Submission 1.2

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1. Jones, feeling upset about the insecurity of the Social Security system, sighs that he faces a dilemma: "If taxes aren't raised, I'll have no money when I'm old. If taxes are raised, I'll have no money now." Smith, ever the even-tempered one, reasons that neither of Jones's contentions is true. Jones answers, "Aha! You've contradicted yourself!" Show that Smith's assertion that both Jones's claims are false is indeed contradictory. (B 133)
   A: Taxes are raised.
   B: Jones will have no money when he's old.
   C: Jones will have no money now.
   D: Both of Jones' claims are false, i.e. $\neg(A \to C) \wedge \neg(\neg A \to B)$

A	B	C	$A \to C$	$\neg A \to B$	D
T	T	T	T	T	F
T	T	F	F	T	F
T	F	T	T	T	F
T	F	F	F	T	F
F	T	T	T	T	F
F	T	F	T	T	F
F	F	T	T	F	F
F	F	F	T	F	F

   There is no case in which both of Jones' claims are false, so Smith's assertion is never true. Thus, Smith's assertion is a contradiction.

2. Consider the statement: If a fetus is a person, it has a right to life. Which of the following sentences follow from this? (B 26)
   Statement C, "If a fetus doesn't have a right to life, it isn't a person" and Statement E, "A fetus isn't a person only if it doesn't have a right to life," follow by contraposition.

3. Consider this statement from IRS publication 17: Your Federal Income Tax: If you are single, you must file a return if you had gross income of $3,560 or more for the year. What follows from this, together with the information listed? (B 26)
   (a) You are single with an income of $2,500.
   No information can be derived from this.
   (b) You are married with an income of $2,500.
   No information can be derived from this.
   (c) You are single with an income of $25,000.
   You must file a tax return.
   (d) You are single but do not have to file a return.
   Your gross income was less than $3,560.
   (e) You are married but do not have to file a return.
   No information can be derived from this.

For problems 4-9, prove that the argument is valid (if applicable) with a proof by natural deduction.

4. I have already said that he must have gone to King's Pyland or to Mapleton. He is not at King's Pyland, therefore he is at Mapleton. (Sir Arthur Conan Doyle) (B 20)
   (a) He is at King's Pyland or at Mapleton. (Premise 1)
   (b) He is not at King's Pyland. (Premise 2)
   (c) He is at Mapleton. (DS)

5. If I'm right, then I'm a fool. But if I'm a fool, I'm not right. Therefore, I'm not right. (B 70)
   (a) If I'm right, then I'm a fool. (Premise 1)
   (b) If I'm a fool, then I'm not right. (Premise 2)
   (c) If I'm not a fool, then I'm not right. (ContraPos, b)
   (d) I'm either a fool or not a fool. (Taut)

(e) I'm not right or I'm not right. (CD, b, c, d)

(f) I'm not right. (Rep, e)

6. Congress will agree to the cut only if the President announces his support first. The President won't announce his support first, so Congress won't agree to the cut. (B 20)

   (a) Congress will agree to the cut only if the President announces his support first. (Premise 1)

   (b) The President will not announce his support first. (Premise 2)

   (c) If the president does not announce his support, Congress will not agree to the cut. (ContraPos, a)

   (d) Congress will not agree to the cut. (MP, b, c)

7. If you are ambitious, you'll never achieve all your goals. But life has meaning only if you have ambition. Thus, if you achieve all your goals, life has no meaning. (B 132)

   (a) If you are ambitious, then you'll never achieve all your goals. (Premise 1)

   (b) Life has meaning only if you have ambition. (Premise 2)

   (c) If you achieve all your goals, you are not ambitious. (ContraPos, a)

   (d) If you are not ambitious, life does not have meaning. (ContraPos, b)

   (e) If you achieve all your goals, life does not have meaning. (HS, c, d)

8. Mittens meows exactly when she is hungry. Mittens is meowing, but she isn't hungry. Therefore the end of the Earth is at hand. (B 70)

   (a) Mittens meows exactly when she is hungry. (Premise 1)

   (b) Mittens is meowing, but she isn't hungry. (Premise 2)

   (c) The world is ending. (ContraPrm, a, b)

9. God is omnipotent if and only if He can do everything. If He can't make a stone so heavy that He can't lift it, then he can't do everything. But if He can make a stone so heavy that He can't lift it, He can't do everything. Therefore, either God is not omnipotent or God does not exist. (B 132)

   (a) God is omnipotent if and only if he can do everything. (Premise 1)

   (b) If he can't make a stone so heavy that he can't lift it, then he can't do everything. (Premise 2)

   (c) If he can make a stone so heavy that he can't lift it, then he can't do everything. (Premise 3)

   (d) He either can or can't make a stone so heavy that he can't lift it. (Taut)

   (e) He can't do everything or he can't do everything. (CD, b, c, d)

   (f) He can't do everything. (Rep, e)

   (g) If God is omnipotent, he can do everything, and if he can do everything, he is omnipotent. (Equiv, a)

   (h) If God is omnipotent, he can do everything. (Simp, g)

   (i) If he can't do everything, he is not omnipotent. (ContraPos, h)

   (j) He is not omnipotent. (MP, i, f)

   (k) He is not omnipotent or he does not exist. (Add, j)

For problems 10-13:

- *If the claim that's made is correct, prove that it's correct.*
- *If the claim that's made is incorrect, prove that it's incorrect.*
- *If the problem asks a question, answer it with a correct claim, and prove that your claim is correct.*

10. A two-place connective, $\circ$, is called associative if $(A \circ B) \circ C$ is logically equivalent to $A \circ (B \circ C)$. Which of $\wedge, \vee, \rightarrow, \leftrightarrow$ are associative? (J 20)
    $\wedge, \vee$, and $\leftrightarrow$ are associative. $\rightarrow$ is not.
    $(A \wedge B) \wedge C$ and $A \wedge (B \wedge C)$ are logically equivalent because each is true if and only if A, B, and C are all true; if at least one is false, the expression is as well.
    $(A \vee B) \vee C$ and $A \vee (B \vee C)$ are logically equivalent because each is false if and only if A, B, and C are all false; if at least one is true, the expression is true.
    $(A \leftrightarrow B) \leftrightarrow C$ and $A \leftrightarrow (B \leftrightarrow C)$ are logically equivalent. I can't explain it, but hey look, I made a truth table!

A	B	C	$(A \leftrightarrow B) \leftrightarrow C$	$A \leftrightarrow (B \leftrightarrow C)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

$(A \to B) \to C$ and $A \to (B \to C)$ are not logically equivalent because $(F \to T) \to F$ is false while $F \to (T \to F)$ is true.

11. Suppose C is a tautology. What can you say about the argument $\dfrac{\begin{array}{c}A\\B\end{array}}{C}$? (M 46)

    The argument is valid. Because C is a tautology, the conclusion of the argument is always true, and thus there can be no counterexample to the argument.

12. Suppose that A and B are logically equivalent. What can you say about $A \vee B$? (M 46)
    A and B are each logically equivalent to $A \vee B$. If A is true, $A \vee B$ is also true. If A is false, B must be false as well (since $A \leftrightarrow B$), and thus $A \vee B$ is also false. This means A is logically equivalent to $A \vee B$, and because B is logically equivalent to A, B is logically equivalent to $A \vee B$ as well.

13. Suppose that A and B are not logically equivalent. What can you say about $A \vee B$? (M 46)
    $A \vee B$ is a tautology. Where A is true, B must be false and $A \vee B$ must be true. Where A is false, B must be true and so $A \vee B$ must be true. $A \vee \neg A$ is a tautology, and because $A \vee B$ follows from it, $A \vee B$ must also be a tautology.

14. There are a number of languages with only two operators that are equivalent to truth-functional logic. Show that it is sufficient to have only the negation ($\neg$) and the conditional ($\to$) by writing sentences (containing only the operators $\neg$ and $\to$) that are logically equivalent to the following: (M 46-7)

    - $A \vee B$
    - $A \wedge B$
    - $A \leftrightarrow B$

    $A \vee B$ is true where A is true or where B is true. $A \to B$ is true where A is false or where B is true. Therefore, $A \vee B$ is equivalent to $\neg A \to B$.
    $A \wedge B$ can be found using DeMorgan's Law. $A \wedge B$ is equivalent to $\neg\neg(A \wedge B)$, which is in turn equivalent to $\neg(\neg A \vee \neg B)$. This can be rewritten using the definition for $\vee$ above to get $\neg(A \to \neg B)$.
    $A \leftrightarrow B$ is defined as $(A \to B) \wedge (B \to A)$. Using the definition above, this can be rewritten as $\neg((A \to B) \to \neg(B \to A))$.

15. Show that there is a language containing only two truth-functional operators, the negation ($\neg$) and the disjunction ($\vee$), that is equivalent to truth-functional logic. (M 47)
    By the same logic as above, $A \to B$ can be written as $\neg A \vee B$.
    Again using DeMorgan's Law, $A \wedge B$ can be written as $\neg(\neg A \vee \neg B)$.
    $A \leftrightarrow B$ is defined as $(A \to B) \wedge (B \to A)$, which can be rewritten as $\neg(\neg(\neg A \vee B) \vee \neg(\neg B \vee A))$.