

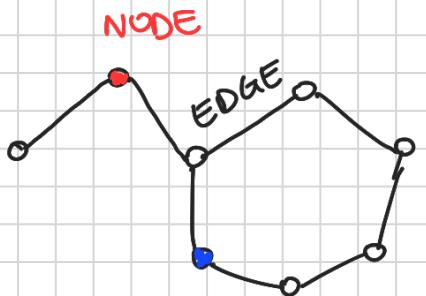
NOTE BOOK

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Algorithms

DATE : 22/2/22

LOL



ALGORITHMS

mathematical procedure
that allows us to tell
what's the shortest path
from the red point to
the blue point.

• OPTIMISATION PROBLEMS

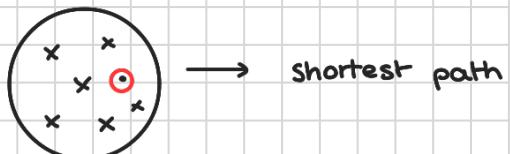
- we will use optimized algorithms.
- we will be looking for the most optimal solutions.

DETERMINISTIC
ALGORITHM

→ doesn't make random decisions.

• OPTIM. PROBLEM (shortest path, minimum spanning tree)

SPANNING TREE → graph with no cycles.

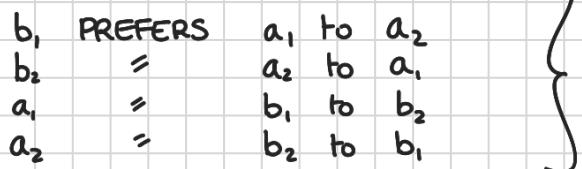
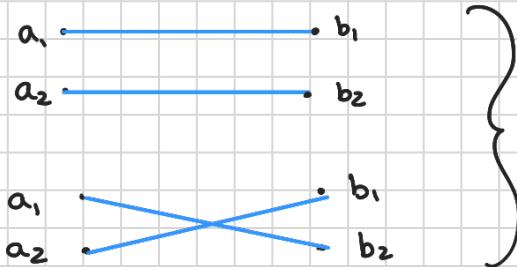


• ALGORITHM TECHNIQUES

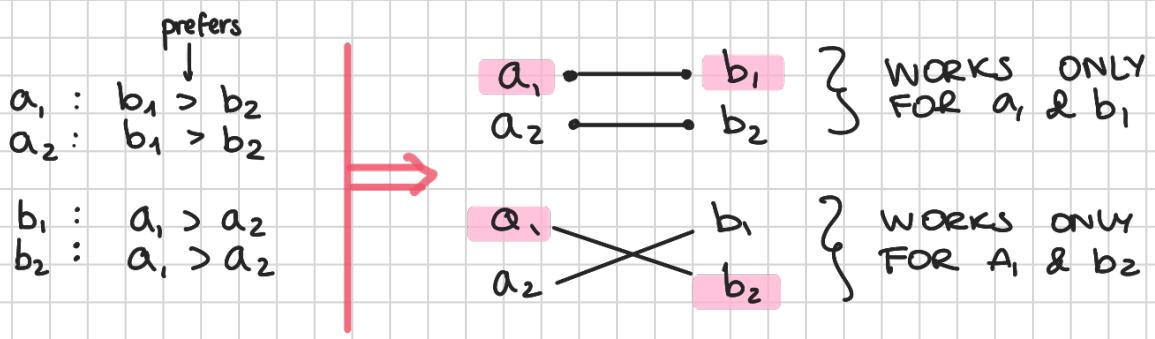
- greedy algorithms
- dynamic programming
- divide - et - impera (divide - and - conquer)

"STABLE MATCHING" PROBLEM

COMPANIES (a_n) APPlicants (b_n)



IF WE CONSIDER THESE CASES,
EVERYONE WILL BE SATISFIED



UNSTABLE MATCHING

Def: Let A and B be two sets of cardinality $|A| = |B| = n$, with $A \cap B = \emptyset$:

A perfect matching between A and B is a pairing of the elements of A with the elements of B . That is, $M \subseteq \{ \{a,b\} \mid a \in A \wedge b \in B \}$

M is a perfect matching if and only if (IFF)

$\forall a \in A$ there exists exactly one $b \in B$ such that $\{a,b\} \in M$ (AND viceversa)

↓ example

One company will have only one applicant and viceversa.

$$A = \{a_1, a_2, a_3\}$$

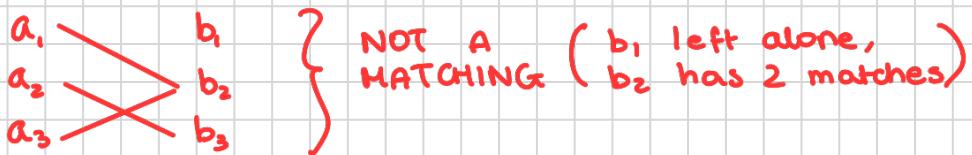
$$B = \{b_1, b_2, b_3\}$$



$$M = \{\{a_1, b_1\}, \{a_2, b_2\}, \{a_3, b_3\}\}$$

M is a matching IFF \exists bijective function

$$f: A \rightarrow B \quad \{\{a, f(a)\} \mid a \in A\} = M$$

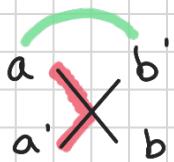


STABLE MATCHING

Let A, B be two sets, $|A| = |B| = n$, $A \cap B = \emptyset$ suppose that each $a \in A$ has a preference order on B and, likewise, each $b \in B$ has a preference order on A .

Given a perfect matching M of A and B , we say that M is unstable if it contains two pairs $\{a, b\}, \{a', b'\} \in M$, such that:

- a PREFERENCES b' TO b AND
- b' PREFERENCES a TO a'



A MATCHING M IS STABLE, IF ITS NOT UNSTABLE (bruh...)

Questions :

- ① Does a stable matching always exist?
- ② How can we find a stable matching (if it exists)?

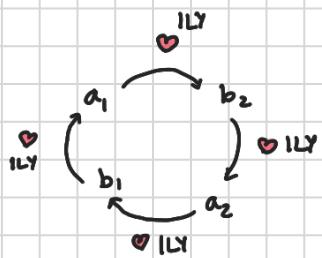
ANOTHER CASE :

$$a_1 : b_1 > b_2$$

$$a_2 : b_2 > b_1$$

$$b_1 : a_2 > a_1$$

$$b_2 : a_1 > a_2$$



INTUITIVE ALGORITHM

- Suppose \underline{a} is unmatched, let \underline{a} propose an engagement to his most preferred $\underline{b} \in B$
 - says "YES" if b is unmatched,
 - says "NO" if b is matched with a' that she likes better than " \underline{a} "
 - says "YES" if b is matched with " \underline{a}' " and she likes " \underline{a}' " more than " \underline{a} "
- \underline{a} NEVER ASKS the same b more than once.

(slowly dying)