1

INTRODUCTION TO CONTROL SYSTEMS

1.1 INTRODUCTION

Control systems are an integral part of modern society. They play a vital role in our day-to-day life. Control systems find applications in manufacturing process industries, satellites, guided missiles, navigation, biomedical engineering etc. The study of control is not only concerned with engineering applications but extends to other areas like economics, business, political systems and so on.

Automatic control systems also exist in nature. Within our own bodies there are numerous control systems.

1.2 BASIC DEFINITIONS

System

An arrangement or combination of different physical components that are connected or related together to form an entire unit to achieve a certain objective is called a system.

Control

The meaning of control is to regulate, direct or command a system so that a desired objective is obtained.

Control System

A control system is an arrangement of physical components connected or related in such a manner as to command, direct or regulate itself or another system to obtain a certain objective.

Input

The excitation or stimulus applied to a control system from an external energy source is usually known as input.

Output

The actual response that is obtained from a control system due to the application of the input is called output.

Plant or Process

It is defined as the portion of a system which is to be controlled or regulated. It is also called process.

Controller

It is an element within the system itself, or external to the system, and it controls the plant or the process.

In its simplest form, a control system is shown in Fig. 1.1.

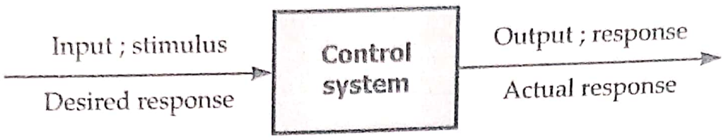


Fig. 1.1

The input variable is generally known as **reference input** and output is generally called **controlled output.**

Disturbances

Disturbance is a signal which tends to adversely affect the value of the output of the system. If such a disturbance is generated within the system itself, it is called an internal **disturbance.** The system generated outside the system acting as an extra input to the system in addition to its normal input, affecting the output adversely is called the external **disturbance.**

Automatic Control System

An automatic control or automatic regulator system is one that maintains the actual output at the desired value in the presence of disturbance, or for a slowly varying reference input.

Process Control System

An automatic control system in which the output is a variable such as temperature, pressure, flow, liquid level, is called a process control system.

Transducer

A transducer is a device which converts a signal from one form to another. Usually, the transducers employed in control systems convert a signal from any form to electrical form. This is because the electrical signal can be handled, amplified and transmitted easily.

1.3 CONTROL SYSTEM CONFIGURATIONS

There are *two* major configurations of control systems :

1. Open-Loop System
2. Closed-Loop System

1.3.1 Open-Loop System

An open-loop control system is one in which the output is dependent on input, but controlling action or input is totally independent of the output or changes in output of the system.

The open-loop control systems represent the simplest form of controlling devices.

The open-loop system is also called the **nonfeedback system.**

Figure 1.2 shows an open-loop system. Reference input $r(t)$ is applied to the controller which generates the actuating signal $u(t)$ required to control the process which is to be controlled. Process gives the desired controlled output $c(t)$.

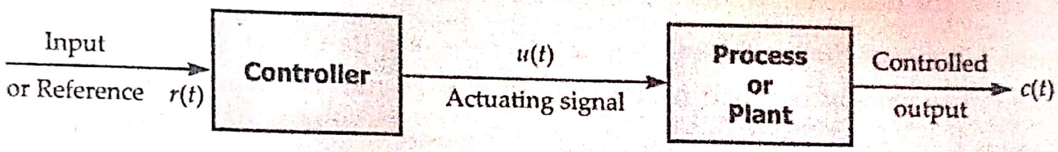


Fig. 1.2 The basic open-loop control system

For a given input the system produces a certain output. If there are any disturbances, the output changes and there is no adjustment of the input to bring back the output to the original value. A perfect calibration is required to get accuracy. No measurements are made at the output.

Some examples of open-loop systems are bread toaster, an automatic washing machine, electric lift, traffic signals etc.

Advantages of Open-Loop Systems

1. Open-Loop systems are simple in construction and design.
2. These systems are economical.
3. These systems are easy from maintenance point of view.
4. There is no stability problem. Generally they are stable.

Disadvantages of Open-Loop Systems

1. Open-Loop systems are less accurate and unreliable because accuracy of such systems is dependent on the controller.
2. If there are any disturbances, the output changes and there is no adjustment of the input to bring back the input to the original value.
3. Recalibration of the controller is required from time to time for maintaining quality and accuracy.

1.3.2 Closed-Loop System

A system in which the controlling action or input is somehow dependent on the output or changes in output is called **closed-loop system.**

In order to have dependence of input on the output, a closed-loop system uses the feedback property.

A system which maintains a prescribed relationship between the controlled variable and the reference input, and uses the difference between them as a signal to activate the control, is known as a **feedback control system.** In such a system, output or part of the output is feedback to the input for comparison with the reference input and an actuating signal is generated.

Closed-loop control systems are also called **feedback control systems.**

The disadvantages of open-loop systems, namely sensitivity to disturbances and inability to correct for these disturbances, may be overcome in closed-loop systems.

Figure 1.3 shows the block diagram of a closed-loop (feedback control) system. The input to the entire system is called **reference input** or **command input,** $r(t)$.

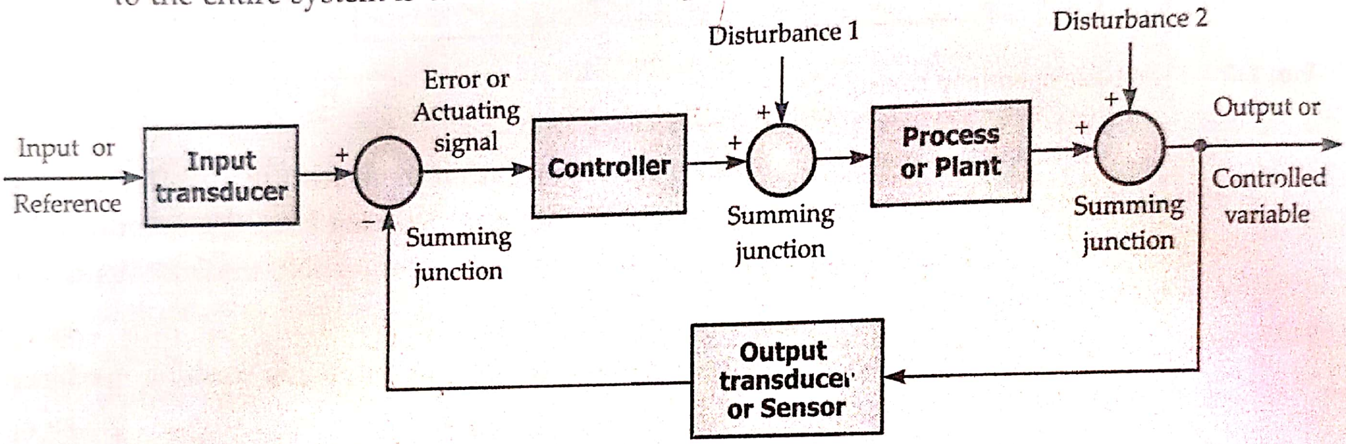


Fig. 1.3 Block diagram of a closed-loop system

The **input transducer** converts the form of the input to the form used by the controller. An **output transducer,** or **sensor** measures the output response and converts it into the form used by the controller.

The first summing junction algebraically adds the signal from the input to the signal from the output, which arrives via the **feedback path.**

The feedback path is the return path from the output to the summing junction.

In Fig. 1.3, the output signal is subtracted from the input signal. The result is generally called the **actuating signal.**

However, in systems where both input and output transducers have *unity gain* (that is, the transducer amplifies its input by 1), the actuating signal value is equal to the actual difference between the input and the output. Under this condition, the actuating signal is called the **error.** The actuating signal is the input to the controller.

The closed-loop system compensates for disturbance by measuring the system response, feeding that measurement back through a feedback path, and comparing that response to the input at the summing junction.

If there is any difference between the two responses, the system drives the plant, via the actuating signal, to make a correction. If there is no difference, the system does not drive the plant, since the plant's response is already the desired response.

Thus, closed-loop systems monitor the output and compare it to the input. If an error is detected, the system corrects the output and hence corrects the effects of disturbances.

Advantages of Closed-Loop Systems

1. Closed-loop systems are more accurate than open-loop systems because of the presence of feedback.
2. Closed-loop systems reduce the effect of noise and disturbance on the system performance.
3. The sensitivity of the closed-loop systems for parameter variations is made small by increasing the feedback loop gain.
4. The range of frequencies over which the system responds is increased because of the increased bandwidth.
5. If an open-loop system is unstable, it is possible to make the system stable by providing feedback.
6. There is reduced effect of nonlinearities in these systems.

Disadvantages of Closed-Loop Systems

1. Closed-loop systems are more complex and costlier than open-loop systems because of additional components requried for providing feedback.
2. Feedback can be harmful to stability if it is not properly applied. The system tries to correct the error time to time. Tendency to overcorrect the error may cause oscillations in the system.

However, the advantages of feedback far outweigh the disadvantages.

1.4 FEEDBACK CONTROL

The **feedback control** is an operation in which the output is sampled and a proportional signal is fedback to the input. The feedback output may be in phase or out-of-phase with respect to the input. When the feedback output is in phase with the input it is termed *positive*.

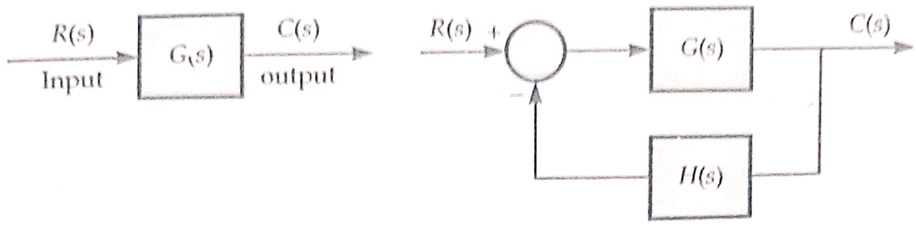
When the feedback output is out-of-phase with respect to the input it is termed *negative*.

All control systems are usually negative feedback systems. In a negative feedback system, the difference between the reference input and the output produces an error which is reduced gradually and bring the output of the system to a desired level.

The **positive feedback** output gets added to the reference input and increases the error signal and drives the output to instability, but sometimes positive feedback is used in minor loops in control systems to amplify certain internal signals or parameters.

1.5 EFFECTS OF FEEDBACK

The error between the system input and output can be reduced by using a feedback system as shown in Fig. 1.4.



(a) Open loop

(b) Closed loop

Fig. 1.4

The effects of feedback are as follows :

1. Effect of Feedback on Overall Gain

Consider an open-loop system with overall gain $G(s)$ as shown in Fig. 1.4.

If the feedback with transfer function $H(s)$ is introduced in such a system, the overall gain with negative feedback becomes $\frac{G(s)}{1+G(s)\,H(s)}$.

Therefore, for a negative feedback the gain $G(s)$ is reduced by a factor $\frac{1}{1+G(s)\,H(s)}$.

However, gain can be increased by gain amplification.

2. Effect of Feedback on Stability

The feedback can improve stability or be harmful to stability if it is not properly applied.

3. Effect of Feedback on Sensitivity

In general, a good control system should be very insensitive to parameter variations but sensitive to input commands. Feedback can increase or decrease the sensitivity of the system.

4. Effect of Feedback on External Disturbance or Noise

All physical systems are subject to some external disturbance signals or noise during operation.

Therefore, control systems should be designed so that they are insensitive to noise and disturbance and sensitive to input commands. In general, feedback can reduce the effects of noise and disturbance on system performance.

5. Effect of Feedback on Bandwidth, Impedance, Transient Response and Frequency Response

In general, feedback also has effects on such performance characteristics as bandwidth, impedance, transient response and frequency response.

Thus, the presence of feedback imparts in a system the following most important features :

1. Increases accuracy, by reducing the steady-state error in tracking a reference.
2. Reduces sensitivity to parameter variations, that is, makes system insensitive to parameter variations.
3. Reduces the effects of nonlinearities and distortion.
4. Increases bandwidth.
5. Reduces the effects of noise and disturbance on the system performance.
6. Stabilizes an unstable system.

2

TRANSFER FUNCTION

2.1 INTRODUCTION

The **transfer function** is defined as the ratio of the Laplace transform of the output response (effect) to the Laplace transform of input (excitation) provided that all the initial conditions (ICs) are zero.

If $G(s)$ is the transfer function of the system, we can write mathematically

$$G(s) \triangleq \left.\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}\right|_{\substack{\text{all initial conditions}\\ \text{are zero}}} \quad \text{...(2.1.1)}$$

$$G(s) \triangleq \left.\frac{C(s)}{R(s)}\right|_{\substack{\text{all initial conditions}\\ \text{are zero}}}$$

The transfer function can be represented as a block diagram as shown in Fig. 2.1, with the input on the left and the output on the right and the system transfer function inside the block.

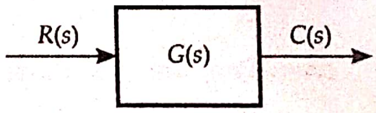


Fig. 2.1 Block diagram of a linear system.

The input may be regarded as the "**cause**" and the output as the "**effect**". The block diagram is "unidirectional" since the "effect" cannot produce the "**cause**".

2.2 PROCEDURE FOR FINDING TRANSFER FUNCTIONS OF ELECTRIC NETWORKS

The following procedure is used to find transfer function of an electric network :

1. Draw the given network in the s domain with each inductance L replaced by sL and each capacitance by $\frac{1}{sC}$.
2. Replace all sources and time variables with their Laplace transforms. That is, replace $v(t)$ and $i(t)$ by their Laplace transforms $V(s)$ and $I(s)$ respectively.
3. Use KCL, KCL, mesh analysis, node analysis to write network equations.
4. Solve the simultaneous equations for the output.
5. Form the transfer function.

Example 2.1 *Determine the transfer function of the phase lag network shown in Fig. 2.2(a).*

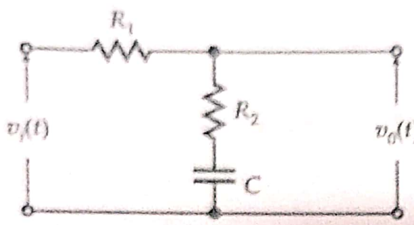


Fig. 2.2 (a)

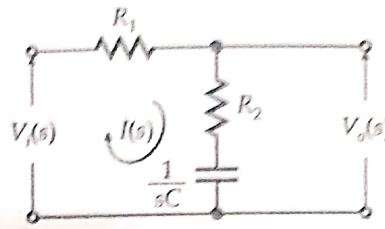


Fig. 2.2 (b)

Solution. Figure 2.2(*b*) shows the network in *s*-domain.

By KVL in the left-hand mesh,

$$V_i(s) = R_1 I(s) + R_2 I(s) + \frac{1}{sC} I(s)$$

$$V_i(s) = \left[\frac{(R_1 + R_2)sC + 1}{sC}\right] I(s)$$

By KVL in the right-hand mesh,

$$V_o(s) = R_2 I(s) + \frac{1}{sC} I(s)$$

$$V_o(s) = \left[\frac{(R_2 Cs + 1)}{sC}\right] I(s)$$

Therefore, the transfer function is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2\, Cs + 1}{(R_1 + R_2)Cs + 1}$$

or

$$\frac{V_o(s)}{V_i(s)} = \frac{1 + T_1 s}{1 + T_2 s}$$

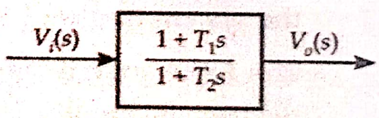


Fig. 2.2 (c)

where $T_1 = R_2 C$ and $T_2 = (R_1 + R_2)C$.

The block diagram representation is shown in Fig. 2.2(c).

Example 2.2 *Determine the transfer function for the simple lag network shown in Fig. 2.3(a), assuming no external load.*

Solution. Using the differential equation method and applying Kirchhoff's voltage law,

$$v_i(t) = Ri + v_0(t) \qquad \text{...(E2.2.1)}$$

$$v_r(t) = \frac{1}{C}\int i\, dt \qquad \text{...(E2.2.2)}$$

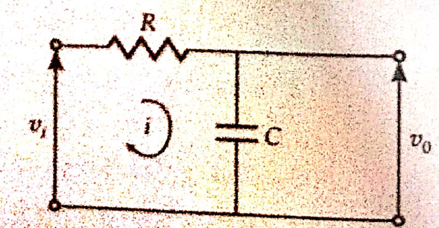


Fig. 2.3 (a)

Taking the Laplace transform on both sides of these two equations and assuming zero initial conditions

$$V_i(s) = RI(s) + V_o(s) \qquad \text{...(E2.2.3)}$$

$$V_o(s) = \frac{1}{sC} I(s) \qquad \text{...(E2.2.4)}$$

Eliminating $I(s)$ between equations (E2.2.3) and (E2.2.4), the transfer function of the network is given by

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC} = \frac{1}{1+sT} \qquad \text{...(E2.2.5)}$$

where $T = RC$

ALTERNATIVE METHOD

The s-domain network of Fig. 2.3(*a*) is shown in Fig. 2.3(*b*). By KVL in the left-hand mesh of Fig. 2.3(*b*),

$$V_i(s) = RI(s) + \frac{1}{sC} I(s)$$

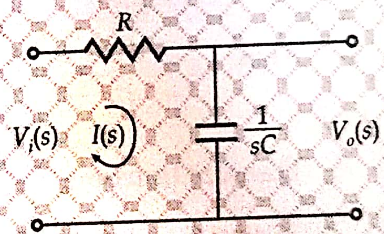


Fig. 2.3 (*b*)

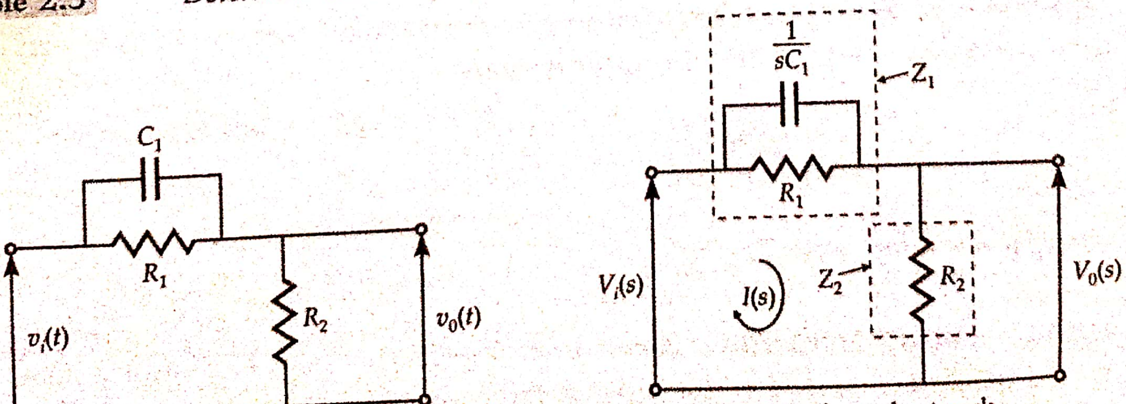
By KVL in the right-hand mesh

$$V_o(s) = \frac{1}{sC} I(s)$$

Therefore,

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1+sRC}$$

Example 2.3 *Determine the transfer function of the network shown in Fig. 2.4(a).*



(*a*) Original network

(*b*) Transformed network

Fig. 2.4

Solution. The transformed network of Fig. 2.4(a) is shown in Fig. 2.4(b).

From Fig. 2.4(b),

$$Z_1(s) = R_1 \parallel \frac{1}{sC_1} = \frac{R_1 \cdot \dfrac{1}{sC_1}}{R_1 + \dfrac{1}{sC_1}} = \frac{R_1}{sR_1C_1 + 1}$$

$$Z_2(s) = R_2$$

Let $I(s)$ be the mesh current. By KVL in the left-hand mesh

$$V_i(s) = Z_1(s)\,I(s) + Z_2(s)\,I(s)$$

$$= \frac{R_1}{s\,R_1C_1 + 1} I(s) + R_2 I(s)$$

$$= \left[\frac{R_1}{sR_1C_1 + 1} + R_2\right] I(s) \qquad \text{...(E2.3.1)}$$

Also,

$$V_o(s) = R_2 I(s) \qquad \text{...(E2.3.2)}$$

Transfer function,

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 I(s)}{\left[\dfrac{R_1}{sR_1C_1 + 1} + R_2\right] I(s)}$$

or

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2(1 + sR_1C_1)}{R_1 + R_2 + s\,R_1R_2C} \qquad \text{...(E2.3.3)}$$

Equation (E2.3.3) is often rearranged to give the standard form of a phase lead network used in compensation work for control systems.

$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1 + sT)}{1 + s\alpha T} \qquad \text{...(E2.3.4)}$$

where $\alpha = \dfrac{R_2}{R_1 + R_2}$ and $T = R_1C_1$

It is to be noted that the simple circuit when $R_1 = 0$ is not used in control systems since it would block d.c. signals and could not therefore be used in the forward path of the systems.

Example 2.4 *Determine the transfer function* $\frac{V_o(s)}{V_i(s)}$ *of the RC network of the lag-lead compensator shown in Fig. 2.5(a).*

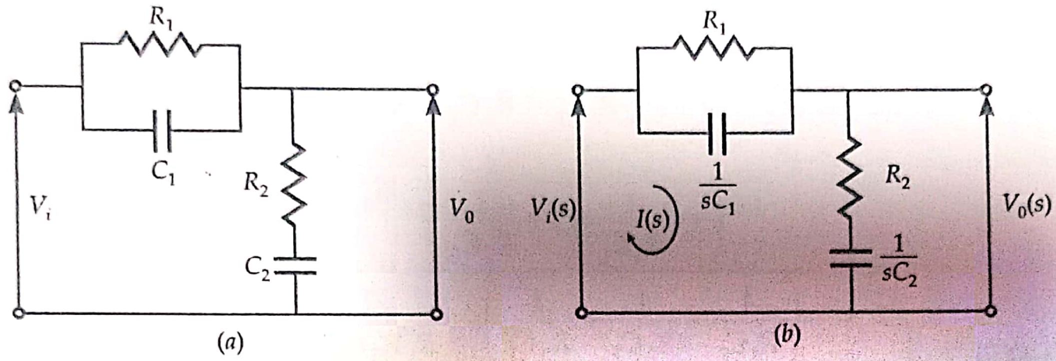


Fig. 2.5

Solution. The transform network of Fig. 2.5(*a*) is shown in Fig. 2.5(*b*). By KVL in the left-hand mesh of Fig. 2.5(*b*),

$$V_i(s) = \left[\left(R_1 \,||\, \frac{1}{sC_1}\right) + \left(R_2 + \frac{1}{sC_2}\right)\right] I(s)$$

$$= \left[\frac{\frac{R_1}{sC_1}}{R_1 + \frac{1}{sC_1}} + \left(R_2 + \frac{1}{sC_2}\right)\right] I(s)$$

$$= \left[\frac{R_1}{R_1 C_1 s + 1} + R_2 + \frac{1}{sC_2}\right] I(s)$$

Also, $$V_o(s) = \left(R_2 + \frac{1}{sC_2}\right) I(s)$$

Therefore,

$$\frac{V_o(s)}{V_i(s)} = \left(R_2 + \frac{1}{sC_2}\right) \div \left(\frac{R_1}{R_1 C_1 s + 1} + R_2 + \frac{1}{sC_2}\right)$$

$$= \frac{R_2 C_2 s + 1}{sC_2} \div \frac{R_1 C_2 s + R_2 C_2 s + R_1 R_2 C_1 C_2 s^2 + 1 + R_1 C_1 s}{(R_1 C_1 s + 1) sC_2}$$

$$= \frac{(R_1 C_1 s+1)(R_2 C_2 s+1)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2)s + 1}$$

$$= \frac{R_1 R_2 C_1 C_2 \left(s+\frac{1}{R_1 C_1}\right)\left(s+\frac{1}{R_2 C_2}\right)}{R_1 R_2 C_1 C_2 \left[s^2 + \left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}+\frac{1}{R_2 C_1}\right)s + \frac{1}{R_1 R_2 C_1 C_2}\right]}$$

$$= \frac{\left(s+\frac{1}{R_1 C_1}\right)\left(s+\frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}+\frac{1}{R_2 C_1}\right)s + \frac{1}{R_1 R_2 C_1 C_2}}$$

Example 2.5 *Determine the voltage ratio transfer function* $\frac{V_o(s)}{V_i(s)}$ *for the ladder network shown in Fig. 2.6(a).*

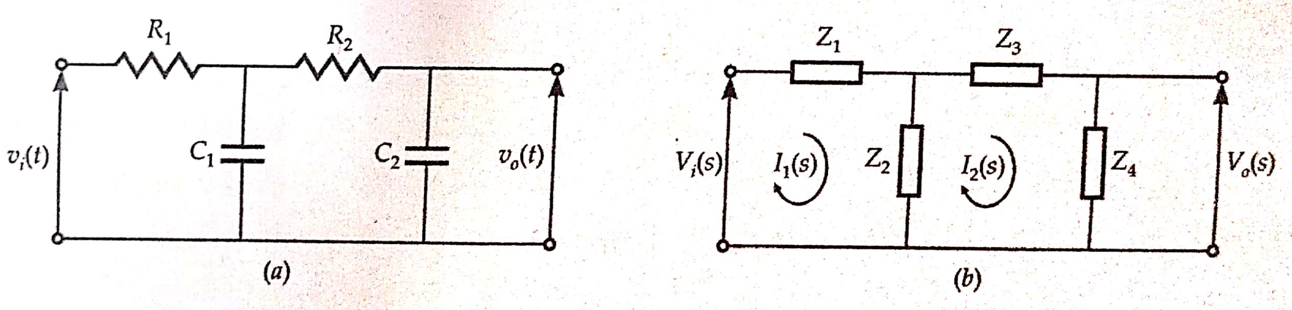


Fig. 2.6

Solution. Let us consider the general network shown in Fig. 2.6(*b*).

By KVL

$$V_i(s) = (Z_1 + Z_2) I_1(s) - Z_2 I_2(s)$$

$$0 = -Z_2 I_1(s) + (Z_2 + Z_3 + Z_4) I_2(s)$$

Elimination of $I_2(s)$ gives

$$\frac{V_i(s)}{I_2(s)} = \frac{[(Z_1 + Z_2)(Z_2 + Z_3 + Z_4) - Z_2^2]}{Z_2}$$

But $$V_o(s) = Z_4 I_2(s)$$

Therefore $$\frac{V_o(s)}{V_i(s)} = \frac{Z_4 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_1 Z_4 + Z_2 Z_3 + Z_2 Z_4}$$

Here

$$Z_1 = R_1,\ Z_3 = R_2,\ Z_2 = \frac{1}{sC_1},\ Z_4 = \frac{1}{sC_2}$$

Substitution of these values gives,

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2)s + 1}$$

EXERCISES

1. What do you mean by transfer function ?
2. Find the transfer function of the network shown in Fig. P2.1

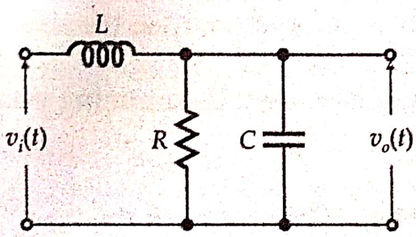


Fig. P2.1

[Ans. $\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + \frac{L}{R}s + 1}$]

3. Derive the transfer function of the network shown in Fig. P2.2

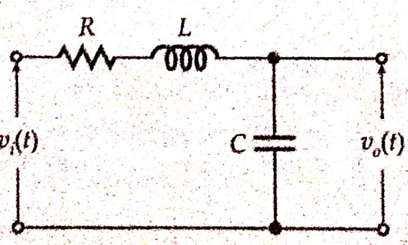


Fig. P2.1

[Ans. $\frac{V_o(s)}{V_i(s)} = \frac{1}{LCs^2 + RCs + 1}$