Writing LLE in matrix Form

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The normalized Lugiato Lefever Equation:

$$\frac{\partial\psi}{\partial\tau} = -(1+i\alpha)\psi + i|\psi|^2\psi - i\frac{\beta}{2}\frac{\partial^2\psi}{\partial\theta^2} + F[1]$$

Taking the Fourier transform by ignoring the nonlinearity, as FT is a linear operation.

$$\frac{\partial\tilde{\psi}}{\partial\tau} = -(1+i\alpha)\tilde{\psi} + im^2\frac{\beta}{2}\tilde{\psi} \quad [2]$$

$$Or, \quad \frac{\partial\tilde{\psi}}{\partial\tau} = -(1+i\alpha - im^2\frac{\beta}{2})\tilde{\psi}$$

$$Let\ \tilde{\psi}\ be\ a\ 1\times N\ column\ vector.$$

$$Then, -\left(1+i\alpha - im^2\frac{\beta}{2}\right)$$

$$can\ be\ represented\ by\ a\ diagonal\ matrix\ of\ order\ N\times N$$

So, the term becomes:

$$\begin{bmatrix} -(1+i\alpha-im^2\frac{\beta}{2}) & 0 & \dots & 0 \\ 0 & -(1+i\alpha-im^2\frac{\beta}{2})\tilde{\psi} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -(1+i\alpha-im^2\frac{\beta}{2})\tilde{\psi} \end{bmatrix} \begin{bmatrix} \tilde{\psi}_1 \\ \dots \\ \dots \\ \tilde{\psi}_N \end{bmatrix}$$

Again, let the differentiation from the left side is performed as a matrix operation:

$$\begin{bmatrix} \frac{\partial\widetilde{\psi_1}}{\partial\tau} \\ \dots \\ \dots \\ \frac{\partial\widetilde{\psi_N}}{\partial\tau} \end{bmatrix} = \begin{bmatrix} \widetilde{\psi_1}' \\ \dots \\ \dots \\ \widetilde{\psi_2}' \end{bmatrix}$$