

Munkres 1.3

1. Define two points (x_0, y_0) and (x_1, y_1) of the plane to be equivalent if $y_0 - x_0^2 = y_1 - x_1^2$. Check that this is an equivalence relation and describe the equivalence classes.

Reflexivity: Let (x_0, y_0) . $y_0 - x_0^2 = y_0 - x_0^2$ so $(x_0, y_0) \sim (x_0, y_0)$ for every (x_0, y_0) .

Symmetry: Let (x_0, y_0) and (x_1, y_1) such that $(x_0, y_0) \sim (x_1, y_1)$. $y_0 - x_0^2 = y_1 - x_1^2$. Equality is symmetric, so $y_1 - x_1^2 = y_0 - x_0^2$, so $(x_1, y_1) \sim (x_0, y_0)$. Thus, $(x_0, y_0) \sim (x_1, y_1) \Rightarrow (x_1, y_1) \sim (x_0, y_0)$.

Transitivity: Let (x_0, y_0) , (x_1, y_1) , and (x_2, y_2) such that $(x_0, y_0) \sim (x_1, y_1)$ and $(x_1, y_1) \sim (x_2, y_2)$. $y_0 - x_0^2 = y_1 - x_1^2$ and $y_1 - x_1^2 = y_2 - x_2^2$. By the transitive property of equality, $y_0 - x_0^2 = y_2 - x_2^2$, so $(x_0, y_0) \sim (x_2, y_2)$. Thus $((x_0, y_0) \sim (x_1, y_1) \wedge (x_1, y_1) \sim (x_2, y_2)) \Rightarrow (x_0, y_0) \sim (x_2, y_2)$. The relation satisfies all three properties of an equivalence relation.

An equivalence class of this relation determined by an element (x_0, y_0) is the set of all points (x, y) such that $y - x^2 = y_0 - x_0^2$. In other words, let $a = y_0 - x_0^2$. For any value of x , $(x, x^2 + a)$ is in the equivalence class. Note that for any point (x, y) in the equivalence class, $(-x, y)$ is also in the equivalence class.

2.