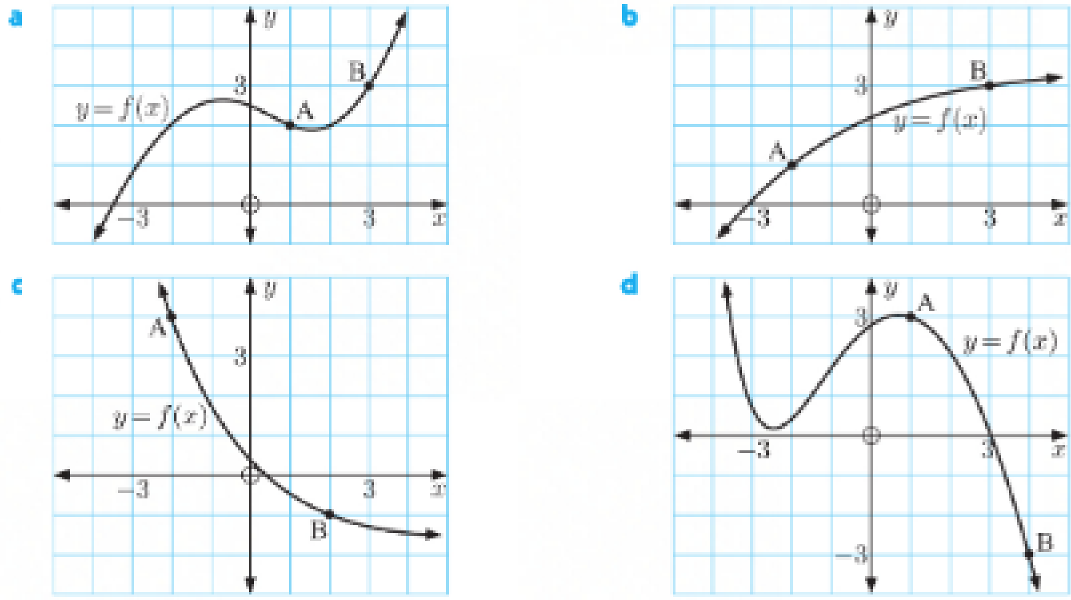
Introduction to Differential Calculus

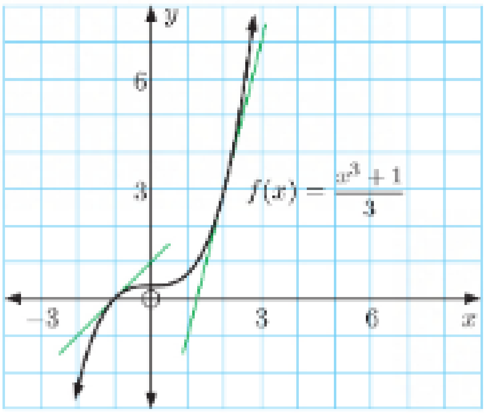
Exercise 16A2 (pg. 419) - #3

3 For each function, find the average rate of change in $f(x)$ from A to B:



Exercise 16B (pg. 422) - #2

2



The graph of $f(x) = \frac{x^3+1}{3}$ is shown alongside.

Use the tangents drawn to find the instantaneous rate of change in $f(x)$ at:

a $x = -1$

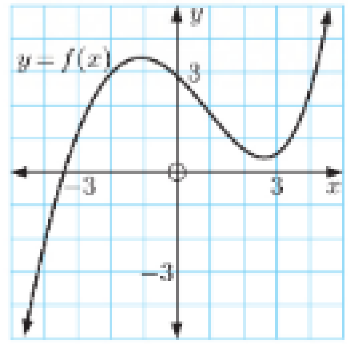
b $x = 2$.

Exercise 16D (pg. 425) - #3,4

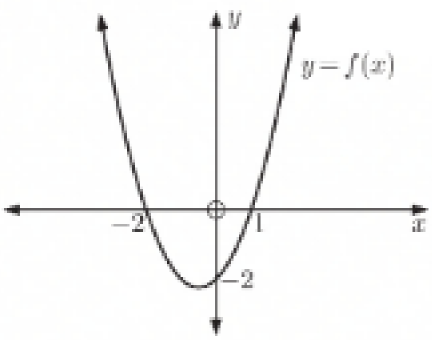
3 For the graph of $y = f(x)$ alongside, decide whether the following are positive or negative:

a $f(3)$ **b** $f'(1)$

c $f(-4)$ **d** $f'(-2)$



4



For the graph of $y = f(x)$ alongside, the derivative function is $f'(x) = 2x + 1$.

a Find and interpret: **i** $f'(-2)$ **ii** $f'(0)$

b Copy the graph, and include the information in **a**.

Exercise 16E (pg. 428, 429) - #2, 5, 10

2 Find $f'(x)$ from first principles, given that $f(x)$ is:

a $2x+5$ **b** x^2-3x **c** $-x^2+5x-3$

5 **a** Find $f'(x)$ given $f(x)=\frac{1}{x}$. **b** Find $f'(-1)$ and $f'(3)$, and interpret your answers.

10 Given $y=\sqrt{x}$, find $\frac{dy}{dx}$ from first principles. Comment on the differentiability of $y=\sqrt{x}$.

Exercise 16F (pg. 431) - #6

6 Let $f(x) = \begin{cases} ax^2, & x \geqslant 2 \\ x+b, & x < 2 \end{cases}$. Find a and b such that $f(x)$ is differentiable at $x = 2$.

Exercise R16A (pg. 433) - #4,5,8

5 **a** Given $y = 2x^2 - 1$, find $\frac{dy}{dx}$ from first principles.

b Hence state the gradient of the tangent to $y = 2x^2 - 1$ at the point where $x = 4$.

c For what value of x is the gradient of the tangent to $y = 2x^2 - 1$ equal to -12?

8 Explain why $f(x) = \frac{2}{x^2 - 2x}$ is not differentiable at $x = 2$.

4 Find, from first principles, the derivative of:

a $f(x) = x^2 + 2x$

b $y = 4 - 3x^2$

Exercise 17A (pg. 438, 439, 440) - #1cnpt, 2bfkp, 3chkp, 4h, 5b, 7cfh, 9, 13

1 Find $f'(x)$ given that $f(x)$ is:

a x^3	**b** x^8	**c** x^{11}	**d** $6x$
e $2x^3$	**f** $7x^2$	**g** $3x^5$	**h** $5x^6$
i $5x-2$	**j** x^2+3	**k** x^2+x	**l** x^2+3x-5
m $2x^2+x-1$	**n** $3x^2-7x+8$	**o** $4-2x^2$	**p** $\frac{1}{2}x^4-6x^2$
q x^3-4x^2+6x	**r** $2x^3+x-1$	**s** $7-x-4x^3$	**t** $\frac{1}{5}x^3-\frac{7}{2}x^2-2$

2 Differentiate with respect to x:

a $\frac{1}{x^2}$	**b** $\frac{1}{x^5}$	**c** $\frac{3}{x}$	**d** $\frac{4}{x^3}$
e $-\frac{7}{x^4}$	**f** $2x+\frac{3}{x^2}$	**g** $x^2-\frac{6}{x}$	**h** $9-\frac{2}{x^3}$
i $\frac{2}{x^2}+\frac{9}{x^4}$	**j** $3x-\frac{1}{x}+\frac{2}{x^2}$	**k** $5-\frac{8}{x^2}+\frac{4}{x^3}$	**l** $\frac{1}{5x^2}$
m $4x-\frac{1}{4x}$	**n** $\frac{x^2-3}{x}$	**o** $\frac{x^3+4}{x}$	**p** $\frac{2x-5}{x^2}$

3 Find the gradient function for $f(x)$ where $f(x)$ is:

a $\sqrt{x}$ **b** $\sqrt[3]{x}$ **c** $\frac{1}{\sqrt{x}}$ **d** $x^3 - \frac{1}{2}\sqrt{x}$

e $\frac{1}{x^2} + 6\sqrt{x}$ **f** $2x - \sqrt{x}$ **g** $x\sqrt{x}$ **h** $\frac{1}{x\sqrt{x}}$

i $2x^2 - \frac{3}{\sqrt{x}}$ **j** $\frac{\sqrt{x} - 4}{x}$ **k** $\frac{x+5}{\sqrt{x}}$ **l** $\frac{7 - x^2}{\sqrt{x}}$

m $3x^2 - x\sqrt{x}$ **n** $\frac{4}{x^2\sqrt{x}}$ **o** $2x - \frac{3}{x\sqrt{x}}$ **p** $\frac{x^2 - x + 2}{\sqrt[3]{x}}$

4 Find $\frac{dy}{dx}$ for:

a $y = \pi x^2$ **b** $y = 3x^2 - \frac{8}{x^2}$ **c** $y = 6\sqrt{x} + \frac{5}{x}$

d $y = 4\pi x^3$ **e** $y = 2.5x^3 - 1.4x^2 - 1.3$ **f** $y = 10(x+1)$

g $y = (x+1)(x-2)$ **h** $y = (2x+1)(3x-2)$ **i** $y = (5-x)^2$

j $y = (2x-1)^2$ **k** $y = x(x+1)(2x-5)$ **l** $y = \frac{(x-3)^2}{\sqrt{x}}$

5 Use a binomial expansion to find the derivative of:

a $f(x) = (1-x)^3$ **b** $f(x) = \left(3x - \frac{1}{\sqrt{x}}\right)^3$

7 Find the gradient of the tangent to:

a $y = x^2$ at $x = 2$

b $y = x^3 - 5x + 2$ at the point $(3, 14)$

c $y = \frac{8}{x^2}$ at the point $(9, \frac{8}{81})$

d $y = 2x^2 - 3x + 7$ at $x = -1$

e $y = 3\sqrt{x}$ at the point $(1, 3)$

f $y = 2x - \frac{5}{x}$ at the point $(2, \frac{3}{2})$

g $y = \frac{x^2 - 4}{x^2}$ at the point $(4, \frac{3}{4})$

h $y = \frac{x^3 - 4x - 8}{x^2}$ at $x = -1$

9 Determine whether $f(x) = \begin{cases} 4x^2 - 3, & x \geqslant 2 \\ x^3 + 2x + 1, & x < 2 \end{cases}$ is differentiable at $x = 2$.

13 The cost of producing x toasters each week is given by $C = 1785 + 3x + 0.002x^2$ pounds. Find the value of $\frac{dC}{dx}$ when $x = 1000$, and interpret its meaning.

Exercise 17B2 (pg. 443, 444) - #2, 3acg, 4cf

2 Differentiate $y = (2x+3)^2$ by:

- **a** using the chain rule with $u = 2x+3$
- **b** expanding $y = (2x+3)^2$ then differentiating term-by-term.

3 Find the derivative function $\frac{dy}{dx}$ for:

- **a** $y = (4x-5)^2$
- **b** $y = \frac{1}{5-2x}$
- **c** $y = \sqrt{3x - x^2}$
- **d** $y = (1-3x)^4$
- **e** $y = 6(5-x)^3$
- **f** $y = \sqrt[3]{2x^3 - x^2}$
- **g** $y = \frac{6}{(5x-4)^2}$
- **h** $y = (x^2 - 5x + 8)^5$
- **i** $y = 2\left(x^2 - \frac{2}{x}\right)^3$

4 Find the gradient of the tangent to:

- **a** $y = \sqrt{1-x^2}$ at $x = \frac{1}{2}$
- **b** $y = (3x+2)^6$ at $x = -1$
- **c** $y = \frac{1}{(2x-1)^4}$ at $x = 1$
- **d** $y = 6 \times \sqrt[3]{1-2x}$ at $x = 0$
- **e** $y = \frac{4}{x + 2\sqrt{x}}$ at $x = 4$
- **f** $y = \left(x + \frac{1}{x}\right)^3$ at $x = 1$.

Check your answers using technology.

Exercise 17C (pg. 445, 446) - #1bef, 2e, 3d, 5

1 Use the product rule to differentiate:

a $f(x) = x(x-1)$
b $f(x) = 2x(x+1)$
c $f(x) = x^2\sqrt{x+1}$
d $f(x) = (x+3)(x-1)$
e $f(x) = x\sqrt{x^2-1}$
f $f(x) = x(x+1)^2$

2 Find $\frac{dy}{dx}$ using the product rule:

a $y = x^2(2x-1)$
b $y = 4x(2x+1)^3$
c $y = x^2\sqrt{3-x}$
d $y = \sqrt{x}(x-3)^2$
e $y = 5x^2(3x^2-1)^2$
f $y = \sqrt{x}(x-x^2)^3$

3 Find the gradient of the tangent to:

a $y = x^4(1-2x)^2$ at $x = -1$
b $y = \sqrt{x}(x^2-x+1)^2$ at $x = 4$
c $y = x\sqrt{1-2x}$ at $x = -4$
d $y = x^3\sqrt{5-x^2}$ at $x = 1$.

5 Suppose $y = -2x^2(x+4)$. For what values of x does $\frac{dy}{dx} = 10$?

Exercise 17D (pg. 448) - #1bef, 2f, 3d, 6, 8

1 Use the quotient rule to find $\frac{dy}{dx}$ if:

a $y = \frac{1+3x}{2-x}$

b $y = \frac{x^2}{2x+1}$

c $y = \frac{x}{x^2-3}$

d $y = \frac{\sqrt{x}}{1-2x}$

e $y = \frac{x^2-3}{3x-x^2}$

f $y = \frac{x}{\sqrt{1-3x}}$

2 Find:

a $\frac{d}{dx}\left(\frac{x+1}{3-x}\right)$

b $\frac{d}{dx}\left(\frac{3x}{x^2-1}\right)$

c $\frac{d}{dx}\left(\frac{x^3}{2x-1}\right)$

d $\frac{d}{dx}\left(\frac{4x}{\sqrt{x-5}}\right)$

e $\frac{d}{dx}\left(\frac{\sqrt{x}}{3-x^2}\right)$

f $\frac{d}{dx}\left(-\frac{x^2}{\sqrt{x^2+3}}\right)$

3 Find the gradient of the tangent to:

a $y = \frac{x}{1-2x}$ at $x = 1$

b $y = \frac{x^3}{x^2+1}$ at $x = -1$

c $y = \frac{\sqrt{x}}{2x+1}$ at $x = 4$

d $y = \frac{x^2}{\sqrt{x^2+5}}$ at $x = -2$.

6 a If $y = \dfrac{2\sqrt{x}}{1-x}$, show that $\dfrac{dy}{dx} = \dfrac{x+1}{\sqrt{x}(1-x)^2}$.

b For what values of x is $\dfrac{dy}{dx}$: i zero ii undefined?

8 a If $y = \dfrac{x^2-3x+1}{x+2}$, show that $\dfrac{dy}{dx} = \dfrac{x^2+4x-7}{(x+2)^2}$.

b For what values of x is $\dfrac{dy}{dx}$: i zero ii undefined?