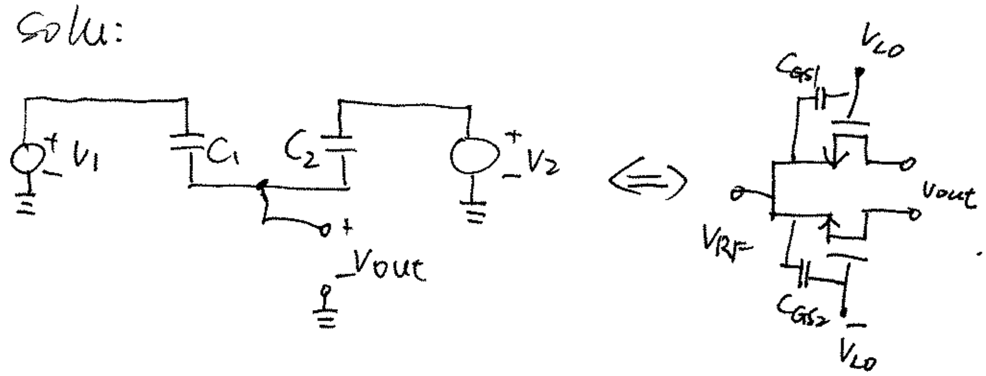
6.2 Solu:



the same as the 6.1's analysis.

$$NF_{tot} = 1 + \frac{1}{4KTR_S} \frac{(NF_{LNA}-1)A_0^2 \cdot 4KTR_S + 2\overline{V_{n,in,max\ I\ or\ Q}^2}}{A_0^2}$$

6.3 soln:



(a) $C_1 = C_2 = C_0(1 + \gamma_1 V)$

Vout @ RF input port

$$= V_1 \cdot \frac{C_1}{C_1+C_2} + V_2 \frac{C_2}{C_1+C_2}$$

$$= V_0 \cos w_{LO} t \frac{C_1 - C_2}{C_1 + C_2}$$

$$= 0$$

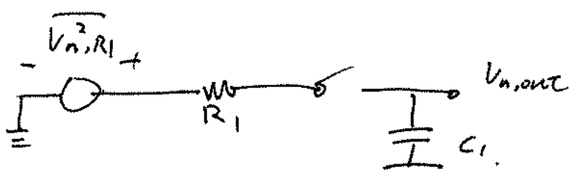
∴ So for single-balanced mixer like Fig 6.5 (b), the LO-RF feedthrough at w_{LO} vanishes if.

the circuit is symmetric.

(b). the result is the same as (a)

because of symmetry.

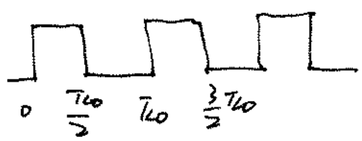
Solu: 6.4



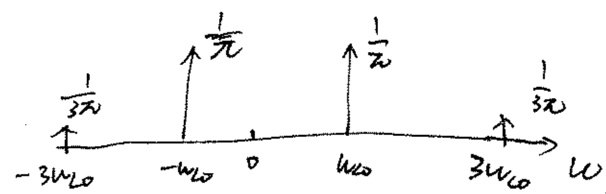
the $V_{n1}(t)$ is the product of $V_{n,LPF}(t)$ and a square wave between 0 and 1

Assume. $\frac{1}{R_1C_1} << 3w_{LO}$.

$$\overline{V_{n,LPF}^2} = \overline{V_{nR_1}^2}\frac{1}{1+(R_1C_1w)^2}.$$



$\Longleftrightarrow$.



$$V_{n1}(f) = V_{n,LPF}(f) * \text{Square}(f)$$

$$= V_{n,LPF}(f) * \left[\frac{1}{jw}(1-e^{-jwT_{LO}/2})\frac{1}{T_{LO}}\sum_{k=-3}^{+3}\delta(f-\frac{k}{T_{LO}})\right]$$

$$= V_{n,LPF}(f) * \left[\frac{1}{j\pi}\delta(f-\frac{1}{T_{LO}}) + \frac{1}{j\pi}\delta(f+\frac{1}{T_{LO}}) + \frac{1}{j3\pi}\delta(f-\frac{3}{T_{LO}}) + \frac{1}{j3\pi}\delta(f-\frac{3}{T_{LO}})\right]$$

$$= V_{n,LPF}(f-\frac{1}{T_{LO}})\cdot\frac{1}{j\pi} + V_{n,LPF}(f)(f+\frac{1}{T_{LO}})\cdot\frac{1}{j\pi}.$$
$$+ V_{n,LPF}(f-\frac{3}{T_{LO}})\cdot\frac{1}{j3\pi} + V_{n,LPF}(f)(f+\frac{3}{T_{LO}})\cdot\frac{1}{j3\pi}.$$

$$\therefore \overline{V_{n1}^2}(f) = 2\times\left(\frac{1}{\pi^2}+\frac{1}{9\pi^2}\right)\frac{2KTR_1}{1+(2\pi R_1C_1f)^2}.$$

6.5 Solu:

Eq. (6.54)

$$I_{IF}(t) = \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot I_{RF_0} \cos \omega_{IF} t.$$

V_{RF_0}.

$$V_{IF}(t) = \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot I_{RF_0} \cdot \cos_{IF}(t) \times \underbrace{2}_{\text{by differential output.}} \times Z_{BB}$$

So $\Rightarrow$ votlage conversion gain

$$= \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot 2 .$$

where d stands for duty cycle.

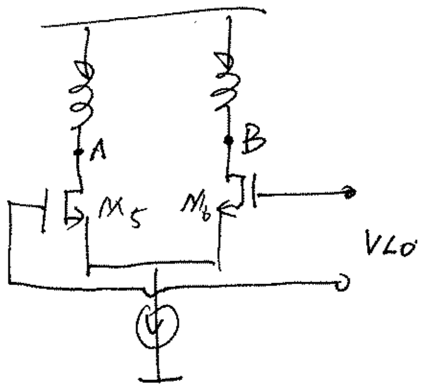
$$\lim_{d \to 0} \frac{2}{\pi} \cdot \frac{\sin \pi d}{2d} \cdot 2 = 2.$$

So votlage conversion gain

$$= 20 \log_{10} (2)$$

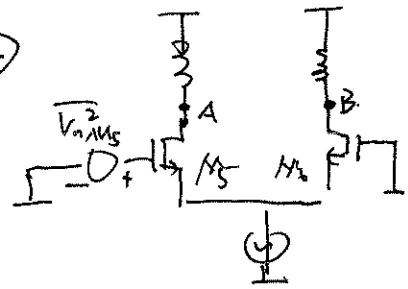
$$= 6 \text{ dB} .$$

6.6 Solu:



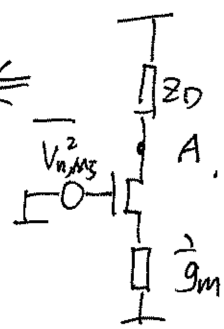
Assume the buffer's MOS are in saturation region. and the M_5 and M_6 are the same.

⇒



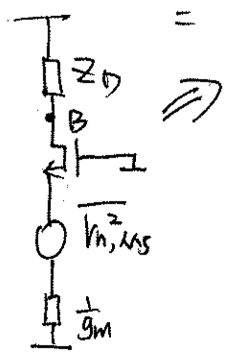
$V_{n,A}$ by M_5

$= -V_{n,M5} \cdot g_m \cdot Z_D.$



$V_{n,B}$ by M_5

$= + V_{n,M5} \cdot g_m \cdot Z_D$

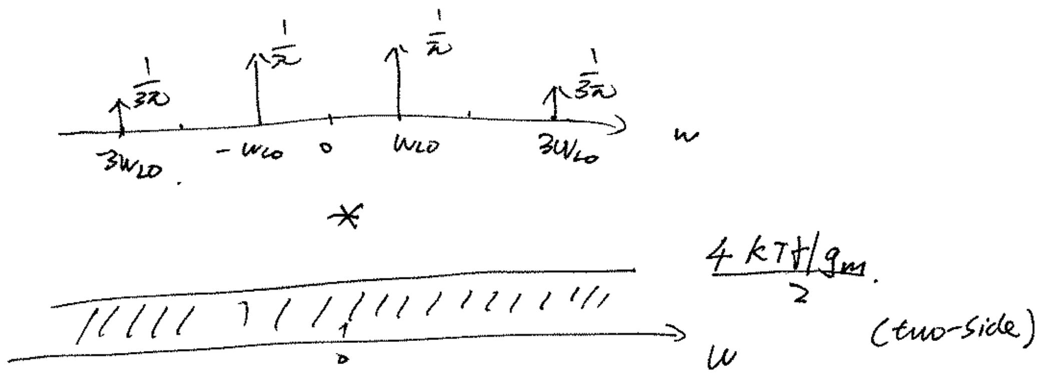


the situation of M_6's noise is the same.

So we can easily say that. the noise of M_5 & M_6 apprears differentially at nodes A & B.

6.7 Solu:

LO: 50% duty cycle.



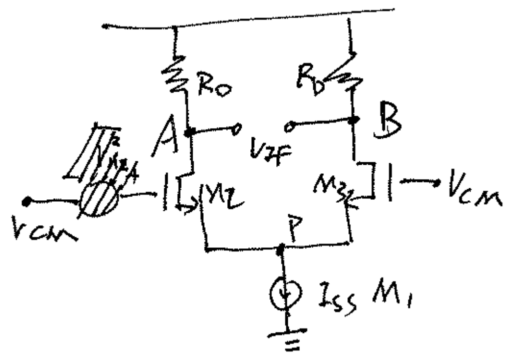
$\Downarrow$

$$\frac{4KT\gamma/g_m}{2}\left[\frac{1}{\pi^2}+\frac{1}{(3\pi)^2}+\frac{1}{(5\pi)^2}+\cdots\right]\times 2$$

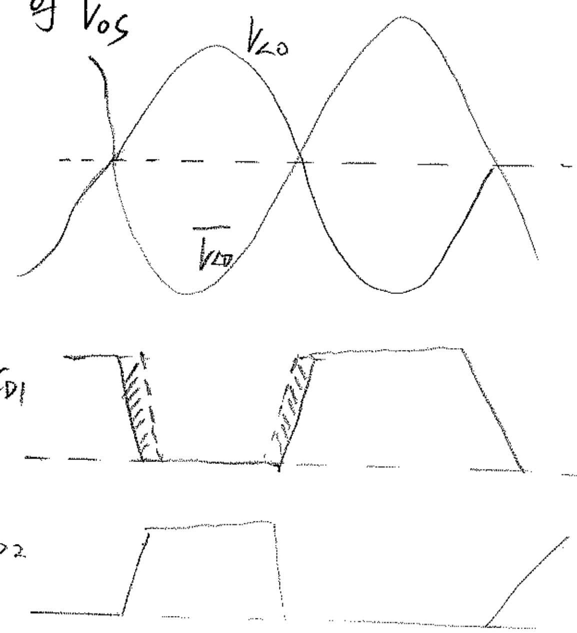
$$= 4KT\gamma/g_m \cdot \frac{1}{\pi^2}\cdot\frac{\pi^2}{8} \qquad \text{(two side)}$$

$$\Rightarrow \quad \frac{4KT\gamma}{g_m}\cdot\frac{1}{\pi^2}\cdot\boxed{\frac{\pi^2}{4}}$$

6.8 Solu:



Consider a threshold mismatch of V_{OS}



From the Figures on the righ, we find that the mistach change the zero crossing of M_2, M_3 current.

$$V_{CM} + V_{P,LO}\sin w_{LO}t + V_{OS} = V_{CM} - V_{P,LO}\sin w_{LO}t.$$

$$2V_{P,LO}\sin w_{LO}t = -V_{OS}.$$

In the vicinity of $t=0$.

$$\Rightarrow \quad 2V_{P,LO}\cdot w_{LO}t \approx -V_{OS}$$

$$\Rightarrow \quad |\Delta T| = \frac{|V_{OS}|}{2V_{P,LO}w_{LO}}$$

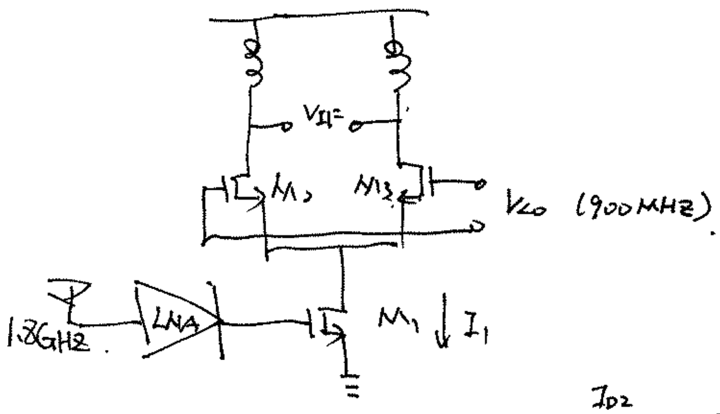
@ B $\Rightarrow \overline{V_{n,B}^2} = \frac{1}{2}\overline{I_{n,M1}^2}R_D^2$

@ A $\Rightarrow \overline{V_{n,A}^2} = \overline{I_{n,M1}^2}\left(\frac{1}{2} - \frac{2\Delta T}{T_{LO}}\right)R_D^2 = \overline{I_{n,M1}^2}\left(\frac{1}{2} - \frac{|V_{OS}|}{2\pi\cdot V_{P,LO}}\right)R_D^2$

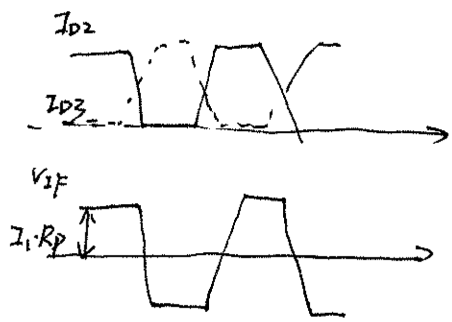
Output noise due to I_{SS}'s flicker noise

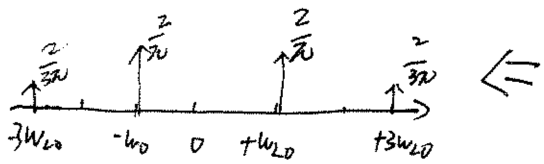
$$= \frac{|V_{OS}|R_D^2}{2\pi V_{P,LO}}\cdot\underbrace{\overline{I_{n,M1}^2}}_{\text{Flicker noise}}.$$

6.9 Solu:



(a) If V_{RF} is zero,





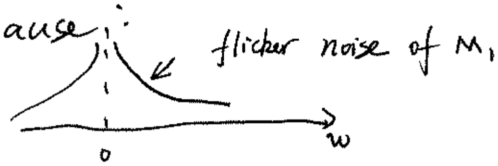
LO-IF feedthrough

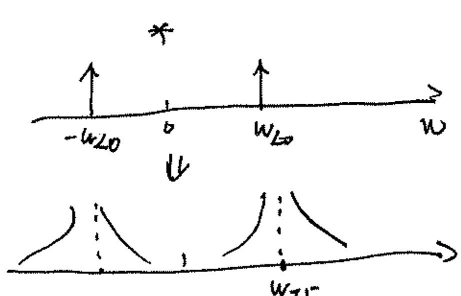
$$= I_1 \cdot R_p \cdot \frac{4}{\pi} \qquad \left(\frac{R_p}{\omega_0 L_1} = Q\right)$$

$$= I_1 \, \omega_0 L_1 \cdot Q \cdot \frac{4}{\pi}.$$

(b). flicker noise of M_1 is critical.

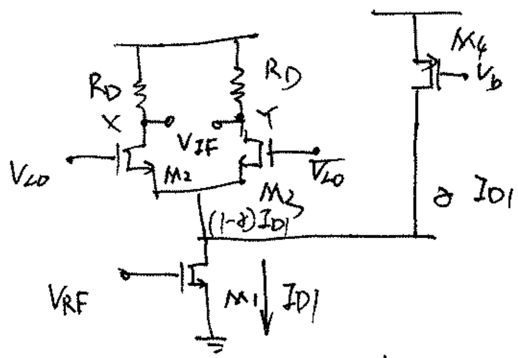
because:





$\Rightarrow$ We notice that flicker noise is transferred to IF band.

6.10 Soln:



$$I_2 = I_3 = \frac{1-\gamma}{2} I_{D1} = \frac{1}{2} I'_{2,org} = \frac{1}{2} I'_{3,org}$$

$$\Rightarrow \gamma = \frac{1}{2}$$

$$V_{n,out,org}(f) = \left(\frac{I_{D1} \cdot R_D}{\pi V_{p,LO}}\right) \cdot V_{n2}(f)$$

$\Rightarrow$ R_D & gain hence can be doubled.

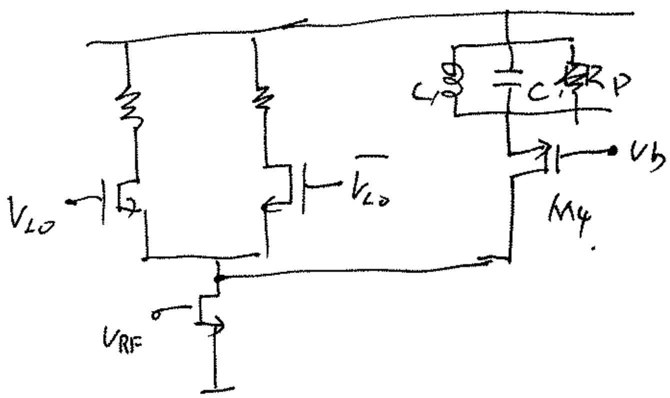
$$\therefore \overline{V^2_{n,out}} = \frac{1}{4} \overline{V^2_{n,out,org}}$$

$$gain = 2\, gain_{org}.$$

$$\therefore \overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{gain} = \frac{1}{16} \cdot \overline{V^2_{n,in,org}}.$$

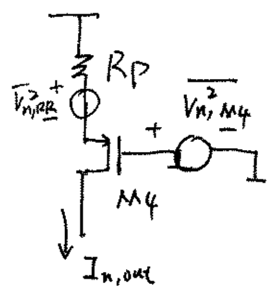
So. the input-refered flicker noise falls to sixteenth. of original one.

6.11 Soln:



Let me analyse the noise current we saw from V_{RF} MOS drain.

$$\frac{g_{m4}}{1+g_{m4}R_p} \approx \frac{1}{R_p} \quad (g_{m4}R_p \gg 1)$$



$$\Rightarrow \overline{I^2_{n,out}} = \frac{4KT\gamma}{g_{m4}}\left(\frac{1}{R_p}\right)^2 + 4KTR_p\left(\frac{1}{R_p}\right)^2$$

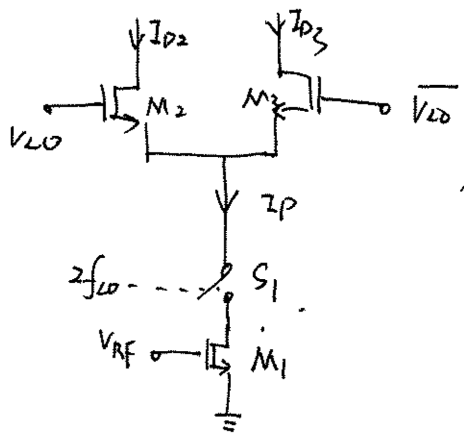
$$= \frac{4KT\gamma}{g_{m4}R_p^2} + \frac{4KT}{R_p}.$$

So Eq. (6.116) should be re-writed as

$$\overline{I^2_{n,M1}} + \overline{I^2_{n,M4}} + \overline{I^2_{n,RP}}$$

$$= 4KT\gamma \frac{2I_{D1}}{(V_{GS}-V_{th})_1} + \frac{4KT\gamma}{R_p^2}\frac{(V_{GS}-V_{th})_4}{2\partial I_{D1}} + \frac{4KT}{R_p}.$$

6.12 Solu:



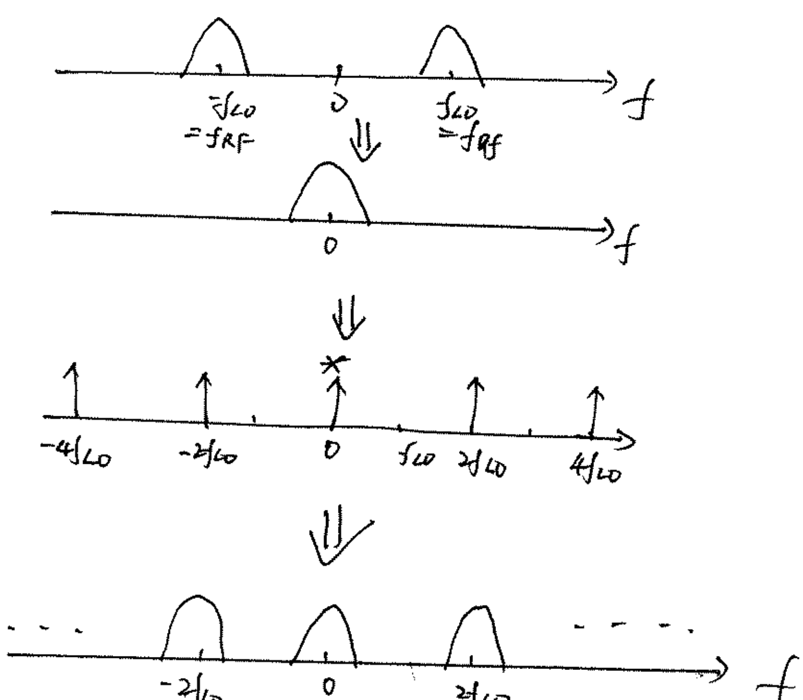
Yes. we can view this scheme as a differential pair whose tail current is modulated at a rate of $2f_{LO}$.

$$I_{D2} - I_{D3}(t) = \sum_{k=-\infty}^{+\infty} g_m \cdot V_{RF}(t) \cos(\omega_{LO} t) \cdot \delta\left(t - k \cdot \frac{T_{LO}}{2}\right)$$

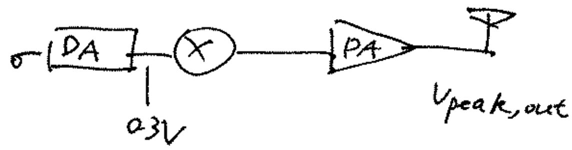
F.T.

$$I_{D2} - I_{D3}(f) = \sum_{k=-\infty}^{\infty} g_m V_{RF}(f) * \delta(f - f_{LO}) \cdot \delta(f - 2k f_{LO}).$$

We can appreciate from spectrum.



6.13 Soln:



1° gain

assume 50Ω attenna system.

$$V_{peak,out} = \sqrt{1W \cdot 50\Omega \cdot 2}$$

$$= 10V.$$

$$gain = \frac{V_{peak,out}}{V_{peak,in}} = \frac{10}{0.3} = 33.3$$

2°. noise power = −155 dBm.

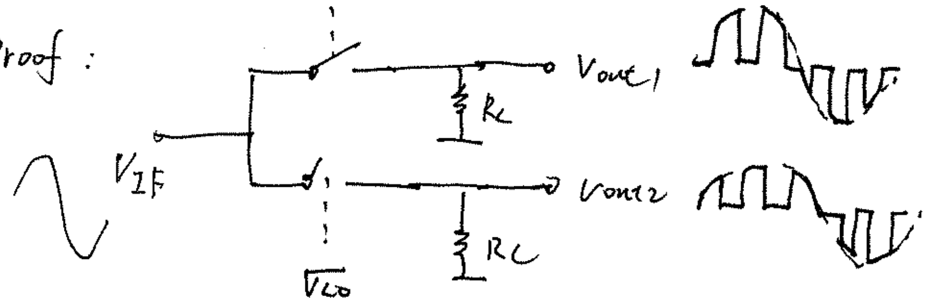
$$\Rightarrow 10^{-15.5} mW = 3.16 \times 10^{-16} mW.$$

$$V_{noise,rms,out} = 3.98\ nV.$$

$$3^{\circ} \Rightarrow \overline{V^2_{nois,rms,in}} = \frac{(3.98\ nV)^2}{gain^2} = 1.43 \times 10^{-20}\ V^2/Hz.$$

6.14. Solu:

Proof:



$$V_{out1}(t) = V_{IF}(t) \cdot \text{Square}(t)$$

$$V_{out2}(t) = V_{IF}(t) \cdot \text{Square}\left(t - \frac{T_{LO}}{2}\right).$$

only consider the fundamental freq.

$$V_{out1}(t) \approx V_{IF}(t) \cdot \frac{2}{\pi} \cos w_{LO} t.$$

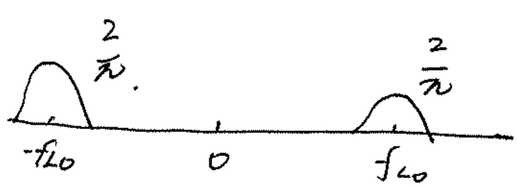
$$V_{out2}(t) \approx - V_{IF}(t) \cdot \frac{2}{\pi} \cos w_{LO} t.$$

$$\therefore V_{out}(t) = (V_{out1} - V_{out2})(t)$$

$$= V_{IF}(t) \frac{4}{\pi} \cdot \cos w_{LO}(t).$$

F.T.

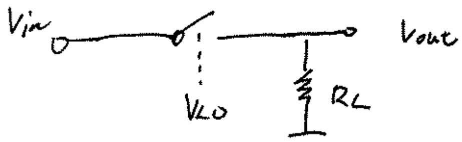
$$V_{out}(f) = \frac{2}{\pi} V_{IF}(f) * [\delta(f + f_{LO}) + \delta(f - f_{LO})]$$



(assume $w_{IF} = 0$)

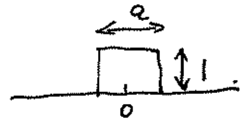
$\therefore$ voltage conversion gain of a single-balanced return-to-zero mixer is equal to $\frac{2}{\pi}$.

6.15 Solu:



Let's study the affect on the spectrum of V_{LO} about duty cycle's distortion.

$$V_{LO}(t) = \sum_{k=-\infty}^{+\infty} rec(at) \cdot \delta(t - k \cdot T_{LO})$$

where $rec(at) \Rightarrow$ 

So duty cycle $= \frac{a}{T_{LO}}$.

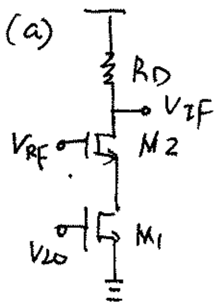
That means, if T_{LO} is unchangeable, duty cycle distortion is equivalent to a's distortion.

$$V_{LO}(f) = \frac{1}{T_{LO}} \cdot \sum_{k=-\infty}^{+\infty} \operatorname{sinc}\left(\frac{f}{a}\right) \cdot \frac{1}{|a|} * \delta(f - k \cdot f_{LO})$$

From the above equation, we can find. duty cycle only affect the gain of mixer, but doesn't produce any feedthrough. at the output.

6.16 Solu:

(a)



$$V_{out}(t) = I_{RF}(t) \cdot R_D \frac{2}{\pi} \cdot \cos w_{LO}t + \cdots$$

$$I_{RF}(t) = \frac{g_{m2}}{1+g_{m2}R_{on1}} R_D V_{RF} \cdot \cos w_{RF}(t)$$

$$V_{IF}(t) = \frac{1}{\pi} \cdot \frac{g_{m2}}{1+g_{m2}R_{on1}} R_D V_{RF} \cdot \cos(w_{RF} - w_{LO})t$$

$$\therefore \frac{V_{IF \cdot p}}{V_{RF \cdot p}} = \frac{1}{\pi} \cdot \frac{g_{m2}}{1+g_{m2}R_{on1}} \cdot R_D.$$

(b). If $R_{on} << \frac{1}{g_{m1}}$

$$\therefore gain = \frac{1}{\pi} \cdot g_{m2} \cdot R_D.$$

$$V_{RF} = V_m \cos w_1 t + V_m \cos w_2 t + V_{GS0}.$$

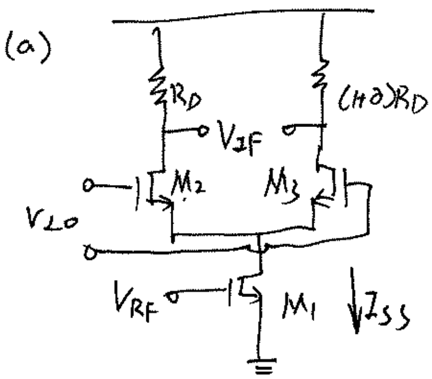
$$I_{IM_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_m^2 \cdot \cos(w_1 - w_2)t$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L} V_{IIP_2}^2 R_D = \frac{1}{\pi} \cdot g_{m2} \cdot R_D \cdot V_{IIP2}$$

$$\Rightarrow V_{IIP_2} = \frac{2}{\pi} (V_{GS0} - V_{th})_2$$

6.17 Solu:

(a)



$V_{RF} = 0.$

$I_2 = I_{ss} \cdot S(t)$

$I_3 = I_{ss} \cdot S\left(t - \frac{T_{LO}}{2}\right)$

$V_{out} = I_3 R_D (1+\partial) - I_2 R_D$

$$= \underbrace{I_{ss}\partial \cdot R_D\, S\left(t - \frac{T_{LO}}{2}\right)}_{\text{output offset}} + \underbrace{I_{ss} R_D \left[S\left(s - \frac{T_{LO}}{2}\right) - S(t) \right]}_{\text{original output}}$$

$\Rightarrow I_{ss} R_D \cdot \partial \cdot \frac{2}{\pi}.$

$=$ output offset.

(b). $V_{RF} = V_m \cos w_1 t + V_m \cos w_2 t + V_{GS0}$

$I_{M_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_m^2 \cos(w_1 - w_2)t.$

$V_{out, IM_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_m^2 \cdot \cos(w_1 - w_2)t\, R_D \cdot \partial \cdot \frac{2}{\pi}.$

$\frac{1}{2}\mu_n C_{ox} \frac{W}{L} V_{IIP_2}^2 R_D \partial \cdot \frac{2}{\pi} = \frac{2}{\pi} \cdot g_{m1} R_D \cdot V_{IIP_2}$

$= \frac{2}{\pi} \cdot \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{th})_1 \cdot R_D V_{IIP_2}$

$\Rightarrow V_{IIP_2} = \frac{2(V_{GS} - V_{th})_1}{\partial}$

7.1 Solu:

N - turn spirals.

$$L_{tot} = L_1 + L_2 + \cdots + L_N$$

$$\left.\begin{array}{l} + M_{12} + M_{13} + M_{14} + \cdots + M_{1N} \\ + M_{23} + M_{24} + \cdots + M_{2N} \\ + M_{34} + \cdots \\ \vdots \\ + M_{N-1\,N} \end{array}\right\} C_N^2 \cdot M_{ij}.$$

$C_N^2 = \dfrac{N(N-1)}{2}$ terms for mutual inductance.

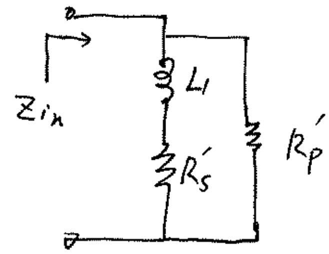
Total terms Number

$$= N + \frac{N(N-1)}{2}$$

$$= \frac{N^2 - N + 2N}{2} = \frac{N(N+1)}{2}.$$

7.2 Solu:

Proof:



$$Z_{in}(s) = (sL_1 + R_s') \parallel R_p'$$

$$= \frac{sL_1R_p' + R_s' \cdot R_p'}{sL_1 + R_s' + R_p'}$$

$$Z_{in}(j\omega) = \frac{R_s' \cdot R_p' + j\omega L_1 R_p'}{R_s' + R_p' + j\omega L_1} = \frac{(R_s' R_p' + j\omega L_1 R_p')(R_s' + R_p' - j\omega L_1)}{(R_s' + R_p')^2 - \omega^2 L_1^2}$$

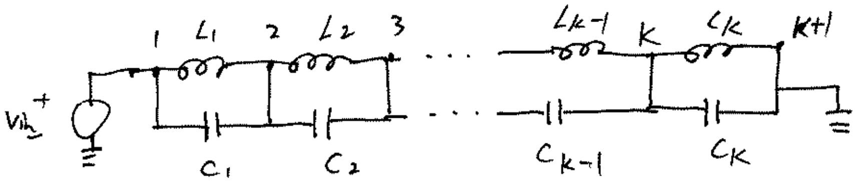
$$Q \triangleq \frac{Im(Z_{in})}{Re(Z_{in})} = \frac{\omega L_1 R_p'(R_s + R_p') - \omega L_1 (R_s' \cdot R_p')}{R_s'^2 R_p' + R_p'^2 R_s' + \omega^2 L_1^2 R_p'}$$

$$= \frac{\omega L_1 \cdot R_p'^2}{L_1^2 \omega^2 R_p' + R_s' R_p' (R_s' + R_p')}$$

$$= \frac{L_1 \omega}{L_1^2 \omega^2 + R_s'(R_s' + R_p)}$$

7.3 Solu:

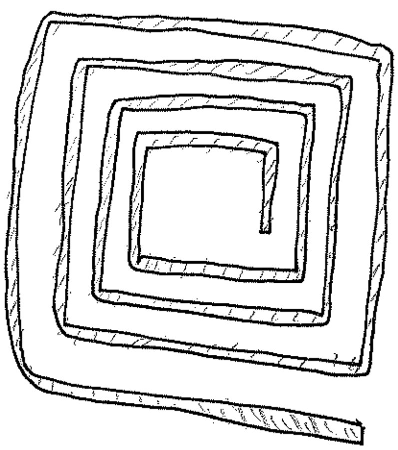
Proof. model of interwinding capacitance



For N-turn spiral inductor.

the equivalent interwinding capacitance

is $$C_{eq} = \frac{C_1 + C_2 + \cdots C_{4(N-1)}}{\cancel{(N^2-1)^2}\, 4(N-1)}$$



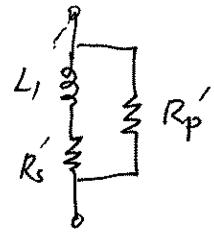
For example: 4-turn spiral inductor.

$$K = 4(N-1)$$

$$C_{eq} = \frac{C_1 + C_2 + \cdots + C_{4(N-1)}}{[4(N-1)]^2}.$$

7.4 Soln:

Eq. (7.62) $$Q = \frac{L_1 \omega R_p'}{L_1^2 \omega^2 + R_s'(R_s' + R_p')} \Leftarrow$$



For Fig. 7.37 (b) $$Q_1 = \frac{L \omega R_1}{L\omega^2 + R_s(R_s + R_1)}$$

For Fig. 7.37 (d)

$$Q_2 = \frac{\frac{L}{2}\omega \cdot R_1}{\frac{L}{2}\omega^2 + \frac{R_s}{2}\left(\frac{R_s}{2} + R_1\right)}$$

$$= \frac{L \omega R_1}{L\omega^2 + R_s\left(\frac{R_s}{2} + R_1\right)}.$$

We can find the differences between Q_1 & Q_2.

the denominator of Q_2 is smaller than that of Q_1.

7.5 Solu:

For Fig. 7.41 (a).

Using the righ-hand rule, we observe that the magnetic field due to L_1 points into page.

So does the magnetic field due to L_2. (at far-from point)

On the other hand, for Fig. 7.41 (b)

Using the right-hand rule, we also observe that the magnetic field due to $L1$ points out of the page but that ~~due~~ to $L2$ points into the page (at far-from point).

So Fig 7.41 (b)'s topology can cancel the magnetic field at a point far from the circuit. and has the less net magnetic field.

7.6 Soln:

$L = 5nH$, $W = 5\mu m$, $S = 0.5\mu m$. $N = 4$.

$R_\square = 22 m\Omega/\square$

$$f_{crit} \approx \frac{3.1}{2\pi\mu} \cdot \frac{W+S}{W^2} \cdot R_\square$$

$$= \frac{3.1}{2\pi \cdot 4\pi\times10^{-7}} \cdot \frac{5\mu + 0.5\mu}{(5\mu)^2} \cdot 22 m\Omega/\square$$

$$= 1.9 GHz.$$

$$R_{eff} = R_0 \left[1 + \frac{1}{10}\left(\frac{900M}{1.96}\right)^2\right] = 1.0224 R_0 = 16.1\Omega.$$

$(\text{For } R_0 = 15.75\Omega \Leftarrow L = 5nH).$

From Eq. (7.15) we can calculate the length.

So with length, width, space and no. of turn, the outer diameter is determinate.

7.7 Solu:

$$Y_{11}(s) = \frac{I_{in}(s)}{V_{in}(s)} = \frac{R_{sub} + L_2 s}{L_1 R_{sub} s + (L_1 L_2 - M^2) s^2}$$

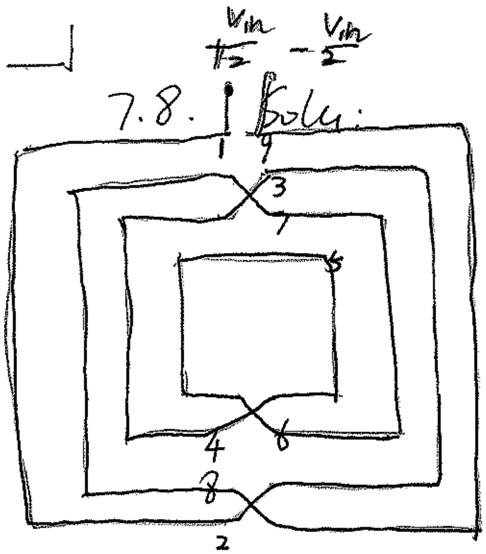
$$Y_{11}(j\omega) = \frac{R_{sub} + jL_2\omega}{jL_1 R_{sub}\omega + (M^2 - L_1L_2)\omega^2}$$

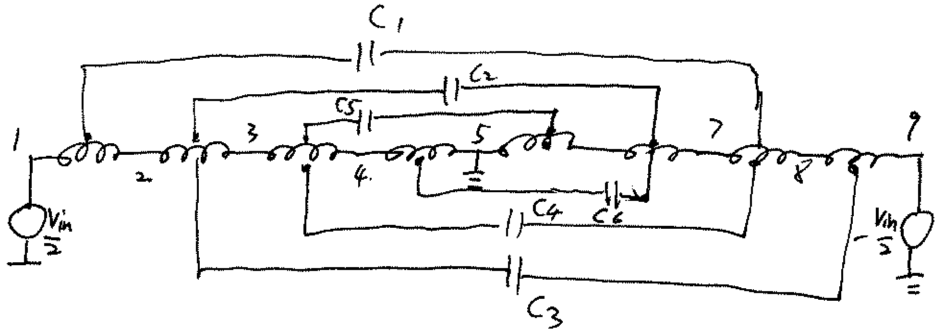
$$= \frac{(R_{sub} + jL_2\omega)[(M^2 - L_1L_2)\omega^2 - jL_1R_{sub}\omega]}{[(M^2 - L_1L_2)\omega^2]^2 + L_1^2 R_{sub}^2 \omega^2}.$$

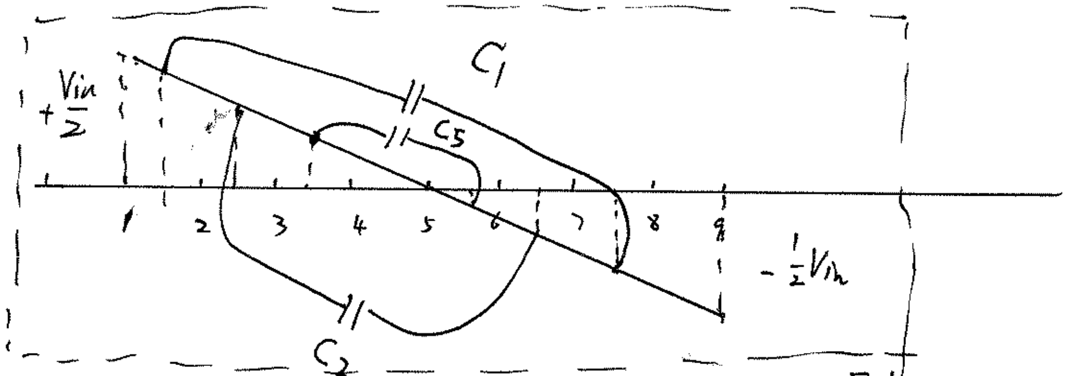
$$= \underbrace{\frac{R(M^2 - L_1L_2)\omega^2 + L_1L_2R_{sub}}{(M^2 - L_1L_2)^2\omega^2 + L_1^2 R_{sub}^2}}_{\frac{1}{R_p'}} + \underbrace{\frac{1}{j\omega}\left(\frac{L_1 R_{sub}^2 - L_2(M^2 - L_1L_2)\omega^2}{L_1^2 R_{sub}^2 + (M^2 - L_1L_2)^2\omega^2}\right)}_{\frac{1}{j\omega L'}}$$

We can find that the result is not the same at that shown in Eq. (7.55).

7.8. Sols:







C_1 & C_3 sustains $\frac{6}{8}V_{in}$

C_2 & C_4 sustain $\frac{4}{8}V_{in}$

C_5 & C_6 sustain $\frac{2}{8}V_{in}$

$$\Rightarrow E_{tot} = 2\cdot\left[\frac{1}{2}C\left(\frac{6}{8}V_{in}\right)^2 + \frac{1}{2}C\cdot\left(\frac{4}{8}V_{in}\right)^2 + \frac{1}{2}C\left(\frac{2}{8}V_{in}\right)^2\right]$$

$$= \frac{7}{8}\cdot C\cdot V_{in}^2$$

$$\because\ C_1 + C_2 + \cdots + C_6 = C_{tot} \Rightarrow c = \frac{C_{tot}}{6}$$

$$\therefore E_{tot} = \frac{7}{8}\cdot\frac{C_{tot}}{6}\cdot V_{in}^2$$

$$\Rightarrow \boxed{C_{eq} = \frac{7}{24}C_{tot}.}$$

For N-turn $\Rightarrow$

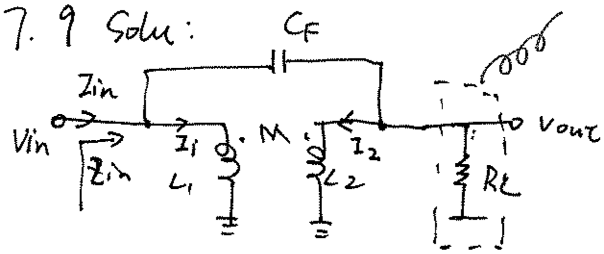
$$E_{tot} = 2\cdot\frac{1}{2}\cdot\frac{C_{tot}}{2(N-1)}V_{in}^2\cdot\left[\left(\frac{N-1}{N}\right)^2 + \left(\frac{N-2}{N}\right)^2 + \left(\frac{N-3}{N}\right)^2 + \cdots + \left(\frac{1}{N}\right)^2\right]$$

$$= \frac{C_{tot}}{2(N-1)}\cdot V_{in}^2\,\frac{1}{N^2}\cdot\frac{(N-1)N\cdot(2N-1)}{6}$$

$$= \frac{C_{tot}}{12}\cdot V_{in}^2\left(\frac{2N-1}{N}\right)$$

$$\boxed{C_{eq} = \frac{2N-1}{6N}\cdot C_{tot}}$$

7.9 Solu:



$$\begin{cases} V_{in} = L_1 s I_1 + M s I_2 \\ V_{out} = L_2 s I_2 + M s I_1 \\ I_{in} = I_1 + (V_{in} - V_{out}) s C_F \\ (V_{in} - V_{out}) s C_F = I_2 \end{cases} \Rightarrow \begin{cases} I_1 = V_{in} \cdot \dfrac{1 - \dfrac{s^2 M C_F (1 - \frac{M}{L_1})}{1 + L_2 s^2 C_F - \frac{M^2}{L_1} s^2 C_F}}{L_1 s} \\ I_2 = \dfrac{V_{in} \cdot s C_F (1 - \frac{M}{L_1})}{1 + L_2 s^2 C_F - \frac{M^2}{L_1} s^2 C_F} \end{cases}$$

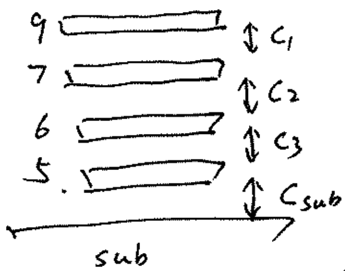
$$\Rightarrow Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)} = \frac{L_1 s I_1 + M s I_2}{I_1 + (V_{in} - L_2 s I_2 - M s I_1) \cdot s C_F}$$

Substitute I_1, I_2 into $Z_{in}(s)$, then we can get the result.

$$Z_{in}(s) = \frac{L_1 s - \dfrac{M \cdot C_F (L - M_1) s^3}{1 + s^2 C_F (L_2 - M)}}{1 + s^2 C_F (L_1 - M) - \dfrac{s^4 C_F^2 (L_2 - M)(L_1 - M)}{1 + s^2 C_F (L_2 - M)}}$$

Soln: 7.10.

Assume we choose to use Metal 9, Metal 7, metal 6 and metal 5.



each spiral must provide an inductance of $\frac{4.96nH}{16} = 0.31nH$.

choose $N = 3$, $W = 4\,um$, $S = 0.5\,um$.

From Eq. (7.15) yeilds

$$l_{tot} \approx 167\,um$$

$$D_{out} = \frac{l_{tot}}{4N} + W + (N-1)(W+S)$$

each spiral has an area of $167\,um \times 4\,um = 667\,um^2$

$$\therefore C_1 = 16 \times 667\,um^2 \times 10^{-18} = 10.7\,fF$$

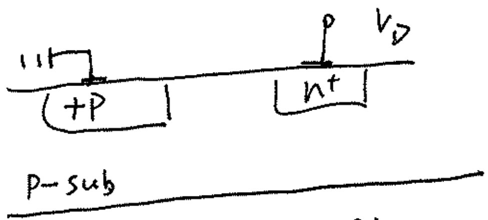
$$C_2 = 88 \times 667 \times 10^{-18} = 58.7\,fF \qquad C_{sub} = 5.7\,fF$$

$$C_3 = 88 \times 667 \times 10^{-18} = 58.7\,fF.$$

$$\therefore C_{eq} = \frac{4 \cdot (C_1 + C_2 + C_3) + C_{sub}}{3 \cdot 4^2} = 10.8\,fF.$$

7.11. Solu:

For pn-junction varactor.



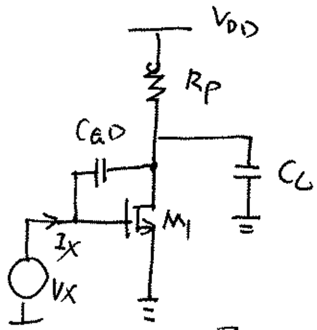
$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_D}{V_0}\right)^m}$$

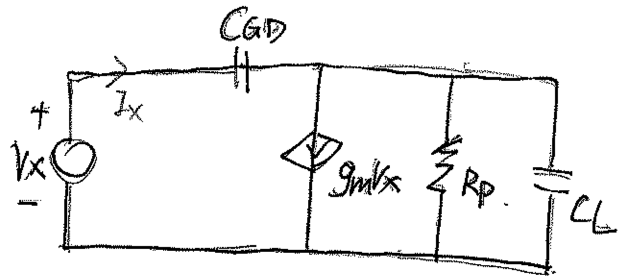
Range of control voltage:

$$V_D \in [0, V_{DD}].$$

the output swing should be as mush as the supply voltage, however the actual output swing depends on the LC VCO design.

8.1. Soln:





$$I_X = g_m V_X + \frac{V_X - I_X \cdot \frac{1}{sC_{GD}}}{R_P \parallel \frac{1}{sC_L}}$$

$$\Rightarrow \frac{I_X}{V_X} = Y_{in} = \frac{g_m + \frac{1}{R_P \parallel \frac{1}{sC_L}}}{1 + \frac{1}{sC_{GD}} \cdot \frac{1}{R_P \parallel \frac{1}{sC_L}}}$$

$$= \frac{(R_P \parallel \frac{1}{sC_L}) g_m + 1}{(R_P \parallel \frac{1}{sC_L}) + \frac{1}{sC_{GD}}}$$

$$= \frac{s^2 C_L C_{GD} R_P + (R_P g_m + 1) C_{GD} \cdot s}{1 + s R_P (C_{GD} + C_L)}$$

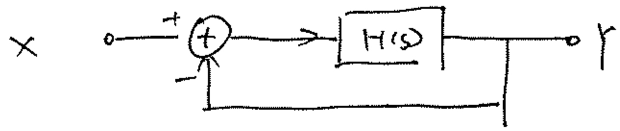
$s = j\omega$

$$Y_{in}(j\omega) = \frac{-\omega^2 C_L C_{GD} R_P + j\omega C_{GD}(R_P g_m + 1)}{1 + j\omega R_P (C_{GD} + C_L)}$$

$$\therefore Re[Y_{in}(j\omega)] = \frac{\omega^2 C_{GD} \cdot R_P (R_P g_{m1} + 1)(C_{GD} + C_L) - \omega^2 C_L C_{GD} \cdot R_P}{1 + \omega^2 R_P^2 (C_{GD} + C_L)^2}$$

$$= \frac{\omega^2 R_P C_{GD} [(1 + R_P g_{m1}) C_{GD} + R_P g_{m1} C_L]}{1 + \omega^2 R_P^2 (C_{GD} + C_L)^2}.$$

8.2 Solu:



$|H(j\omega_1)| = 1$

$\angle H(j\omega_1) = 170^\circ$

$$\frac{Y}{X}(s) = \frac{H(s)}{1 + H(s)}$$

$$\left|\frac{Y}{X}(j\omega_1)\right| = \frac{1}{|1 + e^{+j\frac{170}{180}\pi}|} = 1/0.174 = 5.73$$

$$\angle \frac{Y}{X}(j\omega_1) = \frac{170}{180}\pi - \angle(1 + H(j\omega_1))$$

$$= \frac{17}{18}\pi - 1.4835 = 1.4835 \Rightarrow 85^\circ$$

So if $x(t)$ is a sinsoidal signal at ω_1,

the amplitude of output will be multiplied by $1/0.174 = 5.73$ & the phase

will change by 85°.

8.3 Solu:

$$|H(j\omega_1)| = A > 1$$

$$\angle H(j\omega_1) = 180^\circ$$

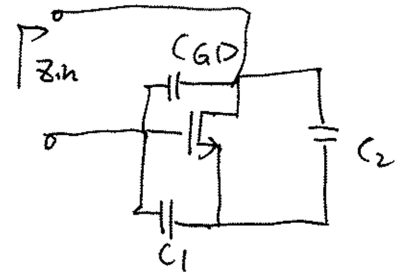
$$\therefore H(j\omega_1) = A \cdot e^{j\pi} = -A.$$

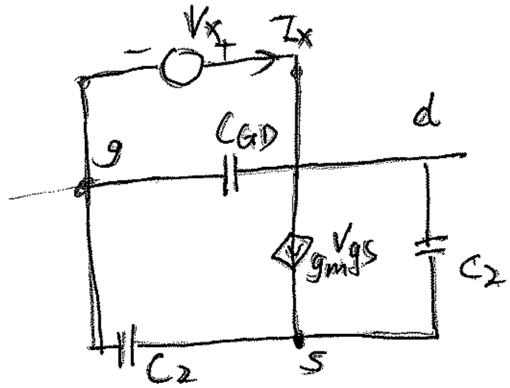
$$\left|\frac{Y}{X}(j\omega_1)\right| = \frac{A}{-(1-A)} = \frac{A}{A-1} > 1.$$

$$\angle \frac{Y}{X}(j\omega_1)| = \pi - \pi = 0.$$

So if the input of the sysmem is a sinsoidal signal at ω_1, the amplitude of output will be multiplied by $\frac{A}{A-1}$ and the phase of output will not change, compared with input.

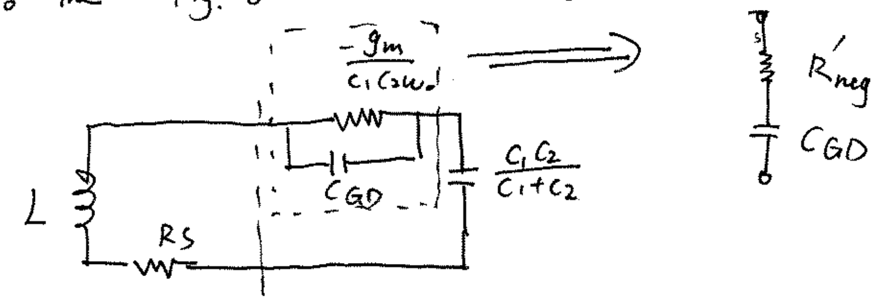
8.4. Solu:





$$Z_{in} = \frac{1}{sC_{GD}} \,//\, -\frac{g_m}{C_1C_2\omega^2}.$$

So the Fig. 8.15 will change to



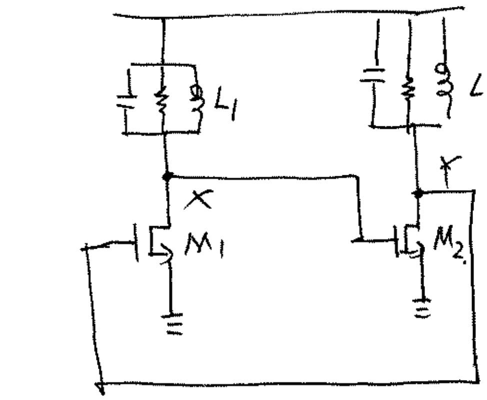
So the $$\omega_{osc} = \frac{1}{\sqrt{L_1 \cdot \frac{C_1C_2C_{GD}}{C_1+C_2} \Big/ \left(\frac{C_1C_2}{C_1+C_2} + C_{GD}\right)}}$$

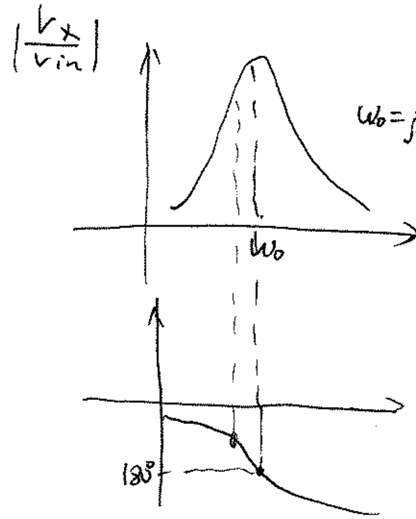
8.5 Soln:

Yes. Any feedback oscillator that employs a lossy resonator be viewed as one-port system of Figure 8.13 (c).

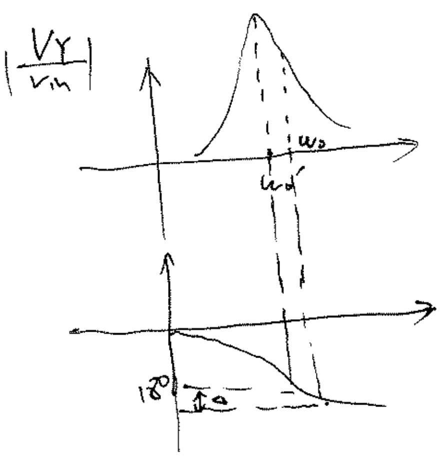
Only in this way, the feedback system can satisfy the Barkhausen's Criteria $|H(s=j\omega)| = 1$, $\angle H(s=j\omega) = -180°$. at resonate freq.

8.6 Solu:





$$\omega_0=\frac{1}{\sqrt{L_1C_1}}$$



$$\omega_0'=\frac{1}{\sqrt{C_1(L_1+\Delta L)}}$$

$$\frac{V_{out}}{V_{in}}=(-g_m)\left(R_p \,//\, \frac{1}{C_1 s} \,//\, sL_1\right)\cdot(-g_m)\left[R_p \,//\, \frac{1}{C_1 s} \,//\, s(L_1+\Delta L)\right]$$

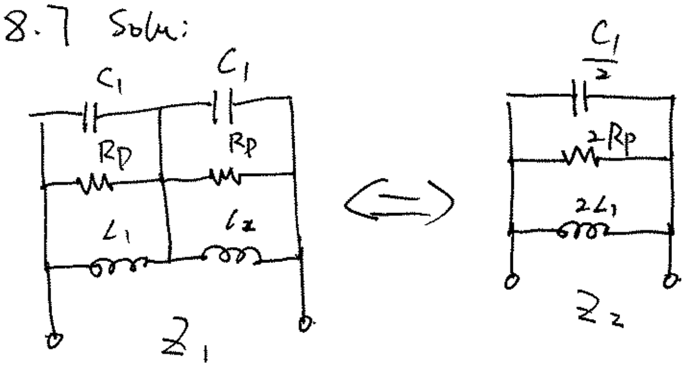
$$=g_m^2\left(\underbrace{R_p \,//\, \frac{1}{C_1 s}}_{①} \,//\, sL_1\right)\left(\underbrace{R_p \,//\, \frac{1}{C_1 s} \,//\, s(\Delta L+L_1)}_{②}\right)$$

phase contribution by ① & ②

$$\pi-\left[\arctan\frac{\omega L_1}{R_p(1-L_1C_1\omega^2)}+\arctan\frac{\omega(L_1+\Delta L)}{R_p(1-\omega^2(L_1+\Delta L)C_1)}\right]=0$$

$\Rightarrow$ Solve the ω.

8.7 Soln:



Proof: $Z_1 = 2 \cdot \frac{1}{C_1 s} // R_p // L_1 s$

$$= 2 \cdot \frac{L_1 R_p s}{L_1 s + R_p(1 + s^2 L_1 C_1)}$$

For $Z_2 = \frac{1}{\frac{sC_1}{2}} // 2R_p // 2L_1 s$

$$= \frac{2L_1^2 R_p s}{2L_1 s + 2R_p(1 + s^2 L_1 C_1)}$$

$$= \frac{2 L_1 R_p s}{L_1 s + R_p(1 + s^2 L_1 C_1)}$$

$\therefore \quad Z_1 = Z_2.$

8.8 soln:

If C_b is placed at node P & Q.

$$1/C'_{var} = 1/C_{var} + 1/C_b$$

$$C'_{var} = \frac{C_{var} C_b}{C_{var} + C_b}.$$

Without C_s & C_b. Eq. (8.73) shows

$$\Delta\omega_{osc} \approx \frac{1}{\sqrt{L_1 C_1}} \frac{0.5\, C_{max}}{2 C_1}$$

For this range to reach 10% of centre freq.

$$C_{max} = \frac{2}{5} C_1.$$

With effect of C_s & C_b. From Eq. 8.(69)

$$\Delta\omega_{oc} = \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{1}{2C_1} \cdot \frac{C_s^2 \left(\frac{C_{max} C_b}{C_{max}+C_b} - \frac{C_{min} C_b}{C_{min}+C_b} \right)}{\left(C_s + \frac{C_{max} C_b}{C_{max}+C_b} \right)\left(C_s + \frac{C_{min} C_b}{C_{min}+C_b} \right)}$$

$C_s = 10\, C_{max}$

$C_b = 0.5 C_{max}$

$C_{min} = \frac{1}{2} C_{max}$

$$= \frac{1}{\sqrt{L_1 C_1}} \cdot \frac{100\, C_{max}^2 \left(\frac{0.5\, C_{max}^2}{1.5 C_{max}} - \frac{0.25\, C_{max}^2}{1 \cdot C_{max}} \right)}{\left(10 C_{max} + \frac{1}{3} C_{max} \right)\left(70\, C_{max} - \frac{1}{4} C_{max} \right)} \cdot \frac{1}{2C_1}$$

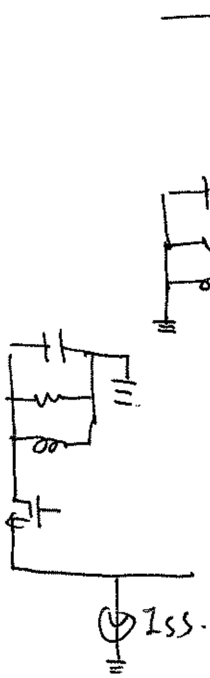
$$= \frac{1}{\sqrt{L_1 C_1}} \quad \frac{100}{31 \cdot 39.2} \cdot \frac{C_{max}}{C_1}.$$

$$= 100/(393|2) \cdot \frac{2}{5} \cdot \omega_0$$

$$= 1.65\%\ \omega_0$$

So the turning range there falls to 1.65% around $(L_1 C_1)^{-1/2}$.

8.9. Soln:



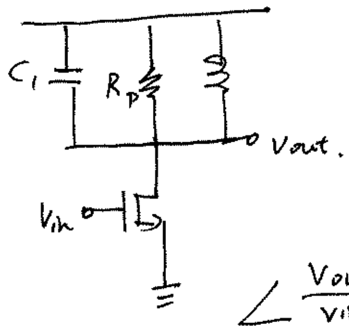
Why do the PMOS devices in Fig 8.36 carry a current of I_{SS}?

Because we know the ground in the middle is only ac ground. The dc path at this time is the right PMOS & the left NMOS. That's why PMOS carries I_{SS}.

Solu: 8.10.

Proof: For a CS stage loaded by a second-order parallel RLC tank, prove that

$$\frac{R_p}{L\omega_0} = \frac{\omega_0}{2}\left|\frac{d\phi}{d\omega}\right|.$$



$$\frac{V_{out}}{V_{in}}(j\omega) = -\frac{j g_m R_p L_1 \omega}{R_p(1-L_1C_1\omega^2) + jL_1\omega}$$

$$\angle\frac{V_{out}}{V_{in}}(j\omega) = \left[-\frac{\pi}{2} - \tan^{-1}\frac{L_1\omega}{R_p(1-L_1C_1\omega^2)}\right]$$

$$\frac{d\angle\frac{V_{out}}{V_{in}}(j\omega)}{d\omega} = -\frac{1}{1+\left[\frac{L_1\omega}{R_p(1-L_1C_1\omega^2)}\right]^2}\left[\frac{L_1}{R_p(1-L_1C_1\omega^2)} + L_1\omega R_p\frac{(-1)\cdot(-L_1C_1\cdot 2\omega)}{R_p^2[(1-L_1C_1)\omega^2]^2}\right]$$

$$= -\frac{1}{1+\left[\frac{L_1\omega}{R_p(1-L_1C_1\omega^2)}\right]^2}\left[\frac{L_1}{R_p(1-L_1C_1\omega^2)} + \frac{2L_1^2C_1\omega^2R_p}{R_p^2[(1-L_1C_1\omega^2)]^2}\right]$$

$$= -\frac{L_1R_p(1-L_1C_1\omega^2) + 2L_1^2C_1\omega^2R_p}{[R_p(1-L_1C_1\omega^2)]^2 + (L_1\omega)^2}$$

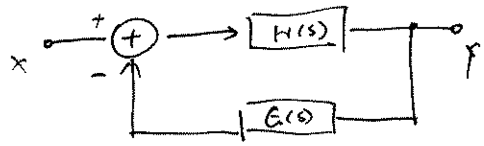
$$\left|\frac{\omega_0}{2}\,\frac{d\angle\frac{V_{out}}{V_{in}}(j\omega)}{d\omega}\right|_{\omega=\omega_0} = \frac{\omega_0^2}{2}\,\frac{2C_1R_p\cdot\omega_0^2}{\cancel{\omega_0^2}} = \omega_0^2 C_1 R_p$$

$$= \frac{1}{\sqrt{C_1L_1}}C_1R_p$$

$$= \frac{R_p}{L_1\omega_0}.$$

So $\dfrac{R_p}{L\omega_0} = \dfrac{\omega_0}{2}\left|\dfrac{d\phi}{d\omega}\right|.$

3.11 Solu:



$$[X(s) - Y\cdot(G(s))]\cdot H(s) = Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+G(s)\cdot H(s)}.$$

$$H(s) \Rightarrow H(j(w_0+\Delta w)) \approx H(jw_0) + \Delta w\frac{dH}{dw}.$$

$$G(s) \Rightarrow G(j(w_0+\Delta w)) \approx G(jw_0) + \Delta w\frac{dG}{dw}.$$

$$\frac{Y}{X}(jw_0+j\Delta w) \approx \frac{-1+\cancel{w}\frac{dH}{dw}}{1+G\cdot H(jw_0+j\Delta w)}$$

$$\approx \frac{-1}{1+G(jw_0)H(jw_0)+\Delta w\frac{dGH}{dw}}.$$

$$\approx -\frac{1}{\Delta w\frac{dGH}{dw}}. \quad (\text{assume } G(jw_0)H(jw_0)\approx -1)$$

$$\left|\frac{dGH}{dw}\right|^2 = \left|\frac{d|GH|}{dw}\right|^2 + \left|d\frac{\phi}{dw}\right|^2\cdot|GH|^2$$

$$\therefore \left|\frac{Y}{X}(jw_0+j\Delta w)\right|^2 = \frac{1}{\Delta w^2}\cdot\frac{1}{\left|\frac{d\angle GH}{dw}\right|^2|G(jw_0)|^2}$$

$$= \frac{1}{4\left(\frac{w_0}{2}\right)^2\left|\frac{d\angle GH}{dw}\right|^2}\cdot\frac{4\left(\frac{w_0}{2}\right)^2}{\Delta w^2}\cdot\frac{1}{|G(jw_0)|^2}$$

$$= \frac{1}{4\cdot Q^2}\cdot\left(\frac{w_0}{\Delta w}\right)^2\cdot\frac{1}{|G(jw_0)|^2}.$$

Where $Q = \frac{w_0}{2}\cdot\left|\frac{d\angle GH}{dw}\right|.$

8.12 Solu:

$$x(t) = A\cos\omega_0 t + n_I(t)\cdot\cos\omega_0 t - n_Q(t)\sin\omega_0 t.$$

$$x(t) = \sqrt{[A+n_I(t)]^2 + n_Q^2(t)}\,\cos\left[\omega_0 t + \tan^{-1}\frac{n_Q(t)}{A+n_I(t)}\right]$$

$$\approx \sqrt{[A+n_I(t)]^2 + n_Q^2(t)}\,\cos\left[\omega_0 t + \frac{n_Q(t)}{A}\right]$$

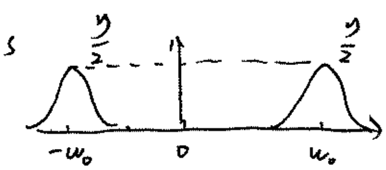
For PM,

$$x_1(t) = A\cos\left(\omega_0 t + \frac{n_Q(t)}{A}\right)$$

$$\approx A\cos\omega_0 t - n_Q(t)\cdot\sin\omega_0 t.$$

Noise Power in PM sidebands

$= n_Q(t)\cdot\sin\omega_0 t.$



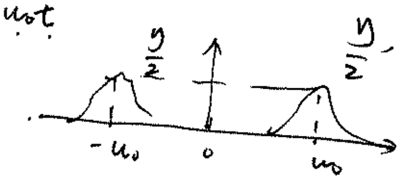
Fo AM .

$$x_2(t) = \sqrt{(A+n_I(t))^2 + n_Q^2(t)}\,\cos\omega_0 t.$$

Noise Power in AM sidebands

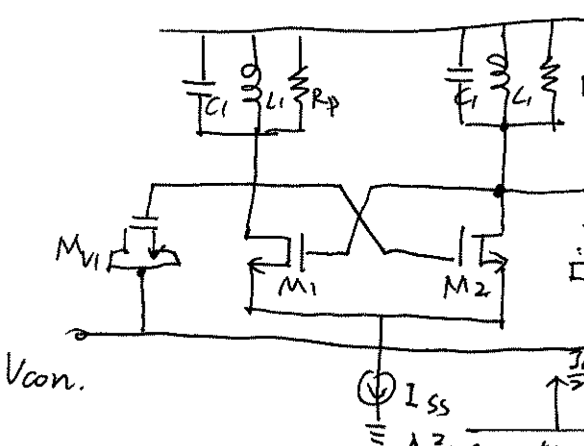
$$x_2(t) \approx A\left[1 + \frac{1}{2A}\left[n_I(t) + n_Q(t)\right]\right]\cdot\cos\omega_0 t$$

$\Rightarrow \quad n_I(t)\cdot\cos\omega_0 t.$



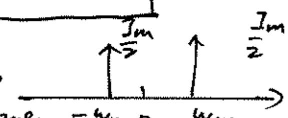
So. the power carried by AM sidebands is equal to that carried by the PM sidebands . and equal to the half power of $n(t)$.

8.13 Solu: part(2)

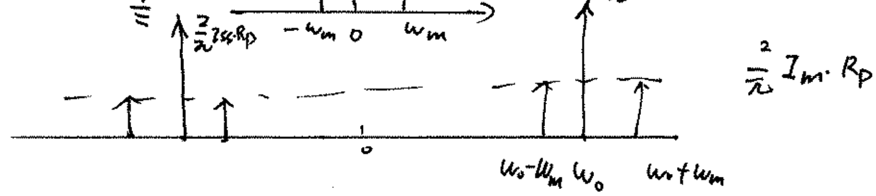


$C_{var} = C_0(1+\partial_1 V + \partial_2 V^2)$

$I_n = I_m \cos w_m t$, in I_{ss}



(a)



$$\therefore V_{n,out}\Big|_{@ w_0 \text{ near}} = \frac{2}{\pi} I_m \cdot R_p \cdot [\cos(w_0 - w_m)t + \cos(w_0+w_m)t]$$

$$= \frac{4}{\pi} I_m R_p \cdot \cos w_m t \cdot \cos w_0 t \quad \text{(only near the } w_0\text{)}.$$

(b). $C_{var} = C_0(1+\partial_1 V + \partial_2 V^2)$

$(1+\partial_1 V)$ don't bring non-zero component, so only consider $\partial_2 V^2$ term.

$$V_{out,total}\Big|_{@\text{ near } w_0} = \frac{4}{\pi} I_m R_p \cdot \cos w_m t \cdot \cos w_0 t + \frac{4}{\pi} I_{ss} \cdot R_p \cdot \cos w_0 t$$

$$V^2_{out,total} = \left(\frac{4}{\pi} I_m R_p\right)^2 (\cos w_m t \cdot \cos w_0 t)^2 + \left(\frac{4}{\pi} I_{ss} \cdot R_p\right)^2 \cos^2 w_0 t + \left(\frac{4}{\pi} R_p\right)^2 2 I_m \cdot I_{ss} \cdot \cos w_m t \cdot \cos w_0 t$$

take the DC part:

$$V_{out,total}\Big|_{DC\ part} = \left(\frac{4}{\pi} I_{ss} R_p\right)^2 \cdot \frac{1}{2} + \frac{1}{4} \cdot \left(\frac{4}{\pi} I_m R_p\right)^2$$

$$= \left(\frac{4}{\pi} \cdot R_p\right)^2 \left(\frac{I_{ss}^2}{2} + \frac{I_m^2}{4}\right)$$

$$\therefore C_{avg} = C_0 \partial_2 \left(\frac{4}{\pi} R_p\right)^2 \left(\frac{I_{ss}^2}{2} + \frac{I_m^2}{4}\right)$$

8.13 Solu: Part II

(b). (continue). If the definition of Cavg is above mentioned, the result is meaningless. That require us to compute the avarage value in the $\frac{2\pi}{\omega_0}$.

So. Cavg is revised to

$$\Rightarrow C_{avg} = C_0 + \left(\frac{4}{\pi}R_P\right)^2 \left[I_{SS}\cdot I_m \cos\omega_m t + \frac{1}{4} I_m^2 \cdot \cos 2\omega_m t \right].$$

(c). Compute ~~the noise~~ PM. because of. tank freq. modulation.

From the reference paper [13],

the conversion coefficient

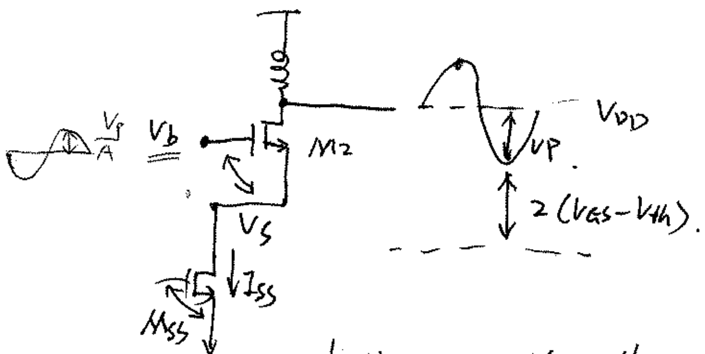
$$K_{AM/PM} = \left|\frac{\partial \omega}{\partial A}\right| \frac{A}{\alpha}.$$

$$\begin{cases} A \text{ is amplitude} \Rightarrow \frac{4}{\pi} I_{SS} R_P ; \\ \alpha \text{ is } \omega_m ; \\ \omega \text{ is oscillation freq} ; \end{cases}$$

8.14 Solu:

In Fig. 8.86 (b).

the peak drain voltage swing max.



Let's consider the critical point, which let M_{SS} & M_2 are on the edge of triode region.

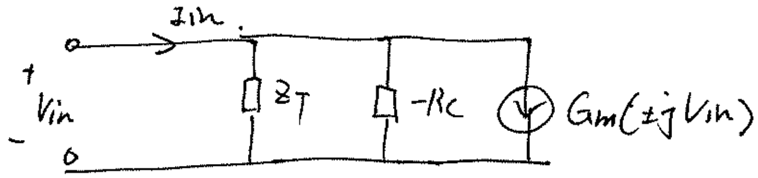
In this situation. the peak drain voltage swing is maximized as. $V_{DD} - 2(V_{GS} - V_{th})$.

In this situation, V_b, the gate voltage, cannot. change too much, that's because when the M2 is on the edge of. triode region, $V_{DS2} = V_b - V_s - V_{th}$.

$V_{DS1} = V_b - V_s - V_{th}$. ($V_s = V_{GS} - V_{th}$) where $V_b = V_b + \frac{V_P}{A}$

If $\frac{V_P}{A}$ is too large, the stress for M_{SS} will be too large.

8.5 Solu:



$$Z_T = (L_1 s) \,//\, (C_1 s)^{-1} \,//\, R_p.$$

assume $\begin{cases} G_{m1} = G_{m2} = G_m. \\ V_x = \pm\, V_x \end{cases}$

$$V_{in} = [\,\cancel{I_{in}} \cdot [Z_T \,//\, (-R_C)]\,] \cdot [\,I_{in} - G_m(\pm j V_{in})]$$

$$V_{in}(1 + G_m(\pm j)(-R_C) // Z_T) = I_{in} \cdot (Z_T \,//\, (-R_C))$$

$$\frac{I_{in}}{V_{in}} = \frac{1 \pm j\, G_m (Z_T \,//\, (-R_C))}{Z_T \,//\, (-R_C)}$$

$$= \frac{1}{Z_T // (-R_C)} + j\, G_m.$$

$$= \frac{1}{j w_0 L_1} + j w_0 C_1 + \frac{1}{R_p} - \frac{1}{R_C} \pm G_m.$$

So. $\frac{I_{in}}{V_{in}}$ can be zero even $\frac{1}{R_p} - \frac{1}{R_C} \neq 0$.

⇒ Because of each oscillator receiving energy from each other the start-up condition need not to be as stringent as $Z_T(s = w_0) = R_C$.

8.16 Solu:

$$\Delta w = \frac{w_0}{2Q\tan k}\tan^{-1}\frac{g_{m3}}{g_{m1}}.$$

$$g_{m3} = \sqrt{2\mu_n C_{ox}(\frac{W}{L})_3 \cdot (I_{T1}+I_n)/2}.$$

assume $K = \mu_n C_{ox}(\frac{W}{L})_3$

$$g_{m3} = \sqrt{K(I_{T1}+I_n)} \simeq \sqrt{KI_{T1}} + 2\sqrt{\frac{K}{I_{T1}}}\cdot I_{n1}.$$

$$\tan^{-1}\frac{g_{m1}}{g_{m3}} \approx \tan^{-1}\left(\sqrt{KI_{T1}} + 2\sqrt{\frac{K}{I_{T1}}}\cdot I_{n1}\right)$$

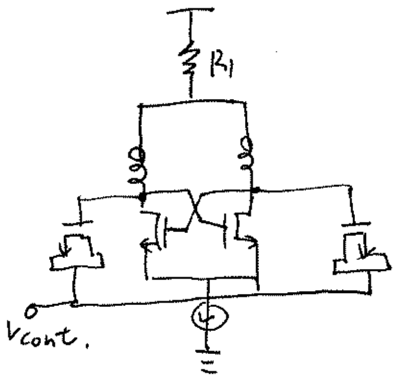
$$\approx \tan^{-1}(\sqrt{KI_{T1}}) + \frac{2\sqrt{\frac{K}{I_{T1}}}}{1+KI_{T1}}\cdot I_{n1}$$

The approximation is done according to Taylar Seriers, we choose the first and second terms.

Note:

$$f(x) \approx f(a) + f'(a)\cdot(x-a).$$

8.17 Soln:



(a) $$\omega_{out} = \frac{1}{\sqrt{L_1 C_0 (1+\gamma \cdot V_{var})}}$$

V_{var} is the average voltage across the varactor.

$$V_{var} = V_{DD} - R_1 \cdot I_{SS} - V_{cont}$$

$$\omega_{out} = \frac{1}{\sqrt{L_1 C_0 [1+\gamma (V_{DD} - R_1 I_{SS} - V_{cont})]}}$$

$$\simeq \frac{1}{\sqrt{L_1 C_0 [1+\gamma (V_{DD}-V_{cont})]}} + \frac{-L_1 C_0 \gamma R_1}{2\sqrt{L_1 C_0 [1+\gamma (V_{DD}-V_{cont})]}} \cdot I_{SS}$$

$\Rightarrow$ the "gain" from I_{SS} to ω_{out}

$$= -\frac{L_1 C_0 \gamma R_1}{2\sqrt{L_1 C_0 [1+\gamma (V_{DD}-V_{cont})]}}$$

(b). $$\omega_{out} = \omega_0 + \frac{L_1 C_0 \gamma R_1}{2\sqrt{L_1 C_0 (1+\gamma (V_{DD}-V_{cont}))}} \cdot I_n \cdot \cos \omega_m t$$

The frequency is modulated by $I_n \cos \omega_n t$.

Using narrowband FM approximation.

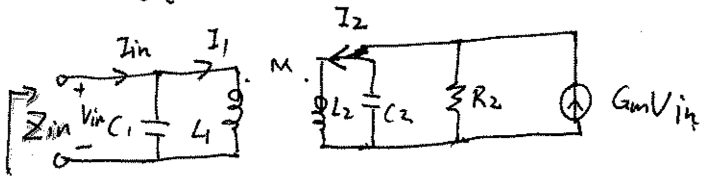
$\Rightarrow$ $$X(t) = A \cdot \cos(\omega_0 t + B(t)) \qquad B(t) = I_n \cos \omega_n t$$

$$\approx A \cos \omega_0 t - A \cdot B(t) \cdot \sin \omega_0 t$$

$$\approx A \cos \omega_0 t - A I_n \cos \omega_n \cdot \sin \omega_0 t$$

So the sidebands relative magnitude is $\frac{I_n}{2}$.

8.18 Solu:



(a)

$$\begin{cases} V_{in} = I_1 L_1 S + I_2 M S & ① \\ I_1 = I_{in} - V_{in}\cdot S C_1 & ② \\ I_2 + \dfrac{(I_2 L_2 S + I_1 M S)}{\frac{R_2}{S R_2 C_2 + 1}} = G_m \cdot V_{in} & ③ \end{cases}$$

$$V_{in} = (I_{in} - V_{in}\cdot SC_1) L_1 S + I_2 M S$$

$$\Rightarrow \quad I_2 = \frac{V_{in}(1 + s^2 L_1 C_1) - I_{in} L_1 S}{MS} \qquad ④$$

substitute I_1, I_2 into ③.

$$V_{in}\frac{1+s^2L_1C_1}{MS} - I_{in}\frac{L_1}{M} + \left[V_{in}\frac{L_2(1+s^2L_1C_1)}{M} - I_{in}\frac{L_1L_2S}{M} + I_{in}\cdot MS - V_{in}S^2C_1M\right]\frac{SR_2C_2+1}{R_2} = G_mV_{in}$$

$$V_{in}\left(\frac{1+s^2L_1C_1}{M} + \frac{L_2(1+s^2L_1C_1)}{M}\cdot\frac{SR_2C_2+1}{R_2} - S^2C_1M\frac{SR_2C_2+1}{R_2} - G_m\right)$$

$$= I_{in}\left(\frac{L_1}{M} + \frac{L_1L_2S}{M} - MS\right)$$

$$Z_{in} = \frac{V_{in}}{I_{in}}$$

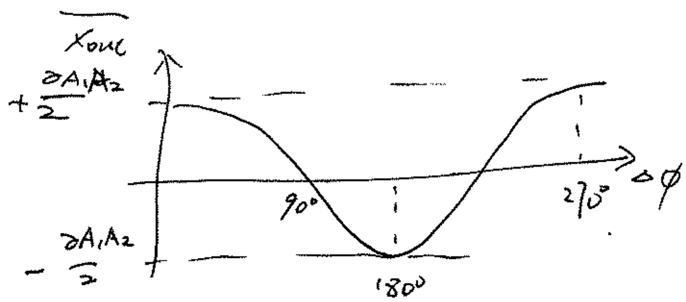
(b). when $\left.\dfrac{1+s^2L_1C_1}{M} + \dfrac{L_2(1+s^2L_1C_1)}{M}\cdot\dfrac{SR_2C_2+1}{R_2} - s^2C_1M\dfrac{SR_2C_2+1}{R_2} - G_m\right|_{s=j\omega} = 0$

$$\Rightarrow \quad \omega_1 = \sqrt{\frac{R_2 + L_2 - G_mMR_2}{L_1C_1R_2 + L_1L_2C_1 - C_1M^2}}$$

$$\omega_2 = \sqrt{\frac{L_1}{L_1L_2C_1 - C_1M^2}}$$

9.1 Soln:

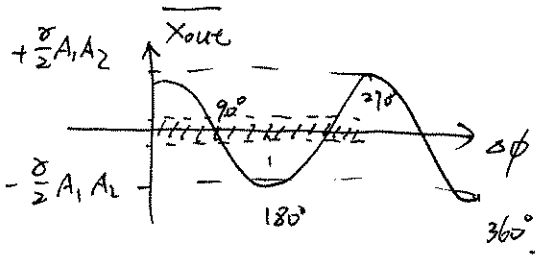
$$\overline{x_{out}(t)} = \frac{\gamma A_1 A_2}{2} \cos \Delta\phi.$$

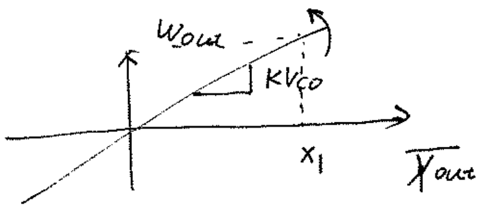


The "gain" should be difined carefully in Phase Detector.

The zero gain at $\Delta\phi = 180°$ & $0°$, only means when $\Delta\phi$ is very near to $0°$ or $180°$, the average of x_{out} will not charge or change very slowly. However, with the accumulation of $\Delta\phi$, the gain cannot remain zero.

9.2 Soln:





$\omega_{out} = \omega_0 + K_{VCO} \cdot \overline{X_{out}}$

$\because$ K_{VCO} is very high

$\therefore$ $\overline{X_{out}}$ is very small.

From the Figure above,

we can find $\Delta\phi$ should be near $\underline{90° + 180° \cdot k}$ $(k = 0, 1, 2 \cdots)$

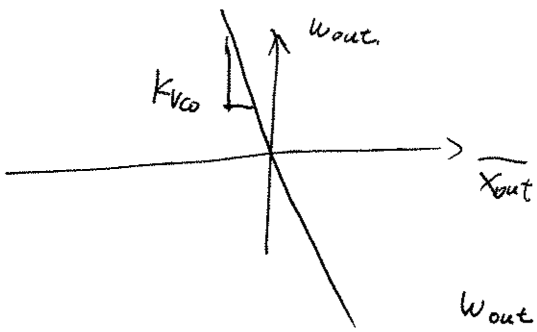
$\Rightarrow$ $\Delta\phi \approx 90° + 180° \times k$

$(k = 0, 1, 2 \cdots)$.

9.3 Solu:

For Problem 2, if the K_{VCO}'s sign is change.

the V_{out} v.s. $\overline{X_{out}}$ diagram should be



$$\omega_{out} = \omega_0 + K_{VCO} \cdot \overline{X_{out}}$$

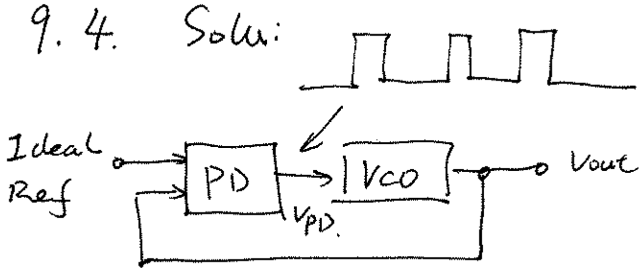
$\overline{X_{out}}$ will be also changing its sign.

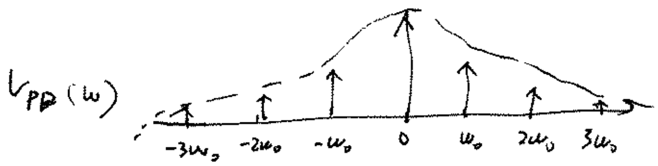
However, our result won't change.

$$\Delta\phi \approx 90^\circ + 180^\circ \cdot k$$

$$(k = 0, 1, 2, \cdots)$$

9.4. Solu:



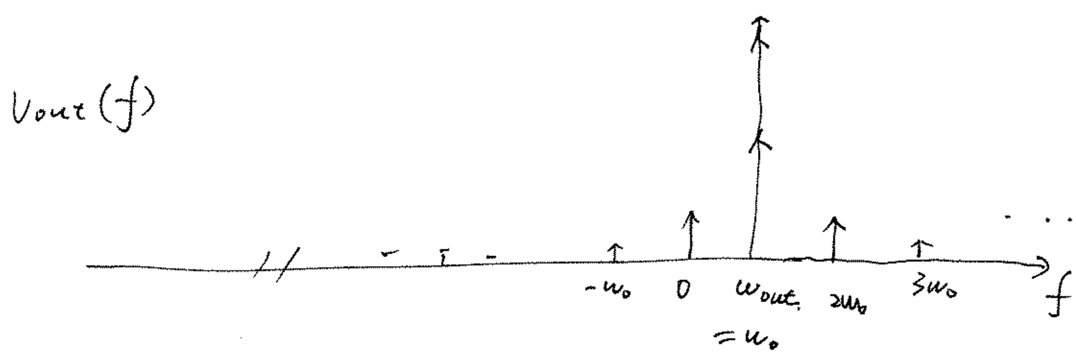


Vco $\Rightarrow$ Vout.

$$\omega_{out} = \omega_0 + V_{PD} K_{VCO}.$$

$$V_{out} = V_0 \cos\left[\omega_0 t + K_{VCO} \int V_{PD}(t)\, dt \right]$$

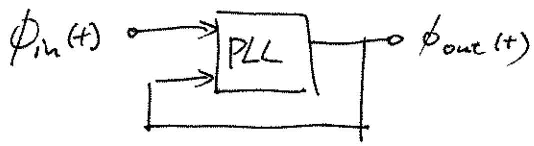
$$\approx V_0 \cos\omega_0 t - \underline{V_0 \cdot \int V_{PD}(t)\, dt}\, K_{VCO} \cdot \sin\omega_0 t.$$

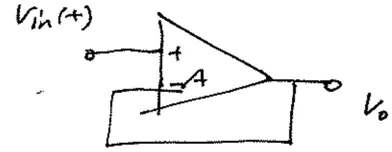


$\Rightarrow$ So,

output sidesbands. are located as the figure above.

9.5. Solu:





$$\because \frac{d\phi_{out}}{dt} = \frac{d\phi_{in}}{dt}$$

$$\Rightarrow \Delta\phi_{out} = \Delta\phi_{in}.$$

$\cancel{\Delta V_{out}}$ ΔV_{in}

$$\Delta V_{out} = \frac{\Delta V_{in}}{A_0+1}$$

The statement of unity buffer is not correct.

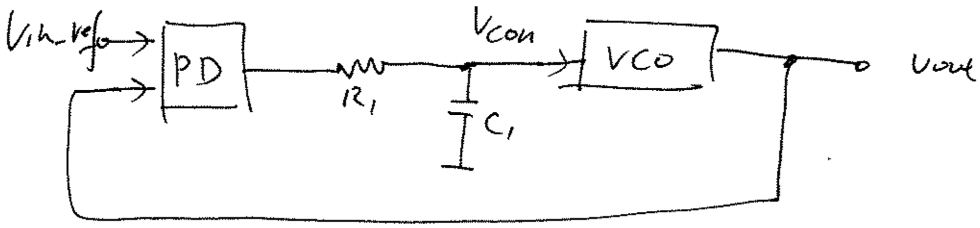
We can compute the transfer function of unity-gain buffer.

$$H(s) = \frac{A(s)}{1+A(s)}$$

$S \approx 0$ 时 $H(s) = \frac{A_0}{1+A_0} \approx 1$

So $\Delta V_{in} \Rightarrow \Delta V_{out} = \Delta V_{in}$.

9.6 Solu:



(I) VCO noiseless

If we break the R_1,

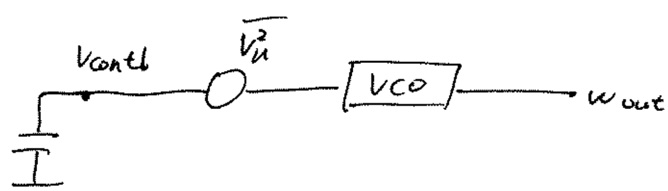
V_{con} won't change because the charge is conserved on capactor C_1.

So the w_{out} will also be stable.

(II) VCO noisy.

If we break the R_1.

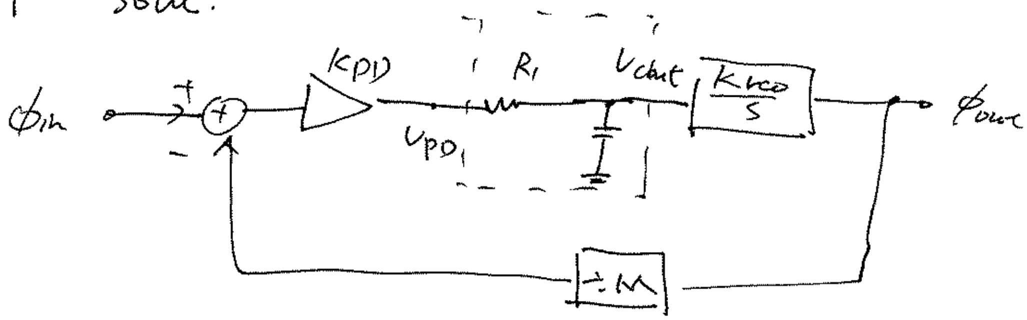
V_{con} on C_1 will be in series with a input-referred noise source.



So. the w_{out} ~~will~~ be modulated by $\overline{V_n^2}$.

$$H_1(s) = \frac{\frac{1}{sC_1}}{\frac{1}{sC_1}+R_1} = \frac{1}{sR_1C_1+1}.$$

9.7 Soln:



$$\left(\phi_{in} - \frac{\phi_{out}}{M}\right) \cdot K_{PD} \cdot \frac{1}{sC_1R_1+1} \cdot \frac{K_{VCO}}{s} = \phi_{out}.$$

$$\phi_{in} \cdot \frac{K_{PD} \cdot K_{VCO}}{s(sR_1C_1+1)} = \phi_{out}\left[1 + \frac{K_{PD}\cdot K_{VCO}}{s(sR_1C_1+1)M}\right]$$

$$\frac{\phi_{out}}{\phi_{in}}(s) = \frac{K_{PD}\cdot K_{VCO} M}{s(sR_1C_1+1)M + K_{PD}\cdot K_{VCO}}$$

$$= \frac{K_{PD}\cdot K_{VCO}\cdot M}{s^2 \cdot R_1C_1M + sM + K_{PD}\cdot K_{VCO}}$$

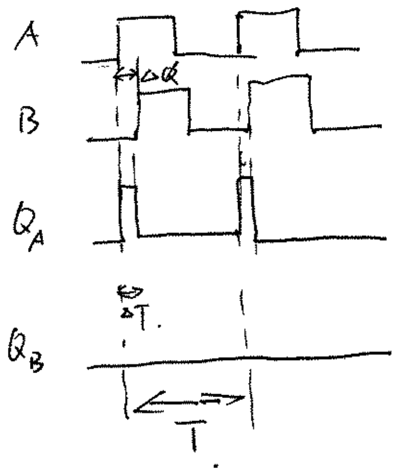
$$= \frac{K_{PD}\cdot K_{VCO}\cdot M / R_1C_1M}{s^2 + \frac{M}{R_1C_1M}\cdot s + \frac{K_{PD}\cdot K_{VCO}}{R_1C_1M}}$$

$$\omega_n = \sqrt{K_{PD}\cdot K_{VCO}\cdot \omega_{LPF}/M} \qquad \left(\omega_{LPF} = \frac{1}{R_1C_1}\right)$$

$$\zeta = \frac{1}{2}\sqrt{\frac{\omega_{LPF} M}{K_{PD}\cdot K_{VCO}}}.$$

9.8 Solu:

(I)



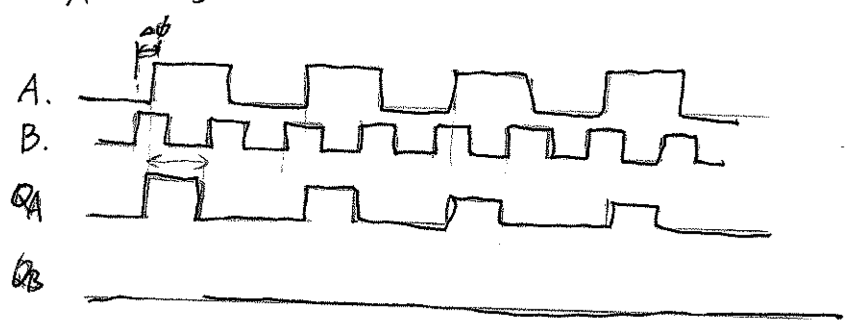
assume the magnitude of all signall will be 1.

$$\frac{\Delta\phi_{AB}}{2\pi}\cdot T = \Delta T$$

$$\overline{Q_A - Q_B} = \frac{\Delta T\cdot 1}{T} = \frac{\Delta\phi_{AB}}{2\pi}.$$

$\Rightarrow$ $\overline{Q_A - Q_B}$ is a linear function of input phase error.

(II)



assume $f_B = 2f_A$. $Q_A = f_A$.

$$\overline{Q_A - Q_B} = \frac{\pi - \Delta\phi}{2\pi}\cdot\frac{1}{f_A}\cdot\Big/\frac{1}{f_A} = \frac{\pi - \Delta\phi}{2\pi}.$$

$\Rightarrow$ $\overline{Q_A - Q_B}$ is a linear function of input freq. difference.

(It's a special case to demonstrate, but not a strict pro

9.9. Solu.

Eq. (9.19)

$$H(s) = \frac{\frac{I_p \cdot K_{vco}}{2\pi C_1} \cdot (R_1 C_1 s + 1)}{s^2 + \frac{I_p}{2\pi} K_{vco} R_1 s + \frac{I_p}{2\pi} K_{VCO}}.$$

$$= \frac{\frac{I_p \cdot K_{VCO} R_1}{2\pi} \left(s + \frac{\omega_n}{2\zeta} \right)}{s^2 + 2\zeta \omega_n s + \omega_n^2}.$$

peak is at ω_n.

$$|H(s = j\omega_n)| = \frac{I_p \cdot K_{vco} \cdot R_1}{2\pi} \left| \frac{j\omega_n + \frac{\omega_n}{2\zeta}}{-\omega_n^2 + \omega_n^2 + 2\zeta \omega_n^2 j} \right|$$

$$= 2\zeta \omega_n \left| \frac{\sqrt{\omega_n^2 + \frac{\omega_n^2}{4\zeta^2}}}{2\zeta \omega_n^2} \right|$$

$$= 2\zeta \sqrt{1 + \frac{1}{4\zeta^2}}.$$

$$= \sqrt{4\zeta^2 + 1}.$$

7

9.10 Solu:

$$\because \zeta = \frac{R_1}{2}\sqrt{\frac{I_P C_1 K_{VCO}}{2\pi}}$$

$$\omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_1}}$$

$\zeta = 1$

time constant

$$= 1/\zeta \cdot \omega_n = \frac{25}{\omega_{in}}$$

$$\omega_n = \frac{\omega_{in}}{25}$$

$$K_{VCO}(V_{con\,max} - V_{con\,min}) = 10\% \cdot \omega_{in}.$$

$$K_{VCO} \cdot V_{DD} = 0.1 \cdot \omega_{in}$$

$$\Rightarrow \omega_n = \frac{R_1}{2} \cdot \frac{I_P \cdot K_{VCO}}{2\pi}$$

$$\Rightarrow \omega_n = \frac{R_1}{2} \cdot \frac{I_P}{2\pi} \cdot \frac{0.1 \cdot \omega_{in}}{V_{DD}}$$

$$\Rightarrow \frac{\omega_{in}}{25} = R_1 \cdot I_P \cdot \frac{0.1}{4\pi} \frac{\omega_{in}}{V_{DD}}$$

$$\Rightarrow R_1 \cdot I_P = 1 / \frac{2.5}{4\pi} \cdot V_{DD}$$

$$= 1.6\pi V_{DD}.$$

9.11. Solu:

$M_1 = 1000$

(a) If output freq. remain

$M_2 = 500$.

$$\frac{A_{side}}{A_{carrier}} = \frac{1}{2\pi} \cdot \frac{\Delta T\, I_P}{C_2} T_{res} \cdot K_{VCO}$$

the ration doesn't change.

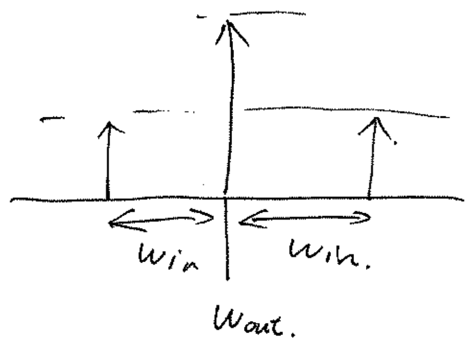
(b) If output freq. doubled, K_{VCO} doubled.

$M_2 = 1000$

$$\frac{A_{side}}{A_{carrier}} = \frac{1}{2\pi} \cdot \frac{\Delta T\, I_P}{C_2} T_{res} \cdot K_{VCO} \cdot 2$$

The ration also doubles.

9.12 Solu:

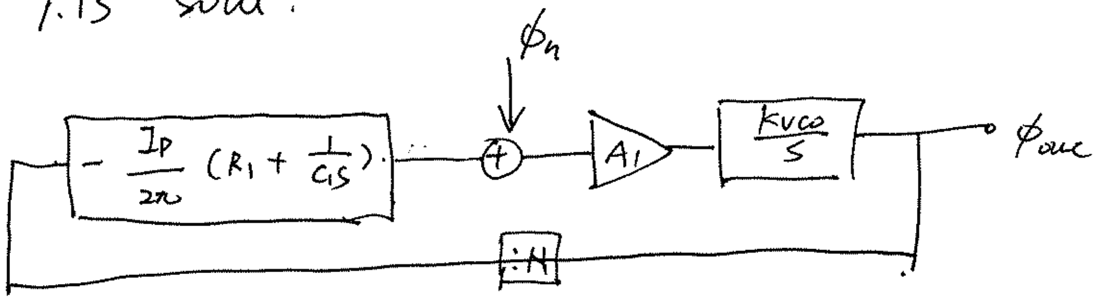


loop bandwidth << w_{in} to ensure the continuous-time approximation's validity.

So the sidebands is located at w_{in} away from w_{out}. PLL cannot suppress high-freqency noise.

⇒ That's why PLL suppress VCO phase noise but not the sidebands due to ripple.

9.13 Solu:



$$\left[\frac{\phi_{out}}{N}\left(-\frac{I_P}{2\pi}\left(R_1+\frac{1}{C_1 s}\right)\right)+\phi_n\right]\cdot A_1\cdot\frac{K_{VCO}}{s}=\phi_{out}.$$

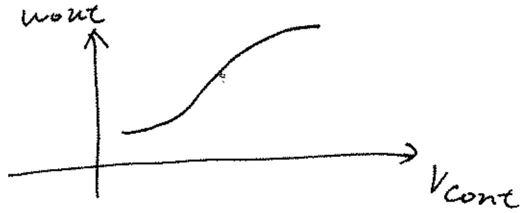
$$\left(1+\frac{I_P}{2\pi}\left(R_1+\frac{1}{C_1 s}\right)\frac{A_1 K_{VCO}}{N\,s}\right)\phi_{out}=A_1\cdot\frac{K_{VCO}}{s}\cdot\phi_n.$$

$$\phi_{out}=\frac{\frac{A_1\cdot K_{VCO}}{s}}{1+\frac{I_P}{2\pi}\left(R_1+\frac{1}{C_1 s}\right)\frac{A_1 K_{VCO}}{N\,s}}\phi_n.$$

$$\phi_{out,n}^2=\frac{(A_1 K_{VCO})^2}{\sqrt{\left(\frac{I_P A_1 K_{VCO}}{2\pi N C_1}-\omega^2\right)+\left(\frac{I_P A_1 K_{VCO} R_1}{2N\pi}\right)^2}}\cdot\frac{\sigma^2}{f^2}.$$

9.14. Solu:

14.



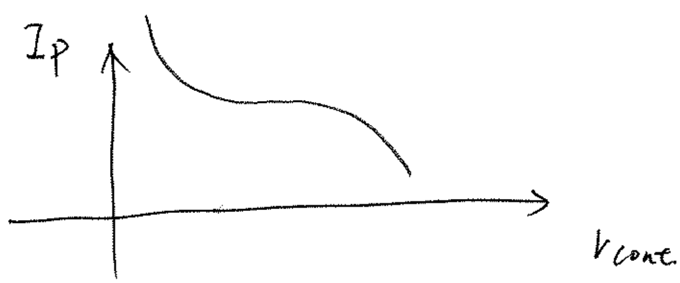
VCO characteristics

Characteristic equation :

$$: s^2 + \frac{I_p}{2\pi} K_{VCO} R_1 s + \frac{I_p}{2\pi C} \cdot K_{VCO} = 0.$$

$I_p \cdot K_{VCO}$ should be constant.

$$\Rightarrow I_p \propto \frac{1}{K_{VCO}}, \qquad \left(K_{VCO} = \frac{d\omega_{out}}{dV_{cont}} \right).$$



9.15 Solu:

(a) PFD now make half as many phase comparisons per second.

pumping half as much as charge into the loop filter.

Thus, loop is less stable.

Not correct.

Because of unchangable all loop parameters,

the stability is unchanged.

(b). Equation $\zeta = \frac{R_P}{2}\sqrt{\frac{I_P K_{VCO} C_P}{2\pi}}$ indicates that ζ remains

constant and the loop is as stable as before.

Correct.

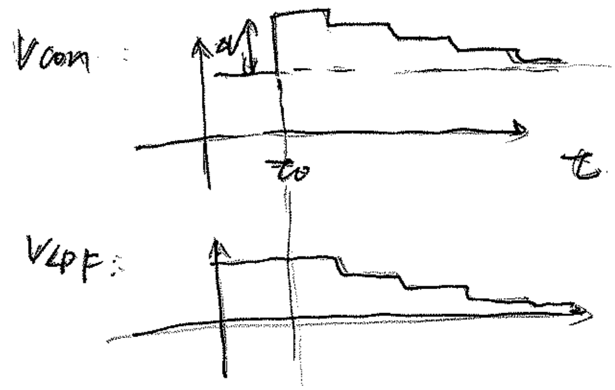
$$H(s) = \frac{2\zeta\omega_n \left(s + \frac{\omega_n}{2\zeta}\right)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ is unchanged.

So is poles & zero. That means the Phase

margin of this loop is unchanged. Only operation

point is different.

9.16 Soln:



assume non-idealities are all neglected.

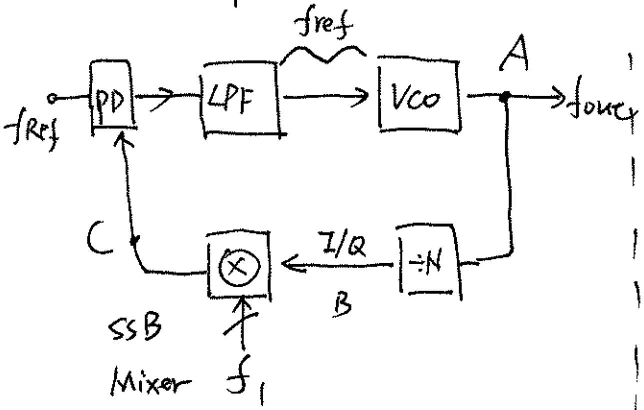
V_{cont} total change should be zero.

V_{LPF} total change should be $-\Delta V$.

output frequency won't change. when phase-locked.

The input & output phase differente :

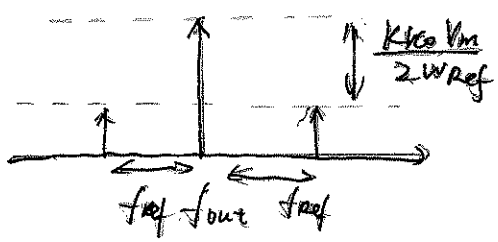
9.17 Solu:



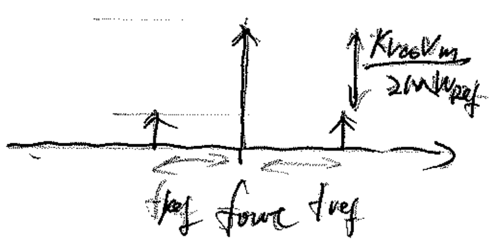
(a) $\frac{f_{out}}{N} + f_1 = f_{Ref}$

$f_{out} = N(f_{Ref} - f_1)$

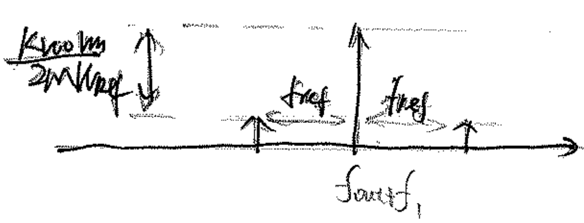
(b)

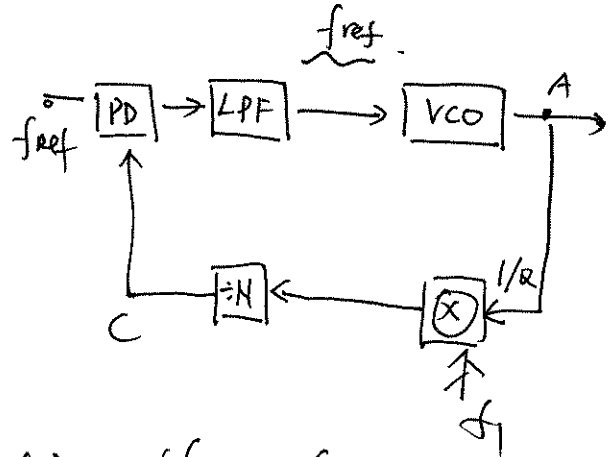


(c) ① B node



② C node



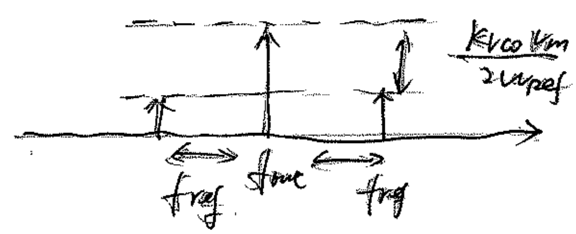


(a) $\frac{(f_{out} + f_1)}{N} = f_{Ref}$

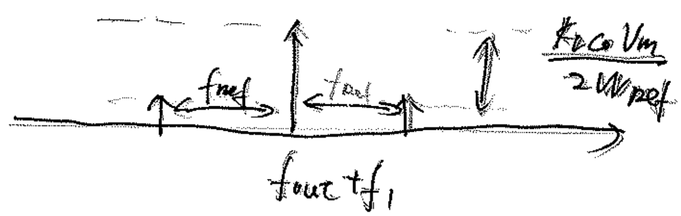
$f_{out} = N \cdot f_{Ref} - f_1$

* (V_m is the magnitude of ripple).

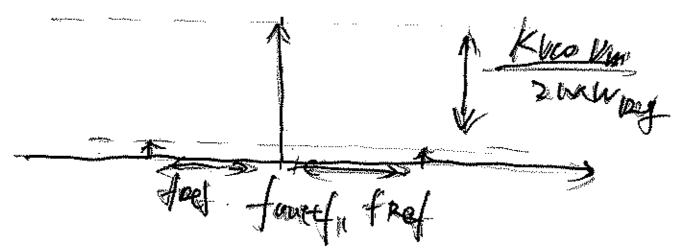
(b)



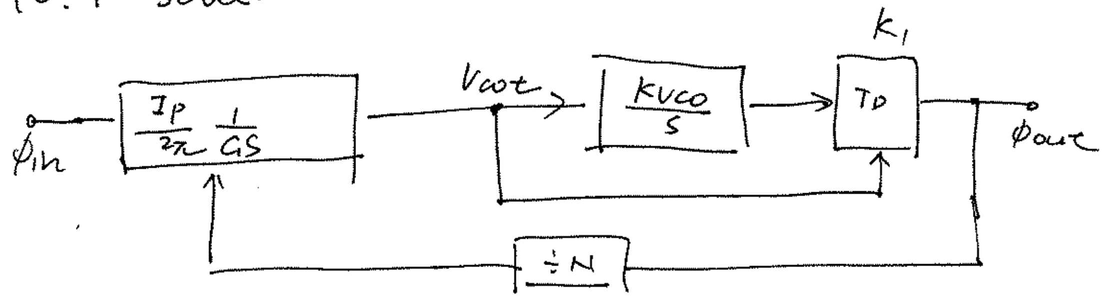
(c) ① B node



② C node



10.1 Solu:



$$H_{open}(s) = \frac{I_P}{2\pi}\frac{1}{C_1S}\left(\frac{K_{VCO}}{S} + K_1\right)$$

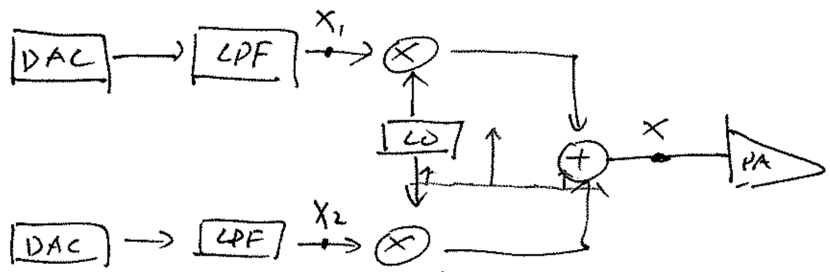
$$H_{close}(s) = \frac{H_{open}(s)}{1 + 1/N \cdot H_{open}(s)}$$

$$= \frac{\frac{I_P}{2\pi}\frac{1}{C_1S}\left(\frac{K_{VCO}}{S} + K_1\right)}{1 + \frac{1/N\, I_P}{2\pi C_1 S}\left(\frac{K_{VCO}}{S} + K_1\right)}$$

$$= \frac{\frac{I_P K_{VCO}}{2\pi C_1} + \frac{I_P K_1}{2\pi C_1}\cdot S}{S^2 + \frac{1/N\, I_P K_1}{2\pi C_1}\cdot S + \frac{1/N\, I_P \cdot K_{VCO}}{2\pi C_1}}$$

$$\begin{cases} \frac{1/N\, I_P K_1}{2\pi C_1} = 2\zeta w_n \\ w_n^2 = \frac{1/N\, I_P \cdot K_{VCO}}{2\pi C_1} \end{cases} \Rightarrow \begin{cases} w_n = \sqrt{\frac{I_P \cdot K_{VCO}}{2\pi C_1 N}} \\ \zeta = \frac{K_1}{2}\sqrt{\frac{I_P}{2\pi C_1 K_{VCO} N}} \end{cases}$$

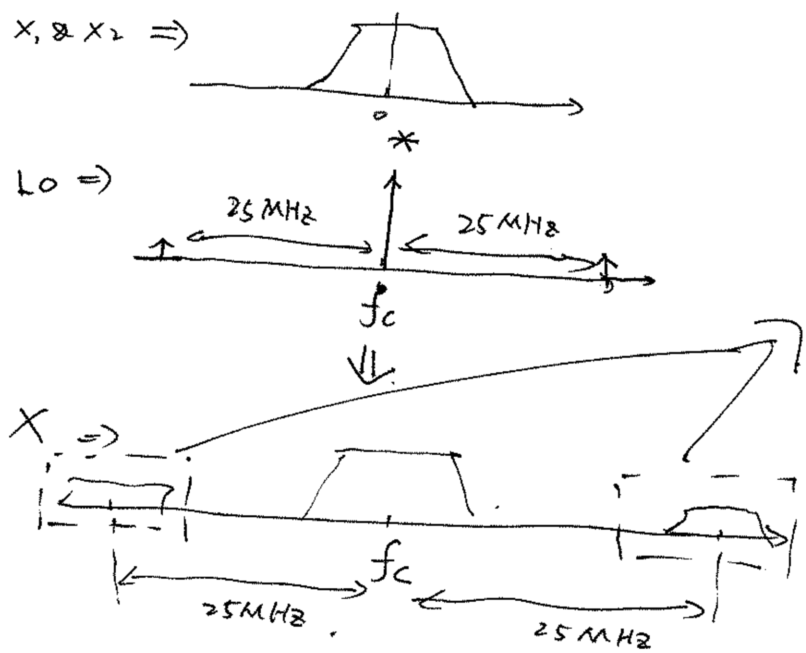
10.2 Soln:



Prove that far-out phase noise of LO also apprease as noise in RX band.

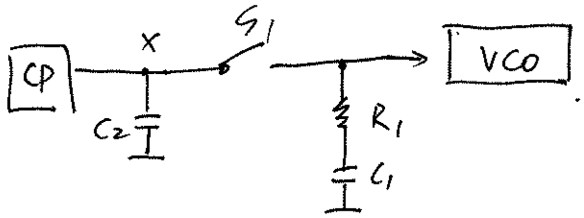
Determine the phase noise @ 25 MHz offset for GSM.

Model the far-out ~~phose~~ phase of LO as impulse.



will apprear in the RX band.

10.3 Solu:



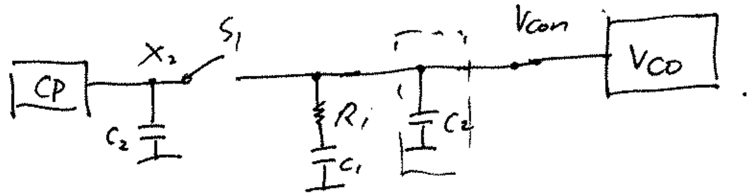
No. the sampling filter cannot remove the effect of the mismatch between the up & down current.

When ~~there~~ is an $\Delta I = I_{up} - I_{down}$, duration time ΔT

$\cdot Q = \Delta I \cdot \Delta T$, this amount of charge will be stored in capaitor C_2 when S_1 is off.

when S_1 is on, $Q = \Delta I \cdot \Delta T$ will share between C_1 and C_2, which will affect the V_{con}.

Soln 10.4.

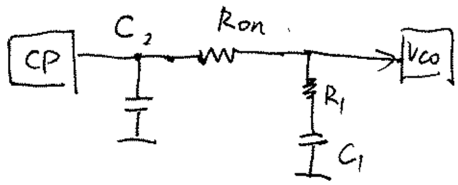


We still need a C_2 at the node of Vcon.

because the switch S_1 truly helps us to remove the most of ripple by PFD/CP circuit, but it also brings charge injection and clock feed through to the node of Vcon.

The purpose of C_2 tied to V_{con} is to suppress the charge injection and clock feedthrough of S_1.

10.5 Solu:



This Ron suppress the ripple at node Vcont,

However, it degrades the phase Margin of the loop.

From Eq. (9.42) and Appendix [I].

we know.

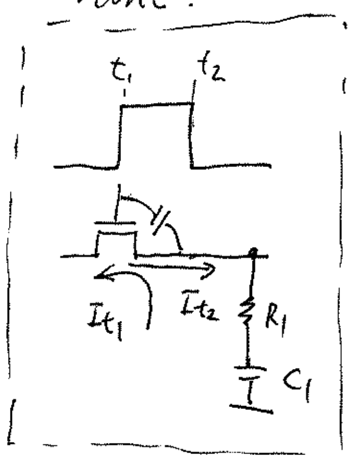
$$PM \approx \tan^{-1}(4\zeta^2) - \tan^{-1}\left(4\zeta^2 \frac{R_{on}C_2}{R_1 C_1}\right).$$

Which means $(R_{on}C_2)^{-1}$ must remain 5 to 10 times higher than ω_z. So Ron cannot be too large.

10.6 Solu:

Sure. Even when the PLL is Locked, the charge injection and clock feedthrough of S_1 still produce ripple on the Vcont. (Even Neglect the CP/PFD nonideality).

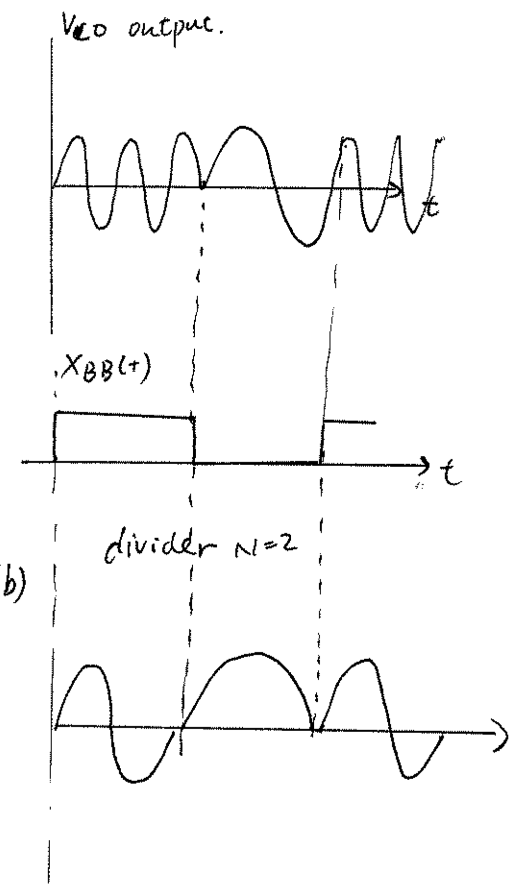
Because. when the switch is on I_{t_1} discharges and I_{t_2} charges the C_1, which will affect the Vcont.



10.7. Solu:

(I) The baseband bit period much shorter that loop time constant.

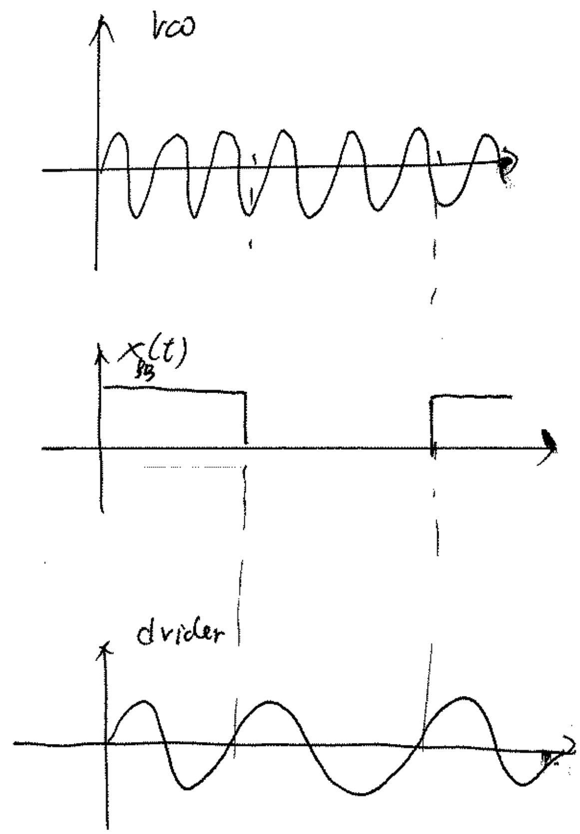
(a). output of VCO.



(b)

(II) The baseband bit period much longer then the loop time constant

(a) output of VCO



10.8 Solu:

If the modulus control of the prescalar.

assume the change happens at prescaler is M (< N)

at the beginning.

Ideally, the modulus should be N+1, now it changes

to N.

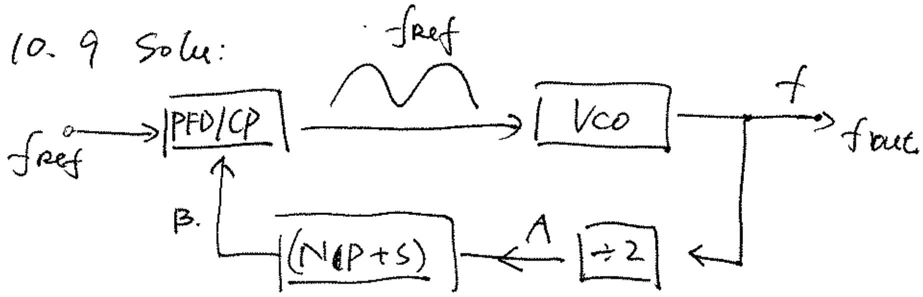
$$NS + (N+1)(P-S)$$

$$= (N+1)P.$$

So the result is that the modulus of the

overall pulse swallow divider is change to $(N+1)P$.

10.9 Solu:

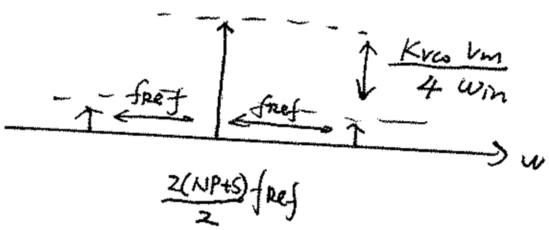


$$f_{out} = 2\cdot(NP+S) f_{Ref} + \frac{K_{vco}}{V_m} \sin f_{Ref}\cdot 2\pi\cdot t.$$

(2) node A.

$$V_A = V_0 \cos\left[\frac{1}{2} f_{out}\cdot 2\pi\cdot t + \frac{K_{vco}}{2}\int V_m\cdot \sin f_{Ref}\cdot 2\pi\cdot t\right]$$

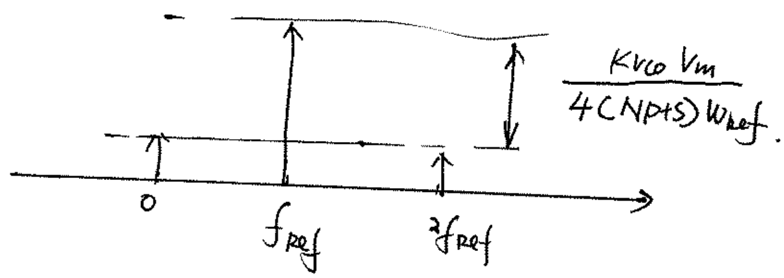
$$\approx V_0\cos\left[2\pi\frac{f_{out}}{2}t\right] - \frac{K_{vco}V_m}{2\cdot 2\cdot w_{in}} V_0 \cos\left(\frac{w_{out}}{2}+w_{in}\right)t \quad \frac{K_{vco}V_m}{2\cdot 2\cdot w_{in}} V_0 \cos\left(\frac{w_{out}}{2}-w_{in}\right)$$



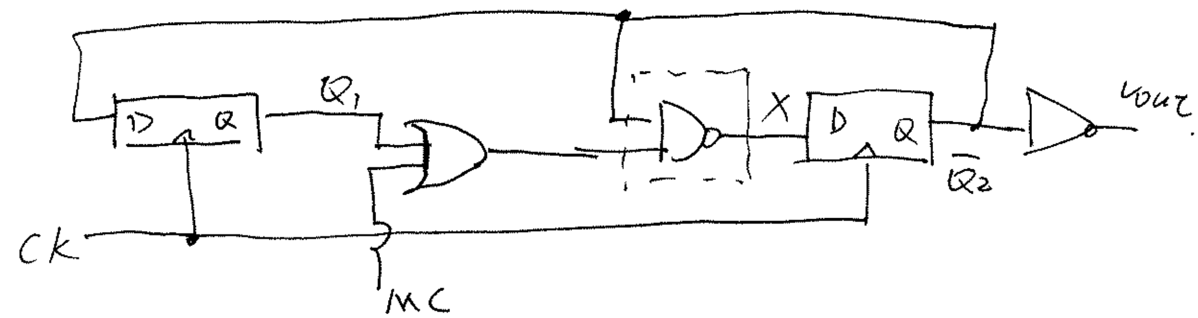
relitively ratio.

(2) node B.

V_B:



10.10. Soln:



If G_1 is the NAND Gate,

When $MC = 1$, X is always 0.

So the $\div 2$ divider cannot work correctly

When $MC = 0$,

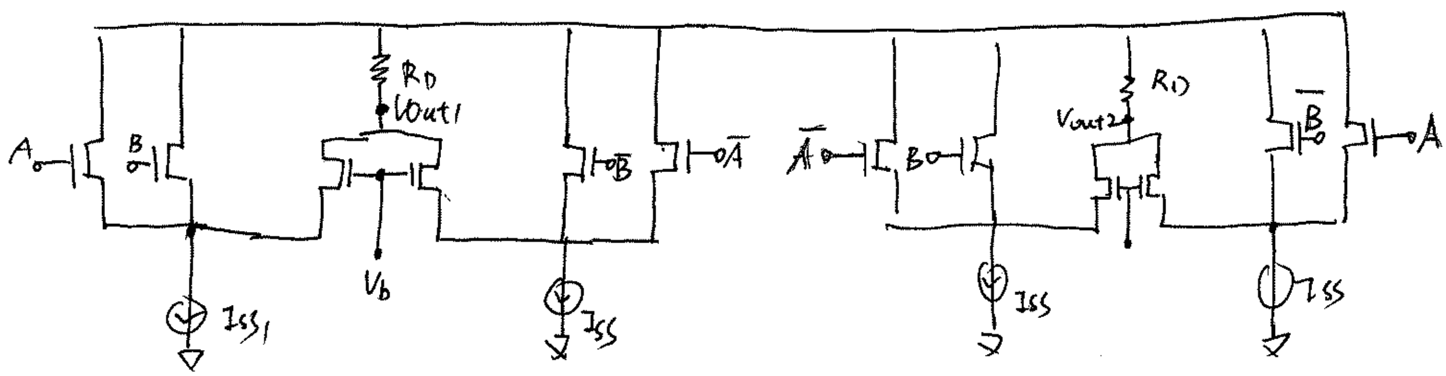
Q_1	Q_2	$\overline{Q_2}$	X.	Vout
0	0	1	0	0
1	0	1	0	0
1	0	$\cancel{0}$ 1	0	0.

X is alway 0 also.

So the $\div 3$ divider can't work correctly too.

10.11. Solu:

Modify Fig. 10.42 to provide differential outputs.



$\therefore V_{out1} = \bar{A}B + A\bar{B}$;

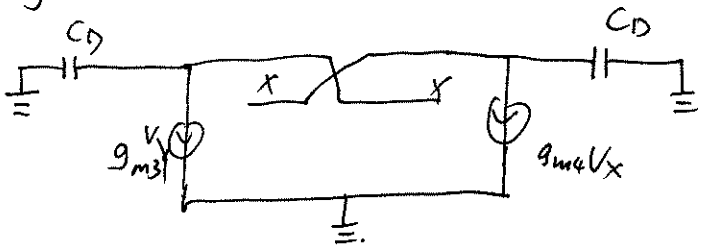
$\overline{V_{out1}} = \overline{\bar{A}B + A\bar{B}} = (A+\bar{B})\cdot(\bar{A}+B)$

$= A\cdot B + \bar{B}\cdot\bar{A}$.

$\therefore V_{out2} = A\cdot B + \bar{A}\cdot\bar{B}$

10.12 Solu:

If Fig 10.44 (a) changes into



$$C_D \cdot \frac{dV_X}{dt} + g_{m3,4} V_Y = 0$$

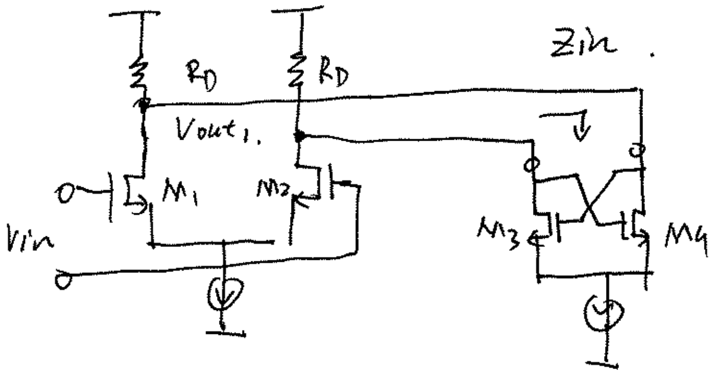
$$C_D \cdot \frac{dV_Y}{dt} + g_{m3,4} V_X = 0$$

$$C_D \left(\frac{d(V_X - V_Y)}{dt} \right) = + (g_{m3,4})(V_X - V_Y)$$

$$\Rightarrow V_{XY} = V_{XY0} \exp\left(\frac{g_{m3,4}}{C_D} \cdot t \right)$$

$$\Rightarrow \tau = \frac{C_D}{g_{m3,4}}$$

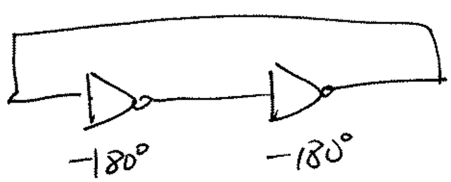
10.13 Solu:



$$Z_{in} = -\frac{2}{g_{m3,4}}$$

From stage 1 operation, we can find that there are 180° phase shift., and also produces some gain.

So for two-stage consideration, we can model this as



So Barkhausen condition is satisfied.

10.14. Soln:

From example 10.18.

$$\tau_{reg} = \frac{R_D C_D}{g_{m3,4} R_D - 1} \qquad \text{Eqation } (10.44)$$

If $g_{m3,4} R_D \gg 1 \Rightarrow \tau_{reg} = \frac{C_D}{g_{m3,4}}$

independent of R_D.

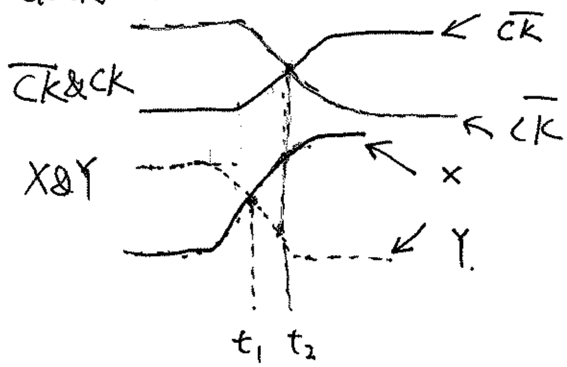
When R_D is very large. the current generated by M_3 & M_4 will mainly charge the capactor C_D. That means the current flowing into R_D is neglected.

So τ_{reg} is ~~dep~~ determined by the C_D and $g_{m3,4}$.

10.15 Solu:

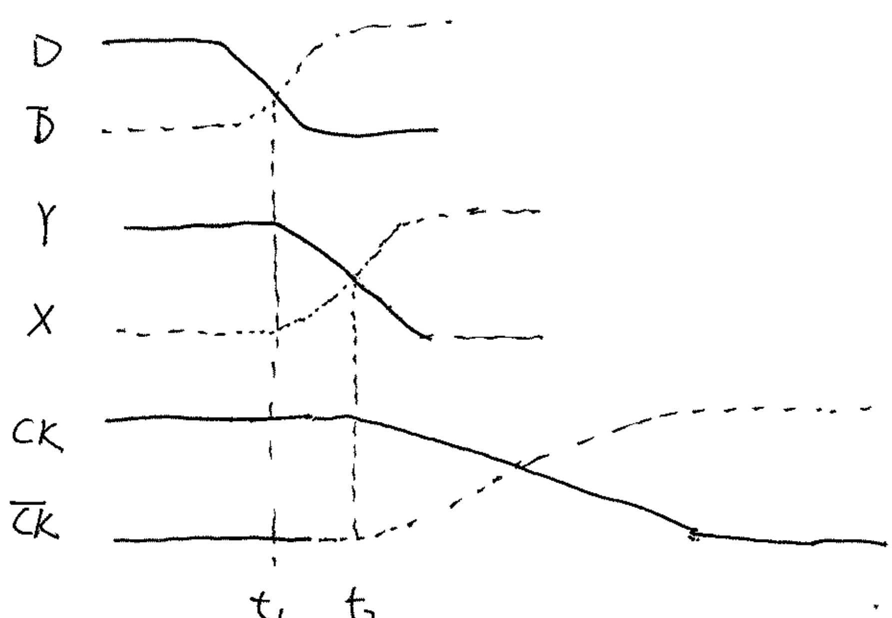
In Fig. 10.43.

(a) clock transition time on the order of X.Y time constant.



From the above figure, when clock transition time is on the order of X.Y time constant, the D latch can be working correctly.

(b) clock transition time is much longer.



From the above, figure, we know that even the clock transition is much longer, the operation will be still correct.

10.16. Solu:

Fig. 10.68.

loop gain = mixer conversion gain * amplify.

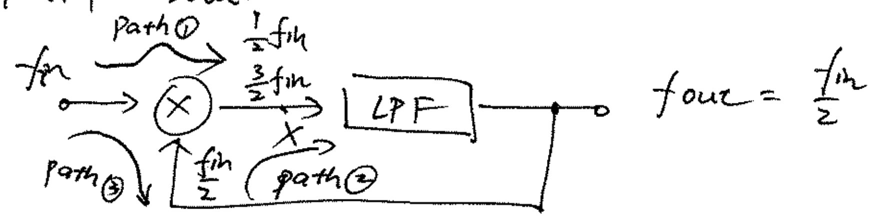
$$= \frac{\cancel{4}2}{\pi} \cdot g_{m5,6} \cdot \left| R_P \,//\, \left(-\frac{2}{g_{m7,8}}\right) \right|$$

$$= \frac{\cancel{4}2}{\pi} \cdot g_{m5.6} \cdot \left| \frac{R_P \cdot \frac{2}{g_{m7.8}}}{\frac{2}{g_{m78}} - R_P} \right|$$

$$= \frac{\cancel{8}4}{\pi} \frac{g_{m5.6}}{g_{m78}} \left| \frac{g_{m7.8} \cdot R_P}{2 - R_P \cdot g_{m7.8}} \right|$$

$$= \frac{4 g_{m56} R_P}{\pi \left| 2 - R_P \cdot g_{m7.8} \right|}$$

10.17 Solu:



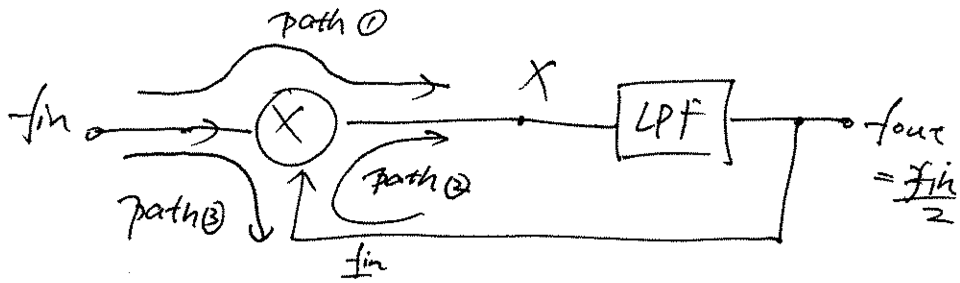
Path ① f_{in} to X node, will be attanuated by LPF.

path ②. the frequency is f_{out} itself, which doesn't contribute spurs.

Path ③.
Feed through f_{in} signal with mixering with the input f_{in} signal, which results at output is just DC. signal.

In summary, even if the mixer suffers from port to port feed throughs, there are not spurs at output.

10.18 Solu:



Assume mode nonlinearity by $y(x) = ax + bx^2 + cx^3$;

path ① at X node.

$\Rightarrow$ f_{in}, $2f_{in}$, $3f_{in}$ components.

path ② at X node

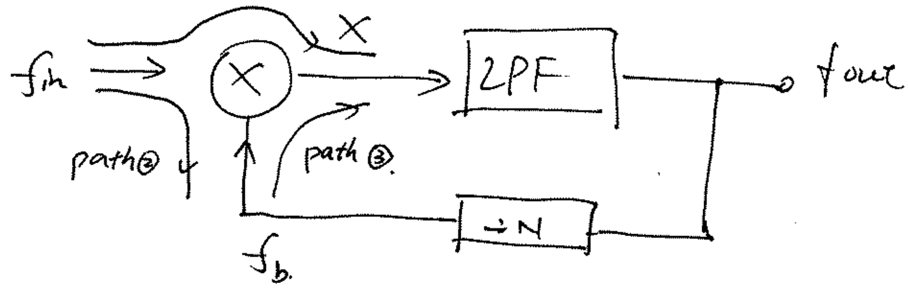
$\Rightarrow$ $\frac{f_{in}}{2}$, f_{in}, $\frac{3}{2}f_{in}$ components.

path ③ at X node.

$\Rightarrow$ 0, f_{in}, $2f_{in}$ components.

In summary, if the Lowpass filter has enough orders to attenuate the component at f_{in}, then the results are the same with the previous problem.

10.19 Solu: path①



$$f_{out} = \frac{N}{N+1} f_{in}, \quad f_b = \frac{1}{N+1} f_{in}.$$

(a) port-to-port feed through.

path ① ⇒ ② X node.

f_{in}, $f_{in} + \frac{1}{N+1} f_{in}$, $f_{in} - \frac{1}{N+1} f_{in}$

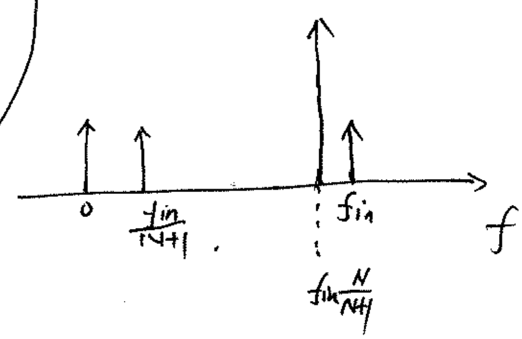
path ② ⇒ ③ X node

DC,

path ③ ⇒ ② X node

$\frac{1}{N+1} f_{in}$

⇒ output spectrum



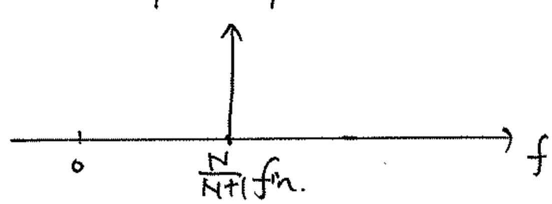
In sum, there are three spurs.

(b). port nonlinearity

② X node.

$f_{in} + \frac{1}{N+1} f_{in}$, $\frac{N}{N+1} f_{in}$, $\frac{2N}{N+1} f_{in}$, ...

output spectrum.



port linearity can product high-order harmonics of $\frac{N}{N+1} f_{in}$, However, they can be filterd by LPF.

10.20 Solu:

Which mixer topologies are suited to the Miller Divider?

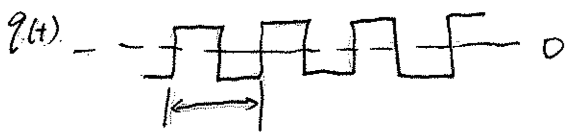
In order to achiever more gain, I'd like to choose avctive mixer. Because before staady state the loop need enough loop gain to startup.

If passive mixer must be used, some gain enhencement techinique should be used like Fig. 10.68. a coupled pair M7 & M8.

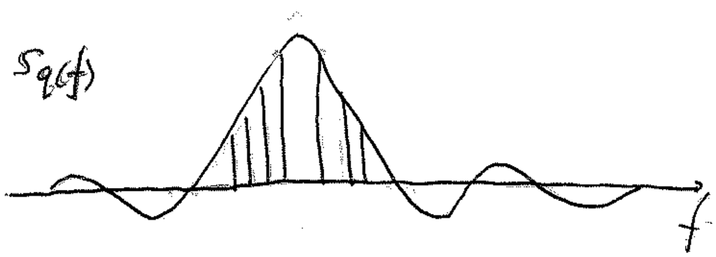
11.1 Solu:

$$f_{FB}(t) = \frac{f_{out}}{N + b(t)} \simeq \frac{f_{out}}{N}\left(1 - \frac{b(t)}{N}\right)$$

$b(t) = a + q(t)$, $a = 0.1$. periodic.

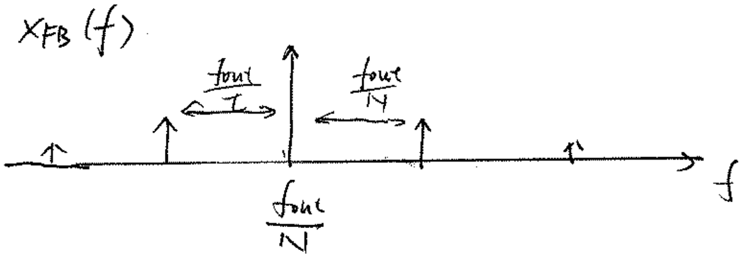


$10\,T_{Ref}$ $\qquad f_{q(t)} = 0.1 f_{Ref}$



$$X_{FB}(t) = V_0 \cos\left(\frac{f_{out}}{N}\left(1 - \frac{b(t)}{N}\right) t \right)$$

with narrowband FM approximation



11.2 Soln:

$$f_{FB}(t) \approx \frac{f_{out}}{N}\left(1 - \frac{b(t)}{N}\right)$$

$$\phi_{out}(t) = \frac{f_{out}}{\cancel{N}} t + \phi_0$$

$$= \frac{f_{FB}(t) N}{1 - \frac{b(t)}{N}} t. + \phi_0$$

$$= \frac{f_{FB}(t) N^2}{N - b(t)} \cdot t + \phi_0 .$$

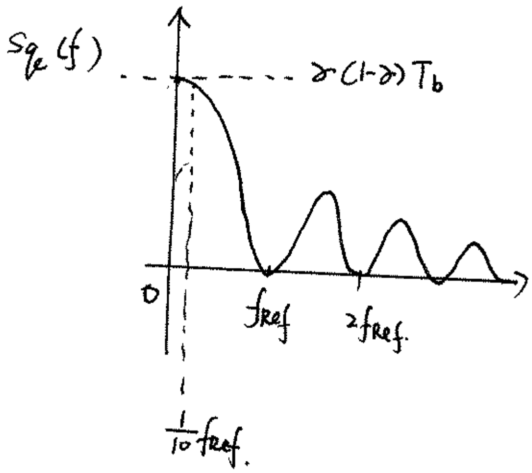
From the equation above, we can conclude that $f_{FB}(t)$ and $b(t)$ are periodic $\Rightarrow$ $\frac{f_{FB}(t) N^2}{N - b(t)}$ are also periodic.

$$\phi_{out}(t) = A(t) \cdot t + \phi_0.$$

$$\frac{d\phi_{out}(t)}{dt} = A(t) + A'(t)(t).$$

Solu 11.3.

If $T_b = T_{ref}$.



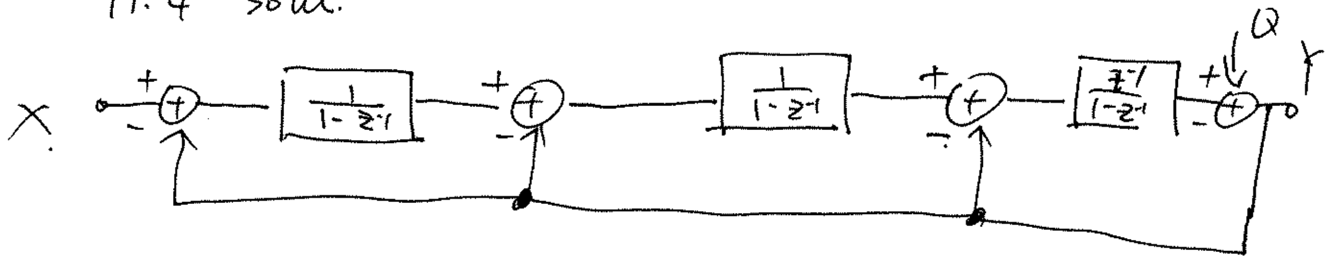
It implies that the low-frequency part ($f < \frac{1}{10} f_{ref}$) of $S_q(f)$ can be suppressed by PLL, The components with higher frequencies will the critical part of $S_q(f)$

$$S_q\left(\frac{1}{10} f_{ref}\right) = \gamma(1-\gamma) \cdot f_{ref} \left(\frac{\sin \pi \frac{f}{f_{ref}}}{\pi f} \right) \Big|_{f=\frac{1}{10} f_{ref}}$$

$$= \gamma(1-\gamma) \cancel{f_{ref}} \frac{\sin \pi \frac{1}{10}}{\pi \cdot \frac{1}{10} f_{ref}}$$

$$= 0.98\,\gamma(1-\gamma).$$

11.4 Soln:



assum $X = 0$.

$$\left\{\left[\left[0 - Y(z)\right]\frac{1}{1-z^{-1}} - Y(z)\right]\frac{1}{1-z^{-1}} - Y(z)\right\}\frac{z^{-1}}{1-z^{-1}} + Q = Y(z)$$

$$\Rightarrow \quad \frac{Y}{Q}(z) = (1-z^{-1})^3.$$

$$\Rightarrow \quad S_y(f) = S_q(f)\,\left|2\sin(\pi f T_{ck})\right|^6.$$

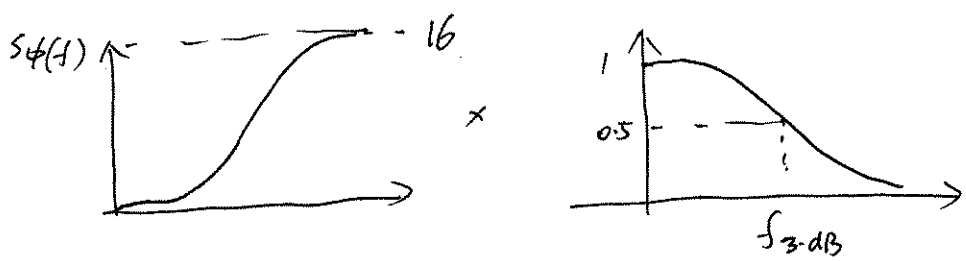
$$\because \quad \phi(z) = Y(z)/(1-z^{-1})$$

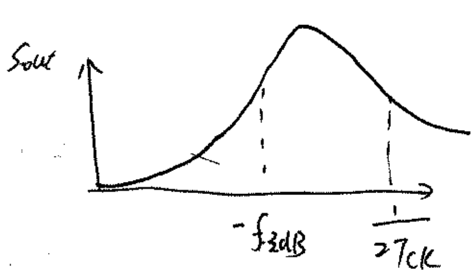
$$\therefore \quad \phi(z) = (1-z^{-1})^2\,Q(z)$$

$$\therefore \quad S_\phi(f) = \left|1-z^{-1}\right|^4 S_q(f)$$

$$= \left|2\sin\pi f T_{ck}\right|^4 \quad S_q(f)$$

$$\therefore \quad S_{out}(f) = S_\phi(f)\cdot N^2\,\frac{4\zeta^2\omega_n^2\omega^2 + \omega_n^2}{(\omega^2-\omega_n^2)^2 + 4\zeta^2\omega_n^2\omega^2},$$





11.5 Soln:

For a second order:

$$S_y(f) = S_q(f)\,|2\sin(\pi f T_{ck})|^4$$

For a fourth-order:

$$S_y(f) = S_q(f)\,|2\sin(\pi f T_{ck})|^8.$$

$|H_{PLL}(f)|^2$ is the same.

For 2nd order

$$S_{out}(f)_{2nd} = S_q(f)\,|2\sin\pi f T_{ck}|^2 \cdot |H_{PLL}(f)|^2$$

For 4th order

$$S_{out}(f)_{4th} = S_q(f)\,|2\sin\pi f T_{ck}|^6\,|H_{PLL}(f)|^2.$$

$$\frac{f_{ref}}{10} \approx f \ll f_{ck}.$$

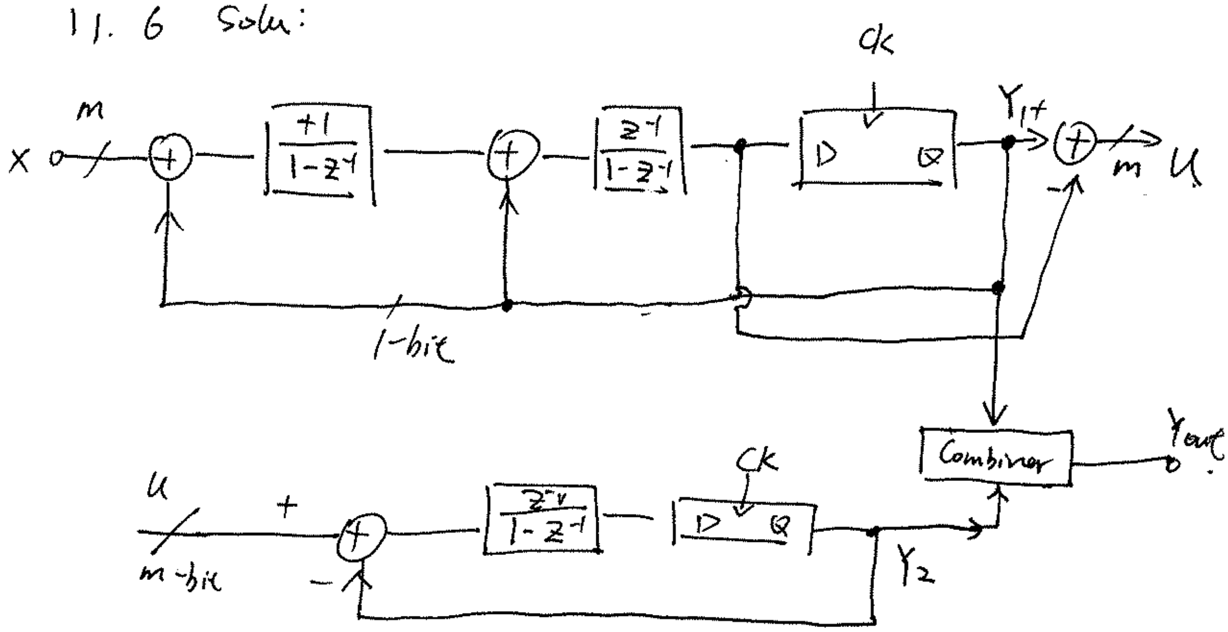
$$\therefore \quad \frac{S_{out}(f)_{4th}}{S_{out}(f)_{2nd}} = |2\sin\pi f T_{ck}|^4 \approx 16 f^4 (\pi T_{ck})^4.$$

assume $f = \frac{f_{ck}}{10}$

$$10\lg\left(16\,\frac{f_{ck}^4}{10^4}\cdot\pi^4\frac{1}{f_{ck}^4}\right)^{-1} = 10\log_{10}\left(\frac{16\pi^4}{10^4}\right)^{-1}$$

$$= 8.1\text{ dB}.$$

11.6 Solu:



$$Y_1(z) = (1-z^{-1})^2 Q(z)$$

$$Y_2(z) = z^{-1}Q(z) + (1-z^{-1})Q'(z)$$

$$Y_{out}(z) = Y_1(z)\cdot z^{-1} - (1-z^{-1})^2 Y_2(z)$$

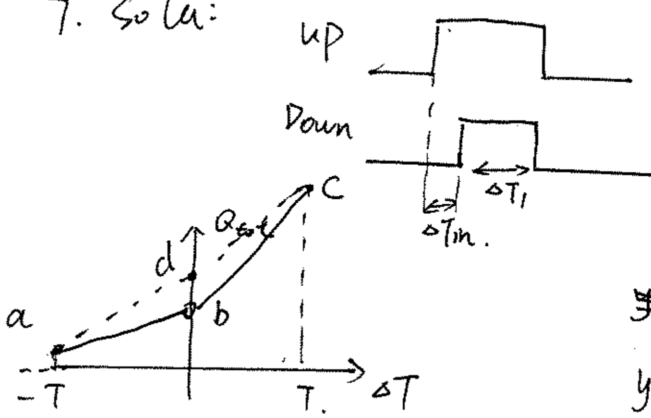
$$= z^{-1}(1-z^{-1})^2 Q(z) - z^{-1}(1-z^{-1})^2 Q(z) - (1-z^{-1})^3 Q'(z)$$

$$= -(1-z^{-1})^3 Q'(z)$$

$\Downarrow$

This is what combiner should do.

7. Solu:



b $(0,\ (I_1 - I_2)\Delta T_1)$

a $(-T,\ -I_2T + (I_1-I_2)\Delta T_1)$

c $(+T,\ +I_1T + (I_1-I_2)\Delta T_1)$

assume $(I_1-I_2)\Delta T_1 = A$

$$\frac{y - I_1T - (I_1-I_2)\Delta T_1}{x - T} = \frac{I_1+I_2}{2}$$

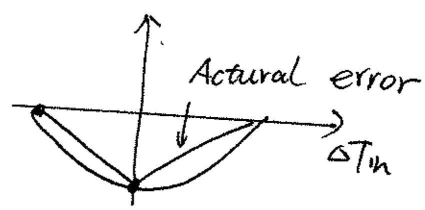
$$\boxed{y = \frac{I_1+I_2}{2}x + \frac{I_1-I_2}{2}\cdot T + A}.$$

$$-b + \frac{I_1-I_2}{2}\cdot T + A = A$$

$$\Rightarrow \quad b = \frac{I_1-I_2}{2}\cdot T.$$

$$a\Delta T_{in}^2 - b = 0$$

$$\Rightarrow \quad \Delta T_{in} = \pm\sqrt{\frac{a}{b}}$$



Actural error:

$$\text{Error} = \frac{I_1-I_2}{2}\Delta T_{in} - \frac{I_1-I_2}{2}T. = 0$$

$$\Rightarrow \quad \Delta T_{in} = T.$$

$$\therefore \quad \sqrt{\frac{a}{b}} = T.$$

$$\Rightarrow \quad a = T^2 b = T^3\frac{I_1-I_2}{2} = 2.5\%\,T^3.$$

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11.8 Solu:

(a) unequal Up and Down pulsewidths.

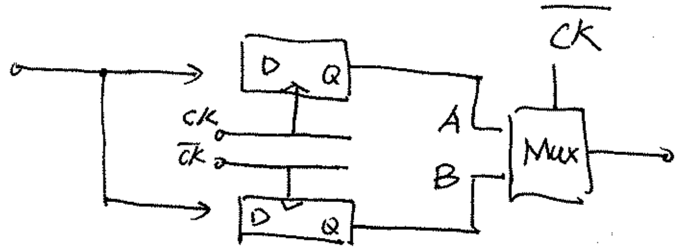
Unequal up and down pulse widths are equivalene to the I_1 and I_2 mismatch in CP. So it's noise folding behavior can be modele as the same as Fig. 11.30.

(b) charge injection mismatch between up & Down switch in CP.

For charge injection mismatch, it contribute to the $(I_1 - I_2)$ mismatch every cycle. That means. In equation $Q_{tot} \approx I_{avg} \Delta T_{in} + a \Delta T_{in}^2 - b$, a will be larger.

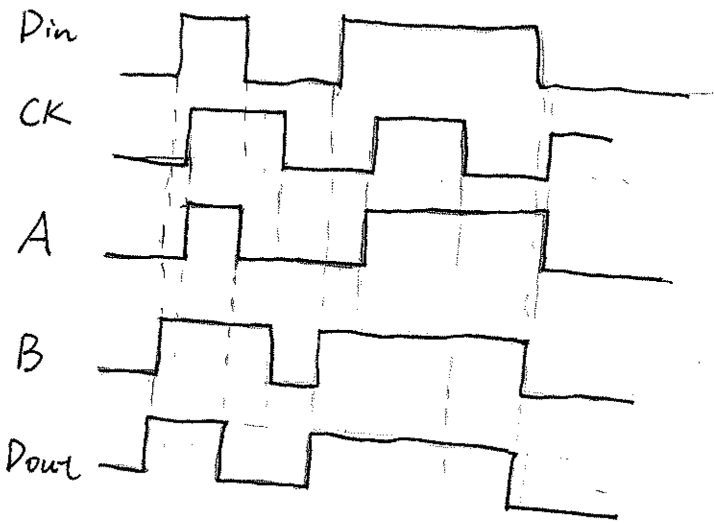
So the mismatch makes noise folding worse.

11.9 Soln:



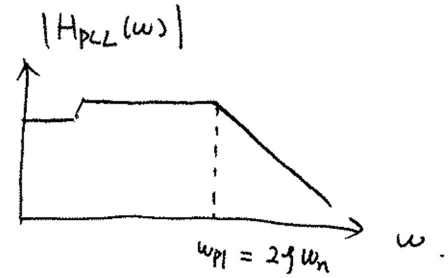
Previously, when CK is high B is selected.

Now, when CK is high. A is selected.



11.10 Solu:

PLL



Model as one-pole system.

$$H_{PLL}(s) = \frac{2\zeta\omega_n}{s + 2\zeta\omega_n}$$

$$= \frac{0.1 \cdot \frac{2\pi}{T_1}}{s + 0.1\frac{2\pi}{T_1}}$$

$$\therefore \; |H_{PLL}(j\omega)| = \left| \frac{0.1}{j\frac{10f}{f_1} + 0.1} \right| \quad \left(f_1 = \frac{1}{T_1}\right)$$

$$= \frac{0.1}{\sqrt{0.1^2 + \frac{10f}{f_1}}}$$

$$\left|H_{PLL}\left(j2\pi\frac{f_1}{2}\right)\right| = \frac{0.1}{\sqrt{0.1^2 + 5^2}} = 0.02$$

$$= -16.9\,dB.$$

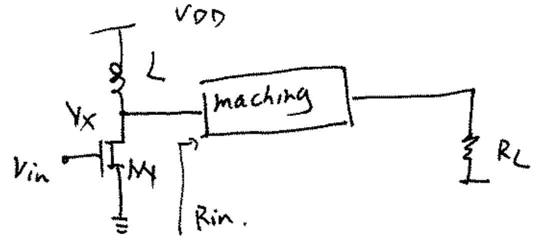
If. $f_{out} = N \cdot f_{in}$. (assume, ω_n, ζ remain the same).

$$\zeta = \frac{R_1}{2}\sqrt{\frac{I_P C_1 K_{vco}}{2\pi M}} \; ; \; \omega_n = \sqrt{\frac{I_P K_{vco}}{2\pi M}}.$$

If ζ, ω_n is the same, then $I_P \cdot K_{vco}$ will be change to M times larger.

So the attunation is not changed, but the circuit design is more difficult.

12.1 Solu:



$V_x(t) = V_{DD}\,(1 + \cos\omega t)$

$i_d(t) = \frac{V_{DD}}{R_{in}}\,(1 + \cos\omega t)$

assume a current bias

$= \frac{V_{DD}}{R_{in}}$

only on this condition

$\eta = 50\%$

P_{FET}

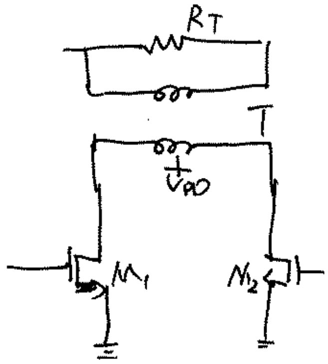
$$= \frac{1}{2T}\int_0^T V_x(t)\cdot i_d(t)\,dt$$

$$= \frac{1}{2T}\int_0^T \cdot\frac{V_{DD}^2}{R_{in}}\,(1 + 2\cos\omega t + \cos^2\omega t)\,dt$$

$$= \frac{V_{DD}^2}{2R_{in}}.$$

∴ The other 50% of supply power is dissipated by M_1 itself.

12.2 Solu:

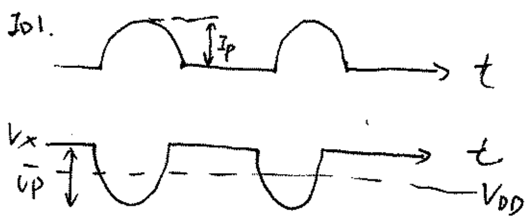


This class B amplifier seems very symmetric like differential structure. However, when it works, M_1, M_2 is ~~spea~~ seperately operating just like

single-end. Structure.

So it's still sensitive to bond wire inductance in series with V_{DD}.

12.3 Soln:



$$I_{D1} = I_p \sin \omega_0 t \qquad \left(0 < t < \frac{\pi}{\omega_0}\right)$$

$$\frac{1}{T}\left(\int_0^{\frac{T}{2}} (V_x - V_p \sin \omega_0 t)\, dt + \int_0^{\frac{T}{2}} V_x \, dt\right) = V_{DD}$$

$$\Rightarrow \quad V_X = V_{DD} + \frac{V_p}{\pi}.$$

The voltage swing above V_{DD} is:

$$V_X - V_{DD} = \frac{V_p}{\pi} \approx 0.32\, V_p$$

The voltage swing below V_{DD} is:

$$V_p - \frac{V_p}{\pi} = V_p\left(1 - \frac{1}{\pi}\right) \approx 0.68\, V_p$$

So the swing above V_{DD} is approx. half that below V_{DD}.

12.4 solu:

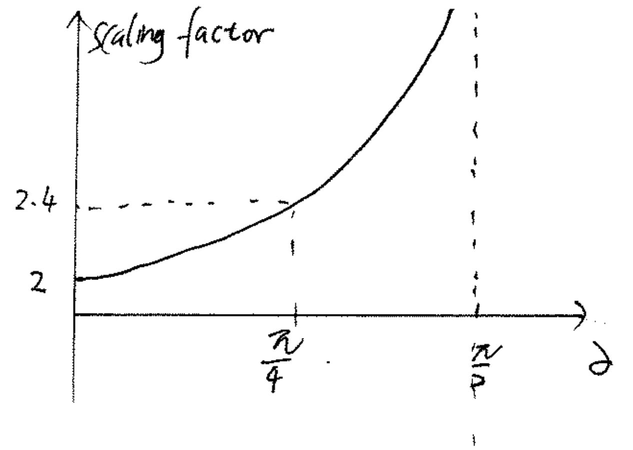
From Eq 12.39 & Eq. 12.40.

$$a_1 = I_p \frac{\pi - 2\gamma}{2\pi} + \frac{I_p}{2\pi} \sin 2\gamma.$$

$$(\gamma \in (0, \frac{\pi}{2})).$$

scaling factor

$$= \frac{1}{a_1/I_p} = \frac{1}{\frac{\pi-2\gamma}{2\pi} + \frac{\sin 2\gamma}{2\pi}}$$



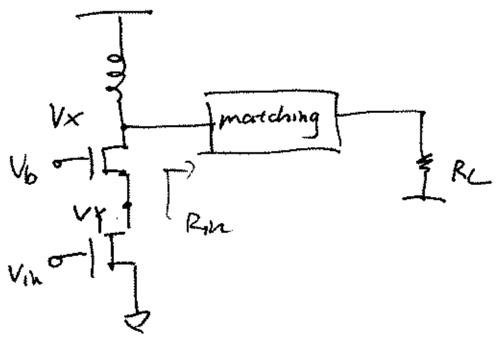
From the figure above, we can conclude that.

only when the transistor is infinitely large, the efficiency

can be 100% on the condition of providing

an comparable output power to that of A class.

12.5 soln:



when V_x reaches to $2V_{DD}$ and nearly zero.

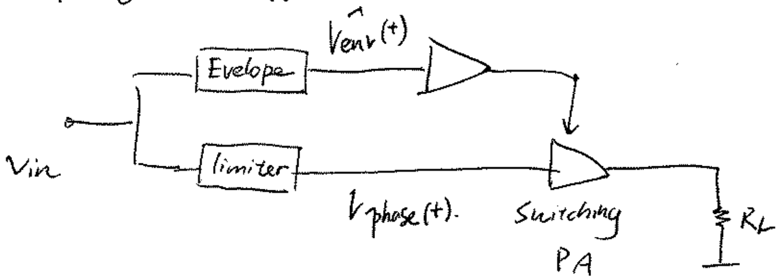
$$P_L = \left(\frac{2V_{DD}}{2}\right)^2 / 2R_{in}.$$

$$\text{The } I_D = \frac{V_{DD}}{R_{in}}.$$

$$\therefore \eta_{max.} = \frac{P_L}{I_D \cdot V_{DD}} = \frac{V_{DD}^2 / 2R_{in}}{V_{DD}^2 / R_{in}} = 50\%$$

In sum, the efficiency is the same as class A PA.

12.6 Solu:



$$V_{in} = V_{env}(t)\cos(\omega_0 t + \phi(t))$$

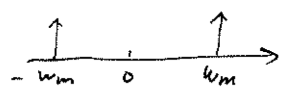
$$\begin{cases} \hat{V}_{env}(t) = V_{env}(t) + \frac{1}{3}\gamma V_{env}^3(t). \\ V_{phase}(t) = V_0 \cos(\omega_0 t + \phi(t)) \end{cases}$$

$$V_{out} = \hat{V}_{env}(t) \cdot V_{phase}(t)$$

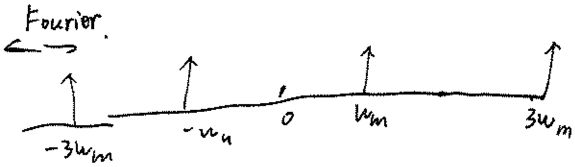
$$= V_0 V_{env}(t) \cdot \cos(\omega_0 t + \phi(t)) + \frac{1}{3} V_0 \gamma V_{env}^3(t) \cos(\omega_0 t + \phi(t)).$$

Assume

$V_0 V_{env}(t) \xleftrightarrow{\text{Fourier}}$



$V_{env}^3(t) \xleftrightarrow{\text{Fourier.}}$



This part of spectrum will also convert to the vancinity of ω_0.

So the output spectrum exhibits growth in the adjacent channels.

12.7 Soln:

Assuming the phase signal experiences a delay mismatch of ΔT.

$$V_{out} = A_0 V_{env}(t) \cos[\omega_0 (t - \Delta T) + \phi(t - \Delta T)]$$

$$\simeq A_0 V_{env}(t) \cos\left[\omega_0 t - \omega_0 \Delta T + \phi(t) - \Delta T \frac{d\phi(t)}{dt}\right]$$

$$\simeq A_0 V_{env}(t) \cos[\omega_0 t + \phi(t)] \cos\left[\left(\omega_0 + \frac{d\phi(t)}{dt}\right)\Delta T\right]$$

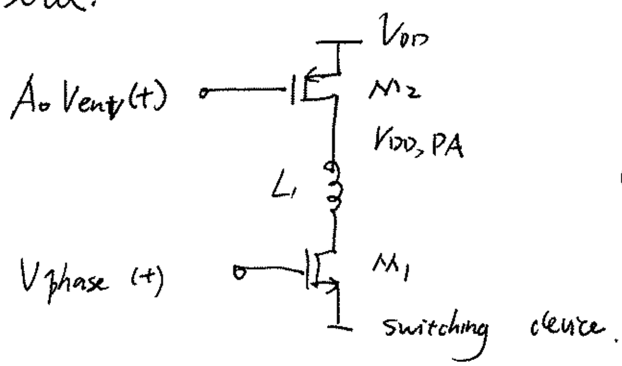
$$+ A_0 V_{env}(t) \sin[\omega_0 t + \phi(t)] \sin\left[\left(\omega_0 + \frac{d\phi(t)}{dt}\right)\Delta T\right]$$

assume $\Delta T << \dfrac{1}{\omega_0 + \frac{d\phi(t)}{dt}}$.

$$V_{out} \approx A_0 V_{env}(t) \cos[\omega_0 t + \phi(t)] + \underbrace{\Delta T\left(\omega_0 + \frac{d\phi(t)}{dt}\right)}\, A_0 V_{env}(t) \sin[\omega_0 t + \phi(t)]$$

From the second term, we can also conclude that this mismatch ΔT leads to substantial spectral regrowth.

12.8 Solu:



(neglect the R_{on} of M_1)

In this stage, L_1 does not consume power, neither M_1

So the only dissipated power is consumed by M_2.

$$\text{effeciency} = 1 - \frac{I_0 V_0}{I_0 \cdot V_{DD}}$$

$$\boxed{= 1 - \frac{V_0}{V_{DD}}}.$$

12.9 Soln:

$v_1(t)$ $v_2(t)$ are defined by Eq (12.109) (12.110)

$$v_1(t) + v_2(t)$$

$$= \left(\frac{V_0}{2} + \Delta V\right) \sin[w_0 t + \phi(t) + \theta(t) + \Delta\theta) - \frac{V_0}{2} \sin[w_0 t + \phi(t) - \theta(t))$$

$$= \frac{V_0}{2}\left\{\sin[w_0 t + \phi(t) + \theta(t) + \Delta\theta] - \sin[w_0 t + \phi(t) - \theta(t)]\right\}$$

$$+ \Delta V \sin[w_0 t + \phi(t) + \theta(t) + \Delta\theta]$$

$$= \frac{V_0}{2}\left\{\sin[w_0 t + \phi(t) + \theta(t)] \cdot \cos\Delta\theta - \sin[w_0 t + \phi(t) - \theta(t)]\right\}$$

$$+ \frac{V_0}{2} \cdot \sin\Delta\theta \cos[w_0 t + \phi(t) + \theta(t)]\} + \Delta V\{\sin[w_0 + \phi(t) + \theta(t)] \cos\Delta\theta$$

$$+ \cos[w_0 + \phi(t) + \theta(t)] \cdot \sin\Delta\theta\}$$

$$\approx \frac{V_0}{2} \cdot 2 \cdot \cos[w_0 t + \phi(t)] \sin\theta(t) + \Delta V \sin[w_0 + \phi(t) + \theta(t)]$$

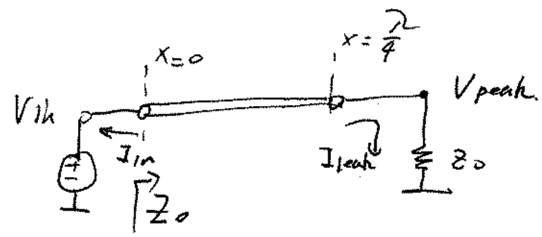
$$+ \left(\frac{V_0}{2} + 1\right) \Delta\theta \cos[w_0 t + \phi(t) + \theta(t)]$$

$$\approx \frac{V_0}{V_a} v_{env}(t) \cdot \cos[w_0 t + \phi(t)] + \Delta V \sin[w_0 + \phi(t) + \theta(t)]$$

$$+ \left(\frac{V_0}{2} + 1\right) \Delta\theta \cos[w_0 t + \phi(t) + \theta(t)].$$

12.10 Soln:

If the input is driven by an ideal voltage source.



$$V(t,x) = V^{+}\cos(\omega_0 t - \beta x) + V^{-}\cos(\omega_0 t + \beta x)$$

$$I(t,x) = \frac{V^{+}}{Z_0}\cos(\omega_0 - \beta x) - \frac{V^{-}}{Z_0}\cos(\omega_0 t + \beta x).$$

$$V(t,0) = (V^{+}+V^{-})\cos\omega_0 t = V_{in}.$$

$$I(t,0) = \left(\frac{V^{+}}{Z_0} - \frac{V^{-}}{Z_0}\right)\cos\omega_0 t = I_{in}$$

$$V(t,\tfrac{\lambda}{4}) = (-V^{+}+V^{-})\sin\omega_0 t = V_{peak}$$

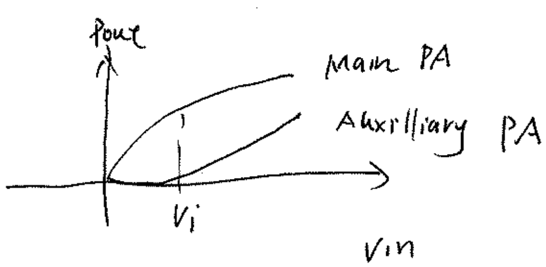
$$I(t,\tfrac{\lambda}{4}) = \left(-\frac{V^{+}}{Z_0} - \frac{V^{-}}{Z_0}\right)\sin\omega_0 t = I_{peak}.$$

$$\frac{V_{peak}}{Z_0} = I(t,\tfrac{\lambda}{4})$$

$$\frac{(-V^{+}+V^{-})\sin\omega_0 t}{Z_0} = \left(\frac{-V^{+}}{Z_0} - \frac{V^{-}}{Z_0}\right)\sin\omega_0 t \Rightarrow -V^{+}+V^{-} = -(V^{+}+V^{-})$$

$$\frac{V_{in}}{I_{in}} = \frac{V^{+}+V^{-}}{V^{+}-V^{-}}\cdot Z_0 = Z_0$$

The inputs of peaking PA and carrier PA are the exactly same. So it cannot satisfy the figure below.

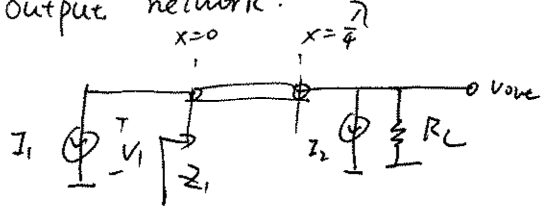


12.11. Soln:

$\gamma = 0.5$. The waveform at $x=0$ & $x=\lambda/4$.

$Z_0 = R_L$.

output network:



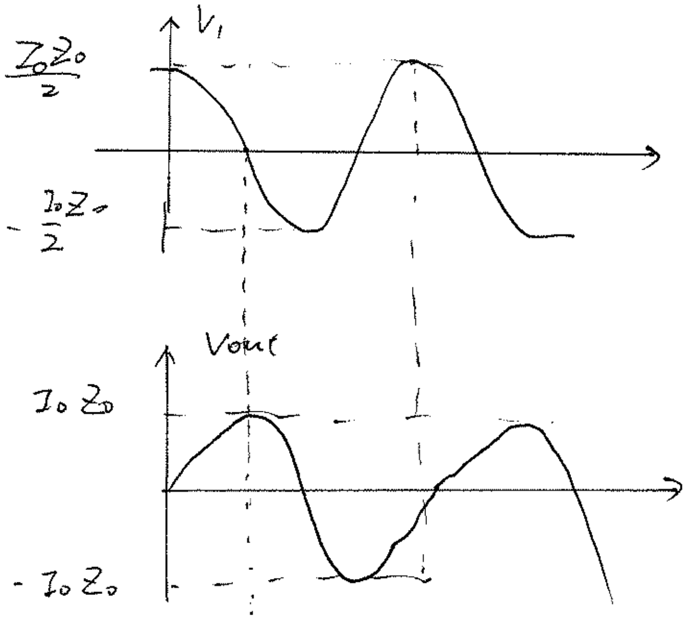
$$Z_1 = Z_0\left(\frac{Z_0}{R_L} - \gamma\right) = \frac{Z_0}{2}$$

assume $I_1 = -I_0 \cos w_0 t$.

$$V_1 = I_0 \cos w_0 t \cdot \frac{Z_0}{2}.$$

$$V_{out} = 2 \cdot I_0 \cos\left(w_0 t + \frac{\pi}{2}\right) \cdot \frac{Z_0}{2}. \qquad I(t, \tfrac{\lambda}{4}) = -I_0 \frac{Z_0}{2} \cdot \sin w_0 t$$

$$V(t, \tfrac{\lambda}{4}) = I_0 Z_0 \sin w_0 t$$



13.1. Solu:

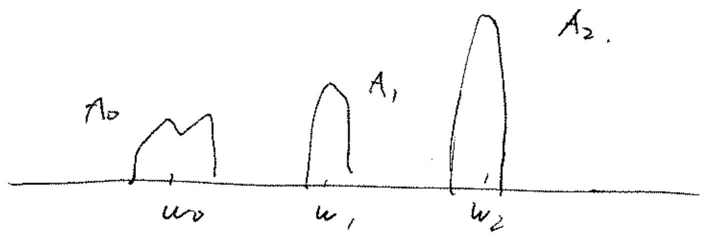
data rate $= 54\,\text{Mb/s}$

sensitivity of -65 dBm.

desire signal : $A_0 \cos \omega_0 t$

adjacent : : $A_1 \cos \omega_1 t$

alternate : : $A_2 \cos \omega_1 t$



$$20\log A_0 = -62\,\text{dBm}$$
$$20\log A_1 = -46\,\text{dBm}$$
$$20\log A_2 = -30\,\text{dBm}.$$

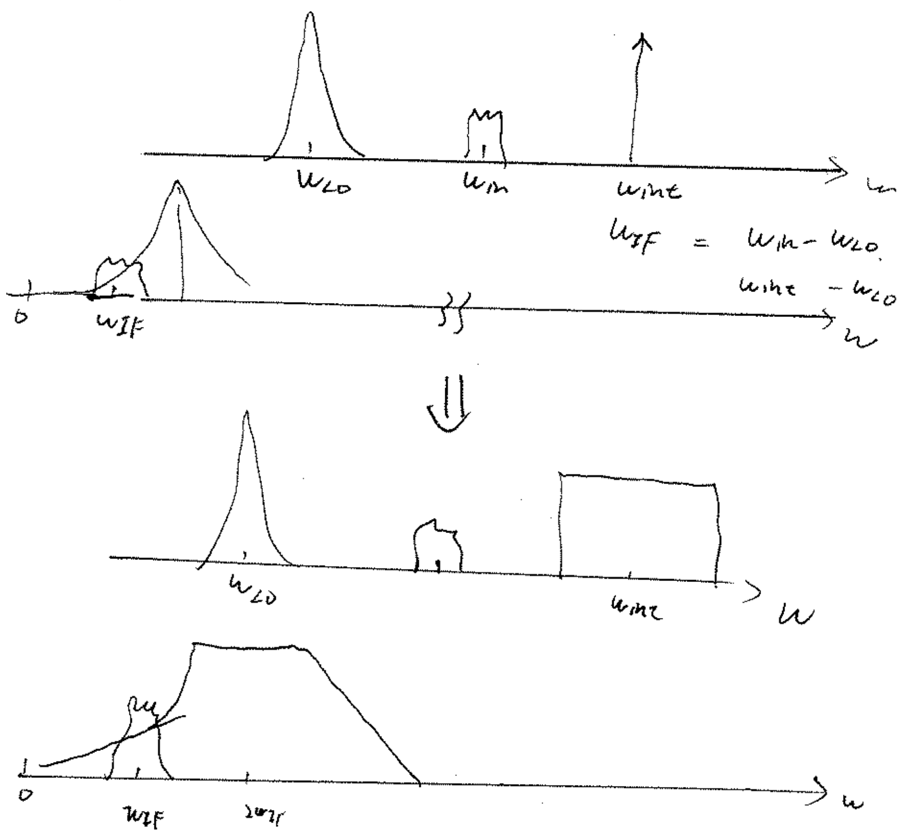
$$20\log\left|\frac{3a_3}{4a_1}\right| = -15\,\text{dB} - 40\log A_1 - 20\log A_2 + 20\log A_0$$
$$= \left(-15 + 92 + 60 - \frac{124}{2}\right)\text{dBm}$$
$$= +75\,\text{dBm}.$$

$$IIP_3\,\text{dBm} = 20\log\sqrt{\left|\frac{4a_1}{3a_3}\right|} = -37.5\,\text{dBm}.$$

13.2 Solu:

If interfers are not approximated by narrow-band sy signal.

The corruption due to reciprocal mixing is larger.



13.3 Solu:

desired input -65 dBm

~~SR~~ SNR = -35 dB

desired input = -62 dBm

adjacent signal = -46 dBm

alternative : = -30 dBm.

$$\frac{P_{PN,tot}}{P_{sig}} = a_1 \gamma \left(\frac{1}{f_1} - \frac{1}{f_2}\right) + a_2 \gamma \left(\frac{1}{f_3} - \frac{1}{f_4}\right)$$

$$= 39.8 \gamma \left(\frac{1}{10M} - \frac{1}{30M}\right) + 1585 \gamma \left(\frac{1}{30M} - \frac{1}{50M}\right)$$

$$= -35 \text{dB} \qquad \boxed{-35 = 10 \log_{10} \frac{P_{N,tot}}{P_{sig}}}$$

$$\Rightarrow \quad 10^{-5} \cdot 2.3787 \gamma = 10^{-\frac{35}{10}}$$

$$\gamma \approx 13.3$$

$$\therefore \quad S_n(f) = \frac{13.3}{f^2}.$$

13.4 Soln:

From Eq (6.51)

$$Z_{in,SB} = \frac{1}{2}\left[R_1 + \frac{1}{\frac{jC_1\omega}{2} + 2fC_1}\right] \quad \omega \approx \omega_{LO}.$$

R_1 : on resistance of switch
C_1 : the load of the switch capcitance.

In the Fig. 13.19.

$$C_1 \approx \frac{2}{3} WLC_{ox} = 130fF.$$

$$R_1 \approx 100\Omega.$$

(a) 2.4G

$$Z_{in,SB} = \frac{1}{2}\left(100 + \frac{1}{48G \cdot 130f + j\frac{\pi \cdot 2 \cdot 2.4G \cdot 130f}{2}}\right)$$

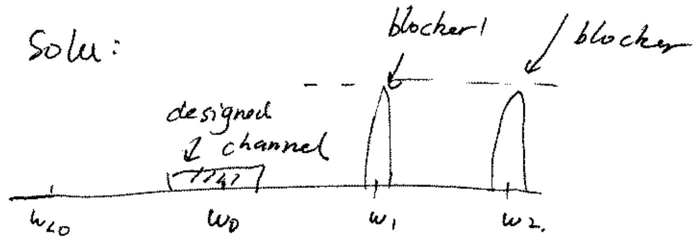
$$= \frac{1}{2}\left(100 + \frac{1}{4.8 \cdot 130 \times 10^{-6} + j\,\pi \cdot 2.4 \cdot 130\, 10^{-6}}\right)$$

$$= 459\, e^{-j0.91}.$$

(b) 6G

$$Z_{in,SB} = 203.4\, e^{-j0.795}.$$

13.5 Solu:



$S_n(f) = \frac{\gamma}{f^2}$ assume $-100\,dBc/Hz$ @ 1MHz.

$10\log\frac{\gamma}{f^2} = -100$ $\qquad \frac{\gamma}{10^{12}} = 10^{-5\times 2} \Rightarrow \gamma = 100$

$$\therefore \frac{P_{NN,tot}}{P_{sig}} = a\,100\left(\frac{1}{f_1} - \frac{1}{f_2}\right) + a\cdot 100\left(\frac{1}{f_3} - \frac{1}{f_4}\right)$$

$$100a\left(\frac{1}{10M} - \frac{1}{30M} + \frac{1}{30M} - \frac{1}{50M}\right)$$

$$= \frac{8a}{10^6}$$

$$\therefore \quad 10\log\frac{8a}{10^6} = -30$$

$$\frac{8a}{10^6} = 10^{-3}$$

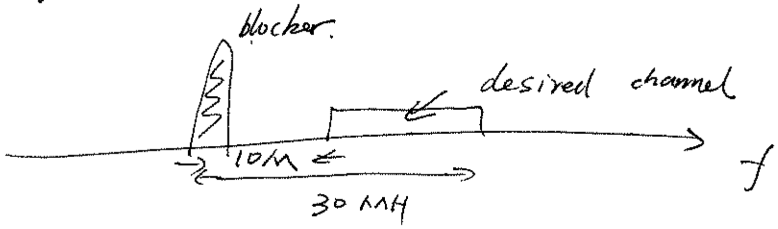
$$a = 125$$

$$10\log a = 21\,dB$$

So the highese blocker can be stronger than designed signal than 21dB.

13.6 Solu:

If only one blocker is located in the adjacent channel. $\partial = 100$ (from previous problem)



$$\frac{P_{PN}}{P_{sig}} = a\int_{f_1}^{f_2} \frac{a}{f^2}\, df$$

$$= a\partial \left(\frac{1}{f_1} - \frac{1}{f_2}\right) = -30\text{dB}$$

$$= 100\, a \cdot \left(\frac{1}{10M} - \frac{1}{30M}\right) = 10^{-3}$$

$$6.67\, a \times 10^{-6} = 10^{-3}$$

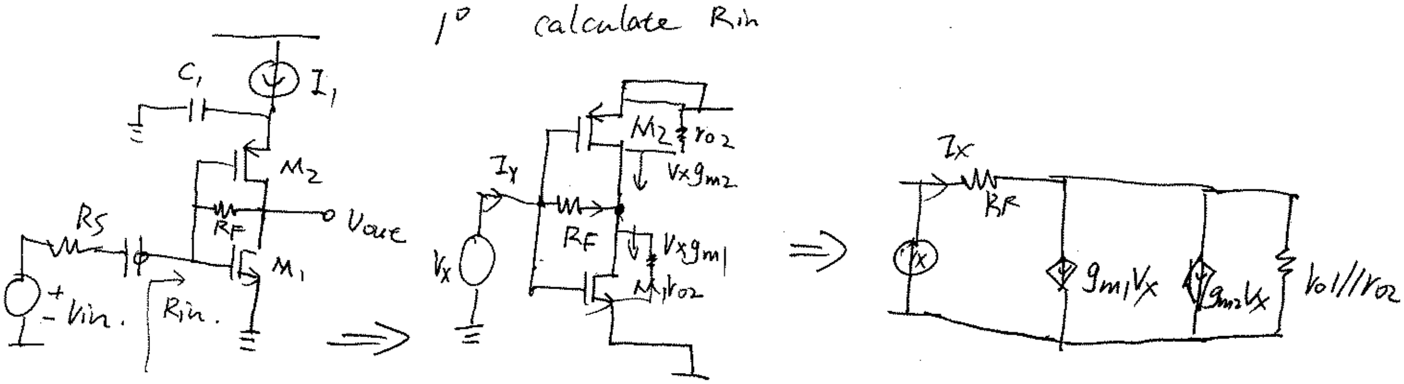
$$a = 150$$

$$\Rightarrow \quad 10 \log_{10} a = 21.76 \text{ dB}$$

Compared with the result of ~~Pb~~ Problem 5, 21 dB, we can conclude ~~that the~~ main effect is because of the blocker in the adjacent channel.

13.7 Solu:

1° calculate R_{in}



$$\frac{V_X - I_X R_F}{r_{o1} // r_{o2}} + g_{m1} V_X + g_{m2} V_X - I_X = 0$$

$$\Rightarrow \quad R_{in} = \frac{V_X}{I_X} = \boxed{\frac{r_{o1} // r_{o2} + R_F}{1 + (g_{m1} + g_{m2})(r_{o1} // r_{o2})}}$$

2° Calculate $\frac{v_{out}}{v_{in}}$

$$\frac{V_{out}}{V_X} = \frac{(r_{o1} // r_{o2})\,[1 - (g_{m1} + g_{m2}) R_F]}{R_F + (r_{o1} // r_{o2})}$$

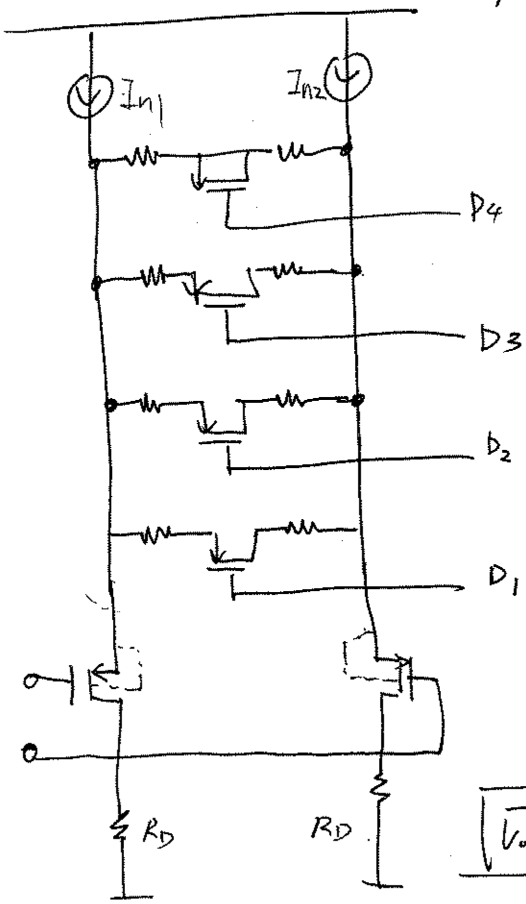
$$\frac{R_{in}}{R_S + R_{in}} = \frac{(r_{o1} // r_{o2}) + R_F}{[1 + (g_{m1} + g_{m2})(r_{o1} // r_{o2})] R_S + r_{o1} // r_{o2} + R_F}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_{in}}{R_S + R_{in}} \cdot \frac{V_{out}}{V_X}$$

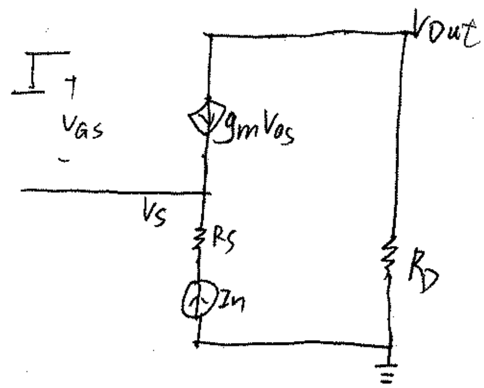
$$= \boxed{\frac{[1 - (g_{m1} + g_{m2}) R_F] \cdot (r_{o1} // r_{o2})}{[1 + (g_{m1} + g_{m2}) R_S](r_{o1} // r_{o2}) + R_F + R_S}}$$

13.8 Solu:

neglect on-resistance of switch, channel-length modulation, body effect.



$\Rightarrow$



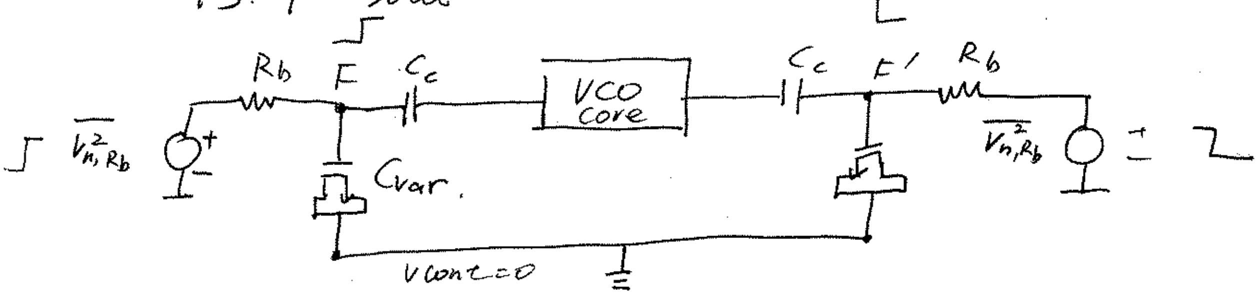
$$\frac{V_{out}}{R_D} = I_n$$

$$V_{out} = I_n \cdot R_D$$

$$\boxed{\overline{V_{out}^2} = (\overline{I_{n1}^2} + \overline{I_{n2}^2}) \cdot R_D^2}$$

So the I_{n1} and I_{n2} contribute the output derectly, which is very bad.

13.9 Soln:



$\Delta V_{con} \cdot K_{VCO} = \Delta \omega.$

The noise of R_b directly modulates the varactor, as if it were in series with Vcont., offset freq. below $\omega_{-3dB} \approx \frac{1}{R_b C_c}$

$C_c >> C_{var}$, $\omega \approx \frac{1}{R_b C_1}$

$$\Delta V_F = \frac{\frac{1}{C_{var} \cdot s}}{R_b + \frac{1}{C_{var} \cdot s}} \cdot V_{n,R_b}$$

$$= \frac{1}{R_b \cdot C_{var} \cdot s + 1} \cdot V_{n,R_b}$$

$$\approx V_{n,R_b}$$

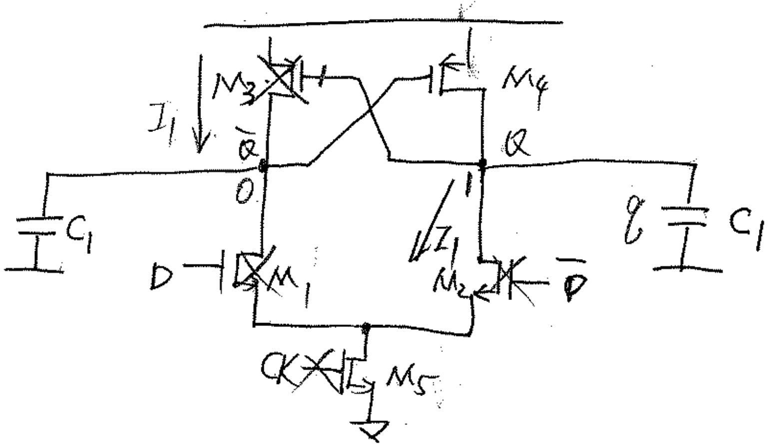
$$\Delta V_{F'} \approx V_{n,R_b}.$$

$$\Delta V_{cont} \approx \Delta V_F = V_{n,R_b}.$$

$$\therefore \quad \Delta \omega = V_{n,R_b} \cdot K_{VCO}.$$

$\Rightarrow$ the gain from the noise voltage of each resistor to VCO output frequency is equal to K_{VCO}.

13.10 Soln:



Assume last state is $Q=1$, M_3 is off, M_4 is on, C_1 in the ~~left~~ right hand have q_e charge in it. (M_3 is off)

And $\bar{Q}$ node has a leakage from M_3, which can charge the C_1 in the left hand. Q node has a leakage I_1 through M_3 & M_5.

when the voltage $V_Q < V_{\bar{Q}}$, the state will be corrupted.

Estimate Time $\Rightarrow$ $\frac{I_1 \cdot t}{C_1} > \frac{q_e - I_1 t}{C_1}$

Critital Time $\Rightarrow$ $t = \frac{q_e}{2I_1}$.

$$q_e = V_{DD} \cdot C_1$$

$$\Rightarrow \quad t = \frac{V_{DD} \cdot C_1}{2I_1}$$