Question 1.

$$
\begin{aligned}
V_1(X) &= A_1 e^{-X} + B_1 e^{X} \quad & 0 \le X \le L_1 \\
V_{21}(X) &= A_{21} e^{-X} + B_{21} e^{X} \quad & L_1 \le X \le L_{21} \\
V_{22}(X) &= A_{22} e^{-X} + B_{22} e^{X} \quad & L_1 \le X \le L_{22}
\end{aligned} \tag{2}
$$

$$
\left.\frac{dV_1}{dX}\right|_{X=0} = -(r_i\lambda_c)_1 I_{app} \tag{3}
$$

$$
V_{21}(L_{21}) = V_{22}(L_{22}) = 0 \tag{4}
$$

$$
V_1(L_1) = V_{21}(L_1) = V_{22}(L_1) \tag{5}
$$

$$
\frac{-1}{(r_i\lambda_c)_1}\left.\frac{dV_1}{dX}\right|_{X=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\left.\frac{dV_{21}}{dX}\right|_{X=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\left.\frac{dV_{22}}{dX}\right|_{X=L_1} \tag{6}
$$

$$V_1(x) = A_1 e^{-x} + B_1 e^{x}$$

$$\frac{d}{dx} V_1(x) = -A_1 e^{-x} + B_1 e^{x}$$

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -A_1 + B_1 = -(r_i\lambda_c)_1 I_{app} \qquad \text{(using equation 3)}$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \text{ — } ①$$

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0 \qquad \text{(using equation 4)}$$

$$A_{21} e^{-L_{21}} + B_{21} e^{L_{21}} = 0 \text{ — } ②$$

$$A_{22} e^{-L_{22}} + B_{22} e^{L_{22}} = 0 \text{ — } ③$$

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1) \qquad \text{(using equation 5)}$$

$$A_1 e^{-L_1} + B_1 e^{L_1} = A_{21} e^{-L_1} + B_{21} e^{L_1} = A_{22} e^{-L_1} + B_{22} e^{L_1}$$

$$(A_1 - A_{21}) e^{-L_1} + (B_1 - B_{21}) e^{L_1} = 0 \text{ — } ④$$

$$(A_1 - A_{22}) e^{-L_1} + (B_1 - B_{22}) e^{L_1} = 0 \text{ — } ⑤$$

$$\frac{dV_1}{dx} = -A_1 e^{-x} + B_1 e^{x} \qquad \frac{dV_{21}}{dx} = -A_{21} e^{-x} + B_{21} e^{x} \qquad \frac{dV_{22}}{dx} = -A_{22} e^{-x} + B_{21} e^{x}$$

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1}{dx}\bigg|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_1}\left(-A_1e^{-L_1}+B_1e^{L_1}\right)$$

$$\frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dx}\bigg|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21}e^{-L_1}+B_{21}e^{L_1}\right)$$

$$\frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dx}\bigg|_{x=L_1} = \frac{-1}{(r_i\lambda_c)_{22}}\left(-A_{22}e^{-L_1}+B_{21}e^{L_1}\right)$$

$$\frac{-1}{(r_i\lambda_c)_1}\left(-A_1e^{-L_1}+B_1e^{L_1}\right) = \frac{-1}{(r_i\lambda_c)_{21}}\left(-A_{21}e^{-L_1}+B_{21}e^{L_1}\right)\frac{-1}{(r_i\lambda_c)_{22}}\left(-A_{22}e^{-L_1}+B_{22}e^{L_1}\right)$$

$$-\frac{A_1}{(r_i\lambda_c)_1}e^{-L_1}+\frac{B_1}{(r_i\lambda_c)_1}e^{L_1}+\frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1}-\frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1}+\frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1}-\frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1}=0$$

$\therefore$ Equation (7) is valid.

Question 2

$$\mathrm{Ax} = \mathrm{b} \tag{9}$$

where

$$\mathrm{b} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{10}$$

and

$$\mathrm{A} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix}$$

$$Ax = b$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{bmatrix} = \begin{bmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \quad \text{—— ①}$$

$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0 \quad \text{—— ②}$$

$$A_{22}e^{-L22}+B_{22}e^{L22}=0 \quad \text{--- (3)}$$

$$A_1e^{-L1}+B_1e^{L1}-A_{21}e^{-L1}-B_{21}e^{L1}=0 \quad \text{--- (4)}$$

$$A_{21}e^{-L1}+B_{21}e^{L1}-A_{22}e^{-L1}-B_{22}e^{L1}=0 \quad \text{--- (5)}$$

$$-\frac{A_1}{(r_i\lambda_s)_1}e^{-L1}+\frac{B_1}{(r_i\lambda_c)_1}e^{L1}+\frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L1}-\frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L1}+\frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L1}-\frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L1}=0 \quad \text{--- (6)}$$

All 6 equations are obtainable from $Ax=B$.

Load the definition of matrix A by loading the cable.m file to MATLAB, and the cable parameters.

```
cable;
```

Question 3

By defining b determine the values of the coefficients of equation (2) assuming the boundary and nodal conditions of sections 1 and 2.

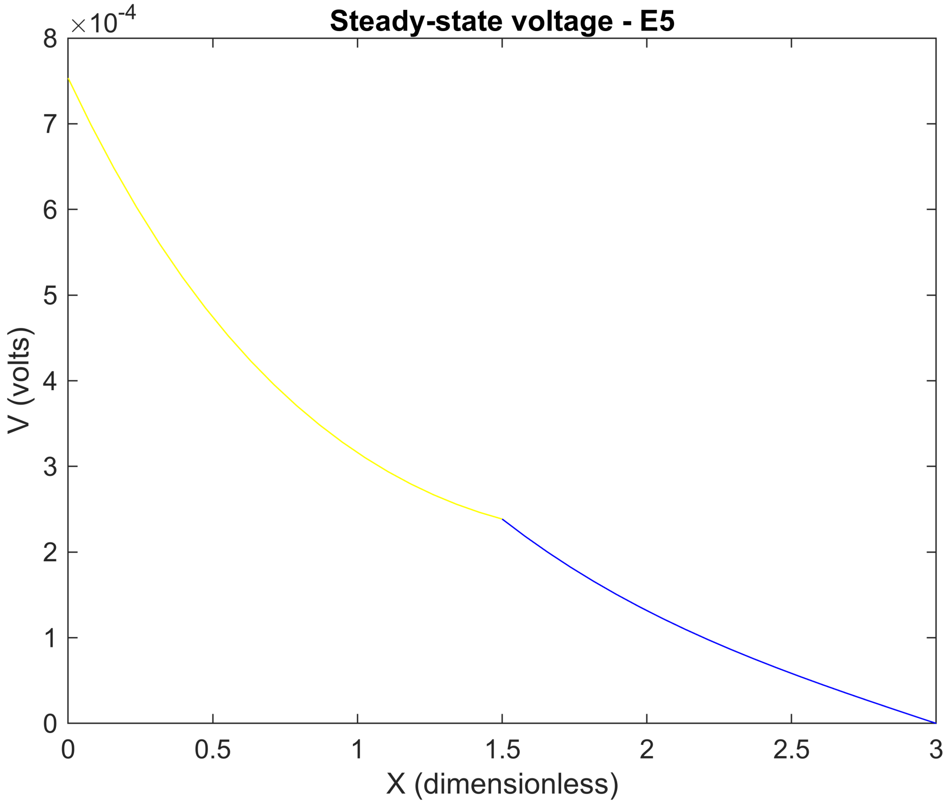
```
x=A\b
```

```
x = 6×1
    0.0007
    0.0000
    0.0011
   -0.0000
    0.0011
   -0.0000
```

Question 4

By using the coefficients found above and assuming that the coefficient array is stored in the variable � ordered according to equation (8) plot the steady-state voltage profile in each branch.

```
y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



What do you note about the steady state voltage profile in the two daughter branches?

The red line representing one of the daughter branches is overlapped by the blue line. This means the steady state voltage profile of both daugther branches are equal. This is verified by the resulting column vector in Question 3 where A21 = A22 and B21 = B22.

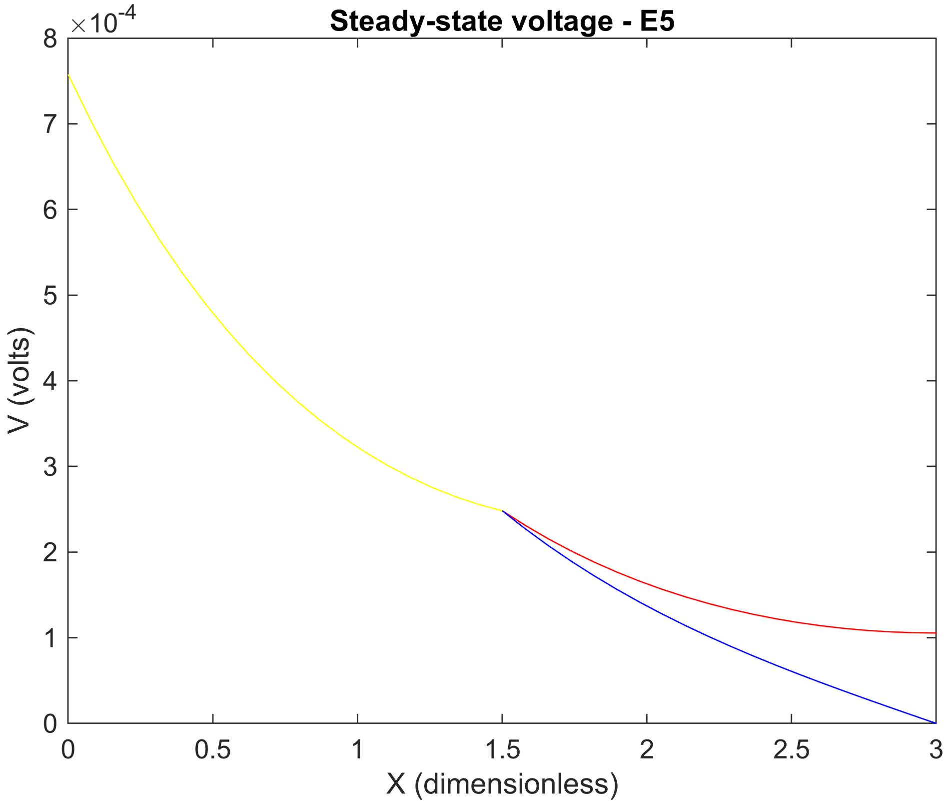
Plot the steady-state voltage profiles for each of the boundary conditions in Figure 2.

2 (a)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];

x=A\b;

y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```

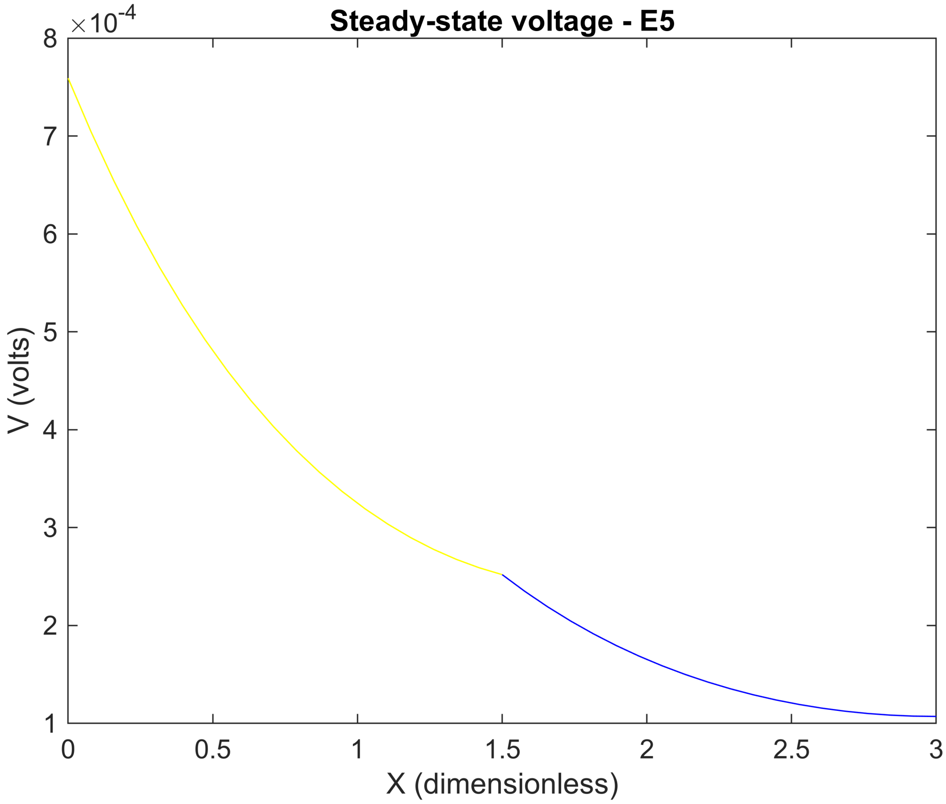


2 (b)

```
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x=A\b;

y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```

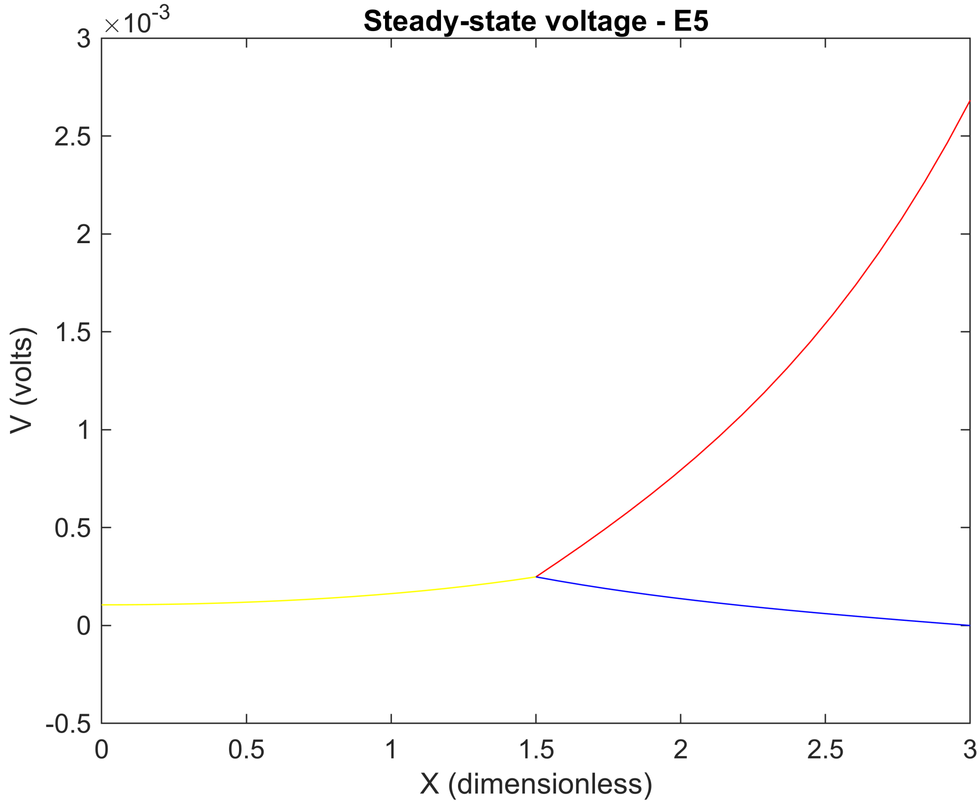


2 (c)

```
cable;
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
b(1) = 0; b(2) = rl21*iapp;

x=A\b;

y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```

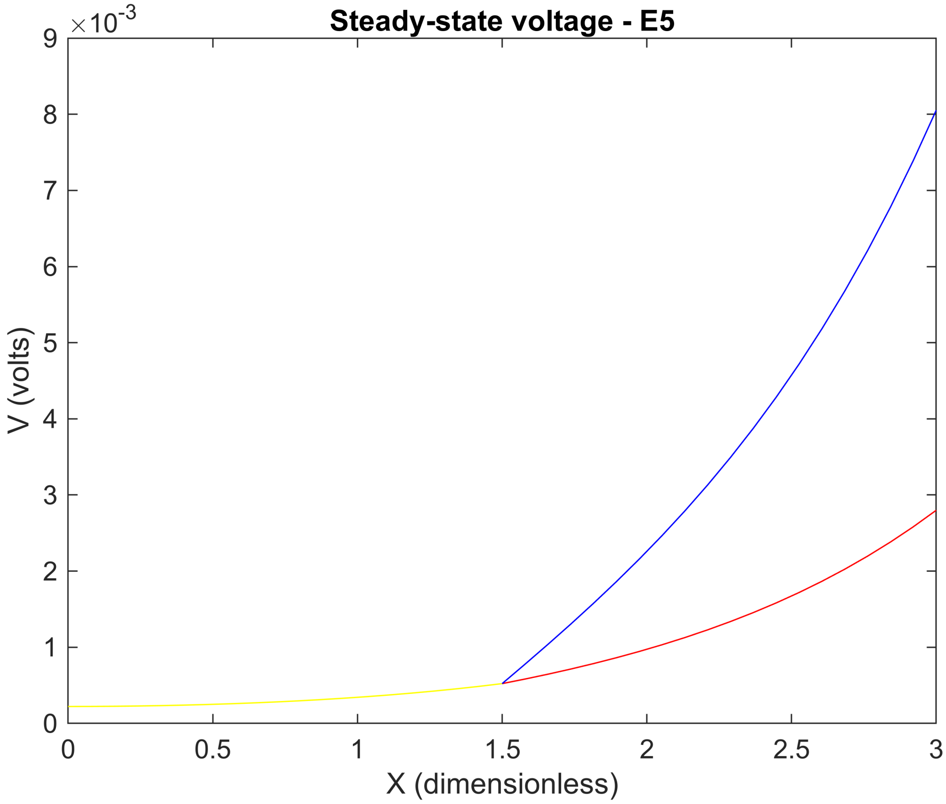


2 (d)

```
b(3) = rl22*iapp;

x=A\b;

y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



Question 5

What is the meaning of the positive right hand sides of dV21/dx|x=L21 and dV22/dx|x=L22 in 2(c) and 2(d)?

In parts 2(c) and 2(d), the current is flowing out from the right side of the branch. As this current moves through the impedance (resistance in the branch), it creates a positive voltage. This is similar to how dendrites in a neuron send electrical signals to other neurons. Because of the current, the right side of the branch becomes more positively charged, meaning the voltage is higher on that side.

Question 6

Recalculate the coefficients of equation (2) and replot the steady-state voltage profile for the boundary conditions of Figure 2(b) and 2(d) for d21 = d22 = 47.2470 * 10^(−4) cm.

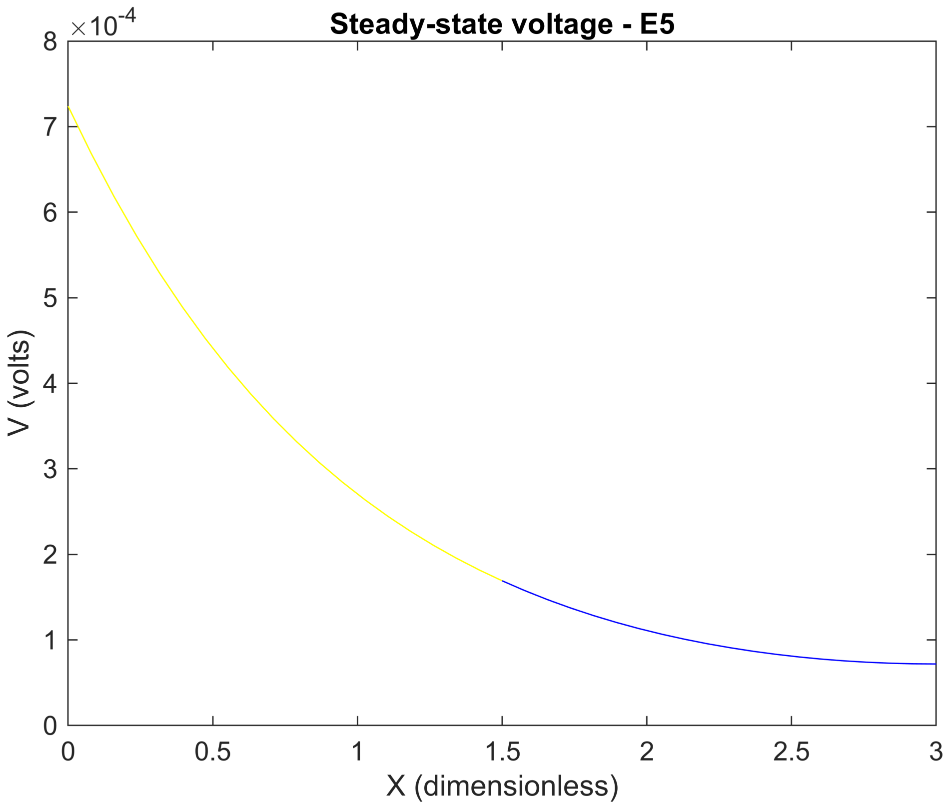
```
cable_modified;
```

2 (b)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x=A\b;
```

```
y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



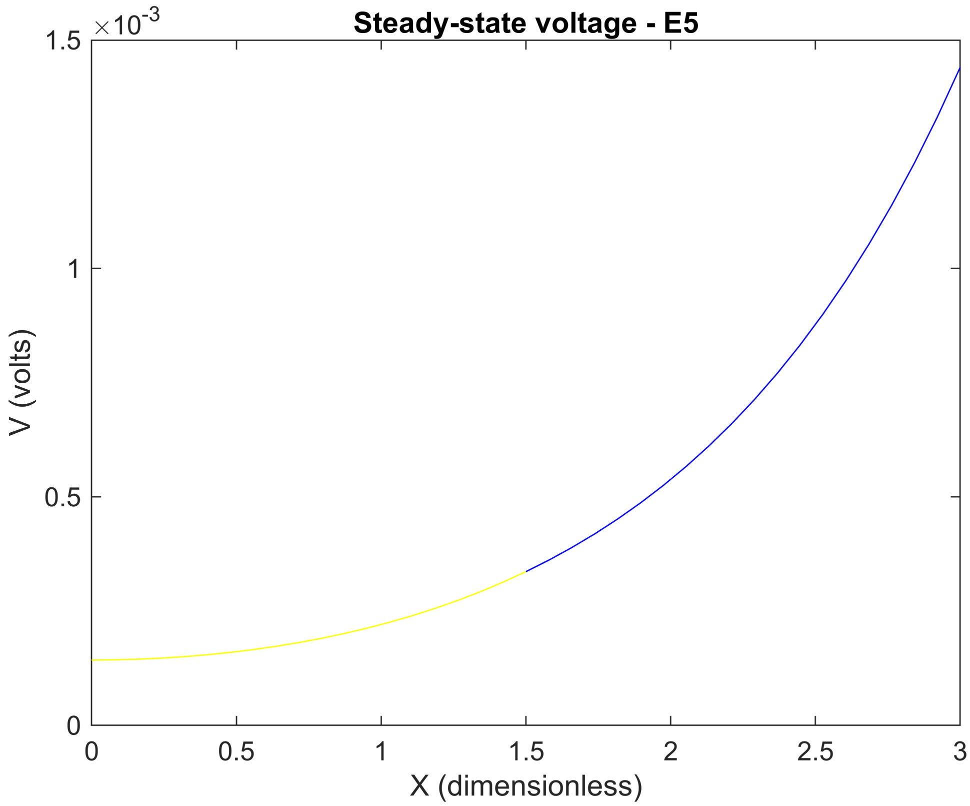
2 (d)

```
cable_modified;
b(1) = 0; b(2) = rl21*iapp;
b(3) = rl22*iapp;

x=A\b;

y1 = linspace(0, l1, 20);
y21 = linspace(l1, l21, 20);
y22 = linspace(l1, l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b-');
```

```
xlabel('X (dimensionless)');
ylabel('V (volts)');
title('Steady-state voltage - E5');
```



What do you notice?

The graphs have become smoother at the branching point. There are no sharp changes. The voltage profile for both daughter branches are approximately equal in both cases. This is due to the equal diameter of both daughter branches which carry equal amounts of current.