Linear Algebra 2

Assignments 2: Change of Basis (§2.4)

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Change of basis (§2.4)

Assignment 2.1

Prove Theorem 2.11 (§2.4, p.27) from the book:

If $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are bases for the same vector space, then $P_{\mathcal{B}\mathcal{D}} = P_{\mathcal{B}\mathcal{C}} P_{\mathcal{C}\mathcal{D}}$.

Assignment 2.2

Exercise (5) from page 28 of the book:

Let $\mathcal{B} = \left((1,1,1)^\top, (2,3,1)^\top, (1,2,1)^\top\right)$ be a basis for $\mathbb{R}^3$ and $\boldsymbol{v} = (5,-2,3)^\top$. Find $P_{\mathcal{E}\mathcal{B}}$, $P_{\mathcal{B}\mathcal{E}}$ and $[\boldsymbol{v}]_{\mathcal{B}}$.

Assignment 2.3

Exercise (10) from page 29 of the book:

In $\mathbb{R}_2[t]$, let $\mathcal{B} = \left(t^2, (t-1)^2, (t+1)^2\right)$ be a basis, and $\boldsymbol{v}(t) = t^2 - 4t + 1$. Let $\mathcal{E} = (1, t, t^2)$ be the standard basis.

Find $P_{\mathcal{E}\mathcal{B}}$, $P_{\mathcal{B}\mathcal{E}}$ and $[\boldsymbol{v}]_{\mathcal{B}}$.

Assignment 2.4

Exercise (12) from page 29 of the book:

In $M_{2,2}$*, let* $\mathcal{E}$ *be the standard basis, and let*

$$\mathcal{D} = \left(\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \right)$$

be another basis. Let $\boldsymbol{v} = \begin{pmatrix} 5 & 6 \\ 5 & 4 \end{pmatrix}$.

Find $P_{\mathcal{D}\mathcal{E}}$*,* $P_{\mathcal{E}\mathcal{D}}$ *and* $[\boldsymbol{v}]_{\mathcal{D}}$.

Assignment 2.5 (for submission)

Consider the bases

$$\mathcal{B} = (t^2 - t + 1, t + 1, t^2 + 1)$$

and

$$\mathcal{D} = (t^2 + t + 4, 4t^2 - 3t + 2, 2t^2 + 3)$$

for the vector space $\mathbb{R}_2[t]$*. Determine the change of basis matrix that changes* $\mathcal{D}$ *coordinates into* $\mathcal{B}$ *coordinates.*