Differentiation

Eg:

(1) Power rule

1. $y = x^3 \rightarrow \frac{dy}{dx} = 3x^{3-1} = 3x^2$

2. $y = x^{1/2} \rightarrow \frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}}$

3. $y = \frac{1}{x^2} = x^{-2} \rightarrow \frac{dy}{dx} = -2x^{-2-1} = -2x^{-3} = \frac{-2}{x^3}$

4. $y = \frac{1}{x^{1/3}} = x^{-1/3} \rightarrow \frac{dy}{dx} = -\frac{1}{3}x^{-1/3-1} = -\frac{1}{3}x^{-4/3} = \frac{-1}{3x^{4/3}}$

---

(2) Constant multiple rule

1. $y = 2x^5 \rightarrow \frac{dy}{dx} = 2\left[5x^4\right] = 10x^4$

---

(3) Sum rule

1. $y = 3x^2 - 10x + 5$

$\frac{dy}{dx} = 3\left[2x^{2-1}\right] - 10\left[x^{1-1}\right] + 0$

$= 6x - 10$

(4) Product rule

$$y = 2x^2 (10x^3 - 7)$$

$$\frac{dy}{dx} = 2x^2 \times [10(3x^2) - 0] + (10x^3 - 7)\ (2(2x^{2-1}))$$

$$= 2x^2 (30x^2) + (10x^3 - 7)\ 4x$$

$$= 60x^4 + 40x^4 - 28x$$

$$= 100x^4 - 28x$$

$$\cancel{= 2x(50x^3 - 14}$$

$$= 4x (25x^3 - 7)$$

---

(5) Quotient rule

$$y = \frac{3x^2}{(2x+5)}$$

$$\frac{dy}{dx} = \frac{(2x+5)(3(2x)) - 3x^2(2+0)}{(2x+5)^2}$$

$$= \frac{(2x+5)\ 6x - 6x^2}{(2x+5)^2}$$

$$= \frac{12x^2 + 30x - 6x^2}{(2x+5)^2}$$

$$= \frac{6x^2 + 30x}{(2x+5)^2} = \frac{6x(x+5)}{(2x+5)^2}$$

⑥ Chain rule

$y = (2x^2+5)^6$

$\frac{dy}{dx} = 6(2x^2+5)^5 \times [2(2x)+0]$

$= 6(2x^2+5)^5 \, 4x = 24x(2x^2+5)^5$ //

Elementary functions

① Trigonometry

1. $y = \sin x$

$\frac{dy}{dx} = \cos x$ //

2. $y = \sin 2x$

$\frac{dy}{dx} = (\cos 2x)\, 2$

$= 2\cos 2x$ //

3/ $y = 3\sin x^2$

$\frac{dy}{dx} = 3(\cos x^2) \cdot 2x = 6x \cos x^2$ //

② Exponential function

1. $y = e^x$

$\frac{dy}{dx} = e^x$ //

2. $y = e^{2x}$

$\frac{dy}{dx} = e^{2x} \cdot 2$

$= 2e^{2x}$

3/ $y = e^{x^2}$

$\frac{dy}{dx} = e^{x^2} \cdot 2x$

$= 2x e^{x^2}$

③ Logarithmic function

1) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

2) $y = \ln 2x$

$\frac{dy}{dx} = \frac{1}{2x} \times 2$

$= \frac{1}{x}$

3) $y = \ln x^2$

$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x$

$= \frac{2}{x}$

Turning points

$$y = \frac{x^3}{3} - \frac{x^2}{2} - 6x + 10$$

step 1 $\frac{dy}{dx} = \frac{\cancel{3}x^2}{\cancel{3}} - \frac{\cancel{2}x}{\cancel{2}} - 6 + 0 = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

step 2 $(x-3)(x+2) = 0$

$x = 3$ or $x = -2$

step 3 $\frac{d^2y}{dx^2} = 2x - 1$

step 4 $\left.\frac{d^2y}{dx^2}\right|_{x=3} = 2(3) - 1 = 5 > 0 \rightarrow$ min

$\left.\frac{d^2y}{dx^2}\right|_{x=-2} = 2(-2) - 1 = -5 < 0 \rightarrow$ max

step 5 $y\Big|_{x=3} = \frac{3^3}{3} - \frac{3^2}{2} - 6(3) + 10$

$= 3^2 - \frac{9}{2} - 18 + 10$

$= 1 - \frac{9}{2} = \frac{-7}{2}$

$(3, \frac{-7}{2})$ is a minimum point

$y\Big|_{x=-2} = \frac{(-2)^3}{3} - \frac{(-2)^2}{2} - 6(-2) + 10$

$= \frac{-8}{3} - \frac{\cancel{4}^2}{\cancel{2}} + 12 + 10$

$= -\frac{8}{3} + 20$

$= \frac{52}{3}$

$(-2, \frac{52}{3})$ is a maximum point

Integration

Eg:

(1) $\int 5\,dx = 5x + C$

(2) $\int -6\,dx = -6x + C$

(3) $\int x^2\,dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$ //

(4) $\int x^{-6}\,dx = \frac{x^{-6+1}}{-6+1} + C = \frac{x^{-5}}{-5} + C = -\frac{1}{5x^5} + C$ //

(5) $\int \frac{1}{x^{3/2}}\,dx = \int x^{-3/2}\,dx = \frac{x^{-3/2+1}}{-3/2+1} + C = \frac{x^{-1/2}}{-1/2} + C$

$= \frac{-2}{x^{1/2}} + C$ //

(6) $\int 5x^{1/2}\,dx = 5\int x^{1/2}\,dx = \frac{5x^{1/2+1}}{1/2+1} + C = \frac{5x^{3/2}}{3/2} + C$

$= \frac{2 \times 5\, x^{3/2}}{3} + C = \frac{10x^{3/2}}{3} + C$ //

(7) $\int -x\,dx = -\int x\,dx = -\frac{x^2}{2} + C$ //

(8) $\int (x^2 + x^{-2})\,dx = \int x^2\,dx + \int x^{-2}\,dx$

$= \frac{x^3}{3} + \frac{x^{-1}}{-1} + C = \frac{x^3}{3} - \frac{1}{x} + C$ //

(9) $\int (2x^{1/4} + 3x^5)\,dx = 2\int x^{1/4}\,dx + 3\int x^5\,dx$

$= \frac{2x^{5/4}}{5/4} + \frac{3x^6}{6} + C = \frac{8x^{5/4}}{5} + \frac{x^6}{2} + C$ //

Definite integration

1. $\int_1^2 x\,dx = \frac{x^2}{2}\Big|_1^2 = \frac{1}{2}(2^2) - \frac{1}{2}(1^2) = 2 - \frac{1}{2} = \frac{3}{2}$

2. $\int_{-1}^{2} \left(x^2 - \frac{1}{x^3}\right) dx = \int_{-1}^{2} x^2\,dx - \int_{-1}^{2} x^{-3}\,dx$

$$= \frac{x^3}{3}\Big|_{-1}^{2} - \frac{x^{-2}}{-2}\Big|_{-1}^{2}$$

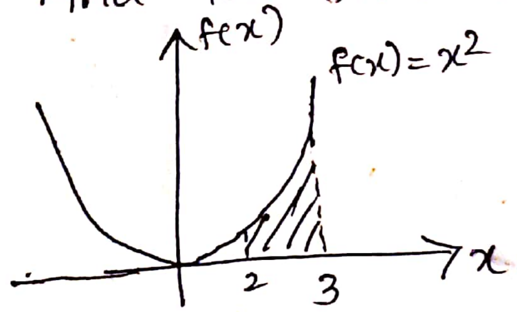
$$= \frac{1}{3}\left(2^3 - (-1)^3\right) + \frac{1}{2}\left(2^{-2} - (-1)^{-2}\right)$$

$$= \frac{1}{3}(8+1) + \frac{1}{2}\left(\frac{1}{4} - 1\right)$$

$$= 3 + \frac{1}{2}\left(\frac{-3}{4}\right) = 3 - \frac{3}{8}$$

$$= \frac{21}{8}$$

Find the shaded area.



Area = A

$$A = \int_2^3 x^2\,dx = \frac{x^3}{3}\Big|_2^3$$

$$= \frac{3^3}{3} - \frac{2^3}{3} = 3^2 - \frac{8}{3}$$

$$= \frac{19}{3}$$

Scanned with CamScanner