Brusselator Jacobian for N=2

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The Brusselator equation has the form:

$$u_i^{'} = f_i = 1 + u_i^2 v_i - 4u_i + \alpha(N+1)^2(u_{i-1} - 2u_i + u_{i+1})$$
$$v_i^{'} = g_i = 3u_i - u_i^2 v_i + \alpha(N+1)^2(v_{i-1} - 2v_i + v_{i+1})$$

Taking $\alpha = 1/50$ and the initial conditions:

$$u_j(0) = 1 + \sin(2\pi x_j)$$

and

$$v_j(0) = 3$$

Here, $x_j = i/N + 1$ Now placing N=2, the equivalent equations are:

$$f_1 = 1 + u_1^2 v_1 - 4u_1 + \alpha(N+1)^2(u_0 - 2u_1 + u_2)$$

$$g_1 = 3u_1 - u_1^2 v_1 + \alpha(N+1)^2(v_0 - 2v_1 + v_2)$$

$$f_2 = 1 + u_2^2 v_2 - 4u_2 + \alpha(N+1)^2(u_1 - 2u_2 + u_3)$$

$$g_2 = 3u_2 - u_2^2 v_2 + \alpha(N+1)^2(v_1 - 2v_2 + v_3)$$

Independent variables: u_1, v_1, u_2 and v_2. Then the Jacobian:

$$
\mathrm{J}= \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial v_1} & \frac{\partial f_1}{\partial u_2} & \frac{\partial f_1}{\partial v_2} \\ \frac{\partial g_1}{\partial u_1} & \frac{\partial g_1}{\partial v_1} & \frac{\partial g_1}{\partial u_2} & \frac{\partial g_1}{\partial v_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial v_1} & \frac{\partial f_2}{\partial u_2} & \frac{\partial f_2}{\partial v_2} \\ \frac{\partial g_2}{\partial u_1} & \frac{\partial g_2}{\partial v_1} & \frac{\partial g_2}{\partial u_2} & \frac{\partial g_2}{\partial v_2} \end{pmatrix}
$$

$$
= \begin{pmatrix} 2\mathrm{u}_1 v_1 - 4 - 2\alpha(N+1)^2 & \mathrm{u}_1^2 & \alpha(N+1)^2 & 0 \\ 3\text{-}2\mathrm{u}_1 v_1 & -u_1^2 - 2\alpha(N+1)^2 & 0 & \alpha(N+1)^2 \\ \alpha(N+1)^2 & 0 & 2\mathrm{u}_2 v_2 - 4 - 2\alpha(N+1)^2 & \mathrm{u}_2^2 \\ 0 & \alpha(N+1)^2 & 3\text{-}2\mathrm{u}_2 v_2 & \text{-}\mathrm{u}_i^2 - 2\alpha(N+1)^2 \end{pmatrix}
$$