Data-Driven Hierarchical Runge-Kutta and Adams Methods for Nonlinear Dynamical Systems

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Abstract

This paper presents a comprehensive implementation of the Runge-Kutta 3rd order method and Adams methods for solving nonlinear differential equations. We introduce a novel data-driven hierarchical architecture inspired by transformer networks that enhances traditional numerical integration methods. The framework is implemented in C/C++ with Objective-C visualization capabilities, making it suitable for macOS and VisionOS platforms.

1 Introduction

Numerical methods for solving ordinary differential equations (ODEs) are fundamental tools in scientific computing. The Runge-Kutta family of methods, particularly the 3rd order variant, provides a good balance between accuracy and computational efficiency.

2 Runge-Kutta 3rd Order Method

The Runge-Kutta 3rd order method (RK3) is defined by the following stages:

$$k_1 = f(t_n, y_n) \tag{1}$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \tag{2}$$

$$k_3 = f(t_n + h, y_n - hk_1 + 2hk_2) \tag{3}$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 4k_2 + k_3) \tag{4}$$

where h is the step size, f is the ODE function, and y_n is the state at time t_n.

3 Adams Methods

Adams-Bashforth and Adams-Moulton methods are multi-step methods that use information from previous steps.

3.1 Adams-Bashforth 3rd Order

The predictor step:

$$y_{n+1} = y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2}) \tag{5}$$

3.2 Adams-Moulton 3rd Order

The corrector step:

$$y_{n+1} = y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1}) \tag{6}$$

4 Hierarchical Data-Driven Architecture

We propose a hierarchical architecture inspired by transformer networks that processes ODE solutions through multiple layers with attention mechanisms. Each layer applies transformations to the state space, enabling adaptive refinement of the numerical solution.

The hierarchical solver consists of:

- Multiple processing layers with learnable weights
- Attention mechanisms for state-space transformations
- Adaptive step size control based on hierarchical features

5 Implementation

The framework is implemented in C/C++ for core numerical methods, with Objective-C wrappers for visualization and integration with Apple platforms.

6 Results

[Results section would contain experimental validation]

7 Conclusion

We have presented a comprehensive framework for solving nonlinear ODEs using traditional and data-driven hierarchical methods, suitable for deployment on Apple platforms.

References

[1] Butcher, J. C. (2008). *Numerical Methods for Ordinary Differential Equations*. Wiley.

[2] Gear, C. W. (1971). *Numerical Initial Value Problems in Ordinary Differential Equations*. Prentice-Hall.