STATISITCS & PROBABILITY P1

4.2	Interquartile range	$\text{IQR} = Q_3 - Q_1$
4.3	Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$, where $n = \sum_{i=1}^{k} f_i$
4.5	Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
	Complementary events	$P(A) + P(A') = 1$
4.6	Combined events	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
	Mutually exclusive events	$P(A \cup B) = P(A) + P(B)$
	Conditional probability	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
	Independent events	$P(A \cap B) = P(A)\,P(B)$
4.7	Expected value of a discrete random variable X	$E(X) = \sum_{i=1}^{k} x_i\, P(X = x_i)$
4.8	Binomial distribution $X \sim B(n, p)$	
	Mean	$E(X) = np$
	Variance	$\text{Var}(X) = np(1-p)$
4.12	Standardized normal variable	$z = \frac{x - \mu}{\sigma}$
4.13	Bayes' theorem	$P(B \mid A) = \frac{P(B)\,P(A \mid B)}{P(B)\,P(A \mid B) + P(B')\,P(A \mid B')}$
		$P(B_i \mid A) = \frac{P(B_i)P(A \mid B_i)}{P(B_1)P(A \mid B_1) + P(B_2)P(A \mid B_2) + P(B_3)P(A \mid B_3)}$

4.14	Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$
	Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i (x_i - \mu)^2}{n}}$
	Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $\text{Var}(aX + b) = a^2\,\text{Var}(X)$
	Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$
	Variance	$\text{Var}(X) = E\left[(X - \mu)^2\right] = E(X^2) - [E(X)]^2$
	Variance of a discrete random variable X	$\text{Var}(X) = \sum (x - \mu)^2\, P(X = x) = \sum x^2\, P(X = x) - \mu^2$
	Variance of a continuous random variable X	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

1. [Maximum mark: 13]

Two regular six-sided dice are rolled and their outcomes are added together.

(a) **Copy and complete** the following table showing the sample space. [3]

+	1	2	3	4	5	6
1	2					
2				6		
3						
4						
5			8			
6						

Let events A, B and C be defined as follows:

A: the first die shows a 1

B: the second die shows an even number

C: the sum is even

(b) Show that the following pairs of events are independent [6]

(i) A and B

(ii) B and C

(iii) C and A

The three events are independent if any two combination of events are independent, and $P(A \cap B \cap C) = P(A)P(B)P(C)$.

(c) Determine whether the three events are independent. [4]

2. [Maximum mark: 6]

In a class of 20 students eight have visited (S)eoul, six have visited (B)eijing, and ten have visited neither of these cities.

(a) Display this information in a Venn diagram. [3]

(b) A student is chosen at random. Find the following probabilities. [3]

(i) $P(S \cap B)$

(ii) $P(B \mid S')$

3. [Maximum mark:6]

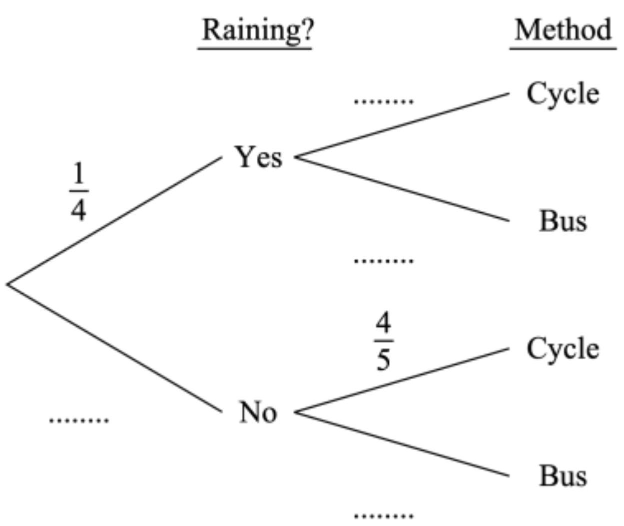
Events A and B are such that $P(A \cap B') = 0.2$, $P(B) = 0.4$ and $P(A') = 0.5$.

(a) Determine $P(A \cup B)$. [2]

(b) Determine $P(A \mid B)$. [4]

4. [Maximum Mark: 16]

If it rains the probability I cycle to school is 1/3. If it doesn't rain the probability I cycle to school is 4/5. If I do not cycle then I take the bus. The probability of it raining on any given day is 1/4. Some of this information is shown in the tree diagram below.



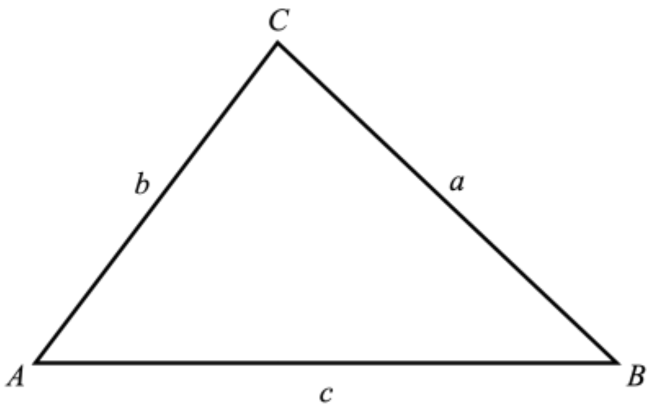
(a) Write down the missing probabilities on the dotted lines above. [2]

(b) Determine the probability I cycle to school on any given day. [2]

(c) Given that I cycled to school yesterday determine the probability it was raining. [2]

5. [Maximum mark; 16]

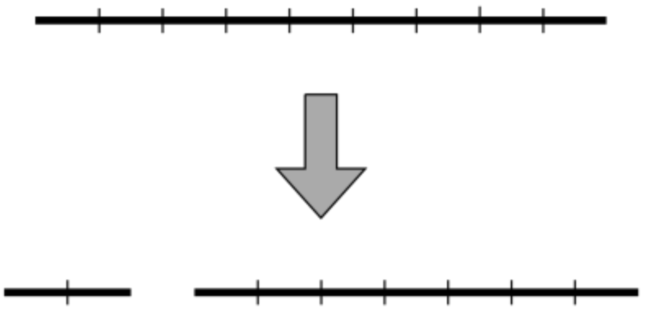
The diagram below shows $\triangle ABC$ where c is the longest side.



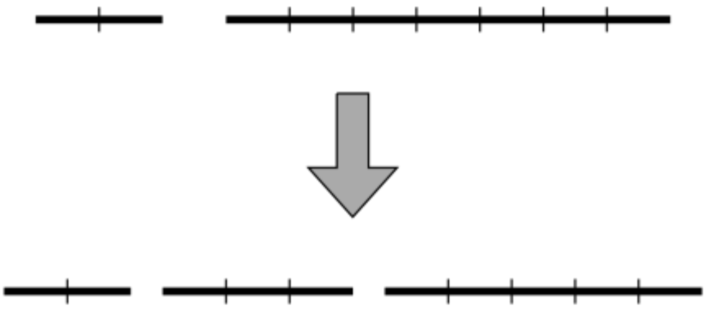
(a) Complete the following inequality by replacing $\blacksquare$ with $\leq$, $\geq$, $<$ or $>$. [1]

$$a + b \blacksquare c$$

A stick of length 9 units has 8 equally spaced points marked on it. One of these points is randomly chosen and the stick is broken at that point. An example of this is shown in the diagram below.



One of the points on the longest piece is them randomly chosen and this piece is broken into two pieces. An example of this, continuing from the previous example, is shown below.



Sometimes the three pieces can be arranged to form a triangle and other times they can not.

(b) Explain why in the example above the three pieces cannot be arranged to form a triangle. [1]

Let event A_k be defined as *after the first break the shortest stick has a length of k units*.

(c) Write down the possible values of k. [2]

Let event B be defined as *after the second break the three pieces can be arranged to form a triangle*.

(d) Write down the probability of A_k. [2]

(e) Show that $P(B \cap A_1) = \frac{1}{28}$. [2]

(f) Find the other values of $P(B \cap A_k)$ for all other possible values of k. [6]

(g) Hence find $P(B)$. [2]

6. [Maximum mark:12]

Two numbers are randomly chosen from the set $\{0,1,2,3\}$ with repetition allowed. Let X represent the mean of these two numbers.

The table below shows the probability distribution of X.

x	0	0.5	1	1.5	2	2.5	3
$P(X=x)$	1/16	2/16	3/16	4/16	a	b	c

(a) Explain why [4]

(i) $P(X=0)=1/16$

(ii) $P(X=1)=3/16$

(b) Find the values of a, b and c. [6]

(c) Hence or otherwise find $E(X)$. [2]

7. [Maximum mark: 5]

Let $X \sim B(n,p)$. If $E(X)=2$ and $\mathrm{Var}(X)=1.6$ find the values of n and p.

8. [Maximum mark:4]

Consider the continuous random variable X where $X \sim N(8,4)$. Write down the mean and standard deviation of the following random variables.

(a) $X-1$ [2]

(b) $\frac{X+3}{2}$ [2]

9. [Maximum mark: 5]

Let $X \sim B(n,p)$. If $E(X) = 2$ and $\mathrm{Var}(X) = 1.6$ find the values of n and p.

10. A coin is flipped three times with each flip showing either heads (H) or tails (T).

(a) Show that $P(HHT) = P(HTT)$. [2]

The coin is now flipped repeatedly until either HHT appears, or HTT appears. For example, if the first five flips show $TTHTT$ then HTT has appeared.

Let event A be defined as *HHT appears before HTT*.

(b) Explain why if the first two flips show HH then event A must occur. [1]

Let the notation $P(A \mid T)$ be defined as *The probability of event A happening given that the current flip shows T.*

(c) Explain why $P(A \mid T) = P(A)$. [2]

(d) Show that $P(A \mid H) = \frac{1}{2} + \frac{1}{4}P(A \mid H)$. [3]

(e) Hence find the value of $P(A \mid H)$. [2]

(f) Find the value of $P(A)$. [3]