L35
Binary Search : Problem Solving 1

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RECAP

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Today is the class where we get a deeper understanding of Binary Search, by practicing a few problems.

Given a non-negative integer N,
find integral part of sqrt(N)

$\sqrt{4} = 2$ ✓

$\sqrt{5} = \underline{2}.23$ 2

$\sqrt{6} = \underline{2}.45$ 2

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$\sqrt{8}$

$\sqrt{18}$

Intuition

$1^2 = 1 \leq 8$

$2^2 = 4 \leq 8$

$3^2 = 9 \not\leq 8$

$1^2 = 1 \leq 18$

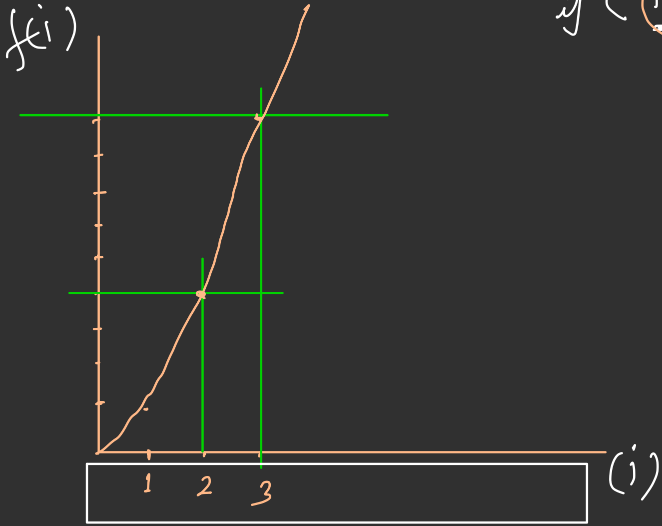
$2^2 = 4 \leq 18$

$3^2 = 9 \leq 18$

$4^2 = 16 \leq 18$

$5^2 = 25 \not\leq 18$

Let's implement



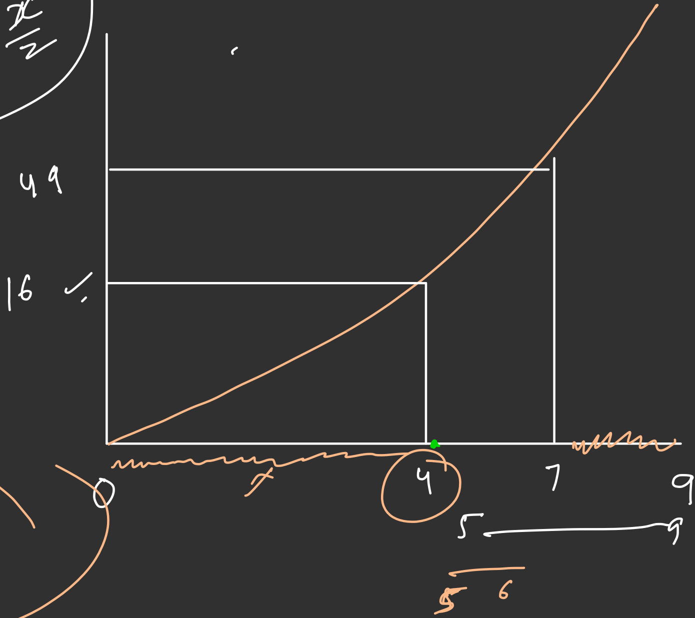
if (i*i <= x)

~~f(x) = x*x~~

~~= x^2~~

f(i) = i*i

$0 - \frac{x}{2}$



$x = 18$

$f(i) = i^2$

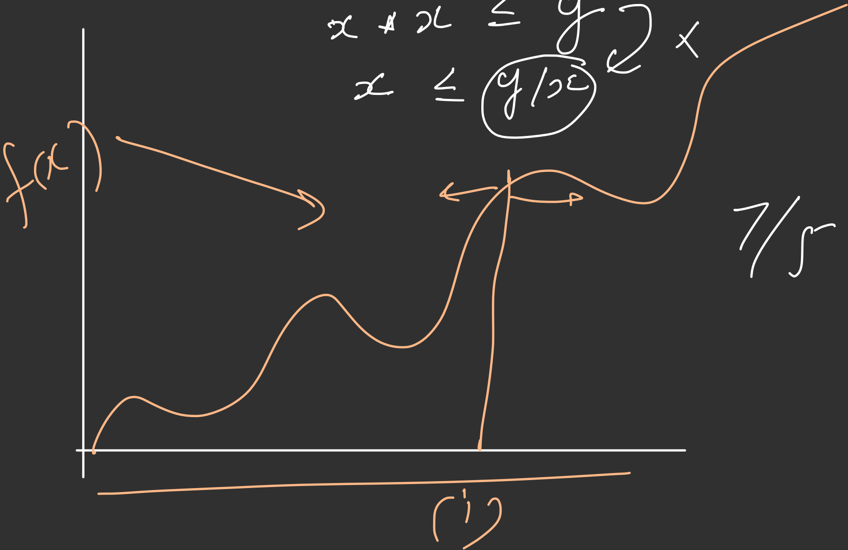
4^2

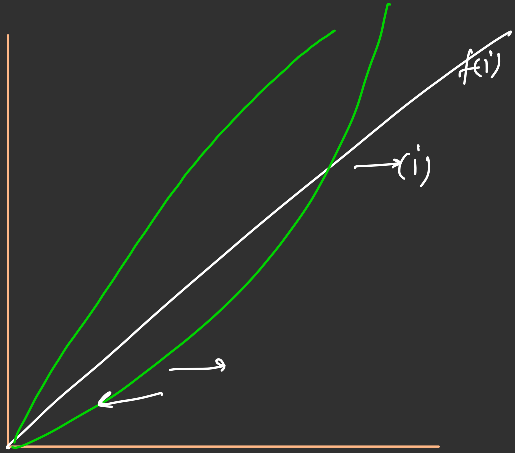
16

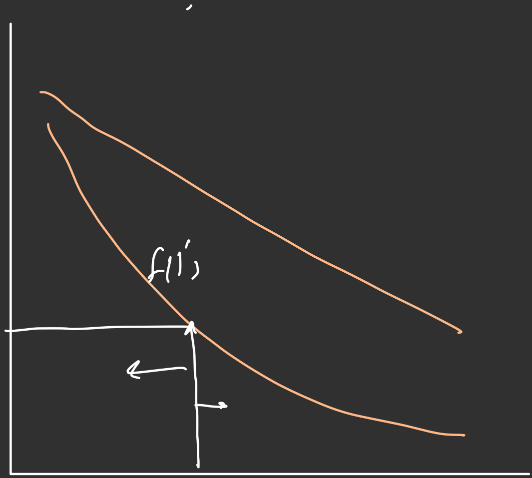
ans = 4

$x * x \leq y$

$x \leq y/x$




f(i)
(i)



0 $\quad x/2$

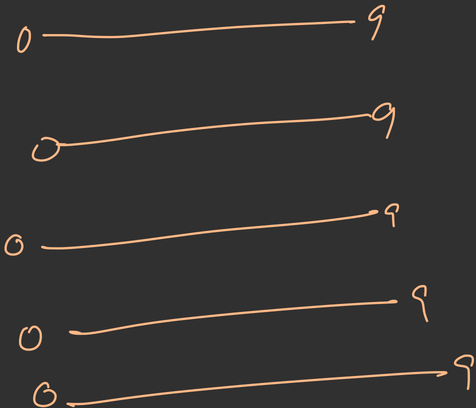
Now, we're not only interested in the integral part.

Given a value of N & P, we need to
find sqrt(N) accurate up to p decimal places.

$N = 3 \quad P = 2$

$\sqrt{3} = 1.73$

$p = 1$



$p = 0$ 10 code $p = 2$

$p = 1$ 100 code

$p = 2$ 1000 code

$p = 3$ 10^4

$p = 4$ 10^5

S —— e

0 $x/2$

3.125 3.375

S —— e

0

s —— e

s —— e

while (s<=e)

s>e X stop

e - s

.1

p=1

0.01

0 —— 9

0 —+— 4.5

2.25

2.25 —— 4.5

s e

3.375

2.25 —— 3.375

2.8125 - 3.375

$e - s < 0.1$ ✗ $\quad p = 1$

$e - s < 0.01$ ✗ $\quad p = 2$

$pow(10, -1) = \frac{1}{10}$

$pow(10, -2) \quad \frac{1}{100}$

Intuition

Let’s go code

$$n \quad \frac{n}{2} \quad \frac{n}{4} \quad \cdots \quad 1 \quad \cdots \quad \frac{1}{10^p} \quad 10^{-p}$$

$$\frac{n}{2^0} \quad \frac{n}{2^1} \quad \frac{n}{2^2} \quad \frac{n}{2^k}$$

Time Complexity?

$$\frac{1}{2^k} = \frac{1}{10^p}$$

$$1 = \frac{n}{2^k}$$

$$O(\log n + p) \quad \boxed{\log_2 n}$$

$$2^k = 10^p$$

$$k \log 2 = p \log 10$$

$$k = p \frac{\log 10}{\log 2} \quad \log_2 10$$

$$k = 3.3p$$

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Okay, let’s move to the next problem now. It’s an interesting one.
[Was also asked in a nice company’s interview]

Given a function $f(x) = x + x/10 + x/100 + x/1000 \ldots$

Note : the division is integer division.

Examples:

1. $f(1234) = 1234 + 123 + 12 + 1 = 1370$
2. $f(214) = 214 + 21 + 2 = 237$
3. $f(50) = 50 + 5 = 55$
4. $f(8) = 8$

123

$f(x)$

$f(x+1)$

124

$$f(x) = x + \frac{x}{10} + \frac{x}{100} \ldots \frac{x}{1000} \ldots$$

$$f(x+1) = (x+1) + \frac{(x+1)}{10} + \frac{x+1}{100} \cdot \frac{x+1}{1000}$$

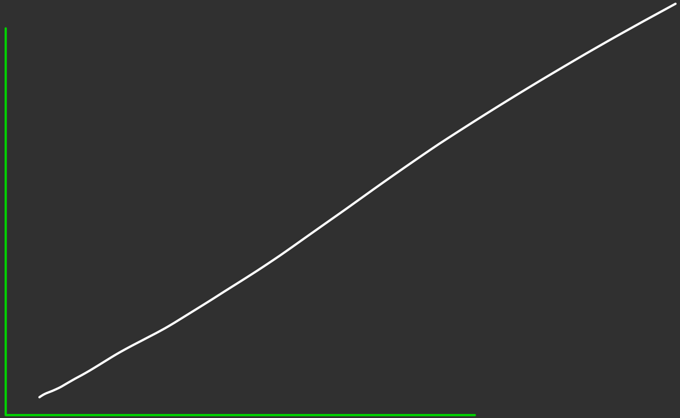
$$= 1 + x + \left(\frac{x+1}{10}\right) + \left(\frac{x+1}{100}\right) + \frac{x+1}{1000} \ldots$$

$$f(x) \qquad f(x+1)$$

1st $\quad x \qquad x$

$$\left(\frac{x}{10}\right) \qquad \left(\frac{x+1}{10}\right) \qquad \frac{x}{10} + \frac{1}{10}$$

$$\frac{x}{100} \qquad \frac{x+1}{100}$$



Now, given a value K,
it should be pretty simply to find f(K), right?

But the problem is not that xD

$$f(K) = \left\lfloor K + \frac{K}{10} + \frac{K}{100} \cdots \right\rfloor$$

1370

The problem is that given a number N,
we need to find fInverse(N).

In other words, we need to find a valid value K, such that f(K) = N

In yet another words, we're basically given the output (N) of the function, and we need to find what to give as an input (K) so that we get the given output (N).

A couple of things:

1. We have to print -1 if there is no valid answer.
2. It is guaranteed that if there is a valid answer, then that answer will be uni

Examples

Input : 1370
Output : 1234

Input : 237
Output : 214

Input : 55
Output : 50

Input : 8
Output : 8

Input : 243
Output : -1

Let’s think now

For now, assume that 1 <= N <= 10^5

Now, 1 <= N <= 10^9

$n * \log n$

$\log n * \log n$

$O\left(\log n\right)^2$

Is there a possibility of binary search?
What do you folks think?

When/where can we apply binary search?
Let’s try to dive deeper

The property of functions we just discussed
is called monotonicity.

Binary Search can be applied if the function is monotonic.

Is the given function monotonic?

Let’s implement

Time Complexity?

Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!