

L54  
Number Theory Final Problem Solving

Join Discord - <https://bit.ly/ly-discord>

## RECAP

1. Modulo Inverse Intro & Need
2. Extended Euclid's Algorithm
3. Connected the dots
4. Number of distinct anagrams problem

Last day of Number Theory

Let's get going

A N

# 1. Addition modulo m

$$a = 10^{15} - 1$$

$$b = 10^{15} - 2$$

$$m = 10^{15}$$

$$(a * b) \% m$$

$$X = ((a \% m) + (b \% m)) \% m$$

$$\begin{array}{r} \hline 10^{15} - 1 \\ + 10^{15} - 2 \\ \hline \end{array}$$

Given a, b & m (where  $1 \leq a, b, m \leq 10^{15}$ ),

Find  $(a * b) \% m$

$$a = 10$$

$$b = 7$$

$$m = 3$$

$$(10 \times 7) \% 3$$

$$70 \% 3 = 1$$

$$\left. \begin{array}{l} a = 10^{15} \\ b = 10^{15} \end{array} \right\} \underline{a * b}$$

$$\begin{array}{c}
 \textcircled{a * b} \rightarrow \underbrace{a + a + a + a + \dots}_{b \text{ times}} \\
 \downarrow \\
 a + (a * (b-1)) \% m
 \end{array}$$

$$6 \times 8$$

$$6 + 6 + 6 + 6 + 6 + 6 + 6 + 6$$


---

Intuition

sum = 0;

10 15 ✓

for (int i = 1; i <= b; i++)

{  
     sum += a;  
     sum % m = m; ✓  
 }

$$\begin{aligned}
 x^n &\rightarrow x^{n/2} * x^{n/2} & n \text{ is even} \\
 &\rightarrow x^{n/2} * x^{n/2} * x & n \text{ is odd.}
 \end{aligned}$$

$$a * b \rightarrow \left( \left( a * \frac{b}{2} \right) \% n + \left( a * \frac{b}{2} \right) \% n \right) \% n \quad b \text{ is even}$$

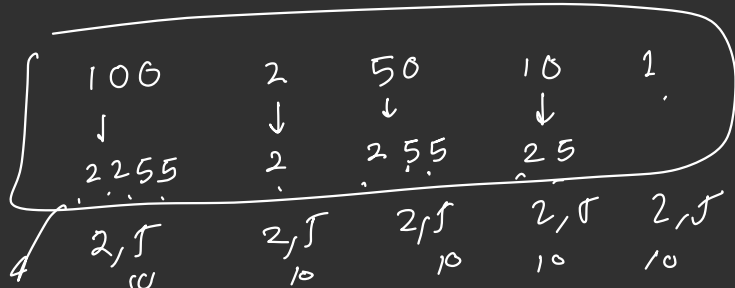
Solution

$$a * \frac{b}{2} + a * \frac{b}{2} + a \quad b \text{ is odd.}$$

Let's  
implement







Intuition

$2 \rightarrow 5$   
 $5 \rightarrow 5$

Solution

Let's  
implement

### 3. Strange Number

$$A = \underbrace{2^2}_{(2)} \cdot 5^1$$

$$(2+1)(1+1)$$

$$6$$

$$1.$$

$$\begin{bmatrix} 1. \\ 2. \\ 4. \\ 5. \\ 16 \\ 20 \end{bmatrix}$$

$$A = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdots p_k^{a_k}$$

$$X = (a_1+1) \cdot (a_2+1) \cdot (a_3+1) \cdots (a_k+1)$$

$$2 \geq$$

$$2 \geq$$

Intuition

$$2 \geq$$

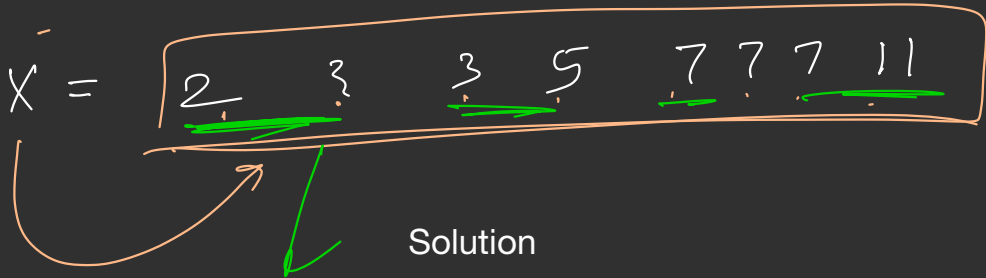
$$2 \geq$$

$$K$$

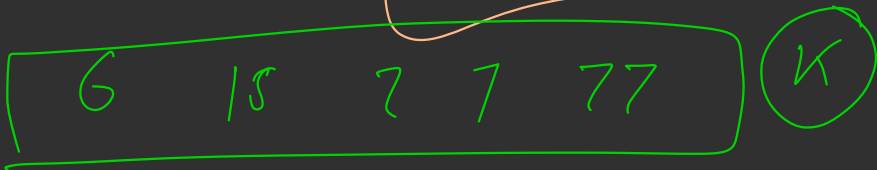
prime factors

$$K$$

$$k = 5$$



$$2 = k$$



Let's  
implement



# Thank You!

Reminder: Going to the gym & observing the trainer work out can help you know the right technique, but you'll muscle up only if you lift some weights yourself.

So, PRACTICE, PRACTICE, PRACTICE!