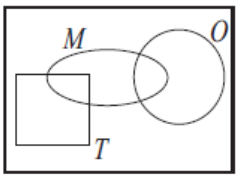
MT206 Probability and Statistics

Assignment No. 2

Deadline: November 06; 2019

Q1.

A pizza at Gerlanda's is either regular (R) or Tuscan (T). In addition, each slice may have mushrooms (M) or onions (O) as described by the Venn diagram at right. For the sets specified below, shade the corresponding region of the Venn diagram.



(1) R

(2) $M \cup O$

(3) $M \cap O$

(4) $R \cup M$

(5) $R \cap M$

(6) $T^c - M$

Q2. You enter a chess tournament where your probability of winning a game is 0.3 against half of the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. What is the probability of winning?

Q3. Consider an experiment involving two successive rolls of a 4-sided die in which all 16 possible outcomes are equally likely and have probability 1/16. Are the events $A_i = \{1st\ roll\ results\ in\ i\}$,

$B_j = \{2nd\ roll\ results\ in\ j\}$ independent?

Q4. There are two types of celllular phones, handheld phones (H) that you carry and mobile phone (M) that are mounted in vehicles. Phone calls can be classified by the traveling speed of the user as fast (F) or slow (W). Monitor a cellular phone call and observe the type of telephone and the speed of the user. The probability model for this experiment has the following information:
$P[F] = 0.5, P[HF] = 0.2, P[MW] = 0.1$. What is the sample space of the experiment? Calculate the following probabilities:
(a) $P[W]$ (b) $P[MF]$ (c) $P[H]$

Q5. In an experiment, *A*, *B*, *C* and *D* are events with probabilities $P[A \cup B] = 5/8$, $P[A] = 3/8$, $P[C \cap D] = 1/3$, and $P[C] = 1/2$.

Furthermore, *A* and *B* are disjoint, while *C* and *D* are independent.

(a) Find $P[A \cap B]$, $P[B]$, $P[A \cap B^c]$, and $P[A \cup B^c]$.

(b) Are *A* and *B* independent?

(c) Find $P[D]$, $P[C \cap D^c]$, $P[C^c \cap D^c]$, and $P[C|D]$.

(d) Find $P[C \cup D]$ and $P[C \cup D^c]$.

(e) Are *C* and D^c independent?

Q6. For independent events *A* and *B*, Prove that

(a) *A* and B^c are independent.

(b) A^c and *B* are independent.

(c) A^c and B^c are independent.