BT6270-Computational Neuroscience
Assignment 2
FitzHugh-Nagumo Model Simulation

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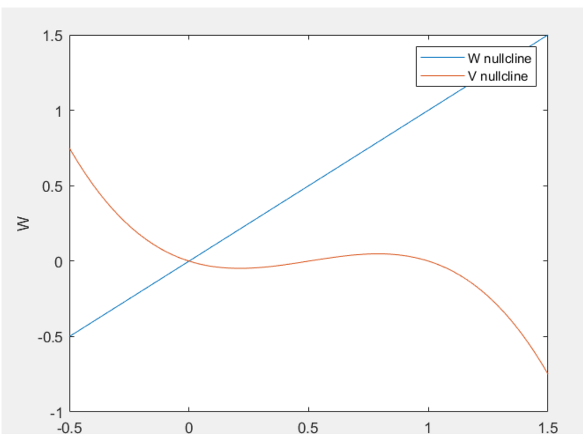
BS20B032

- **All plots were generated using MATLAB.**
- **The codes corresponding to each of the cases is attached in the zip file.**
- **Numerical Integration was carried out using Euler Integration Scheme with a delta value of 0.01.**

Case 1: Iext = 0

(a) Draw a Phase Plot superimposed (use hold on command in MATLAB)

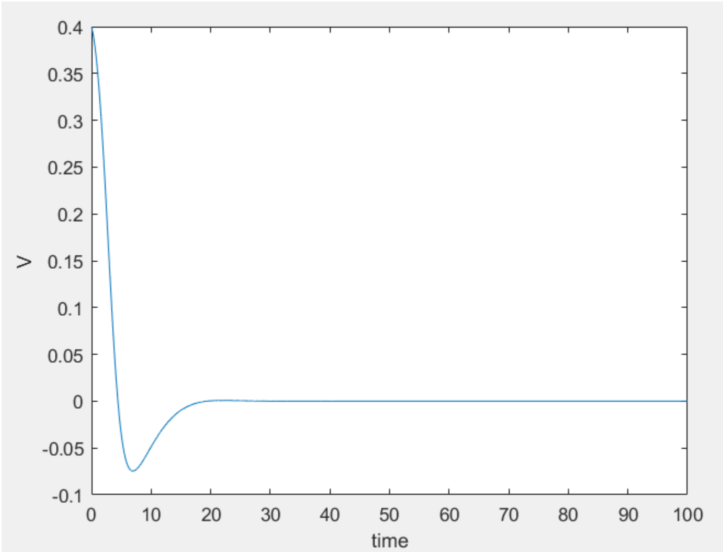
a=0.5,b=0.1,γ=0.1,Im=0



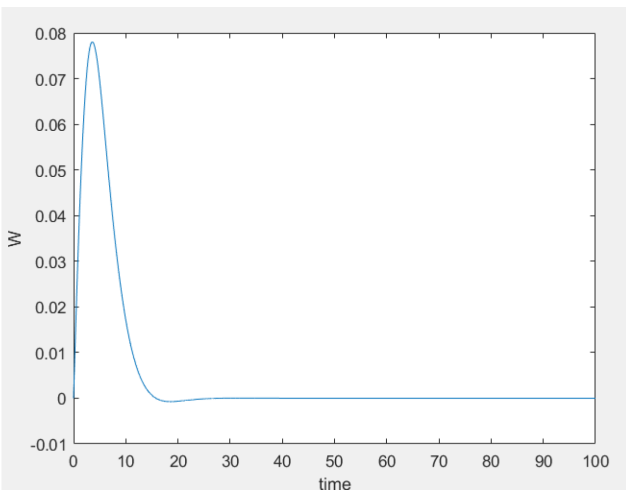
(b) Plot V(t) vs t and W(t) vs t and also show the trajectory on the phase plane for the both cases

(i) V(0) < a and W (0)= 0

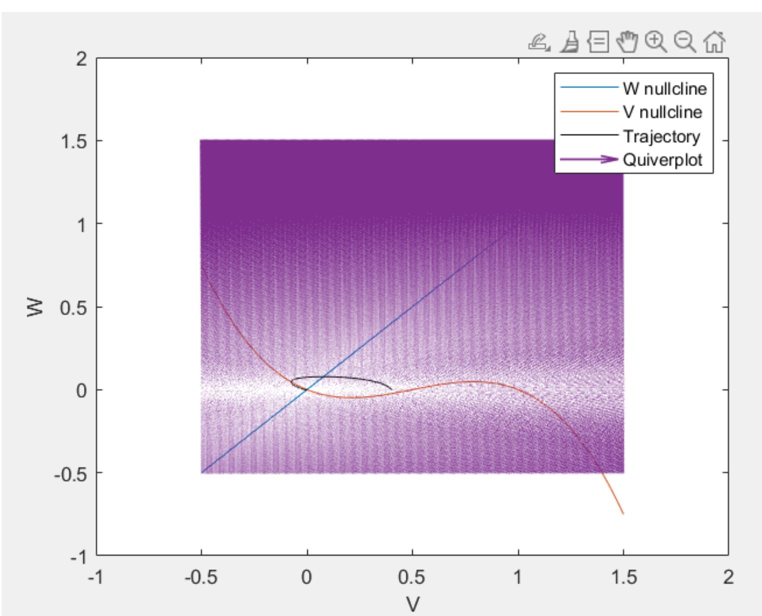
V(1)=0.4



V
time



W
time

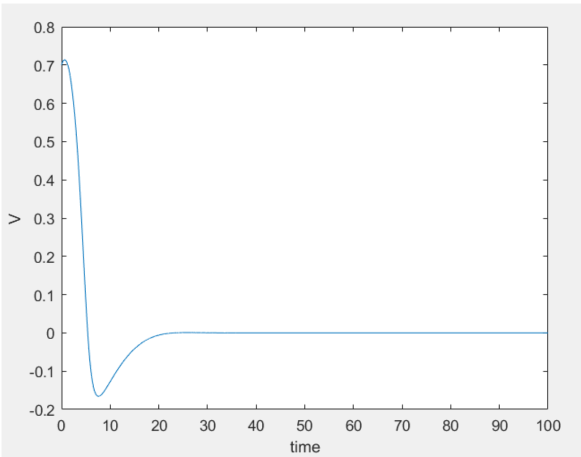


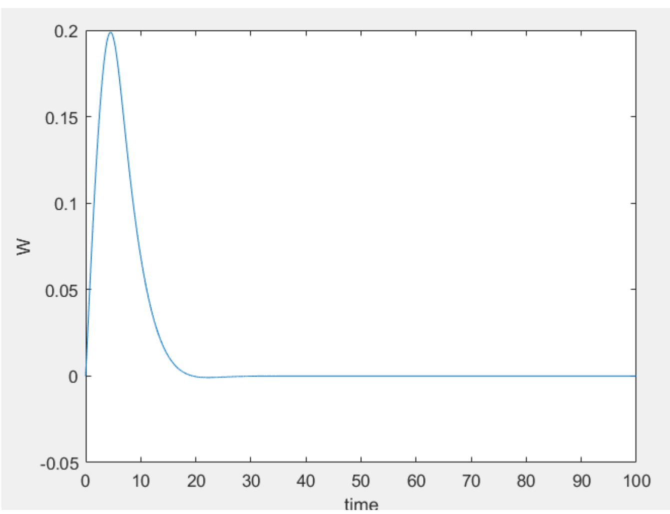
W nullcline
V nullcline
Trajectory
Quiverplot
W
V

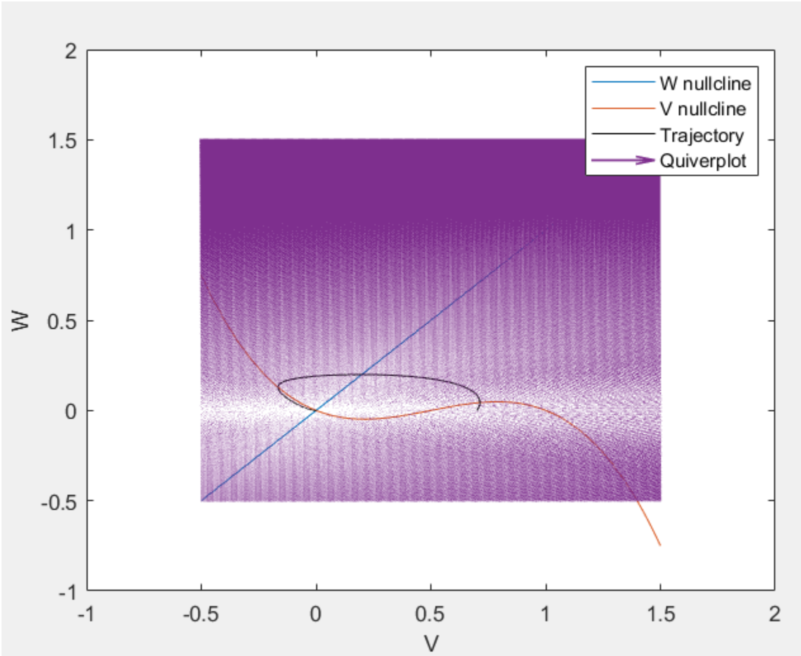
Inference: It is evident from the above plots that the origin which is the point of intersection of the 2 null clines (V and W) is a stable fixed point.When the initial values of (V,W) is set to (0.4,0), from the trajectory we can see that the curve eventually reaches and settles down at the origin.

(ii) V(0) > a and W (0)= 0

V(1)=0.7

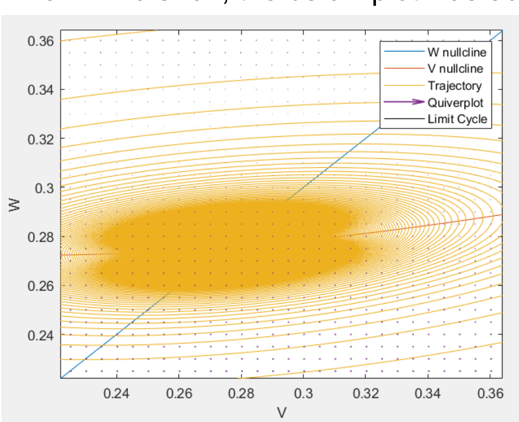




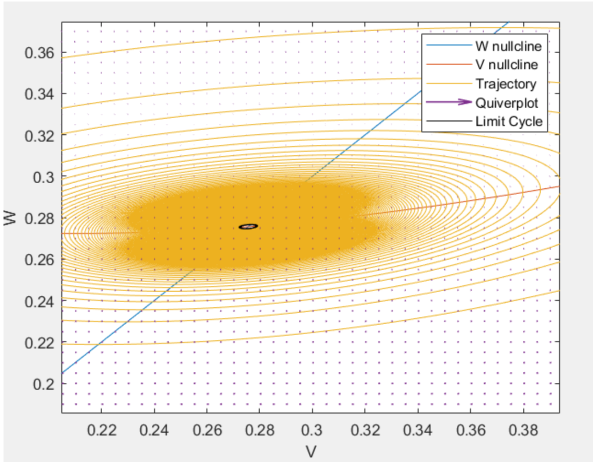


Inference: In these above plots , the initial V value ,ie V(1) is set to 0.7 >a which is 0.5.In this case, the V values attain a maxima (0.713) and then falls sharply and eventually settles down to 0. Similarly to case (i) , we can see that W attains a maxima in time before settling down to 0. Overall , it is evident that the origin here acts as a stable fixed point.

Case 2: Choose some current value I1 < Iext < I2 where it exhibits oscillations. Find the values of I1 and I2.
I1=0.3204
I2=0.6795
The above values were obtained by iteratively changing the values of Im to find the points where the system exhibits bifurcation from limit cycle behavior.
When Im=0.3204, the below plot was obtained

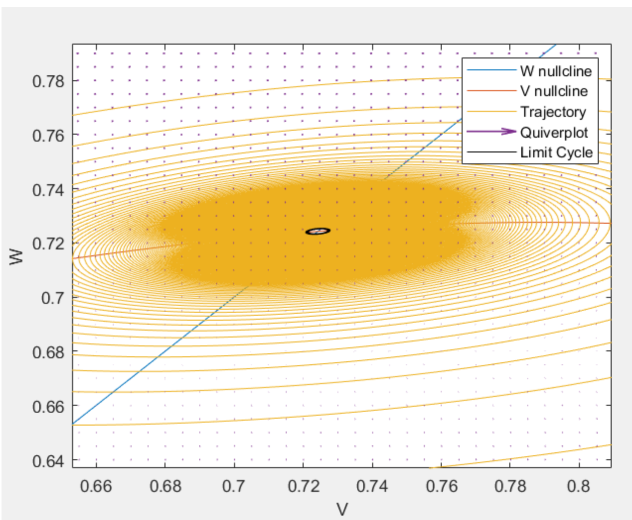


When Im=0.3205, the below plot was obtained

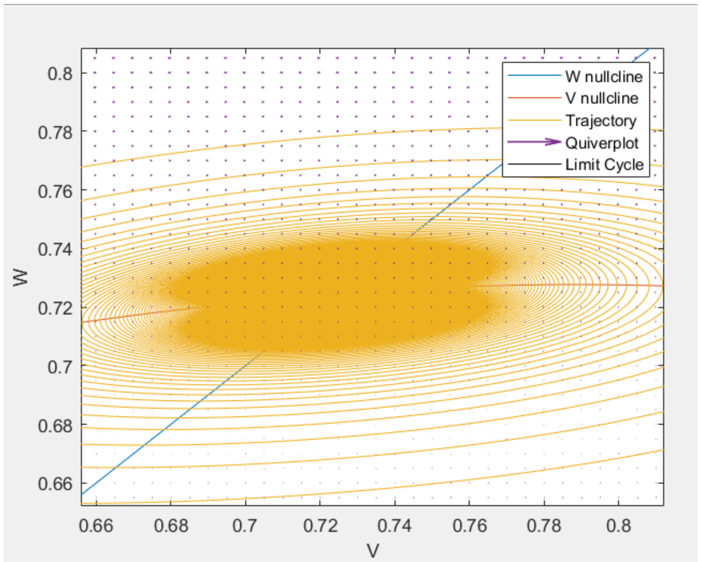


From the above plots, it is evident that there is a bifurcation in the dynamic behavior for I>0.3204

Similarly, when I set current to 0.6795 the below plot was obtained



When Im=0.6796, the below plot was obtained



From the above plots, it is evident that there is a bifurcation in the dynamic behavior for I<0.6796

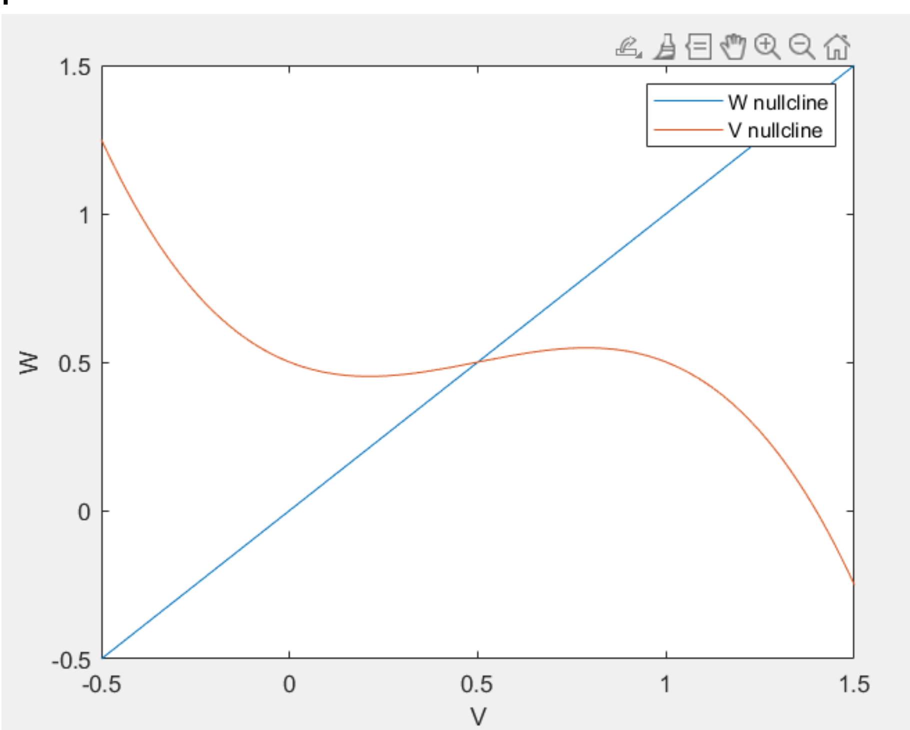
(a) Draw a Phase Plot for some sample value of Iext

Im=0.5

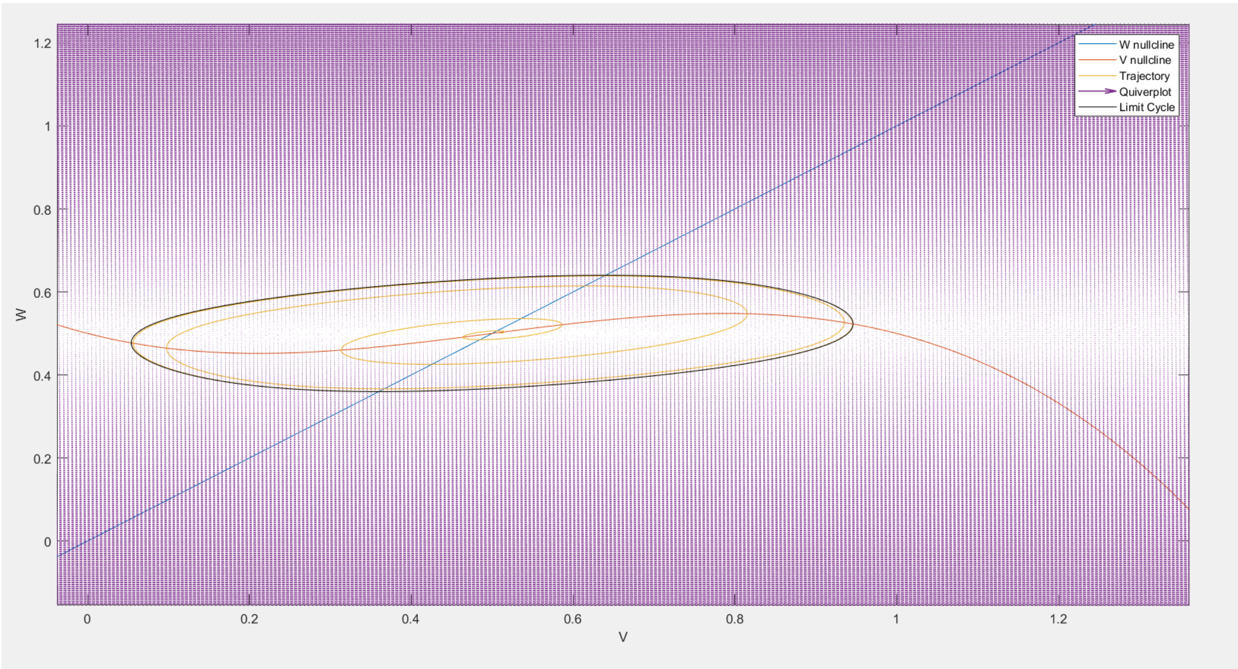
a=0.5

b=0.1

γ=0.1



(b) Show that the fixed point is unstable i.eFor a small perturbation there is no return to the fixed point (show the trajectory on the phase plane) – also show a limit cycle on the phase plane .



As we can see above, the null clines intersect at (0.5,0.5) for the initial conditions, Im=0.5,b=0.1,**γ**=0.1 and a=0.5.

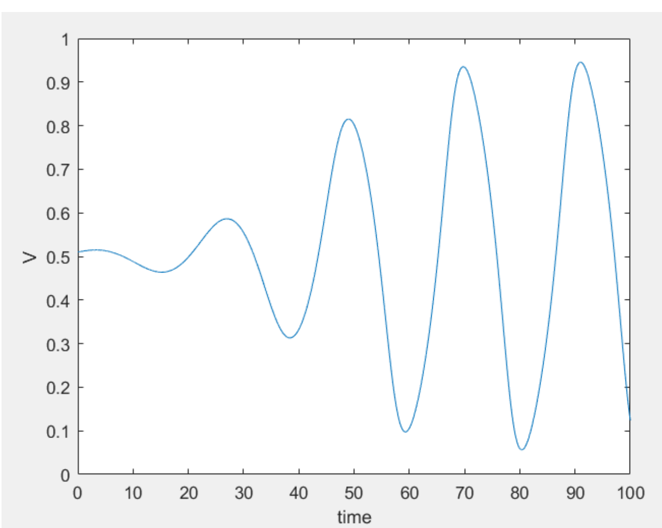
When I set initial voltage to 0.51 and initial W to 0,5, we can see through the trajectory that the system eventually attains a limit cycle as indicated by the above plot.

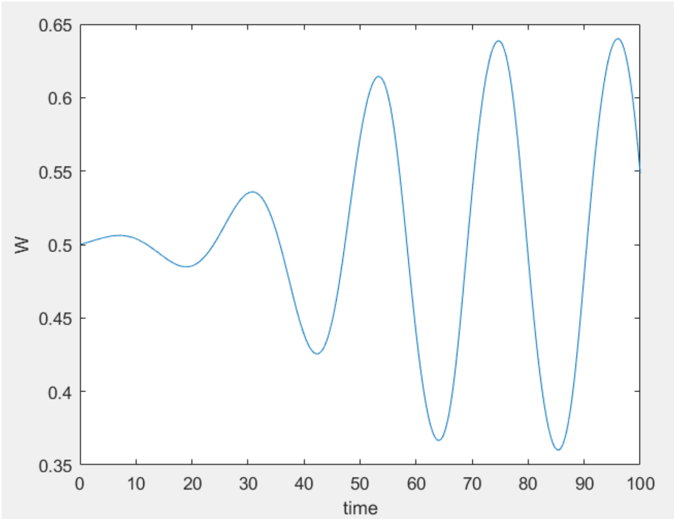
(c) Plot V(t) vs t and W(t) vs t

V(1)=0.51

W(1)=0.5

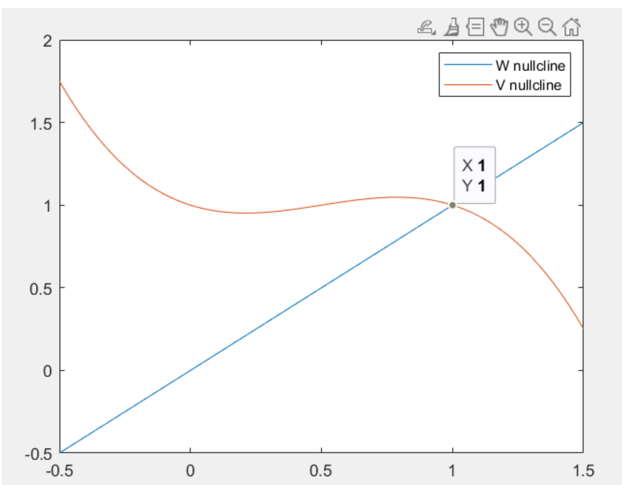
Im =0.5



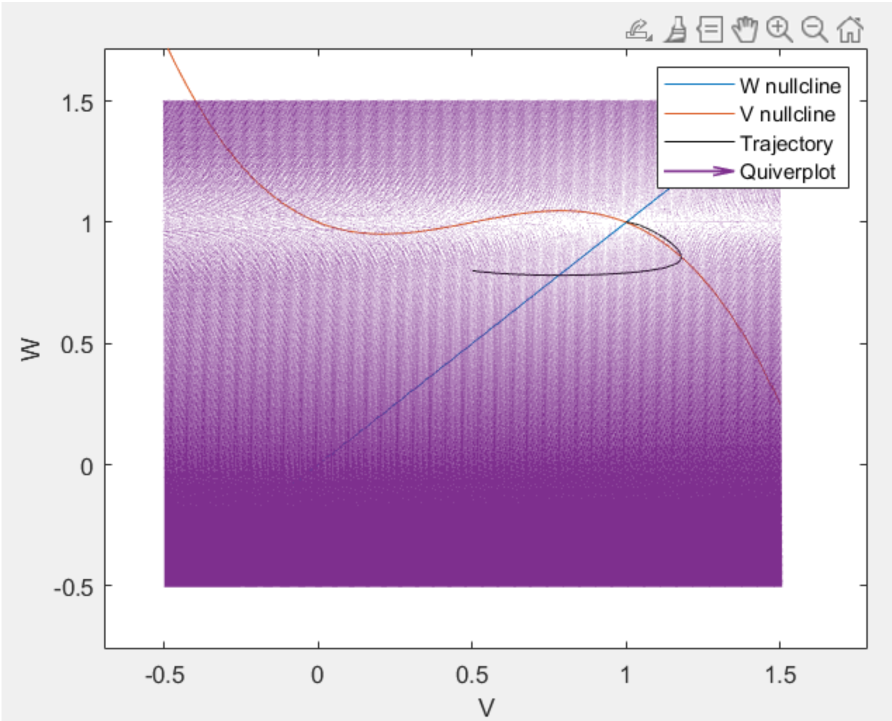


Inference: Through the above plots, it is evident that the system displays a limit cycle behavior when the W null cline intersects the V null cline at the region where f'(V)=(a-v)(v)(v-1) >0. This shows that the point of intersection of the 2 null clines is an unstable fixed point.

Case 3: Choose some Iext > I2
(a) Draw a Phase Plot for some sample value of Iext
Iext=1.0



(b) Show that the fixed point is stable i.e., for a small perturbation there is a return to the fixed point (show the trajectory on the phase plane)



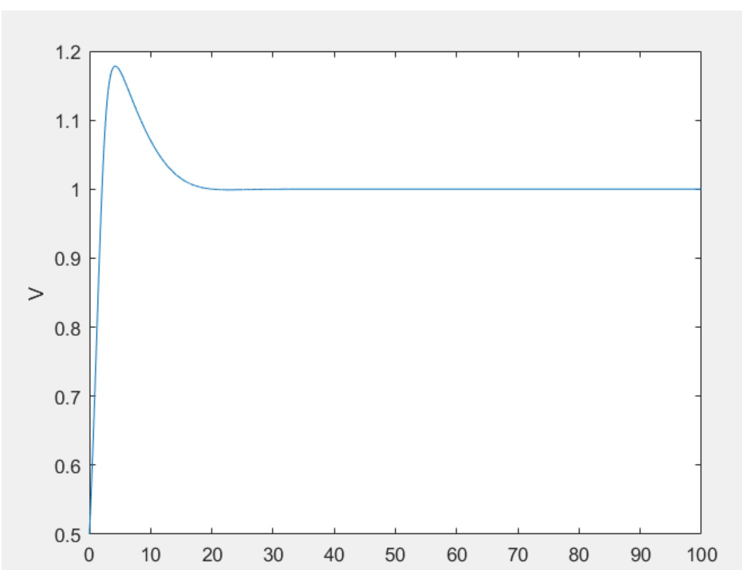
The V & W nullclines intersect at (1,1). When I set the initial V and W values to be (0.5,0.8) and simulate the model , I observed that the (V,W) attained a steady state value of (1,1) which coincides with the intersection point of the null clines.This shows that the fixed point is stable.

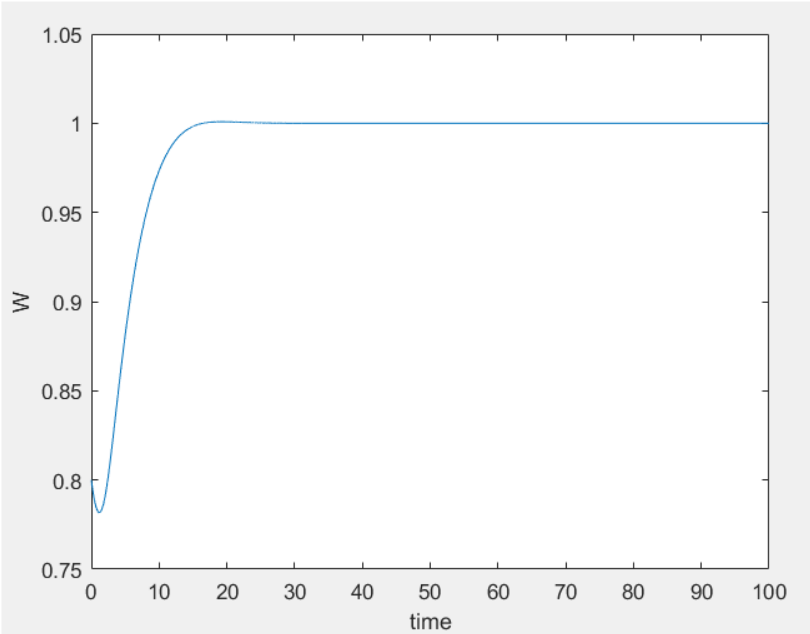
(c) Plot V(t) vs t and W(t) vs t

V(1)=0.5

W(1)=0.8

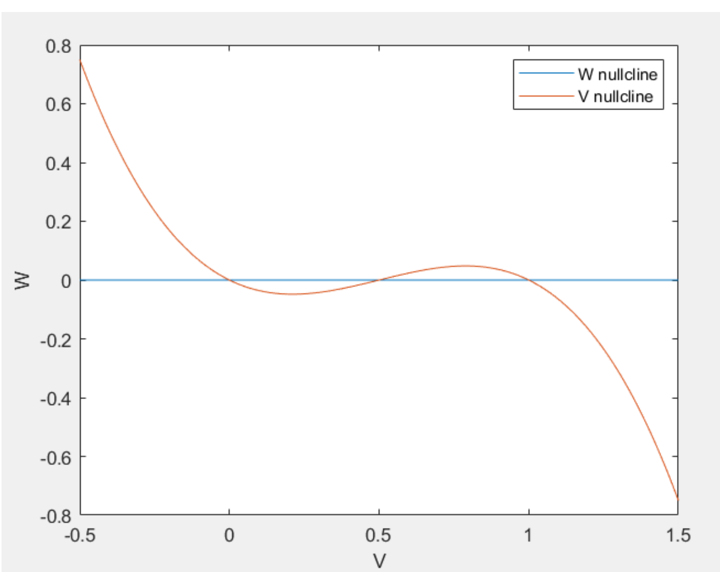
Im=1





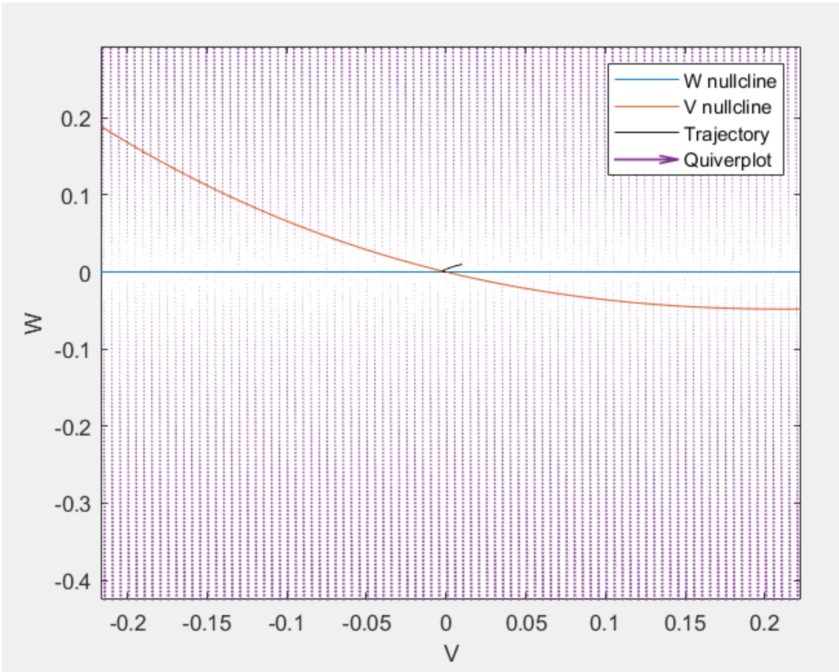
Inference: It is evident from the above plots that the point of intersection of the 2 null clines (V and W) (1,1) is a stable fixed point.When the initial values of (V,W) is set to (0.5,0.8), from the trajectory we can see that the V(t) and W(t) curves settle at 1 respectively.

Case 4: Find suitable values of Iext and (b/r) such that the graph looks as phase plot shown as
below.
(a) Redraw the Phase plot
b=0.0, γ=0.8 , Im=0.0 ,b/γ=0.0
Note: For the sake of convenience of getting P1,P2,P3 as integral points , the b value has been set to 0.



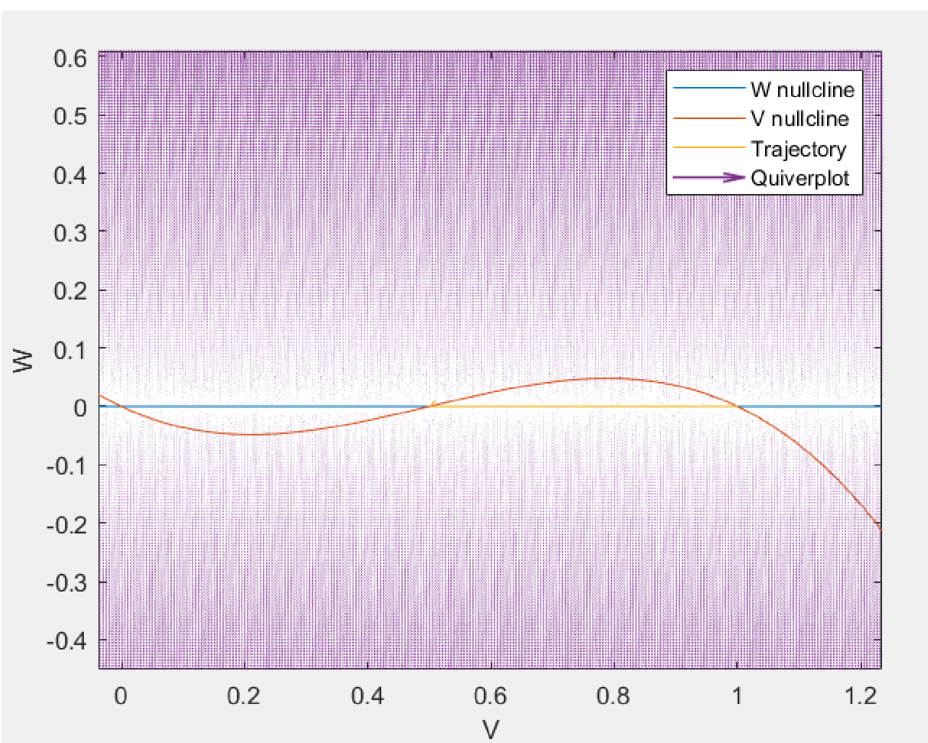
(b) Show stability of P1, P2, P3

Perturbing P1 state by setting V(1)=0.01, W(1)=0.04

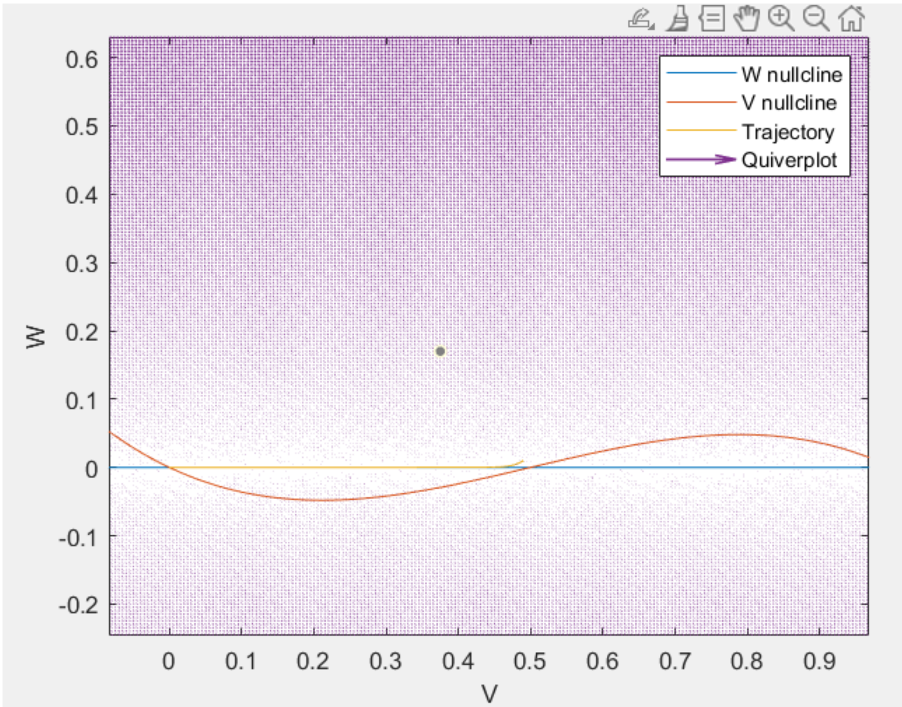


From the trajectory it is evident that P1 is a stable fixed point in the neighborhood of P1, as V and W return back to the origin which coincides with P1.

Perturbing P2 state by setting V(1)=0.51, W(1)=0.01

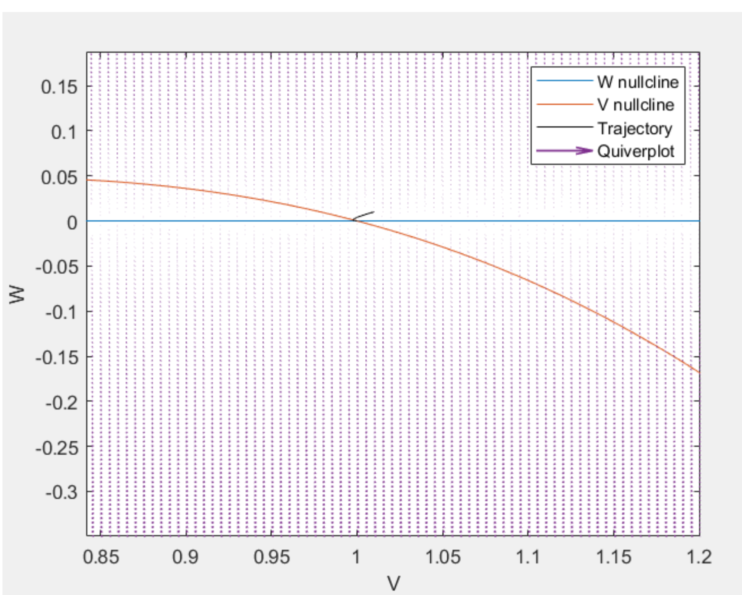


Perturbing P2 state by setting V(1)=0.49, W(1)=0.01



From the above trajectories it is evident that P2 is a saddle fixed point, as V and W return back to the origin which coincides with P1 or to the point (1,0) which coincides with P3 depending on the initial conditions of (V,W), thus acting as a switch that induces bi stability.

Perturbing P3 state by setting V(1)=1.1, W(1)=0.01

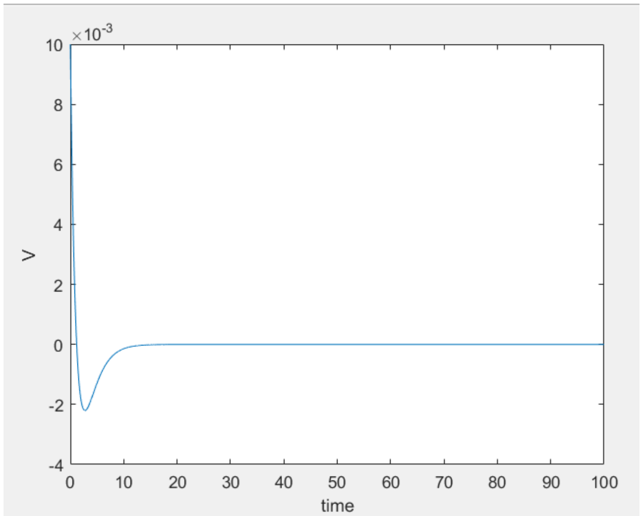


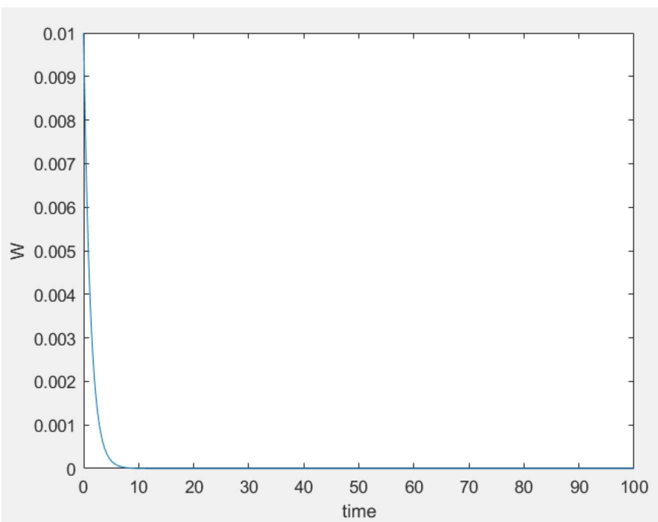
From the above trajectory it is evident that P3 is a stable fixed point in the neighborhood of P3, as V and W return back to the origin which coincides with P1.

c) Plot V(t) vs t and W(t) vs t

Initial Conditions used for analyzing stability of P1

V(1)=0.01,W(1)=0.01,Im=0,b=0,γ=0.8,a=0.5

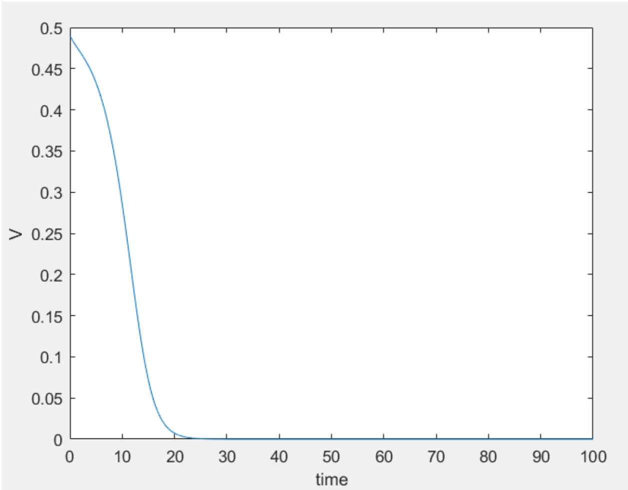


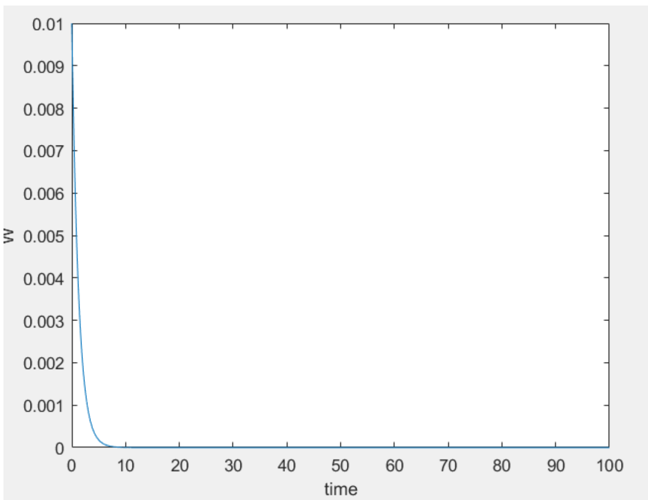


Inference: The above plots indicate that P1 is a stable fixed point. The (V,W) values reach (0,0) when the initial conditions are set in the neighborhood of P1

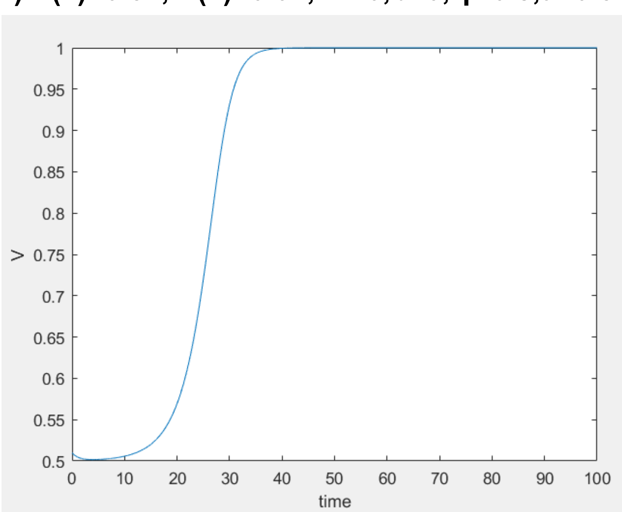
Initial Conditions used for analyzing stability of P2

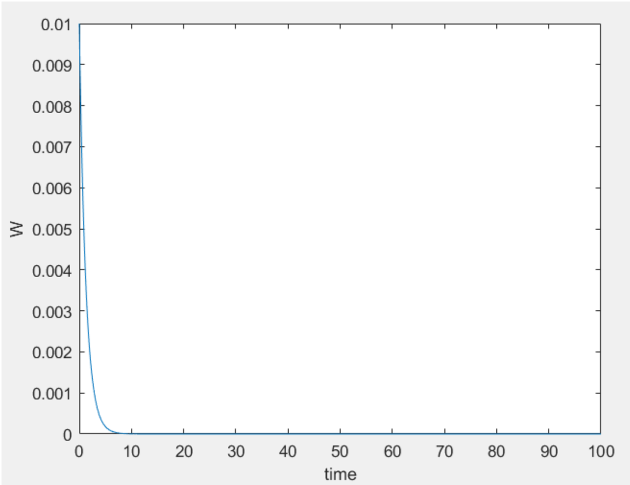
i)V(1)=0.49,W(1)=0.01,Im=0,b=0, γ=0.8,a=0.5





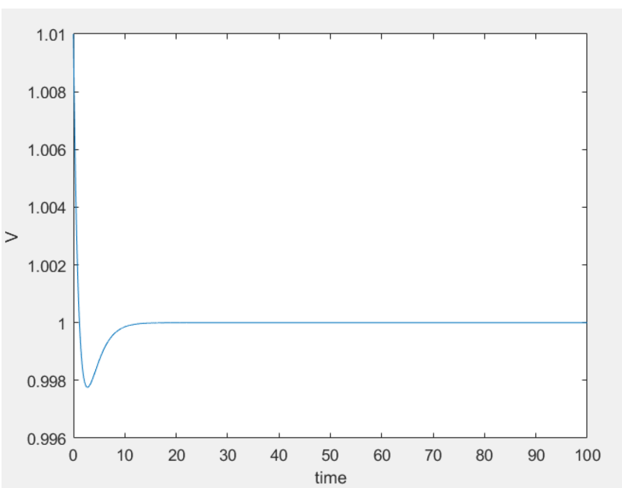
ii) V(1)=0.51,W(1)=0.01,Im=0,b=0, γ=0.8,a=0.5

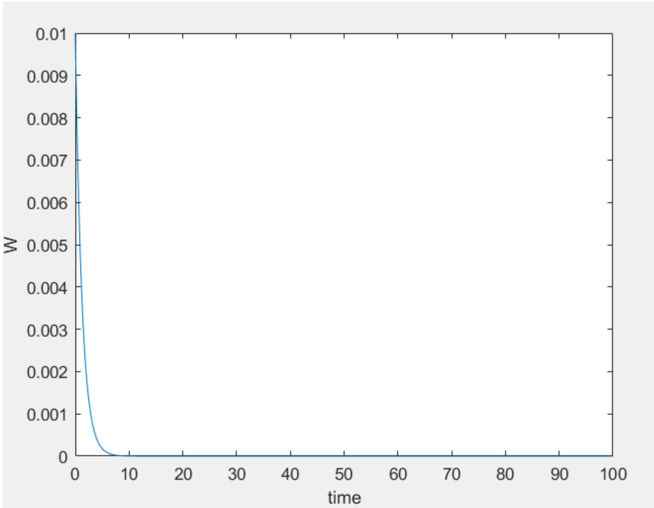




Inference: The above plots indicate that P2 is a saddle node. The (V,W) values reach (0,0) which is P1 when the initial conditions are set in such a way that V(1)<a and the (V,W) values reach (1,0) which is P3 when V(1)>a.

Initial Conditions used for analyzing stability of P3
V(1)=1.01,W(1)=0.01,Im=0,b=0, γ=0.8,a=0.5





Inference: The above plots indicate that P3 is a stable fixed point. The (V,W) values reach (1,0) which is P3 when the initial conditions are set in the neighborhood of P3.

The W(t) vs t plot for the above plots in Case 4 are identical because the b value has been set to 0. Thus, the W(t) curve depends only on the value of γ and the initial W value. Since the W value has been taken as 0.01 for all the plots in Case 4, identical plots have been obtained.