Question 1, Equation Derivation:

Forward Propagation :

$$z_i = \sum_{j=1}^{n} \theta_{ij}\, a_j + b_i$$

$$a_1(2) = w_{11}^{(1)} * x_1 + w_{12}(1) * x_2 + b_{11}(1)$$

$$a_2(2) = w_{21}(1) * x_1 + w_{22}(1) * x_2 + b_{21}(1)$$

$$h_1(2) = \frac{1}{1 + \exp(-a_1(2))}$$

$$h_2(2) = \frac{1}{1 + \exp(-a_2(2))}$$

$$a_1(3) = w_{11}(2) * h_1(2) + w_{12}(2) * h_2(2) + b_{11}(2)$$

$$h_1(3) = \frac{1}{1 + \exp(-a_1(3))}$$

$$del_1(3) = h_1(3) - y$$

# Backpropagation

$$\frac{\partial E^{(3)}}{\partial \Theta_{1j}^{(2)}} = \left(a_1^{(3)} - y^{(t)}\right) a_1^{(3)} \left(1 - a_1^{(3)}\right) a_j^{(2)}$$

$$= \frac{\partial E^{(3)}}{\partial a_1^{(3)}} \cdot \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \cdot a_j^{(2)} = \delta_1^{(3)} \cdot a_j^{(2)}$$

$$\frac{\partial E^{(3)}}{\partial b_1^{(2)}} = \left(a_1^{(3)} - y^{(t)}\right) \cdot a_1^{(3)} \cdot \left(1 - a_1^{(3)}\right)$$

$$= \frac{\partial E^{(3)}}{\partial a_1^{(3)}} \cdot \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} = \delta_1^{(3)}$$

$$\frac{\partial E^{(2)}}{\partial \Theta_{1j}^{(1)}} = \frac{\partial E^{(3)}}{\partial a_1^{(3)}} \frac{\partial a_1^{(3)}}{\partial z_1^{(3)}} \frac{\partial z_1^{(3)}}{\partial a_1^{(2)}} \frac{\partial a_1^{(2)}}{\partial z_1^{(2)}} \frac{\partial z_1^{(2)}}{\partial \Theta_{1j}^{(1)}}$$

$$= \delta_1^{(3)} \cdot \Theta_{11}^{(2)} \cdot a_1^{(2)} \left(1 - a_1^{(2)}\right) * a_j^{(1)}$$

$$\frac{\partial E^{(2)}}{\partial b_i^{(1)}} = \delta_1^{(3)} \cdot \Theta_{1j}^{(2)} a_i^{(2)} \cdot \left(1 - a_i^{(2)}\right)$$

## Output layer

$$w_{11}(2) = w_{11}(2) - 0.007 * (h_1(3) - y) * h_1(2)$$

$$w_{12}(2) = w_{12}(2) - 0.007 * (h_1(3) - y) * h_2(2)$$

$$b_{11}(2) = b_{11}(2) - 0.007 * (h_1(3) - y)$$

## Hidden layer

$$w_{11}(1) = w_{11}(1) - 0.007 * (h_1(3) - y) * w_{11}(2) * h_1(2) * (1 - h_1(2)) * x_1$$

$$w_{12}(1) = w_{12}(1) - 0.007 * (h_1(3) - y) * w_{11}(2) * h_1(2) * (1 - h_1(2)) * x_2$$

$$b_{11}(1) = b_{11}(1) - 0.007 * (h_1(3) - y) * w_{11}(2) * h_1(2) * (1 - h_1(2))$$

$$w_{21}(1) = w_{21}(1) - 0.007 * (h_1(3) - y) * w_{12}(2) * h_2(2) * (1 - h_2(2)) * x_1$$

$$w_{22}(1) = w_{22}(1) - 0.007 * (h_1(3) - y) * w_{12}(2) * h_2(2) * (1 - h_2(2)) * x_2$$

$$b_{21}(1) = b_{21}(1) - 0.007 * (h_1(3) - y) * w_{12}(2) * h_2(2)(1 - h(2))$$

# Question 2, Comparison Between Hand Written Calculation and Matlab Calculation

Random Data Weights:

W11(1) = 9, W12(1) = 7, b11(1) = -6,

W21(1) = 3, W22(1) = 1, b21(1) = -2,

W11(2) = -8, W12(2) = -10, b11(2) = -9,

Data Set Used = [0 1 0]

## Hand Calculation

Forward Propagation:

a1(2) = 1,

a2(2) = - 1,

h1(2) = 0.7311,

h2(2) = 0.2689,

a1(3) = - 12.1591,

h1(3) = $5.3*10^{(-6)}$

del1(3) = 2.7465e-11

Back Propagation:

W11(1) = -8, W12(1) = 10, b11(1) = -9,

W21(1) = 9, W22(1) = 7, b21(1) = -6,

W11(2) = 3, W12(2) = 1, b11(2) = -2,

## Matlab Calculation:

Forward Propagation:

a1(2) = 1,

a2(2) = - 1,

h1(2) = 0.7311,

h2(2) = 0.2689,

a1(3) = - 12.1591,

h1(3) = 5.2407e-6

del1(3) = 2.7465e-11

Back Propagation:

W11(1) = -8, W12(1) = 10, b11(1) = -9,

W21(1) = 9, W22(1) = 7, b21(1) = -6,

W11(2) = 3, W12(2) = 1, b11(2) = -2,