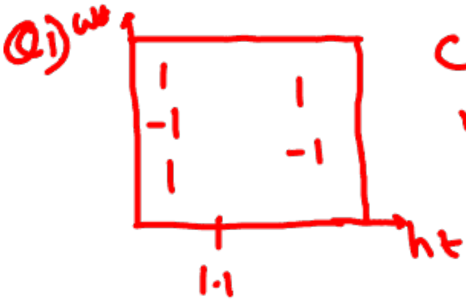
Q1)



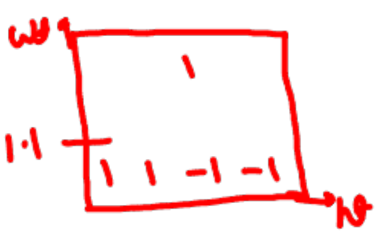
Cut at ht = 1.1
mean for left node = 1/3
" " right " = 0

$MSE_{left} = \left(1-\frac{1}{3}\right)^2 + \left(-1-\frac{1}{3}\right)^2 + \left(1-\frac{1}{3}\right)^2$
$= \frac{4}{9} + \frac{16}{9} + \frac{4}{9} = \frac{24}{9}$

$MSE_{right} = 1+1$

Total $MSE = 2 + \frac{24}{9}$ 1

Cut at wt = 1.1



mean for top node = 1
" " bottom " = 0

$MSE_{top} = 0$

$MSE_{bot} = 1+1+1+1 = 4$

Total error for wt is smaller than ht. 1

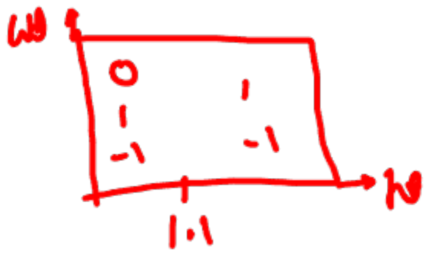
$F(x)$

once we obtain $F(x)$, we need to obtain residuals, $y_i - F(x_i)$

ht	wt.	$y_i - F(x_i)$
1	2	1 - 1
0	1	1 - 0
3	0	1 - 0
1	1	-1 - 0
2	1	-1 - 0

1

$h(x)$ will be trained on this data.



Mean left = 0
" right = 0

MSE = 0 + 1 + 1 + 1 + 1

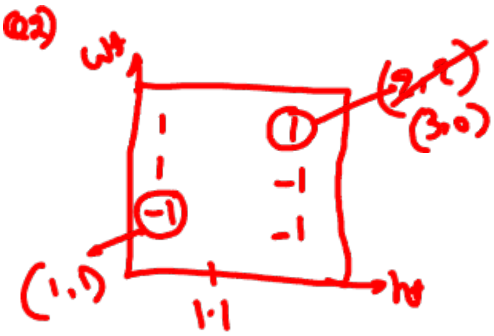
1

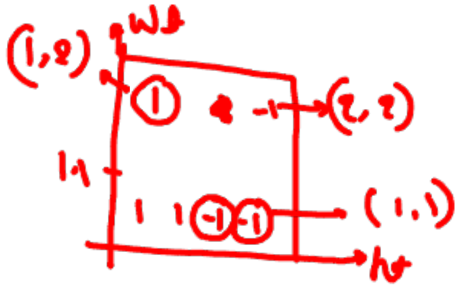
mean top = 0
bot = 0

∴ MSE = 1+1+1+1 = 4

∵ both cuts give equal error, we can choose either

Q2)





$$L = \frac{\sum_i w_i \, \mathbb{I}(y_i \neq h(x_i))}{\sum w_i}$$

$L = 3/6$

1

$w_i = 1/6$

For left, $L = 2/6$ 1

circled points are error

$h_1(x)$ will be cut across ht.

$$\alpha_1 = \frac{1}{2}\log\frac{1\cdot L}{L} = \frac{1}{2}\log\frac{1-1/3}{1/3} = \frac{1}{2}\log 2$$ .5

Updated weights for circled points.

$$\omega_{new} = \frac{1}{6}e^{2\alpha_1} = \frac{1}{6}e^{\log 2} = \frac{1}{3}$$ .5

(Taking log base e)

Since the cuts are at same points.

error for cut at ht.

$$= \frac{2\cdot 1/3}{2\cdot 1/3 + 4\cdot 1/6}$$

{two error points have weight: $\frac{1}{3}$
reds have weight: $1/6$}

1

$$= \frac{2/3}{8/6} = \frac{1}{2}$$

$\text{Error for cut at } w_1: \dfrac{\frac{1}{3}+\frac{1}{3}+1/6}{4/3} = \dfrac{1}{2}$ 1

Since errors if same, take $h_1(x)$

Boosted, $f(x) = \text{sign}(\alpha_1 h_1 + \alpha_2 h_2)$

$\alpha_2 = \frac{1}{2} \log 1 = 0$

$f(x) = \text{sign}(\alpha_1 h_1) \equiv \text{sign}(h_1(x))$

$f((x=3,0)) = -1$ 1

Q3)

~~Q4)~~ Since this is binary, we can model

$P(x|Gui)$ & $P(x|Mum)$ as multivariate Bernoulli

for Gui, so θ_1 be $P(x_1=1) = \dfrac{\sum_i x_i}{n} = \dfrac{1+0+1+0+0}{5}$

θ_2 be $P(x_2=1) = \dfrac{0+1+1+0+1}{5} = \dfrac{3}{5}$ .5

for Mum

$\theta_1 = \dfrac{0+1+1+1+0}{5} = \dfrac{3}{5}$

$\theta_2 = 2/5$ .5

$$P(X|Gw) = \theta_1^{x_1}(1-\theta_1)^{1-x_1}\,\theta_2^{x_2}(1-\theta_2)^{1-x_2}$$
$$= \left(\frac{2}{5}\right)^{x_1}\left(\frac{3}{5}\right)^{1-x_1}\left(\frac{3}{5}\right)^{x_2}\left(\frac{2}{5}\right)^{1-x_2}$$
$$= \left(\frac{2}{5}\right)^{x_1+1-x_2}\left(\frac{3}{5}\right)^{1-x_1+x_2}$$

.5

$$P(x|Mum) = \left(\frac{3}{5}\right)^{x_1}\left(\frac{2}{5}\right)^{1-x_2}\left(\frac{2}{5}\right)^{x_2}\left(\frac{3}{5}\right)^{1-x_2}$$
$$= \left(\frac{3}{5}\right)^{x_1+1-x_2}\left(\frac{2}{5}\right)^{1-x_1+x_2}$$

.5

Since other conditions are equal, that is equal prior

$$P(x|Gw) = \left(\frac{2}{5}\right)^{1+1-0}\left(\frac{3}{5}\right)^{1-1+0} = \left(\frac{2}{5}\right)^{4}$$
$$P(x|Mum) = \left(\frac{3}{5}\right)^{1+1-0}\left(\frac{2}{5}\right)^{1-1+0} = \left(\frac{3}{5}\right)^{4}$$

1

Since ~~P(Gw)~~ priors are equal, we can use likelihood.
P(X|M) is large & will win.

Q4) Class 1 1 $w_1=1, w_2=0, w_3=-1$

x_1	x_2	x_3	$y = w_1x_1+w_2x_2+w_3x_3$
1	0	-1	2
0	0	0	0
-1	1	1	-2

class 2

x_1	x_2	x_3	o/p
1	1	0	1
0	1	0	0
0	0	0	0
-1	0	0	-1

b. $\mu_y = 0$ for both.

$\sigma_y^2 = \frac{1}{2}(2^2+2^2) = 4$ class 1

$\sigma_y^2 = 2/3$ 1

Prior for class 1 $= \frac{3}{7} = P(\text{class1})$

D.B. $P(X|\text{class1})P(\text{class1}) = P(X|\text{class2})P(\text{class2})$

$$\Rightarrow \frac{1}{\sqrt{2\pi\sigma_{y_1}^2}} e^{-y^2/2\sigma_{y_1}^2} \cdot \frac{3}{7} = \frac{1}{\sqrt{2\pi\sigma_{y_2}^2}} e^{-y^2/2\sigma_{y_2}^2} \cdot \frac{4}{7}$$ .5

$$= \frac{1}{\sqrt{2\pi\cdot 4}} e^{-y^2/2\cdot 4} \cdot 3 = \frac{1}{\sqrt{2\pi\cdot\frac{2}{3}}} e^{-y^2/2\cdot 2/3} \cdot 4$$

$$= \frac{3}{8}\cdot\sqrt{\frac{2}{3}} = e^{-y^2\left[\frac{3}{4}-\frac{1}{8}\right]} = e^{-y^2 5/8}$$

$$-y^2\sqrt{\frac{5}{8}} = \ln\left(\frac{7}{8}\sqrt{\frac{2}{3}}\right)$$

$$y = \pm\left\{-\sqrt{\frac{8}{5}}\ \ln\left(\frac{7}{8}\sqrt{2/3}\right)\right\}^{1/2}$$ 1.5

$$= \pm 1.2236$$