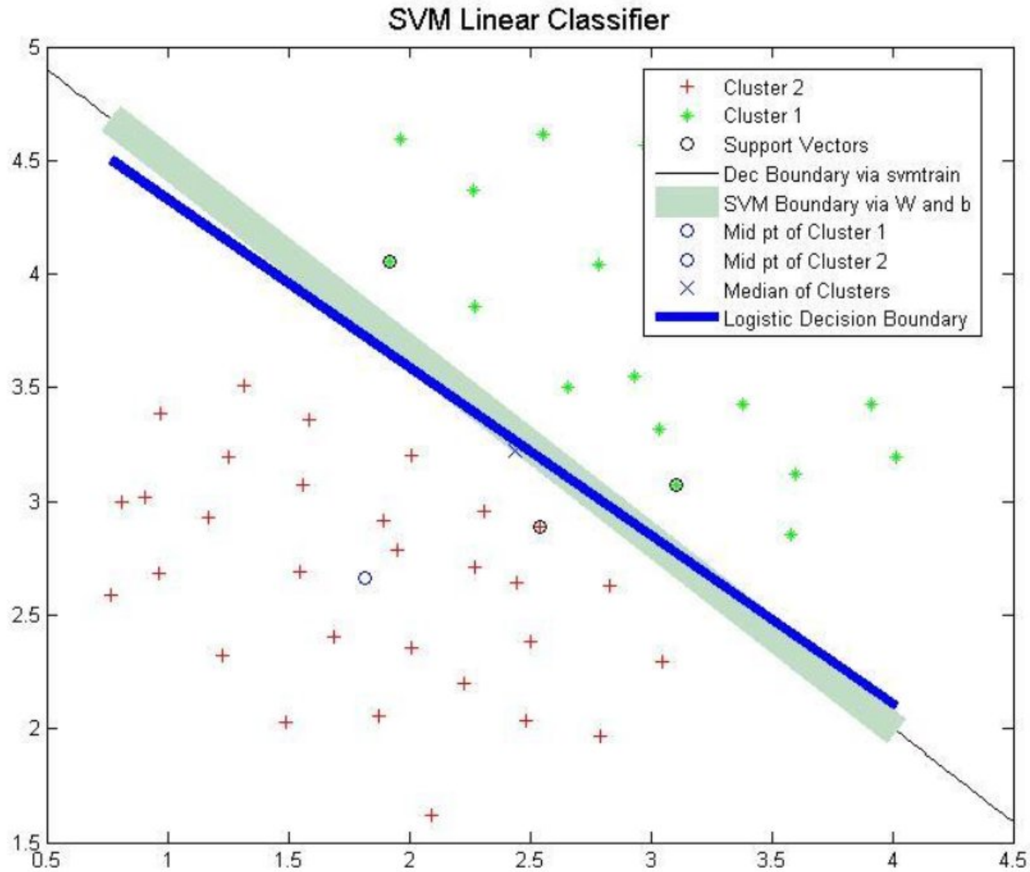
Assignment 4 – ECE 7650 Applied Computational Intelligence

Answer to Question Number 1

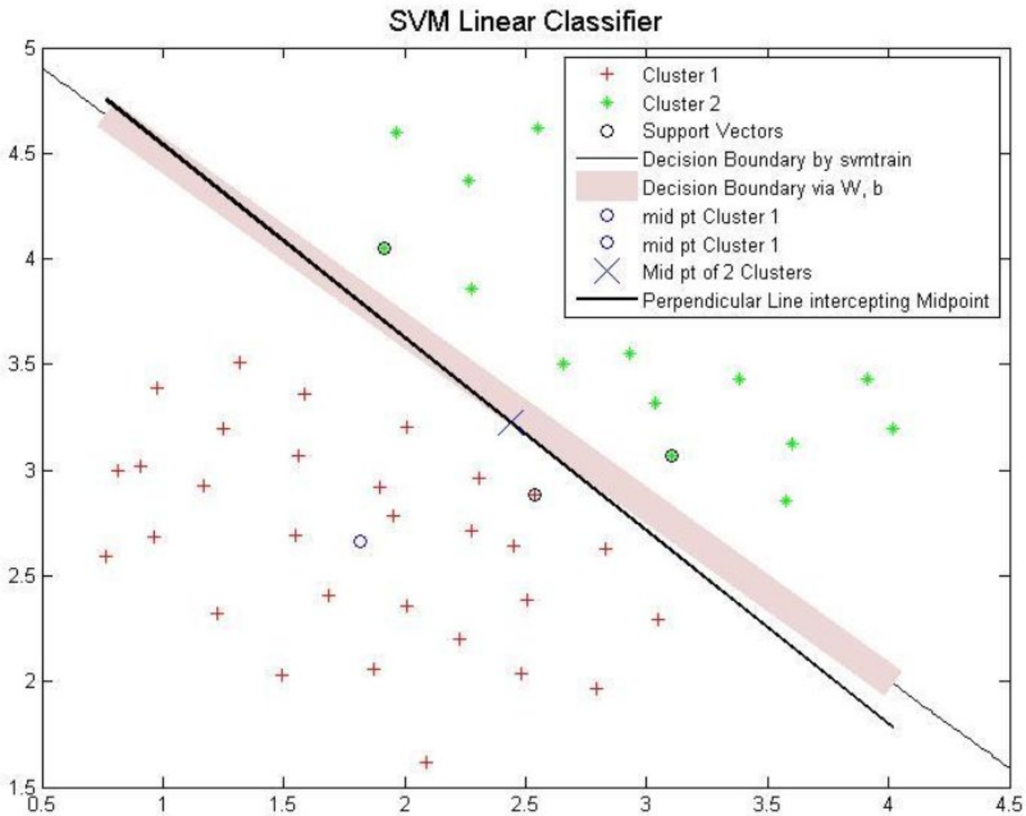


c. w = Support Vectors' * Weights = [-2.5403 -3.0680], b = bias = 16.3089

d.

SVM Theta: Theta = [-2.5403 -3.0680 16.3089] Elapsed Time : 2.733106 seconds. SVM has better error margin while having more computational time.	Logistic Regression Theta: Theta = [110.5019 16.0947 21.8199] Elapsed Time : 0.001570 seconds. Logistic Regression has lower error margin while being less computationally expensive.

e.



Midpoint of 2 Clusters does not intercept SVM Boundary by W and b.

f. Mean of Cluster 1 = (Sum of Vectors of Cluster 1) / Number of Feature Vectors of Cluster 1

Mean of Cluster 2 = (Sum of Vectors of Cluster 2) / Number of Feature Vectors of Cluster 2

Mid Point of Clusters = (Mean of Cluster 1 + Mean of Cluster 2) / 2

Gradient of Straight Line joining Mean points of Clusters = g1

Normal to Gradient of Line joining Mean points of Clusters = g2

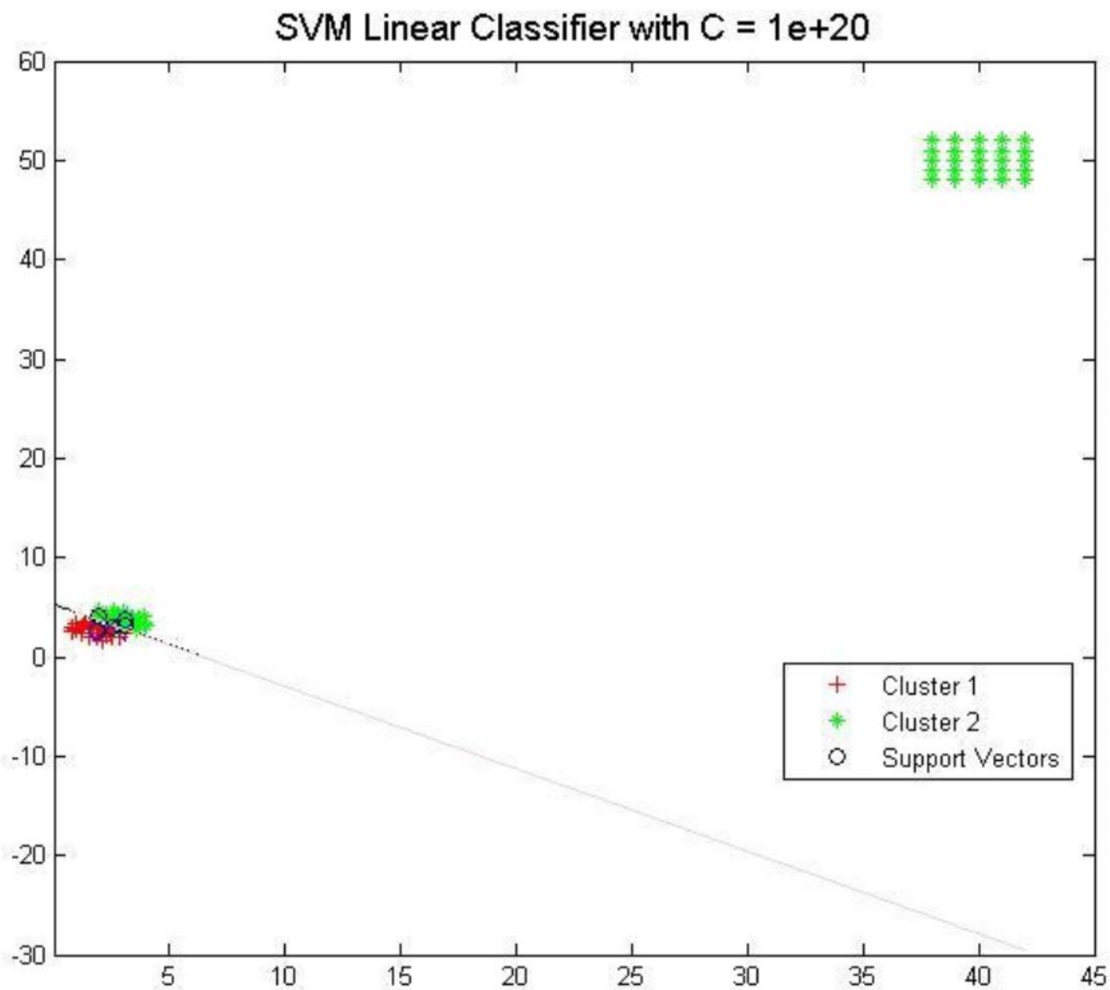
g1 * g2 = -1

g2 = -1/g1

Y – Y1 = m (X – X1), where m = g2;

Y = - 0.9145 * X + 5.4538

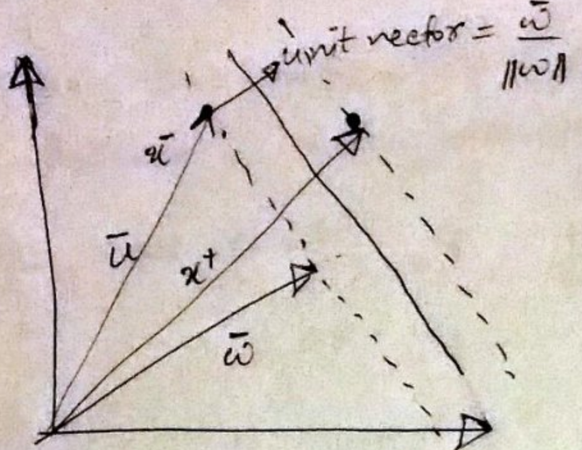
i.



With 1000 Training Sets, Theta = [-2.5403 -3.0680 16.3089]	With 50 Training Sets, Theta = = [-2.5403 -3.0680 16.3089]

Both Decision Boundary are the same. Decision Boundary depends on support vectors which lies nearest to Decision margin.

Answer to Question Number 2



$\bar{\omega} \cdot \bar{u} \geqslant c$

where $\bar{\omega}$ is a vector perpendicular to Decision Boundary

If projection $\bar{u}$ on $\bar{\omega}$ is greater than c,

ie. $\bar{\omega} \cdot \bar{u} - c \geqslant 0$, then it belongs to positive samples.

$\therefore$ $\bar{\omega} \cdot \bar{u} + b \geqslant 0$, is decision Rule.

where $b = -c$,

In taking a positive sample x_+,

$\bar{\omega} \cdot \bar{x}_+ + b \geqslant +1$

likewise, taking a negative sample x_-

$\bar{\omega} \cdot \bar{x}_- + b \leq -1$

For mathematical convenience,
introduce a new variable y_i,
y_i such that $y_i = +1$ for + samples
or $y_i = -1$ for − samples.

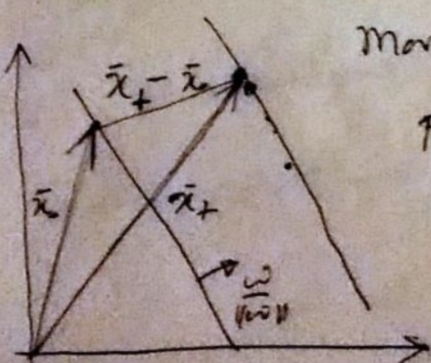
$\therefore \quad \bar{\omega} \cdot \bar{x}_+ + b \geq +1$

$\Rightarrow \quad y_i(\bar{\omega} \cdot \bar{x}_+ + b) \geq +1$, where $y_i = +1$

and $\quad \bar{\omega} \cdot \bar{x}\_ + b \leq -1$

$\Rightarrow \quad y_i(\bar{\omega} \cdot \bar{x}_+ + b) \geq +1$, where $y_i = -1$

In SVM, we try to maximize margins
as much as possible.



Margin width $= x_+ - x_- \cdot \frac{\omega}{\|\omega\|}$

For +ve samples, $y = +1$

$y_i(\bar{x}\bar{\omega} + b) - 1 = 0$

$\bar{x}\bar{\omega} = 1 - b$

For −ve samples, $y = -1$

$y_i(\bar{x}\bar{\omega} + b) - 1 = 0 \Rightarrow -\bar{x}\bar{\omega} = 1 + b$

$\therefore$ Margin width becomes

$$\max \frac{1-b+1+b}{\|w\|} = \frac{2}{\|w\|}$$

$\Rightarrow \min \|w\|$

For mathematical convenience

$\min \frac{1}{2}\|w\|^2$

$\therefore$ For at least 2 classes,

Support vector margin maximizes

margin width $\frac{2}{\|w\|}$,

or minimizes $\frac{1}{2}\|w\|^2$

with constraint $y_i(\bar{w}\cdot\bar{x}_i + b) - 1$