Submission 2.1

1. In school, a curve is a function, usually from [0, 100] to [0, 100]. The following are some made-up definitions:

   - A curve f is called fair if $\forall x \forall y (x \geq y \rightarrow f(x) \geq f(y))$.
   - A curve f is called totally unfair if $\forall x \forall y (x > y \rightarrow f(x) < f(y))$.
   - A curve f is called progressive if $\forall x \forall y (x < y \rightarrow f(x) - x > f(y) - y)$.

   (a) Say (both in English and in Quantificational Logic) what it means for a curve to be unfair.
   A curve is unfair if a grade x greater than or equal to a grade y before the curve ends up lower than grade y after the curve. $\exists x \exists y \neg (x \geq y \rightarrow f(x) \geq f(y))$

   (b) Say (both in English and in Quantificational Logic) what it means for a curve to be not totally unfair.
   A curve is not totally unfair if a score x greater than score y does not end up lower after the curve. $\exists x \exists y \neg (x > y \rightarrow f(x) < f(y))$

   (c) Conjure an example of a curve that is unfair but not totally unfair.
   An example of this would be f(x) = (x % 2) * 100. This is not fair, because when a = 2 and b = 1 (i.e. $a \geq b$), f(a) = 0 and f(b) = 100 (i.e. $f(a) < f(b)$). However, this curve is also not totally unfair, because when a = 3 and b = 2 (i.e. $a > b$), f(a) = 100 and f(b) = 0 (i.e. $f(a) \geq f(b)$).

   (d) Say in English what it means for a curve to be progressive. Say (both in English and in Quantificational Logic) what it means for a curve to be not progressive.
   A curve is progressive if the difference between the post- and pre- curve scores is higher for lower scores (for most intents and purposes, if lower scores are "bumped" more than higher scores.)
   A curve is not progressive if there is a case where a lower score is bumped the same amount or less than a higher score. $\exists x \exists y (x < y \rightarrow f(x) - x \leq f(y) - y)$

   (e) Classify the curve "everyone gets 5 points" as fair or unfair and progressive or not progressive.
   This curve is fair but not progressive.

2. The limit of a function, f(x), at a point c is l if and only if for all $\varepsilon > 0$ there is a $\delta > 0$ such that, for all x, if the distance between x and c is less than δ, then the distance between f(x) and l is less than ε. Suppose f is a function, and the limit of f at 0 is not 1. Given this definition of a limit, how can you prove that the limit of f at 0 is not 1? That is, what evidence would you need to provide in order to prove that the limit of f at 0 is not 1?
   You would need to find an ε such that for all δ, the distance between x and 1 being less than δ does not guarantee that the distance between f(x) and 1 will be less than ε.

3. A function, f, is continuous at a point c if and only if for all sequences of real numbers, x_n, that converge to c, the sequence $f(x_n)$ converges to f(c). Suppose f is a function, and f (0) = 1. Given this definition of pointwise continuity above, what would you need to do to prove that f is discontinuous at 0? That is, what evidence would you need to provide in order to prove that f is discontinuous at 0?
   You would need to find a sequence of real numbers x_n that converges to 0 but where $f(x_n)$ does not converge to 1.

For problems 4-12:

(a) *Translate the argument into quantificational logic. Be sure to delineate the extensions you give to predicates and names, and write the argument (now in quantificational logic) in standard form.*

(b) *Claim whether the argument is valid or sound. In some cases, soundness will be difficult to determine, so "soundness is difficult to determine" is an appropriate answer.*

(c) *Prove that the argument is valid or invalid (as appropriate) with an informal proof.*

(d) *Prove that the argument is valid (if applicable) with a proof by natural deduction.*

4. Everything has a cause. Therefore something is the cause of everything. (Some people think St. Thomas Aquinas advocated this.) (B 183)

   (a) Cxy: x is the cause of y

   $\forall x \exists y (Cyx)$

   ———————————————

   $\exists y \forall x (Cyx)$

(b) This argument is invalid, and therefore unsound.

(c) Although each thing may have a cause, they may not be the same cause. Therefore, it does not follow that all things share the same cause. For example, if A is caused by A, and B is caused by B, there is not a single thing that causes both A and B.

(d) Because the argument is invalid, it cannot be proven with natural deduction.

5. Fred hates everyone who hates Al. Al hates everyone. So Al and Fred hate each other. (B213)

   (a) Hxy: x hates y
   a: Al
   f: Fred

   $\forall x(Hax)$
   $\forall y(Hya \rightarrow Hfy)$
   _____

   $Haf \wedge Hfa$

   (b) This argument is valid. Soundness is difficult to determine.

   (d)   i. $\forall x(Hax)$ (Premise 1)
      ii. $\forall y(Hya \rightarrow Hfy)$ (Premise 2)
      iii. Haf (UI, i)
      iv. Haa (UI, i)
      v. $Haa \rightarrow Hfa$ (UI, ii)
      vi. Hfa (MP, v, iv)
      vii. $Haf \wedge Hfa$ (Conj, iii, vi)

6. All insects in this house are large and hostile. Some insects in this house are impervious to pesticides. Thus, some large, hostile insects are impervious to pesticides. (B 213)

   (a) Lx: x is large
   Hx: x is hostile
   Px: x is impervious to pesticides

   $\forall x(Lx \wedge Hx)$
   $\exists x(Px)$
   _____

   $\exists x(Lx \wedge Hx \wedge Px)$

   (b) This argument is valid and (unfortunately) quite likely sound.

   (d)   i. $\forall x(Lx \wedge Hx)$ (Premise 1)
      ii. $\exists x(Px)$ (Premise 2)
      iii. Pa (EI, ii)
      iv. $La \wedge Ha$ (UI, i)
      v. $La \wedge Ha \wedge Pa$ (Conj, iii, iv)
      vi. $\exists x(Lx \wedge Hx \wedge Px)$ (EG, v)

7. Some students cannot succeed at the university. All students who are bright and mature can succeed. It follows that some students are either not bright or immature. (B 213)

   (a) Sx: x can succeed
   Bx: x is bright
   Mx: x is mature

   $\exists x(\neg Sx)$
   $\forall x((Bx \wedge Mx) \rightarrow Sx)$
   _____

   $\exists x(\neg Bx \vee \neg Mx)$

   (b) This argument is valid. I would argue that it's unsound.

   (d)   i. $\exists x(\neg Sx)$ (Premise 1)
      ii. $\forall x((Bx \wedge Mx) \rightarrow Sx)$ (Premise 2)
      iii. $\neg Sa$ (EI, i)
      iv. $(Ba \wedge Ma) \rightarrow Sa$ (UI, ii)
      v. $\neg Sa \rightarrow \neg(Ba \wedge Ma)$ (ContraPos, iv)

    vi. $\neg Sa \rightarrow (\neg Ba \lor \neg Ma)$ (DeM, v)

    vii. $\neg Ba \lor \neg Ma$ (MP, iii, vi)

    viii. $\exists x(\neg Bx \lor \neg Mx)$ (EG, vii)

8. There are at least 3 pigs. So there are at least two pigs.

    (a) Px: x is a pig

       $\exists x \exists y \exists z(Px \land Py \land Pz \land (x \neq y) \land (x \neq z) \land (y \neq z))$

       $\overline{}$

       $\exists x \exists y(Px \land Py \land (x \neq y))$

    (b) This argument is valid and sound.

    (d)   i. $\exists x \exists y \exists z(Px \land Py \land Pz \land (x \neq y) \land (x \neq z) \land (y \neq z))$ (Premise 1)

       ii. $\exists y \exists z(Pa \land Py \land Pz \land (a \neq y) \land (a \neq z) \land (y \neq z))$ (EI, i)

       iii. $\exists z(Pa \land Pb \land Pz \land (a \neq b) \land (a \neq z) \land (b \neq z))$ (EI, ii)

       iv. $Pa \land Pb \land Pc \land (a \neq b) \land (a \neq c) \land (b \neq c)$ (EI, iii)

       v. $Pa \land Pb \land Pc \land (a \neq b) \land (a \neq c)$ (Simp, iv)

       vi. $Pa \land Pb \land Pc \land (a \neq b)$ (Simp, v)

       vii. $Pa \land Pb \land (a \neq b)$ (Simp, vi)

       viii. $\exists y(Pa \land Py \land (a \neq y))$ (EG, vii)

       ix. $\exists x \exists y(Px \land Py \land (x \neq y))$ (EG, viii)

9. Popeye and Olive Oyl like each other since Popeye likes everyone who likes Olive Oyl, and Olive Oyl likes everyone. (B 218)

    (a) Lxy: x likes y

       p: Popeye

       o: Olive Oyl

       $\forall x(Lxo \rightarrow Lpx)$

       $\forall x(Lox)$

       $\overline{}$

       $Lpo \land Lop$

    (b) This argument is valid and sound.

    (d)   i. $\forall x(Lxo \rightarrow Lpx)$ (Premise 1)

       ii. $\forall x(Lox)$ (Premise 2)

       iii. $Loo \rightarrow Lpo$ (UI, i)

       iv. Loo (UI, ii)

       v. Lpo (MP, iii, iv)

       vi. Lop (UI, ii)

       vii. $Lpo \land Lop$ (Conj, v, vi)

10. This argument is unsound, for its conclusion is false, and no sound argument has a false conclusion. (J 49)

    (a) U: x is unsound

       F: x has a false conclusion

       a: this argument

       Fa

       $\forall x(Fx \rightarrow Ux)$

       $\overline{}$

       Ua

    (b) This argument is valid. Soundness is (very) difficult to determine.

    (c) The second premise states that any argument with a false conclusion is unsound. The first premise states that "this argument" has a false conclusion, so it must follow that "this argument" is unsound.

    (d)   i. Fa (Premise 1)

       ii. $\forall x(Fx \rightarrow Ux)$ (Premise 2)

       iii. $Fa \rightarrow Ua$ (UI, ii)

       iv. Ua (MP, iii, i)

11. Everyone likes Mandy. Mandy likes nobody but Andy. Therefore Mandy and Andy are the same person. (B 238)

(a) Lxy: x likes y
   m: Mandy
   a: Andy

   $\forall x(Lxm)$
   $\forall x(x \neq a \rightarrow \neg Lmx)$
   _____

   $m = a$

(b) This argument is valid and sound.

(d)   i. $\forall x(Lxm)$ (Premise 1)
      ii. $\forall x(x \neq a \rightarrow \neg Lmx)$ (Premise 2)
      iii. Lmm (UI, i)
      iv. $m \neq a \rightarrow \neg Lmm$ (UI, ii)
      v. $Lmm \rightarrow m = a$ (ContraPost, iv)
      vi. $m = a$ (MP, v, iii)

12. Everyone is afraid of Mr. Hyde. Mr. Hyde is afraid only of Dr. Jekyll. Therefore, Dr. Jekyll is Mr. Hyde. (B 234)

   (a) Axy: x is afraid of y
      h: Mr. Hyde
      j: Dr. Jekyll

      $\forall x(Axh)$
      $\forall x(x \neq j \rightarrow \neg Ahx)$
      _____

      $h = j$

   (b) This argument is valid and sound.

   (d)   i. $\forall x(Axh)$ (Premise 1)
         ii. $\forall x(x \neq j \rightarrow \neg Ahx)$ (Premise 2)
         iii. Ahh (UI, i)
         iv. $h \neq j \rightarrow \neg Ahh$ (UI, ii)
         v. $Ahh \rightarrow h = j$ (ContraPos, iv)
         vi. $h = j$ (MP, v, iii)

For problems 13-15:

(a) *Claim whether the argument is valid or invalid.*

(b) *Prove that the argument is valid or invalid (as appropriate) with an informal proof.*

13. $\forall x(Fx \rightarrow Gx)$
      $\forall x(Fx \rightarrow Hx)$ (B 204)
      _____
      $\forall x(Gx \rightarrow Hx)$

   (a) The argument is invalid.

   (b) Let y be an instance of x such that Fy is false, Gy is true, and Hy is false. In this case, both $Fy \rightarrow Gy$ and $Fy \rightarrow Hx$ are true, but $Gy \rightarrow Hy$ is not. This means that the conclusion is not true for all x.

14. $\forall x(Fx \rightarrow Gx)$
      _____   (B 204)
      $\forall x(\neg Gx \rightarrow \neg Fx)$

   (a) This argument is valid.

   (b) This is a contrapositive.

15. $\neg \exists x(Fx \wedge Gx)$
      $\forall x(Gx \rightarrow Hx)$ (B 204)
      _____
      $\forall x(Fx \rightarrow \neg Hx)$

   (a) This argument is invalid.

   (b) Let y be an instance of x such that Fy is true and Hy is true. Since Fy is true and there is no x such that both Fx and Gx are true, Gy must be false. However, this does not imply anything abuot Hy. Since this case does not pose any contradictions but renders the conclusion false, the argument is invalid.