Linear Algebra 2

Assignments 8: Metrics, Orthogonal Decompositions, and the Gram-Schmidt Process (§6.3-6.6)

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Assignment 8.1

Exercise (4) from page 156 of the book:

- *On* $\mathbf{C}^3$ *with the standard inner product, compute* $\langle \boldsymbol{x}|$ *and* $|\boldsymbol{y}\rangle$*, where* $\boldsymbol{x} = (1, 2i, 3+i)^\top$ *and* $\boldsymbol{y} = (-i, 2, 5-i)^\top$.
- *Is the inner product* $\langle \boldsymbol{x}|\boldsymbol{y}\rangle$ *the same thing as the product of the row* $\langle \boldsymbol{x}|$ *and the column* $|\boldsymbol{y}\rangle$*?*
- *Compute the product of the row* $\langle \boldsymbol{x}|$ *with the column* $|\boldsymbol{y}\rangle$.

Assignment 8.2

Exercise (6) from page 157 of the book:

- *On* $\mathbb{R}_2[t]$ *with the inner product* $\langle \boldsymbol{f}|\boldsymbol{g}\rangle = \int_0^1 f(t)g(t)dt$*, and with the standard basis* $\mathcal{E} = (1, t, t^2)$*, compute the metric matrix* $\boldsymbol{G}_{\mathcal{E}}$.
- *What are* ${}_{\mathcal{E}}\langle 1+t+t^2|$ *and* $|1+t+t^2\rangle_{\mathcal{E}}$*?*
- *Show that the product of the row* ${}_{\mathcal{E}}\langle 1+t+t^2|$ *and the column* $|1+t+t^2\rangle_{\mathcal{E}}$ *is indeed the norm squared of the vector* $1+t+t^2$.

Assignment 8.3 (for submission)

Exercise (16) from page 157 of the book.

If $\mathcal{B}$ and $\mathcal{D}$ are bases for a complex vector space, derive a formula, analog to $\boldsymbol{G}_{\mathcal{D}} = P_{\mathcal{B}\mathcal{D}}^{\top} \boldsymbol{G}_{\mathcal{B}} P_{\mathcal{B}\mathcal{D}}$, for $\boldsymbol{G}_{\mathcal{D}}$ in terms of $\boldsymbol{G}_{\mathcal{B}}$ and $P_{\mathcal{B}\mathcal{D}}$.

Assignment 8.4

Exercise (2) from page 161 of the book:

In Eucledian $\mathbb{R}^3$, let $\boldsymbol{b}_1 = (1, 1, 1)^\top$, $\boldsymbol{b}_2 = (-2, 1, 1)^\top$, and $\boldsymbol{b}_3 = (0, 1, -1)^\top$. Let $\boldsymbol{d}_i = \boldsymbol{b}_i / |\boldsymbol{b}_i|$. For each i, compute the matrix $|\boldsymbol{d}_i\rangle\langle\boldsymbol{d}_i|$ and confirm that $\sum_i |\boldsymbol{d}_i\rangle\langle\boldsymbol{d}_i|$ is the identity.

Assignment 8.5

Exercise (3) from page 161 of the book:

- *With $\boldsymbol{d}_i$ as in the previous exercise, decompose the vectors $\boldsymbol{x} = (-3, 7, 2)^\top$, $\boldsymbol{y} = (1, 3, 4)^\top$, and $\boldsymbol{y} = (1, -1, 0)^\top$ in the $\mathscr{D}$ basis.*
- *How do the inner products of $\boldsymbol{x}$ and $\boldsymbol{y}$, etc., compare to the inner products of $[\boldsymbol{x}]_{\mathscr{D}}$, with $[\boldsymbol{y}]_{\mathscr{D}}$, etc.?*

Assignment 8.6

Exercise (4) from page 161 of the book:

With $\boldsymbol{d}_i$ *as in the previous exercise, decompose the matrix* $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ *as a linear combination of* $|\boldsymbol{d}_i\rangle\langle\boldsymbol{d}_i|$.

Projections and the Gram-Schmidt Process (§6.5)

Assignment 8.7 (for submission)

Exercise (4) from page 166 of the book.

Assignment 8.8 (for submission)

Exercise (11) from page 166 of the book.

Orthogonal Complements and Projections onto Subspaces (§6.6)

Assignment 8.9

Exercise (3) from page 169 of the book.

Assignment 8.10

Exercise (6) from page 169 of the book.