BT6270: Computational Neuroscience
Assignment-3

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BS20B032

1.Complex Coupling of 2 Hopf Oscillators at same frequency

Calculations Involved :
Objective is to achieve a phase difference of **ψ** between the Hopf Oscillators.
The equations of the 2 oscillators are given by
Note: Bold notation below denotes the derivative of the variable
$\mathbf{z_1}= (\mu-|z_1|^2)z_1 +iw_1z_1+Ae^{i\varphi}z_2$
$\mathbf{z_2}= (\mu-|z_2|^2)z_2 +iw_2z_2+Ae^{-i\varphi}z_1$

Representing in polar coordinates, we obtain

$\mathbf{r_1}=(\mu-r_1^2)r_1+ Ar_2\cos(\theta_2-\theta_1+\varphi)$
$\mathbf{r_2}=(\mu-r_2^2)r_2+ Ar_1\cos(\theta_2-\theta_1+\varphi)$
$\boldsymbol{\theta_1}=w_1+A(r_2/r_1)\sin(\theta_2-\theta_1+\varphi)$
$\boldsymbol{\theta_1}=w_2-A(r_1/r_2)\sin(\theta_2-\theta_1+\varphi)$
$\boldsymbol{\psi=\theta_1-\theta_2}$

Setting **ψ=0,** we obtain the equation,

$(w_1-w_2)+A((r_1^2+r_2^2)/r_1r_2)\sin(\theta_2-\theta_1+\varphi)$

This implies that $\theta_2-\theta_1+\varphi=0$ or $\varphi= \psi$.

Thus the value of A is not of much significance as long as it is set to low enough value. The coupling angle φ must be set to the desired value ψ to be achieved.
Given that:
$w_1=w_2 =5$
Assumptions:
A=0.2
μ=1
$\theta_1(0)=45^o$
$\theta_2(0)=0^o$
$r_1(0)=1$
$r_2(0)=1$
The complex coupling coefficients for $\psi=-47^o$ and $\psi=98^o$ are thus given by $0.2(e^{i(-47)})$,$0.2(e^{i(47)})$ and $0.2(e^{(98)})$,$0.2(e^{(-98)})$ respectively.

Phase Difference=-47^{O}

Fig (a): Plot of Real(z) of the 2 oscillators

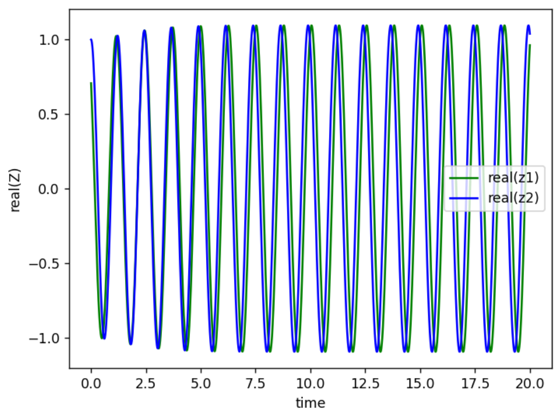
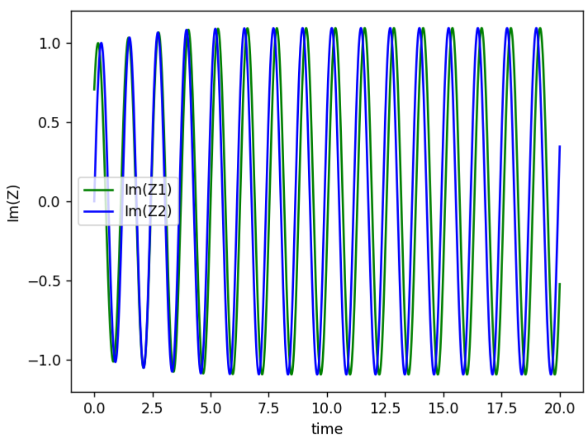
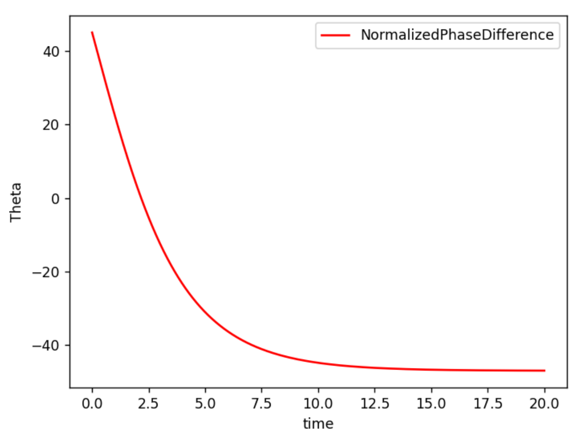


Fig (b): Plot of Imag(z) of the 2 oscillators



Plot Showing the trajectory of Psi: ψ ,i.e Phase difference between the 2 oscillators in time



Phase Difference=98^{O}

Fig (a): Plot of Real(z) of the 2 oscillators

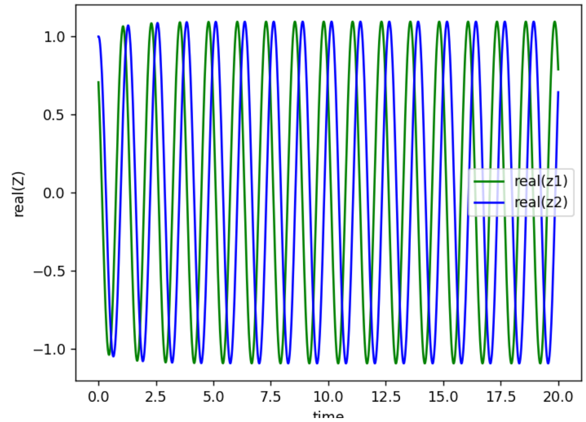
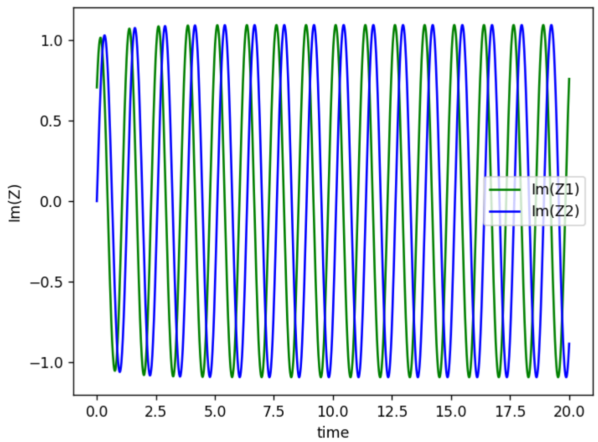
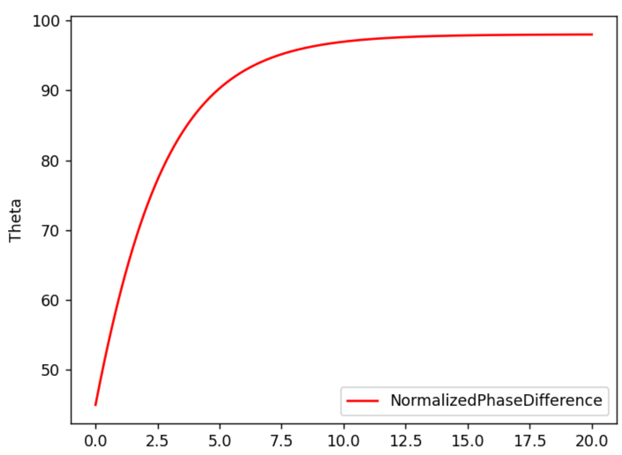


Fig (b): Plot of Imag(z) of the 2 oscillators



Plot Showing the trajectory of Psi: ψ ,i.e Phase difference between the 2 oscillators in time



2.Power Coupling of 2 Hopf Oscillators at different frequencies

Calculations Involved:

Objective is to achieve a normalized phase difference of **ψ** between the Hopf Oscillators.
The equations of the 2 oscillators are given by

Note: Bold notation below denotes the derivative of the variable

$\mathbf{z_1}$= $(\mu-|z_1|^2)z_1 + iw_1z_1 + Ae^{i\varphi/w2}z_2^{(w1/w2)}$

$\mathbf{z_2}$= $(\mu-|z_2|^2)z_2 + iw_2z_2 + Ae^{-i\varphi/w1}z_1^{(w2/w1)}$

Representing in polar coordinates, we obtain

$\mathbf{r_1}=(\mu-r_1^2)r_1+ Ar_2^{(w1/w2)}\cos(w_1((\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2)))$

$\mathbf{r_2}=(\mu-r_2^2)r_2+ Ar_1^{(w2/w1)}\cos(w_2(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2))$

$\boldsymbol{\theta_1}=w_1+A(r_2^{(w1/w2)}/r_1)\sin(w_1(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2))$

$\boldsymbol{\theta_2}=w_2-A(r_1^{(w2/w1)}/r_2)\sin(w_2(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2)$

$\boldsymbol{\psi=(\theta_1/w_1)-(\theta_2/w_2)}$

Setting **ψ=0,** we obtain the equation,

$(1-1)+(A/w_1)(r_2^{(w1/w2)}/r_1)\sin(w_1(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2))+(A/w_2)(r_1^{(w2/w1)}/r_2)\sin(w_2(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1 w_2)=0$

This implies that $(\theta_2/w_2)-(\theta_1/w_1)+(\varphi/w_1w_2)=0$ or $\varphi/(w_1w_2)= \psi$.

Thus the value of A is not of much significance as long as it is set to low enough value. The coupling angle φ must be set to the desired value ψ to be achieved multiplied by w_1 and w_2.

Given that:

$w_1=5, w_2=15$

Assumptions:

A=0.2

μ=1

φ=ψ*75

ψ=-47°,	**ψ=98°**
$\theta_1(0)=-60^o$	$\theta_1(0)=360^o$
$\theta_2(0)=0^o$	$\theta_2(0)=0^o$
$r_1(0)=1$	$r_1(0)=1$
$r_2(0)=1$	$r_2(0)=1$

The initial conditions have been chosen to avoid undesired solutions

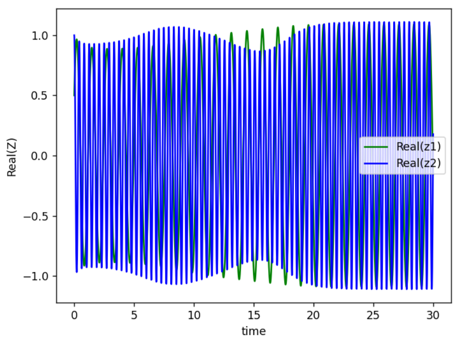
The power coupling coefficients for ψ=-47° and ψ=98° are thus given by $0.2(e^{i(-47/15)})$,$0.2(e^{i(47/5)})$and $0.2(e^{(98/15)})$,$0.2(e^{(-98/15)})$ respectively.

Normalized Phase Difference: -47°

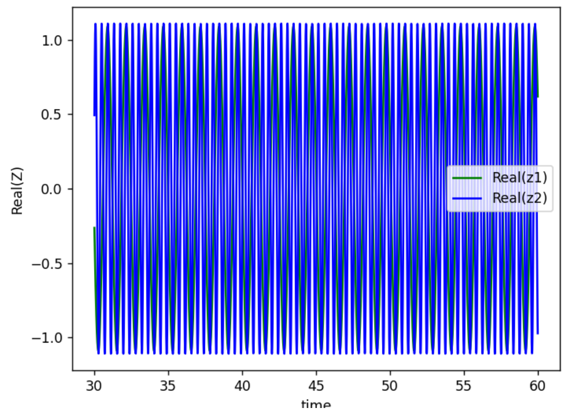
Plot of Oscillator response in time (0-60 secs)

Real Part:

A) 0-30 secs

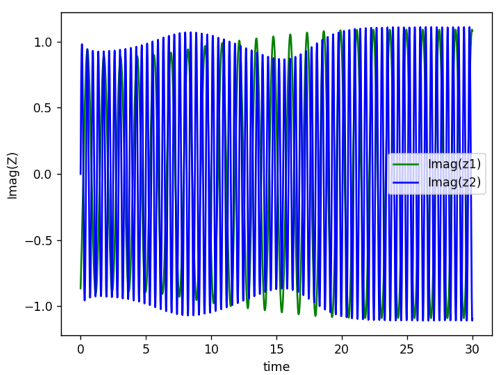


B) 30-60 secs

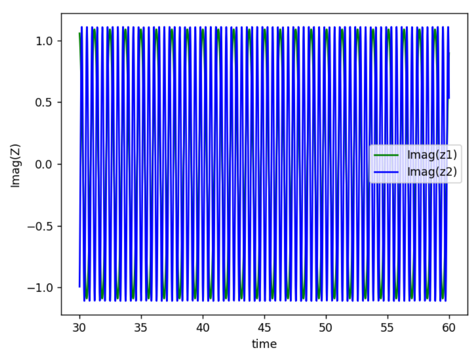


Imaginary Part:

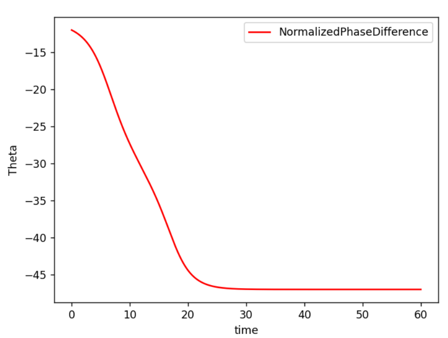
A) 0-30 secs



B) 30-60 secs



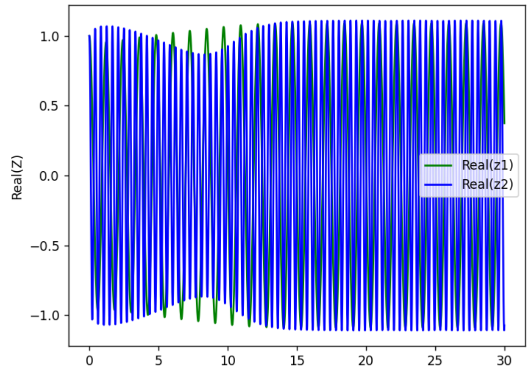
Plot Showing the trajectory of Psi: ψ between the 2 oscillators in time



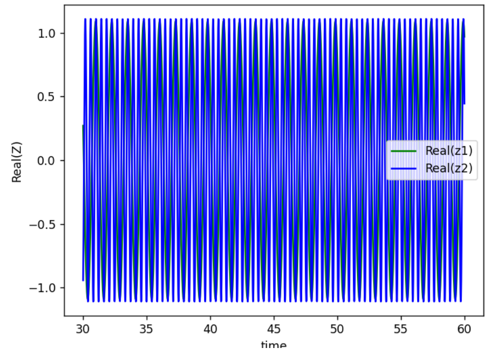
Normalized Phase Difference: 98°

Plot of Oscillator response in time (0-60 secs)
Real Part

A) 0-30 sec

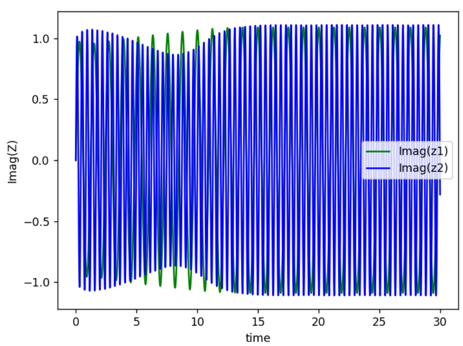


B) 30-60 Sec

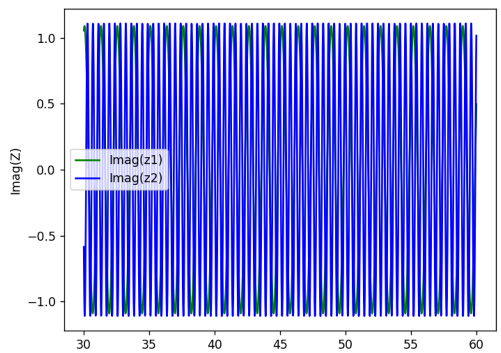


Imaginary Part:

A)0-30 Sec



B)30-60 Sec



Plot Showing the trajectory of Psi in time