DSC TIET - Basics of Quantum Computing

SWAP GATE

This gate swaps the state of 2 qubits

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Decomposing the SWAP gate into a combination of CNOT gates.

→ $|00\rangle$

After 1st CNOT

q_0 and q_1 remain $|0\rangle$, as the control qubit is not $|1\rangle$

After 2nd CNOT

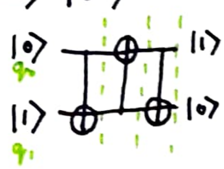
$q_0 = |0\rangle$
$q_1 = |0\rangle$
as control qubit $\neq |1\rangle$

After 3rd CNOT

$q_0 = |0\rangle$
$q_1 = |0\rangle$
as control qubit $\neq |1\rangle$

No change

→ $|01\rangle$



control qubit $= |0\rangle$ (q_0)
so $q_0 = |0\rangle$
$q_1 = |1\rangle$

control qubit $= |1\rangle$ (q_1)
So, $q_0 = |1\rangle$
$q_1 = |1\rangle$

control qubit $= |1\rangle$ (q_0)
So $q_0 = |1\rangle$
$q_1 = |0\rangle$

Therefore state of the qubits is swapped.

→ $|10\rangle$

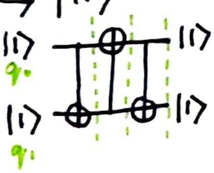
control qubit $= |1\rangle$ (q_0)
so $q_0 = |1\rangle$
$q_1 = |1\rangle$

control qubit $= |1\rangle$ (q_1)
so $q_0 = |0\rangle$
$q_1 = |1\rangle$

control qubit $= |0\rangle$ (q_0)
so, $q_0 = |0\rangle$
$q_1 = |1\rangle$

swapped

→ $|11\rangle$



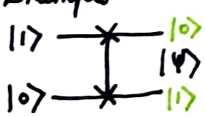
control qubit $= |1\rangle$ (q_0)
so $q_0 = |1\rangle$
$q_1 = |0\rangle$

control qubit $= |0\rangle$ (q_1)
so $q_0 = |1\rangle$
$q_1 = |0\rangle$

control qubit $= |1\rangle$ (q_0)
so, $q_0 = |1\rangle$
$q_1 = |1\rangle$

swapped

Example



$$|10\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$|\psi\rangle = SWAP\,|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow |01\rangle$$

So $|\psi\rangle = |01\rangle$

Hermitian

$$SWAP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad SWAP^{+} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Conjugate transpose

We can see that
$SWAP = SWAP^{+}$
Therefore it is Hermitian

Unitary

$$(SWAP)(SWAP^{+}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

hence SWAP gate is Unitary.