Answers of the extra exercises for chapter 1

Complex Analysis (EE2M11-2021v1)

(b)
$$-\frac{3}{5} - \frac{4}{5}i$$

(c)
$$-11 - 2i$$

(d)
$$-8 + 8i\sqrt{3}$$

(e)
$$e^3 \cos 2 + ie^3 \sin 2$$

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- 7.	
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	$\operatorname{Re} z$	$\operatorname{Im} z$	\bar{z}	$z\bar{z}$	z	$\operatorname{Arg} z$
(a)	3	-4	3+4i	25	5	$-\arctan\frac{4}{3}$
(b)	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2}i$	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$-\frac{\pi}{4}$
(c)	-2	14	-2 - 14i	200	$10\sqrt{2}$	$\pi - \arctan 7$
(d)	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\tfrac{1}{4} + \tfrac{1}{4}i$	$\frac{1}{8}$	$\frac{1}{4}\sqrt{2}$	$-\frac{3}{4}\pi$
(e)	-64	0	-64	4096	64	π
(f)	0	-1	i	1	1	$-\frac{1}{2}\pi$

3. 2.

4. (a)
$$2^{\frac{n+2}{2}} \cdot \cos(\frac{n\pi}{4})$$
.

(b)
$$\frac{3}{2} - \frac{1}{2}i$$
.

5. (a) the open half-plane
$$y > x + 1$$

(b) the closed square with corners
$$1, i, -1$$
 and $-i$

(c) the open half-plane
$$x > 0$$

(d) the closed sector
$$-x \le y \le x$$
 on or outside of the circle $x^2 + y^2 = 1$.

6. the closed half-plane
$$y \leq x$$

7.
$$7-2\pi$$
.

8.
$$-5 + i\sqrt{5}$$
, $-5 - i\sqrt{5}$

9. (a)
$$\frac{4\pi}{3} + 2k\pi \ (k \in \mathbb{Z}).$$

10.
$$\frac{1}{2} \ln 2 + i \left(\frac{\pi}{4} + 2k\pi \right)$$
.

11. (a)
$$\sqrt{3} + i$$
, $-\sqrt{3} - i$.

(b)
$$\exp\left(\frac{3\pi i}{8}\right)$$
, $\exp\left(\frac{7\pi i}{8}\right)$, $\exp\left(\frac{11\pi i}{8}\right)$, $\exp\left(\frac{15\pi i}{8}\right)$

12. 0, 2,
$$-1 + i\sqrt{3}$$
, $-1 - i\sqrt{3}$

13. All
$$z_1$$
 and z_2 where $\overline{z_2} = z_1$.

- **17.** (a) -1.
 - (b) $2 \arctan a$.
- **18.** The straight line through a parallel with the line through 0 and b.
- **20.** $I_1 = -\frac{2}{27}\pi^2$ and $I_2 = \frac{2}{9}\pi\sqrt{3}$.
- **22.** $\operatorname{Im}(-\frac{1}{z}) = \operatorname{Im}(-\frac{\overline{z}}{z\overline{z}}) = \frac{\operatorname{Im} z}{z\overline{z}} > 0.$
- 23. Opmerking: punten waarvan het product van de afstanden toe twee vaste punten constant is vormen een zogenaamde Cassini ovaal. Google maar eens.
 - (a) Not connected
 - (b) Yes
 - (c) Not connected
 - (d) Yes
 - (e) Yes
- **B-C 9.2.** (a) $2\pi/3$;
 - (b) π .
- **B-C** 11.2. $\pm \sqrt{3} i$, 2*i*.
- **B-C 11.3.** $\pm(\sqrt{3}-i), \pm(1+\sqrt{3}i).$
- **B-C 11.4.** (b) $\pm \sqrt{2}$, $\pm \frac{1+\sqrt{3}i}{\sqrt{2}}$, $\pm \frac{1-\sqrt{3}i}{\sqrt{2}}$,
- **B-C 11.6.** $(z^2 + 2z + 2)(z^2 2z + 2)$.
- **B-C 12.1.** Only (b) and (c) are domains.
- B-C 12.2. Only (e).
- B-C 12.3. Only (a).
- B-C 12.5. No.