Q1. Discriminant $g_i(x) = -R(\alpha_i|x)$

$$= -\sum_{j=1}^{c} \lambda_{ij} P(\omega_j|x)$$

$$= -[\lambda_{i1} P(\omega_1|x) + \dots + \lambda_{ii} P(\omega_i|x) + \dots \lambda_{ic} P(\omega_c|x)]$$

$\because$ $\lambda_{ii} = 0$, & $\lambda_{ij} = 1$ for 0-1 loss.

$$g_i(x) = -[P(\omega_1|x) + \dots + P(\omega_c|x)] = -\sum_{j \neq i} P(\omega_j|x)$$

$$= -1 + P(\omega_i|x)$$

[0.5]

For generic loss:

$$g_i(x) = -\lambda_s \left[P(\omega_1|x) + \dots + P(\omega_c|x) \right]$$

$$= -\lambda_s \left(1 - P(\omega_i|x) \right)$$

[0.5]

Discriminant can be taken in log.

0-1 loss: $g_i(x) = \ln P(\omega_i|x)$

generic: $g_i(x) = \ln \left(\lambda_s P(\omega_i|x) \right) = \ln \lambda_s + \ln P(\omega_i|x)$

Since $\ln \lambda_s$ is constant for all classes, discriminants are same for both cases.

[0.5]

Q2. Compute Posteriors.

Decide class '$\cancel{i}$' 'k', when $P(\omega_k|X) > P(\omega_j|X)$ $\forall j \neq k$

$$P(\omega_1|X) = P(X|\omega_1)\,P(\omega_1)$$

~~As~~ priors are same, factor $P(\omega_1)$ won't play a role.

$$P(X|\omega_1) = \prod_{i=1}^{3} P_{ij}^{x_i}(1-P_{ij})^{1-x_i}, \quad \text{here 'j'} = 1$$

$\boxed{0.5}$

$$= \prod_{i=1}^{3} P_{i1}^{x_i}(1-P_{i1})^{1-x_i}$$

$$= P_{11}^{x_1}(1-P_{11})^{1-x_1}\,P_{21}^{x_2}(1-P_{21})^{1-x_2}\,P_{31}(1-P_{31})^{1-x_3}$$

$$= P_{11}^{x_1+x_2+x_3}(1-P_{11})^{3-x_1-x_2-x_3}$$

$$= (0.8)(0.2)^2 = \cancel{0.16}\ 0.032$$

for $P(\omega_2|x) = P_{12}^{x_1+x_2+x_3}(1-P_{12})^{3-x_1-x_2-x_3}$

$$= 0.6(0.4)^2 = 0.096$$

for $P(\omega_3|x) = \cancel{.9(.9)}\ .9(.1)^2 = .009$

$$x \in \omega_2$$

floor of x_0 is also fine and will give $x_0 = (0,0,0)$

$[1]$

Q3: Chernoff bound:

$$P(\text{error}) \leq P^{\beta}(\omega_1)\, P^{1-\beta}(\omega_2)\, e^{-k(\beta)}$$

$$k(\beta) = \frac{\beta(1-\beta)}{2}\, 2\mu \left[\beta\sigma^2 + (1-\beta)\sigma^2\right]^{-1} 2\mu + 0$$

$$= 2\beta(1-\beta)\,\mu^2/\sigma^2 \qquad [0.5]$$

for min/max of $k(\beta)$, $\frac{\partial k(\beta)}{\partial \beta} = 0 \Rightarrow \beta = 1/2$ $[0.5]$

$$P(\text{error}) \leq \sqrt{P(\omega_1)P(\omega_2)}\, e^{-k(1/2)} = \sqrt{P(\omega_1)P(\omega_2)}\, e^{-\mu^2/2\sigma^2} \quad [0.5]$$

Q4.) $X = \begin{bmatrix} a \\ b \end{bmatrix}$ $a \sim N(\mu, \sigma^2)$ $b \sim N(\mu, \sigma^2)$

$\because$ a & b are Gaussian & statistical ind.

$$P(X) = P(a)P(b)$$

$$= \frac{1}{2\pi\sigma^2} e^{\left\{-\frac{1}{2}(a-\mu)^2 - \frac{1}{2}(b-\mu)^2\right\}/\sigma^2}$$ [.5]

likelihood: $$\prod_{i=1}^{N} P(X_i) = \prod_{i=1}^{N} \frac{1}{(2\pi\sigma^2)} e^{-\frac{1}{2\sigma^2}\left\{a_i^2 - 2\mu(a_i+b_i) + \mu^2\right\}}$$ [.5]

$\because$ constant won't play a role in determining μ_{ML}, taking log-likelihood.

$$= -\frac{1}{2\sigma^2}\sum_{i=1}^{N} [a_i^2 - 2\mu(a_i+b_i) + \mu^2]$$

Taking derivative & setting to '0':

$$\sum_{i=1}^{N} -2(a_i+b_i) + 2\mu = 0$$

$$\mu_{ML} = \frac{1}{N}\sum_{i=1}^{N}(a_i+b_i)$$

[0.5]

Q5. $g_i(x) = \ln p(x \mid \mu_i, \Sigma) + \ln p(\mu_i \mid \mu_0, \Sigma_0)$

Since Σ is same & so is Σ_0

[0.5] $g_i(x) = -\frac{1}{2}\left\{(x-\mu_i)^T \Sigma^{-1} (x-\mu_i)\right\} - \frac{1}{2}\left\{(\mu_i-\mu_0)^T \Sigma_0^{-1} (\mu_i-\mu_0)\right\}$

$$= -\frac{1}{2}\left[\underbrace{x^T\Sigma^{-1}x} - 2\mu_i^T\Sigma^{-1}x + \mu_i^T\Sigma^{-1}\mu_i\right] - \frac{1}{2}\left[\mu_i^T\Sigma_0^{-1}\mu_i - 2\mu_0^T\Sigma_0^{-1}\mu_i + \underbrace{\mu_0^T\Sigma_0^{-1}\mu_0}\right]$$

$$= \mu_i^T\Sigma^{-1}x - \frac{1}{2}\mu_i^T\Sigma^{-1}\mu_i - \frac{1}{2}\mu_i^T\Sigma_0^{-1}\mu_i + \mu_0^T\Sigma_0^{-1}\mu_i$$

other terms are same for all classes

[0.5] $W = \Sigma^{-1}\mu_i$,