NOTE BOOK

Rokshana Ahmed

CALCULUS 2

Date: 10 / 10 / 2022

① **CYLINDER**:
- → circular
- → elliptical
- → parabolic

PARABOLIC CYLINDER

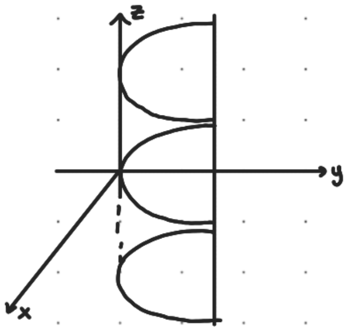
$\boxed{(\text{variable}) = (\text{variable})^2}$ $\xrightarrow{\text{it could be}}$ $\begin{cases} x = y^2 \\ y = x^2 \\ x = (y-1)^2 \\ z = x^2 \end{cases}$

REMINDER (parabola)
$y = a(x-h)^2 + k$
(h, k)

EX : $y = x^2$ ⟶ parabolik cylinder around Z-axis

$z = 0$ → z plane

$y = x^2$ → parabola in the (x,y) plane whose vertex is $(0,0)$ and opens in the y-axis



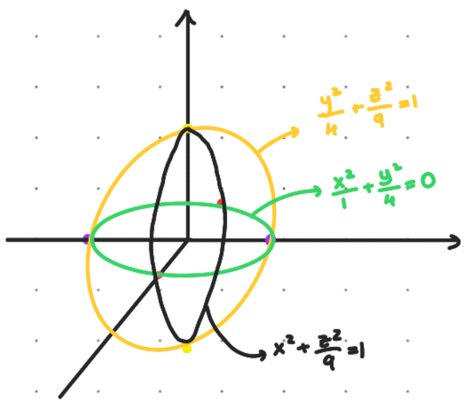
② **ELLIPSOID**

$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1}$ ⟶ ellipsoid of center $(0,0,0)$

→ The intercepts are $(\pm a, 0, 0)$, $(0, \pm b, 0)$, $(0, 0, \pm c)$

EX1: $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$

$(\pm 1, 0, 0)$
$(0, \pm 2, 0)$
$(0, 0, \pm 3)$



CALCULUS 2

Date: 10 / 10 / 2022

EX2: $\frac{(x-2)}{9} + \frac{(y+1)^2}{4} + z^2 = 1 \longrightarrow$ ellipsoid centered at $(2,-1,0)$

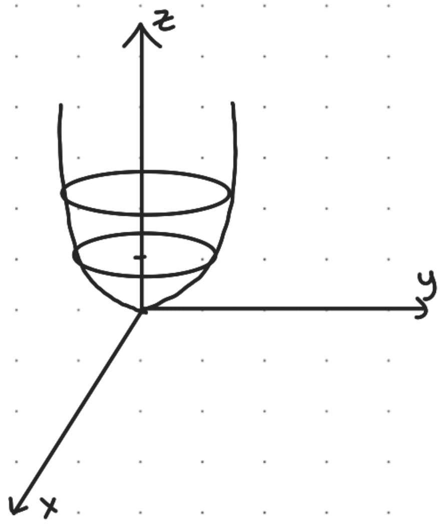
- $x=2$ and $y=-1 \Rightarrow z=\pm 1$ $(2,-1,1)$, $(2,-1,-1)$
- $x=2$ and $z=0 \Rightarrow (y+1)^2=4 \Rightarrow y+1=2$ or $y+1=-2$
  $\Rightarrow y=1$ or $y=-3$
  $(2,1,0)$ and $(2,-3,0)$
- $y=-1$ and $z=0 \Rightarrow (x-2)=\pm 3$ so $x=5$ or $x=-1$
  $(-1,-1,0)$, $(5,-1,0)$

3 PARABOLOID

$$\boxed{\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}}$$

ex. 1 $z = x^2+y^2$

- $z=0$
  $x^2+y^2=0$
  $(0,0,0)$
- $z=1$
  $x^2+y^2=1$
  $(0,0,1)$
  of radius 1
- $z=4$
  $x^2+y^2=4$
  $(0,0,4)$
  of radius 2



EX: 2 $x^2 + \frac{y^2}{9} = \frac{z}{2}$

$\downarrow$

$2x^2 + \frac{2}{9}y^2 = z$

$\downarrow$

$\frac{x^2}{1/2} + \frac{y^2}{9/2} = z$

so this is actually an elliptic parabula

STEP 1. $x=0 \Rightarrow z = \frac{2}{9}y^2$ parabula around z-axis with vertex $(0,0,0)$

STEP 2. $y=0 \Rightarrow z = 2x^2$ parabola around z-axis in the (xz) plane

CALCULUS 2

Date: 10 / 10 / 2022

4 CONE

$$\boxed{(\text{variable})^2 = a(\text{variable})^2 + b(\text{variable})^2}$$

GENERAL FORMULA

this determines our axis

ex1: $z^2 = x^2 + y^2$ → a circular cone around z-axis

For $x=0$:

$z^2 = y^2$

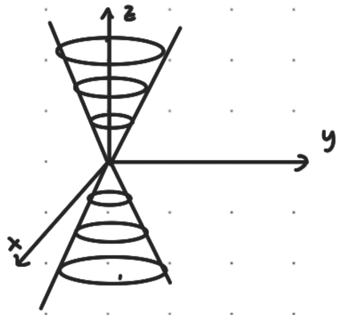
$z = \pm y$

(eq. straigth line)

For $y=0$:

$z^2 = x^2$

$z = \pm x$



ex2: $z = \sqrt{x^2 - y^2} \longrightarrow z > 0 \longrightarrow z^2 = x^2 + y^2$ circular cone with $z \geq 0$

$\Downarrow$

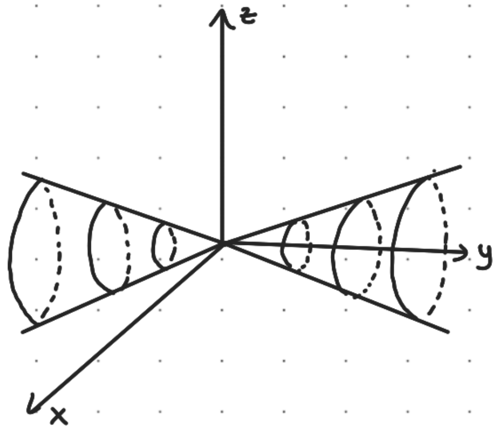
so circular cone upper plane

$z = -\sqrt{x^2 + y^2} \rightarrow z \leq 0$

EX3: $4x^2 + 9z^2 = 9y^2$

$y^2 = \frac{4}{9}x^2 + z^2$

$\Longrightarrow$ elliptic cone around y-axis



MULTIVARIABLE FUNCTION

D R

$(x, y) \longrightarrow z \quad f(x, y)$

$(x, y, z) \longrightarrow$

$\vdots$

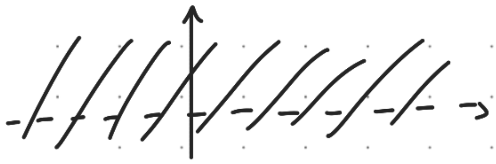
$\mathbb{R}^n$

DOMAIN AND RANGE

① $f(x,y) = x^2 + y^2$
Domain : (x,y) plane
Range: $[0, +\infty)$

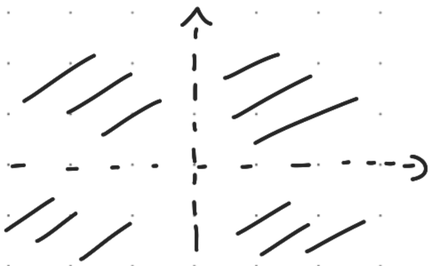
② $f(x,y) = \frac{x}{y}$
Domain: (x,y) plane except the x-axis

she could ask to draw the domain at the exam.



Range: $(-\infty; +\infty)$

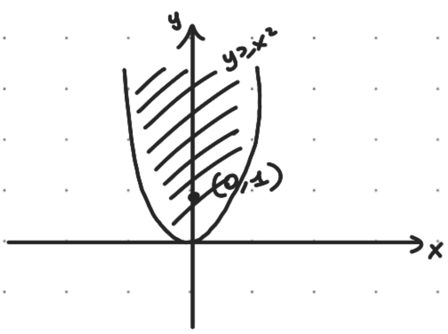
③ $f(x,y) = \frac{3}{xy}$
Domain: (xy)plane $- \{$x-axis and y-axis$\}$



Range: $\mathbb{R}^*$ $\{$z-axis without 0$\}$

④ $f(x,y) = \sin(xy)$
Domain: (xy)plane
Range : $[-1, 1]$ along z-axis

⑤ $f(x,y) = \sqrt{y - x^2}$
Domain: $y \geq x^2$



Range: $[0 ; +\infty]$