Submission 1.2

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For problems 10-13:

- *If the claim that's made is correct, prove that it's correct.*
- *If the claim that's made is incorrect, prove that it's incorrect.*
- *If the problem asks a question, answer it with a correct claim, and prove that your claim is correct.*

10. A two-place connective, $\circ$, is called associative if $(A \circ B) \circ C$ is logically equivalent to $A \circ (B \circ C)$. Which of $\wedge, \vee, \rightarrow, \leftrightarrow$ are associative? (J 20)
$\wedge, \vee$, and $\leftrightarrow$ are associative. $\rightarrow$ is not.
$(A \wedge B) \wedge C$ and $A \wedge (B \wedge C)$ are logically equivalent because each is true if and only if A, B, and C are all true; if at least one is false, the expression is as well.
$(A \vee B) \vee C$ and $A \vee (B \vee C)$ are logically equivalent because each is false if and only if A, B, and C are all false; if at least one is true, the expression is true.
$(A \leftrightarrow B) \leftrightarrow C$ and $A \leftrightarrow (B \leftrightarrow C)$ are logically equivalent. I can't explain it, but hey look, I made a truth table!

A	B	C	$(A \leftrightarrow B) \leftrightarrow C$	$A \leftrightarrow (B \leftrightarrow C)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	T	T
F	T	T	F	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

$(A \rightarrow B) \rightarrow C$ and $A \rightarrow (B \rightarrow C)$ are not logically equivalent because $(F \rightarrow T) \rightarrow F$ is false while $F \rightarrow (T \rightarrow F)$ is true.

11. Suppose C is a tautology. What can you say about the argument $\dfrac{\begin{array}{c}\text{A}\\ \text{B}\end{array}}{\text{C}}$? (M 46)

The argument is valid. Because C is a tautology, the conclusion of the argument is always true, and thus there can be no counterexample to the argument.

12. Suppose that A and B are logically equivalent. What can you say about $A \vee B$? (M 46)
A and B are each logically equivalent to $A \vee B$. If A is true, $A \vee B$ is also true. If A is false, B must be false as well (since $A \leftrightarrow B$), and thus $A \vee B$ is also false. This means A is logically equivalent to $A \vee B$, and because B is logically equivalent to A, B is logically equivalent to $A \vee B$ as well.

13. Suppose that A and B are not logically equivalent. What can you say about $A \vee B$? (M 46)
$A \vee B$ is a tautology. Where A is true, B must be false and $A \vee B$ must be true. Where A is false, B must be true and so $A \vee B$ must be true. $A \vee \neg A$ is a tautology, and because $A \vee B$ follows from it, $A \vee B$ must also be a tautology.