

# Enhancing Risk Parity: A Factor-Based Approach & Automatic Update

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## Research Goal and Novelty

### **Traditional Approach**

Most previous studies focus on traditional risk parity models that allocate portfolio weights based on static asset-level risk contributions. These models are primarily suited for low-frequency trading and lack adaptability to dynamic market conditions.

#### **Our Innovation**

We enhance the traditional risk parity framework by introducing a factor-based approach. This method shifts the focus from static asset-level risk parity to dynamic factor-level risk parity, enabling monthly recalibration of portfolio weights. By incorporating macroeconomic factors like growth, interest rates, and inflation, our approach offers more robust, automated, and diversified returns across various trading environments.

### Data Overview: The 3Vs

### Volume

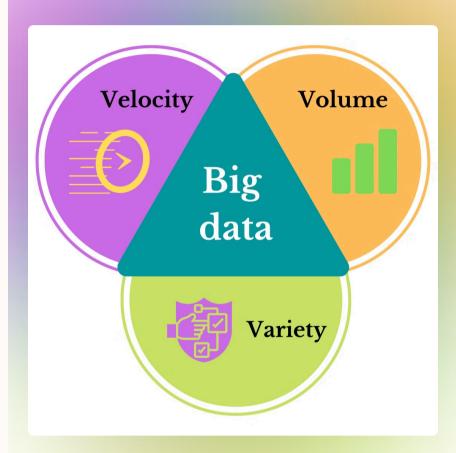
Our dataset includes historical factor and asset data from reliable platforms, such as Yahoo Finance and Wind Database, covering both U.S. and international markets.

### Variety

We utilize five macroeconomic factors (growth, inflation, credit, interest, and exchange rates) and diverse asset classes, including stock indices, treasury futures, and commodity indices.

### Velocity

While our current dataset uses static, high-frequency data, our framework is designed for future integration with real-time data APIs, allowing seamless, automated updates for dynamic portfolio management.



## Data Overview: The 3Vs

#### **Asset Data**

The table includes the following assets:

- 1. **SPX.GI** S&P 500 Index (U.S. stock market).
- 2. **TY.CBT** U.S. Treasury Bond Futures (U.S. bond market).
- 3. **CRB.RB** CRB Index (U.S. commodity market).
- 4. **00001.SH** Shanghai Composite Index (Chinese stock market).
- 5. **T.CFE** Bond-related futures (Chinese bond market).
- NH0100.NHF Chinese Commodity Index (Chinese commodity market).

Date	Asset_000001.SH	Asset_T.CFE	Asset_NH0100.NHF	Asset_SPX.GI	Asset_TY.CBT	Asset_CRB.RB
Date	000001.SH	T.CFE	NH0100.NHF	SPX.GI	TY.CBT	CRB.RB
2018-01-02	3348	93	1423	2,695.81	123.69	194.72
2018-01-03	3369	93	1418	2,713.06	123.81	195.35
2018-01-04	3386	92	1426	2,723.99	123.66	195.37
2018-01-05	3392	92	1421	2,743.15	123.48	193.45
2018-01-08	3409	93	1428	2,747.71	123.48	192.92
2018-01-09	3414	93	1431	2,751.29	123.05	194.16
2018-01-10	3422	92	1426	2,748.23	123.00	194.86
2018-01-11	3425	92	1429	2,767.56	123.06	195.16
2018-01-12	3429	92	1421	2,786.24	122.95	196.06
2018-01-15	3410	92	1420	2,786.24	122.95	196.06
2018-01-16	3437	92	1405	2,776.42	122.97	195.11
2018-01-17	3445	92	1402	2,802.56	122.58	196.40
2018-01-18	3475	91	1405	2,798.03	122.42	196.34
2018-01-19	3488	92	1410	2,810.30	122.13	195.50
2018-01-22	3501	92	1413	2,832.97	122.19	196.22
2018-01-23	3547	92	1408	2,839.13	122.50	196.94
2018-01-24	3559	92	1405	2,837.54	122.22	199.51
2018-01-25	3548	92	1424	2,839.25	122.42	199.34
2018-01-26	3558	92	1416	2.872.87	122.06	200.52

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2023-10-17	3083	102	2532	4,373.20	106.25	283.08
2023-10-18	3059	102	2534	4,314.60	105.80	285.83
2023-10-19	3005	101	2540	4,278.00	105.44	287.34
2023-10-20	2983	101	2524	4,224.16	106.00	286.01
2023-10-23	2939	102	2507	4,217.04	106.39	283.40
2023-10-24	2962	101	2518	4,247.68	106.39	281.85
2023-10-25	2974	102	2524	4,186.77	105.64	283.10
2023-10-26	2988	102	2533	4,137.23	106.33	281.52
2023-10-27	3018	102	2545	4,117.37	106.47	285.10
2023-10-30	3022	102	2557	4,166.82	106.09	280.76
2023-10-31	3019	102	2534	4,193.80	105.88	281.15

#### Factor data

The table includes the following factors:

- 1. **Factor\_Growth** Captures risks related to global economic growth.
- 2. **Factor\_Inflation** Accounts for risks arising from changes in nominal prices.
- 3. **Factor\_Credit** Measures risks associated with financial distress or credit spread widening.
- 4. Factor\_Interest Represents risks from real interest rate fluctuations.
- 5. **Factor\_Exchange** Reflects risks with exchange rate movements.

Date	Factor_Growth	Factor_Inflation	Factor_Credit	Factor_Interest	Factor_Exchange
Date	Growth	Inflation	Credit	Interest	Exchange
2018-01-02	-0.000369	-0.031677	-0.034472	0.003780	-0.024532
2018-01-03	0.004056	-0.035244	-0.059402	0.005203	-0.024851
2018-01-04	0.001852	-0.045118	-0.075499	0.002406	-0.026824
2018-01-05	-0.003903	-0.056739	-0.118130	-0.000652	-0.023526
2018-01-08	-0.003149	-0.057694	-0.139317	-0.004549	-0.016235
2018-01-09	-0.003243	-0.062057	-0.175931	-0.004315	-0.016874
2018-01-10	-0.003462	-0.062344	-0.210498	-0.003302	-0.019182
2018-01-11	0.000692	-0.062321	-0.212843	-0.003466	-0.031626
2018-01-12	0.006297	-0.054093	-0.211357	0.000219	-0.031110
2018-01-15	0.006789	-0.057685	-0.179500	0.000689	-0.029290
2018-01-16	0.010383	-0.060721	-0.251861	0.000881	-0.033829
2018-01-17	0.014075	-0.057351	-0.389381	0.001473	-0.028392
2018-01-18	0.017174	-0.051110	-0.488521	0.000996	-0.036899
2018-01-19	0.014770	-0.051299	-1.399767	0.001056	-0.037427
2018-01-22	0.016504	-0.060893	3.775451	0.000173	-0.033168
2018-01-23	0.019022	-0.036391	0.880010	-0.001030	-0.023931
2018-01-24	0.016809	-0.028232	0.310439	-0.001814	-0.051656
2018-01-25	0.016780	-0.017993	0.255983	-0.001886	-0.053317
2018-01-26	0.017253	-0.011998	0.197039	-0.001178	-0.051519

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2023-10-17	-0.039079	-0.006654	0.006613	-0.003051	-0.030985
2023-10-18	-0.020047	0.000105	0.005926	-0.004320	-0.043603
2023-10-19	0.005451	-0.006951	0.004121	-0.004954	-0.027643
2023-10-20	0.017606	-0.004546	0.001886	-0.003397	-0.008046
2023-10-23	0.012203	0.000277	0.001232	-0.003159	-0.004841
2023-10-24	-0.002207	-0.000981	-0.001072	-0.002517	-0.025632
2023-10-25	-0.010721	-0.011180	-0.000478	-0.002150	-0.042186
2023-10-26	-0.103039	-0.006468	-0.000705	-0.002857	-0.030618
2023-10-27	-0.120757	-0.011706	-0.002258	-0.002097	-0.022787
2023-10-30	-0.082376	-0.007555	-0.002904	-0.000531	-0.007732
2023-10-31	-0.109025	-0.004920	-0.001205	0.000133	-0.016604

## **Asset Data Coverage**

### **U.S.** Markets

Stock market indices, 10-year treasury futures, and commodity indices

# Foreign Markets (Especially China)

Stock market indices, 10-year treasury futures, and commodity indices



## **Factor Data Deep Dive**

Macro factors in this project are primarily low-frequency in nature, making them unsuitable for monthly portfolio weight adjustments. To address this, **proxy variables** are used to transform **low-frequency data** into **high-frequency factors**. This approach ensures compatibility with the monthly recalibration process required by the factor-based risk parity model. Below is the mapping of macro factors to their high-frequency proxies:

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Economic Growth Risk associated with global economic growth

Broad-market equity index returns

Real Rates Risk of bearing exposure to real interest rate changes

Inflation-linked bond returns

Inflation Risk of bearing exposure to changes in nominal prices

Return of portfolio long nominal bonds, short inflation-linked bonds

Credit Risk of default or spread widening associated with financial distress

Return of portfolio long corporate bonds, short nominal bonds

Emerging Markets Risk that emerging sovereign governments will change capital market rules or

general political risk in emerging markets

Basket of EM equity premiums, EM CDX, and EM FX

Commodity Risk associated with commodity markets

Weighted GSCI Commodity Index returns

## Methodology: Factor-Risk Parity Model History

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#### **Asset Allocation**

The Mean-Variance Optimization (MVO) model is a classic asset allocation framework, but its improved version, the Black-Litterman (BL) model, has practical limitations, including sensitivity to the accuracy of views and a lack of standardized parameter selection. In 1996, Bridgewater introduced the 'All Weather Strategy', often considered the precursor to the Risk Parity model.

#### **Factor Allocation**

Boudt (2013) introduced a factor-based risk attribution method to break down portfolio volatility into individual factor contributions, which can also guide asset allocation. Kelly (2014) used principal component analysis (PCA) to identify three key factors—growth, interest rates, and inflation—from asset prices. Greenberg (2016) and Bass (2017) from BlackRock replicated macro factor trends with long-short portfolios, creating high-frequency, investable factor proxies. Bender (2019) classified risk factors into two categories: macro factors (e.g., growth, interest rates, inflation, credit, and liquidity), which drive major asset class price movements, and **style factors**, which explain variations within asset classes.

### **Risk Allocation**

The traditional Risk Parity strategy focuses on balancing the risk allocation of assets, aiming for equal risk contributions from all asset classes within the portfolio. Qian (2005) pointed out that Risk Parity portfolios represent an efficient beta portfolio family, distributing market risk evenly across asset classes, including equities, bonds, and commodities. However, the Risk Parity model has limitations: 1. Low Portfolio Returns; 2. Limited Risk Diversification; 3. Risk Measurement Issues; 4. Sensitivity to Short-Term Shocks

## **Methodology: Construction of Macro Factor**

What are appropriate underlying factors?

How are our asset classes exposed to these factors?

What factor exposures are desired?

What asset class portfolios match our desired factor exposures?

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#### **Selection of Macro Factor**

Principal Component Analysis (PCA) is a method for reducing data dimensions by capturing key directions of variance. Applied to asset returns, PCA identifies principal components that explain price fluctuations.

While it helps define factors through economic analysis, **results can be unstable**, **and some components may lack clear economic meaning**. Factors like interest rates, credit, and exchange rates typically have clear economic indicators.

#### **Factor Construction**

	Original Macroeconomic Factors	High-Frequency Factors
Growth Factors	PMI year-over-year difference, comparison of fixed asset investment completion, retail sales, export and import ratios	Growth indices, CRB commodity price indices, steel rebar, crude oil, and other high-frequency price indices
Inflation Factors	Weighted year-over-year fluctuations of CPI and PPI	Inflation expectations, core inflation indices
Interest Rate Factors	10-year government bond yield	Medium-term government bond yields (1–5 years)
Credit Factors	Weighted average yields of mid-to-long-term AA bonds over 3 years, yields of 3-year government bonds	Medium-term AA corporate bond yields (3–5 years), 3-year government bond yields
Exchange Rate Factors	U.S. Dollar Index	U.S. Dollar Index
Liquidity Factors	M2 growth rate compared to social financing	Growth rates of large and medium-sized financial institutions' loans and small and medium-sized enterprise loans

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## Methodology: Calculate Factor Exposure by Month

What are appropriate underlying factors?

How are our asset classes exposed to these factors?

What factor exposures are desired?

What asset class portfolios match our desired factor exposures?

### Factor Exposure on Assets (Preparation)

$$R_t = B_t F_t + e_t$$

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- Before implementing the factor-based risk parity model, it is essential to calculate the factor exposures (beta) of various assets to different factors. In this section, we employ **multiple linear regression** to estimate the beta values for each factor.
- Using the Newton method and iterative training, we derive the regression coefficients for each asset. By applying multivariate linear regression, we estimate the exposure of asset classes to macro factors, represented by the coefficients. Combining the coefficients from all assets, we construct the factor exposure matrix, which serves as the foundation for subsequent calculations.

### **Exposure Matrix**

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# Methodology: Automatic Factor-Risk Parity Model

Multiple Linear Regression - Find the Factor Exposure Monthly

Compute the exposure matrix of factors to assets using linear regression techniques.

2 Problem Reformulation

Utilize the exposure matrix to reformulate the traditional risk parity optimization problem.

**3** Factor-Level Optimization

Solve the optimization problem to achieve risk parity across factors instead of assets.

Portfolio Construction and Automatically update

Allocate weights based on the factor-risk parity solution, aiming for more stable and diverse returns.

## Methodology: Factor-Risk Parity Model Fomula

#### Traditional Risk Parity Model

Based on the formula, the portfolio risk under a given asset weight vector can be calculated as:

$$R(w) = \sqrt{w^T \Sigma w} = \sqrt{\sum_{i,j} 
ho_{i,j} w_i w_j \sigma_i \sigma_j}$$

#### where:

- R(w): Total portfolio risk
- . w: Weight vector of assets
- Σ: Covariance matrix of assets
- $ho_{i,j}$ : Correlation coefficient between assets i and j
- $\sigma_i$ : Standard deviation of asset i

The marginal risk contribution (MRC) for each asset is:

$$\mathrm{MRC}_i = rac{\partial R(w)}{\partial w_i} = rac{1}{2} \cdot rac{\partial (w^T \Sigma w)}{\partial w_i} = rac{(\Sigma w)_i}{\sqrt{w^T \Sigma w}}$$

The total risk contribution (TRC) for asset i is:

$$ext{TRC}_i = w_i \cdot ext{MRC}_i = w_i \cdot rac{(\Sigma w)_i}{\sqrt{w^T \Sigma w}}$$

In matrix form, the total risk contribution for the entire portfolio is

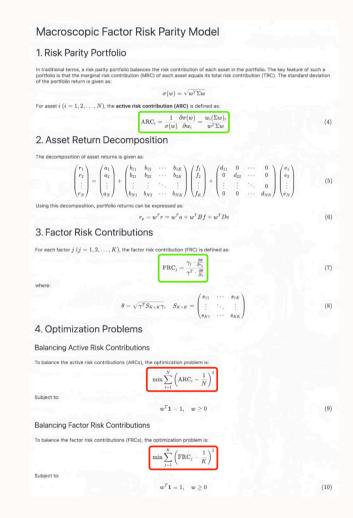
$$\mathrm{TRC} = \sum_{i=1}^{n} \mathrm{TRC}_i = \sum_{i=1}^{n} w_i \cdot \frac{(\Sigma w)_i}{\sqrt{w^T \Sigma w}} = w^T \frac{\Sigma w}{\sqrt{w^T \Sigma w}} = R(w)$$

The optimization problem is then formulated as:

$$\min_{w} \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \operatorname{TRC}_{i}(w) - \operatorname{TRC}_{j}(w) \right)^{2}$$

Subject to the following constraints:

$$\sum_{i=1}^n w_i = 1, \quad 0 \leq w_i \leq 1$$



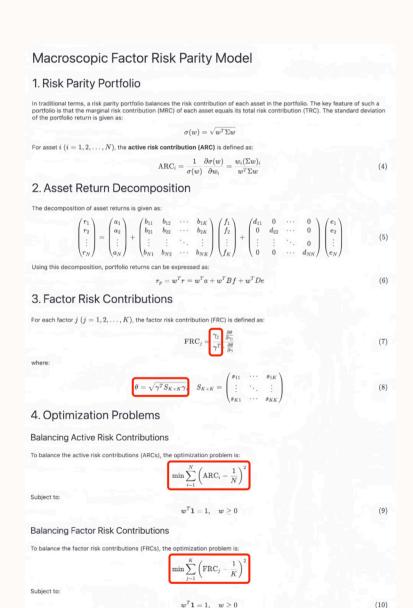
#### **Enhanced Risk Parity Model with Factor**

#### It involves:

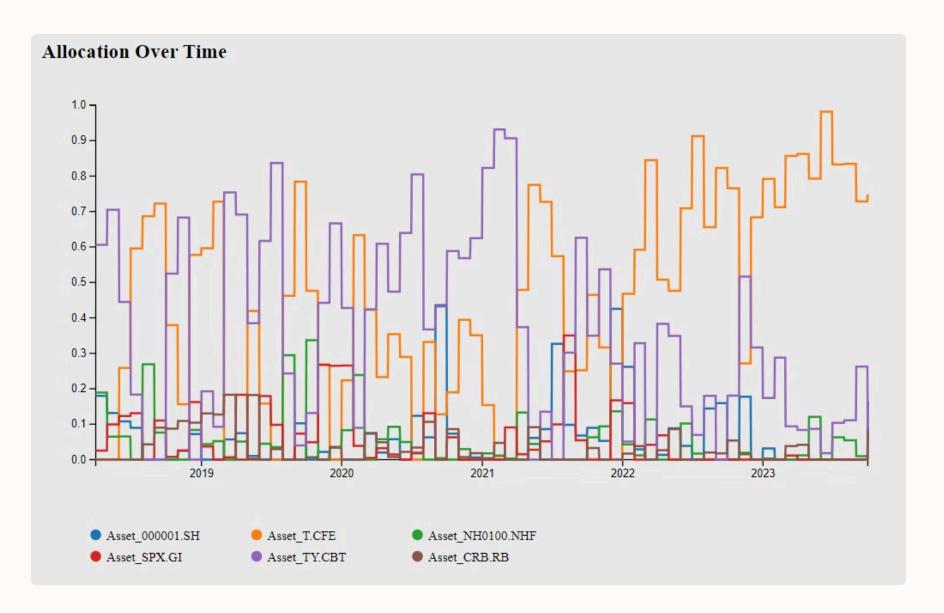
- Substituting the risk and return values of the factors themselves into asset allocation models such as risk parity and mean-variance optimization to derive the target factor exposures through optimization, as demonstrated by Niu Xiaojian (2021) and others.
- Using the asset portfolio generated by traditional asset allocation models as a benchmark, certain factor deviations are incorporated based on subjective macroeconomic views, as discussed by Blyth (2016), Boudt (2013), and others.

# Methodology: Factor-Risk Parity Model Code

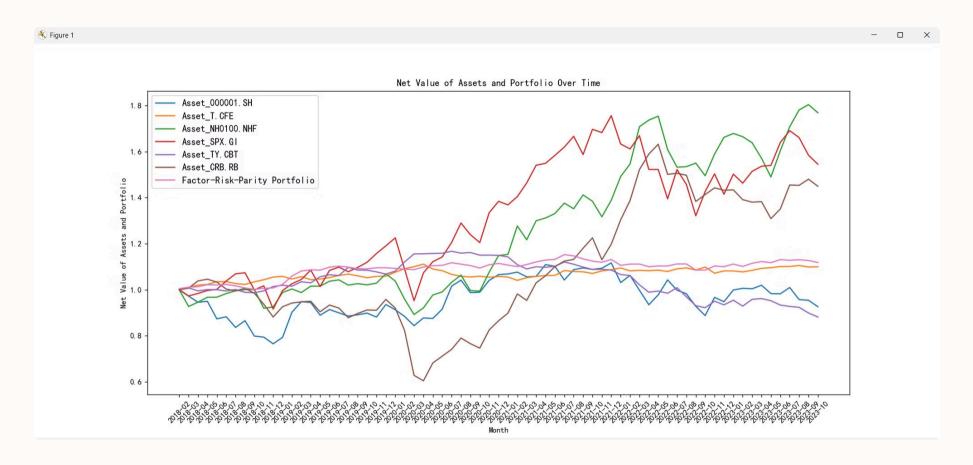
```
2. Macro Factor Model Equations --
# Equation 1: Calculate the variance of risk contribution
def risk budget objective(weights, cov beta, gama):
   weights = np.array(weights) # weights is a one-dimen
    # (1) Calculate intermediate values
    beta new = np.dot(weights, beta) # Intermediate valu
   gama new = np.dot(weights, gama) # Intermediate valu
    result = np.concatenate((beta new, gama new)) # New
   # (2) Calculate factor risk contributions
    sigma = np.sqrt(np.dot(result, np.dot(cov, result)))
   MRC = np.dot(cov, result) / sigma # Marginal Risk Co
   FRC = result * MRC # Factor Risk Contribution (FRC)
   # (3) Derive the optimization condition
   delta FRC = [sum((i - FRC)**2) for i in FRC] # Varia
   return sum(delta FRC)
# Equation 2: Factors cannot be shorted, and the sum of w
def total weight constraint(x):
   return np.sum(x) - 1.0
# Equation 3: Convert decimals to percentages
def percent formatter(x, pos):
   return f'{x*100:.0f}%'
```



# Methodology: Factor-Risk Parity Model Result



## Methodology: Factor-Risk Parity Model Result





## **Advantages of Factor-Risk Parity**

### **Enhanced Stability**

By focusing on factor-level risk parity, our model potentially offers more stable returns across various market conditions.

### **Improved Diversification**

Factor-based approach allows for better diversification across underlying economic drivers rather than just asset classes.

### Adaptability

The model can be applied to both low and high-frequency trading environments, offering greater flexibility than traditional risk parity models.

### Scalability

Our framework supports easy integration of additional asset and factor data, allowing for expansion to global markets.

## Demo

jiapeng-li-columbia.github.io

#### peng-ii-columbia.giti lub.io

### **Risk Parity Demo**



	000001.SH	T.CFE	NH0100.NHF	SPX.GI	TY.CBT	CRB.RB
000001.SH	1.000	0.137	0.417	0.704	0.243	0.271
T.CFE	0.137	1,000	0.649	0.624	-0.083	0.435
NH0100.NHF	0.417	0.649	1.000	0.846	-0.503	0.915
SPX.GI	0.704	0.624	0.846	1.000	-0.120	0.677
TY.CBT	0.243	-0.083	-0.503	-0.120	1.000	-0.688
CRB.RB	0.271	0.435	0.915	0.677	-0.688	1,000

#### Factors

Asset	Factor Growth	Factor Inflation	Factor Credit	Factor Interest	Factor Exchange	Factor Interest A
Asset_300	-0.100	1.146	2.248	-0.364	-0.045	-0.295
Asset_1000	-0.006	1.141	3.054	0.367	0.054	-0.217
Asset_cyb	-0.711	1.831	3.242	-0.295	0.026	-0.163
Asset_500	-0.145	1.278	2.623	0.097	-0.027	-0.256
Asset_guozhai	-0.106	-0.011	0.008	-0.098	-0.016	-0.011
Asset_qiyezhai	-0.044	0.003	0.040	-0.030	-0.023	-0.016
Asset_gongyepin	0.637	2.138	2.369	-0.721	0.380	-0.529
Asset_nongchanpin	0.307	1.155	1.593	-0.616	0.610	-0.641

	Growth	Inflation	Credit	Interest	Exchange
Growth	1.000	0.000	-0.001	0.007	-0.005
Inflation	0.000	1.000	-0.001	0.003	0.001
Credit	-0.001	-0.001	1.000	-0.004	-0.009
Interest	0.007	0.003	-0.004	1.000	-0.003
Exchange	-0.005	0.001	-0.009	-0.003	1.000

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# **Future Directions and Implications**

Factor Optimization
Incorporating better-performing factors to refine predictions.

Global Expansion
Integrate data from more countries and markets.

**Industry Adoption** 

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Partner with financial institutions for real-world implementation.

## References

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