

# L34

## Binary Search : Introduction

Join Discord - <https://bit.ly/ly-discord>

## RECAP

1. Ferris Wheel Problem
2. Stick Lengths Problem
3. Another problem given as homework (did you try?)

# The Secret Number Game

I'll select a number b/w 1 and  $10^6$  (i.e. 1 mil)

Your target is to ask me some questions and then  
guess the number finally.

What question can you ask?

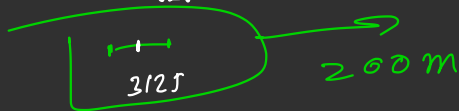
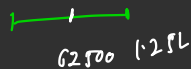
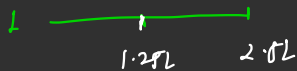
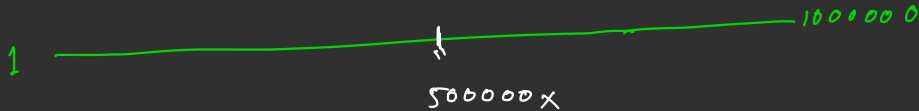
In 1 operation, you can choose a number X and ask me:  
“Is the secret number greater than X or not?”

1 — 1000000 500000

Two rules :

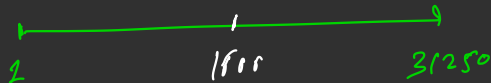
1. I can't change the number once chosen. So, it'll be same always.
2. I won't lie xD

Understanding the strategy for the game



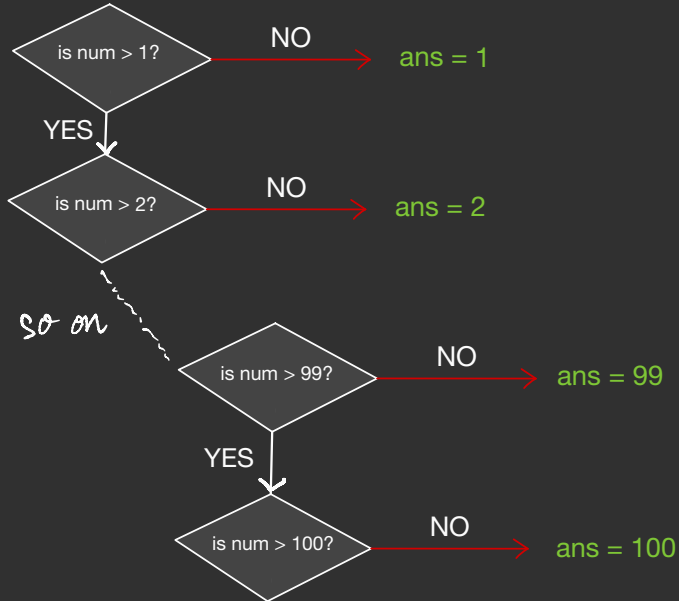
$$\begin{array}{r} 250 \\ \hline \end{array} \downarrow 500$$

$$\begin{array}{r} 273 - 281 \\ \hline 265 - 281 \\ \hline \end{array}$$

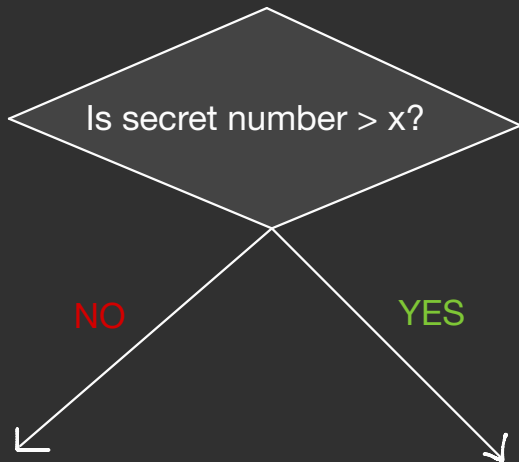


$$\begin{array}{r} 250 \downarrow 375 \\ \hline \end{array}$$

# Linear Approach



Let's try to analyze a better approach





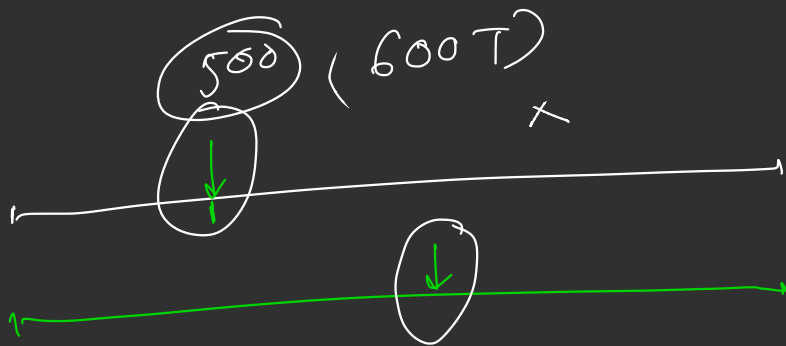
What are the number of operations required  
in the worst case scenario?

$$\left. \begin{array}{c} n \\ n-1 \\ n-2 \\ n-3 \\ \vdots \\ 1 \end{array} \right\}$$

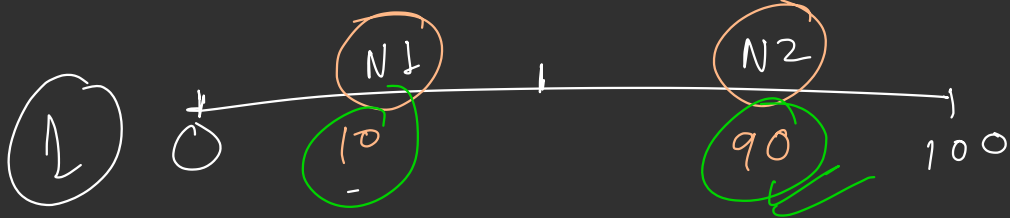
$$\underline{\underline{1 - 10^6}}$$

$$\left. \begin{array}{c} 2 \\ 2/3 \\ 2/3 \\ 5/3 \\ 8/3 \\ \vdots \\ 1 \end{array} \right\}$$

$$\underline{\log_2 n}$$

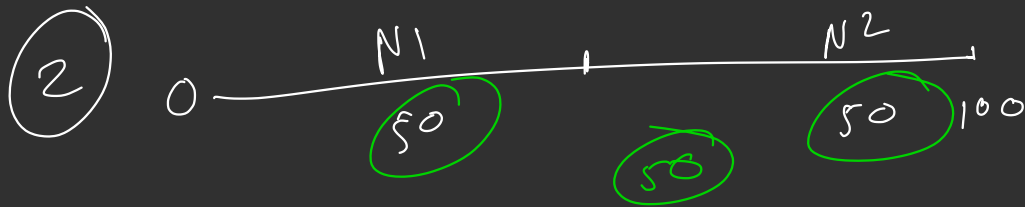


1	20	18	12	30	10	6	15	20
0	1	2	3	4	5	6	7	8

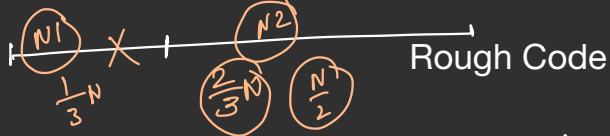


How to choose  $x$ ?

Target : Minimise  $\max(N1, N2)$  where  
we know that  $N1 + N2 = N$ :



What we just discovered is  
exactly what BINARY SEARCH is!



```
int findSecretNum(int l, int r) {  
    if(l == r)  
        return l;
```

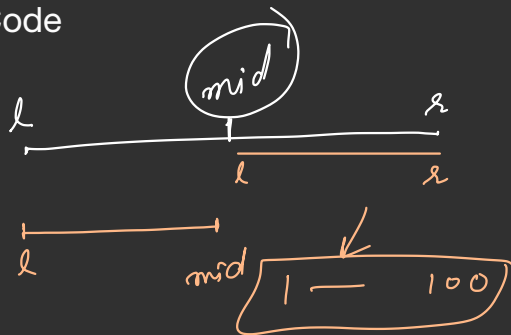
```
    int mid = (l + r)/2;
```

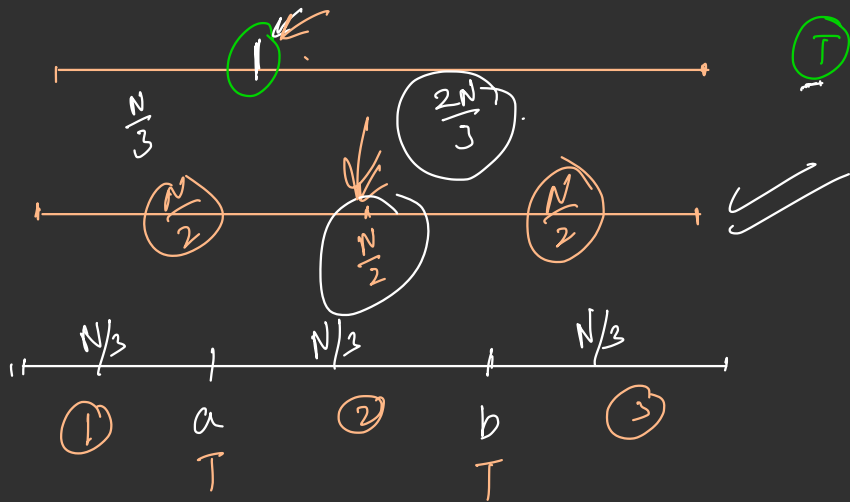
```
    // Assume that isSNGreater() is a function that takes O(1) time  
    bool checkForGreater = isSNGreater(mid);
```

```
    if(checkForGreater)  
        return findSecretNum(mid + 1, r);
```

```
    return findSecretNum(l, mid);
```

```
}
```





$\frac{\text{INT\_MAX}}{2} + \frac{\text{INT\_MAX}}{2}$

$\text{nums}[\text{mid}] > \text{target}$

$l$

$r$

$\text{mid}$

$l + \left( \frac{r-l}{2} \right)$

$\frac{2l + r - l}{2}$

$\frac{3 \text{ INT\_MAX}}{2}$

$\text{INT\_MAX}/2$

$\text{INT\_MAX}$

$\frac{l + r}{2}$

$10^8 + 10^8$

$\frac{2 \times 10^8}{2}$

$\text{mid} =$



$$l = -\frac{\infty}{2} \quad r = -\infty$$

$$-\infty - (-\infty/2)$$

$$-\infty + \infty/2$$

$$= \left( \frac{-\infty}{2} \right)$$

$$\frac{2/b}{2} + \frac{(2/b)}{2}$$

$$\frac{l+r}{2}$$

$$\left( \frac{3\infty}{2} \right)$$

$$\frac{2/b}{2} + \frac{2/b}{2} = \left( \frac{2/b}{1} \right)$$

$$l + \left( \frac{r-l}{2} \right)$$

$$\frac{2l+r-l}{2}$$

$$= \left( \frac{l+r}{2} \right)$$

Time Complexity?

## Contacts Example

## Dictionary Example

## Assembly Queue Example

## Git Bisect Example

Searching for a given target in a sorted Array

# Power of Binary Search

Time Comp Input Size	$O(\log N)$	$O(\sqrt{N})$	$O(N)$	$O(N \log N)$	$O(N^2)$	$O(N^3)$	$O(2^N)$
20	~20 ns	~22 ns	~100 ns	~450 ns	~2 $\mu$ s	~40 $\mu$ s	~2 ms
50	~30 ns	~35 ns	~250 ns	~2 $\mu$ s	~12 $\mu$ s	~625 $\mu$ s	~ 2 months
500	~45 ns	~111 ns	~3 $\mu$ s	~25 $\mu$ s	~1 ms	~650 ms	Out of Syllabus xD
5000 ( $5 \cdot 10^3$ )	~60 ns	~350 ns	~25 $\mu$ s	~300 $\mu$ s	125 ms	~11 mins	Out of Syllabus xD
1 million ( $10^6$ )	~100 ns	~5 $\mu$ s	~5 ms	~100 ms	~1.5 hours	~159 years	Out of Syllabus xD
100 mil ( $10^8$ )	~135 ns	~50 $\mu$ s	~500 ms	~13 secs	~1.6 years	~159 megayears	Out of Syllabus xD
1 billion ( $10^9$ )	~150 ns	~0.2 ms	5 secs	~2.5 mins	~159 years	~159 eons	Out of Syllabus xD
1 trillion ( $10^{12}$ )	~200 ns	~5 ms	~1.5 hours	~5.5 hours	~159 megayears	~159 billion eons	Out of Syllabus xD
$10^{15}$	~250 ns	~200 ms	~2 months	~8 years	~159K eons	Out of Syllabus xD	Out of Syllabus xD
$10^{18}$	~300 ns	~ 5 secs	~159 years	~95 centuries	~159 billion eons	Out of Syllabus xD	Out of Syllabus xD

Approximate Time Taken  
(assuming  $2 \cdot 10^8$  operations per second)



Can we do better than  $\text{LogN}$  (base 2)?

Let's implement binary search now

↓

1	2	4	4	6	6	7
0	1	2	3	4	5	6



> Upper Bound & Lower Bound >=



3

target  
4

## Searching in a rotated sorted Array

# Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!

