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Assigment - Parameter Estimation
Subject - UCS654

Q1 Let $(x_1, x_2 - - -)$ be a random sample of size n taken from a normal population with parameter mean $= \theta_1$ and variance θ_2.
Find the MLE of these two parameters.

Sol

$\theta_1 =$ mean
$\theta_2 -$ Variance

$$PDF = f(x) = \frac{1}{\sqrt{\theta_2 2\pi}} e^{-1/2 \left(\frac{x_1 - \theta_1}{\sqrt{\theta_2}}\right)^2}$$

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-1/2 \left(\frac{x_i - \theta_1}{\theta_2}\right)^2}$$

Take log on both sides

$$\log(L) = \log\left(\sqrt{2\pi\theta_2}^{\,-n} \prod_{i=1}^{n} e^{-1/2 \left(\frac{x_i - \theta_1}{\theta_2}\right)^2}\right)$$

$$= -\frac{n}{2}\log(2\pi\theta_2) + \left(\frac{-1}{2\theta_2}\right) \sum_{i=1}^{n} (x_i - \theta_1)^2$$

$\log(AB) = \log A + \log B$

Differentiate on both sides w.r.t θ_1

$$\frac{\partial L}{\partial \theta_1} = \frac{-1}{2\theta_2} \sum_{i=1}^{n} 2(x_i - \theta_1)(-1)$$

$$= \frac{1}{2\theta_2} \sum_{i=1}^{n} 2(x_i - \theta_1)$$

Equate $\frac{\partial L}{\partial \theta_1} = 0$

Either $L = 0$ or $\frac{1}{2\theta_2} \sum_{i=1}^{n} 2(x_i - \theta_1) = 0$

$L = 0$ can't be possible as probability density of already happened can't be 0

$$\frac{-1}{2\theta_2} \sum_{i=1}^{n} 2(\theta_1 - x_i) = 0$$

$$\sum_{i=1} 2\theta_1 = \sum_{i=1}^{n} 2x_i$$

$$n\theta_1 = \sum_{i=1}^{n} x_i$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\theta_1 = \frac{\text{Sample}}{\text{mean}}$$

Differentiate L wrt θ_2

$$\frac{\partial L}{\partial \theta_2}\left(\frac{1}{L}\right) = \frac{-n}{2} \frac{2\pi}{2\pi\theta_2} + \sum_{i=1} (x_i - \theta_1)^2 \frac{1}{2\theta_2^2}$$

Put $\frac{\partial L}{\partial \theta_2} = 0$

L can't be zero

$$\frac{-n}{2\theta_2} + \sum_{i=1}^{n} (x_i - \theta_1)^2 \frac{1}{2\theta_2^2} = 0$$

$$\sum_{i=1}^{n} (x_i - \theta_1)^2 = n\theta_2$$

$$\theta_2 = \frac{\sum (x_i - \theta_1)^2}{n}$$

sample variance

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Q2 Let $X_1, X_2 \dots X_n$ be a random sample from $B(m, \theta)$ distribution where $\theta \in (0,1)$ is unknown and m is a known positive integer compute value of θ using the MLE

Sol PMF of $B(M,\theta)$

$$P(X=K) = {}^{m}C_{k}\, \theta^{k} (1-\theta)^{m-k}$$

Let $X_1 \dots X_n$ be random sample from $B(m,\theta)$ dist where for a X_i, it represent number of successes in ith trial of experiment

So likelihood function becomes

$$l(\theta) = \prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking log on both sides

$$\log L = \log\left(\prod_{i=1}^{n} {}^{m}C_{x_i}\, \theta^{x_i} (1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n}\left[\log {}^{m}C_{x_i} + x_i \log\theta + (m-x_i)\log(1-\theta)\right]$$

Performing differenciation wrt θ

$$\frac{1}{L}\frac{dL}{d\theta} = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n}(m-x_i)$$

$$\frac{dL}{d\theta} = 0$$

$$L = \frac{1}{\theta}\sum_{i=1}^{n} x_i - \frac{1}{1-\theta}\sum_{i=1}^{n} m-x_i) = 0$$

L cant be zero

$$\frac{1}{\theta}\sum_{i=1}^{n} x_i = \frac{1}{1-\theta}\sum_{i=1}^{n} m-x_i$$

$$1-\theta \sum_{i=1} x_i = \theta n m - \theta\sum_{i=1}^{n} x_i$$

$$\theta = \frac{\sum x_i}{nm} \quad \text{i goes from 1 to n}$$

$$\theta = \frac{\text{Sample mean}}{m}$$