B-Tree Insert Complexity Proof

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1 Theorem: B-Tree Insert Complexity

Statement: Inserting into a B-Tree with n keys takes $O(\log n)$ time.

2 Proof

1. Find insertion point: $O(\log n)$ (same as search)
2. Insert into leaf: $O(1)$
3. If node splits:
   - Split operation: $O(t) = O(1)$ if t is constant
   - Propagate split upward: at most $O(\log n)$ levels
4. Total time: $O(\log n) + O(\log n) \times O(1) = O(\log n)$

Conclusion: B-Tree insert has $O(\log n)$ time complexity.