NOTE BOOK

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CALCULUS 2

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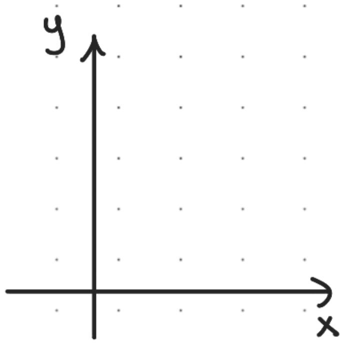
REVISION

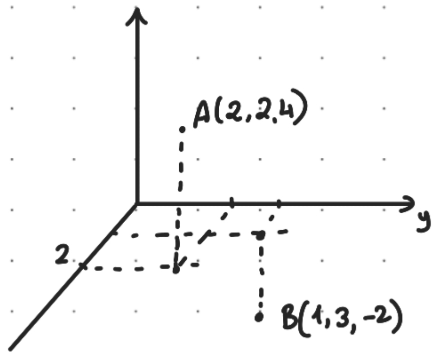
① A point A in space is represented as :

$$A(x,y,z)\begin{cases} x = \text{abscissa of } A \\ y = \text{ordinate of } A \\ z = \text{elevation} \end{cases}$$



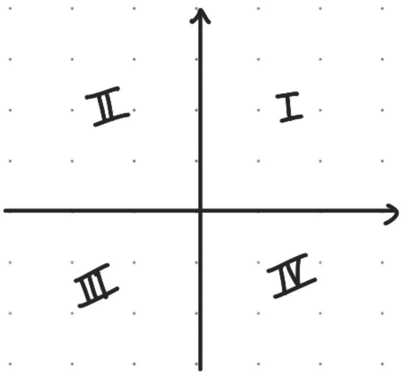
EXAMPLE :

Plot $A(2,2,4)$ $B(1,3,-2)$



1. REGIONS ON SPACE

In space $\begin{cases} z=0 \text{ is the } (x,y) \text{ plane} \\ y=0 \text{ is the } (x,z) \text{ plane} \\ x=0 \text{ is the } (y,z) \text{ plane} \end{cases}$ Coordinate planes



1st octant

xyz	$x'yz$
xyz'	$x'yz'$
$xy'z$	$x'y'z$
$xy'z'$	$x'y'z$

OCTANTS (8 regions)

EX: Specify the regions determined by the following equations and inequalities:

1. $z \geq 0$ → upper half-space including (x,y) plane

2. $z = 1$ → plane parallel to xy plane passing $(0,0,1)$

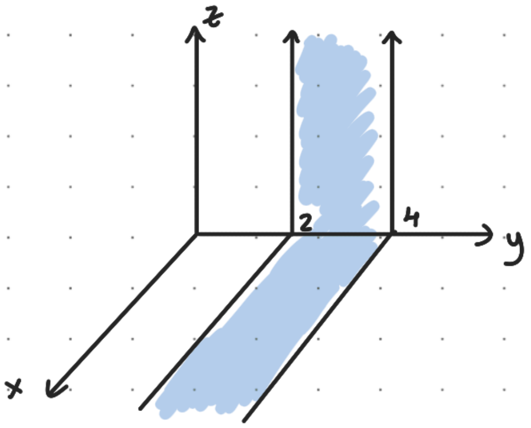
3. $z=0,\ x \geq 0,\ y \geq 0$

   ↳ 1^{st} quadrant of the xy plane

4. $2 \leq y \leq 4$

   $y=2$ → plane // to (x,z) plane thru $(0,2,0)$

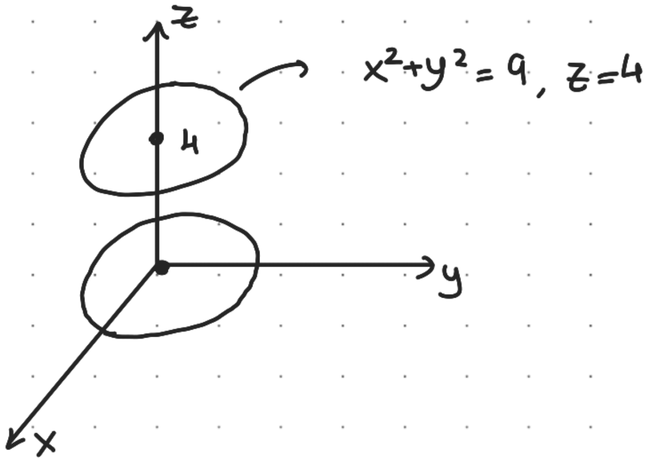
   $y=4$ → plane // to (x,z) plane thru $(0,4,0)$



5. $x^2+y^2=9$ $\quad yz=0$

   ↓

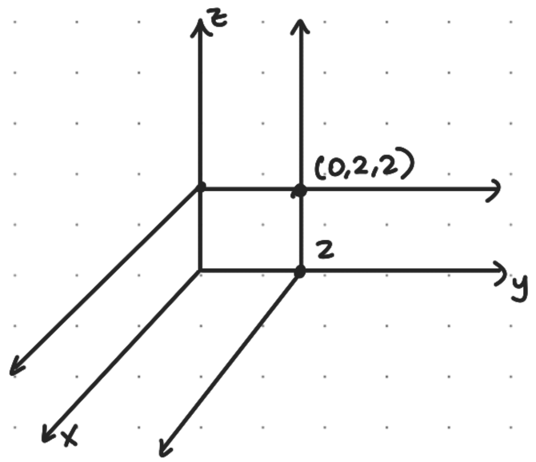
   circle centered at $(0,0,0)$ in the (xy) plane of radius 3



6 $y=2$ and $z=2$

$y=2 \rightarrow$ plane // to (x,z) plane thru $(0,2,0)$

$z=2 \rightarrow$ plane // to (x,y) plane thru $(0,0,2)$



2. DATES AND SPHERE IN SPACE

The distance between $A(x,y,z)$ and $B(x_2,y_2,z_2)$ then $|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)}$

distance between A and B

ex : $A(2,1,5)$ and $B(-2,3,0)$

$|AB| = \sqrt{4+(-2)^2+5^2} = \sqrt{45} \simeq$

2.1 EQUATION OF A SPHERE

A point $A(x,y,z)$ lies on a sphere of radius $\underline{a}$ and center $C(x_0,y_0,z_0)$ $\Longrightarrow$ $|AC| = a$

$$\boxed{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = a^2}$$

ex: $(x+1)^2 + y^2 + (z-2)^2 = 9$

$\downarrow$

sphere centered at $(-1,0,2)$ and radius $r=3$)

Sketch

$x^2+y^2+z^2=1$

$x=0$ $\quad y^2+z^2=1 \longrightarrow$ circle center $(0,0,0)$ $r=1$ in the (yz) plane

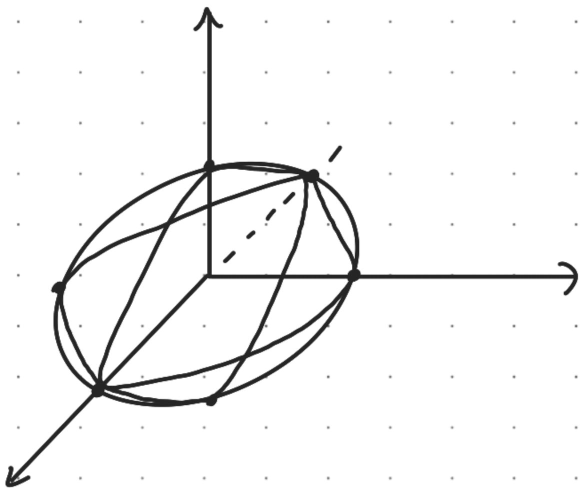
$y=0 \Rightarrow z=\pm 1 \quad (0,0,1) \quad (0,0,-1)$

$z=0 \Rightarrow y^2=1 \Rightarrow y=\pm 1 \quad (0,1,0) \quad (0,-1,0)$

$y=0$ $\quad x^2+z^2=1$

$x=0 \Rightarrow z=\pm 1$

$z=0 \Rightarrow x=\pm 1$



$$x^2 + (y-2)^2 + z^2 = 1$$

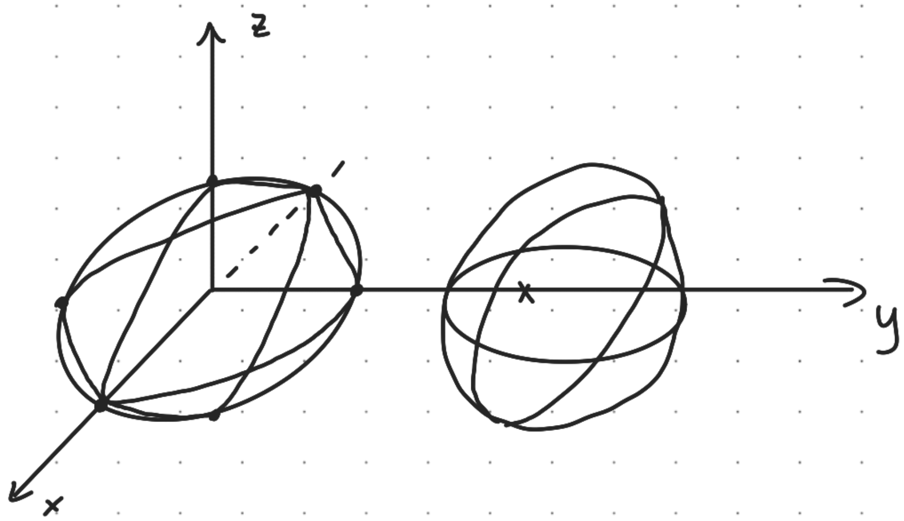
$(0,2,0)$ $\quad + \; r = 1$

$x = 0 \quad (y-2)^2 + z^2 = 1$

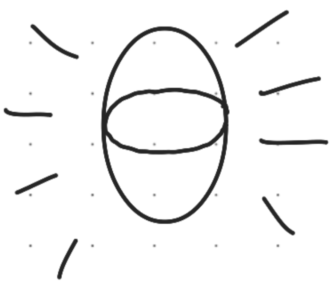
$(y-2)^2 = 1$

$y - 2 = 1$ or $y - 2 = -1$

$y = 3$ or $y = 1$



1. $x^2 + y^2 + z^2 > 1 \longrightarrow$ exterior of the unit sphere



$< 1 \rightarrow$ interior

$\leq 1 \rightarrow$ sphere together with interior

Wtf are you kidding me?

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3. VECTORS IN R^3

$\vec{v} = (v_1, v_2, v_3)$ $\quad$ $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$

- if $\|\vec{u}\| = 1 \Rightarrow \vec{u}$ is a unit vector

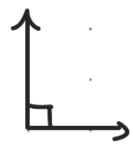
  NORM

standard unit : $\vec{i} = (1,0,0)$ $\vec{j} = (0,1,0)$ $\vec{k} = (0,0,1)$

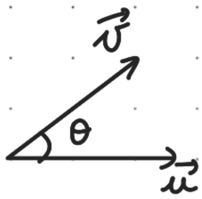
3.1 DOT PRODUCT

$\vec{u} = (u_1, u_2, u_3)$

$\vec{v} = (v_1, v_2, v_3)$



$\theta = 90°$



$u \cdot v = \|u\| \times \|v\| \cos\theta$

$\Rightarrow \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$ $\quad$ $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|u\| \cdot \|v\|}$

$\vec{u}$ and $\vec{v}$ are orthogonal if $\vec{u} \cdot \vec{v} = 0$

PROPERTIES

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2. $\vec{u} \cdot \vec{u} = \|u\|^2$
3. $0 \cdot \vec{u} = 0$
4. $\vec{u}(\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

3.2 CROSS PRODUCT

1. $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
2. $0 \times u = 0$
3. $u \times v = 0$ iff they are parallel

$\vec{u} = (2,1,1)$ $\vec{v}\ (-4,3,1)$

$$\det\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = i(1-3) - j(2+4) + k(6+4)$$

$$= -2i - 6j + 10k$$

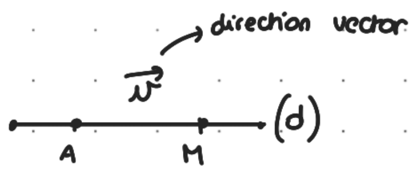
3.3 TRIPLE SCALAR PRODUCT

$$(\vec{u} \times \vec{v}) \cdot w$$

$$\det\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = u_1(v_2w_3 - v_3w_2) - u_2(v_1w_3 - v_3w_1) + u_3(v_1w_2 - v_2w_1)$$

3.4 LINES IN SPACE

$A = (x_A : y_A, z_A) \in (d)$



$\vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ direction vector

Let $M(x, y, z) \in d$

$$\underbrace{\overrightarrow{AM}} = k\vec{v}$$

$$\begin{pmatrix} x - x_A \\ y - y_A \\ z - z_A \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \longrightarrow \begin{cases} x - x_A = k \cdot a \\ y - y_A = kb \\ z - z_a = kc \end{cases}, \quad k \in \mathbb{R}$$

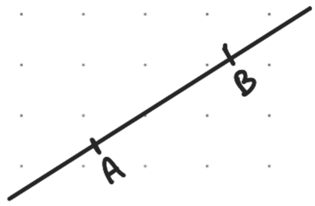
parametric

$$\frac{x-x_a}{a} = \frac{y-y_A}{b} = \frac{z-z_A}{c}$$ } cartesian equation

EX: $A(-3,2,-3)$

$B(1,-1,4)$

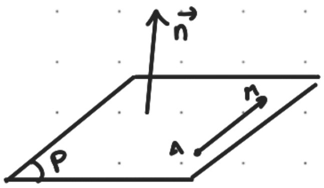
$$\vec{AB} = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix} \begin{array}{l} \to a \\ \to b \\ \to c \end{array}$$



$$\begin{cases} x = -4t+3 \\ y = 3t+2 \\ z = -7t-3 \end{cases} \quad t \in \mathbb{R}$$ equation of a line

3.5 PLANE IN SPACE

To find an equation of a plane (P), we need $A \in (P)$ and a normal vector $\vec{n}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$



EX:

$$\vec{AM} \cdot \vec{n} = 0$$

$$\begin{pmatrix} x-x_A \\ y-y_A \\ z-z_A \end{pmatrix} \quad \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$a(x-x_a) + b(y-y_A) + c(z-z_A) = 0$$
$$Ax + By + Cz + ax + by + cz - ax_A - by_A - cz_A = 0$$

(P) $A(0,0,1)$ $B(2,0,0)$ $C(0,3,0)$

$\vec{n} = \vec{AB} \times \vec{AC}$

$M(x, y, z) \in (P)$

$\vec{AM} \cdot \vec{n} = 0$

$AM \cdot (\vec{AB} \times \vec{AC}) = 0$

$$\vec{AM} = \begin{pmatrix} x \\ y \\ z-1 \end{pmatrix} \qquad \vec{AB} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \qquad \vec{AC} = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$$

note: $AM = \begin{pmatrix} x_M - x_A \\ y_M - y_A \\ z_M - z_A \end{pmatrix} = \begin{pmatrix} x \\ y \\ z-1 \end{pmatrix}$

$$\begin{vmatrix} x & y & z-1 \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = 0$$

$$x(3) - y(-2) + (z-1)(6) = 0$$
$$3x + 2y + 6z - 6 = 0 \quad (P)$$

SHAPES

I. Cylinders
- circular
- eliptical
- parabolic

① Circular cylinder

$$1(\text{variable}①)^2 + 1(\text{variable}②)^2 = a^2$$

EX:

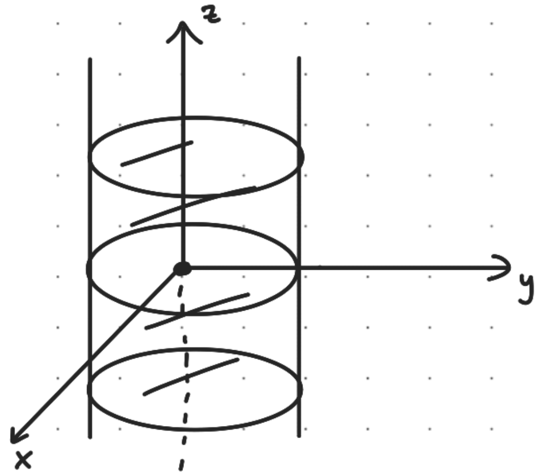
1. $x^2+y^2=4 \longrightarrow$ is a circular cylinder along z-axis

$\underline{z=0}$

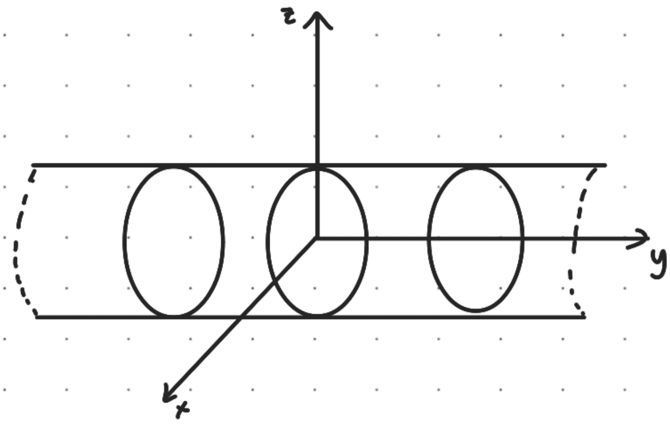
circle $(0,0,0)$
$r=2$

$\underline{z=3}$

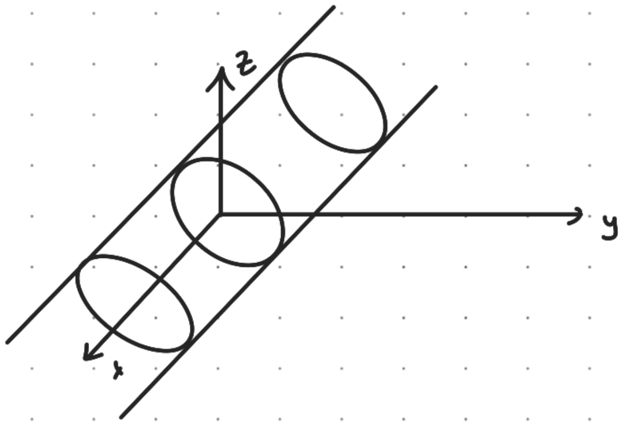
circle $(0,0,3)$
$r=3$



2. $x^2+z^2=4 \longrightarrow$ circular cylinder around y-axis



3. $y^2+z^2=1$



② Elliptical Cylinder

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = c$$

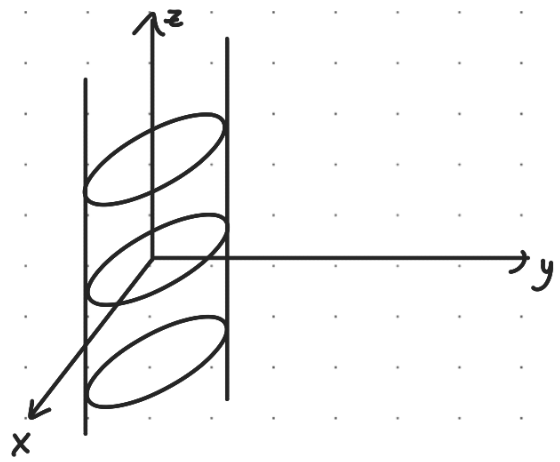
$a \neq b$, o.w it will be circular cylinder

Ex: $\frac{x^2}{4} + y^2 = 1 \rightarrow$ it is an elliptical cylinder along z-axis

$z=0$: $\frac{x^2}{4} + y^2 = 1$, ellipse of center $(0,0,0)$

$(\pm 2,0,0) \longrightarrow y=0 \rightarrow \frac{x^2}{4} = 1 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

$(0, \pm 1, 0) \longrightarrow x=0 \rightarrow y^2 = 1 \rightarrow y = \pm 1$



$$\frac{(x-2)^2}{4} - y^2 = 1$$

$\downarrow$

ellipse of center $(2,0,0)$ along z-axis