# 算法模板

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# 1 计算几何

# 1.1 自适应辛普森

```
typedef double db;
struct Simpson {
 /* 系数 */
 db F(db x) { return /* 表达式 */; }
 db Simpson(db l, db r) {
   db m = (1 + r) / 2.0;
   return (F(1) + 4 * F(m) + F(r)) * (r - 1) / 6.0;
 db Asr(db 1, db r, db ans, db eps) {
   db m = (l + r) / 2.0;
   db l_ans = Simpson(l, m), r_ans = Simpson(m, r);
   if (fabs(l_ans + r_ans - ans) \le 15.0 * eps) return
         l_ans + r_ans + (l_ans + r_ans - ans) / 15.0;
    return Asr(l, m, l_ans, eps / 2.0) + Asr(m, r, r_ans,
          eps / 2.0);
 }
};
```

# 1.2 立体几何

```
typedef double db;
const db inf = 1e100;
const db eps = 1e-9;
const db pi = acos(-1.);
const db delta = 0.98;
int Sgn(db k) { return std::fabs(k) < eps ? 0 : (k < 0 ?</pre>
    -1:1);}
int Cmp(db k1, db k2) { return Sgn(k1 - k2); }
int Min(db k1, db k2) { return Cmp(k1, k2) < 0 ? k1 : k2;</pre>
int Max(db k1, db k2) { return Cmp(k1, k2) > 0 ? k1 : k2;
struct point { db x, y, z; }
bool operator == (point k1, point k2) { return !Sgn(k1.x
    - k2.x) && !Sgn(k1.y, k2.y) && !Sgn(k1.z, k2.z); }
point operator + (point k1, point k2) { return (point){k1
     x + k2.x, k1.y + k2.y, k1.z + k2.z; }
point operator - (point k1, point k2) { return (point){k1
     x - k2.x, k1.y - k2.y, k1.z - k2.z;
point operator * (point k1, db k2) { return (point){k1.x
     * k2, k1.y * k2, k1.z * k2; }
point operator / (point k1, db k2) { return (point){k1.x
     / k2, k1.y / k2, k1.z / k2}; }
db operator * (point k1, point k2) { k1.x * k2.x + k1.y *
      k2.y + k1.z * k2.z;}
```

```
point operator ^ (point k1, point k2) { return (point){k1
            y * k2.z - k1.z * k2.y, k1.z * k2.x - k1.x * k2.z,
            k1.x * k2.y - k1.y * k2.x; }
db GetLen(point k) { return std::sqrt(k * k); }
db GetLen2(point k) { return k * k; }
point GetUnit(point k) { return k / GetLen(k); }
db GetDis(point k1, point k2) { return GetLen(k2 - k1); }
db GetDis2(point k1, point k2) { return GetLen2(k2 - k1);
db GetMinSphereR(std::vector<point> p) {
    point cur = p[0];
     db pro = 10000, ret = inf;
    while (pro > eps) {
        int idx = 0;
         for (int i = 0; i < p.size(); ++i)</pre>
             if (Cmp(GetDis(cur, p[i]), GetDis(cur, p[idx])) >
                        0)
                  idx = i;
        db r = GetDis(cur, p[idx]);
         ret = Min(ret, r);
         cur = cur + (p[idx] - cur) / r * pro;
        pro *= delta;
    }
    return ret;
}
struct line { point s, t; };
struct seg: public line {};
db GetLen(seg k) { return GetDis(k.s, k.t); }
db GetLen2(seg k) { return GetDis2(k.s, k.t); }
db GetDis(point k1, line k2) { return std::fabs((k1 - k2.
           s) ^ (k2.t - k2.s)) / GetLen(k2); }
db GetDis(point k1, seg k2) {
    if (Sgn((k1 - k2.s) * (k2.t - k2.s)) < 0 | | Sgn((k1 - k2.s)) < 0 | | Sgn((k1 - k2.s)) 
                k2.t) * (k2.s - k2.t)) < 0)
         return Min(GetDis(k1, k2.s), GetDis(k1, k2.t));
     return GetDis(k1, seg); // point to line dis
}
struct sphere { point o; db r; };
db GetV(sphere k) { return 4. / 3. * pi * k.r * k.r * k.r
           ; }
db GetInterV(sphere k1, sphere k2) {
    db dis = GetDisP2P(k1.o, k2.o);
     if (Sgn(dis - k1.r - k2.r) >= 0) return ret;
    if (Sgn(k2.r - (dis + k1.r)) >= 0) return GetV(k1);
     else if (Sgn(k1.r - (dis + k2.r)) >= 0) return GetV(k2)
     db len1 = ((k1.r * k1.r - k2.r * k2.r) / dis + dis) /
                2;
     db len2 = dis - len1;
     db x1 = k1.r - len1, x2 = k2.r - len2;
     db v1 = pi * x1 * x1 * (k1.r - x1 / 3.0);
     db v2 = pi * x2 * x2 * (k2.r - x2 / 3.0);
     return v1 + v2;
```

# 1.3 皮克定理

```
polygon: S = in + (on / 2) - 1
```

# 1.4 平面几何

```
typedef double db; // typedef long double db;
const db inf = 1e100;
const db eps = 1e-9;
const db delta = 0.98;
int Sgn(db k) { return std::fabs(k) < eps ? 0 : (k < 0 ?</pre>
     -1:1);
int Cmp(db k1, db k2) { return Sgn(k1 - k2); }
bool IsInMid(db k1, db k2, db k3) { return Sgn(k1 - k3) *
      Sgn(k2 - k3) \leftarrow 0; 
db Max(db k1, db k2) { return Cmp(k1, k2) > 0 ? k1 : k2;
db Min(db k1, db k2) { return Cmp(k1, k2) < 0 ? k1 : k2;
struct point { db x, y; };
bool operator == (point k1, point k2) { return !Cmp(k1.x,
      k2.x) && !Cmp(k1.y, k2.y); }
point operator + (point k1, point k2) { return (point){k1
     x + k2.x, k1.y + k2.y; }
point operator - (point k1, point k2) { return (point){k1
     .x - k2.x, k1.y - k2.y; }
point operator * (point k1, db k2) { return (point){k1.x
     * k2, k1.y * k2; }
point operator / (point k1, db k2) { return (point){k1.x
     / k2, k1.y / k2}; }
db operator * (point k1, point k2) { return k1.x * k2.x +
      k1.y * k2.y; }
db operator ^ (point k1, point k2) { return k1.x * k2.y -
      k1.y * k2.x; }
bool IsInMid(point k1, point k2, point k3) { return
     IsInMid(k1.x, k2.x, k3.x) && IsInMid(k1.y, k2.y, k3.
db GetLen(point k) { return std::sqrt(k * k); }
db GetLen2(point k) { return k * k; }
point GetUnit(point k) { return k / GetLen(k); }
db GetDis(point k1, point k2) { return GetLen(k2 - k1); }
db GetDis2(point k1, point k2) { return GetLen2(k2 - k1);
      }
db GetAng(point k1, point k2) { return std::atan2((k1 ^
     k2), (k1 * k2)); }
point Rotate(point k, db ang) { return (point)\{k.x * std
     ::cos(ang) - k.y * std::sin(ang), k.x * std::sin(ang)
     ) + k.y + std::cos(ang)}; }
```

```
point Rotate90(point k) { return (point){-k.y, k.x}; }
struct line { point s, t; };
struct seg: public line {}; // typedef line seg
db GetLen(seg k) { return GetDis(k.s, k.t); }
bool IsOn(point k1, seg k2) { return !Sgn((k1 - k2.s) ^ (
     k2.t - k2.s) && Sgn((k1 - k2.s) * (k1 - k2.t)) <=
     0: }
point Proj(point k1, line k2) { point k = k2.t - k2.s;
     return k2.s + k * (((k1 - k2.s) * k) / GetLen(k)); }
point Reflect(point k1, line k2) { return Proj(k1, k2) *
     2 - k1: 
bool IsParallel(line k1, line k2) { return !Sgn((k1.s -
     k1.t) ^ (k2.s - k2.t)); }
bool IsInter(seg k1, seg k2) {
   Max(k1.s.x, k1.t.x) >= Min(k2.s.x, k2.t.x) &&
   Max(k2.s.x, k2.t.x) >= Min(k1.s.x, k1.t.x) &&
   Max(k1.s.y, k1.t.y) >= Min(k2.s.y, k2.t.y) &&
   Max(k2.s.y, k2.t.y) >= Min(k1.s.y, k1.t.y) &&
    Sgn((k2.s - k1.t) \wedge (k1.s - k1.t)) * Sgn((k2.t - k1.t))
         ) ^{(k1.s - k1.t)} \le 0 \& 
    Sgn((k1.s - k2.t) ^ (k2.s - k2.t)) * Sgn((k1.t - k2.t
         ) ^{(k2.s - k2.t)} = 0;
bool IsInter(line k1, seg k2) {
  return Sgn((k2.s - k1.t) \wedge (k1.s - k1.t)) * Sgn((k2.t - k1.t))
        k1.t) ^ (k1.s - k1.t)) <= 0;
bool IsInter(line k1, line k2) {
  if (!IsParallel(k1, k2)) return true;
  return !Sgn((k1.s - k2.s) \wedge (k2.t - k2.s));
db GetDis(point k1, line k2) {
  point k = Proj(k1, k2);
  return GetDis(k1, k);
db GetDis(point k1, seg k2) {
 point k = Proj(k1, k2);
  if (IsInMid(k2.s, k2.t, k)) return GetDis(k1, k);
  return Min(GetDis(k1, k2.s), GetDis(k1, k2.t));
point Cross(line k1, line k2) {
 db w1 = (k1.s - k2.s) \land (k2.t - k2.s), w2 = (k2.t - k2.s)
       s) ^{(k1.t-k2.s)};
  return (k1.s * w2 + k1.t * w1) / (w1 + w2);
// 平面直线图(PSLG)
struct edge { int u, v; db ang; };
struct PSLG {
 int n, m, face_cnt; // 多边形数
  point p[maxn];
 std::vector<edge> e;
  std::vector<int> g[maxn];
```

bool vis[maxn \* 2];

```
int left[maxn * 2], prev[maxn * 2];
std::vector<polygon> faces; // 多边形
db area[maxn]; // 多边形面积
void Init() {
 n = m = 0;
 for (int i = 0; i < n; ++i) g[i].clear();</pre>
 e.clear();
  faces.clear();
// 有向线段pt.x->pt.y的极角
db GetAng(point pt) {
 return std::atan2(pt.y, pt.x);
void AddEdge(int u, int v) {
  e.push_back((edge)\{u, v, GetAng(p[v] - p[u])\});
  e.push_back((edge)\{v, u, GetAng(p[u] - p[v])\});
 m = e.size();
 g[u].push_back(m - 2);
 g[v].push_back(m - 1);
// 找出faces并计算面积
void Build() {
 for (int u = 0; u < n; ++u) {
    int sz = g[u].size();
    for (int i = 0; i < sz; ++i)
      for (int j = i + 1; j < sz; ++j)
        if (e[g[u][i]].ang > e[g[u][j]].ang) std::swap(
             g[u][i], g[u][j]);
    for (int i = 0; i < sz; ++i) prev[g[u][(i + 1) % sz
         ]] = g[u][i];
  face cnt = 0:
  memset(vis, false, sizeof(vis));
  for (int u = 0; u < n; ++u) {
   int sz = g[u].size();
    for (int i = 0; i < sz; ++i) {
     int v = g[u][i];
      // 卷包裹逆时针找圈
      if (!vis[v]) {
        ++face_cnt;
        polygon poly;
        while (true) {
         vis[v] = 1;
          left[v] = face_cnt;
          int f = e[v].u;
          poly.push_back(p[f]);
          v = prev[v \land 1];
          if (v == g[u][i]) break;
          assert(vis[v] == 0);
        faces.push_back(poly);
```

```
for (int i = 0; i < face_cnt; ++i) area[i] = GetArea(</pre>
         facesΓil):
 }
};
struct circle { point o; db r; };
// 切点
std::vector<point> TagentCP(circle k1, point k2) {
  db a = GetLen(k2 - k1.0), b = k1.r * k1.r / a, c = std
       ::sqrt(Max(0., k1.r * k1.r - b * b));
  point k = GetUnit(k2 - k1.o), m = k1.o + k * b, del =
       Rotate90(k) * c;
  return {m - del, m + del};
}
// 公切线数量
int CheckPosCC(circle k1, circle k2) {
  if (Cmp(k1.r, k2.r) == -1) std::swap(k1, k2);
  double dis = k1.o.Dis(k2.o);
  int w1 = Cmp(dis, k1.r + k2.r), w2 = Cmp(dis, k1.r - k2)
       .r);
  if (w1 > 0) return 4;
  if (w1 == 0) return 3;
  else if (w2 > 0) return 2;
  else if (w2 == 0) return 1;
  return 0;
// 交点
std::vector<point> GetCC(circle k1, circle k2) {
  int pd = CheckPosCC(k1, k2);
  if (pd == 0 || pd == 4) return {};
  double a = (k2.0 - k1.0).Abs2();
  double cosA = (k1.r * k1.r + a - k2.r * k2.r) / (2 * k1)
       .r * sqrt(std::max(a, 0.0)));
  double b = k1.r * cosA, c = sqrt(std::max(0.0, k1.r *
       k1.r - b * b));
  point k = (k2.0 - k1.0).Unit(), m = k1.0 + k * b, del =
        k.Turn90() * c;
  return {m - del, m + del};
circle GetCircle(point k1, point k2, point k3) {
  db a1 = k2.x - k1.x, b1 = k2.y - k1.y, c1 = (a1 * a1 +
       b1 * b1) * 0.5;
  db a2 = k3.x - k1.x, b2 = k3.y - k1.y, c2 = (a2 * a2 +
       b2 * b2) * 0.5;
  db d = a1 * b2 - a2 * b1;
  point o = (point)\{k1.x + (c1 * b2 - c2 * b1) / d, k1.y\}
       + (a1 * c2 - a2 * c1) / d;
  return (circle){o, GetDis(k1, o)};
db GetMinCircleR(std::vector<point> p) {
  point cur = p[0];
  db pro = 10000, ret = inf;
  while (Sgn(pro) > 0) {
    int idx = 0;
    for (int i = 0; i < p.size(); ++i)</pre>
```

```
if (GetDis(cur, p[i]) > GetDis(cur, p[idx]))
          db r = GetDis(cur, p[idx]);
         ret = Min(ret, r);
          cur = cur + (p[idx] - cur) / r * pro;
         pro *= delta;
    }
     return ret;
circle GetMinCircle(std::vector<point> p) {
     std::random_shuffle(p.begin(), p.end());
     circle ret = (circle){p[0], 0.};
     for (int i = 1; i < p.size(); ++i) {</pre>
          if (Cmp(GetDis(ret.o, p[i]), ret.r) <= 0) continue;</pre>
          ret = (circle){p[i], 0.};
          for (int j = 0; j < i; ++j) {
               if (Cmp(GetDis(ret.o, p[j]), ret.r) <= 0) continue;</pre>
               ret.o = (p[i] + p[j]) * 0.5;
               ret.r = GetDis(ret.o, p[i]);
               for (int k = 0; k < j; ++j) {
                     if (Cmp(GetDis(ret.o, p[k]), ret.r) <= 0)</pre>
                                   continue:
                     ret = GetCircle(p[i], p[j], p[k]);
              }
         }
     }
     return ret;
typedef std::vector<point> polygon;
db GetArea(polygon &poly) {
     db ret = 0.;
     for (int i = 0; i < poly.size(); ++i) ret += poly[i] ^</pre>
                  poly[(i + 1) % poly.size()];
     return ret * 0.5;
polygon GrahamScan(std::vector<point> p) {
     polygon ret;
     if (p.size() < 3) {</pre>
         for (point &v : p) ret.push_back(v);
          return ret:
     int idx = 0;
     for (int i = 0; i < p.size(); ++i)</pre>
          if (Cmp(p[i].x, p[idx].x) < 0 \mid | (!Cmp(p[i].x, p[idx)) \mid | (!Cmp(p[
                       ].x) && Cmp(p[i].y, p[idx].y) < 0))
               idx = i;
     std::swap(p[0], p[idx]);
     std::sort(p.begin() + 1, p.end(),
          [&](const point &k1, const point &k2) {
               db tmp = (k1 - p[0]) \wedge (k2 - p[0]);
               if (Sgn(tmp) > 0) return true;
               else if (!Sgn(tmp) && Cmp(GetDis(k1, p[0]), GetDis(
                             k2, p[0]) \ll 0) return true;
               return false;
```

```
}
     ret.push_back(p[0]);
     for (int i = 1; i < p.size(); ++i) {</pre>
         while (ret.size() > 1 && Sgn((ret.back() - ret[ret.
                        size() - 2]) \land (p[i] - ret[ret.size() - 2])) <=
                        0) ret.pop_back();
          ret.push_back(p[i]);
     return ret;
bool IsIn(point p, const polygon &ch) {
    point base = ch[0];
     if (Sgn((p - base) \land (ch[1] - p)) > 0 \mid | Sgn((p - base))
                     ^ (ch.back() - base)) < 0) return false;
     if (!Sgn((p - base) \land (ch[1] - p)) \&\& Cmp(GetLen(p - p)) \&\& Cmp(
                 base), GetLen(ch[1] - base)) <= 0) return true;</pre>
     int idx = std::lower_bound(ch.begin(), ch.end(), p,
         [&](const point &k1, const point &k2) {
               return Sgn((k1 - base) \wedge (k2 - base)) > 0;
    ) - ch.begin() - 1;
     return Sgn((ch[idx + 1] - ch[idx]) \land (p - ch[idx])) >=
polygon Minkowski(const polygon &k1, const polygon &k2) {
     int sz1 = k1.size(), sz2 = k2.size();
     std::queue<point> buf1, buf2;
     for (int i = 0; i < sz1; ++i) buf1.push(k1[(i + 1) %</pre>
                  sz1] - k1[i]);
     for (int i = 0; i < sz2; ++i) buf2.push(k2[(i + 1) %
                 sz2] - k2[i]);
     polygon ret;
     ret.push_back(k1[0] + k2[0]);
     while (!buf1.empty() && !buf2.empty()) {
         point tmp1 = buf1.front(), tmp2 = buf2.front();
          if (Sgn(tmp1 ^ tmp2) > 0) {
               ret.push_back(ret.back() + tmp1);
               buf1.pop();
         }
          else {
               ret.push_back(ret.back() + tmp2);
               buf2.pop();
         }
    }
    while (!buf1.empty()) {
          ret.push_back(ret.back() + buf1.front());
         buf1.pop();
    while (!buf2.empty()) {
          ret.push_back(ret.back() + buf2.front());
          buf2.pop();
```

return GrahamScan(ret);

## 1.5 动态凸包

```
// CodeForces 70D 动态凸包
#include <bits/stdc++.h>
typedef double db;
const int maxn = 1e5 + 5;
const db eps = 1e-9;
int Sgn(db \ k) \ \{ return \ fabs(k) < eps ? 0 : (k < 0 ? -1 : 
     1); }
int Cmp(db k1, db k2) { return Sgn(k1 - k2); }
struct point { db x, y; };
point operator - (point k1, point k2) { return (point){k1
     .x - k2.x, k1.y - k2.y; }
point operator + (point k1, point k2) { return (point){k1
    x + k2.x, k1.y + k2.y; }
db operator * (point k1, point k2) { return k1.x * k2.x +
     k1.y * k2.y; }
db operator ^ (point k1, point k2) { return k1.x * k2.y -
      k1.y * k2.x; }
db GetLen(point k) { return sqrt(k * k); }
int n;
point basic;
point p[maxn];
std::set<point> set;
bool operator < (point k1, point k2) {</pre>
 k1 = k1 - basic; k2 = k2 - basic;
 db Ang1 = atan2(k1.y, k1.x), Ang2 = atan2(k2.y, k2.x);
 db Len1 = GetLen(k1), Len2 = GetLen(k2);
 if (Cmp(Ang1, Ang2) != 0) return Cmp(Ang1, Ang2) < 0;</pre>
 return Cmp(Len1, Len2) < 0;</pre>
```

```
std::set<point>::iterator Prev(std::set<point>::iterator
  if (k == set.begin()) k = set.end();
  return —k;
std::set<point>::iterator Next(std::set<point>::iterator
     k) {
  ++k;
  return k == set.end() ? set.begin() : k;
bool Query(point k) {
  std::set<point>::iterator it = set.lower_bound(k);
  if (it == set.end()) it = set.begin();
  return Sgn((k - *(Prev(it))) \wedge (*(it) - *(Prev(it))))
       <= 0:
void Insert(point k) {
  if (Query(k)) return;
  set.insert(k):
  std::set<point>::iterator cur = Next(set.find(k));
  while (set.size() > 3 && Sgn((k - *(Next(cur))) ^ (*(
       cur) - *(Next(cur)))) <= 0) {
    set.erase(cur);
    cur = Next(set.find(k));
  cur = Prev(set.find(k));
  while (set.size() > 3 && Sgn((k - *(cur)) \wedge (*(cur) - cur)
       *(Prev(cur)))) >= 0) {
    set.erase(cur);
    cur = Prev(set.find(k));
 }
}
int main() {
  scanf("%d", &n);
  basic.x = basic.y = 0.0;
  for (int i = 1, T; i \le 3; ++i) {
    scanf("%d%lf%lf", &T, &p[i].x, &p[i].y);
    basic.x += p[i].x; basic.y += p[i].y;
  basic.x /= 3.0; basic.y /= 3.0;
  for (int i = 1; i <= 3; ++i) set.insert(p[i]);</pre>
  for (int i = 4, T; i \le n; ++i) {
    scanf("%d%lf%lf", &T, &p[i].x, &p[i].y);
    if (T == 1) Insert(p[i]);
    else {
      if (Query(p[i])) printf("YES\n");
      else printf("NO\n");
    }
  }
  return 0;
```

# 2 数论

# 2.1 高斯消元

```
const int Mod = 1e9 + 7;
const int maxn = 1e3 + 5;
const double eps = 0.0000001;
const int INF = 0x3f3f3f3f;
int n, m;
double a[maxn][maxn], x[maxn];
bool manySolutionFlag = false, noSolution = false;
void Swap(int i, int j) {
   for (int k = 1; k \le n + 1; k ++)
        swap(a[i][k], a[j][k]);
bool Check(int i) {
   bool vis = false;
   for (int j = 1; j \ll n; j \leftrightarrow j) {
       if(fabs(a[i][j]) >= eps) vis = true;
   if(!vis && fabs(a[i][n + 1]) >= eps) return false;
   return true;
void GS() {
    for (int i = 1; i <= n; i ++) {
        bool flag = false;
        for (int j = i; j \le m; j ++) {
            if(a[j][i] != 0) {
                Swap(j, i);
                flag = true;
                break;
            }
        if(!flag) {
            manySolutionFlag = true;
        for (int j = i + 1; j \le m; j ++)
            for (int k = n + 1; k >= i; k ---)
                a[j][k] = a[j][k] * 1. - a[i][k] * (a[j][
                     i] * 1./a[i][i] * 1.) * 1.;
    for (int i = 1; i <= m; i ++) {
        if(!Check(i)) {
            noSolution = true;
            return ;
        }
    for (int i = n; i >= 1; i ---) {
        for (int j = i + 1; j \le n; j ++) {
            a[i][n + 1] = a[i][n + 1] - a[i][j] * x[j];
            a[i][j] = 0;
```

```
x[i] = a[i][n + 1] * 1./a[i][i] * 1.;
    }
}
int main()
    cin >> n >> m:
    for (int i = 1; i <= m; i ++)</pre>
        for (int j = 1; j <= n + 1; j ++)
            cin >> a[i][j];
    GS():
    if(noSolution) cout << "No solutions" << endl;</pre>
    else if(manySolutionFlag) cout << "Many solutions" <<</pre>
    else {
        for (int i = 1; i <= n; i ++)
            cout \ll (int)(x[i] +0.5) \ll endl;
    }
    return 0;
```

# 2.2 逆元

```
int inv[maxn];// 递推打表
void getInv(int n, int m) {
   inv[1] = 1;
    for (int i = 2; i <= n; i ++)
        inv[i] = (long long) (m - m/i) * inv[m%i] % m;
long long ex_gcd(long long a, long long b, long long &x,
    long long &y) { //扩展欧几里德求逆元
    if(!b) {
       x = 1; y = 0;
       return a;
   long long d = ex_gcd(b, a\%b, x, y);
    long long t = x;
   x = y; y = t - (a/b) * y;
   return d;
long long getInv(long long a, long long p) {
   long long x, y;
   ex_gcd(a, p, x, y);
   return (x \% p + p) \% p;
long long Ksm(long long a, long long b, long long mod) {
    // 递推打阶乘逆元表
   long long res = 1;
   while(b) {
        if(b \& 1) res = res * a % mod;
        a = a \% a \% mod;
```

# 2.3 线性基

```
struct LB{
   long long b[35], nb[35], tot;
   bool flag;
   LB () { // 初始化
       memset(b, 0, sizeof(b));
       flag = false;
   LB(const LB& a) {
       for (int i = 0; i < 35; i ++)
           b[i] = a.b[i];
       flag = a.flag;
   void Ins(long long x) { //插入
       for (int i = 34; i >= 0; i ---)
           if(x & (1ll << i)) {</pre>
               if(!b[i]) { b[i] = x; return; }
               x \sim b[i];
           }
       flag = true; //能xor出0
   bool Fin(long long x) {
       if(x == 0 && flag) return true;
       for (int i = 34; i >= 0; i ---)
           if(x >> i) x ^= b[i];
       return x == 0;
   }
   long long getMax(long long x) { // 得到最大值
       long long res = x;
       for (int i = 34; i >= 0; i ---)
           res = max(res, res ^ b[i]);
       return res;
   }
   long long getMin(long long x) { // 得到最大值
       long long res = x;
       for (int i = 0; i <= 34; i ++)
```

```
if(b[i]) res ^= b[i];
        return res;
    long long ReBuild() { // 重新Build 为下面的Kth
        for (int i = 34; i >= 0; i ---) {
            if(b[i] == 0) continue;
            for (int j = i - 1; j >= 0; j ---) {
                if(b[j] == 0) continue;
                if(b[i] & (1ll << j)) b[i] ^= b[j];</pre>
        for (int i = 0; i <= 34; i ++)
            if(b[i]) nb[tot++] = b[i];
    long long Kth_Max(long long k) { // 得到第k小的数, k
        if(flag) k —;
        if(k == 0) return 0;
        long long res = 0;
        if(k >= (111 << tot)) return -1;
        for (int i = 34; i >= 0; i ---)
           if(k & (1ll << i)) res ^= nb[i];</pre>
        return res;
    LB Corss(LB k) { //求交集
        LB res, tmp = k;
        for (int i = 0; i < 35; i ++) {
            long long x = b[i], y = 0;
            bool vis = false;
            for (int j = 34; j >= 0; j ---) {
                if(x \gg j) {
                    if(k.b[j]) \times ^= k.b[j], y ^= tmp.b[j]
                         ];
                    else {
                        k.b[j] = x;
                        tmp.b[j] = y;
                        vis = true;
                        break;
                    }
                }
            }
            if(!vis) res.b[i] = y;
        }
        return res;
    LB Merge(LB u) { //合并两个线性集
        LB w = *this;
        for (int i = 34; i >= 0; i—) {
            if(u.b[i] == 0) continue;
            w.Ins(u.b[i]);
        }
        return w;
};
```

# 2.4 约瑟夫环

# 2.5 欧拉函数

```
int getPhi(int n) {
   int rea = n;
    for (int i = 2; i * i <= n; i ++) {
       if (n % i == 0) {
           rea = rea - rea / i;
           while(n % i == 0) n /= i;
   if(n > 1) rea = rea - rea / n;
   return rea;
int phi[maxn];//递推
void getPhi() {
   for (int i = 1; i < maxn; i ++)
       phi[i] = i;
   for (int i = 2; i < maxn; i ++)
       if(phi[i] == i)
           for (int j = i; j < maxn; j += i)
               phi[j] = (phi[j] / i) * (i-1);
```

# 2.6 杜教筛

```
const int M = 5e6;
const int Mod = 1e9 +7;
```

```
const double eps = 0.000000001;
long long phi[M + 30], prim[M + 30], tot;
bool mark[M + 30];
map<long long, long long> m;
#define inv_2 (Mod+1)/2
long long Add(long long a, long long b) {
   long long c = (a + b) \% Mod;
   if(c >= Mod) return c - Mod;
   if(c < 0) return c + Mod;</pre>
   return c;
void init() {
   phi[1] = 1;
    for(long long i = 2; i <= M; i ++) {
        if(!mark[i]) {
            prim[++tot] = i;
            phi[i] = i - 1;
        for (long long j = 1; j \ll tot; j ++) {
            if(i * prim[j] > M) break;
            mark[i * prim[j]] = 1;
            if(i % prim[j] == 0) {
                phi[i * prim[j]] = phi[i] * prim[j];
            }
            phi[i *prim[j]] = phi[i] * phi[prim[j]];
    for (int i = 1; i <= M; i ++) phi[i] = Add(phi[i-1],</pre>
         phi[i]);
long long getPhi(long long n) {
   if(n <= M) return phi[n];</pre>
   if(m[n]) return m[n];
   long long ans;
   ans = 1LL * n % Mod * (n % Mod + 1) % Mod * inv_2 %
        Mod;
    for (long long l = 2, r; l <= n; l = r + 1) {
        r = n/(n/l);
        ans = (ans - (r - l + 1) \% Mod * getPhi(n/l) \%
             Mod + Mod) % Mod;
   return m[n] = ans;
long long solve(long long n) {
   long long ans = 0;
    for (long long l = 1, r; l <= n; l = r + 1) {
        r = n/(n/l);
        ans = Add(ans, 1ULL * (n/l) % Mod * (n/l) % Mod *
              (Add(getPhi(r) % Mod, -getPhi(l-1) % Mod)
             ) % Mod);
    return ans;
```

```
int main()
{
    init();
    long long n;
    cin >> n;
    m.clear();
    cout << solve(n) << endl;
    return 0;
}
/*
gcd ≥ №
gcd(i,j)(1<=i<=n)(1<=j<=m)
* /</pre>
```

# 2.7 数论函数打表

```
int phi[maxn], prime[maxn], tot;//线性打欧拉函数
bool vis[maxn];
void init() {
   phi[1] = 1;
    for (int i = 2; i < maxn; i ++) {</pre>
       if(!vis[i]) {
           prime[++tot] = i;
           phi[i] = i - 1;
        for (int j = 1; j <= tot; j ++) {
           if(i * prime[j] >= maxn) break;
           vis[i*prime[j]] = 1;
            if(i % prime[j] == 0) {
               phi[i*prime[j]] = phi[i] * prime[j];
               break;
           phi[i*prime[j]] = phi[i] * phi[prime[j]];
       }
   }
bool vis[maxn];
int mu[maxn], prime[maxn];
void Mobius() {//线性打莫比乌斯函数
   mu[1] = 1;
   int tot = 0;
   for (int i = 2; i < maxn; i ++) {
       if(!vis[i]) {
           prime[tot++] = i;
           mu[i] = -1;
       for (int j = 0; j < tot; j ++) {
           if(i * prime[j] >= maxn) break;
           vis[i*prime[j]] = true;
            if(i % prime[j] == 0) {
               mu[i*prime[j]] = 0;
```

# 2.8 扩展卢卡斯

```
long long Pre[maxn];
long long extend_gcd(long long a, long long b, long long
     &x, long long &y) {
    if(!b) {
        x = 1; y = 0;
        return a;
    long long d = extend\_gcd(b, a \% b, x, y);
    long long t = x;
    x = y; y = t - (a / b) * y;
    return d;
long long mul(long long a, long long b, long long P){
    long long L = a * (b >> 25LL) % P * (1LL << 25) % P;
    long long R = a * (b & ((1LL << 25) - 1)) % P;
    return (L + R) % P;
long long Pow(long long a, long long b, long long P) {
    long long ans = 1; a \% = P;
    while(b) {
       if(b & 1) ans = mul(ans, a, P);
       a = mul(a, a, P);
       b >>= 1;
    }
    return ans;
long long getInv(long long a, long long p) {
    long long x, y;
    extend_gcd(a, p, x, y);
    x = (x \% p + p) \% p;
    return x;
long long CRT(long long m, long long p, long long P) {
    return mul(mul(m, (P / p), P), getInv(P / p, p), P);
void init(long long pi, long long pk) {
    Pre[0] = 1;
    for (int i = 1; i <= pk; i ++) {
        Pre[i] = Pre[i - 1];
        if(i % pi) Pre[i] = mul(Pre[i], i, pk);
```

```
long long Mul(long long n, long long pi, long long pk) {
    if(n <= 1) return 1;</pre>
    long long ans = Pow(Pre[pk], n / pk, pk);
    if(n % pk) ans = mul(ans, Pre[n % pk], pk);
    return mul(ans, Mul(n / pi, pi, pk), pk);
long long C(long long n, long long m, long long pi, long
     long pk) {
    if(n < m) return 0;</pre>
    init(pi, pk);
    long long r = 0;
    for(long long i = n; i; i \neq pi) r += i \neq pi;
    for(long long i = m; i; i \neq pi) r = i \neq pi;
    for(long long i = n - m; i; i \neq pi) r = i \neq pi;
    long long a = Mul(n, pi, pk);
    long long b = getInv(Mul(m, pi, pk), pk);
    long long c = getInv(Mul(n - m, pi, pk), pk);
    long long ans = mul(mul(a, b, pk), c, pk);
    return mul(ans, Pow(pi, r, pk), pk);
long long ex_lucas(long long n, long long m, long long P)
     {
//C_n^m %p
    long long ans = 0;
    long long p = P;
    for (int i = 2; i \Leftarrow P; i \leftrightarrow +) {
        if(p % i == 0) {
            long long pi = i, pk = 1;
            while(p \% i == 0) {
                p /= i;
                pk *= i;
            }
            ans = (ans + CRT(C(n, m, pi, pk), pk, P)) % P
        }
   }
    return ans;
```

```
long long gcd(long long a, long long b) {
   return !b ? a : gcd(b, a %b);
long long exgcd(long long a, long long b, long long &x,
    long long &y) {
    if(!b) {
       x = 1;
       y = 0;
        return a;
   long long d = exgcd(b, a \% b, x, y);
   long long t = x;
   x = y;
   y = t - (a / b) * y;
   return d;
long long getInv(long long a, long long p) {
   long long x, y;
    exgcd(a, p, x, y);
   x = (x \% p + p) \% p;
   return x;
long long exCrt() {
    for (long long i = 2; i <= n; i ++) {
        long long M1 = M[i - 1], M2 = M[i];
        long long C1 = C[i - 1], C2 = C[i];
        long long T = gcd(M1, M2);
        long long t = (C2 - C1 \% M2 + M2) \% M2;
        if(t % T) return −1;
        M[i] = M1 / T * M2;
        C[i] = mul(getInv(M1 / T, M2 / T), t / T, (M2 / T)
             ));
        C[i] = C[i] * M1 + C1;
        C[i] = (C[i] \% M[i] + M[i]) \% M[i];
   }
   return C[n];
```

#### 2.9 扩展中国剩余定理

```
long long M[maxn], C[maxn];//模数, 余数
long long mul(long long a, long long b, long long p) {
    if(b < 0) b = -b;
    long long ans = 0;
    while(b) {
        if(b & 1) ans = (ans + a) % p;
        a = (a + a) % p;
        b >>= 1;
    }
    return ans;
```

# 2.10 快速幂

```
long long Ksm(long long a, long long b, long long mod) {
    long long res = 1;
    while(b) {
        if(b & 1) res = Ksc(res, a, mod);
        a = Ksc(a, a, mod);
        b >>= 1;
    }
    return res;
}
```

# 2.11 快速傅里叶变换

```
const int maxn = 5e5 + 5;
const int inf = 0x3f3f3f3f;
const int mod = 1e9 + 7;
typedef complex<double> cp;
const double PI = acos(-1);
char sa[maxn], sb[maxn];
int n = 1, lena, lenb, res[maxn];
cp a[maxn], b[maxn], omg[maxn], inv[maxn];
void init() {
   for (int i = 0; i < n; i ++) {
       omg[i] = cp(cos(2*PI*i/n), sin(2*PI*i/n));
        inv[i] = conj(omg[i]);
   }
void fft(cp *a, cp *omg) {
   int lim = 0;
   while((1<<li>1 < n) lim++;</pre>
   for (int i = 0; i < n; i ++) {
       int t = 0;
        for (int j = 0; j < lim; j ++)
            if((i>>j) & 1) t |= (1<<(lim-j-1));
        if(i < t) swap(a[i], a[t]);</pre>
    for (int l = 2; l <= n; l *= 2) {
        int m = 1 / 2;
        for (cp *p = a; p != a + n; p += 1)
            for (int i = 0; i < m; i ++) {
                cp t = omg[n/l*i] * p[i+m];
                p[i+m] = p[i] - t;
                p[i] += t;
            }
   }
int main() {
   scanf("%d", &n);
   scanf("%s%s", sa, sb);
   lena = lenb = n;
   n = 1;
   while(n < lena + lenb) n <<= 1;</pre>
   for (int i = 0; i < lena; i ++)
       a[i].real(sa[lena-1-i] - '0');
    for (int i = 0; i < lenb; i ++)</pre>
       b[i].real(sb[lenb-1-i] - '0');
    init();
    fft(a, omg);fft(b, omg);
    for (int i = 0; i < n; i ++)
       a[i] *= b[i];
   fft(a, inv);
    for (int i = 0; i < n; i ++) {</pre>
        res[i] += floor(a[i].real()/n + 0.5);
        res[i+1] += res[i] / 10;
```

```
res[i] %= 10;
}
int pos = n - 1;
while(!res[pos]) pos—;
for (int i = pos; i >= 0; i —) printf("%d", res[i]);
puts("");
return 0;
}
```

#### 2.12 快速乘

```
long long Ksc(long long a, long long b, long long mod) {
     //普通版
    long long ans = 0;
    while(b) {
       if(b \& 1) ans = (ans + a) \% mod;
       b >>= 1:
       a = (a + a) \% mod;
    return ans;
long long Ksc(long long a, long long b, long long mod){//
    long long L = a * (b >> 2511) \% mod * (111 << 25) %
        mod;
    long long R = a * (b & ((111 << 25) - 1)) % mod;
    return (L + R) % mod;
long long Ksc(long long a, long long b, long long mod) {
    //精确版
    a %= mod, b %= mod;
    return ((a * b - (long long)((long long)((long double
         )a / mod * b + 1e-3) * mod)) % mod + mod) % mod;
}
```

## 2.13 卢卡斯定理

```
long long exgcd(long long a, long long b, long long &x,
    long long &y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    long long ans = exgcd(b, a % b, x, y);
    long long temp = x;
    x = y;
    y = temp - (a / b) * y;
    return ans;
```

```
long long inv(long long a) {
   long long x, y;
   long long t = exgcd(a, M, x, y);
   if (t != 1) {
       return -1;
   return (x % M + M) % M;
long long fac[maxn];
void getfac() {
   fac[0] = 1;
   for (int i = 1; i < maxn; i++) {
        fac[i] = fac[i - 1] * i % M;
long long C(long long n, long long m) {
   if (n < 0 || m < 0 || n < m) {</pre>
       return 0:
   return fac[n] * inv(fac[m]) % M * inv(fac[n - m]) % M
long long lucas(long long n, long long m) {
   if (m == 0) {
       return 1;
   return (lucas(n / M, m / M) * C(n % M, m % M)) % M;
```

# 2.14 十进制快速幂

```
const int maxn = 1e6 + 5; //矩阵
long long mod;
struct Matrix{
   long long mat[2][2];
   Matrix() {memset(mat, 0, sizeof(mat));};
   void init() {
       mat[0][0] = mat[1][1] = 1;
   void init(long long a, long long b) {
       mat[0][0] = 0; mat[0][1] = b;
       mat[1][0] = 1; mat[1][1] = a;
   void operator = (Matrix x) {
       for (int i = 0; i <= 1; i ++)
            for (int j = 0; j <= 1; j ++)
               mat[i][j] = x.mat[i][j];
};
void Print(Matrix x) {
```

```
for (int i = 0; i <= 1; i ++) {
        for (int j = 0; j <= 1; j ++)
            cout << x.mat[i][j] << " ";</pre>
        cout << endl;</pre>
    }
}
Matrix operator * (Matrix x, Matrix y) {
    Matrix t;
    for (int i = 0; i <= 1; i ++)
        for (int j = 0; j <= 1; j ++)
            for (int k = 0; k \le 1; k ++)
                t.mat[i][j] = (t.mat[i][j] + x.mat[i][k]
                     * y.mat[k][j]) % mod;
    return t;
Matrix Ksm(Matrix x, long long b) {
    Matrix t; t.init();
    while(b) {
        if(b \& 1) t = t * x;
        X = X * X;
        b >>= 1;
    }
    return t;
}
int main() {
    long long x0, x1, a, b;
    scanf("%11d %11d %11d", &x0, &x1, &a, &b);
    char s[maxn];
    scanf("%s%11d", s, &mod);
    int len = strlen(s);
    reverse(s, s+len);
    Matrix t, ans; t.init(a, b);
    ans.mat[0][0] = x0; ans.mat[0][1] = x1;
    Matrix res;
    res.init();
    for (int i = 0; i < len; i ++) {
        res = res * Ksm(t, s[i]-'0');
        t = Ksm(t, 10);
    ans = ans * res;
    printf("%11d\n", ans.mat[0][0]);
    return 0:
```

#### 2.15 二次剩余

```
struct T{
    long long p, d;
};
long long Ksm(long long a, long long b, long long p) {
    long long res = 1;
```

```
while(b) {
       if(b & 1) res = res * a % p;
       a = a * a % p;
       b >>= 1;
   return res;
long long w;
T Mul_er(T a, T b, long long p) {//二次域乘法
   T ans:
   ans.p = (a.p * b.p + a.d * b.d % p * w % p) % p;
   ans.d = (a.p * b.d \% p + a.d * b.p \% p) \% p;
   return ans;
T Ksm_er(T a, long long b, long long p) {//二次域快速幂
   ans.p = 1; ans.d = 0;
   while(b) {
       if(b & 1) ans = Mul_er(ans, a, p);
       a = Mul_er(a, a, p);
       b >>= 1;
   }
   return ans;
long long Legendre(long long a, long long p) {//求勒让德
   return Ksm(a, (p-1)>>1, p);
long long Recever(long long a, long long p) {
   a %= p;
   if(a < 0) a += p;
   return a;
long long solve(long long n, long long p) {
   if(n % p == 0) return 0;
   if(p == 2) return 1;
   if(Legendre(n, p) + 1 == p) return -1;
   long long a = -1, t;
   while(1) {
       a = rand() \% p;
       t = a * a - n;
       w = Recever(t, p);
       if(Legendre(w, p) + 1 == p) break;
   }
   T tmp;
   tmp.p = a; tmp.d = 1;
   T ans = Ksm_er(tmp, (p+1)>>1, p);
   return ans.p;
```

# 2.16 三分

```
double Com(double X) {
   return sqrt((X - x) * (X - x) + (a * X * X + b * X +
         (c - y) * (a * X * X + b * X + c - y);
void Binary(double 1, double r) {
   if(l + eps \ll r) {
        double lm, rm;
        double k = r - 1;
        lm = l + (1./3) * k;
        rm = r - (1./3) * k;
        if(fabs(Com(lm) - Com(rm)) <= eps) {</pre>
             printf("%.31f\n", Com(lm));
             return ;
        if(Com(lm) < Com(rm))</pre>
            Binary(l, rm);
        else Binary(lm, r);
   }
}
```

## 2.17 min25 筛

```
#define inv_2 (Mod+1)/2
#define inv_6 (Mod+1)/6
long long sqr, m, w[maxn], g[maxn], h[maxn];
long long sumg[maxn], sumh[maxn], id1[maxn], id2[maxn];
long long prim[maxn], tot;
bool mark[maxn];
long long Add(long long a, long long b) {
    return (a + b) % Mod;
long long Sup(long long a, long long b) {
    return (a - b + Mod) \% Mod;
long long Pow(long long a, long long b) {
    long long res = 1;
    while(b) {
       if(b \& 1) res = res * a % Mod;
       a = a * a % Mod;
       b >>= 1;
    }
    return res;
void init(long long n) {
    mark[1] = 1;
    for (long long i = 2; i <= n; i ++) {
        if(!mark[i]) {
            prim[++tot] = i;
            sumg[tot] = (sumg[tot-1] + i * i) % Mod;
            sumh[tot] = (sumh[tot-1] + i) \% Mod;
        }
```

```
for (long long j = 1; j \ll tot; j \leftrightarrow j
            if(i * prim[j] > n) break;
            mark[i * prim[j]] = 1;
            if(i % prim[j] == 0) break;
   }
void GetW(long long n) {
    for (long long i = 1, j; i \le n; i = j + 1) {
        j = n / (n / i);
        w[++m] = n / i;
        long long t = w[m] % Mod;
        g[m] = t * (t + 1) % Mod * ((2LL * t + 1) % Mod)
             % Mod * inv_6 % Mod;
        g[m] —;
        h[m] = t * (t + 1) % Mod * inv_2 % Mod;
        h[m] \longrightarrow;
        if(w[m] \leftarrow sqr) id1[w[m]] = m;
        else id2[n/w[m]] = m;
   }
void GetG(long long n) {
    for (long long i = 1; i <= tot; i ++) {</pre>
        for (long long j = 1; j \leftarrow m \&\& prim[i] * prim[i]
              <= w[j]; j ++) {
            long long d = w[j] / prim[i];
            long long id = d \le sqr ? id1[d] : id2[n/d];
            g[j] = Sup(g[j], prim[i] * prim[i] % Mod * ((
                  g[id] - sumg[i-1] + Mod) % Mod) % Mod);
            h[j] = Sup(h[j], prim[i] * ((h[id] - sumh[i]))
                  -1] + Mod) % Mod) % Mod);
        }
   }
long long S(long long x, long long y, long long n) {
    if(x <= prim[y-1] \mid \mid x <= 1) return 0;
    long long id = x \le sqr ? id1[x] : id2[n/x];
    long long res = (g[id] - h[id] + Mod - sumg[y-1] +
         sumh[y-1] + Mod) % Mod;
    for (long long i = y; i <= tot && prim[i] * prim[i]</pre>
         <= x; i ++) {
        long long t = prim[i];
        for (long long j = 1; t <= x; j ++, t = t * prim[</pre>
             i]) {
            long long p1 = t % Mod;
            res = Add(res, p1 * (p1 - 1) % Mod * (S(x/t),
                 i+1, n) + (j != 1)) % Mod);
   }
    return res % Mod;
int main(int argc, char *args[]) {
    long long n;
    scanf("%11d", &n);
```

```
sqr = sqrt(n);
init(sqr);
GetW(n);
GetG(n);
printf("%11d\n", (S(n, 1, n) + 1) % Mod);
return 0;
}
```

## 2.18 SG 函数

```
int SG[maxn], S[maxn];
int f[maxn];
void ff() {//f是每一次走的步数
    f[0] = 1;
    for (int i = 1; i <= 10; i ++)
        f[i] = f[i-1] * 2;
void getSG(int n) {
    ff();
    for (int i = 1; i \ll n; i \leftrightarrow ++) {
        memset(S, 0, sizeof(S));
        for (int j = 0; f[j] <= i && j <= 10; j ++)
            S[SG[i-f[j]]] = 1;
        for (int j = 0; ; j ++)
             if(!S[j]) {
                 SG[i] = j;
                 break;
    }
}
int main() {
   int n;
    getSG(1000);
    while(cin >> n) {
        if(SG[n]) cout << "Kiki\n";</pre>
        else cout << "Cici\n";</pre>
    return 0;
```

#### 2.19 Pollard Rho

```
long long Ksm(long long a, long long b, long long mod) {
   long long res = 1;
   while(b) {
       if(b & 1) res = Ksc(res, a, mod);
       a = Ksc(a, a, mod);
       b >>= 1;
   return res;
bool check(long long a, long long n, long long x, long
    long t) {
   long long ret = Ksm(a, x, n);
   long long last = ret;
    for(int i = 1; i <= t; i ++) {</pre>
        ret = Ksc(ret, ret, n);
        if(ret == 1 && last != 1 && last != n-1) return
            true;//是合数
        last = ret;
   }
   if(ret != 1) return true;
   return false;
bool Miller_Rabin(long long n) {//判素数
   if(n < 2) return false;</pre>
   if(n == 2) return true;
   if((n&1) == 0)return false;
   long long x = n - 1;
   long long t = 0;
   while((x&1) == 0) \{
       x >>= 1;
        t ++;
    for(int i = 0; i < S; i ++) {</pre>
        long long a = rand() \% (n-1) + 1;
        if(check(a, n, x, t))//如果检查出来是合数
       return false;
   }
   return true;
long long gcd(long long a,long long b) {
   if(a == 0) return 1;
   if(a < 0) return gcd(-a, b);</pre>
   while(b) {
       long long t = a % b;
       a = b;
       b = t;
   }
   return a;
long long pollard_rho(long long x,long long c) {
    long long i = 1, k = 2;
    long long x0 = rand() \% x;
   long long y = x0;
```

```
while(1) {
        x0 = (Ksc(x0, x0, x) + c) % x;
        long long d = gcd(y-x0, x);
       if(d != 1 && d != x) return d;
        if(y == x0) return x;
        if(i == k) {
           y = x0;
            k += k;
        }
   }
void findphi(long long n) {
   if(Miller_Rabin(n)) {
        factor[tot++] = n;
        return;
   long long p = n;
   while(p >= n) {
       p=pollard_rho(p, rand()%(n-1)+1);
   findphi(p);
   findphi(n/p);
int main() {
   long long n;
   while(scanf("%I64d",&n)!=E0F) {
        tot=0;
        findphi(n);
        for(int i=0;i<tot;i++)</pre>
        printf("%I64d",factor[i]),printf("\n");
        if(Miller_Rabin(n))printf("yes\n");
        else printf("no\n");
   }
   return 0;
```

#### 2.20 NTT

```
const int maxn = 1e5 + 5;
const int inf = 0x3f3f3f3f;
const int mod = 998244353;
#define Mod(x) ((x)>=mod?(x)-mod:(x))
#define g 3
int rnk[maxn];
long long a[maxn], b[maxn];
long long Ksm(long long a, long long b) {
    long long res = 1;
    while(b) {
        if(b & 1) res = res * a % mod;
        a = a * a % mod;
}
```

```
b >>= 1;
   return res;
void init(int n) {
   memset(rnk, 0, sizeof(rnk));
   int lim = 0;
   while((1<<li>1 < n) lim ++;</pre>
   for (int i = 0; i < n; i ++)
        rnk[i] = (rnk[i>>1]>>1) | ((i&1) << (lim-1));
void ntt(long long *a, int op, int n) {
   for (int i = 0; i < n; i ++)
       if(i < rnk[i]) swap(a[i], a[rnk[i]]);</pre>
    for (int i = 2; i <= n; i <<= 1) {
        int nw = Ksm(g, (mod-1)/i);
        if(op == -1) nw = Ksm(nw, mod-2);
        for (int j = 0, m = i >> 1; j < n; j += i)
            for (int k = 0, w = 1; k < m; k ++) {
                int t = 111 * a[j+k+m] * w % mod;
                a[j+k+m] = Mod(a[j+k]-t+mod);
                a[j+k] = Mod(a[j+k]+t);
                w = 111 * w * nw % mod;
            }
    if(op == -1)
        for (int i = 0, inv = Ksm(n, mod-2); i < n; i ++)
            a[i] = 111 * a[i] * inv % mod;
char s1[maxn], s2[maxn];
long long ans[maxn];
int main() {
   scanf("%s", s1);
   scanf("%s", s2);
   int len1 = strlen(s1), len2 =strlen(s2);
   int n = 1:
   while(n < len1 + len2) n <<= 1;</pre>
   init(n):
    for (int i = 0; i < len1; i ++) a[len1-i-1] = s1[i]-'
    for (int i = 0; i < len2; i ++) b[len2-i-1] = s2[i]-'
        0';
   ntt(a, 1, n); ntt(b, 1, n);
    for (int i = 0; i < n; i ++)
       a[i] = (111 * a[i] * b[i]) % mod;
   ntt(a, -1, n);
    for (int i = 0; i < n; i ++)
       cout << a[i] << " ";
   cout << endl;</pre>
    for (int i = 0; i < n; i ++) {
        ans[i+1] += ans[i] / 10;
        ans[i] %= 10;
    int pos = n-1;
```

```
while(!a[pos]) pos—;
    for (int i = pos; i \rightarrow 0; i \longrightarrow cout << a[i];
    cout << endl;</pre>
    return 0;
//三模NTT
#define long long long long
const long long maxn = 3 * 1e6 + 10;
#define swap(x,y) x \stackrel{}{} = y, y \stackrel{}{} = x, x \stackrel{}{} = y
using namespace std;
long long a[maxn], b[maxn];
long long Mul(long long a, long long b, long long mod) {
   a %= mod, b %= mod;
    return ((a * b - (long long)((long long)((long double
         )a / mod * b + 1e-3) * mod)) % mod + mod) % mod;
long long Ksm(long long a, long long p, long long mod) {
   long long base = 1;
    while(p) {
       if(p & 1) base = 1ll * a * base % mod;
        a = 111 * a * a % mod; p >>= 1;
    return base % mod:
}
namespace NTT{
    const long long P1 = 469762049, P2 = 998244353, P3 =
         1004535809, g = 3;
    const long long PP = 1ll * P1 * P2;
    long long n, m, p, len = 1, lim;
    long long tmp1[maxn], tmp2[maxn], ans[3][maxn], r[
    long long res[maxn], tmp[maxn], base[maxn];
        传的参数n,m都比实际个数少一
        n-;m;
        输入两个数n=1
        输入一个数n=0;
    void init(long long n) { //初始化, 传入alen+blen,得到
         最小的len
        len = 1; \lim = 0;
        while(len <= n) len <<= 1, lim++;</pre>
        for(long long i = 0; i \leftarrow len; i++) r[i] = (r[i
             >> 1] >> 1) | ((i & 1) << (lim - 1));
    void ntt_Mod(long long *a, const long long n, const
         long long type, const long long mod) { //ntt
        for(long long i = 0; i < n; i++) if(i < r[i])
             swap(a[i], a[r[i]]);
        for(long long mid = 1; mid < n; mid <<= 1) {</pre>
            long long W = Ksm(type == 1 ? g : Ksm(g, mod)
                 -2, mod), (mod - 1) / (mid << 1), mod)
```

```
for(long long j = 0; j < n; j += (mid << 1))
            long long w = 1;
            for(long long k = 0; k < mid; k++, w = 111
                  * w * W % mod) {
                long long x = a[j + k], y = 111 * w *
                       a[j + k + mid] \% mod;
                a[j + k] = (x + y) \% mod,
                a[j + k + mid] = (x - y + mod) \% mod;
            }
        }
    }
    if(type == -1) {
        long long inv = Ksm(n, mod - 2, mod);
        for(long long i = 0; i < n; i++)
            a[i] = 111 * a[i] * inv % mod;
}
void Out(long long *a, long long len) {
    for (int i = 0; i <= len; i ++)</pre>
        cout << a[i] << " ";
    cout << endl;</pre>
int ntt_Mul(long long *a, long long *b, long long
     alen, long long blen, long long mod) {
    init(alen + blen);
    memcpy(tmp1, a, sizeof(tmp1)); memcpy(tmp2, b,
         sizeof(tmp2));
    ntt_Mod(tmp1, len, 1, P1); ntt_Mod(tmp2, len, 1,
         P1):
    for(long long i = 0; i \leftarrow len; i++) ans[0][i] = 1
         ll * tmp1[i] * tmp2[i] % P1;
    memcpy(tmp1, a, sizeof(tmp1)); memcpy(tmp2, b,
         sizeof(tmp2));
    ntt_Mod(tmp1, len, 1, P2); ntt_Mod(tmp2, len, 1,
         P2):
    for(long long i = 0; i \leftarrow len; i++) ans[1][i] = 1
         ll * tmp1[i] * tmp2[i] % P2;
    memcpy(tmp1, a, sizeof(tmp1)); memcpy(tmp2, b,
         sizeof(tmp2));
    ntt_Mod(tmp1, len, 1, P3); ntt_Mod(tmp2, len, 1,
         P3):
    for(long long i = 0; i \le len; i++) ans[2][i] = 1
         ll * tmp1[i] * tmp2[i] % P3;
    ntt_Mod(ans[0], len, -1, P1);
    ntt_Mod(ans[1], len, -1, P2);
    ntt_Mod(ans[2], len, -1, P3);
    for(long long i = 0; i \le alen + blen; i++) {
        long long t = (Mul(111 * ans[0][i] * P2 % PP,
               Ksm(P2 \% P1, P1 - 2, P1), PP) +
                Mul(111 * ans[1][i] * P1 % PP, Ksm(P1
                       % P2, P2 - 2, P2), PP) ) % PP;
        long long K = ((ans[2][i] - t) \% P3 + P3) \%
             P3 * Ksm(PP \% P3, P3 - 2, P3) \% P3;
```

```
a[i] = (t \% mod + ((K \% mod) * (PP \% mod)) \%
                 mod ) % mod;
        }
        return alen + blen;
    int ntt_Ksm(long long *a, long long b, int blen, long
          long mod) {
        memcpy(base, a, sizeof(base));
        memset(a, 0, maxn*sizeof(a));
        a[0] = 1; int alen = 0;
        while(b) {
            if(b & 1) alen = ntt_Mul(a, base, alen, blen,
                  mod):
            memcpy(tmp, base, sizeof(tmp));
            blen = ntt_Mul(base, tmp, blen, blen, mod);
            b >>= 1;
        }
        return alen;
    }
}
int main() {
    long long n, m, p;
    scanf("%11d %11d %11d", &n, &m, &p);
    for(long long i = 0; i \le n; i++) scanf("%11d", &a[i
         1);
    for(long long i = 0; i \leftarrow m; i++) scanf("%11d", &b[i
         ]);
    NTT::ntt_Mul(a, b, n, m, p);
    for (int i = 0; i \le n + m; i ++)
        printf("%11d ", a[i]);
    printf("\n");
    return 0;
```

# 2.21 Miller Rabin

```
bool Miller_Rabin(long long n) {
    if(n <= 2) {
        if(n == 2) return true;
        return false;
    if(n % 2 == 0) return false;
    long long u = n - 1;
    while(u % 2 == 0) u /= 2;
    int S = 100;
    srand((long long)time(0));
    for (int i = 1; i \le S; i ++){
        long long a = rand() \% (n - 2) + 2;
        long long x = Ksm(a, u, n);
        while(u < n) {</pre>
            long long y = Ksm(x, 2, n);
            if(y == 1 \&\& x != 1 \&\& x != n - 1)
                return false;
            x = y;
            u = u * 2;
        }
        if(x != 1) return false;
    }
    return true;
```

# 2.22 BigInteger

```
#define MAXN 999
#define MAXSIZE 100240
#define DLEN 3
struct BigInt{
   int a[MAXSIZE],len;
   bool flag;
   BigInt() {
       len = 1;
        memset(a, 0, sizeof(a));
        flag = 0;
   BigInt (const int b) {
        int c, d = b;
        len = 0;
        memset(a, 0, sizeof(a));
        if(!b) {
            len = 1;
            return ;
        while(d) {
            a[len++] = d \% (MAXN + 1);
            d = (MAXN+1);
   }
```

```
BigInt(const char *s) {
    int t, k, index, 1;
    memset(a, 0, sizeof(a));
    l = strlen(s);
    len = 1/DLEN;
    if(1 % DLEN) len ++;
    index = 0:
    for (int i = l - 1; i >= 0; i -= DLEN) {
        t = 0;
        k = i - DLEN + 1;
        if(k < 0) k = 0;
        for (int j = k; j \leftarrow i; j \leftrightarrow t = t * 10 + s[
             j] - '0';
        a[index ++] = t;
    }
}
BigInt (const BigInt& T) {
    memset(a, 0, sizeof(a));
    len = T.len;
    for (int i = 0; i < len; i ++) a[i] = T.a[i];
bool operator < (const BigInt &T) const {</pre>
    int ln:
    if(len < T.len) return 233;</pre>
    if(len == T.len) {
        ln = len - 1;
        while(ln >= 0 \& a[ln] == T.a[ln]) — ln;
        if(ln >= 0 && a[ln] < T.a[ln]) return 233;</pre>
        return 0;
    }
    return 0;
inline bool operator < (const int &t) const {</pre>
    BigInt tee(t);
    return *this < tee;</pre>
BigInt& operator = (const BigInt &T) {
    memset(a, 0, sizeof(a));
    len = T.len;
    for (int i = 0; i < len; i ++) a[i] = T.a[i];
    return *this;
BigInt operator + (const BigInt &T) const {
    BigInt t(*this);
    int big = len;
    if(T.len > len) big = T.len;
    for (int i = 0; i < big; i ++) {
        t.a[i] += T.a[i];
        if(t.a[i] > MAXN) {
            ++t.a[i + 1];
            t.a[i] -= MAXN + 1;
        }
    if(t.a[big]) t.len = big + 1;
```

```
else t.len = big;
    return t;
BigInt operator - (const BigInt &T) const {
    int big;
    bool ctf;
    BigInt t1, t2;
    if(*this < T) {</pre>
        t1 = T;
        t2 = *this;
        ctf = 1;
    }else {
        t1 = *this;
        t2 = T;
        ctf = 0;
    big = t1.len;
    int j = 0;
    for (int i = 0; i < big; i ++) {</pre>
        if(t1.a[i] < t2.a[i]) {</pre>
            j = i + 1;
            while(t1.a[j] == 0) ++j;
            — t1.a[j—];
            while(j > i) t1.a[j --] += MAXN;
            t1.a[i] += MAXN + 1 - t2.a[i];
        }else t1.a[i] -= t2.a[i];
    t1.len = big;
    while(t1.len > 1 && t1.a[t1.len - 1] == 0) {
        — t1.len;
        -- big;
    if(ctf) t1.a[big - 1] = -t1.a[big - 1];
    return t1;
BigInt operator * (const BigInt &T) const {
    BigInt res;
    int up;
    int te, tee;
    for (int i = 0; i < len; i ++) {
        up = 0;
        for (int j = 0; j < T.len; <math>j ++) {
            te = a[i] * T.a[j] + res.a[i + j] + up;
            if(te > MAXN) {
                tee = te - te / (MAXN + 1) * (MAXN +
                     1);
                up = te / (MAXN + 1);
                res.a[i + j] = tee;
            }else {
                up = 0;
                res.a[i + j] = te;
        if(up) res.a[i + T.len] = up;
```

```
}
        res.len = len + T.len;
        while(res.len > 1 && res.a[res.len - 1] == 0) —
             res.len:
        return res;
    }
    BigInt operator / (const int &b) {
        BigInt res;
        int sum = 0, newlen = 0;
        for (int i = len-1; i >= 0; i ---) {
            sum = sum * (MAXN+1) + a[i];
            if(sum < b) res.a[i] = 0;
            else {
                if(!newlen) newlen = i + 1;
                res.a[i] = sum / b;
                sum \%= b;
            }
        }
        res.len = max(newlen, 1);
        return res;
    int operator % (const int &b) const {
        int d = 0:
        for (int i = len - 1; i >= 0; i \longrightarrow)
            d = (d * (MAXN + 1) % b + a[i]) % b;
        return d;
    BigInt operator ^ (const int &n) const {
        BigInt t(n), res(1);
        int y = n;
        while(y) {
            if(y & 1) res = res * t;
            t = t * t;
            y >>= 1;
        }
        return res;
    }
    inline void print() {
        printf("%d", a[len - 1]);
        for (int i = len - 2; i >= 0; i \longrightarrow)
            printf("%03d", a[i]);
        printf("\n");
    }
};
```

#### 2.23 BSGS

```
map<long long, long long> Hash;
long long Mul(long long a, long long b, long long p) {
   long long L = a * (b >> 25LL) % p * (1LL << 25) % p;
   long long R = a * (b & ((1LL << 25) - 1)) % p;</pre>
```

```
return (L + R) % p;
long long Pow(long long a, long long b, long long p) {
   long long res = 1;
    while(b) {
        if(b & 1) res = Mul(res, a, p);
        a = Mul(a, a, p);
        b >>= 1:
    return res;
}
get ans for a^ans = b % p
A^{iS-j} = B \mod p \quad A^{iS} = B*A^{j} \mod p
A^{iS+j} = B \mod p
long long BSGS(long long a, long long b, long long p) {
   long long m = sqrt(p) + 1;
    long long res = 1;
    for (int j = 0; j <= m; j ++) {
        Hash[Mul(b, res, p)] = j;
        res = Mul(res, a, p);
    for (int i = 1; i <= m; i ++) {
        long long k = Pow(a, i * m, p);
        if(Hash.count(k))
            return i * m - Hash[k];
```

#### 2.24 BM

```
//BM模板
//a[n] = f[1]a[n-1] + f[2]a[n-2] + f[3]a[n-3] + ...+ f[k]
    a[n-k]
//有限项
const long long mod = 998244353;
#define sz(x) ((int)(x).size())
typedef vector<long long> VI;
long long Ksm(long long a, long long b) {
   long long res = 1; a %= mod;
   assert(b >= 0);
    while(b) {
       if(b & 1) res = res * a % mod;
       a = a * a \% mod;
       b >>= 1;
    return res;
int _, n;
```

```
namespace Linear_Seq{
   const int N = 10010;
   long long res[N], base[N], _c[N], _md[N];
   vector<int> Md;
   void Mul(long long *a, long long *b, int k) {
        for (int i = 0; i < k+k; i ++) _c[i] = 0;
        for (int i = 0; i < k; i ++)
            if(a[i]) for (int j = 0; j < k; j ++)
                _{c[i+j]} = (_{c[i+j]} + a[i]*b[j]) % mod;
        for (int i = k + k - 1; i >= k; i ---)
            if(_c[i]) for (int j = 0; j < sz(Md); j ++)
                _c[i-k+Md[j]] = (_c[i-k+Md[j]] - _c[i] *
                     _md[Md[j]]) % mod;
        for (int i = 0; i < k; i ++)
            a[i] = _c[i];
   int solve(long long n, VI a, VI b) {
        long long ans = 0, pnt = 0;
        int k = sz(a);
        assert(sz(a) == sz(b));
        for (int i = 0; i < k; i ++) _md[k-1-i] = -a[i];
        _{md[k]} = 1; Md.clear();
        for (int i = 0; i < k; i ++)
            if(_md[i]) Md.push_back(i);
        for (int i = 0; i < k; i ++) res[i] = base[i] =</pre>
             0;
        res[0] = 1;
        while((1ll<<pnt) <= n) pnt ++;</pre>
        for (int p = pnt; p \ge 0; p \longrightarrow ) {
           Mul(res, res, k);
            if((n>>p) & 1) {
                for (int i = k-1; i >= 0; i ---) res[i+1]
                     = res[i];
                res[0] = 0;
                for (int j = 0; j < sz(Md); j ++)
                    res[Md[j]] = (res[Md[j]] - res[k] *
                         _md[Md[j]]) % mod;
           }
        }
        for (int i = 0; i < k; i ++) ans = (ans + res[i]
             * b[i]) % mod;
        if(ans < 0) ans += mod;
        return ans;
   VI BM(VI s) {
        VI C(1, 1), B(1, 1);
        int L = 0, m = 1, b = 1;
        for (int n = 0; n < sz(s); n ++) {
            long long d = 0;
            for (int i = 0; i < L + 1; i ++) d = (d + (
                 long long)C[i] * s[n-i]) % mod;
            if (d == 0) ++m;
            else if(2 * L \le n) {
                VI T = C;
```

```
long long c = mod - d * Ksm(b, mod-2) %
                while(sz(C) < sz(B) + m) C.push_back(0);
                for (int i = 0; i < sz(B); i ++) C[i+m] =</pre>
                      (C[i+m] + c * B[i]) % mod;
               L = n + 1 - L; B = T;
               b = d; m = 1;
            }else {
               long long c = mod - d * Ksm(b, mod-2) %
                while(sz(C) < sz(B) + m) C.push_back(0);
                for (int i = 0; i < sz(B); i ++) C[i+m] =
                      (C[i+m] + c * B[i]) \% mod;
           }
       }
       return C;
   }
   int Gao(VI a, long long n) { //得到第n项
       VI c = BM(a);
       c.erase(c.begin());
        for (int i = 0; i < sz(c); i ++) c[i] = (mod-c[i])
            ]) % mod;
       return solve(n, c, VI(a.begin(), a.begin()+sz(c))
            );
   }
};
using namespace Linear_Seq;
void solve() { //预处理前3k项
   long long n, k;
   scanf("%11d %11d", &n, &k);
   VI v, f;
   f.push_back(0);
    for (int i = 0; i < k; i ++) { //f只有k项
       long long x;
       scanf("%11d", &x);
       f.push_back(x);
   for (int i = 0; i < k; i ++) { //a的前k项
       long long x;
       scanf("%11d", &x);
       v.push_back(x);
   for (int i = k; i \le 2 * k; i ++) {//a的前3k项
       long long x = 0;
        for (int j = 1; j <= k; j ++)
           x = (x + f[j] * v[i-j]) % mod;
       v.push_back(x);
   printf("%11d\n", Gao(v, n));
int main() {
    solve();
    return 0;
```

```
//另一个板子
#define maxk 100005
#define maxn 200005
const int mod = 998244353;
#define mul(x, y) static_cast<long long> (x) * (y) % mod
namespace Math {
        inline int pw(int base, int p) {
                static int res; res = 1;
        while(p) {
            if(p & 1) res = mul(res, base);
            base = mul(base, base);
            p >>= 1:
        }
                return res;
        }
        inline int inv(int x) { return pw(x, mod - 2); }
inline void reduce(int &x) { x += x >> 31 & mod; }
namespace Poly {
#define N maxn
        int lim, s, rev[N], Wn[N];
        inline void init(const int n) { //初始化
                \lim = 1, s = -1;
        while (lim < n) lim <<= 1, ++s;</pre>
                for (register int i = 1; i < lim; ++i)</pre>
            rev[i] = rev[i >> 1] >> 1 | (i & 1) << s;
                const int t = Math::pw(3, (mod - 1) / lim)
                      ); *Wn = 1;
        for (register int *i = Wn + 1; i != Wn + lim; ++i
            *i = mul(*(i - 1), t);
        }
        inline void FFT(int *A, const int op = 1) { //FFT
                for (register int i = 1; i < lim; ++i)</pre>
            if (i < rev[i]) std::swap(A[i], A[rev[i]]);</pre>
                for (register int mid = 1; mid < lim; mid</pre>
                       <<= 1) {
                         const int t = lim / mid >> 1;
                         for (register int i = 0; i < lim;</pre>
                               i += mid << 1)
                                 for (register int j = 0;
                                      j < mid; ++j) {</pre>
                                         const int X = A[i
                                               + j], Y =
                                               mul(A[i + j
                                               + mid], Wn[t
                                               * j]);
                                         reduce(A[i + j]
                                              += Y - mod),
                                               reduce(A[i
                                               + j + mid] =
                                                X - Y);
                                 }
```

```
}
        if (!op) {
                const int ilim = Math::inv(lim);
                for (register int *i = A; i != A
                     + lim; ++i) *i = mul(*i,
                     ilim):
                std::reverse(A + 1, A + lim);
        }
void INV(int *A, int *B, int n) { //多项式A求逆
     到B, [0,n-1]
        if (n == 1) { *B = Math::inv(*A); return
             ; }
        static int C[N], D[N];
        const int len = n + 1 \gg 1;
        INV(A, B, len), init(len * 3);
        std::memcpy(C, A, n << 2), std::memset(C</pre>
             + n, 0, lim - n << 2);
        std::memcpy(D, B, len << 2), std::memset(</pre>
             D + len, 0, lim - len << 2);
        FFT(C), FFT(D);
        for (int i = 0; i < lim; ++i) D[i] = (2 -
              mul(D[i], C[i]) + mod) * D[i] % mod
        FFT(D, 0);
        std::memcpy(B + len, D + len, n - len <<
             2);
int G[N], INVG[N];
void DIV(int *A, int *Q, int n, int m) {
        static int C[N];
        const int len = n - m + 1;
        std::reverse_copy(A, A + n, C), std::
             memset(C + len, 0, lim - len \ll 2);
        FFT(C);
        for (int i = 0; i < lim; ++i) Q[i] = mul(</pre>
             C[i], INVG[i]);
        FFT(Q, 0), std::reverse(Q, Q + len);
void DIV_MOD(int *A, int *R, int n, int m) {
        static int Q[N];
        const int len = n - m + 1;
        DIV(A, Q, n, m), std::memset(Q + len, 0,
             lim - len << 2);
        FFT(Q);
        for (int i = 0; i < lim; ++i) R[i] = mul(</pre>
             G[i], Q[i]);
        FFT(R, 0);
        for (int i = 0; i < m; ++i) reduce(R[i] =
              A[i] - R[i];
void POW(int *A, int p, int m) {
        if (!p) return;
        POW(A, p \gg 1, m);
```

```
static int T[N];
                std::memcpy(T, A, m << 2), std::memset(T</pre>
                     + m, 0, \lim - m << 2);
                FFT(T);
                for (int i = 0; i < lim; ++i) T[i] = mul(</pre>
                     T[i], T[i]);
                FFT(T, 0);
                if (p & 1) {
                        for (int i = 2 * m - 1; \sim i; -i)
                             T[i] = T[i - 1];
                        T[0] = 0;
                DIV_MOD(T, A, 2 * m, m + 1);
        }
        int solve(int *f, int *a, int n, int k) { //a为遂
             推式0~k-1项, f为转移数组1~k项
                static int A[maxn], B[maxn];
                for (int i = 1; i \le k; ++i) reduce(G[k - k])
                      i] = -f[i];
                G[k] = A[0] = 1;
                std::reverse_copy(G, G + k + 1, B), B[k]
                     = 0;
                INV(B, INVG, k), init(k << 1);</pre>
                FFT(G), FFT(INVG);
                Poly::POW(A, n, k);
                int ans = 0;
                for (int i = 0; i < k; ++i) reduce(ans +=
                      mul(A[i], a[i]) - mod);
                return ans;
#undef N
}
int n, k;
int f[maxk], a[maxk];
int main() {
        /* 能求线性递推和mod 998244353的多项是求逆, 其他
             的好
        像可以求, 但是不会, 先打个板子, 以后再说把*/
    // int n;
    // scanf("%d", &n);
    // for (int i = 0; i < n; i ++)
          scanf("%d", &a[i]);
    // Poly::INV(a, f, n);
    // for (int i = 0; i < n; i ++)
          printf("%d ", f[i]);
    // printf("\n");
    // a(n)=f(i)*a(n-i) {1<=i<=k}
        std::ios::sync_with_stdio(false), std::cin.tie(0)
             , std::cout.tie(0);
        std::cin >> n >> k;
        for (int i = 1; i <= k; ++i) std::cin >> f[i];
        for (int i = 0; i < k; ++i) std::cin >> a[i],
             reduce(a[i]);
        std::cout << Poly::solve(f, a, n, k) << '\n';</pre>
```

```
return 0;
}
```

# 3 数据结构

# 3.1 线段树套伸展树

```
/* BZOJ 3196 (线段树套伸展树)
1. 查询k在区间内的排名
2. 查询区间内排名为k的值
3.修改某一位值上的数值
4.查询k在区间内的前驱(前驱定义为小于X, 且最大的数)
5.查询k在区间内的后继(后继定义为大于X, 且最小的数) */
#include <bits/stdc++.h>
const int inf = 2147483647;
const int maxn = 5e4 + 5;
const int maxm = maxn * 25;
int n;
int arr[maxn];
namespace SplayTree {
 int rt[maxm], tot;
 int fa[maxm], son[maxm][2];
 int val[maxm], cnt[maxm];
 int sz[maxm];
 void Push(int o) {
   sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];
 bool Get(int o) {
   return o == son[fa[o]][1];
 void Clear(int o) {
   son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[
        o] = 0;
 void Rotate(int o) {
   int p = fa[o], q = fa[p], ck = Get(o);
   son[p][ck] = son[o][ck ^ 1];
   fa[son[o][ck \land 1]] = p;
   son[o][ck \land 1] = p;
   fa[p] = o; fa[o] = q;
   if (q) son[q][p == son[q][1]] = o;
   Push(p); Push(o);
 void Splay(int &root, int o) {
   for (int f = fa[o]; (f = fa[o]); Rotate(o))
     if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
   root = o;
 void Insert(int &root, int x) {
   if (!root) {
     val[++tot] = x;
     cnt[tot]++;
     root = tot;
     Push(root);
     return;
```

```
int cur = root, f = 0;
 while (true) {
   if (val[cur] == x) {
      cnt[cur]++;
      Push(cur); Push(f);
      Splay(root, cur);
      break;
   }
    f = cur;
    cur = son[cur][val[cur] < x];</pre>
    if (!cur) {
      val[++tot] = x;
      cnt[tot]++;
      fa[tot] = f;
      son[f][val[f] < x] = tot;
      Push(tot); Push(f);
      Splay(root, tot);
      break:
   }
 }
int GetRank(int &root, int x) {
 int ans = 0, cur = root;
 while (cur) {
   if (x < val[cur]) {</pre>
      cur = son[cur][0];
      continue;
   ans += sz[son[cur][0]];
   if (x == val[cur]) {
     Splay(root, cur);
      return ans;
   if (x > val[cur]) {
     ans += cnt[cur];
      cur = son[cur][1];
   }
 }
 return ans;
int GetKth(int &root, int k) {
 int cur = root;
 while (true) {
    if (son[cur][0] \&\& k \le sz[son[cur][0]]) cur = son[
         cur][0];
    else {
      k = cnt[cur] + sz[son[cur][0]];
      if (k \ll 0) return cur;
     cur = son[cur][1];
   }
 }
int Find(int &root, int x) {
```

```
int ans = 0, cur = root;
  while (cur) {
    if (x < val[cur]) {</pre>
      cur = son[cur][0];
      continue;
    ans += sz[son[cur][0]];
    if (x == val[cur]) {
      Splay(root, cur);
      return ans + 1;
    ans += cnt[cur];
    cur = son[cur][1];
 }
int GetPrev(int &root) {
  int cur = son[root][0];
  while (son[cur][1]) cur = son[cur][1];
  return cur;
}
int GetPrevVal(int &root, int x) {
  int ans = -inf, cur = root;
  while (cur) {
    if (x > val[cur]) {
      ans = std::max(ans, val[cur]);
      cur = son[cur][1];
      continue;
    cur = son[cur][0];
  return ans;
}
int GetNext(int &root) {
  int cur = son[root][1];
  while (son[cur][0]) cur = son[cur][0];
  return cur:
}
int GetNextVal(int &root, int x) {
  int ans = inf, cur = root;
  while (cur) {
    if (x < val[cur]) {</pre>
      ans = std::min(ans, val[cur]);
      cur = son[cur][0];
      continue;
    }
    cur = son[cur][1];
  return ans;
}
void Delete(int &root, int x) {
  Find(root, x);
  if (cnt[root] > 1) {
    cnt[root]--;
    Push(root);
```

```
return;
   if (!son[root][0] && !son[root][1]) {
     Clear(root);
     root = 0;
     return;
   }
   if (!son[root][0]) {
     int cur = root;
      root = son[root][1];
     fa[root] = 0;
     Clear(cur);
     return;
   }
   if (!son[root][1]) {
      int cur = root;
      root = son[root][0];
      fa[root] = 0;
     Clear(cur);
     return;
   int p = GetPrev(root), cur = root;
   Splay(root, p);
    fa[son[cur][1]] = p;
   son[p][1] = son[cur][1];
   Clear(cur);
   Push(root);
};
namespace SegTree {
 int tree[maxn * 4];
 void Build(int o, int l, int r) {
    for (int i = 1; i <= r; ++i) SplayTree::Insert(tree[o</pre>
         ], arr[i - 1]);
   if (l == r) return;
   int m = (l + r) / 2;
   Build(o * 2, 1, m);
   Build(0 * 2 + 1, m + 1, r);
  void Modify(int o, int l, int r, int ll, int rr, int u,
       int v) {
   SplayTree::Delete(tree[o], u); SplayTree::Insert(tree
        [o], v);
   if (l == r) return;
   int m = (1 + r) / 2;
   if (ll <= m) Modify(o * 2, 1, m, ll, rr, u, v);</pre>
   if (rr > m) Modify(o * 2 + 1, m + 1, r, ll, rr, u, v)
 int QueryRank(int o, int l, int r, int ll, int rr, int
    if (ll <= l && rr >= r) return SplayTree::GetRank(
         tree[o], v);
```

```
int m = (1 + r) / 2, ans = 0;
    if (ll \leftarrow m) ans \leftarrow QueryRank(o * 2, 1, m, ll, rr, v)
    if (rr > m) ans += QueryRank(o * 2 + 1, m + 1, r, ll,
          rr. v):
    return ans;
  }
  int QueryPrev(int o, int l, int r, int ll, int rr, int
    if (ll <= l && rr >= r) return SplayTree::GetPrevVal(
         tree[o], v);
    int m = (l + r) / 2, ans = -inf;
    if (ll <= m) ans = std::max(ans, QueryPrev(o * 2, 1,</pre>
         m, ll, rr, v));
    if (rr > m) ans = std::max(ans, QueryPrev(o * 2 + 1,
         m + 1, r, ll, rr, v));
    return ans;
 }
  int QueryNext(int o, int l, int r, int ll, int rr, int
    if (ll <= l && rr >= r) return SplayTree::GetNextVal(
         tree[o], v);
    int m = (l + r) / 2, ans = inf;
    if (ll <= m) ans = std::min(ans, QueryNext(o * 2, 1,</pre>
         m, ll, rr, v));
    if (rr > m) ans = std::min(ans, QueryNext(o * 2 + 1,
         m + 1, r, ll, rr, v));
    return ans;
  int QueryKth(int ll, int rr, int v) {
    int l = 0, r = 1e8 + 10;
    while (l < r) {
      int m = ((l + r) / 2) + 1;
      if (QueryRank(1, 1, n, ll, rr, m) < v) l = m;
      else r = m - 1;
    }
    return 1;
 }
};
int main() {
  std::ios::sync_with_stdio(false);
  std::cout.tie(0);
  std::cin.tie(0);
  int m:
  std::cin >> n >> m;
  for (int i = 0; i < n; ++i) std::cin >> arr[i];
  SplayTree::tot = 0;
  SegTree::Build(1, 1, n);
  for (int i = 0, op, l, r, pos, k; i < m; ++i) {
    std::cin >> op;
    if (op == 1) {
      std::cin >> l >> r >> k;
      std::cout \ll SegTree::QueryRank(1, 1, n, l, r, k) +
```

 $1 \ll ' n';$ 

```
else if (op == 2) {
    std::cin >> l >> r >> k;
    std::cout << SegTree::QueryKth(1, r, k) << '\n';</pre>
  else if (op == 3) {
    std::cin >> pos >> k;
    SegTree::Modify(1, 1, n, pos, pos, arr[pos - 1], k)
    arr[pos - 1] = k;
  else if (op == 4) {
    std::cin >> l >> r >> k;
    std::cout << SegTree::QueryPrev(1, 1, n, l, r, k)</pre>
         << '\n';
  else if (op == 5) {
    std::cin >> l >> r >> k;
    std::cout << SegTree::QueryNext(1, 1, n, l, r, k)</pre>
         << '\n';
 }
}
return 0;
```

# 3.2 线段树

#### 3.2.1 线段树合并

```
// BZ0J2212: 交换左右子树后最小逆序对
#include <bits/stdc++.h>
const int maxn = 1e7 + 5;
template <typename t>
inline bool Read(t &ret) {
 char c; int sgn;
 if (c = getchar(), c == EOF) return false;
 while (c != '-' && (c < '0' || c > '9')) c = getchar();
 sgn = (c == '-') ? -1 : 1;
 ret = (c == '-') ? 0 : (c - '0');
 while (c = getchar(), c >= '0' && c <= '9') ret = ret *</pre>
       10 + (c - '0');
 ret *= sgn;
 return true:
struct node {
 int sz, lson, rson;
 node() \{ sz = lson = rson = 0; \}
};
int n:
int tot;
node tree[maxn];
```

```
long long ans1, ans2;
long long ans;
int Build(int l, int r, int c) {
  tree[++tot].sz = 1;
  if (l == r) return tot;
  int m = (1 + r) / 2, o = tot;
  if (c <= m) tree[o].lson = Build(l, m, c);</pre>
  else tree[o].rson = Build(m + 1, r, c);
int Merge(int 1, int r, int x, int y) {
  if (!x || !y) return x + y;
  if (l == r) {
    tree[++tot].sz = tree[x].sz + tree[y].sz;
    return tot;
  ans1 += 1ll * tree[tree[x].rson].sz * tree[tree[y].lson
      1.sz;
  ans2 += 1ll * tree[tree[x].lson].sz * tree[tree[y].rson
      ].sz;
  int m = (1 + r) / 2, o = ++tot;
  tree[o].lson = Merge(l, m, tree[x].lson, tree[y].lson);
  tree[o].rson = Merge(m + 1, r, tree[x].rson, tree[y].
  tree[o].sz = tree[x].sz + tree[y].sz;
  return o;
int Dfs() {
  int c = 0;
  Read(c);
  if (c) return Build(1, n, c);
  int o = Merge(1, n, Dfs(), Dfs());
  ans += std::min(ans1, ans2);
  ans1 = ans2 = 0;
  return o;
}
int main() {
  Read(n);
  Dfs();
  printf("%11d", ans);
  return 0;
```

#### 3.2.2 线段树

```
void Pull(int o) {
  sum[o] = Unite(sum[o * 2], sum[o * 2 + 1]);
void Push(int o, int l, int r) {
 int m = (l + r) / 2;
  if (lazy[o] != 0) {
    sum[o * 2] += (m - l + 1) * lazy[o];
    sum[o * 2 + 1] += (r - m) * lazy[o];
    lazy[o * 2] += lazy[o];
    lazy[o * 2 + 1] += lazy[o];
    lazy[o] = 0;
 }
}
void Build(int o, int l, int r, long long arr[]) {
  sum[o] = lazy[o] = 0;
  if (1 == r) {
   sum[o] = arr[l];
   return;
 int m = (l + r) / 2;
  Build(o * 2, 1, m, arr);
 Build(0 * 2 + 1, m + 1, r, arr);
 Pull(o);
void Init(int _n, long long arr[]) {
 n = _n;
 Build(1, 1, n, arr);
void Modify(int o, int l, int r, int ll, int rr, long
     long v) {
  if (ll <= l && rr >= r) {
    sum[o] += (r - l + 1) * v;
    lazy[o] += v;
    return;
 Push(o, 1, r);
  int m = (l + r) / 2;
  if (ll <= m) Modify(o * 2, 1, m, ll, rr, v);</pre>
  if (rr > m) Modify(o * 2 + 1, m + 1, r, ll, rr, v);
 Pull(o);
void Modify(int ll, int rr, long long v) {
 Modify(1, 1, n, ll, rr, v);
long long Query(int o, int l, int r, int ll, int rr) {
  if (ll <= l && rr >= r) return sum[o];
 Push(o, 1, r);
 int m = (l + r) / 2;
 long long ret = 0;
  if (ll <= m) ret = Unite(ret, Query(o * 2, l, m, ll,</pre>
  if (rr > m) ret = Unite(ret, Query(o * 2 + 1, m + 1,
       r, ll, rr));
```

```
return ret;
}
long long Query(int ll, int rr) {
  return Query(1, 1, n, ll, rr);
}
};
```

#### 3.2.3 矩形面积异或并

```
// CodeForces GYM 101982 F 矩形面积异或并
#include <bits/stdc++.h>
std::vector<int> x;
int Get(int k) {
 return std::lower_bound(x.begin(), x.end(), k) - x.
       begin();
}
struct SegTree {
 struct node {
   int v, lazy;
    node() \{ v = lazy = 0; \}
 };
  int n;
  std::vector<node> tree:
  node Unite(const node &k1, const node &k2) {
    node ans:
    ans.v = k1.v + k2.v;
    return ans;
  void Pull(int o) {
    tree[o] = Unite(tree[o * 2], tree[o * 2 + 1]);
  void Push(int o, int l, int r) {
    int m = (l + r) / 2;
    if (tree[o].lazy != 0) {
      tree[o * 2].v = x[m] - x[l - 1] - tree[o * 2].v;
      tree[o * 2 + 1].v = x[r] - x[m] - tree[o * 2 + 1].v
      tree[o * 2].lazy ^= 1;
      tree[o * 2 + 1].lazy ^= 1;
      tree[o].lazy = 0;
  void Build(int o, int l, int r) {
    if (l == r) return;
    int m = (l + r) / 2;
    Build(o * 2, 1, m);
    Build(0 * 2 + 1, m + 1, r);
    Pull(o);
  SegTree(int _n): n(_n) {
    tree.resize(n << 2);</pre>
    Build(1, 1, n);
```

```
void Modify(int o, int l, int r, int ll, int rr) {
   if (ll <= l && rr >= r) {
      tree[o].v = x[r] - x[l - 1] - tree[o].v;
      tree[o].lazy ^= 1;
     return;
   }
   Push(o, 1, r);
    int m = (1 + r) / 2;
    if (ll <= m) Modify(o * 2, 1, m, ll, rr);</pre>
   if (rr > m) Modify(o * 2 + 1, m + 1, r, ll, rr);
   Pull(o);
  void Modify(int ll, int rr) {
   Modify(1, 1, n, ll, rr);
  node Query(int o, int l, int r, int ll, int rr) {
   if (ll <= l && rr >= r) return tree[o];
   Push(o, 1, r);
    int m = (l + r) / 2;
    node ans;
    if (ll <= m) ans = Unite(ans, Query(o * 2, 1, m, 11,</pre>
    if (rr > m) ans = Unite(ans, Query(o * 2 + 1, m + 1,
         r, ll, rr));
    Pull(o);
    return ans;
  node Query() {
    return Query(1, 1, n, 1, n);
};
struct seg { int l, r, h, flag; };
bool operator < (seg k1, seg k2) { return k1.h < k2.h; }</pre>
std::vector<seg> s;
int main() {
 std::ios::sync_with_stdio(false);
  std::cout.tie(nullptr);
  std::cin.tie(nullptr);
  int n; std::cin >> n;
  for (int i = 0, x1, y1, x2, y2; i < n; ++i) {
   std::cin >> x1 >> y1 >> x2 >> y2;
    if (x1 > x2) std::swap(x1, x2);
   if (y1 > y2) std::swap(y1, y2);
   x.emplace_back(x1); x.emplace_back(x2);
   s.emplace_back((seg){x1, x2, y1, 1});
   s.emplace_back((seg)\{x1, x2, y2, -1\});
  sort(s.begin(), s.end());
  sort(x.begin(), x.end());
  x.erase(unique(x.begin(), x.end()), x.end());
  SegTree tree((int)x.size());
  long long ans = 0;
  for (int i = 0, l, r; i < (int)s.size() - 1; ++i) {
```

#### 3.2.4 矩形面积并

```
// HDU 1542 矩形面积并
#include <bits/stdc++.h>
typedef double db;
const int maxn = 1e2 + 5;
const db eps = 1e-9;
int Sgn(db k) {
 return std::fabs(k) < eps ? \emptyset : (k < \emptyset ? -1 : 1);
int Cmp(db k1, db k2) {
  return Sgn(k1 - k2);
struct seg {
  db 1, r, h;
  int flag;
};
bool operator < (seg &k1, seg &k2) {</pre>
  return Cmp(k1.h, k2.h) < 0;</pre>
std::vector<seg> segs;
std::vector<db> pos:
int BinarySearch(db k) {
  int ret = (int)pos.size() - 1, l = 0, r = (int)pos.size
       () - 1;
  while (l \ll r) {
    int m = (l + r) >> 1;
    if (Cmp(pos[m], k) >= 0) {
      ret = m;
      r = m - 1;
    else l = m + 1;
  }
  return ret;
}
struct node {
  int 1, r, cnt;
  db len;
};
node seg_tree[maxn * 10];
void Pull(int o) {
  if (seg_tree[o].cnt) seg_tree[o].len = pos[seg_tree[o].
       r + 1] - pos[seg_tree[o].l];
```

```
else if (seg_tree[o].l == seg_tree[o].r) seg_tree[o].
       len = 0.0;
  else seg_tree[o].len = seg_tree[o << 1].len + seg_tree[</pre>
       o << 1 | 1].len;
void Build(int l, int r, int o) {
  seg_tree[o].l = l; seg_tree[o].r = r;
  seg_tree[o].cnt = 0; seg_tree[o].len = 0.0;
  if (l == r) return;
  int Mid = (l + r) \gg 1;
  Build(l, Mid, o << 1);</pre>
  Build(Mid + 1, r, o << 1 | 1);
  Pull(o);
void Update(int 1, int r, int v, int o) {
 if (l <= seg_tree[o].l && r >= seg_tree[o].r) {
    seg_tree[o].cnt += v;
   Pull(o);
   return;
 }
  int Mid = (seg_tree[o].l + seg_tree[o].r) >> 1;
  if (r <= Mid) Update(l, r, v, o << 1);</pre>
  else if (l > Mid) Update(l, r, v, o << 1 | 1);
  else {
    Update(l, Mid, v, o << 1);</pre>
    Update(Mid + 1, r, v, o \ll 1 | 1);
  Pull(o);
int cas;
int n;
db x1, y1, x2, y2;
db ans;
int main() {
 while (~scanf("%d", &n) && n) {
   segs.clear();
    pos.clear();
    for (int i = 0; i < n; ++i) {
      scanf("%1f%1f%1f%1f", &x1, &y1, &x2, &y2);
      segs.push_back((seg)\{x1, x2, y1, 1\});
      segs.push_back((seg)\{x1, x2, y2, -1\});
      pos.push_back(x1);
     pos.push_back(x2);
    std::sort(segs.begin(), segs.end());
    std::sort(pos.begin(), pos.end(), [&](db k1, db k2) {
          return Cmp(k1, k2) < 0; });</pre>
    int cur = 1;
    for (int i = 1; i < (int)pos.size(); ++i)</pre>
      if (Cmp(pos[i], pos[i-1]) != 0)
        pos[cur++] = pos[i];
    pos.erase(pos.begin() + cur, pos.end());
    Build(0, (int)pos.size(), 1);
    ans = 0.0;
```

```
for (int i = 0; i < (int)segs.size() - 1; ++i) {
    int l = BinarySearch(segs[i].l), r = BinarySearch(
        segs[i].r);
    Update(l, r - 1, segs[i].flag, 1);
    ans += (segs[i + 1].h - segs[i].h) * seg_tree[1].
        len;
    }
    printf("Test case #%d\n", ++cas);
    printf("Total explored area: %.21f\n\n", ans);
}
return 0;
}</pre>
```

#### 3.3 树链剖分

```
const int maxn = "Edit";
int n;
long long val[maxn];
int fa[maxn], dep[maxn];
int sz[maxn], son[maxn];
int rk[maxn], top[maxn];
int id[maxn];
int dfs_clock;
std::vector<int> g[maxn];
void Dfs1(int u, int p, int d) {
  fa[u] = p;
  dep[u] = d;
  sz[u] = 1;
  for (int &v : g[u]) {
    if (v == p) continue;
    Dfs1(v, u, d + 1);
    sz[u] += sz[v];
    if (sz[v] > sz[son[u]]) son[u] = v;
 }
Dfs2(int u, int tp) {
  top[u] = tp;
  id[u] = ++dfs_clock;
  rk[dfs_clock] = u;
  if (!son[u]) return;
  Dfs2(son[u], tp);
  for (int &v : g[u]) {
    if (v == son[u] || v == fa[u]) continue;
    Dfs2(v, v);
 }
}
long long Modify(int u, int v, long long c) {
 while (top[u] != top[v]) {
    if (dep[top[u]] < dep[top[v]]) std::swap(u, v);</pre>
    /* modify c from [id[top[u]], id[u]] in val */
    u = fa[top[u]];
```

```
}
if (id[u] > id[v]) std::swap(u, v);
/* modify c from [id[u], id[v]] in val */
}
long long Query(int u, int v) {
  long long ret = 0;
  while (top[u] != top[v]) {
    if (dep[top[u]] < dep[top[v]]) std::swap(u, v);
    ret += /* query from [id[top[u]], id[u]] in val */
    u = fa[top[u]];
  }
  if (id[u] > id[v]) std::swap(u, v);
  ret += /* query from [id[u], id[v]] in val */
  return ret;
}
```

# 3.4 树状数组

#### 3.4.1 树状数组

```
const int maxn = "Edit";
struct BitTree {
 int tree[maxn];
 void Init() {
   memset(tree, 0, sizeof(tree));
 void Modify(int x, int v) {
   for (int i = x; i < maxn; i += i & (-i))
     tree[i] += v;
 int Query(int x) {
   int ret = 0;
   for (int i = x; i > 0; i = i & (-i))
     ret += tree[i];
   return ret;
 int GetRank(int v) {
   int ret = 1;
   for (int i = v; i > 0; i = i & (-i))
     ret += tree[i];
   return ret;
 int GetKth(int k) { // kth min
   int ret = 0, cnt = 0, max = log2(maxn);
   for (int i = max; i >= 0; —i) {
     ret += (1 << i);
     if (ret >= maxn || cnt += tree[ret] >= k) ret -= (1
            << i);
     else cnt += tree[ret];
```

```
return ++ret;
}
int GetPrev(int v) {
  return GetKth(GetRank(v) - 1);
}
int GetNext(int v) {
  return GetKth(GetRank(v) + 1);
}
};
```

#### 3.4.2 二维树状数组

```
const int maxn = "Edit";
struct BitTree2D {
  int tree[4][maxn][maxn];
  void Modify(int x, int y, int v) {
    for (int i = x; i < maxn; i += i & (-i)) {
      for (int j = y; j < maxn; j += j & (-j)) {
        tree[0][i][j] += v;
        tree[1][i][j] += v * y;
        tree[2][i][j] += v * x;
        tree[3][i][j] += v * x * y;
      }
   }
  }
  int Query(int x, int y) {
   int ret = 0;
    for (int i = x; i > 0; i = i & (-i)) {
      for (int j = y; j > 0; j = j & (-j)) {
       ret += tree[0][i][j] * (x + 1) * (y + 1);
        ret -= tree[1][i][j] * (x + 1);
        ret -= tree[2][i][j] * (y + 1);
        ret += tree[3][i][j];
    return ret;
 }
};
```

# 3.5 最近公共祖先

#### 3.5.1 欧拉序 +RMQ

```
const int maxn = "Edit";
const int maxlog = "Edit";
int n;
std::vector<int> g[maxn];
int ele[maxn * 2], dep[maxn * 2];
int fi[maxn], fa[maxn];
```

```
int tot;
int dp[maxn * 2][maxlog];
void Dfs(int u, int p, int d) {
       ele[++tot] = u;
       fi[u] = tot;
       dep[tot] = d;
       fa[u] = p;
       for (int &v : g[u]) {
            if (v == p) continue;
            Dfs(v, u, d + 1);
             ele[++tot] = u;
             dep[tot] = d;
     }
void Init() {
       for(int i = 1; i \le 2 * n - 1; ++i) dp[i][0] = i;
       for (int j = 1; (1 << j) <= 2 * n - 1; ++j)
            for (int i = 1; i + (1 \ll j) - 1 \ll 2 * n - 1; ++i)
                     dp[i][j] = dep[dp[i][j-1]] < dep[dp[i+(1 << j-1)] < dep[dp[i+(1 <<
                                           - 1))][j - 1];
int Query(int 1, int r) {
     if (l > r) std::swap(l, r);
       int len = log2(r - l + 1);
       return dep[dp[l][len]] \leftarrow dep[dp[r - (1 \leftrightarrow len) + 1][
                         len]] ? dp[l][len] : dp[r - (1 << len) + 1][len];
int GetLCA(int u, int v) {
       return ele[Query(fi[u], fi[v])];
```

#### 3.5.2 倍增

```
const int maxn = "Edit";
const int maxlog = "Edit";
int n, k; // k = log2(n) + 1
std::vecotr<int> g[maxn];
int anc[maxn][maxlog];
int dep[maxn];
// 从根节点开始深搜预处理
void Dfs(int u, int p, int d) {
 anc[u][0] = p;
 dep[u] = d;
 for (int &v : g[u]) {
   if (v == p) continue;
   Dfs(v, u, d + 1);
 }
}
void Swim(int &u, int h) {
 for (int i = 0; h > 0; ++i) {
   if (h & 1) u = anc[u][i];
```

```
h >>= 1;
}
int GetLCA(int u, int v) {
  if (dep[u] < dep[v]) std::swap(u, v);
  Swim(u, dep[u] - dep[v]);
  if (u == v) return v;
  for (int i = k - 1; i >= 0; —i) {
    if (anc[u][i] != anc[v][i]) {
        u = anc[u][i];
        v = anc[v][i];
    }
  }
  return anc[u][0];
}
```

#### 3.5.3 tarjan

```
const int maxn = "Edit";
const int maxm = "Edit";
int n;
int pre[maxn];
int Find(int o) {
 return pre[o] == o ? o : pre[o] = Find(pre[o]);
void Union(int u, int v) {
 if (Find(u) != Find(v)) pre[Find(u)] = Find(v);
std::vector<int> g[maxn];
bool vis[maxn];
struct query { int v, id; };
std::vector<query> qry[maxm];
void Init() {
 for (int i = 1; i <= n; ++i) {
    pre[i] = i;
    vis[i] = false;
 }
void Tarjan(int u) {
  vis[u] = true;
  for (int &v : g[u]) {
    if (vis[v]) continue;
    Tarjan(v);
    Union(v, u);
  for (query &q : qry[u]) {
    if (vis[q.v]) ans[q.id] = Find(q.v);
 }
}
```

# 3.6 伸展树

```
const int inf = "Edit"
const int maxn = "Edit";
struct SplayTree {
 int rt, tot;
 int fa[maxn], son[maxn][2];
 int val[maxn], cnt[maxn];
 int sz[maxn];
 bool lazy[maxn];
 void Pull(int o) {
   sz[o] = sz[son[o][0]] + sz[son[o][1]] + cnt[o];
 void Push(int o) {
   if (lazy[o]) {
     std::swap(son[o][0], son[o][1]);
     if (son[0][0]) lazy[son[0][0]] ^= 1;
     if (son[o][1]) lazy[son[o][1]] ^= 1;
     lazy[o] = 0;
   }
 }
 bool Get(int o) {
   return o == son[fa[o]][1];
 void Clear(int o) {
   son[o][0] = son[o][1] = fa[o] = val[o] = sz[o] = cnt[
        o] = 0;
 void Rotate(int o) {
   int p = fa[o], q = fa[p], ck = Get(o);
   son[p][ck] = son[o][ck ^ 1];
   fa[son[o][ck ^ 1]] = p;
   son[o][ck \land 1] = p;
   fa[p] = o; fa[o] = q;
   if (q) son[q][p == son[q][1]] = o;
   Pull(p); Pull(o);
 }
 void Splay(int o) {
   for (int f = fa[o]; f = fa[o], f; Rotate(o))
     if (fa[f]) Rotate(Get(o) == Get(f) ? f : o);
   rt = 0;
  // 旋转O节点到节点tar
 void Splay(int o, int tar = 0) {
   for (int f = fa[o]; (f = fa[o]) != tar; Rotate(o)) {
     Pull(fa[f]); Pull(f); Pull(o);
     if (fa[f] != tar) {
       if (Get(o) == Get(f)) Rotate(f);
       else Rotate(o);
     }
   }
   if (!tar) rt = 0;
```

```
void Insert(int x) {
 if (!rt) {
    val[++tot] = x;
    cnt[tot]++;
    rt = tot;
    Pull(rt);
    return;
 }
 int cur = rt, f = 0;
 while (true) {
    if (val[cur] == x) {
      cnt[cur]++;
      Pull(cur); Pull(f);
      Splay(cur);
      break;
    f = cur;
    cur = son[cur][val[cur] < x];
    if (!cur) {
      val[++tot] = x;
      cnt[tot]++;
      fa[tot] = f;
      son[f][val[f] < x] = tot;
      Pull(tot); Pull(f);
      Splay(tot);
      break;
int GetRank(int x) {
 int ans = 0, cur = rt;
 while (true) {
    if (x < val[cur]) cur = son[cur][0];</pre>
      ans += sz[son[cur][0]];
      if (x == val[cur]) {
       Splay(cur);
        return ans + 1;
     }
     ans += cnt[cur];
      cur = son[cur][1];
 }
}
int GetKth(int k) {
 int cur = rt;
 while (true) {
    if (son[cur][0] \&\& k \le sz[son[cur][0]]) cur = son[
         cur][0];
    else {
      k = cnt[cur] + sz[son[cur][0]];
      if (k <= 0) return cur;</pre>
      cur = son[cur][1];
```

```
}
// 获取以r为根节点Splay Tree中的第k大个元素在Splay Tree
     中的位置
int Kth(int r, int k) {
 Pull(r);
 int tmp = sz[son[r][0]] + 1;
 if (tmp == k) return r;
 if (tmp > k) return Kth(son[r][0], k);
 else return Kth(son[r][1], k - tmp);
}
// Insert之后求前驱后继
int GetPrev() {
 int cur = son[rt][0];
 while (son[cur][1]) cur = son[cur][1];
 return cur;
int GetNext() {
 int cur = son[rt][1];
 while (son[cur][0]) cur = son[cur][0];
 return cur;
// 获取Splay Tree中以O为根节点子树的最小值位置
int GetMin(int o) {
 Pull(o);
 while (son[o][0]) {
   o = son[o][0];
   Pull(o);
 return o;
}
// 获取Splay Tree中以O为根节点子树的最大值位置
int GetMax(int o) {
 Pull(o);
 while (son[o][1]) {
   o = son[o][1];
   Pull(o);
 }
 return o;
void Delete(int x) {
 GetRank(x);
 if (cnt[rt] > 1) {
   cnt[rt]--;
   Pull(rt);
   return;
 if (!son[rt][0] && !son[rt][1]) {
   Clear(rt);
   rt = 0;
   return;
 if (!son[rt][0]) {
```

int cur = rt;

```
rt = son[rt][1];
      fa[rt] = 0;
      Clear(cur);
      return;
   if (!son[rt][1]) {
      int cur = rt;
      rt = son[rt][0];
      fa[rt] = 0;
      Clear(cur);
      return;
   }
   int p = GetPrev(), cur = rt;
    Splay(p);
    fa[son[cur][1]] = p;
    son[p][1] = son[cur][1];
   Clear(cur);
   Pull(rt);
 }
  /* 维护数组操作 */
  // 翻转Splay Tree中l~r区间
  void Reverse(int 1, int r) {
   int o = Kth(rt, 1), Y = Kth(rt, r);
    Splay(o, 0); Splay(Y, o);
   lazy[son[Y][0]] ^= 1;
  // 建立Splay Tree
  void Build(int l, int r, int o) {
   if (l > r) return;
   int m = (l + r) >> 1;
   Build(l, m - 1, m);
   Build(m + 1, r, m);
    fa[m] = o;
   val[m] = /* 节点权值 */;
   lazy[m] = 0;
   Push(m);
   if (m < o) son[o][0] = m;
   else son[o][1] = m;
  // 输出Splay Tree
  void Print(int o) {
   Pull(o);
   if (son[0][0]) Print(son[0][0]);
   // 哨兵节点判断
   if (val[o] != -inf && val[o] != inf) printf("%d ",
         val[o]);
   if (val[son[o][1]]) Print(son[o][1]);
 }
};
```

#### 3.7 主席树

```
const int maxn = "Edit";
struct FuncSegTree {
 int tot;
 int rt[maxn];
 int lson[maxn * 40], rson[maxn * 40];
 int cnt[maxn * 40];
 int Build(int 1, int r) {
   int o = ++tot, m = (l + r) / 2;
   cnt[o] = 0;
   if (l != r) {
     lson[o] = Build(l, m);
     rson[o] = Build(m + 1, r);
   return o;
 }
 int Modify(int prev, int l, int r, int v) {
   int o = ++tot, m = (l + r) / 2;
   lson[o] = lson[prev];
   rson[o] = rson[prev];
   cnt[o] = cnt[prev] + 1;
   if (l != r) {
     if (v <= m) lson[o] = Modify(lson[o], 1, m, v);</pre>
     else rson[o] = Modify(rson[o], m + 1, r, v);
   }
   return o;
 // 区间[u+1,v]静态第k小
 int Query(int u, int v, int l, int r, int k) {
   if (l == r) return l;
   int m = (l + r) / 2;
   int num = cnt[lson[v]] - cnt[lson[u]];
   if (num >= k) return Query(lson[u], lson[v], l, m, k)
   else return Query(rson[u], rson[v], m + 1, r, k - num
        );
 // 区间[u+1,v]内[s,t]数量
 int Query(int u, int v, int s, int t, int l, int r) {
   if (s <= 1 && t >= r) return cnt[v] - cnt[u];
   int m = (l + r) / 2, ret = 0;
   if (s <= m) ret += Query(lson[u], lson[v], s, t, l, m</pre>
        );
    if (t > m) ret += Query(rson[u], rson[v], s, t, m +
        1, r);
    return ret;
 }
};
```

#### 3.8 dfs 序

```
const int maxn = "Edit";
```

```
std::vector<int> g[maxn];
int in[maxn], out[maxn];
int ele[maxn];
int dfs_clock;
void DfsSeq(int u, int p) {
  in[u] = ++dfs_clock;
  ele[dfs_clock] = u;
  for (int &v : g[u]) {
    if (v == p) continue;
    DfsSeq(v, u);
  }
  out[u] = dfs_clock;
}
```

#### 3.9 ST 表

```
const int maxn = "Edit";
const int maxlog = "Edit";
int max[maxn][maxlog], min[maxn][maxlog];
void Init(int arr[]) {
 int m = log2(n) + 1;
 for (int i = 1; i <= n; ++i) max[i][0] = min[i][0] =</pre>
      arr[i];
 for (int j = 1; j < m; ++j) {
   for (int i = 1; i + (1 << j) - 1 <= n; ++i) {
     max[i][j] = std::max(max[i][j-1], max[i+(1 << (
           j - 1))][j - 1]);
     min[i][j] = std::min(min[i][j - 1], min[i + (1 << (
           j - 1))][j - 1]);
   }
 }
// 区间[1,r]最大值
int QueryMax(int 1, int r) {
 int k = log2(r - l + 1);
 return std::max(max[l][k], max[r - (1 \ll k) + 1][k]);
// 区间[1,r]最小值
int QueryMin(int 1, int r) {
 int k = log2(r - l + 1);
 return std::min(min[l][k], min[r - (1 << k) + 1][k]);</pre>
```

#### 3.10 Link Cut Tree

```
const int maxn = "Edit";
struct LCT {
  int fa[maxn], son[maxn][2];
```

```
int val[maxn], sum[maxn];
int rev[maxn], stk[maxn];
void Init(int n) {
 for (int i = 1; i <= n; ++i) scanf("%d", &val[i]);</pre>
  for (int i = 1; i \le n; ++i) fa[i] = son[i][0] = son[
       i][1] = rev[i] = 0;
bool IsRoot(int o) {
 return son[fa[o]][0] != o && son[fa[o]][1] != o;
bool Get(int o) {
 return son[fa[o]][1] == o;
}
// 更新所需维护的信息
void Pull(int o) {
 sum[o] = val[o] ^ sum[son[o][0]] ^ sum[son[o][1]];
void Push(int o) {
 if (rev[o] != 0) {
    std::swap(son[o][0], son[o][1]);
    if (son[o][0]) rev[son[o][0]] ^= 1;
    if (son[o][1]) rev[son[o][1]] ^= 1;
    rev[o] ^= 1;
 }
}
void Rotate(int o) {
  int p = fa[o], q = fa[p], ck = Get(o);
  if (!IsRoot(p)) son[q][Get(p)] = o;
  fa[o] = q;
  son[p][ck] = son[o][ck ^ 1];
  fa[son[p][ck]] = p;
  son[o][ck \land 1] = p;
  fa[p] = o;
  Pull(p);
  Pull(o);
void Splay(int o) {
  int top = 0;
  stk[++top] = o;
  for (int i = 0; !IsRoot(i); i = fa[i]) stk[++top] =
       fa[i];
  for (int i = top; i; —i) Push(stk[i]);
  for (int f = fa[o]; !IsRoot(o); Rotate(o), f = fa[o])
   if (!IsRoot(f)) Rotate(Get(o) == Get(f) ? f : o);
// 将使O成为一条实路径并在同一棵Splay内
void Access(int o) {
  for (int p = 0; o; p = 0, o = fa[o]) {
    Splay(o);
    son[o][1] = p;
   Pull(o);
// 返回0所在树的根节点编号
```

```
int Find(int o) {
   Access(o);
   Splay(o);
   while (son[o][0]) o = son[o][0];
   return o;
 // 使0成为其所在树的根
 void MakeRoot(int o) {
   Access(o);
   Splay(o);
   rev[o] ^= 1;
 // u,v之间连边,先判不能在同一棵树内
 void Link(int u, int v) {
   MakeRoot(u);
   fa[u] = v;
   Splay(u);
 }
 // 删除U,V之间的边
 void Cut(int u, int v) {
   MakeRoot(u);
   Access(v);
   Splay(v);
   fa[u] = son[v][0] = 0;
 // 0节点单点修改
 void Modify(int o, int v) {
   val[o] = v;
   Access(o);
   Splay(o);
 // u, v路径信息
 int Query(int u, int v) {
   MakeRoot(v);
   Access(u);
   Splay(u);
   return sum[u];
 }
};
```

# 4 字符串

## 4.1 最小表示法

```
int minRepresent(char *s, int len) {
   int i = 0, j = 1, k = 0;
   while (i < len && j < len && k < len) {
        int t = s[(i+k) \% len] - s[(j+k) \% len];
       if (t == 0) k++;
       else {
           if (t < 0) j = max(j+k+1, i+1);
           else i = max(i+k+1, j+1);
            k = 0;
       }
   return min(i, j);
int minRepresent(int start, int end, int len) { // 判断[
    strat, end]是否为最小表示
    int i = 0+start, j = 1+start, k = 0;
   while (i < end && j < end && k < len) \{
       int l = i + k; if (l >= end) l = l - end + start
       int r = j + k; if (r >= end) r = r - end + start
       int t = s[l] - s[r];
        if (t == 0) k++;
       else {
           if (t < 0) j = max(j+k+1, i+1);
           else i = max(i+k+1, j+1);
            k = 0;
   }
   return min(i, j) == start;
```

#### 4.2 扩展 kmp

```
struct exKmp{
    // 字符串下标从0开始
    int nex[maxn], ex[maxn]; //模式串nex, 匹配串ex
    void getNex(char *str, int len) {
        int i = 0, j, pos;
        nex[0] = len;
        while (str[i] == str[i+1] && i+1 < len) ++i;
        nex[1] = i;
        pos = 1;
        for (int i = 2; i < len; ++i) {
            if (nex[i-pos] + i < nex[pos] + pos) nex[i] = nex[i-pos];
```

```
else {
                j = nex[pos] + pos - i;
                 if (j < 0) j = 0;
                while (i+j < len \&\& str[j] == str[j+i])
                nex[i] = j;
                pos = i;
            }
        }
    }
    void getEx(char *s1, char *s2) { // s1匹配s2
        int i = 0, j, pos;
        int len1 = strlen(s1);
        int len2 = strlen(s2);
        getNex(s2, len2);
        while (s1[i] == s2[i] \&\& i < len1 \&\& i < len2) ++
             i;
        ex[0] = i;
        pos = 0;
        for (int i = 1; i < len1; ++i) {</pre>
            if (nex[i-pos] + i < ex[pos] + pos) ex[i] =
                 nex[i-pos];
            else {
                j = ex[pos] + pos - i;
                if (j < 0) j = 0;
                while (i+j < len1 \&\& j < len2 \&\& s1[i+j]
                      == s2[j]) ++j;
                ex[i] = j;
                pos = i;
            }
        }
    }
}exkmp;
```

#### 4.3 字典树

```
struct Trie{
   int nex[maxn][26], cnt[maxn], end[maxn];
   int p, root; // root = 0
   int newnode() {
       memset(nex[p], 0, sizeof(nex[p]));
       cnt[p] = end[p] = 0;
       return p++;
   }
   void init() {
       p = 0;
       root = newnode();
   }
   void add(char *s) {
       int now = root;
       for (int i = 0; s[i]; ++i) {
```

# 4.4 回文树

```
struct Palindrome_Tree{
   int nex[maxn][26];
   int fail[maxn], cnt[maxn], num[maxn]; // num 记录每个
        节点右端点的表示回文串的个数
   int len[maxn], S[maxn];
               // cnt 记录每个节点表示的回文串出现的次
   int last, n, p;
   int newnode(int l) { // 新建节点
       for (int i = 0; i < 26; ++i) nex[p][i] = 0;
       cnt[p] = num[p] = 0;
       len[p] = 1;
       return p++;
   }
   void init() { // 初始化
       p = 0;
       newnode(0), newnode(-1); // 新建奇根和偶根
       last = n = 0;
       S[n] = -1;
       fail[0] = 1; // 偶根指向
   int get_fail(int x) { // *fail
       while (S[n - len[x] - 1] != S[n]) x = fail[x];
       return x;
   void add(int c) { // 添加节点
       C -= 'a';
       S[++n] = c;
       int cur = get_fail(last);
       if (!nex[cur][c]) {
           int now = newnode(len[cur] + 2);
           fail[now] = nex[get_fail(fail[cur])][c];
```

## 4.5 哈希

```
struct Hash{
    // mod 402653189, 805306457, 1610612741, 1e9+7
    // base 131, 233
    long long p[maxn], hash[maxn], base = 131;
    long long getHash(int l, int r) {
        long long ans = (hash[r] - hash[l-1] * p[r-l+1])
             % mod;
        return (ans + mod) % mod;
    }
    void init(string s) {
        int n = s.size();
        p[0] = 1;
        for (int i = 1; i \le n; ++i) p[i] = p[i - 1] *
             base % mod:
        for (int i = 1; i <= n; ++i) {</pre>
            hash[i] = (hash[i - 1] * base % mod + (s[i-1])
                  - 'a' + 1)) % mod;
        }
    }
}hash;
```

#### 4.6 后缀自动机

```
struct SAM{
    int trans[maxn<<1][26], slink[maxn<<1], maxlen[maxn <<1];
    // 用来求endpos
    int indegree[maxn<<1], endpos[maxn<<1], rank[maxn <<1], ans[maxn<<1];
    // 计算所有子单的和(0-9表示)
    long sum[maxn<<1];
    int last, now, root, len;
    inline void newnode (int v) {
        maxlen[++now] = v;
    }
```

```
inline void extend(int c) {
    newnode(maxlen[last] + 1);
    int p = last, np = now;
    // 更新trans
    while (p && !trans[p][c]) {
        trans[p][c] = np;
        p = slink[p];
    if (!p) slink[np] = root;
        int q = trans[p][c];
        if (maxlen[p] + 1 != maxlen[q]) {
            // 将q点拆出nq, 使得maxlen[p] + 1 ==
                 maxlen[q]
            newnode(maxlen[p] + 1);
            int nq = now;
            memcpy(trans[nq], trans[q], sizeof(trans[
                 al));
            slink[nq] = slink[q];
            slink[q] = slink[np] = nq;
            while (p && trans[p][c] == q) {
                trans[p][c] = nq;
               p = slink[p];
           }
        }else slink[np] = q;
    last = np;
    // 初始状态为可接受状态
    endpos[np] = 1;
inline void build(char *s) {
    // scanf("%s", s);
    len = strlen(s);
    root = last = now = 1;
    for (int i = 0; i < len; ++i) extend(s[i] - '0');
          // extend(s[i] - '1');
// 计算所有子串的和 (0-9表示)
inline long getSum() {
    // 拓扑排序
    for (int i = 1; i <= now; ++i) indegree[ maxlen[i</pre>
        ] ]++;
    for (int i = 1; i <= now; ++i) indegree[i] +=</pre>
        indegree[i-1];
    for (int i = 1; i <= now; ++i) rank[ indegree[</pre>
        maxlen[i] ]— ] = i;
    mem(endpos, 0);
    endpos[1] = 1; // 从根节点向后求有效的入度
    for (int i = 1; i <= now; ++i) {</pre>
        int x = rank[i];
        for (int j = 0; j < 10; ++j) {
            int nex = trans[x][j];
            if (!nex) continue;
            endpos[nex] += endpos[x]; // 有效入度
```

```
long num = (sum[x] * 10 + endpos[x] * j)
               sum[nex] = (sum[nex] + num) \% mod; // ‡
                    态转移
           }
       }
       long long ans = 0;
       for (int i = 2; i <= now; ++i) ans = (ans + sum[i</pre>
            ]) % mod;
       return ans;
   inline void getEndpos() {
       // topsort
       for (int i = 1; i <= now; ++i) indegree[ maxlen[i</pre>
            ] ]++; // 统计相同度数的节点的个数
       for (int i = 1; i <= now; ++i) indegree[i] +=</pre>
            indegree[i-1]; // 统计度数小于等于 i 的节点
            的总数
       for (int i = 1; i <= now; ++i) rank[ indegree[</pre>
            maxlen[i]]—] = i; // 为每个节点编号, 节
            点度数越大编号越靠后
       // 从下往上按照slik更新
       for (int i = now; i >= 1; --i) {
           int x = rank[i];
           endpos[slink[x]] += endpos[x];
       }
    // 求不同的子串种类
   inline long long all () {
       long long ans = 0;
       for (int i = root+1; i <= now; ++i) {</pre>
           ans += maxlen[i] - maxlen[ slink[i] ];
       return ans;
   // 长度为K的字符串有多种, 求出现次数最多的次数
   inline void get_Maxk() {
       getEndpos();
       for (int i = 1; i \le now; ++i) {
           ans[maxlen[i]] = max(ans[maxlen[i]], endpos[i
       for (int i = len; i >= 1; --i) ans[i] = max(ans[i
            ], ans[i+1]);
       for (int i = 1; i <= len; ++i) //cout << ans[i]
            << endl;
           printf("%d\n", ans[i]);
}sam;
```

#### 4.7 后缀数组

```
int cntA[maxn], cntB[maxn], A[maxn], B[maxn];
int Sa[maxn], tsa[maxn], height[maxn], Rank[maxn];
char s[maxn];
int n;
void SuffixArray () {
        for (int i = 0; i < 1000; ++i) cntA[i] = 0;</pre>
        for (int i = 1; i <= n; ++i) cntA[(int)s[i]]++;</pre>
        for (int i = 1; i < 1000; ++i) cntA[i] += cntA[i</pre>
              -1];
        for (int i = n; i >= 1; —i) Sa[ cntA[(int)s[i
             ]]-- ] = i;
        Rank[Sa[1]] = 1;
        for (int i = 2; i <= n; ++i) {
                Rank[Sa[i]] = Rank[Sa[i-1]];
                if (s[Sa[i]] != s[Sa[i-1]]) Rank[Sa[i
                      ]]++;
        for (int l = 1; Rank[Sa[n]] < n; l <<= 1) {
                 for (int i = 0; i <= n; ++i) cntA[i] = 0;</pre>
                 for (int i = 0; i <= n; ++i) cntB[i] = 0;</pre>
                 for (int i = 1; i <= n; ++i) {</pre>
                         cntA[A[i] = Rank[i]]++;
                         cntB[B[i] = (i + l \le n) ? Rank[
                              i+l] : 0]++;
                 for (int i = 1; i <= n; ++i) cntB[i] +=</pre>
                      cntB[i-1];
                 for (int i = n; i >= 1; —i) tsa[ cntB[B[
                      i]]--- ] = i;
                 for (int i = 1; i <= n; ++i) cntA[i] +=</pre>
                      cntA[i-1];
                 for (int i = n; i >= 1; —i) Sa[ cntA[A[
                      tsa[i]]— ] = tsa[i];
                Rank[Sa[1]] = 1;
                 for (int i = 2; i <= n; ++i) {
                         Rank[Sa[i]] = Rank[Sa[i-1]];
                         if (A[Sa[i]] != A[Sa[i-1]] || B[
                              Sa[i] != B[Sa[i-1]] Rank[
                              Sa[i]]++;
                }
        for (int i = 1, j = 0; i <= n; ++i) {
                 if (j) —j;
                 int tmp = Sa[Rank[i] - 1];
                while (i + j \le n \&\& tmp + j \le n \&\& s[i+
                      j] == s[tmp+j]) ++j;
                height[Rank[i]] = j;
        }
}
```

## 4.8 kmp

```
struct KMP{ // 下标0
        int nex[maxn];
        void get_nex(char *buf, int len) {
                nex[0] = -1;
                int i = 0, j = -1;
                while (i < len) {</pre>
                        if (j == -1 \mid | buf[i] == buf[j])
                             nex[++i] = ++j;
                        else j = nex[j];
                }
        }
        int get_kmp(char *buf1, char *buf2) { // buf1匹配
             串. buf2模式串
                int len1 = strlen(buf1), len2 = strlen(
                     buf2);
                get_nex(buf2, len2);
                int cnt = 0, i = 0, j = 0;
                while (i < len1) {</pre>
                        if (j == -1 || buf1[i] == buf2[i
                             ]) ++i, ++j;
                        else j = nex[j];
                        if (j == len2) cnt++, j = nex[j];
                return cnt; // 匹配个数
        }
}kmp;
```

# 4.9 AC 自动机

```
struct Trie{
   int nex[maxn][26], fail[maxn], end[maxn];
    int root, p;
    inline int newnode() {
        for (int i = 0; i < 26; ++i) {
           nex[p][i] = -1;
        end[p++] = 0;
        return p - 1;
   inline void init() {
        p = 0;
        root = newnode();
   inline void insert(char *buf) {
        int now = root;
        for (int i = 0; buf[i]; ++i) {
            if (nex[now][buf[i]-'a'] == -1)
                nex[now][buf[i]-'a'] = newnode();
           now = nex[now][buf[i]-'a'];
```

```
}
    end[now]++;
inline void build() {
    queue<int> que;
    fail[root] = root;
    for (int i = 0; i < 26; ++i) {
        if (nex[root][i] == -1)
           nex[root][i] = root;
           fail[nex[root][i]] = root;
            que.push(nex[root][i]);
        }
   }
    while (!que.empty()) {
        int now = que.front();
        que.pop();
        for (int i = 0; i < 26; ++i) {
            if (nex[now][i] == -1)
               nex[now][i] = nex[fail[now]][i];
            else {
               fail[nex[now][i]] = nex[fail[now]][i
               que.push(nex[now][i]);
           }
       }
   }
long long num[maxn], dp[maxn]; // num记录节点i匹配的
     个数, dp辅助得到所有适配数量
long long dfs(int now) {
    if (now == root) return 0;
    if (dp[now] != -1) return dp[now];
    return dp[now] = end[now] + dfs(fail[now]);
inline void solve(char *buf) {
   fill(num, num+maxn, 0);
    fill(dp, dp+maxn, -1);
    int now = root;
    for (int i = 0; buf[i]; ++i) {
        now = nex[now][buf[i]-'a'];
        num[i] = dfs(now);
   }
}
inline long long query(char *buf) {
    int now = root;
    long long cnt = 0;
    for (int i = 0; buf[i]; ++i) {
       now = nex[now][buf[i]-'a'];
        int tmp = now;
        while (tmp != root && end[tmp] != -1) {
            cnt += end[tmp];
            end[tmp] = -1; // 统计种类, 加速
           tmp = fail[tmp];
```

```
}
    return cnt;
}
}L, R;
```

# 5 图论

## 5.1 次小生成树

```
// Kruskal
int n, m;
struct ac{
        int u, v, w, flag;
        bool operator <(ac t) {</pre>
                 return w < t.w;</pre>
        }
}g[maxn*maxn];
vector<int> son[maxn];
int pre[maxn], dis[maxn][maxn];
int find (int x) {
        return (pre[x] == x) ? x : pre[x] = find(pre[x]);
void Kruskal() {
        for (int i = 0; i <= n; ++i) {
                 son[i].clear();
                  son[i].push_back(i);
                 pre[i] = i;
        sort(g, g+m);
        int sum = 0;
         int cnt = 0;
         for (int i = 0; i < m; ++i) {
                 if (cnt == n+1) break;
                 int fx = find(g[i].u);
                  int fy = find(g[i].v);
                 if (fx == fy) continue;
                  g[i].flag = 1;
                  sum += g[i].w;
                  cnt++;
                  int lenx = son[fx].size();
                  int leny = son[fy].size();
                  if (lenx < leny) {</pre>
                           swap(lenx, leny);
                           swap(fx, fy);
                 }
                  // 更新两点的距离最大值
                 for (int j = 0; j < lenx; ++j) {
                           for (int k = 0; k < leny; ++k) {
                                    \label{eq:disconfx} \begin{split} \text{dis}[\text{son}[\text{fx}][\text{j}]][\text{son}[\text{fy}][\text{k}] \end{split}
                                         ]] = dis[son[fy][k
                                         ]][son[fx][j]] = g[i
                           }
                 }
                 pre[fy] = fx;
                  //合并子树
                  for (int j = 0; j < leny; ++j) {
```

```
son[fx].push_back(son[fy][j]);
                son[fy].clear();
        }
        int ans = inf;
        for (int i = 0; i < m; ++i) {
                if (g[i].flag) continue;
                ans = min(ans, sum + g[i].w - dis[g[i].u
                     ][g[i].v]);
        }
        printf("%d %d\n", sum, ans);
}
// Prim
int n, m;
int g[maxn][maxn], val[maxn], vis[maxn], dis[maxn];
int pre[maxn], maxd[maxn][maxn];
bool used[maxn][maxn];
void prim(int s) {
    mem(maxd, 0);
    mem(vis, 0);
    mem(used, 0);
    for (int i = 1; i \le n; ++i) {
        dis[i] = g[s][i];
        pre[i] = s;
    vis[s] = 1;
    int sum = 0, cnt = 0;
    for (int i = 1; i < n; ++i) {
        int u = -1, MIN = inf;
        for (int j = 1; j \le n; ++j) {
            if (vis[j]) continue;
            if (MIN > dis[j]) {
                MIN = dis[j];
                u = j;
            }
        if (u == -1) break;
        vis[u] = 1;
        sum += MIN;
        cnt++;
        used[pre[u]][u] = used[u][pre[u]] = 1;
        maxd[u][pre[u]] = maxd[pre[u]][u] = MIN;
        for (int j = 1; j <= n; ++j) {
            if (j == u) continue;
            if (vis[j]) {
                maxd[u][j] = maxd[j][u] = max(maxd[pre[u]
                     ]][j], MIN);
            if (vis[j] == 0 && dis[j] > g[u][j]) {
                dis[j] = g[u][j];
                pre[j] = u;
            }
        }
```

```
if (cnt != n-1) {
    puts("No way");
}
int ans = inf;
for (int i = 1; i <= n; ++i) {
    for (int j = i+1; j <= n; ++j) {
        if (used[i][j]) continue;
        ans = min(ans, sum + g[i][j] - maxd[i][j]);
    }
}
printf("%a %a\n", sum, ans);
}</pre>
```

## 5.2 最小树形图

```
struct ac{
   int u, v, w;
vector<ac> g(maxn);
int pre[maxn], vis[maxn], id[maxn], in[maxn];
int zhuliu(int rt, int n, int m) {
   int ans = 0, u, v, w;
   while (1) {
        for (int i = 0; i < n; ++i) in[i] = inf;
        for (int i = 0; i < m; ++i) {</pre>
            u = g[i].u; v = g[i].v; w = g[i].w;
            if (u != v && w < in[v]) {</pre>
                pre[v] = u;
                in[v] = w;
                // if (u == rt) pos = i; // 记录前驱, 输
                     出序号最小的根
           }
        }
        for (int i = 0; i < n; ++i) {
           if (i != rt && in[i] == inf) return -1;
        int cnt = 0;
        mem(id, -1);
        mem(vis, -1);
        in[rt] = 0;
        for (int i = 0; i < n; ++i) {
           ans += in[i];
            u = i:
            while (vis[u] != i && id[u] == -1 && u != rt)
                vis[u] = i;
                u = pre[u];
            if (u != rt \&\&id[u] == -1) {
                v = pre[u];
                while (v != u) {
```

```
id[v] = cnt;
                    v = pre[v];
                id[u] = cnt++;
            }
        }
        if (cnt == 0) break;
        for (int i = 0; i < n; ++i) {
            if (id[i] == -1) id[i] = cnt++;
        }
        for (int i = 0; i < m; ++i) {
            v = g[i].v;
            g[i].u = id[g[i].u];
            g[i].v = id[g[i].v];
            if (g[i].u != g[i].v) g[i].w -= in[v];
        }
        n = cnt;
        rt = id[rt];
    }
    return ans;
}
```

# 5.3 拓扑排序

```
vector<int> g[maxn];
struct Topsort{ // 下标1
       priority_queue<int, vector<int>, greater<int>> q1
            ; // 字典序小, 正向建图
       priority_queue<int, vector<int>> q2; // 编号小的
            优先级高, 反向建图
       int in[maxn], order[maxn], n, cnt;
       void init () {
               fill(in, in+n+1, 0);
               for (int i = 1; i \le n; ++i) {
                       for (int j = 0; j < (int)g[i].
                           size(); ++j) {
                               in[g[i][j]]++;
                       }
               }
       int min_lex (int len) {
               n = len;
               cnt = 0;
               init();
               for (int i = 1; i \le n; ++i)
                       if (in[i] == 0) q1.push(i);
               while (!q1.empty()) {
                       int u = q1.top();
                       q1.pop();
                       order[++cnt] = u;
```

5.4 Tarjan

```
for (int j = 0; j < (int)g[u].
                              size(); ++j) {
                                int v = g[u][j];
                                in[v]--;
                                if (in[v] == 0) q1.push(v
                                      );
                        }
                }
                return cnt == n;
        }
        int min_num (int len) {
                cnt = n = len;
                init();
                for (int i = 1; i <= n; ++i)</pre>
                        if (in[i] == 0) q2.push(i);
                while (!q2.empty()) {
                        int u = q2.top();
                        q2.pop();
                        order[cnt—] = u;
                        for (int j = 0; j < (int)g[u].
                              size(); ++j) {
                                int v = g[u][j];
                                if (-in[v] == 0) q2.push
                                      (v);
                        }
                }
                return cnt == 0;
}topsort;
```

# 5.5 Dinic

```
int Stack[maxn], low[maxn], dfn[maxn], inStack[maxn],
    belong[maxn];
int now, len, cnt;
// now:时间戳, len: 栈的大小, cnt强连通的个数
void init() {
       now = len = cnt = 0;
       mem(inStack, 0);
       mem(belong, 0);
       mem(dfn, 0);
       mem(low, 0);
void tarjan(int x) {
   // 打上标记,入栈
   low[x] = dfn[x] = ++now;
   Stack[++len] = x;
   inStack[x] = 1;
   for (int i = 0; i < (int)g[x].size(); ++i) {</pre>
       int y = g[x][i];
```

// 没有访问过,继续递归

```
// 在栈中表示可以形成一个强连通分量, 更新
                  根节点的low,继续找
       if (!dfn[y]) tarjan(y), low[x] = min(low[x], low[
       else if (inStack[y]) low[x] = min(low[x], low[y])
   }
       // 回溯, 如果当前节点的dfn = low 表示栈中形成一个
           强连通分量
   if (dfn[x] == low[x]) {
       ++cnt; // 统计个数
       int top;
       while (Stack[len] != x) {
          top = Stack[len-];
          belong[top] = cnt;
          inStack[top] = 0;
       top = Stack[len-];
       belong[top] = cnt; // 记录每个点的隶属关系
       inStack[top] = 0;
   }
for (int i = 1; i <= n; ++i) {</pre>
   if (!dfn[i]) tarjan(i);
```

```
struct ac{
    int v, c, pre;
}edge[maxn<<6];</pre>
int head[maxn<<1], dis[maxn<<1], curedge[maxn<<1], cnt;</pre>
void init() {
    mem(head, -1);
    cnt = 0;
void addedge(int u, int v, int c) { // 记得双向边
    edge[cnt] = \{v, c, head[u]\};
    head[u] = cnt++;
bool bfs() {
    queue<int> que;
    que.push(s);
    mem(dis, 0);
    dis[s] = 1;
    while (!que.empty()) {
        int f = que.front();
        que.pop();
        for (int i = head[f]; i != -1; i = edge[i].pre) {
```

```
if (dis[edge[i].v] || edge[i].c == 0)
            dis[edge[i].v] = dis[f] + 1;
            que.push(edge[i].v);
       }
   }
   return dis[e] > 0;
int dfs(int now, int flow) {
    if (now == e || flow == 0) return flow;
    for (int &i = curedge[now]; i != -1; i = edge[i].pre)
         { // 当前弧优化
        if (dis[edge[i].v] != dis[now] + 1 || edge[i].c
            == 0) continue;
        int d = dfs(edge[i].v, min(flow, edge[i].c));
        if (d > 0) {
           edge[i].c -= d;
            edge[i^1].c += d;
            return d;
       }
    dis[now] = -1; // // 炸点优化
   return 0;
int Dinic() {
    int sum = 0, d;
    while (bfs()) {
        for (int i = 0; i <= e; ++i) curedge[i] = head[i</pre>
       while (d = dfs(s, inf)) sum += d;
   }
   return sum:
}
```

# 6 其它

## 6.1 闰年

```
bool IsLeepYear(int y) {
  return (!(y % 4) && (y % 100)) || !(y % 400);
}
```

#### 6.2 蔡勒公式

```
// 返回y年m月d日是星期几
int Zeller(int y, int m, int d) {
    if (m == 1 || m == 2) {
        —y;
        m += 12;
    }
    int c = y / 100;
    y %= 100;
    //1582年10月4日之前
    return ((y + y / 4 + c / 4 - 2 * c + 13 * (m + 1) / 5 +
        d + 2) % 7) + 7 % 7;
    //1582年10月4日之后
    return ((y + y / 4 + c / 4 - 2 * c + 26 * (m + 1) / 10
        + d - 1) % 7 + 7) % 7;
}
```

# 6.3 莫队算法

#### 6.3.1 静态莫队

```
const int maxn = "Edit";
// 静态莫队算法求区间不同数字数量
struct MoCap {
 int n, m;
 int block;
 int arr[maxn];
 struct query { int l, r, id; };
 query q[maxn];
 int cnt[maxn << 1];</pre>
 int cur;
 int ans[maxn];
 void Add(int x) {
   cur += (++cnt[arr[x]] == 1);
 }
 void Del(int x) {
   cur = (--cnt[arr[x]] == 0);
```

```
void Solve() {
    scanf("%d%d", &n, &m);
    block = std::sqrt(n);
    for (int i = 1; i <= n; ++i) scanf("%d%d", &arr[i]);</pre>
    for (int i = 1; i \le m; ++i) {
      scanf("%d%d", &q[i].1, &q[i].r);
     q[i].id = i;
    std::sort(q + 1, q + m + 1, [\&](query k1, query k2) {
          return (k1.l / block) == (k2.l / block) ? k1.r
         < k2.r : k1.l < k2.l; \});
    int l = 0, r = 0;
    for (int i = 1; i \le m; ++i) {
     while (l < q[i].l) Del(l++);</pre>
      while (l > q[i].l) Add(---l);
      while (r < q[i].r) Add(++r);
     while (r > q[i].r) Del(r—);
     ans[q[i].id] = cur;
    for (int i = 1; i <= m; ++i) printf("%d\n", ans[i]);</pre>
 }
}mo;
```

#### 6.3.2 带修莫队

```
const int maxn = "Edit";
// 动态莫队算法求区间不同数字数量 (支持单点修改)
struct MoCap {
 int n, m;
 int block;
 int arr[maxn];
 struct query { int l, r, pre, id; };
 int q_tot;
 query q[maxn];
 struct change { int pos, val; };
 int c_tot;
 chanae c[maxn]:
  int cnt[maxn << 7];</pre>
  int cur;
 int ans[maxn];
 void Add(int x) {
   cur += (++cnt[arr[x]] == 1);
 }
 void Del(int x) {
   cur = (--cnt[arr[x]] == 0);
 void Modify(int x, int i) {
   if (c[x].pos >= q[i].l && c[x].pos <= q[i].r) {
     cur = (-cnt[arr[c[x].pos]] == 0);
     cur += (++cnt[c[x].val] == 1);
   std::swap(c[x].val, arr[c[x].pos]);
```

```
}
  void Solve() {
    scanf("%d%d", &n, &m);
   block = std::sqrt(n);
    for (int i = 1; i <= n; ++i) scanf("%d", &arr[i]);</pre>
    for (int i = 1; i <= m; ++i) {
      char op; getchar();
      scanf("%c", &op);
      if (op == 'Q') {
       int l, r; scanf("%d%d", &l, &r);
        q[++q\_tot] = (query)\{l, r, c\_tot, q\_tot\};
      else {
        int p, v; scanf("%d%d", &p, &v);
        c[++c_{tot}] = (change)\{p, v\};
    std::sort(q + 1, q + q_tot + 1, [&](query k1, query
      if ((k1.1 / block) == k2.1 / block) {
        if ((k1.r / block) == (k2.r / block)) return k1.
             pre < k2.pre;</pre>
        return k1.r < k2.r;</pre>
      }
      return k1.l < k2.l;</pre>
   });
    int l = 1, r = 0, t = 0;
    for (int i = 1; i <= q_tot; ++i) {</pre>
      while (l < q[i].l) Del(l++);</pre>
      while (l > q[i].l) Add(--l);
      while (r < q[i].r) Add(++r);
      while (r > q[i].r) Del(r—);
      while (t < q[i].pre) Modify(++t, i);</pre>
      while (t > q[i].pre) Modify(t--, i);
      ans[q[i].id] = cur;
   for (int i = 1; i \leftarrow q_tot; ++i) printf("%d\n", ans[i
         1);
 }
}mo;
```

#### 6.4 快读

```
// 普通快读
template <typename t>
inline bool Read(t &ret) {
   char c; int sgn;
   if (c = getchar(), c == EOF) return false;
   while (c != '-' && (c < '0' || c > '9')) c = getchar();
   sgn = (c == '-') ? -1 : 1;
   ret = (c == '-') ? 0 : (c - '0');
```

```
while (c = getchar(), c >= '0' && c <= '9') ret = ret *</pre>
        10 + (c - '0');
 ret *= sgn;
 return true;
// 牛逼快读
namespace FastIO {
 const int MX = 4e7;
 char buf[MX];
 int c, sz;
 void Begin() {
   c = 0;
   sz = fread(buf, 1, MX, stdin);
  template <class T>
 inline bool Read(T &t) {
   while (c < sz && buf[c] != '-' && (buf[c] < '0' ||</pre>
        buf[c] > '9')) c++;
   if (c >= sz) return false;
   bool flag = 0;
   if (buf[c] == '-') {
     flag = 1;
   }
   for (t = 0; c < sz && '0' <= buf[c] && buf[c] <= '9';</pre>
          ++c) t = t * 10 + buf[c] - '0';
   if (flag) t = -t;
   return true;
using namespace FastIO;
```

```
then
   echo right $n
   n=n+1
else
   exit
fi
done
```

#### 6.6 vimrc

```
svntax on
set nu ts=2 sw=2 et mouse=a cindent
"map <F9> :call Run() <CR>
"func! Run()
  exec "W"
    exec "!g++ % -o %<"
    exec "! %<"
"map <F2> :call SetTitle() <CR>
"func SetTitle()
   let 1 = 0
    let l = l + 1 \mid call setline(l, "#include <bits/stdc
    ++.h>")
    let 1 = 1 + 1 | call setline(1, "")
    let l = l + 1 \mid call setline(l, "int main() {"})
    let 1 = 1 + 1 | call setline(1, " return 0;")
    let l = l + 1 \mid call setline(l, "}")
    let 1 = 1 + 1 | call setline(1, "")
"endfunc
```

#### 6.5 对拍

```
// windows
:loop
data.exe > in.txt
main.exe < in.txt > out.txt
std.exe < in.txt > std.txt
fc out.txt std.txt
if not errorlevel 1 goto loop
pause
:end
// Linux
declare -i n=1
while (true)
 do
 ./dtmk
 ./my < 1.in > my.out
 ./force < 1.in > for.out
 if diff my.out for.out
```

#### 6.7 int128

```
using namespace std;
inline __int128 read(){
    __int128 x=0,f=1;
    char ch=getchar();
    while(ch<'0'||ch>'9'){
        if(ch=='-')
            f=-1;
        ch=getchar();
    }
    while(ch>='0'&&ch<='9'){
        x=x*10+ch-'0';
        ch=getchar();
    }
    return x*f;
}
inline void print(__int128 x){
    if(x<0){</pre>
```

```
putchar('-');
    x=-x;
}
if(x>9)
    print(x/10);
putchar(x%10+'0');
}
int main(){
    __int128 a = read();
    __int128 b = read();
print(a + b);
cout<<endl;
return 0;
}</pre>
```