Evaluation Metrics for Regression

We have the following commonly used evaluation metrics that helps us assess the performance of our model.

1. Mean Square Error (MSE)

Mean Squared Error (MSE) is an evaluation metric that calculates the average of the squared differences between the actual and predicted values for all the data points. The difference is squared to ensure that negative and positive differences don't cancel each other out.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

Here,

- n is the number of data points.
- y_i is the actual or observed value for the i^{th} data point.
- $\hat{y}_i$ is the predicted value for the i^{th} data point.

MSE is a way to quantify the accuracy of a model's predictions. MSE is sensitive to outliers as large errors contribute significantly to the overall score.

2. Mean Absolute Error (MAE)

Mean Absolute Error is an evaluation metric used to calculate the accuracy of a regression model. MAE measures the average absolute difference between the predicted values and actual values.
Mathematically MAE is expressed as:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i - \hat{Y}_i\right|$$

Here,

- n is the number of observations
- Y_i represents the actual values.
- $\bar{Y}_i$ represents the predicted values

Lower MAE value indicates better model performance. It is not sensitive to the outliers as we consider absolute differences.

3. Root Mean Squared Error (RMSE)

The square root of the residuals' variance is the **Root Mean Squared Error**. It describes how well the observed data points match the expected values or the model's absolute fit to the data. In mathematical notation, it can be expressed as:

$$\mathrm{RMSE} = \sqrt{\frac{\mathrm{RSS}}{n}} = \sqrt{\frac{\sum_{i=2}^{n}\left(y_i^{\text{actual}} - y_i^{\text{predicted}}\right)^2}{n}}$$

Where:

- n : Number of observations
- y_i : Actual value
- $\hat{y}_i$: Predicted value

RMSE is in the same unit as the target variable and highlights larger errors more clearly.

4. Coefficient of Determination (R-squared)

R-Squared is a statistic that indicates how much variation the developed model can explain or capture. It is always in the range of 0 to 1. In general, the better the model matches the data, the greater the R-squared number.
In mathematical notation, it can be expressed as:

$$R^2 = 1 - \left(\frac{\mathrm{RSS}}{\mathrm{ISS}}\right)$$

- **Residual sum of Squares (RSS):** The sum of squares of the residual for each data point in the plot or data is known as the residual sum of squares or RSS. It is a measurement of the difference between the output that was observed and what was anticipated.

$$\mathrm{RSS} = \sum_{i=1}^{n} (y_i - b_0 - b_1 x_i)^2$$

- **Total Sum of Squares (TSS):** The sum of the data points' errors from the answer variable's mean is known as the total sum of squares or TSS.

$$\mathrm{TSS} = \sum_{i=1}^{n} (y - \overline{y_i})^2.$$

R squared metric is a measure of the proportion of variance in the dependent variable that is explained the independent variables in the model.

5. Adjusted R-Squared Error

Adjusted R^2 measures the proportion of variance in the dependent variable that is explained by independent variables in a regression model. **Adjusted R-square** accounts the number of predictors in the model and penalizes the model for including irrelevant predictors that don't contribute significantly to explain the variance in the dependent variables.
Mathematically, adjusted R^2 is expressed as:

$$\text{Adjusted } R^2 = 1 - \left(\frac{(1 - R^2) \cdot (n - 1)}{n - k - 1}\right)$$

Here,

- n is the number of observations
- k is the number of predictors in the model
- R^2 is coeeficient of determination

It penalizes the inclusion of unnecessary predictors, helping to prevent overfitting.

Keep Learning & Keep Exploring!