

Training Camp on “Knowledge Graph Completion”

— *Sapienza University, M.Sc. Degree in Data Science* —

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<https://www.kaggle.com/c/unicredittrainingcamp/overview/lectures>

June 30th – July 2nd, 2021

Day 2: Knowledge Graph Embedding (KGE)

- Graph embedding
 - General problem
 - Embedding of vanilla graphs (hints)
- KGE
 - Problem definition
 - Overview of some of the most popular state-of-the-art KGE methods (TransE, TransH, TransR, DistMult)
- KGE-based KGC
 - Prediction by looking solely at the KGE score of a triple (threshold-based, ranking-based)
 - Prediction by training a downstream classifier on KGEs
- Lab
 - Introduction to [PyKEEN](#) (Python KnowlEdge EmbeddiNgs) framework
 - Overview of resources available in PyKEEN (KGE models, loss functions, regularizers, training approaches, negative samplers)
 - Examples of training KGE models in PyKEEN

Capability of:

- Devising KGC methods that consist of training a downstream classifier on vectorial representations of triples that are automatically computed via KGE
- Combining such “KGE + downstream classifier”-like methods with methods arising from Day 1

- [*book*] [Graph Representation Learning](#)
- [*survey paper*] [Knowledge graphs](#)
- [*survey paper*] [A Survey on Knowledge Graphs: Representation, Acquisition and Applications](#)
- [*survey paper*] [Knowledge Graph Embedding: A Survey of Approaches and Applications](#)
- [*survey paper*] [Knowledge Graph Embedding for Link Prediction: A Comparative Analysis](#)
- [*tutorial*] [Knowledge Graph Embeddings: From Theory to Practice](#)
- [*course*] [Machine Learning with Graphs – Lecture 10 \(Knowledge Graph Embeddings\)](#)
- [*framework*] [PyKEEN](#) [[system overview \(paper\)](#)] [[evaluation \(paper\)](#)] [[github](#)] [[docs](#)]

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Graph (or Network) Embedding

a.k.a. Graph (or Network) Representation Learning

High-level formulation

Represent (“embed”) **elements** of a **graph** in a **geometric space**, such that, for any pair of elements of the graph, the similarity of the resulting representations in that geometric space (according to some **similarity measure** $\text{sim}_S(\cdot, \cdot)$) closely reflects the **similarity** $\text{sim}_G(\cdot, \cdot)$ between those elements in the graph.

Specific instantiations of the problem are obtained by defining the **type of graph**, the **elements** to be embedded, the target **geometric space**, and the $\text{sim}_S(\cdot, \cdot)$ and $\text{sim}_G(\cdot, \cdot)$ **similarities** (and some other “hidden” aspects, e.g., the number of output embeddings per element).

Graph Embedding: Our focus

- **Type of graph**

- **simple unweighted undirected graphs**, *directed graphs, bipartite graphs, signed graphs, attributed graphs, knowledge graphs, temporal graphs, hypergraphs, heterogeneous graphs, uncertain graphs, multilayer graphs, hyper-heterogeneous graphs, signed heterogeneous graphs, attributed multilayer graphs, attributed multilayer heterogeneous graphs, ...*

- **Elements to be embedded**

- **nodes**, *edges, knowledge-graph triples, subgraphs, communities, entire graphs, ...*

- **Target geometric space and corresponding $\text{sim}_S(\cdot, \cdot)$ function**

- **dot-product in Euclidean space**, *hyperbolic dot-product in hyperbolic space, Hamming distance in Hamming space (i.e., binary embeddings), ...*

- **Similarity $\text{sim}_G(\cdot, \cdot)$ in the graph, to be preserved in the resulting embedding**

- **proximity** (e.g., k-order proximity, probability of co-appearance in a random walk, ...), **structural equivalence**, **tradeoff between proximity and structural equivalence**, **proximity and/or structural equivalent + other info** (e.g., community memberships, core indices), ...

- **Number of output embedding vectors for every element**

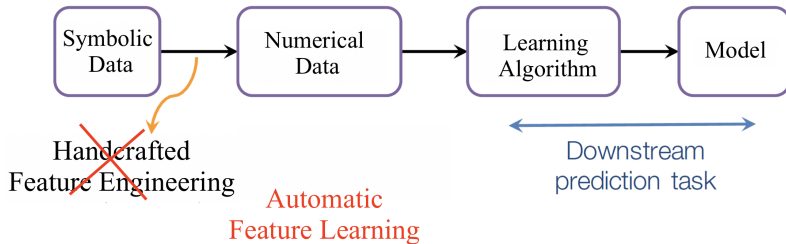
- **single**, *multiple (for capturing, e.g., polysemies)*

- **Methodology**

- **shallow** *vs. deep methods*

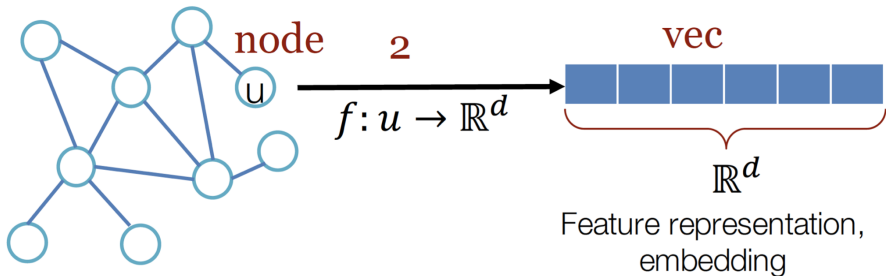
Graph Embedding: Motivations

- Going beyond handcrafted *feature engineering* in machine learning



- Dimensionality reduction
- Data visualization
- Similarity search/detection

Graph Embedding: Motivations



Embedding of vanilla graphs: Random-walk-based methods

- DeepWalk ([Perozzi et al., KDD 2014](#))
 - $sim_G(\cdot, \cdot)$ somehow expresses the probability of a node to be encountered during a (truncated) random walk initiated in the other node
- Node2Vec ([Grover and Leskovec, KDD 2016](#))
 - same as DeepWalk, but with a more general notion of random walk (that may capture structural-equivalence properties too)
- Several more exist
 - e.g., Constrained DeepWalk ([Jin et al., IJCNN 2016](#)), TriDNR ([Pan et al., IJCAI 2016](#)), GenVector ([Yang et al., IJCAI 2016](#)), APP ([Zhou et al., AAAI 2017](#)), Walklets ([Perozzi et al., ASONAM 2017](#)), VERSE ([Tsitsulin et al., WWW 2018](#)), ...
- No rigorous definition of $sim_G(\cdot, \cdot)$

Embedding of vanilla graphs: DeepWalk

[Perozzi *et al.*, KDD 2014]

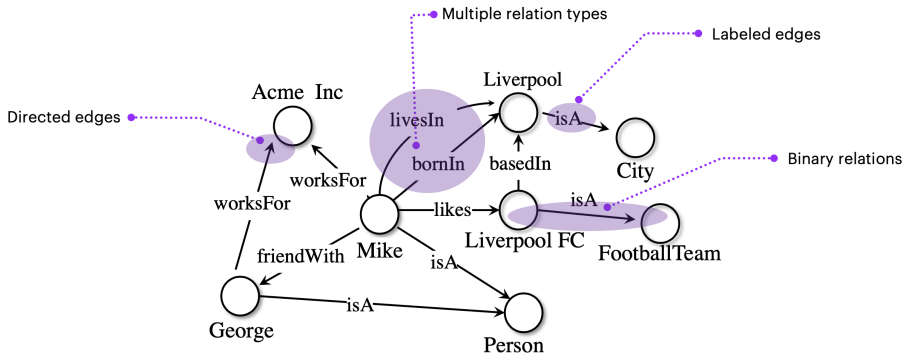
- For every vertex x , generate a set $\mathcal{R}(x) = \{R_1(x), \dots, R_k(x)\}$ of k random walks of length h that start in x
- Interpret $\mathcal{R}(x)$ as a set of **context** vertex sets for x
 - Intuitively, the more two vertices x and y appear in similar contexts, the more similar the embeddings of x and y should be, and vice versa
- Learn for every vertex x an “input” \mathbf{v}_x representation vector and an “output” \mathbf{v}'_x representation vector so as to
 - maximize the average log-probability (defined based on the softmax function) of predicting a vertex in a context $R_i(x)$ of the given x vertex
 - i.e., maximize $\frac{1}{|V|} \frac{1}{k} \frac{1}{h} \sum_{x \in V} \sum_{i=1}^k \sum_{y \in R_i(x)} \log \frac{\exp(\mathbf{v}'_y \cdot \mathbf{v}_x)}{\sum_{z \in V} \exp(\mathbf{v}'_z \cdot \mathbf{v}_x)}$ (V is the input vertex set)
 - such an average log-probability is maximized via **gradient descent**
 - **Hierarchical softmax** is used to approximate the (otherwise computationally very expensive) computation of the posterior distribution
 - $\{\mathbf{v}_x\}_{x \in V}$ are the ultimate **embeddings**

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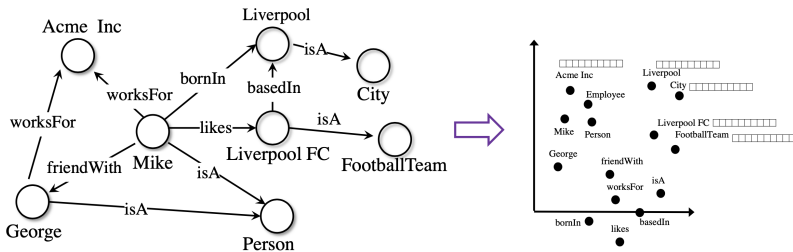
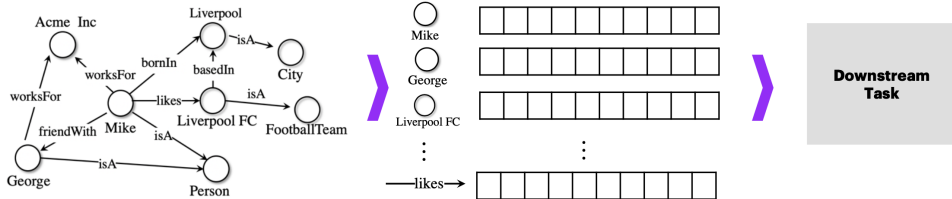
Knowledge Graph Embedding (KGE)

Knowledge Graph: $\mathcal{G} = \{(s, p, o)\} \subseteq \mathcal{E} \times \mathcal{R} \times \mathcal{E}$ (it can be a **multiset**)

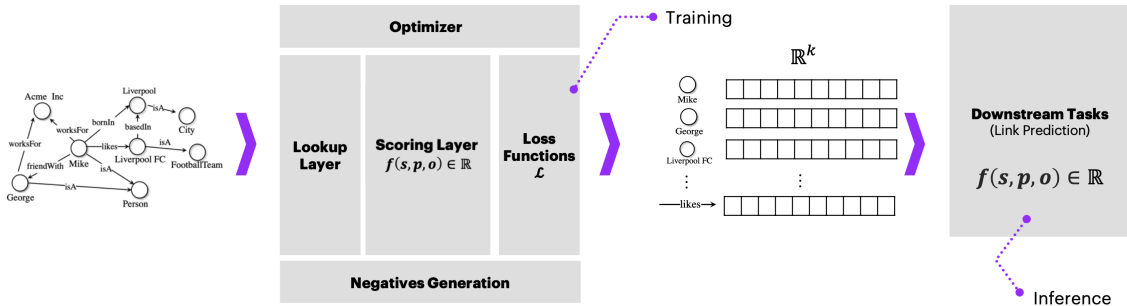
- \mathcal{E} : set of entities of \mathcal{G}
- \mathcal{R} : set of relations of \mathcal{G}



Knowledge Graph Embedding (KGE)



KGE: General approach



- **Scoring function** $f: \mathcal{E} \times \mathcal{R} \times \mathcal{E} \rightarrow \mathbb{R}$
 - It takes the embeddings of the subject, predicate and object of a KG triple
 - It returns a score expressing the **plausibility** of those embeddings to represent a **true fact** (the higher the score, the higher the plausibility)
- **Loss function**
 - It measures the overall error between the (scoring functions of the) embeddings of a triple and the actual existence of that triple in the KG

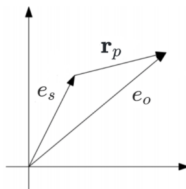
TransE: Translation in the embedding space

[Bordes *et al.*, NIPS 2013]

Idea: sum of subject's embedding and predicate's embedding should be as close as possible to object's embedding

Scoring function:

- $f_{\text{TransE}}(s, p, o) = -\|(\mathbf{e}_s + \mathbf{r}_p) - \mathbf{e}_o\|$



Loss function:

- Generate a set $\bar{\mathcal{G}}$ of negative (i.e., non-existing) triples
- Minimize the **pairwise margin-based Hinge loss**
 - $\mathcal{L}_{\text{TransE}} = \sum_{(s,p,o) \in \mathcal{G}} \sum_{(s',p',o') \in \bar{\mathcal{G}}} \max\{0, (\gamma + f_{\text{TransE}}(s', p', o') - f_{\text{TransE}}(s, p, o))\}$
 - A penalty is paid if the score of a positive triple is no more than the score of a negative triple by a margin $\geq \gamma$ (where γ is a hyperparameter)

TransH: Translation on hyperplanes

[Wang et al., AAAI 2014]

Issue: TransE does not work well with 1-N, N-1, or N-N relations

- If $(s_i, p, o) \in \mathcal{G}$, $\forall i = 1, \dots, k$, then TransE requires $\mathbf{e}_{s_1} = \dots = \mathbf{e}_{s_k}$

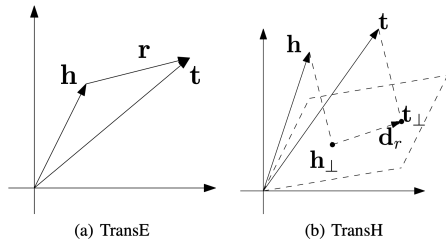
Solution:

- Allow an entity to have different embeddings when involved in different relations
- TransH models a relation p as a vector \mathbf{r}_p on a hyperplane having \mathbf{w}_p as a normal vector

Scoring function:

- For $(s, p, o) \in \mathcal{G}$: $\mathbf{e}_s^\perp = \mathbf{e}_s - \mathbf{w}_p^\top \mathbf{e}_s \mathbf{w}_p$,
 $\mathbf{e}_o^\perp = \mathbf{e}_o - \mathbf{w}_p^\top \mathbf{e}_o \mathbf{w}_p$ (projections of \mathbf{e}_s and \mathbf{e}_o onto a hyperplane with \mathbf{w}_p normal vector)
- $f_{\text{TransH}}(s, p, o) = -\|(\mathbf{e}_s^\perp + \mathbf{r}_p) - \mathbf{e}_o^\perp\|^2$

Loss function: same as TransE



TransR: Translation in relation-specific spaces

[Lin et al., AAAI 2015]

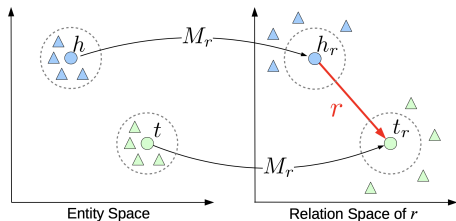
To better capture the **multifaceted** nature of entity representations:

- Relations are assigned to **matrices** that aim at projecting embedding vectors of the entities onto a relation-specific space
- Translation is enforced in the relation-specific space

Scoring function:

$$\bullet f_{\text{TransR}}(s, p, o) = -\|(\mathbf{e}_s \mathbf{M}_p + \mathbf{r}_p) - \mathbf{e}_o \mathbf{M}_p\|^2$$

Loss function: same as TransE and TransH



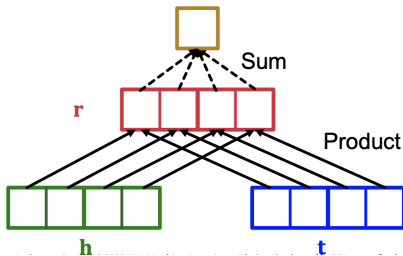
DistMult: A (simplified) bilinear model

[Yang *et al.*, ICLR 2015]

Scoring function:

- $f_{\text{DistMult}}(s, p, o) = \mathbf{e}_s^\top \mathbf{M}_p \mathbf{e}_o$, where \mathbf{M}_p is the matrix of the bilinear form
- In DistMult \mathbf{M}_p is constrained to be **diagonal**

Loss function: same as TransE, TransH, and TransR



- **Negative Log-Likelihood / Cross Entropy**

- $y_t = 1$, if $t \in \mathcal{G}$; $y_t = -1$, if $t \in \overline{\mathcal{G}}$
- $\mathcal{L} = \sum_{t \in \mathcal{G} \cup \overline{\mathcal{G}}} \log(1 + e^{-y_t f(t)})$

- **Binary Cross Entropy**

- $y_t = 1$, if $t \in \mathcal{G}$; $y_t = 0$, if $t \in \overline{\mathcal{G}}$
- $\sigma(z) = \frac{1}{1+e^{-z}}$
- $\mathcal{L} = -\frac{1}{|\mathcal{G} \cup \overline{\mathcal{G}}|} \sum_{t \in \mathcal{G} \cup \overline{\mathcal{G}}} [y_t \log(\sigma(f(t))) + (1 - y_t) \log(1 - \sigma(f(t)))]$

- **Self-adversarial Negative-sampling Loss (NSSA)**

- γ : hyperparameter (fixed margin)
- $p(t') = \frac{e^{\alpha f(t')}}{\sum_{t'' \in \overline{\mathcal{G}}} e^{\alpha f(t'')}} \quad (\alpha: \text{hyperparameter})$
- $\mathcal{L} = \sum_{t \in \mathcal{G}} \left[-\log(\sigma(\gamma - f(t))) - \frac{1}{|\overline{\mathcal{G}}|} \sum_{t' \in \overline{\mathcal{G}}} p(t') \log(\sigma(f(t')) - \gamma) \right]$

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KGE-based Knowledge Graph Completion

- Given a triple (s, p, o) , mark it as a true fact if and only if the $f(s, p, o)$ scoring-function value is more than a certain threshold
- Given a triple (s, p, o)
 - Rank triples (s, p', o) based on non-increasing scoring-function value, for all relations p' in the knowledge graph
 - Mark (s, p, o) as a true fact if and only if it appears in the top- k ranked triples (for a certain k)
- Train a classifier by using, for any triple (s, p, o) , a vectorial representation consisting in the concatenation (or any other proper combination) of \mathbf{e}_s , \mathbf{r}_p , and \mathbf{e}_o

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PyKEEN

A Python Library for Training and Evaluating Knowledge Graph Embeddings

[\[system overview \(paper\)\]](#) [\[evaluation \(paper\)\]](#) [\[github\]](#) [\[docs\]](#)

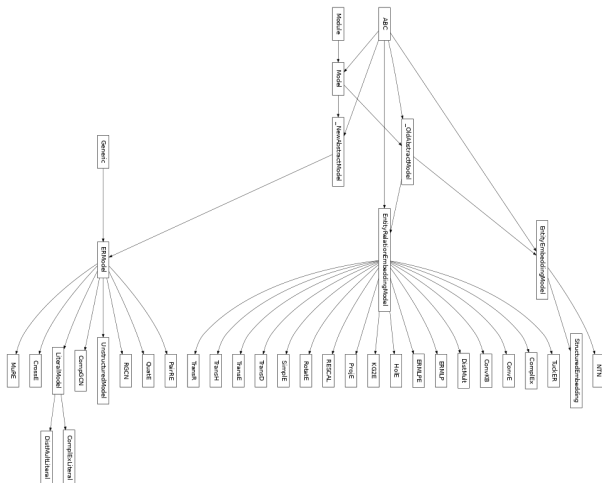


- Support for **composable** KGE (i.e., support for **any** combination of KGE models, loss functions, regularizers, training approaches (optimizers, training loops), and negative samplers)
- Extensive set of resources (28 KGE models, 7 loss functions, 5 regularizers, 6 optimizers, 2 training loops, 3 negative samplers)
- Extensibility
- Support for hyperparameter optimization
- Support for systematic evaluation (16 assessment metrics and 26 built-in benchmarking datasets)
- Community standards (accessible, reusable, reproducible, and maintainable)

PyKEEN: KGE models

Support for 28 KGE models, including TransE, TransH, TransR, and DistMult

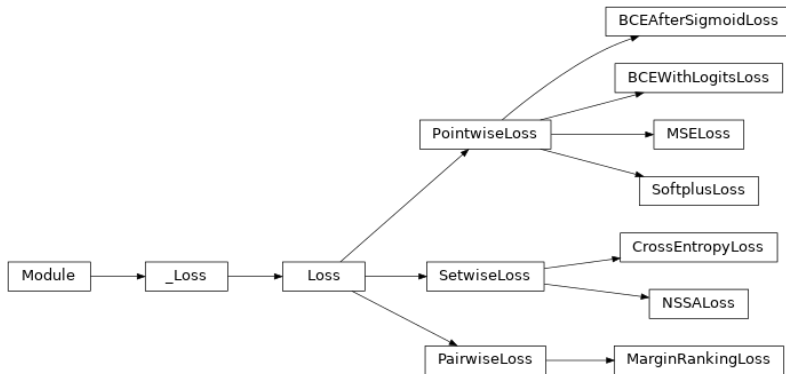
- <https://pykeen.readthedocs.io/en/latest/reference/models.html>



PyKEEN: Loss functions

Support for 7 loss functions, including pairwise Hinge (margin ranking), cross entropy, and NSSA

- <https://pykeen.readthedocs.io/en/latest/reference/losses.html>



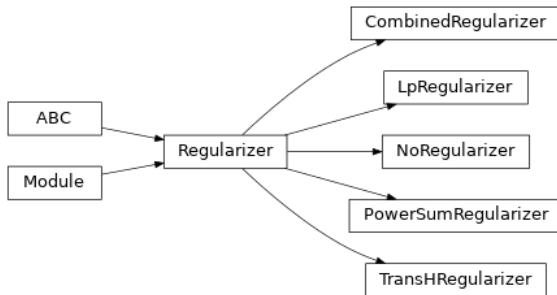
PyKEEN: Regularizers

Typically, regularization means adding a term to the loss function

- e.g., an L_p regularizer adds the $\lambda \|x\|_p$ term

PyKEEN supports 5 regularizers, including an L_p regularizer, and a power-sum regularizer

- <https://pykeen.readthedocs.io/en/latest/reference/regularizers.html>



PyKEEN: Optimizers

Support for 6 optimizers (i.e., algorithms to minimize the given loss) from PyTorch, including Adam, Stochastic Gradient Descent (SGD), Adagrad, Adadelata

- <https://pytorch.org/docs/stable/optim.html#torch.optim>

Optimizers (6)

| Name | Reference | Description |
|-----------|------------------------------------|---|
| adadelata | <code>torch.optim.Adadelata</code> | Implements Adadelata algorithm. |
| adagrad | <code>torch.optim.Adagrad</code> | Implements Adagrad algorithm. |
| adam | <code>torch.optim.Adam</code> | Implements Adam algorithm. |
| adamax | <code>torch.optim.Adamax</code> | Implements Adamax algorithm (a variant of Adam based on infinity norm). |
| adamw | <code>torch.optim.AdamW</code> | Implements AdamW algorithm. |
| sgd | <code>torch.optim.SGD</code> | Implements stochastic gradient descent (optionally with momentum). |

- Typically, negative sampling generates negative triples for every given positive triple (s, p, o) , by corrupting either s , p , or o :
 - Subject (head) corruption: generate a set $\mathcal{S}(s, p, o) = \{(s', p, o) \mid s' \in \mathcal{E}, s' \neq s\}$
 - Predicate (relation) corruption: generate a set $\mathcal{P}(s, p, o) = \{(s, p', o) \mid p' \in \mathcal{R}, p' \neq p\}$
 - Object (tail) corruption: generate a set $\mathcal{O}(s, p, o) = \{(s, p, o') \mid o' \in \mathcal{E}, o' \neq o\}$
- The ultimate overall set of negative triples for a knowledge graph \mathcal{G} will be:
$$\mathcal{N}(\mathcal{G}) = \bigcup_{(s,p,o) \in \mathcal{G}} (\mathcal{S}(s, p, o) \cup \mathcal{P}(s, p, o) \cup \mathcal{O}(s, p, o))$$
- $\mathcal{P}(s, p, o)$ is omitted in PyKEEN negative samplers

PyKEEN provides 3 negative-sampling techniques

(https://pykeen.readthedocs.io/en/stable/reference/negative_sampling.html):

- Uniform (basic) negative sampling:
 - sample corrupted subjects/predicates/objects uniformly at random
- Bernoulli negative sampling:
 - precompute a probability P_p for every relation $p \in \mathcal{R}$
 - every negative sample of a positive (s, p, o) triple is generated via either corrupting s , with probability P_p , or corrupting o , with probability $1 - P_p$
- Pseudotyped negative sampling:
 - it accounts for which entities co-occur with a relation
 - to generate a corrupted subject (object) entity for a positive triple (s, p, o) , only those entities are considered which occur as a subject (object) entity in a triple with the relation p

PyKEEN: Hyper-parameter optimization

PyKEEN provides a hyper-parameter-optimization pipeline through the **Optuna** framework

- <https://pykeen.readthedocs.io/en/stable/reference/hpo.html>

Hyper-parameter Optimization

Samplers (3)

| Name | Reference | Description |
|--------|--|---|
| grid | <code>optuna.samplers.GridSampler</code> | Sampler using grid search. |
| random | <code>optuna.samplers.RandomSampler</code> | Sampler using random sampling. |
| tpe | <code>optuna.samplers.TPESampler</code> | Sampler using TPE (Tree-structured Parzen Estimator) algorithm. |

- Evaluation pipeline (datasets and metrics provided)
 - <https://pykeen.readthedocs.io/en/latest/reference/datasets.html>
 - <https://pykeen.readthedocs.io/en/latest/reference/evaluation.html>
- Results of a large-scale benchmarking study
 - Bringing Light Into the Dark - A Large-scale Evaluation of Knowledge Graph Embedding Models under a Unified Framework