AA27A – Random Variables

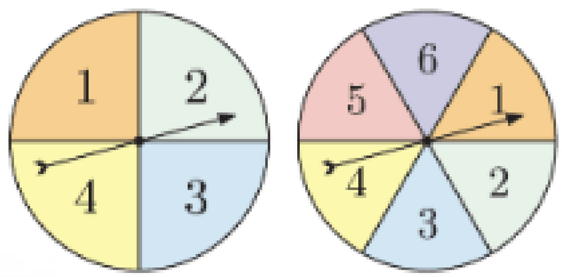
Exercise 27A (pg. 739, 740) - #1, 3, 4, 6

1 Classify each random variable as continuous or discrete:

- a the quantity of fat in a sausage
- b the mark out of 50 for a geography test
- c the weight of a Year 12 student
- d the volume of water in a cup of coffee
- e the number of trout in a lake
- f the number of hairs on a cat
- g the length of a horse's mane
- h the height of a skyscraper.

3 Suppose the spinners alongside are spun, and X is the sum of the numbers.

- a Explain why X is a discrete random variable.
- b State the possible values of X.



4 In the finals series of a baseball championship, the first team to win 4 games wins the championship. Let X represent the number of games played in the finals series.

a State the possible values of X.

b What value(s) of X correspond to the series lasting:

i exactly 5 games **ii** at least 6 games?

6 Suppose three coins are tossed simultaneously. Let X be the number of heads that result.

a State the possible values of X.

b List the possible outcomes and the corresponding values of X.

c Are the possible values of X equally likely to occur? Explain your answer.

Exercise 27B (pg. 742, 743, 744) - #3, 5, 6, 7, 9a, 10b, 11, 13, 15, 16

10 Find k for the following probability mass functions:

a $P(x) = k(x+2)$ for $x = 1, 2, 3$

b $P(x) = \dfrac{k}{x+1}$ for $x = 0, 1, 2, 3.$

11 A discrete random variable X has the probability mass function $P(x) = \dfrac{4x - x^2}{a}$ for $x = 0, 1, 2, 3.$

a Find the value of a.

b Find $P(X = 1)$.

c Find the mode of the distribution.

13 A discrete random variable X has probability mass function $P(x) = a\left(\frac{2}{5}\right)^x$, $x = 0, 1, 2, 3, ...$ Find the value of a.

15 A hat contains 2 red balls and 2 green balls. Balls are randomly selected without replacement until a green ball is selected. Let X denote the total number of balls selected.

a State the possible values of X.

b Find the probability distribution of X.

16 A pair of dice is rolled. Let S denote the sum of the top faces.

a Display the possible results in a table.

b Find the probability distribution of S.

c Find the mode of the distribution.

d Find $P(S \geqslant 10)$.

3 Consider the probability distribution alongside.

x	0	1	2	3
$P(X = x)$	0.1	0.25	0.45	a

a Find the value of a.

b Is X a uniform discrete random variable? Explain your answer.

c State the mode of the distribution.

d Find $P(X \geqslant 2)$.

5 A policeman inspected the safety of tyres on cars passing through a checkpoint. The number of tyres X which needed replacing on each car followed the probability distribution below.

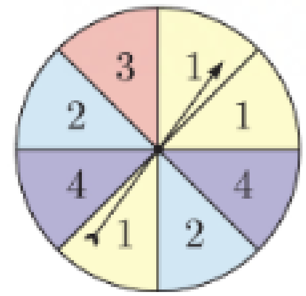
x	0	1	2	3	4
$P(X = x)$	0.68	0.2	0.06	k	0.02

a Find the value of k.

b Find the mode of the distribution.

c Find $P(X > 1)$, and interpret this value.

6 Let X be the result when the spinner alongside is spun.



a Display the probability distribution of X in a table.

b Graph the probability distribution.

c Find the mode and median of the distribution.

d Find $P(X \leqslant 3)$.

7 100 people were surveyed about the number of bedrooms in their house. 24 people had one bedroom, 35 people had two bedrooms, 27 people had three bedrooms, and 14 people had four bedrooms. Let X be the number of bedrooms a randomly selected person has in their house.

a State the possible values of X.

b Construct a probability table for X.

c Find the mode and median of the distribution.

9 Show that the following are valid probability mass functions:

a $P(x) = \frac{x+1}{10}$ for $x = 0, 1, 2, 3$

b $P(x) = \frac{6}{11x}$ for $x = 1, 2, 3.$

Exercise 27C.1 (pg. 746, 747, 748) - #5, 6, 7, 10, 11

5 Each time Pam visits the library, she borrows either 1, 2, 3, 4, or 5 books, with the probabilities shown.

Number of books	1	2	3	4	5
Probability	0.16	0.15	a	0.28	0.16

- **a** Find the value of a.
- **b** Find the mode of the distribution.
- **c** On average, how many books does Pam borrow per visit?

6 Lachlan randomly selects a ball from a bag containing 5 red balls, 2 green balls, and 1 white ball. He is then allowed to take a particular number of lollies from a jar according to the colour of the ball.

Find the average number of lollies that Lachlan can expect to receive.

Colour	*Number of lollies*
Red	4
Green	6
White	10

7 When ten-pin bowler Jenna bowls her first bowl of a frame, she always knocks down at least 8 pins. $\frac{1}{3}$ of the time she knocks down 8 pins, and $\frac{2}{5}$ of the time she knocks down 9 pins.

- **a** Find the probability that she knocks down all 10 pins on the first bowl.
- **b** On average, how many pins does Jenna knock down with her first bowl?

10 Every Thursday, Zoe meets her friends in the city for dinner. There are two car parks nearby, the costs for which are shown below:

Car park A

Time	*Cost*
0 - 1 hour	\$7
1 - 2 hours	\$12
2 - 3 hours	\$15
3 - 4 hours	\$19

Car park B

Time	*Cost*
0 - 1 hour	\$6.50
1 - 2 hours	\$11
2 - 3 hours	\$16
3 - 4 hours	\$18.50

Zoe's dinner takes 1 - 2 hours 20% of the time, 2 - 3 hours 70% of the time, and 3 - 4 hours 10% of the time.

a Which car park is cheapest for Zoe if she stays:

i 1 - 2 hours **ii** 2 - 3 hours **iii** 3 - 4 hours?

b When Zoe parks her car, she does not know how long she will stay. Which car park do you recommend for her? Explain your answer.

11 An insurance policy covers a \$20 000 sapphire ring against theft and loss. If the ring is stolen then the insurance company will pay the policy owner in full. If the ring is lost then they will pay the owner \$8000. From past experience, the insurance company knows that the probability of theft is 0.0025, and the probability of loss is 0.03. How much should the company charge to cover the ring in order that their expected return is \$100?

Exercise 27C.2 (pg. 749) - #3, 6, 7, 8

3 A roulette wheel has 18 red numbers, 18 black numbers, and 1 green number. Each number has an equal chance of occurring. I place a bet of \$2 on red. If a red is spun, I receive my \$2 back plus another \$2. Otherwise I lose my \$2.

a Calculate the expected gain from this bet.

b If the same bet is made 100 times, what is the expected result?

6 A person selects a disc from a bag containing 10 black discs, 4 blue discs, and 1 gold disc. They win \$1 for a black disc, \$5 for a blue disc, and \$20 for the gold disc. The game costs \$4 to play.

a Calculate the expected gain for this game, and hence show that the game is not fair.

b To make the game fair, the prize money for selecting the gold disc is increased. Find the new prize money for selecting the gold disc.

7 At a charity event there is a money-raising game involving a pair of ordinary dice. The game costs $\$a$ to play. When the two dice are rolled, their sum is described by the variable X. A sum which is less than 4 or between 7 and 9 inclusive gives a return of $\$\frac{a}{3}$. A result between 4 and 6 inclusive gives a return of \$7. A result of 10 or more gives a return of \$21.

a Determine $P(X \leqslant 3)$, $P(4 \leqslant X \leqslant 6)$, $P(7 \leqslant X \leqslant 9)$, and $P(X \geqslant 10)$.

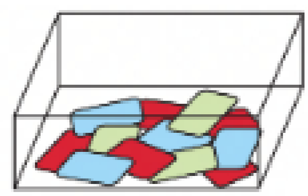
b Show that the expected gain of a player is given by $\frac{1}{6}(35 - 5a)$ dollars.

c What value would a need to have for the game to be "fair"?

d Explain why the organisers would not let a be 4.

e The organisers set $a = 9$ for the event, and the game is played 2406 times. Estimate the amount of money raised by this game.

8 In a fundraising game "Lucky 11", a player selects 3 cards without replacement from a box containing 5 red, 4 blue, and 3 green cards.
The player wins \$11 if the cards drawn are all the same colour *or* are one of each colour.
If the organiser of the game wants to make an average of \$1 per game, how much should they charge to play it?



Exercise 27D (pg. 752, 753) - #3, 4, 6, 8

3 The probability distributions below refer to the number of aces served by Michelle and Amanda in each set of tennis they play.

Michelle:

Number of aces	0	1	2	3	4
Probability	0.1	0.15	0.45	0.25	0.05

Amanda:

Number of aces	0	1	2	3	4
Probability	0.2	0.1	0.35	0.2	0.15

a Show that each player is expected to serve an average of 2 aces per set.

b Calculate the variance and standard deviation of each probability distribution.

c Which player has the greater variation in the number of aces served?

4 A country exports crayfish to overseas markets. The buyers are prepared to pay high prices when the crayfish arrive still alive.

Let X be the number of deaths per dozen crayfish. The probability distribution for X is given by:

x	0	1	2	3	4	5	> 5
$P(X = x)$	0.54	0.26	0.15	k	0.01	0.01	0.00

a Find k.

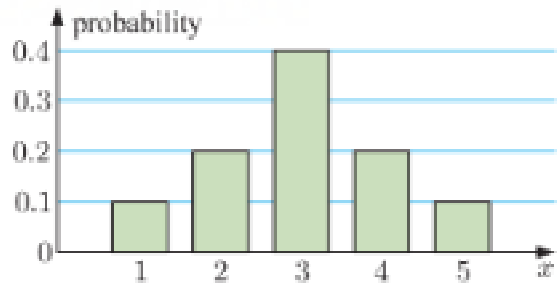
b Over a long period, what is the mean number of deaths per dozen crayfish?

c Find the standard deviation of the distribution.

6 A random variable X has the probability mass function $P(x) = \dfrac{x^2 + x}{20}$, $x = 1, 2, 3$.
For this distribution, calculate the:

a mode **b** median **c** mean μ **d** standard deviation σ.

8 The probability distribution of a random variable X is shown in the graph.



a Copy and complete:

x_i	1	2	3	4	5
p_i					

b Find the mean μ and standard deviation σ of the distribution.

c Find: **i** $\mathrm{P}(\mu - \sigma < x < \mu + \sigma)$ **ii** $\mathrm{P}(\mu - 2\sigma < x < \mu + 2\sigma)$.

Exercise 27E (pg. 755, 756) - #5, 8

5 X has the probability distribution:

x_i	1	2	3	4
p_i	0.4	0.3	0.2	0.1

Find:

a $\mathrm{E}(X)$ **b** $\mathrm{Var}(X)$ **c** $\sigma(X)$ **d** $\mathrm{E}(X+1)$

e $\mathrm{Var}(3X+1)$ **f** $\sigma(5-X)$ **g** $\mathrm{E}\left(\frac{2X+5}{3}\right)$ **h** $\mathrm{Var}(20-4X)$

8 Prove that $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$.

Exercise 27F (pg. 759) - #1

1 For which of these probability experiments does the binomial distribution apply? Explain your answers.

- **a** A coin is thrown 100 times. The variable is the number of heads.
- **b** One hundred coins are each thrown once. The variable is the number of heads.
- **c** A box contains 5 blue and 3 red marbles. I draw out 5 marbles one at a time, replacing each marble before the next is drawn. The variable is the number of red marbles drawn.
- **d** A box contains 5 blue and 3 red marbles. I draw out 5 marbles without replacement. The variable is the number of red marbles drawn.
- **e** A large bin contains ten thousand bolts, 1% of which are faulty. I draw a sample of 10 bolts from the bin. The variable is the number of faulty bolts.

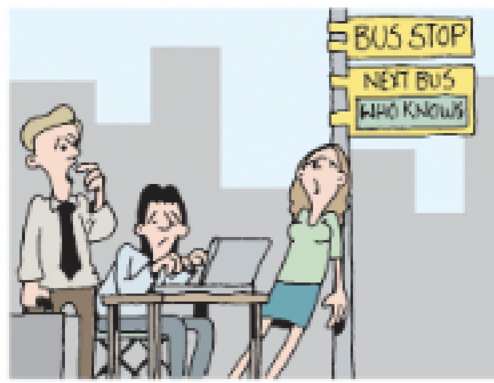
Exercise 27G (pg. 761, 762) - #2, 3, 5, 8, 9, 10, 11

2 Records show that 6% of the items assembled on a production line are faulty. A random sample of 12 items is selected with replacement. Find the probability that:

a none will be faulty
b at most one will be faulty
c at least two will be faulty
d less than four will be faulty.

3 The local bus service does not have a good reputation. The 8 am bus will run late on average two days out of every five. For any week of the year taken at random, find the probability of the 8 am bus being on time:

a all 7 days
b only on Monday
c on any 6 days
d on at least 4 days.



5 An infectious flu virus is spreading through a school. The probability of a randomly selected student having the flu next week is 0.3. Mr C has a class of 25 students.

a Calculate the probability that 2 or more students from Mr C's class will have the flu next week.
b If more than 20% of the students have the flu next week, a class test will have to be cancelled. What is the probability that the test will be cancelled?

8 A fair coin is tossed 200 times. Find the probability of obtaining:

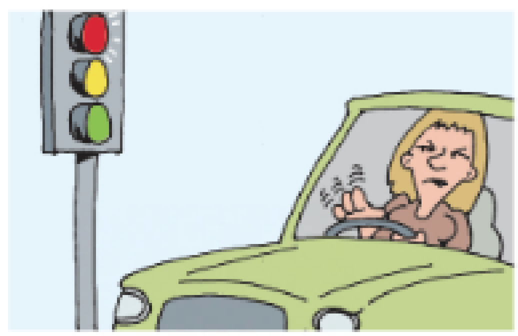
a between 90 and 110 (inclusive) heads

b more than 95 but less than 105 heads.

9 **a** Find the probability of rolling double sixes with a pair of dice.

b Suppose a pair of dice is rolled 500 times. Find the probability of rolling between 10 and 20 (inclusive) double sixes.

10 Shelley must pass through 15 traffic lights on her way to work. She has probability 0.6 of being stopped at any given traffic light. If she is stopped at more than 11 traffic lights, she will be late for work.



a Find the probability that Shelley will be late for work on a given day.

b Find the probability that Shelley is on time for work each day of a 5 day week.

c Shelley wants to increase the probability in **b** to at least 80%. She decides to leave home a little earlier, so she must now be stopped at more than 12 traffic lights in order to be late. Has Shelley achieved her goal? Justify your answer.

11 A hot water unit relies on 20 solar components for its power, and will operate provided at least one of its 20 components is working. The probability that an individual solar component will fail in a year is 0.85, and the failure of each individual component is independent of the others.

a Find the probability that the hot water unit will fail within one year.

b Find the smallest number of solar components required to ensure that a hot water service like this one is operating at the end of one year with a probability of at least 0.98.

Exercise 27H (pg. 764) - #3, 5, 6

3 Bolts produced by a machine vary in quality. The probability that a given bolt is defective is 0.04. Random samples of 30 bolts are taken from the week's production.

- **a** If X is the number of defective bolts in a sample, find the mean and standard deviation of X.
- **b** If Y is the number of non-defective bolts in a sample, find the mean and standard deviation of Y.

5 A new drug has a 75% probability of curing a patient within one week. Suppose 38 patients are treated using this drug. Let X be the number of patients who are cured within a week.

- **a** Find the mean μ and standard deviation σ of X.
- **b** Find $P(\mu - \sigma < X < \mu + \sigma)$.

6 Let X be the number of heads which occur when a coin is tossed 100 times, and Y be the number of ones which occur when a die is rolled 300 times.

- **a** Show that the mean of both distributions is 50.
- **b** Calculate the standard deviation of each distribution.
- **c** Which variable do you think is more likely to lie between 45 and 55 (inclusive)? Explain your answer.
- **d** Find:
  - **i** $P(45 \leqslant X \leqslant 55)$
  - **ii** $P(45 \leqslant Y \leqslant 55)$