SML 2022, Monsoon, Quiz 1, Duration 1 hr 20 mins (inclusive of submission). Late submission will not be accepted. Quiz is open book.

Q1. Consider the two different losses: (i) zero/one loss where $\lambda_{ii} = 0$ and $\lambda_{ij} = 1$ and (ii) generic loss where $\lambda_{ii} = 0$ and $\lambda_{ij} = \lambda_s$. Will the discriminants under both cases be same? Justify your answer by showing all the steps involved. [1.5]

Q2. Let the components of the $d = 3$ dimensional vector $\mathbf{x} = (x_1, ..., x_3)^\top$ be binary valued (0 or 1) and $P(\omega_j)$ be the prior probability for the state of nature (class or category) ω_j and $j = 1, ..., c, c = 3$. Assume prior probabilities are same. Now define

$$p_{ij} = Pr[x_i = 1|\omega_j] \;\; i = 1, ..., d; \;\; j = 1, 2, ...c \tag{1}$$

with the components of x_i being statistically independent. $p_{i1} = .8 \; \forall \; i$, $p_{i2} = .6 \; \forall \; i$, $p_{i3} = .9 \; \forall \; i$. Find the class to which $\mathbf{x}_0 = (1/3, 1/3, 1/3)^\top$ belongs to. [1.5]

Q3. Determine the Chernoff bound for two category case where both the categories follow a Gaussian distribution. Fist category has mean $\mu_1 = -\mu$ and variance σ^2. Second category has mean $\mu_2 = +\mu$ and variance σ^2. Give the answer in terms of priors and μ. [1.5]

Q4. Consider the following distribution for a 2-d vector $\mathbf{x} = [a, b]^\top$. Both a and b are distributed according to Gaussian pdf with mean μ and variance σ^2, and are statistically independent. Find the likelihood and MLE estimate for μ using N iid observations of $\mathbf{x}$. [1.5]

Q5. A discriminant is expressed in terms of likelihood and prior. Consider a case where likelihood is expressed as multivariate Gaussian, $p(\mathbf{x}|\mu_i, \Sigma) \sim N(\mu_i, \Sigma)$, for classes $i = 1, 2, .., c$. Now let us modify this likelihood as,
$p(\mathbf{x}|\mu_i, \Sigma)p(\mu_i|\mu_0, \Sigma_0)$, where $p(\mu_i|\mu_0, \Sigma_0) \sim N(\mu_0, \Sigma_0)$. Consider
$p(\mathbf{x}|\mu_i, \Sigma)p(\mu_i|\mu_0, \Sigma_0)$ as the new likelihood and find the discriminant assuming equal priors. Note that the distribution of mean μ_i is same across all classes. You must express the discriminant as $W^\top \mathbf{x} + b$ and remove the terms that do not impact discriminant for different classes. [1]