Antwoorden van de extra opgaven bij hoofdstuk 4
Complex Analysis (EE2M11-2021v1)

1. (a) $-14 + \frac{32}{3}i$;
(b) $-14 + 12i$.
Notice that these two answers are different.

2. 0 for $k \neq -1$, $2\pi i$ for $k = -1$.

3. $R^2 + 2R + 3$ and $R^2 - 2R - 3$, respectively.

4. (a) $\dfrac{\sqrt{r}}{r\,(1-r)}$.
(b) $\dfrac{2\pi\sqrt{r}}{1-r}$.
(c) Is equal to 0.

6. $\dfrac{f'(z)}{f(z)}$ is analytic on D. Because D is simple and connected, f is certainly analytic in and on every closed contour in D. According to the Cauchy-Goursat theorem, the given integral must then be zero.
An example why this is not true for an open, connected, not necessarily simple domain is $f(z) = z$ on $D = \mathbb{C} \setminus \{0\}$. Namely, then

$$\int_{|z|=1} \frac{f'(z)}{f(z)}\, dz \neq 0.$$

7.

k	integral
0	0
1	2π
2	$-6\pi i$
3	-6π
4	$2\pi i$
≥ 5	0

8. $2\pi i \sinh 1$

9. (a) $\dfrac{\pi R}{R^2 - 1}$.
(b) The solution is $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\, dx = \pi$. You can also solve this integral using techniques learned earlier in your academic career......

10. Branch: for example $-\pi/2 < \arg(z) < 3\pi/2$, but $\operatorname{Log} z$ is okay too!
ML-bound: $\dfrac{\pi R(\ln R + \pi)}{R^2 - 1}$.

11. 0.

12. Such a function would contradict theorem 1 of §52. Thinking of such a function is therefore very difficult. If you do find such a function, you would receive eternal fame.

13. No; as counterexample, take $f(z) = \dfrac{1}{z^2}$.

14. $\cos\left(z^2+z\right)$ is an entire function (analytic on $\mathbb{C}$). If $\cos\left(z^2+z\right)$ on $\mathbb{C}$ would be bounded, then $\cos\left(z^2+z\right)$ would be constant, according to Liouville. But $\cos\left(z^2+z\right)$ is obviously not constant (see for yourself by filling in $z=0$ and $z=1$), thus $\cos\left(z^2+z\right)$ must be unbounded.

15. i, $-\frac{1}{2}\sqrt{3}-\frac{1}{2}i$, $\frac{1}{2}\sqrt{3}-\frac{1}{2}i$.

16. $11e^2$

17. -1.

18. Modify the proof of Liouville.

B-C 42.2. (a) $\dfrac{2}{3}+i$;
(b) $-\dfrac{1}{2}-i\ln 4$;
(c) $\dfrac{\sqrt{3}}{4}+\dfrac{i}{4}$;
(d) $\dfrac{1}{z}$.

B-C 42.4. $-(1+e^{\pi})/2$, $(1+e^{\pi})/2$.

B-C 46.3. $4(e^{\pi}-1)$.

B-C 46.6. $-\dfrac{1+e^{-\pi}}{2}(1-i)$.

B-C 46.7. $i\dfrac{e^{\pi}-1}{2}$.

B-C 49.2. (a) $\dfrac{2}{3}(-1+i)$;
(b) $e+\dfrac{1}{e}$;
(c) 0.

B-C 57.1. (a) 2π;
(b) $\pi i/4$;
(c) $-\pi i/2$;
(d) 0;
(e) $\dfrac{i\pi}{\cos^2(x_0/2)}$.

B-C 57.2. (a) $\pi/2$;
(b) $\pi/16$.