DSC TIET - Basics of Quantum Computing

## QUANTUM CIRCUIT MODEL

A theoretical model for quantum computation.

It consists of 3 Components

1) STATES
2) GATES
3) MEASUREMENT

### STATES

⇒ Classically

states :- 0, 1

bit string :- 01101

⇒ Quantum

states :- $a|0\rangle + \beta|1\rangle = \begin{bmatrix} a \\ \beta \end{bmatrix}$

Quantum Bit String :-

$= |1\rangle \otimes |0\rangle \otimes |0\rangle \otimes |1\rangle$

$= |1001\rangle$

### GATES

⇒ Classically (Boolean logic)

| | |
|---|---|
| AND | $A \cdot B$ |
| OR | $A + B$ |
| NOT | $\bar{A}$ |
| NAND | $\overline{A \cdot B}$ |
| NOR | $\overline{A + B}$ |
| XOR | $A \oplus B$ |

⇒ Quantum (linear algebra)

- quantum operations mathematically represented as matrices.
- state of a qubit can be manipulated by rotating on the Bloch sphere using quantum gates.

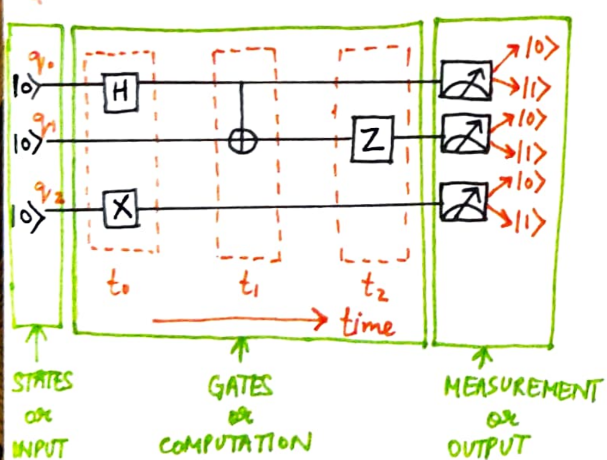
### MEASUREMENT

⇒ classical Measurements :- DETERMINISTIC
quantum Measurements :- PROBABILISTIC

⇒ Measurement can be along any direction. But, z axis is chosen as the frame usually.

⇒ To understand the underlying distribution of a quantum state, the state needs to be created multiple times for repeated Measurements, following the Law of Large Numbers

### QUANTUM CIRCUIT MODEL

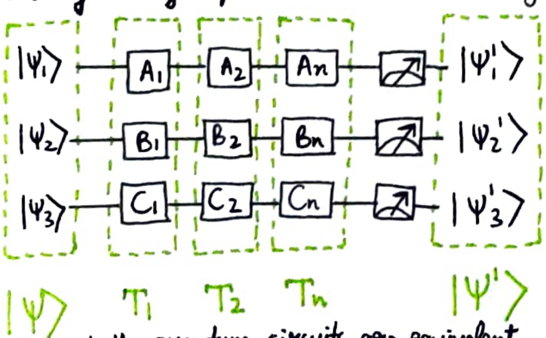



- possible outcomes → $|000\rangle, |001\rangle, |010\rangle, |011\rangle$
$|100\rangle, |101\rangle, |110\rangle, |111\rangle$
- probability of being in a state from the possible outcomes becomes clearer with repeated measurements.

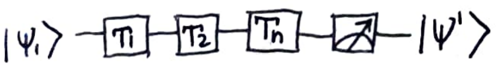
NOTE :- at any time instance t, I gate is considered in place of plain wire for ease of calculation.

## MULTI-QUBIT CIRCUIT MATH

solving n single qubit circuits simultaneously




both quantum circuits are equivalent




So, $|\psi'\rangle = T_n T_2 T_1 |\psi\rangle$ when $|\psi'\rangle$ and $|\psi\rangle$ are $(2^n \times 1)$ AND $T_n$ gates are $(2^n \times 2^n)$ in shape

Tensor product is used to solve multi qubit quantum circuits

* Quantum bit string :- tensor product of qubits
$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_n\rangle$
* similarly for gates
$T = T_1 \otimes T_2 \otimes T_n$

Tensors are n dimensional vectors

state :- 1d tensor or vector

gate :- 2d tensor or matrix

## TENSOR / KRONECKER PRODUCT

If A is $(n \times m)$ matrix

B is $(p \times q)$ matrix

their tensor product is $(pn \times mq)$ matrix

let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$A \otimes B = \begin{bmatrix} a_1 B \\ a_2 B \end{bmatrix} = \begin{bmatrix} a_1 b_{11} & a_1 b_{12} \\ a_1 b_{21} & a_1 b_{22} \\ a_2 b_{11} & a_2 b_{12} \\ a_2 b_{21} & a_2 b_{22} \end{bmatrix}$$

## MULTI QUBIT STATES

$q_0$ — $|0\rangle$

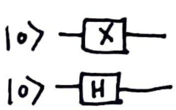
$q_1$ — $|0\rangle$

$q_2$ — $|0\rangle$

$$|q_0 q_1 q_2\rangle = |q_0\rangle \otimes |q_1\rangle \otimes |q_2\rangle$$
$$= |0\rangle \otimes |0\rangle \otimes |0\rangle$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \to |000\rangle \\ \to |001\rangle \\ \to |010\rangle \\ \to |011\rangle \\ \to |100\rangle \\ \to |101\rangle \\ \to |110\rangle \\ \to |111\rangle \end{matrix}$$

- Probability of being in state $|000\rangle$ is 1.
- n qubits are represented by a vector of length $2^n$.

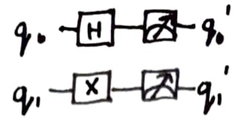
## MULTI QUBIT GATES



$$X \otimes H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 0 & H \\ H & 0 \end{pmatrix}$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

- n qubit gates are $(2^n, 2^n)$ shaped matrix

## MULTI QUBIT QUANTUM CIRCUIT EXAMPLE



$$|q_0 q_1\rangle = |q_0\rangle \otimes |q_1\rangle$$
$$= |00\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H \otimes X = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} X & X \\ X & -X \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$$|q'\rangle = |q_0' q_1'\rangle = (H \otimes X)(|q_0 q_1\rangle)$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \begin{matrix} \to |00\rangle \\ \to |01\rangle \\ \to |10\rangle \\ \to |11\rangle \end{matrix}$$

$$|q'\rangle = q_{00}|00\rangle + q_{01}|01\rangle + q_{10}|10\rangle + q_{11}|11\rangle$$
$$= \frac{1}{\sqrt{2}} \left[ 0\,|00\rangle + 1\,|01\rangle + 0\,|10\rangle + 1\,|11\rangle \right]$$
$$= \frac{1}{\sqrt{2}} \left[ |01\rangle + |11\rangle \right]$$

so $|q'\rangle = \dfrac{|01\rangle}{\sqrt{2}} + \dfrac{|11\rangle}{\sqrt{2}}$

it is consistent with measurement rule

$$|q_{00}|^2 + |q_{01}|^2 + |q_{10}|^2 + |q_{11}|^2 = 1$$

proof:- $0^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$0 + \frac{1}{2} + 0 + \frac{1}{2} = 1$ hence proved

$P(\langle 00|q'\rangle) = |q_{00}|^2 = 0$

$P(\langle 01|q'\rangle) = |q_{01}|^2 = 1/2 \to$ 50% probability

$P(\langle 10|q'\rangle) = |q_{10}|^2 = 0$

$P(\langle 11|q'\rangle) = |q_{11}|^2 = 1/2 \to$ 50% probability.

On measurement:

There is 50% chance of obtaining state $|01\rangle$ and 50% chance of obtaining state $|11\rangle$